Robotics and Control 1
Homework 1

May 20, 2024

1 Robotic Arm

Consider the two-link planar arm with a prismatic joint and a revolute joint given in Figure 1 moving in a vertical plane.

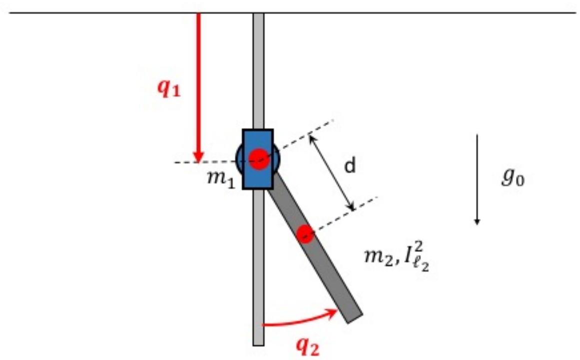

Figure 1: A PR planar arm with the relevant dynamic parameters and variables; specifically, m_1 and m_2 are the masses of link 1 and link 2, respectively, $I_{\ell_2}^2$ is the inertia tensor of link 2 and d is the distance of the center of mass of link 2 with respect to the joint axis around which link 2 rotates.

1. Derive the dynamic model of the robot in the form

$$B(q)\ddot{q} + C(q, \dot{q})\dot{q} + g(q) = \tau.$$

2. Design a PD plus constant gravity compensation law[1] that globally asymptotically stabilizes the robot to the desired configuration $q_d = (0, \pi)$.
   Now assume that K_P and K_D (respectively, the proportional and derivative gains) are diagonal. Which are the minimum values of the proportional and derivative control gains that guarantee global asymptotic stabilization?

3. Is it possible to provide a linear parametrization of the dynamic model of the type $Y(q, \dot{q}, \ddot{q})\pi = \tau$ where

$$\pi = \begin{bmatrix} m_1 + m_2 \\ m_2 d \\ \bar{I} + m_2 d^2 \end{bmatrix}$$

   being $\bar{I}$ is the only component of the inertia tensor $I^2_{\ell_2}$ playing a role in the dynamic model.

4. Design an adaptive controller to track a desired trajectory $(q_d, \dot{q}_d, \ddot{q}_d)$.

[1] That is, the gravity is compensated only with respect to the desired final configuration (see slides).