Robotics and Control 1
Homework 3

June 9, 2024

1 Control of the SCARA robot

Consider the SCARA robot studied in the two laboratory experiences. Using the function of the SCARA dynamics derived with *SCARA_dyn_equations.m*, implement, simulate, and test the two control strategies reported below. Implement and test the two strategies starting from *HW.slx* and *HW_par.m*.

- PD with gravity compensation, namely,

$$\boldsymbol{\tau} = K_p \boldsymbol{e} + K_d \dot{\boldsymbol{e}} + \boldsymbol{g}(\boldsymbol{q})$$

  where $\boldsymbol{e} = \boldsymbol{q}_d - \boldsymbol{q}$ and $\boldsymbol{g}(\boldsymbol{q})$ are, respectively, the tracking error and the vector of gravitational forces. Set the gains of the PD controller to $K_p = 1000I$ and $K_d = 1000I$.

- Feedback linearization controller, namely,

$$\boldsymbol{\tau} = B(\boldsymbol{q}) \left(K_p \boldsymbol{e} + K_d \dot{\boldsymbol{e}} + \ddot{\boldsymbol{q}}_d \right) + \boldsymbol{g}(\boldsymbol{q}) + C(\boldsymbol{q}, \dot{\boldsymbol{q}})\dot{\boldsymbol{q}}$$

  where $C(\boldsymbol{q}, \dot{\boldsymbol{q}})\dot{\boldsymbol{q}}$ corresponds to the vector of forces due to fictitious contributions.

Besides providing the files necessary to run the code, produce a small report discussing the following tasks

1. Starting form initial condition $\boldsymbol{q} = [0\,0\,0\,0]$, verify that the PD controller with gravity compensation is effective in controlling the robot to the target position $\boldsymbol{q} = [\pi/2\;\pi/3\;0.1\;-\pi/4]$, with precision compatible with errors due to delays and numerical approximation

2. Test the PD controller with gravity compensation in the trajectory tracking task, considering as reference a sinusoid. Compare the performance changing the sinusoids angular velocity to $\omega = 0.5$, 3, 6, 10. Try to improve the tracking performance modifying K_p and K_d.

3. Test the Feedback linearization controller in the trajectory tracking task, setting $\omega = 0.5$, 3, 6, 10. Test the controller performance considering as initial value of $\boldsymbol{q}$ $[0\,0\,0\,0]$ and $[\pi/2\;\pi/4\;0\;0]$.

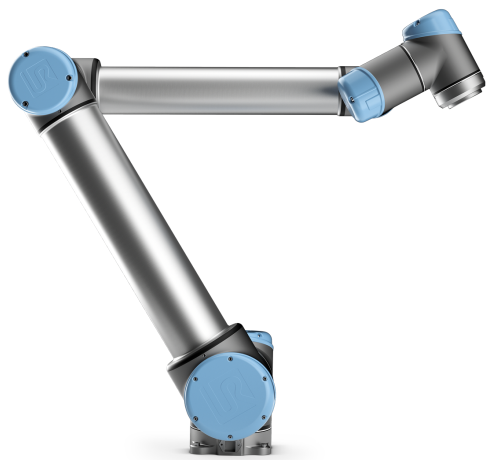

Figure 1: Picture of a UR10 robot.

2 Derivation of the UR10 kinematics and dynamics using *casADi*

As done with the SCARA in the two laboratory experiences, derive the UR10 kinematics and dynamics. The UR10 is a collaborative robot composed of 7 links and 6 revolute joints, see Figure 1. A schematic representation of the robot, together with the Denavit-Hartenberg reference frame, downloaded from the official website, is reported in Figure 2. The numerical values of the Denavit-Hartenberg parameters are reported in the table below.

joint	$\alpha_j[rad]$	$a_j[m]$	$d_j^0[m]$	$\theta_j^0[rad]$
1	$\pi/2$	0	0.1273	0
2	0	-0.612	0	0
3	0	-0.5723	0	0
4	$\pi/2$	0	0.163941	0
5	$-\pi/2$	0	0.1157	0
6	0	0	0.0922	0

Concerning the study of the dynamics, as done with the SCARA, we neglect the contributions of motors. In the table below we reported the numerical values of the mass, and the center of mass of each link. For each link, we have already computed the inertia tensor assuming that each link is a cylinder with radius r_i and length l_i. You can see the computations in the *Matlab* file *UR10_dyn_equations.m*.

link	$m_i[kg]$	$p_x[m]$	$p_y[m]$	$p_z[m]$	$r_i[m]$	$l_i[m]$
1	7.1	0.021	0.0	0.027	0.2	0.1273
2	12.7	0.38	0.0	0.158	0.15	0.612
3	4.27	0.24	0.0	0.068	0.15	0.5723
4	2	0.0	0.007	0.018	0.1	0.163941
5	2	-0.0	0.007	0.018	0.1	0.1157
6	0.365	0.0	0.0	-0.026	0.1	0.0922

Write a *Matlab* file which generates the functions computing the following quantities.

- The position of the last DH reference frame w.r.t. the base frame;
- The orientation of the last DH reference frame w.r.t. the base frame, expressed in yaw, pitch and roll angles;
- The expression of $B(\boldsymbol{q})$, $g(\boldsymbol{q})$, and $C(\boldsymbol{q}, \dot{\boldsymbol{q}})\dot{\boldsymbol{q}}$;

Moreover, we provide also *Q_test.mat*, a *.mat* file containing 4 variables, *Q_kin_test* *Q_dyn_test*, *Q_dot_dyn_test*, and *Q_ddot_dyn_test*. *Q_test* is a $N_1 \times 6$ matrix containing N_1 joint configuration, while *Q_dyn_test*, *Q_dot_dyn_test*, and *Q_ddot_dyn_test* are $N_2 \times 6$ matrices containing a sequence of joints position, velocity and acceleration. Test the kinematics and dynamics functions computed in the configurations contained in *Q_test.mat*. In particular, provide a *.mat* file containing the following variables

- POS, a $N_1 \times 3$ matrix containing the position of the last DH reference frame w.r.t. the base frame, assuming $\boldsymbol{q}$ equals to the values in *Q_kin_test*.
- ANG, a $N_1 \times 3$ matrix containing the orientation of the last DH reference frame w.r.t. the base frame, expressed using the yaw, pitch, and roll angles, assuming $\boldsymbol{q}$ equals to the values in *Q_kin_test*.
- TAU a $N_2 \times 6$ matrix containing the torques related with the trajectory defined by *Q_dyn_test*, *Q_dot_dyn_test*, and *Q_ddot_dyn_test*, with $\tau = B(\boldsymbol{q})\ddot{\boldsymbol{q}} + C(\boldsymbol{q}, \dot{\boldsymbol{q}})\dot{\boldsymbol{q}} + g(\boldsymbol{q})$;

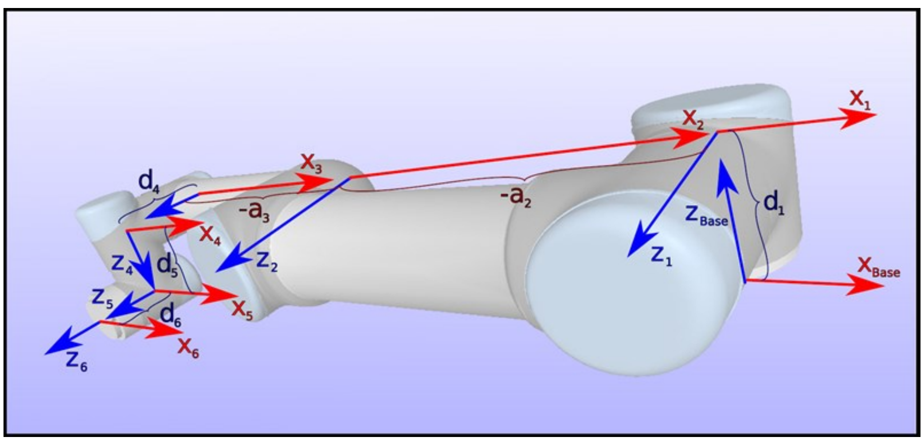

Figure 2: Representation of a UR10 robot with the Denavit-Hartenberg reference frame.