Data Bases II Exercises

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Abstract

The course aims to prepare software designers on the effective development of database applications.

First, the course presents the fundamental features of current database architectures, with a specific emphasis on the concept of transaction and its realization in centralized and distributed systems.

Then, the course illustrates the main directions in the evolution of database systems, presenting approaches that go beyond the relational model, like active databases, object systems and XML data management solutions.

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Exercise session I

1.1 Anomalies classification

Can the following schedules produce anomalies? c_i and a_i indicate the transactional decision commit and abort.

- 1. $r_1(x)w_1(x)r_2(x)w_2(y)$ $a_1 c_2$
- 2. $r_1(x)w_1(x)r_2(y)w_2(y)$ $a_1 c_2$
- 3. $r_1(x)r_2(x)r_2(y)w_2(y)r_1(z)$ $a_1 c_2$
- 4. $r_1(x)r_2(x)w_2(x)w_1(x) c_1 c_2$
- 5. $r_1(x)r_2(x)w_2(x)r_1(y) c_1 c_2$
- 6. $r_1(x)w_1(x)r_2(x)w_2(x) c_1 c_2$

Solution

- 1. We have a serial execution, but with the abort of the first transaction. Since the second transaction reads the modified value of x before the abort, we have a dirty read.
- 2. We have a serial execution and the two transactions require different resources, so there are no anomalies.
- 3. There are no anomalies because the last operation of the first transaction works on a different resource.
- 4. Both transactions first reads in sequence the resource x and then updates it without considering the updated value, so we have a lost update.
- 5. There are no anomalies because the last operation of the first transaction works on a different resource.
- 6. We have a serial execution, so the schedule is correct.

1.2 Anomalies classification

The following schedule may produce 2 anomalies: a lost update and a phantom update. Identify them.

$$r_1(x)r_2(x)r_3(x)w_1(x)r_4(y)w_2(x)r_4(x)w_4(y)r_3(y)w_4(x)r_5(y)w_6(y)w_5(y)w_7(y)$$

Solution

We can write the schedule in the following way:

And, considering both definitions, we can see that there is a lost update with transactions T_1 and T_2 and a phantom update with T_3 and T_4 .

1.3 Schedule classification

Classify the following schedule with respect to CSR and VSR classes:

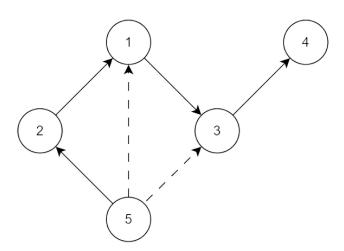
$$r_1(x)r_2(y)w_3(y)r_5(x)w_5(u)w_3(s)w_2(u)w_3(x)w_1(u)r_4(y)w_5(z)r_5(z)$$

Solution

Since CSR contains VSR we check with the conflict graph. To do so we first divide the schedule based on the resources:

- $\bullet \ x: r_1 r_5 w_3$
- $\bullet \ y: r_2 \ w_3 \ r_4$
- \bullet $z: w_5 r_5$
- \bullet $s: w_3$
- $\bullet \ u: w_5 \ w_2 \ w_1$

The nodes are $\{1, 2, 3, 4, 5\}$ and the arcs are found with the write-write or write-read relations found in the previous groups. So we have the following graph:



Some arcs can be omitted if the nodes are connected in another way (in this case we can remove arcs $\{\{5,1\},\{5,3\}\}$).

There are no cycles, so the schedule is CSR (and also VSR).

1.4 Schedule classification

Classify the following schedule with respect to CSR and VSR classes:

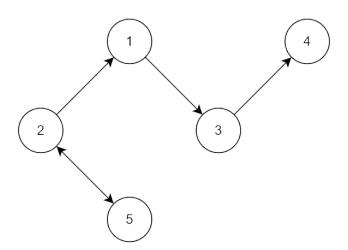
$$r_2(u)w_2(s)r_1(x)r_2(y)w_3(y)r_5(x)w_5(u)w_3(s)w_2(u)w_3(x)w_1(u)r_4(y)w_5(z)r_5(z)$$

Solution

Since CSR contains VSR we check with the conflict graph. To do so we first divide the schedule based on the resources:

- $x : r_1 r_5 w_3$
- $y: r_2 w_3 r_4$
- $\bullet \ z:w_5 r_5$
- \bullet $s: w_2 w_3$
- $\bullet \ u: r_2 \ w_5 \ w_2 \ w_1$

The nodes are $\{1, 2, 3, 4, 5\}$ and the arcs are found with the write-write or write-read relations found in the previous groups. So we have the following graph:



It is possible to see that there is a cycle between two and five. The definition of VSR states that we need to have the same reads-from relations and final writes. So, we try to find a view-equivalent schedule that is also CSR. One possible solution is simply to swap the two writes on the resource u and that is sufficient to eliminate the cycle. So, the schedule:

$$r_2(u)w_2(s)r_1(x)r_2(y)w_3(y)r_5(x)w_5(u)w_2(u)w_3(s)w_3(x)w_1(u)r_4(y)w_5(z)r_5(z)$$

is CSR and also VSR.

1.5 Schedule classification

Classify the following schedule with respect to CSR and VSR classes:

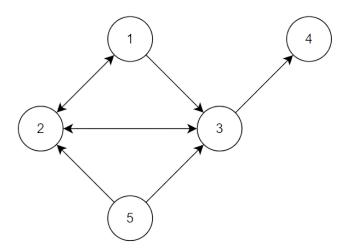
$$r_1(x)r_2(y)w_3(y)r_5(x)w_5(u)w_3(s)w_2(u)w_3(x)w_1(u)r_4(y)w_5(z)r_5(z)r_2(u)w_2(s)$$

Solution

Since CSR contains VSR we check with the conflict graph. To do so we first divide the schedule based on the resources:

- $\bullet \ x: r_1 r_5 w_3$
- $y: r_2 w_3 r_4$
- $z: w_5 r_5$
- \bullet $s: w_3 w_2$
- $u: w_5 w_2 w_1 r_2$

The nodes are $\{1, 2, 3, 4, 5\}$ and the arcs are found with the write-write or write-read relations found in the previous groups. So we have the following graph:



In this case it is not possible to find a VSR schedule because it is impossible to do so without changing the final write on s.

1.6 Schedule classification

Classify the following schedule with respect to CSR and VSR classes:

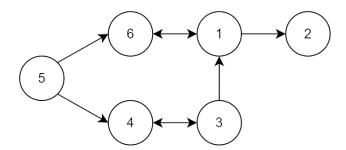
$$r_5(x)r_3(y)w_3(y)r_6(t)r_5(t)w_5(z)w_4(x)r_3(z)w_1(y)r_6(y)w_6(t)w_4(z)w_1(t)w_3(x)w_1(x)r_1(z)w_2(t)w_2(z)$$

Solution

Since CSR contains VSR we check with the conflict graph. To do so we first divide the schedule based on the resources:

- \bullet t: $r_6 r_5 w_6 w_1 w_2$
- $\bullet \ x : r_5 w_4 w_3 w_1$
- $y: r_3 w_3 w_1 r_6$
- \bullet z: $w_5 r_3 w_4 r_1 w_2$

The nodes are $\{1, 2, 3, 4, 5, 6\}$ and the arcs are found with the write-write or write-read relations found in the previous groups. So we have the following graph:



We have two cycles. It is impossible to find a VSR schedule because only the conflict between four and three can be eliminated (the other one changes a read-write relation).

Exercise session II

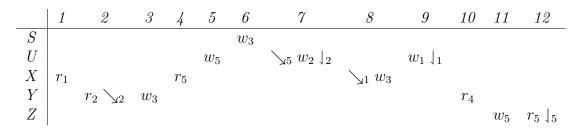
2.1 Schedule classification

Classify the following schedule with respect to 2PL and strict 2PL classes:

$$r_1(x)r_2(y)w_3(y)r_5(x)w_5(u)w_3(s)w_2(u)w_3(x)w_1(u)r_4(y)w_5(z)r_5(z)$$

Solution

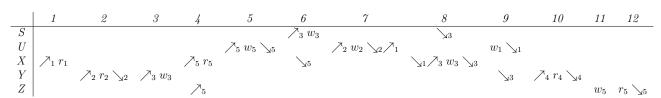
For strict 2PL we assume that all transactions commit and release all locks immediately after their last operation, and check if releases can be executed at commit time.



 ${\cal S}$ clearly cannot be in strict 2PL. The contradictions are:

- T_1 must release X before 8.
- T_2 must release Y before 7.
- T_5 must release U before 12.

For 2PL we have:



It is also not in 2PL: an assignment is not possible for T_2 (which must release Y before locking U).

2.2 Schedule classification

Classify the following schedule with respect to 2PL and strict 2PL classes:

$$r_4(x)r_2(x)w_4(x)w_2(y)w_4(y)r_3(y)w_3(x)w_4(z)r_3(z)r_6(z)r_8(z)w_6(z)w_9(z)r_5(z)r_10(z)$$

Solution

For strict 2PL we assume that all transactions commit and release all locks immediately after their last operation, and check if releases can be executed at commit time.

It is therefore clear that the schedule cannot be in 2PL-strict, due to T_2 and T_4 : T_2 ends after 4, but T_4 wants to write X at 3, and T_2 would thus be required to release X earlier, which is impossible if T_2 has to keep all locks until after 4.

For 2PL we have:

We need to look at those acquisitions that must be anticipated and to those releases that must be delayed to not violate the 2PL rules. T_4 can only get the XL on X only after 2 and on Y after 4 and has to release Y before 6 and X before 7. Thus, the lock on Z must be acquired before 6. T_2 can get all the locks at the beginning and release them immediately after each use. T_3 can acquire X, Y and Z just before using them and release them all before 12. All other transactions (T_6, T_9, T_5, T_{10}) clearly pose no problems.

2.3 Schedule classification

Classify the following schedule with respect to 2PL and strict 2PL classes:

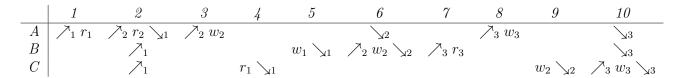
$$r_1(A)r_2(A)w_2(A)r_1(B)w_1(C)w_2(C)r_3(C)w_3(A)w_2(B)w_3(B)$$

Solution

For strict 2PL we assume that all transactions commit and release all locks immediately after their last operation, and check if releases can be executed at commit time.

The schedule is not strict 2PL.

For 2PL we have:



The schedule is 2PL.

2.4 Schedule classification

Classify the following schedule with respect to 2PL and strict 2PL classes:

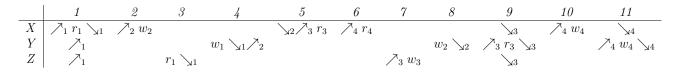
$$r_1(x)w_2(x)r_1(z)w_1(y)r_3(x)r_4(x)w_3(z)w_2(y)r_3(y)w_4(x)w_4(y)$$

Solution

For strict 2PL we assume that all transactions commit and release all locks immediately after their last operation, and check if releases can be executed at commit time.

The schedule is not strict 2PL.

For 2PL we have:



The schedule is 2PL.

2.5. Update locks

2.5 Update locks

Given the schedule:

show the sequence of lock and unlock requests produced by the transactions in a 2PL execution, in a system with update lock (available locks: SL, UL, XL).

Solution

The locking phases with update locks are the following:

X	Y
$\mathrm{UL}_1(x)$	
$r_1(x)$	
$SL_2(x)$	
$r_2(x)$	
	$\mathrm{UL}_3(y)$
	$r_3(y)$
	$XL_3(y)[upgrade]$
	$w_3(y)$
	$\operatorname{rel}(\operatorname{XL}_3(y))$
	$\mathrm{XL}_2(y)$
$rel(SL_2(x))$	
$XL_1(x)[upgrade]$	
$w_1(x)$	
$\operatorname{rel}(\operatorname{XL}_1(x))$	
	$w_2(y)$
	$\operatorname{rel}(\operatorname{XL}_2(y))$

2.6. Update locks

2.6 Update locks

Update lock was introduced to contrast deadlocks. Can we state that deadlocks are impossible in the presence of update locks?

- 1. If so, concisely explain why.
- 2. If not, provide a counter-example.

Solution

- 1. Clearly deadlocks are possible in the presence of UL. Indeed, UL only makes deadlock less likely, by preventing one type of deadlock, due to update patterns, when two transactions compete for the same resource $(r_1(x)r_2(x)w_1(x)w_2(x))$.
- 2. Consider two distinct resources X and Y, and two transactions that want to access them in this order: $r_1(X)r_2(Y)w_1(Y)w_2(X)$. It is likely that they end up in deadlock, especially if the system on which they run applies 2PL. UL is totally irrelevant here, because there is no update pattern.

Exercise session III

3.1 Obermarck's algorithm

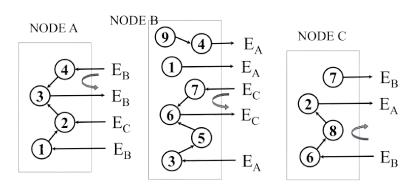
Consider the following waiting conditions:

- Node A: $E_B \to t_1, t_1 \to t_2, E_C \to t_2, t_2 \to t_3, t_3 \to E_B, E_B \to t_4, t_4 \to t_3$
- Node B: $E_A \to t_3, t_3 \to t_5, t_5 \to t_6, t_6 \to E_C, E_C \to t_7, t_7 \to t_6, t_9 \to t_4, t_4 \to E_A, t_1 \to E_A$
- Node $C: E_B \to t_6, t_6 \to t_8, t_8 \to t_2, t_2 \to E_A, t_7 \to E_B$

Simulate the Obermarck algorithm and indicate whether there is a distributed deadlock.

Solution

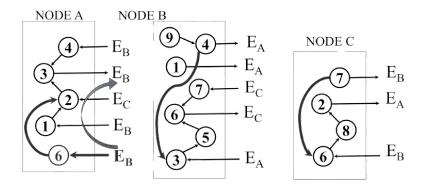
We need to construct the graph with the given constraints, that is:



We have to check all the nodes where the sender has a lower value than the receiver. In this case we have that the update is sent to the other distributed node. The interesting cases are highlighted in the image. So, we now have to add the nodes:

- $4 \rightarrow 3$ in E_B .
- $7 \rightarrow 6$ in E_C .
- $6 \rightarrow 2$ in E_A .

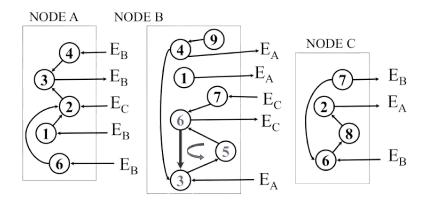
If the numbered node is not present we can add it to the graph of the distributed node. We obtain the following graphs:



We have to check if other messages are sent. We have:

• $6 \rightarrow 3$ in E_B .

So the updated graph is:



We have found a cycle, so there is a deadlock.

3.2 Obermarck's algorithm

The nodes A, B, and C of a distributed transactional system are aware of the following remote and local waiting conditions:

- Node $A: E_B \to t_3, E_C \to t_2, t_1 \to E_C, t_3 \to t_5, t_5 \to t_1$
- Node $B: E_C \to t_2, t_3 \to E_A, t_2 \to t_3$
- Node $C: t_2 \to E_A, t_2 \to E_B, t_1 \to t_4, t_4 \to t_2$

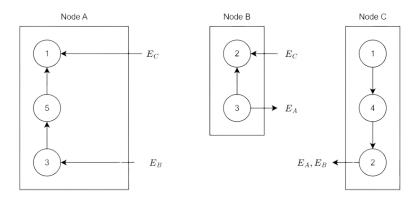
Execute the Obermarck's algorithm twice, with different conventions:

- 1. Sending messages of the form $E_X \to t_i \to t_j \to E_Y$ forward (toward node Y) and only if and only if i > j.
- 2. With the opposite conventions, so if and only if i > j

Discuss the outcome, and explain it, taking into account the properties of the algorithm and the initial conditions.

Solution

The graph is the following:



- 1. We have to add the nodes (connections are distributed):
 - $3 \to 1$ in E_C .
 - $3 \rightarrow 2$ in E_A .
 - $3 \rightarrow 2$ in E_R .

By adding those nodes we found that the third one creates a deadlock.

- 2. We have to add the node (connections are distributed):
 - $2 \rightarrow 3$ in E_A .

No cycles are found, so no deadlocks found.

The algorithm is independent of the conventions but the initial conventions, but the initial conditions must be consistent and complete. On a faulty dataset even the best algorithm returns untrustworthy results. In this case we have a link missing between node A and C.

3.3 Schedule classification

Classify the following schedule with respect to timestamps:

$$r_4(x)r_2(x)w_4(x)w_2(y)w_4(y)r_3(y)w_3(x)w_4(z)r_3(z)r_6(z)r_8(z)w_6(z)w_9(z)r_5(z)r_{10}(z)$$

Solution

We can identify pairs of operations that cause killings:

- $\bullet \ \ X: r_4r_2w_4w_3$
- $Y: w_2w_4r_3$
- $\bullet Z: w_4r_3r_6r_8w_6w_9r_5r_{10}$

On X we have that w_3 is too late with respect to r_4 and w_4 . On Y we have that r_3 is late with respect to w_4 . On Z we have that r_3 is late with respect to w_4 , w_6 with respect to r_8 , r_5 with respect to both w_6 and w_9 . So, the schedule is not in TS-mono.

The schedule is also outside TS-multi, because $w_3(X)$ comes too late $(r_4(X))$ was already given the initial version instead) and also because $w_6(Z)$ is late with respect to $r_8(Z)$. The other five reasons were due to reads (that are always accepted in TS-multi).

3.4 Schedule classification

Classify the following schedule with respect to timestamps:

$$r_1(x)r_2(y)w_3(y)r_5(x)w_5(u)w_3(s)w_2(u)w_3(x)w_1(u)r_4(y)w_5(z)r_5(z)$$

Solution

We can identify pairs of operations that cause killings:

- \bullet $S: w_3$
- $U: w_5w_2w_1$
- $X: r_1r_5w_3$
- $Y: r_2w_3r_4$
- $Z: w_5r_5$

 $S,\ Y$ and Z are ok, U is ok only if the Thomas rule is applied, and X is not ok for both TS-mono and TS-multi.

3.5 Schedule classification

Classify the following schedule:

$$r_1(X)w_1(Y)w_2(Y)w_3(Z)r_1(Z)w_4(X)r_4(Y)w_3(X)r_5(Y)w_5(X)$$

Solution

First, we check if it is CSR:

- $X: r_1w_4w_3w_5$
- $\bullet \ Y: w_1w_2r_4r_5$
- \bullet $Z: w_3r_1$

We found a cycle between the nodes three and one, so it is not CSR. It is also not VSR. We now check for TS using the same list: we find that w_4w_3 are not in the correct order, so it is not in TS-mono, neither in TS-multi (it is a write that causes the problem).

3.6 Schedule classification

Classify the following schedule:

$$r_1(x)r_2(y)w_3(x)r_5(z)w_6(z)w_2(x)w_3(y)r_7(z)w_4(x)$$

Solution

First, we check if it is CSR:

- $\bullet \ \ X: r_1w_3w_2w_4$
- $Y: r_2w_3$
- $Z: r_5 w_6 r_7$

There is a cycle between two and three, so it is not CSR, but by swapping w_3w_2 we can obtain a VSR schedule without changing the schedule.

The schedule is not TS-mono because we have w_3w_2 and so also non TS-multi.

3.7 Schedule classification

Given the resources above and the following transactions:

$$T_1: r_1(C)w_1(B)w_1(C)$$

$$T_2: w_2(A)r_2(C)$$

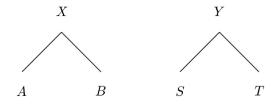
Consider that T_1 and T_2 can only be scheduled in 10 different ways (two serial, eight interleaved). What can be stated about the 2PL-strict compatibility of these schedules?

Solution

We note that the only potential conflict is between $w_1(C)$ and $r_2(C)$. But these are also the last operations of their respective transactions: to check for compatibility, we can always assume that the commit occurs right after the last operation, and that all locks are released in favor of the other one. But this argument is independent of the order of the operations, and is therefore valid for all the eight interleaved schedules. So, all the schedules are strict 2PL.

3.8. Hierarchical lock

3.8 Hierarchical lock



Given the resource hierarchy above, is the following schedule compatible with a 2PL-strict scheduler that applies hierarchical locking?

$$r_1(A)w_1(S)w_2(T)r_2(A)w_1(A)$$

Solution

The lock manager works as follows:

	X	A	В	Y	S	T	
$r_1(A)$	ISL_1	-	-	-	=	-	
	ISL_1	SL_1	-	-	-	-	
$w_1(S)$	ISL_1	SL_1	-	IXL_1	-	-	
	ISL_1	SL_1	-	IXL_1	XL_1	-	
$w_2(T)$	ISL_1	SL_1	-	$IXL_{1,2}$	XL_1	-	
	ISL_1	SL_1	-	$IXL_{1,2}$	XL_1	XL_2	
$r_2(A)$	$ISL_{1,2}$	SL_1	-	$IXL_{1,2}$	XL_1	XL_2	
	$ISL_{1,2}$	$SL_{1,2}$	-	$IXL_{1,2}$	XL_1	XL_2	
commit T_2		,		•			End of T_2
	\mathbf{ISL}_1	SL_1	-	IXL_1	XL_1	-	
$w_1(A)$	\mathbf{IXL}_1	\mathbf{SL}_1	-	IXL_1	XL_1	_	
	IXL_1	\mathbf{XL}_1	-	IXL_1	XL_1	_	
commit T_1							End of T_1

So, it is compatible with a strict 2PL scheduler.

3.9. Hierarchical lock

3.9 Hierarchical lock

Consider the following short schedule occurring on a system with hierarchical lock over a hierarchy where PagA contains tuples t_1 and t_2 :

$$r_1(PagA), w_2(t1), w_1(t2)$$

Show a possible sequence of locks, unlocks, lock upgrades, and lock downgrades performed by transactions T_1 and T_2 such that the schedule is in 2PL.

Solution

We have that:

	PagA	$\mathbf{t2}$	t1
$SIXL_1(PagA)$	$SIXL_1$	-	-
$XL_1(t2)$	SIXL_1	XL_1	-
$r_1(PagA)$			
$U-SL_1(PagA)$	$\mathrm{IXL}_1,\mathrm{IXL}_2$	XL_1	-
$IXL_2(PagA)$	$\mathrm{IXL}_1,\mathrm{IXL}_2$	XL_1	-
$\mathrm{XL}_2(t1)$	IXL_1, IXL_2	XL_1	XL_2
$w_2(t1)$			
$U-XL_2(t1)$	$\mathrm{IXL}_1,\mathrm{IXL}_2$	XL_1	-
U-IXL ₂ $(PagA)$	IXL_1	XL_1	-
Commit T_2			
$w_1(t2)$			
$U-XL_1(t2)$	IXL_1	-	-
U -IXL $_1(PagA)$	-	-	-
Commit T_1			

So the schedule is 2PL.

Exercise session IV

4.1 Ranking and skyline queries

Consider a distributed setting with three data sources, ranking basketball players according to their offensive rating (off), defensive rating (def) and rebounds (reb). An associated score in [0, 1] is indicated (the higher, the better). Sorted access is available.

$R_0 \; ({ m off})$	Player	$R_1 \; (\mathrm{def})$	Player	$R_2 \ ({ m reb})$	Player
0.8	9	0.9	4	0.6	1
0.8	8	0.8	5	0.5	3
0.8	2	0.7	7	0.3	4
0.7	7	0.6	9	0.3	2
0.6	3	0.5	0	0.3	0
0.6	1	0.4	6	0.2	8
0.5	6	0.2	3	0.2	6
0.5	5	0.2	2	0.1	5
0.5	0	0.0	8	0.0	9
0.2	4	0.0	1	0.0	7

- 1. Determine the top-3 players according to their median rank using MedRank.
- 2. Remove the first data source and consider the scoring function

$$MAX(o) = max\{def(o), reb(o)\}$$

Determine the top-2 players according to MAX using the algorithms B_0 and NRA.

- 3. Assume now that random access is also available. Determine the top-2 players according to MAX with the algorithms TA and FA.
- 4. Consider now the scoring function

$$SUM(o) = def(o) + reb(o)$$

equally weighing all partial scores. What are the top-2 players according to SUM?

- 5. Assume now that all the data regarding the players are centralized in a single data source, in which the players are available sortedly according to SUM. Use SFS to determine the skyline of the players.
- 6. Identify the players in the 2-skyband, and in the 3-skyband.

Solution

1. A player is considered when, while doing sorted access it appears in at least two tables. If we check the first row we find the following players: {9,4,1} and the tables are three, so no player appears in at least two tables. For the second iteration we have found {9,4,1,8,5,3}, so again no duplicate player. With the third row we have {9,4,1,8,5,3,2,7,4}. We have found the first player that appears in at least two rankings, but we need two more players, so we can check the fourth row: {9,4,1,8,5,3,2,7,4,7,9,2}. With this iteration we have found other three players, and the algorithm stops. The content of the buffer at this step is the following:

Player	First row	Second row	Third row
4	1	3	?
2	3	4	?
7	3	4	?
9	1	4	?
1	1	?	?
8	2	?	?
5	2	?	?
3	2	?	?

With this algorithm we have found that the best players are: nine, four, two, and seven. In particular, the player four is the best due to the median rank and the other three are equally good.

2. Since we have k = 2, for the B_0 algorithm we need to make two sorted access to the first two rows, and add the objects to the buffer:

\mathbf{Player}	$R_1 \; (\mathrm{def})$	$R_2 \; ({ m reb})$	\mathbf{MAX}
4	0.9	?	0.9
5	0.8	?	0.8
1	?	0.6	0.6
3	?	0.6	0.6

For each object in the buffer we have to make some random accesses to find the missing values. The buffer will be updated like follows.

Player	$R_1 \; (\mathrm{def})$	$R_2 \; ({ m reb})$	MAX
4	0.9	0.3	0.9
5	0.8	0.1	0.8
1	0.0	0.6	0.6
3	0.2	0.6	0.6

The best two players in the buffer are four and five.

For the NRA algorithm we have to make a sorted access to each row until the upper bound of the first element out of the top-k is less or equal than the lower bound of the worst top-k. With the sorted access to the first row we have the following buffer.

Player	$R_1 \; (\mathrm{def})$	$R_2 \; ({ m reb})$	Lower bound	Upper bound
4	0.9	?	0.9	0.9
1	?	0.6	0.6	0.9

And the threshold point has the following coordinates (0.9, 0.6), and since the scoring function is MAX we have that its score is 0.9. After the second sorted access we have:

Player	$R_1 \; (\mathrm{def})$	$R_2 \; ({ m reb})$	Lower bound	Upper bound
4	0.9	?	0.9	0.9
5	0.9	?	0.8	0.8
1	?	0.6	0.6	0.8
3	?	0.6	0.5	0.8

And the threshold point has the following coordinates (0.8, 0.5), and since the scoring function is MAX we have that its score is 0.8. Since the lower bound of three is equal to the upper bound of one the algorithm stops.

3. With FA we have to make sorted accesses until we find k elements that appears in each column. In this case (excluding R_0) we need to make five sorted accesses to find two elements in both tables (four and zero). The elements added to the buffer are:

Player	$R_1 \; (\mathrm{def})$	$R_2 \; ({ m reb})$	Score
4	0.9	0.3	0.9
5	0.8	?	0.8
7	0.7	?	0.7
1	?	0.6	0.6
9	0.6	?	0.6
3	?	0.5	0.5
0	0.5	0.3	0.5
2	?	0.3	0.3

We can now complete the buffer with random accesses to the score, and we obtain the final buffer.

Player	$R_1 \; (\mathrm{def})$	$R_2 \; ({ m reb})$	Score
4	0.9	0.3	0.9
5	0.8	0.1	0.8
7	0.7	0.0	0.7
1	0.0	0.6	0.6
9	0.6	0.0	0.6
3	0.2	0.5	0.5
0	0.5	0.3	0.5
2	0.2	0.3	0.3

The best players are: four and five.

For the TA we have to make sorted access row by row and complete the value with sorted accesses in the other column. We have also to compute the threshold as the scoring function return value of the considered row. For the first row we have the following buffer.

Player	$R_1 \; (\mathrm{def})$	$R_2 \; ({ m reb})$	Score
4	0.9	0.3	0.9
1	0.0	0.6	0.6

The threshold in this case is the maximum of the values of the first row, that is 0.9. So, we need to do another iteration, that creates the following buffer.

Player	$R_1 \; (\mathrm{def})$	$R_2 \; ({ m reb})$	Score
4	0.9	0.3	0.9
5	0.8	0.1	0.8

The threshold in this case is the maximum of the values of the second row, that is 0.8. Since it is less or equal than the worst object's score in the buffer, the algorithm halts. We have found that also with TA the best players are four and five.

4. We decide to use TA algorithm with the given scoring function and k = 2. The steps are the same as the previous point, except for the scoring function. After accessing the first row we have the following buffer:

Player	$R_1 \; (\mathrm{def})$	$R_2 \; ({ m reb})$	Score
4	0.9	0.3	1.2
1	0.0	0.6	0.6

The threshold in this case is the sum of the values of the first row, that is 1.5. Since it is greater than 0.6 we have to do another iteration. If we read the second row we obtain the following buffer.

Player	$R_1 \; (\mathrm{def})$	$R_2 \; ({ m reb})$	Score
4	0.9	0.3	1.2
5	0.8	0.1	0.9

The threshold in this case is the sum of the values of the first row, that is 1.3. Since it is greater than 0.9 we have to do another iteration. If we read the third row we obtain the following buffer.

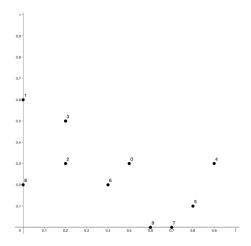
Player	$R_1 \; (\mathrm{def})$	$R_2 \; ({ m reb})$	Score
4	0.9	0.3	1.2
5	0.8	0.1	0.9

The threshold in this case is the sum of the values of the first row, that is 1.0. Since it is greater than 0.9 we have to do another iteration. If we read the fourth row we obtain the following buffer.

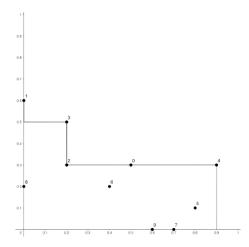
Player	$R_1 \; (\mathrm{def})$	$R_2 \; ({ m reb})$	Score
4	0.9	0.3	1.2
5	0.8	0.1	0.9

The threshold in this case is the sum of the values of the first row, that is 0.9. Since it is equal to 0.9 the algorithm halts. We found that the best player for this scoring function are four and five.

5. It is possible to draw the points with coordinates (R_1, R_2) .



We access the first row and since the window has no elements we add it to it. We check for every tuple if it dominates the inserted one. Only the tuples three and one are not dominated by the tuple four, so the final window contains the points four, three and one. Graphically we have that the skyline of the given dataset is the following.



6. Players zero and five are dominated only by player four, so they are part of the 2-skyband. Players one, three and four are part of the skyline, so they are obviously also part of the 2-skyband. The 3-skyband is made of the players one, three, four, six and seven according to the definition.

4.2 Ranking and skyline queries

Consider the following three ranked lists reporting statistics about European soccer teams:

Goals	Team	Possession	Team	Passes	Team
75	PSG	63.2	BMU	89.9	PSG
73	BMU	62.1	BAR	88.9	MCY
70	ATA	61.8	PSG	88.4	BAR
68	MCY	61.6	MCY	87.5	BMU
68	BDO	59.0	LIV	86.6	BDO
66	LIV	58.5	BDO	84.4	LAZ
63	BAR	55.7	ATA	83.9	LIV
60	LAZ	50.1	LAZ	83.4	ATA

1. Apply an algorithm that does not use random access to determine the best team according to the scoring function:

$$S(\text{team}) = \max\{\text{team.Goals}, \text{team.Possession}, \text{team.Passes}\}$$

- 2. On this dataset, could you do better than the previous algorithm, in terms of access cost, with an algorithm that also uses random access?
- 3. Apply now the TA to determine the best team according to the same scoring function as in one.
- 4. Apply now the TA to determine the best team according to the following scoring function:

$$S(\text{team}) = \text{team.Goals} + \text{team.Possession} + \text{team.Passes}$$

5. Determine the skyline of the dataset.

Solution

1. Since the scoring function is MAX we can use the B_0 algorithm. Find the best team means that we have k = 1, so we need to make only one sorted access. By accessing the first row we obtain the following buffer.

Player	Goals	Possession	Passes	Score
PSG	75	?	89.9	89.9
BMU	?	63.2	?	0.8

The maximum score is given to PSG, that is the best team according to the MAX scoring function.

- 2. No algorithm could do better than reading at least the top scores on each list.
- 3. We have again k = 1. We make a sorted access to the first row to compute the threshold value and the objects to insert in the buffer.

Player	Goals	Possession	Passes	Score
PSG	75	?	89.9	89.9
BMU	?	63.2	?	0.8

The threshold point has a value that is the MAX of the scores in the first row, that is 89.9. We now make random accesses to complete the data in the buffer (remember that the algorithm searches the missing data when it inserts a new object in the buffer).

Player	Goals	Possession	Passes	Score
PSG	75	61.8	89.9	89.9

Note that the passes for PSG are accessed before via random access, and later via sorted access. So at each iteration (except for the third column) the algorithm makes a sorted access and two random accesses. Since the threshold has the same value as the score of PSG, the algorithm halts and returns PSG as the best team.

4. We have to apply again TA with k = 1, but with a different scoring function, so the result may change. The check for the first row is the same as the previous point, except for the score.

Player	Goals	Possession	Passes	Score
PSG	75	61.8	89.9	226.7

The threshold is the sum of the values in the first row 228.1. The threshold is greater than PSG score, so we need to make another iteration. In the next row we find: BMU (223.7), BAR (213.5), and MCY (218.5). All these scores are lower than PSG one, so the buffer remains the same. The threshold point has a value of 224, that is less than PSG score, so the algorithm halts. The best team with SUM is again PSG.

5. It is possible to represent the dataset in a graph with coordinates (Goals, Possession, Passes). By inspecting the ordered table we add PSG to the window and all the other teams that are not dominated by it. The only teams not dominated by PSG are BMU and BAR. We have found that the skyline of the dataset is composed by three teams: PSG, BMU, and BAR.

4.3 Ranking and skyline queries

Consider the following two ranked lists of hotels:

Rating	Hotel	Stars	Hotels
0.8	С	0.8	F
0.4	G	0.6	${ m E}$
0.4	\mathbf{F}	0.5	G
0.3	D	0.4	A
0.3	A	0.2	D
0.2	${ m E}$	0.2	\mathbf{C}
0.1	В	0.2	В

1. Apply FA and TA to determine the top hotel according to the scoring function:

$$MAX(o) = max{Rating(o), Stars(o)}$$

- 2. Indicate the number of sorted accesses and random accesses executed on each source.
- 3. Could FA make fewer sorted accesses than TA?
- 4. Could FA make fewer random accesses than TA?
- 5. TA is instance optimal: can any algorithm cost overall less than TA?

Solution

1. For the FA we have to make sorted access until we found k objects in all tables. In this case we need to find only one element that is in both columns. To do so we need three sorted accesses, and we obtain the following buffer.

Hotel	Rating	Stars	Score
F	0.4	0.8	1.2
\mathbf{C}	0.8	?	0.8
G	0.4	0.5	0.9
\mathbf{E}	?	0.6	0.6

We need to find the missing values with two random accesses, and we have

Hotel	Rating	Stars	Score
F	0.4	0.8	1.2
\mathbf{C}	0.8	0.2	1.0
G	0.4	0.5	0.9
E	0.2	0.6	0.8

The best hotel according to the given scoring function is F.

The TA checks rows until the value of the threshold is greater than the worst score in the top-k. After accessing the first row we have:

Hotel	Rating	Stars	Score
F	0.4	0.8	1.2

With a threshold of 1.6. After accessing the second row we have the same buffer, and a threshold value of 1.0, that is less than F's score, so the algorithm halts. The best hotel found is again F.

- 2. With FA we have made six sorted accesses (three for Rating, and three for Stars) and two random accesses (one for Rating, and one for Stars), so the total is eight. With TA we have made four sorted accesses (two for Rating, and two for Stars) and four random accesses (two for Rating, and two for Stars), so the total is eight.
- 3. No, because FA stops its sorted access phase when all potential top-k objects (according to any possible scoring function) have been seen, so at least k objects are no worse than the threshold according to any scoring function.
- 4. Yes, in this exercise we have an example.
- 5. Yes, it is possible.

4.4 Ranking and skyline queries

Consider the following lists reporting statistics about soccer players.

Goals	Player	Assists	Player
25	LEW	17	DEB
21	RON	15	SAN
19	MES	12	MES
18	MBA	6	NEY
15	ILI	5	MBA
14	SAN	5	ILI
13	NEY	3	RON
8	DEB	3	LEW

1. Apply the TA to determine the top-2 players according to the scoring function:

$$S(player) = player.Goals + player.Assists$$

- 2. Reuse as much as possible the answer of the previous question to indicate the reached depth and the number of sorted and random accesses to determine, with TA, the top-1 player according to the same scoring function as the previous point.
- 3. Discuss whether any algorithm could attain a lower execution cost than the cost incurred by TA to determine the top-1 player.
- 4. Apply now the TA to determine the top-2 players according to the scoring function:

$$S(player) = |player.Goals - player.Assists|$$

5. Determine the skyline of the dataset.

Solution

1. The execution of the TA algorithm needs four rounds to stop on the given dataset. At the fourth round we have the following buffer.

Player	Goals	Assists	Score
MES	19	12	31
SAN	14	15	29

With a threshold value of 24. We found that the best two players for the given scoring function are MES and SAN.

2. The procedure is the same as the previous point, but it stops at the third round since the threshold condition is already verified at this step. The buffer is the following.

Player	Goals	Assists	Score
MES	19	12	31

With a threshold of 31. We have found that the best player is MES.

- 3. FA would stop at depth three as well, but only needs four random accesses to find the top player.
- 4. The given function is not monotone, so the TA can return wrong results. In this case the algorithm will fail, returning DEB instead of RON.
- 5. After sorting the dataset we insert MES in the window. The only players that are not dominated by him are: SAN, LEW, and DEB. So, we have that the skyline is composed by these three players.