

Data Bases II
Theory

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Abstract

The course aims to prepare software designers on the effective development of database applications.

First, the course presents the fundamental features of current database architectures, with a specific emphasis on the concept of transaction and its realization in centralized and distributed systems.

Then, the course illustrates the main directions in the evolution of database systems, presenting approaches that go beyond the relational model, like active databases, object systems and XML data management solutions.

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Chapter 1

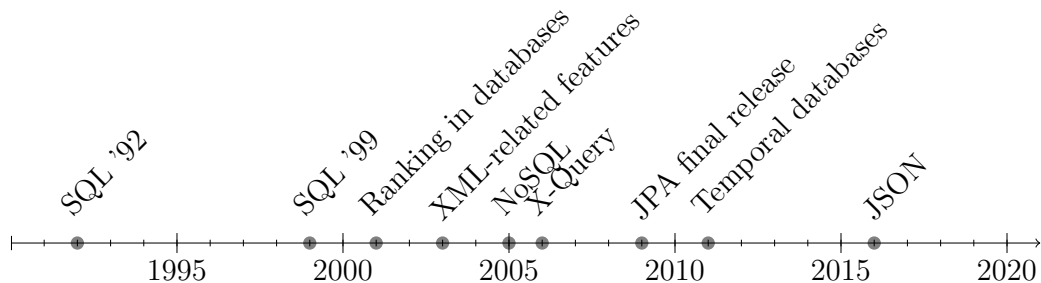
Introduction

1.1 Data Base Management System

Definition

A *Data Base Management System* is a software product capable of managing data collections that are:

- Large: much larger than the central memory available on the computers that run the software.
- Persistent: with a lifetime which is independent of single executions of the programs that access them.
- Shared: used by several applications at a time.
- Reliable: ensuring tolerance to hardware and software failures.
- Data ownership respectful: by disciplining and controlling accesses.



1.2 Transactions

Definition

A *transaction* is an elementary, atomic unit of work performed by an application. Each transaction is conceptually encapsulated within two commands:

- Begin transaction.
- End transaction.

Within a transaction, one of the commands below is executed to signal the end of the transaction: commit or rollback.

Definition

The *On-Line Transaction Processing* (OLTP) is a system that supports the execution of transactions on behalf of concurrent applications.

The application can run many transactions. So, the transactions are part of the application and not vice-versa. The transactions follow the ACID property:

1. Atomicity: a transaction is an indivisible unit of execution. This means that all the operations in the transaction are executed or none is executed. The time in which commit is executed marks the instant in which the transaction ends successfully: an error before should cause the rollback and an error after should not alter the transaction. The rollback of the work performed can be caused by a rollback statement or by the DBMS. In case of a rollback, the work performed must be undone, bringing the database to the state it had before the start of the transaction. It is the application's responsibility to decide whether an aborted transaction must be redone or not.
2. Consistency: A transaction must satisfy the database integrity constraints, that is if the initial state S_0 is consistent then the final state S_f is also consistent. This is not necessarily true for the intermediate states S_i .
3. Isolation: the execution of a transaction must be independent of the concurrent execution of other transactions.
4. Durability: the effect of a transaction that has successfully committed will last forever, independently of any system fault.

Property	Actions	Architectural element
Atomicity	Abort-rollback-restart	Query Manager
Consistency	Integrity checking	Integrity Control System
Isolation	Concurrency control	Concurrency Control System
Durability	Recovery management	Reliability Manager

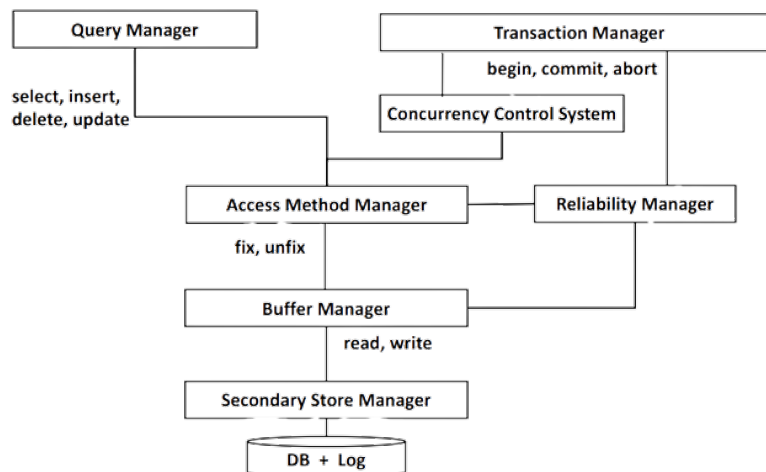


Figure 1.1: Architecture of a Data Base Management System

Chapter 2

Concurrency

2.1 Introduction

A DBMS usually needs to manage multiple applications. A unit of measurement used to evaluate the DBMS workload is the number of transaction per second (*tps*) handled by it. To have an efficient usage of the database the DBMS needs to be able to handle concurrency while avoiding the insurgence of anomalies. The concurrency control system schedules the order of the various transactions.

2.2 Anomalies in concurrent transactions

The anomalies caused by uncorrect scheduling are:

- Lost update: an update is applied from a state that ignores a preceding update, which is lost.

Transaction t_1	Transaction t_2
$r_1(x)$ $x = x + 1$	$r_2(x)$ $x = x + 1$ $w_2(x)$ commit
$w_1(x)$ commit	

- Dirty read: an uncommitted value is used to update the data.

Transaction t_1	Transaction t_2
$r_1(x)$ $x = x + 1$ $w_1(x)$	$r_2(x)$ commit
abort	

- Non-repeatable read: someone else updates a previously read value.

Transaction t_1	Transaction t_2
$r_1(x)$	$r_2(x)$ $x = x + 1$ $w_2(x)$ commit
$r_1(x)$ commit	

- Phantom update: someone else updates data that contributes to a previously valid constraint.

Transaction t_1	Transaction t_2
$r_1(x)$	$r_2(y)$
$r_1(y)$	$y = y - 100$ $r_2(z)$ $z = z + 100$ $w_2(y)$ $w_2(z)$ commit
$r_1(z)$ $s = x + y + z$ commit	

- Phantom insert: someone else inserts data that contributes to a previously read datum.

2.3 Concurrency theory

Definition

A *model* is an abstraction of a system, object or process, which purposely disregards details to simplify the investigation of relevant properties.

Concurrency theory builds upon a model of transaction and concurrency control principles that help understanding the real systems. Real systems exploit implementation level mechanisms which help achieve some desirable properties postulated by the theory.

Definition

An *operation* consist in a reading or in a writing of a specific datum by a specific transaction.

A *schedule* is a sequence of operations performed by concurrent transactions that respects the order of operations of each transaction.

The transactions can be: serial, interleaved or nested. The number of serial schedules for n transaction is equal to:

$$N_S = n!$$

While the total number of distinct schedules given the number of transaction n is equal to:

$$N_D = \frac{(\sum_{i=1}^n k_i)!}{\prod_{i=1}^n (k_i!)}$$

Example : Given two transaction T_1 and T_2 we have six possible different schedules, where only two are serial:

1. $r_1(x)w_1(x)r_2(z)w_2(z)$
2. $r_2(z)w_2(z)r_1(x)w_1(x)$
3. $r_1(x)r_2(z)w_1(x)w_2(z)$
4. $r_2(z)r_1(x)w_2(z)w_1(x)$
5. $r_1(x)r_2(z)w_2(z)w_1(x)$
6. $r_2(z)r_1(x)w_1(x)w_2(z)$

The first two are serial, the third and the fourth are nested, and the last two interleaved.

The concurrency control has to reject all the schedules that causes anomalies.

Definition

The *scheduler* is a component that accepts or rejects operations requested by the transactions.

The *serial schedule* is a schedule in which the actions of each transaction occur in a contiguous sequence.

A serializable schedule leaves the database in the same state as some serial schedule of the same transactions, so it is correct. To introduce other classes we have to initially make two assumptions:

- The transactions are observed a posteriori.
- Commit-projection: consider only the committed transactions.

2.4 View-serializability

Definition

$r_i(x)$ *reads-from* $w_j(x)$ when $w_j(x)$ precedes $r_i(x)$ and there is no $w_k(x)$ in S between $r_i(x)$ and $w_j(x)$.

$w_i(x)$ in a schedule S is a *final write* if it is the last write on x that occurs in S .

Two schedules are said to be *view-equivalent* ($S_i \approx_V S_j$) if they have:

1. The same operations.
2. The same reads-from relationships.
3. The same final writes.

A schedule is view-serializable (VSR) if it is view-equivalent to a serial schedule of the same transactions. The value written by $w_j(x)$ could be uncommitted when $r_i(x)$ reads it, but we are sure that it will be committed (commit-projection hypothesis).

Example : The following schedules are given:

- $S_1 : w_0(x)r_2(x)r_1(x)w_2(x)w_2(z)$
- $S_2 : w_0(x)r_1(x)r_2(x)w_2(x)w_2(z)$
- $S_3 : w_0(x)r_1(x)w_1(x)r_2(x)w_1(z)$
- $S_4 : w_0(x)r_1(x)w_1(x)w_1(z)r_2(x)$
- $S_5 : r_1(x)r_2(x)w_1(x)w_2(x)$

- $S_6 : r_1(x)r_2(x)w_2(x)r_1(x)$
- $S_7 : r_1(x)r_1(y)r_2(z)r_2(y)w_2(y)w_2(z)r_1(z)$

We have that only S_2 and S_3 are serial. S_1 is view-equivalent to serial schedule S_2 (so it is view-serializable). S_3 is not view-equivalent to S_2 (different operations) but is view-equivalent to serial schedule S_4 , so it is also view-serializable.

S_5 corresponds to a lost update, S_6 corresponds to a non-repeatable read, and S_7 corresponds to a phantom update. All these schedules are non view-serializable.

The following schedules are given:

- $S_a : w_0(x)r_1(x)w_0(z)r_1(z)r_2(x)w_0(y)r_3(z)w_3(z)w_2(y)w_1(x)w_3(y)$
- $S_b : w_0(x)w_0(z)w_0(y)r_2(x)w_2(y)r_1(x)r_1(z)w_1(x)r_3(z)w_3(z)w_3(y)$
- $S_c : w_0(x)w_0(z)w_0(y)r_2(x)w_2(y)r_3(z)w_3(z)w_3(y)r_1(x)r_1(z)w_1(x)$

S_a and S_b are view-equivalent because all the reads-from relationship and final writes are the same. In fact, we have:

- Reads-from: $r_1(x)$ from $w_0(x)$, $r_1(z)$ from $w_0(z)$, $r_2(x)$ from $w_0(x)$, $r_3(z)$ from $w_0(z)$.
- Final writes: $w_1(x)$, $w_3(y)$, $w_3(z)$.

S_a and S_c are not view-equivalent because not all the reads-from relationship are the same.

Deciding if a generic schedule is in VSR is a NP-complete problem. Therefore, we need to find a stricter definition that is easier to check. The new definition may lead to rejecting some schedule that would be acceptable under view-serializability but not under the stricter criterion.

2.5 Conflict-serializability

Definition

Two operations o_i and o_j ($i \neq j$) are in *conflict* if they address the same resource and at least one of them is write. There are two possible cases:

1. Read-write conflicts ($r - w$ or $w - r$).
2. Write-write conflicts ($w - w$).

Two schedules are *conflict-equivalent* ($S_i \approx_C S_j$) if S_i and S_j contain

the same operations and in all the conflicting pairs the transactions occur in the same order.

A schedule is *conflict-serializable* (CSR) if and only if it is conflict equivalent to a serial schedule of the same transactions.

We have that VSR is a strict subset of CSR and that CSR implies VSR.

Proof VSR is a subset of CSR: Schedule $S = r_1(x)w_2(x)w_1(x)w_3(x)$ is:

- View-serializable.
- Not conflict-serializable

It is possible to check that there is no conflict-equivalent serial schedule. ■

Proof CSR implies VSR: We assume $S_1 \approx_C S_2$ and prove that $S_1 \approx_V S_2$.

S_1 and S_2 must have:

- The same final writes: if they didn't, there would be at least two writes in a different order, and since two writes are conflicting operations, the schedules would not be \approx_C .
- The same reads-from relations: if not, there would be at least one pair of conflicting operations in a different order, and therefore, again, \approx_C would be violated. ■

The testing of view-serializability is done with a conflict graph that has one node for each transaction T_i , and one arc from T_i to T_j if there exists at least one conflict between an operation o_i of T_i and an operation o_j of T_j such that o_i precedes o_j .

Theorem

A schedule is *conflict-serializable* if and only if its conflict graph is acyclic.

Example: We are given the schedule

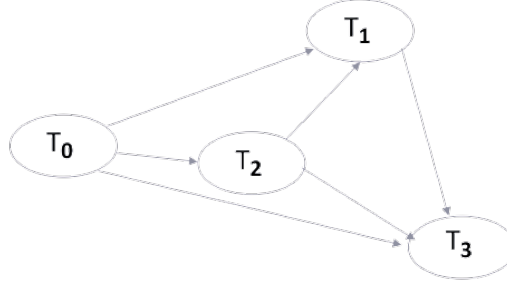
$$S : w_0(x)r_1(x)w_0(z)r_1(z)r_2(x)w_0(y)r_3(z)w_3(z)w_2(y)w_1(x)w_3(y)$$

To test the conflict serializability we have to do the following steps:

1. Create all the nodes based on the number of transactions of the schedule.
2. Divide the operation based on resource requested.
3. Check all the write-write and read-write relationships in each subset, and add the arcs based on these.

In the given example we obtain:

- $x : w_0 r_1 r_2 w_1$
- $y : w_0 w_2 w_3$
- $z : w_0 r_1 r_3 w_3$



There are no cycles in the graph, so this means that the schedule is CSR.

Proof CSR implies acyclicity of the conflict graph: Consider a schedule S in *CSR*. As such, it is \approx_C to a serial schedule. Without loss of generality we can label the transactions of S to say that their order in the serial schedule is: $T_1 T_2 \dots T_n$. Since the serial schedule has all conflicting pairs in the same order as schedule S , in the conflict graph there can only be arcs (i, j) , with $i < j$. Then the graph is acyclic, as a cycle requires at least an arc (i, j) with $i > j$. ■

Proof acyclicity of the conflict graph implies CSR: If S 's graph is acyclic then it induces a topological (partial) ordering on its nodes. The same partial order exists on the transactions of S . Any serial schedule whose transactions are ordered according to the partial order is conflict-equivalent to S , because for all conflicting pairs (i, j) it is always $i < j$. ■

2.6 Concurrency control in practice

Conflict-serializability checking would be efficient if we knew the graph from the beginning, but usually we don't. Therefore, a scheduler must rather work online. So, it is not feasible to maintain the conflict graph, update it, and check its acyclicity at each operation request. At the same time, the assumption that concurrency control can work only with the commit-projection of the schedule is unrealistic because aborts do occur. We need

some simple online decision criterion for the scheduler, which must avoid as many anomalies as possible, and have negligible overhead.

When dealing with online concurrency control, it is important also to consider arrival sequences. The concurrency control system maps an arrival sequence into an effective a posteriori schedule. To implement this online scheduling we use two main families of techniques:

- Pessimistic (locks): if a resource is taken, make the requester wait or pre-empt the holder.
- Optimistic (timestamps and versions): serve as many requests as possible, possibly using out-of-date versions of the data.

Usually, commercial systems take the best of both worlds.

2.7 Locking

The method called locking is the most used in commercial systems. A transaction is well-formed with respect to locking if:

- Read operations are preceded by *r_lock* (shared) and followed by unlock.
- Write operations are preceded by *w_lock* (exclusive) and followed by unlock.

In both cases unlocking can be delayed with respect to the end of the operations. So, every object can be: free, r-locked or w-locked.

Transactions that first read and then write an object may acquire a *w_lock* already when reading or acquire a *r_lock* first and then upgrade it into a *w_lock* (escalation).

The lock manager receives requests from the transactions and grants resources according to the conflict table:

Request	Resource status		
	<i>FREE</i>	<i>R_LOCKED</i>	<i>W_LOCKED</i>
<i>r_lock</i>	✓ R_LOCKED	✓ R_LOCKED(<i>n</i> + +)	✗ W_LOCKED
<i>w_lock</i>	✓ W_LOCKED	✗ R_LOCKED	✗ W_LOCKED
<i>unlock</i>	ERROR	✓ <i>n</i> - -	✓ FREE

Example: Given a schedule with three transactions and the following operations:

$$r_1(x)w_1(x)r_2(x)r_3(y)w_1(y)$$

We have the following locks:

- $r_1(x)$: $r_1_lock(x)$ request \rightarrow Ok $\rightarrow x$ is r -locked with $n_x = 1$.
- $w_1(x)$: $w_1_lock(x)$ request \rightarrow Ok $\rightarrow x$ is w -locked.
- $r_2(x)$: $r_1_lock(x)$ request \rightarrow No, because x is w -locked $\rightarrow T_2$ waits.
- $r_3(y)$: $r_3_lock(y)$ request \rightarrow Ok $\rightarrow y$ is r -locked with $n_y = 1$ and then T_3 unlocks y .
- $w_1(y)$: $w_1_lock(y)$ request \rightarrow Ok $\rightarrow y$ is w -locked and then x and y are freed.

So, the schedule a posteriori will become:

$$r_1(x)w_1(x)r_3(y)w_1(y)r_2(x)$$

and we have that the transaction two is delayed.

The locking system is implemented via lock tables, which are hash tables indexing the lockable items via hashing and where each locked item has a linked list associated with it. Every node in the linked list represents the transaction which requested for lock, the lock mode and the current status. Every new lock request for the data item is appended to the list.

2.8 Two-phase locking

With the locking showed before we do not eliminate the anomalies caused by non-repeatable reads. To avoid this problem we can use a two-phase rule which requires that a transaction cannot acquire any other lock after releasing one. So, we have a phase where the transaction acquires all the locks, a phase where it executes all operations and a final phase of unlocking.

Definition

The class of *two-phase locking* is the set of all schedules generated by a scheduler that:

- Only processes well-formed transactions.
- Grant locks according to the conflict table.
- Checks that all transactions apply the two-phase rule.

We have that $2PL$ is a strict subset of CSR and also that $2PL$ implies CSR .

Proof 2PL implies CSR: We assume that a schedule S is $2PL$. Consider, for each transaction, the moment in which it holds all locks and is going to release the first one. We sort the transactions by this temporal value and consider the corresponding serial schedule S' . We want to prove by contradiction that S is conflict-equivalent to S' :

$$S' \approx_C S, \dots$$

Consider a generic conflict $o_i \rightarrow o_j$ in S' with $o_i \in T_i$, $o_j \in T_j$, and $i < j$. By definition of conflict, o_i and o_j address the same resource r , and at least one of them is write. The two operations cannot occur in reverse order of S . This proves that all $2PL$ schedules are view-serializable. ■

The anomalies that remain in this state are only the phantom inserts (needs predicate locks) and the dirty reads.

2.9 Strict two-phase and predicate locking

Definition

In *strict two-phase locking* (or long duration locks) we also have that locks held by a transaction can be released only after commit or rollback.

This version of locking is used in most commercial DBMS whenever a high level of isolation is required.

To prevent phantom inserts a lock should be placed also on future data using the predicate locks. If the predicate lock is on a resource, other transactions cannot insert, delete, or update any tuple satisfying this predicate.

2.10 Isolation levels in SQL '99

SQL defines transaction isolation levels which specify the anomalies that should be prevented by running at that level:

	Dirty read	Non-repeatable read	Phantoms
Read uncommitted	✓	✓	✓
Read committed	×	✓	✓
Repeatable reads	×	×	✓ (insert)
Serializable	×	×	×

The four levels are implemented respectively with: no read locks, normal read locks, strict read locks, and strict locks with predicate locks. Serializable is not the default because its strictness can arise the following problems:

- Deadlock: two or more transactions in endless mutual wait.
- Starvation: a single transaction in endless wait.

2.11 Deadlocks

A deadlock occurs because concurrent transactions hold and in turn request resources held by other transactions.

Definition

A *lock graph* is a bipartite graph in which nodes are resources or transactions and arcs are lock requests or lock assignments.

A *wait-for graph* is a graph in which nodes are transactions and arcs are waits for relationships.

A deadlock is represented by a cycle in the wait-for graph of transactions.

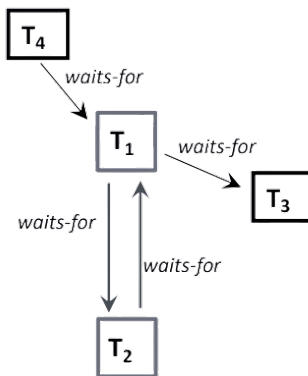


Figure 2.1: An example of deadlock in the wait-for graph

It is possible to solve deadlocks in three different ways:

- Timeout: a transaction is killed and restarted after a given amount of waiting, to be determined by the system manager.
- Deadlock prevention: kills transactions that could cause cycles. It is implemented in two ways:

1. Resource-based prevention puts restrictions on lock requests. The idea is that every transaction requests all resources at once, and only once. The main problem is that it's not easy for transactions to anticipate all requests.
 2. Transaction-based prevention puts restrictions on transactions' IDs. Assigning IDs to transactions incrementally allows to give an age to each one. It is possible to choose to kill the holding transaction (preemptive) or the requesting one (non-preemptive). The main problem is that the number of killings is too big.
- Deadlock detection: it can be implemented with various algorithms and used for distributed resources.

Definition

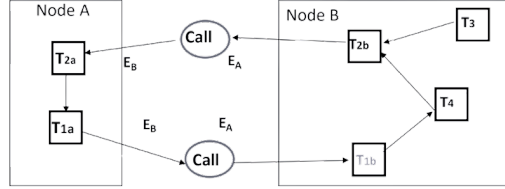
The *distributed dependency graph* is a wait-for graph where external call nodes represent a sub-transaction activating another sub-transaction at a different node.

The arrow shows a wait-for relation among local transactions. If one term is an external call, either the source is waited for by a remote transaction or waits for a remote transaction. The Obermarck's algorithm needs the following assumptions:

- Transactions execute on a single main node.
- Transactions may be decomposed in sub-transactions running on other nodes.
- When a transaction spawns a sub-transaction it suspends work until the latter completes.
- Two wait-for relationships:
 - T_i waits for T_j on the same node because T_i needs a datum locked by T_j .
 - A sub-transaction of T_i waits for another sub-transaction of T_i running on a different node.

The goal of this algorithm is to detect a potential deadlock looking only at the local view of a node. Nodes exchange information and update their local graph based on the received information. Node A sends its local info to a node B only if: it contains a transaction T_i that is waited for from another remote transaction and waits for a transaction T_j active on B and $i > j$.

Example: Given the following distributed dependency graph:



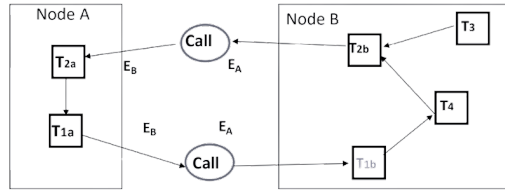
We can see that the potential deadlock is given by cycles. In this case, we have that T_{2a} waits for T_{1a} (data lock) that waits for T_{1b} (call) that waits for T_{2b} (data locks) that waits for T_{2a} (call).

In this case the node A dispatches information to B , in fact we have $E_b \rightarrow T_2 \rightarrow T_1 \rightarrow E_b$ and node B cannot dispatch information to A , because the forwarding rule is not respected: $E_a \rightarrow T_1 \rightarrow T_2 \rightarrow E_a$.

The Obermarck's algorithm runs periodically at each node and consists in four steps:

1. Get graph info from the previous nodes.
2. Update the local graph by merging the received information.
3. Check the existence of cycles among transactions denoting potential deadlocks: if found, select one transaction in the cycle and kill it.
4. Send updated graph info to the next nodes.

Example: Given the following distributed system:



We want to apply the Obermarck's algorithm:

1. Use the forwarding rule, in this case we have:
 - At Node A : $E_b \rightarrow T_2 \rightarrow T_1 \rightarrow E_b$ info sent to Node B
 - At Node B : $E_a \rightarrow T_1 \rightarrow T_2 \rightarrow E_a$ info not sent ($i < j$).
2. At node B there is the updated info $E_b \rightarrow T_2 \rightarrow T_1 \rightarrow E_b$ and it is added to the wait-for graph.

3. At node B a deadlock is detected (cycle between T_1 and T_2) and T_1 , T_2 or T_4 are killed.
4. Updated information are sent to all nodes.

There are four variants of this algorithm, based on various conventions.

	Message condition	Message receiver
A	$i > j$	Following node
B	$i > j$	Preceding node
C	$i < j$	Following node
D	$i < j$	Preceding node

In practice, the probability of deadlocks (n^{-2}) is much less than the conflict probability (n^{-1}). There are techniques to limit the frequency of deadlocks:

- Update lock: the most frequent deadlock occurs when two concurrent transactions start by reading the same resources and then decide to write and try to upgrade their lock to write on the resource. To avoid this situation, systems offer the update lock, that is used by transactions that will read and then write. The lock table become:

Request	Resource status			
	<i>FREE</i>	<i>SHARED</i>	<i>UPDATE</i>	<i>EXCLUSIVE</i>
<i>Shared lock</i>	✓	✓	✓	×
<i>Update lock</i>	✓	✓	×	×
<i>Exclusive lock</i>	✓	×	×	×

- Hierarchical lock: locks can be specified with different granularities. The objective of this is to lock the minimum amount of data and recognize conflicts as soon as possible. The method used to do so consists in asking locks on hierarchical resources by requesting resources top-down until the right level is obtained and releasing locks bottom-up. This is done by using five locking modes: shared, exclusive, ISL (intention of locking a sub-element of the current element in shared mode), IXL (intention of locking a sub-element of the current element in exclusive mode), and SIXL (lock of the element in shared mode with intention of locking a sub-element in exclusive mode). The lock table become:

Request	Resource status					
	<i>FREE</i>	<i>ISL</i>	<i>IXL</i>	<i>SL</i>	<i>SIXL</i>	<i>XL</i>
<i>ISL</i>	✓	✓	✓	✓	✓	×
<i>IXL</i>	✓	✓	✓	×	×	×
<i>SL</i>	✓	✓	×	✓	×	×
<i>SIXL</i>	✓	✓	×	×	×	×
<i>XL</i>	✓	×	×	×	×	×

Example : Given a table X with eight tuples divided in two pages:

P1	P2
$t1$	$t5$
$t2$	$t6$
$t3$	$t7$
$t4$	$t8$

And two transactions with the following schedules:

$$T_1 = r(P1) \ w(t3) \ r(t8)$$

$$T_2 = r(t2) \ r(t4) \ w(t5) \ w(t6)$$

We can see that they are not in a read-write conflict (because they are independent of the order). Without hierarchical locking both transactions needs to operate on the same table, so the concurrency will be almost useless in this case. But with this technique, calling X the table, we have that the transaction acquires the following locks:

$$T_1 : IXL(root) \ SIXL(P1) \ XL(t3) \ ISL(P2) \ SL(t8)$$

$$T_2 : IXL(root) \ ISL(P1) \ SL(t2) \ SL(t4) \ IXL(P2) \ XL(t5) \ XL(t6)$$

2.12 Timestamps

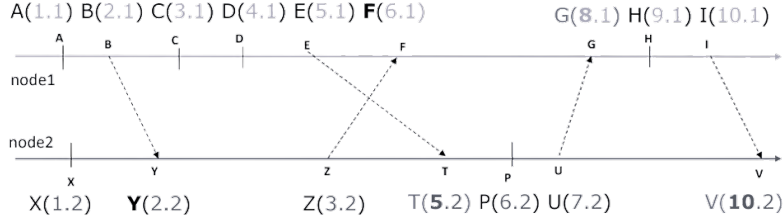
Locking is also named pessimistic concurrency control because it assumes that collisions will arise. In reality collisions are rare. So, it is possible to use optimistic concurrency control methods like timestamps, which are identifier that defines a total ordering of the events of a system. Each transaction has a timestamp representing the time at which the transaction begins so that transactions can be ordered: the smaller is the index the older is the transaction. A schedule is accepted only if it reflects the serial ordering of

the transactions induced by their timestamps. The timestamps are given by a system's function on request. The syntax of a timestamp is the following:

$$\text{event-id.node-id}$$

The algorithm's synchronization is based on send-receive of messages, and it is called Lamport method. It is based on the following rule: it is not possible to receive a message from the future, if this happens the bumping rule is used to bump the timestamp of the receive event beyond the timestamp of the send event.

Example: An example of timestamps assignation at two different nodes can be the following.



The scheduler uses two counters: one for writes $WTM(x)$ and another for reads $RTM(x)$. The scheduler receives read/write requests tagged with the timestamp of the requesting transaction. In case of a read operation we can have:

- If $ts < WTM(x)$ the request is rejected and the transaction is killed.
- Else, access is granted, and we set $RTM(x) = \max(RTM(x), ts)$.

In case of a write operation we can have:

- If $ts < RTM(x)$ or $ts < WTM(x)$ the request is rejected, and the transaction is killed.
- Else, access is granted, and we set $WTM(x) = ts$.

The problem that arise using this technique is that too many transactions are killed.

Example: Let us assume $RTM(x) = 7$ and $WTM(x) = 4$ and the following schedule:

$$S = r_6(x)r_8(x)r_9(x)w_8(x)w_{11}(x)r_{10}(x)$$

Using the timestamps we obtain:

Request	Response	New value
$r_6(x)$	✓	-
$r_8(x)$	✓	RTM(x) = 8
$r_9(x)$	✓	RTM(x) = 9
$w_8(x)$	✗	T_8 killed
$w_{11}(x)$	✓	WTM(x) = 11
$r_{10}(x)$	✗	T_{10} killed

It is not possible to compare 2PL to TS (this means that there is no subset between the categories). We have only that TS implies CSR.

Proof TS implies CSR: Let S be a TS schedule of T_1 and T_2 . Suppose S is not CSR, which implies that it contains a cycle between T_1 and T_2 . S contains $op_1(x)$, $op_2(x)$ where at least one is a write. S contains also $op_2(y)$, $op_1(y)$ where at least one is a write. When $op_1(y)$ arrives:

- If $op_1(y)$ is a read, T_1 is killed by TS because it tries to read a value written by a younger transaction, so it is a contradiction.
- If $op_1(y)$ is a write, T_1 is killed no matter what $op_2(y)$ is, because it tries to write a value already read or written by a younger transaction, so it is a contradiction. ■

Basic TS-based control considers only committed transactions in the schedule, aborted transactions are not considered. If aborts occur, dirty reads may happen. To cope with dirty reads, a variant of basic TS must be used: a transaction T_i that issues $r_{ts}(x)$ or $w_{ts}(x)$ such that $ts > \text{WTM}(x)$ has its read or write operation delayed until the transaction T' that wrote the value of x has committed or aborted. This is similar to long duration write locks, so it introduces delays.

Action	2PL	TS
Transaction	Wait	Killed and restarted
Serialization	Imposed by conflicts	Imposed by timestamp
Delay	Long (strict version)	Long
Deadlocks	Possible	Prevented

Since that restarting a transaction is costlier than waiting, 2PL is better if used alone. Commercial system mediates between those techniques to get the better feature from both. To reduce the number of killings it is possible to use the Thomas rule, that changes the rule for the write operations:

- If $ts < \text{RTM}(x)$ the request is rejected and the transaction is killed.
- Else, if $ts < \text{WTM}(x)$ then our write is obsolete: it can be skipped.
- Else, access is granted, and we set $\text{WTM}(x) = ts$.

2.13 Multi-version concurrency control

The idea of multi-version is that writes generate new versions, and that reads access the right version. Writes generate new copies, each one with a new WTM, so each object x always has $N \geq 1$ active versions. There is a unique global $\text{RTM}(x)$. Old versions are discarded when there are no transactions that need their values. In theory, it is possible to use the following rule:

- $r_{ts}(x)$ is always accepted. A copy x_k is selected for reading such that:
 - If $ts \geq \text{WTM}_N(x)$, then $k = N$.
 - Else take k such that $\text{WTM}_k(x) \leq ts < \text{WTM}_{k+1}(x)$.
- $w_{ts}(x)$:
 - If $ts < \text{RTM}(x)$ the request is rejected.
 - Else a new version is created for timestamp ts (N is incremented). $\text{WTM}_1(x), \dots, \text{WTM}_N(x)$ are the new versions, kept sorted from oldest to youngest.

Example: Let us assume $\text{RTM}(x) = 7$, $N = 1$ and $\text{WTM}_1(x) = 4$ and the following schedule:

$$S = r_6(x)r_8(x)r_9(x)w_8(x)w_{11}(x)r_{10}(x)r_{12}(x)w_{14}(x)w_{13}(x)$$

Using the multi-version we obtain:

Request	Response	New value
$r_6(x)$	✓	-
$r_8(x)$	✓	$\text{RTM}(x) = 8$
$r_9(x)$	✓	$\text{RTM}(x) = 9$
$w_8(x)$	✗	T_8 killed
$w_{11}(x)$	✓	$\text{WTM}_2(x) = 11, N = 2$
$r_{10}(x)$	✓ on $x_{(1)}$	$\text{RTM}(x) = 10$
$r_{12}(x)$	✓ on $x_{(2)}$	$\text{RTM}(x) = 12$
$w_{14}(x)$	✓	$\text{WTM}_3(x) = 14, N = 3$
$w_{13}(x)$	✓	$\text{WTM}_4(x) = 14, N = 4$

In practice, it is possible to use the following rule:

- $r_{ts}(x)$ is always accepted. A copy x_k is selected for reading such that:
 - If $ts \geq \text{WTM}_N(x)$, then $k = N$.
 - Else take k such that $\text{WTM}_k(x) \leq ts < \text{WTM}_{k+1}(x)$.
- $w_{ts}(x)$:
 - If $ts < \text{RTM}(x)$ or $ts < \text{WTM}_N(x)$ the request is rejected.
 - Else a new version is created for timestamp ts (N is incremented)
 - $\text{WTM}_1(x), \dots, \text{WTM}_N(x)$ are the new versions, kept sorted from oldest to youngest.

Example: Let us assume $\text{RTM}(x) = 7$, $N = 1$ and $\text{WTM}_1(x) = 4$ and the following schedule:

$$S = r_6(x)r_8(x)r_9(x)w_8(x)w_{11}(x)r_{10}(x)r_{12}(x)w_{14}(x)w_{13}(x)$$

Using the multi-version we obtain:

Request	Response	New value
$r_6(x)$	✓	-
$r_8(x)$	✓	$\text{RTM}(x) = 8$
$r_9(x)$	✓	$\text{RTM}(x) = 9$
$w_8(x)$	✗	T_8 killed
$w_{11}(x)$	✓	$\text{WTM}_2(x) = 11, N = 2$
$r_{10}(x)$	✓ on $x_{(1)}$	$\text{RTM}(x) = 10$
$r_{12}(x)$	✓ on $x_{(2)}$	$\text{RTM}(x) = 12$
$w_{14}(x)$	✓	$\text{WTM}_3(x) = 14, N = 3$
$w_{13}(x)$	✗	T_{13} killed

The final structure for the sets is the following.

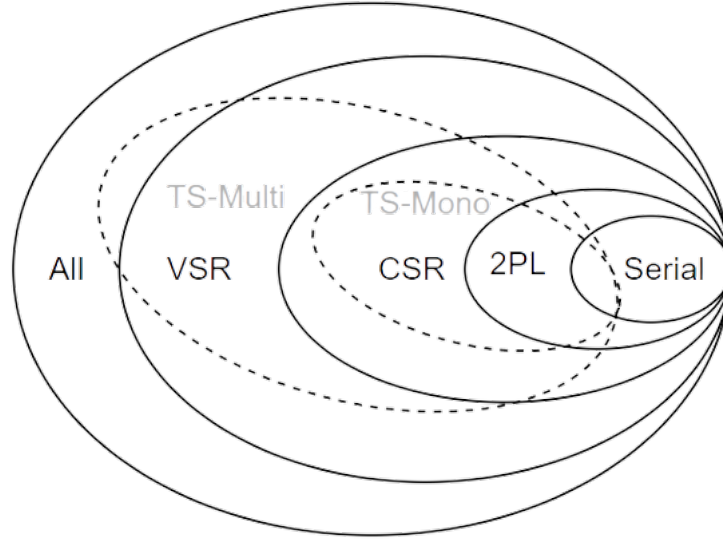


Figure 2.2: Set structure for concurrency classification

The realization of TS-Multi gives the opportunity to introduce into the DBMS another isolation level called snapshot isolation. In this level only $WTM(x)$ is used. The rule used in this level states that every transaction reads the version consistent with its timestamp, and defers writes to the end. If the scheduler detects that the writes of a transaction conflict with writes of other concurrent transactions after the snapshot timestamp, it aborts. Snapshot isolation does not guarantee serializability: it is possible to have a new anomaly called write skew (the order of the operation can change in different executions changing the final result).

Chapter 3

Ranking

3.1 Introduction

Data are: available in large quantities, represented under many models, queried with many languages, and processed with many algorithms. We can use ranking to get the best result out of the data. To do this we have to use the so called multi-objective optimization, that consist in the simultaneous optimization of different criteria. A general multi-objective problem can be formulated as follows: given N objects described by d attributes, we need to find the best k objects.

This problem is relevant in many fields, for instance: search engines, e-commerce, recommender systems, and Machine Learning.

The main approaches to solve the multi-objective optimization are:

- Ranking queries: selects the top k objects according to a given scoring function.
- Skyline queries: selects the set of non-dominated objects.

3.2 History

Rank aggregation is the problem of combining several ranked lists of objects in a robust way to produce a single consensus ranking of the objects.

In the 13th century started this problem was introduced with the elections polls. Borda proposed to give a number of penalty points equivalent to the position of the electors in a voter's ranking, so the winner will be the candidate with the lowest overall penalty. Condorcet, instead, proposed to consider as winner the candidate who defeats every other candidate in pairwise majority rule election. The winner with two methods can be different.

Example: Given ten voters and three candidates with the following votes:

1	2	3	4	5	6	7	8	9	10
A	A	A	A	A	A	C	C	C	C
C	C	C	C	C	C	B	B	B	B
B	B	B	B	B	B	A	A	A	A

For Borda we have:

- $A : 1 \cdot 6 + 3 \cdot 4 = 18$
- $B : 3 \cdot 6 + 2 \cdot 4 = 26$
- $C : 2 \cdot 6 + 1 \cdot 4 = 16$ (winner)

While for Condorcet we have that A beats both B and C in pairwise majority, so it is the winner. So, the winner is different and depends on the method used.

In 1950 Arrow proposed the axiomatic approach. He defined the aggregation as axioms and understood that a small set of natural requirements cannot be simultaneously achieved by any nontrivial aggregation function. So, he stated the Arrow's paradox: no rank-order electoral system can be designed that always satisfies these fairness criteria:

- No dictatorship (nobody determines, alone, the group's preference).
- If all prefer X to Y , then the group prefers X to Y .
- If, for all voters, the preference between X and Y is unchanged, then the group preference between X and Y is unchanged.

To solve this paradox, in the later years researchers tried to measure the values of all analyzed objects and created the metric approach. This consist in finding a new ranking R whose total distance to the initial rankings R_1, \dots, R_n is minimized. The distance between rankings can be found in several ways:

- Kendall tau distance $K(R_1, R_2)$, defined as the number of exchanges in a bubble sort to convert R_1 to R_2 .
- Spearman's foot-rule distance $F(R_1, R_2)$, which adds up the distance between the ranks of the same item in the two rankings.

Finding an exact solution is computationally hard for Kendall tau (NP-complete), but tractable for Spearman's foot-rule (P time). These distances are related:

$$K(R_1, R_2) \leq F(R_1, R_2) \leq 2K(R_1, R_2)$$

And it is possible to find efficient approximation for $F(R_1, R_2)$.

3.3 Opaque rankings

The technique that works on opaque rankings are those which use only the position of the elements in the ranking and no other associated score. The algorithm called MedRank is based on the notion of median and provides an approximation of the Foot-rule optimal aggregation. The inputs of this algorithm are: an integer k and a ranked list R_1, \dots, R_m of N elements. The output is the top k elements according to median ranking. The idea of the algorithm is the following:

1. Use sorted accesses in each list, one element at a time, until there are k elements that occur in more than $m/2$ lists.
2. These are the top k elements.

Definition

The maximum number of sorted accesses made on each list is also called the *depth reached* by the algorithm.

Example: Suppose we have to sort the hotels based on three rankings criteria: price, rating, and distance. Using the MedRank algorithm we can make one sorted access at a time in each ranking and then look for hotels that appear in at least two rankings. We assume that price, rating and distance are opaque, so only the rank's position matters. The ranks of hotels are the following:

Price	Rating	Distance
Ibis	Crillon	Le Roch
Etap	Novotel	Lodge In
Novotel	Sheraton	Ritz
Mercure	Hilton	Lutetia
Hilton	Ibis	Novotel
Sheraton	Ritz	Sheraton
Crillon	Lutetia	Mercure
...

If we use MedRank with $k = 3$, we will obtain the following rank:

Top k hotels	Median rank
Novotel	$\text{median}\{2, 3, 5\} = 3$
Hilton	$\text{median}\{4, 5, ?\} = 5$
Ibis	$\text{median}\{1, 5, ?\} = 5$

The depth in this case is equal to five.

Definition

An algorithm is *optimal* if its execution cost is never worse than any other algorithm on any input.

MedRank is not optimal, but it is instance optimal. This means that among the algorithms that access the lists in sorted order, this is the best possible algorithm on every input instance. In other words, its cost cannot be arbitrarily worse than any other algorithm on any problem instance. The instance optimality is a form of optimality aimed at when standard optimality is unachievable.

Definition

Let \mathbf{A} be a family of algorithms, \mathbf{I} a set of problem instances. Let cost be a cost metric applied to an algorithm-instance pair. Algorithm A^* is instance-optimal with respect to \mathbf{A} and \mathbf{I} for the cost metric cost if there exist constants k_1 and k_2 such that, for all $A \in \mathbf{A}$ and $I \in \mathbf{I}$:

$$\text{cost}(A^*, I) \leq k_1 \text{cost}(A, I) + k_2$$

If A^* is instance-optimal, then any algorithm can improve with respect to A^* by only a constant factor r , which is therefore called the optimality ratio of A^* . Instance optimality is a much stronger notion than optimality in the average or worst case.

3.4 Ranking queries

Ranking queries, also called top- k queries, aim to retrieve only the k best answers from a potentially very large result set. Ranking is based on ordering objects based on their relevance.

Assume a scoring function S that assigns to each tuple t a numerical score for ranking tuples. The algorithm needs this inputs: cardinality k , dataset

R , scoring function S . The output is the k highest-scored tuples with respect to S . The idea is the following:

1. For all tuples t in R compute $S(t)$.
2. Sort tuples based on their scores.
3. Return the first k highest-scored tuples.

This naïve approach is expensive for large dataset because it requires to sort a large amount of data. It is even worse if more than one relation is involved because it needs to join all tuples. So, we now know that two abilities are required:

- Ordering the tuples according to their scores. This is taken care of by

`ORDER BY`

- Limiting the output cardinality to k tuples. This is taken care (since 2008) of by

`FETCH FIRST k ROWS ONLY`

Example: Consider the following queries:

```
a) SELECT *
   FROM UsedCarsTable
   WHERE Vehicle = 'Audi/A4'
   AND Price <= 21000
   ORDER BY 0.8*Price+0.2*Miles
b) SELECT *
   FROM UsedCarsTable
   WHERE Vehicle = 'Audi/A4'
   ORDER BY 0.8*Price+0.2*Miles
```

The values 0.8 and 0.2, also called weights, are a way to normalize our preferences on price and mileage. Query a will likely miss some relevant answers (near-miss). Query b will return all Audi/A4 in the dataset (information overload).

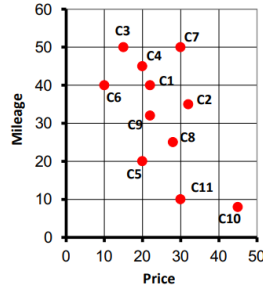
Only the first k tuples become part of the result. If more than one set of k tuples satisfies the ordering directive, any of such sets is a valid answer (non-deterministic semantics).

To evaluate a top- k query it is possible to consider two basic aspects:

- Query type: one relation, many relations, and aggregate results.
- Access paths: no index, indexes on all/some ranking attributes.

In the simplest case, that is a top- k selection query with only one relation, if the input is sorted according to S it is possible to read only the first k tuples; if the tuples are not sorted we need to perform an in-memory sort (through an *heap*), that has a complexity of $O(N \log k)$.

Example: Consider the two-dimensional attribute space (Price, Mileage) of the previous example:



In the image each tuple is represented by a two-dimensional point (p, m) , where p is the Price and m is the Mileage. Intuitively, minimizing $0.8 * Price + 0.2 * Mileage$ is equivalent to looking for points close to $(0, 0)$. Our preferences are essential to determine the result. Consider the line of equation:

$$0.8 * Price + 0.2 * Mileage = v$$

where v is a constant. This can also be written as:

$$Mileage = -4 * Price + 5 * v$$

from which we see that all the lines have a slope -4 . By definition, all the points of the same line are equally good. If we change the weights, it is possible that the best choice also varies. The target of a query is not necessarily $(0, 0)$, rather it can be any point $q = (q_1, q_2)$.

In general, in order to determine the goodness of a tuple t , we compute its distance from the target point q : the lower the distance from q , the better t . If we consider the distances the model become:

- An m -dimensional ($m \geq 1$) space $A = (A_1, A_2, \dots, A_m)$ of ranking attributes.

- A relation $R(A_1, A_2, \dots, A_m, B_1, B_2, \dots)$, where B_1, B_2, \dots are other attributes.
- A target (query) point $q = (q_1, q_2, \dots, q_m)$, $q \in A$.
- A function $d : A \times A \rightarrow \mathbb{R}^+$, measuring the distance between points in A .

Definition

The top- k query is called *k-nearest neighbors* if given a point q , a relation R , an integer $k \geq 1$, and a distance function d it determines the k tuples in R that are closest to q according to d .

The distance function called *L_p -norms* is defined as:

$$L_p(t, q) = \left(\sum_{i=1}^m |t_i - q_i|^p \right)^{\frac{1}{p}}$$

Relevant cases of the L_p -norm are:

- Euclidean distance (ellipsoids): $L_2(t, q) = \sqrt{\sum_{i=1}^m |t_i - q_i|^2}$
- Manhattan distance (rhomboids): $L_1(t, q) = \sum_{i=1}^m |t_i - q_i|$
- Čebyšëv distance (rectangles): $L_\infty(t, q) = \max_i \{|t_i - q_i|\}$

The shape of the attribute space depends on the distance function and the weight associated to each coordinate:

- Euclidean distance (hyper-ellipsoids):

$$L_2(t, q; W) = \sqrt{\sum_{i=1}^m w_i |t_i - q_i|^2}$$

- Manhattan distance (hyper-rhomboids):

$$L_1(t, q; W) = \sum_{i=1}^m w_i |t_i - q_i|$$

- Čebyšëv distance (hyper-rectangles):

$$L_\infty(t, q; W) = \max_i \{w_i |t_i - q_i|\}$$

Definition

In a *top-k join query* we have $n > 1$ input relations and a scoring function S defined on the result of the join. The general formula is:

```
SELECT <attributes>
FROM R1,R2,...,Rn
WHERE <conditions>
ORDER BY S(p1,p2,...,pm) [DESC]
FETCH FIRST k ROWS ONLY
```

where p_1, p_2, \dots, p_m are the scoring criteria.

In the top- k 1 – 1 join queries all the joins are on a common key attribute. It is possible to have two main scenarios:

- There is an index for retrieving tuples according to each preference.
- The relation is spread over several sites, each providing information only on part of the objects.

We make the following assumptions:

- Each input list supports sorted access: this means that each access returns the identifier of the next best object, its partial score p_j .
- Each input list supports random access: this means that each access returns the partial score of an object, given its identifier.
- The identifier of an object is the same across all inputs.
- Each input consists of the same set of objects.

Example : Given the following reviews on two different sites:

EatWell

Name	Score
The old mill	9.2
The canteen	9.0
Cheers!	8.3
Da Gino	7.5
Let's eat!	6.4
Chez Paul	5.5
Los pollos hermanos	5.0

BreadAndWine

Name	Score
Da Gino	9.0
Cheers!	8.5
The old mill	7.5
Chez Paul	7.5
The canteen	7.0
Los pollos hermanos	6.5
Let's eat!	6.0

If we aggregate the two tables with the following query:

```

SELECT *
FROM EatWell EW, BreadAndWine BW
WHERE EW.Name = BW.Name
ORDER BY EW.Score + BW.Score DESC
FETCH FIRST 1 ROW ONLY

```

We will obtain the following result:

Name	Global Score
Cheers!	16.8
The old mill	16.7
Da Gino	16.5
The canteen	16.0
Chez Paul	13.0
Let's eat!	12.4
Los pollos hermanos	11.5

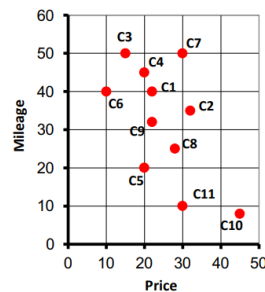
Note that the winner is not the best locally.

Each object o returned by the input L_j has an associated local/partial score $p_j(o) \in [0, 1]$. For convenience, scores are normalized. The hypercube $[0, 1]^m$ is called the score space. The point $p(o) = (p_1(o), p_2(o), \dots, p_m(o)) \in [0, 1]^m$ is the map of o into the score space. The global score $S(o)$ of o is computed by means of a scoring function S that combines in some way the local scores of o :

$$S(o) \equiv S(p(o)) = S(p_1(o), p_2(o), \dots, p_m(o))$$

with $S : [0, 1]^m \rightarrow \mathbb{R}^+$.

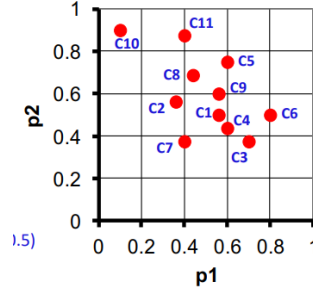
Example : Consider the attribute space $A = (Price, Mileage)$



If we choose $MaxP = 50,000$ and $MaxM = 80,000$, we have that a generic point o is mapped with these formulas:

$$p_1(o) = 1 - \frac{o.Price}{MaxP} \quad p_2(o) = 1 - \frac{o.Mileage}{MaxM}$$

Graphically we have;



The most common scoring functions are:

- Average (SUM), that weighs preferences equally

$$SUM(o) \equiv SUM(p(o)) = p_1(o) + p_2(o) + \dots + p_m(o)$$

- Weighted sum (WSUM), that weighs the preferences differently

$$WSUM(o) \equiv WSUM(p(o)) = w_1p_1(o) + w_2p_2(o) + \dots + w_mp_m(o)$$

- Minimum (MIN), that considers the worst partial score

$$MIN(o) \equiv MIN(p(o)) = \min\{p_1(o), p_2(o), \dots, p_m(o)\}$$

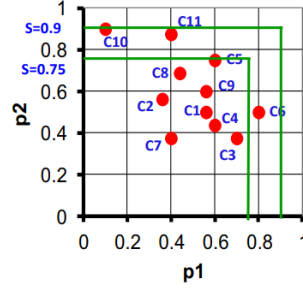
- Maximum (MAX), that considers the best partial score

$$MAX(o) \equiv MAX(p(o)) = \max\{p_1(o), p_2(o), \dots, p_m(o)\}$$

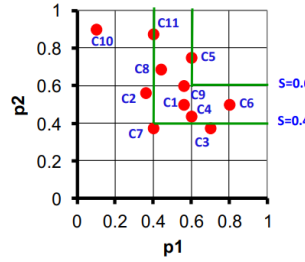
Note that even with the minimum scoring function we always want to retrieve the k objects with the highest global scores.

It is possible to define e-score curves in the score space, which are set of points with the same global score.

Example: In the previous graph we have that the iso-score curves for the maximum function is:



In the previous graph we have that the iso-score curves for the minimum function is:



3.4.1 B-zero algorithm

It is possible to efficiently compute the result of a top-k 1-1 join query using a maximum scoring function S it is possible to use the B_0 algorithm. The inputs of this algorithm are: an integer $k \geq 1$, and a ranked list R_1, \dots, R_m . The idea is:

1. Make k sorted accesses on each list and store objects and partial scores in a buffer B .
2. For each object in B , compute the MAX of its (available) partial scores.
3. Return the k objects with maximum score.

This algorithm is instance-optimal.

Example: Given the following reviews on two different sites:

EatWell

Name	Score
The old mill	9.2
The canteen	9.0
Cheers!	8.3
Da Gino	7.5
Let's eat!	6.4
Chez Paul	5.5
Los pollos hermanos	5.0

BreadAndWine

Name	Score
Da Gino	9.0
Cheers!	8.5
The old mill	7.5
Chez Paul	7.5
The canteen	7.0
Los pollos hermanos	6.5
Let's eat!	6.0

If we want to use the algorithm B_0 with $k = 3$ we have to select the first three row from both tables (that are sorted), and by summing all the found values by identifier we obtain the final rank with the maximum value. In this case we obtain the following table:

Name	S
The old mill	9.2
Da Gino	9.0
The canteen	9.0
Cheers!	8.5

3.4.2 A-zero algorithm

The A-zero algorithm, also called Fagin's algorithm, works with every monotonic scoring function and not only with maximum. The inputs of this algorithm are: an integer k , and a monotone function S combining ranked lists R_1, \dots, R_m . The idea of this algorithm is:

1. Extract the same number of objects by sorted accesses in each list until there are at least k objects in common.
2. For each extracted object, compute its overall score by making random accesses wherever needed.
3. Among these, output the k objects with the best overall score.

The complexity of this algorithm is $O(N^{\frac{m-1}{m}} k^{\frac{1}{m}})$, that is sublinear in the number N of objects. The stopping criterion is independent of the scoring function, and it is not instance optimal.

Example: Given the following hotel's review on different sites:

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We decide to use the following scoring function:

$$0.5 \cdot \textit{cheapness} + 0.5 \cdot \textit{rating}$$

and $k = 2$. We iterate on the rows until k hotels are found in all columns. In this case we need to inspect 5 column to find at least two hotels (we found: Ibis, Novotel and Hilton). Now we have to complete the score of the remaining incomplete hotels, so we make random access to find the missing values. After summing completing the information about those hotels we can compute the final ranking with the give score function, and we obtain:

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The main drawback of this algorithm is that it is dependent from the accesses, so the specificity of some scoring function is not exploited at all, and the memory requirements can become prohibitive because of the buffer that keeps the accessed objects. It is possible to make some improvements, but only changing the stopping condition can really improve the complexity.

3.4.3 Threshold algorithm

The inputs of the threshold algorithm are: an integer k , and a monotone function S combining ranked lists R_1, \dots, R_m . The idea of this algorithm is:

1. Do sorted access in parallel in each list R_i .
2. For each object o , do random accesses in the other lists R_j , thus extracting score s_j .
3. Compute overall score $S(s_1, \dots, s_m)$. If the value is among the k highest seen so far, and remember o .
4. Let s_{L_i} be the last score seen under sorted access for R_i .
5. Define threshold $T = S(s_{L_1}, \dots, s_{L_m})$.
6. If the score of the k -th object is worse than T , go to step one.
7. Return the current top- k objects.

The threshold algorithm is instance optimal among all algorithms that use random and sorted accesses, and the stopping criterion depends on the function and not on the number of accesses.

Example: Given the following hotel's review on different sites:

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We decide to use the following scoring function:

$$0.5 \cdot \text{cheapness} + 0.5 \cdot \text{rating}$$

and $k = 2$. We do a sorted access, and we put the hotels in the first row in the buffer with the mean value of both variables (the second can be found with a random access). The threshold point is $(0.92, 0.9)$ and has a value of 0.91, found with the scoring function. We continue to do this procedure, and we update the top k hotels only if the found hotel has a better score of at least one of the hotels in the buffer (note that we keep in the buffer only the top k hotels, while we delete the others). We stop the iteration when the value of all the objects in the buffer is greater or equal to the threshold value. In this example this happens after three iteration. In fact, we have that the buffer contains the following values:

and the threshold point has a value of 0.825

In general, TA performs much better than FA, since it can adapt to the specific scoring function. In order to characterize the performance of TA, we consider the middleware cost:

$$\text{cost} = SA * c_{SA} + RA * c_{RA}$$

where:

- SA is the total number of sorted accesses.
- RA is the total number of random accesses.
- c_{SA} is the base cost of sorted accesses.
- c_{RA} is the base cost of random accesses.

3.4.4 No random accesses algorithm

The NRA applies when random accesses cannot be executed. It returns the top- k objects, but their scores might be uncertain. The idea at the base of this algorithm is to maintain, for each selected object o a lower bound $S^-(o)$ and an upper bound $S^+(o)$ on its score. So, we need an unlimited buffer to store the objects like in the B_0 algorithm, sorted by decreasing lower bound values. This algorithm stops when the new object upper bound is less or equal to the worst lower bound of the best objects. The inputs are: an

integer $k \geq 1$, a monotone function S combining ranked lists R_1, \dots, R_m and the output is the ranking without the score of the objects. The idea is the following:

1. Make sorted access to each list.
2. Store in B each retrieved object o and maintain $S^-(o)$ and $S^+(o)$ and a threshold τ .
3. Repeat from step one as long as $S^-(B[k]) < \max\{\max\{S^+(B[i]), i > k\}, S(\tau)\}$