Computing Infrastructures
Exercises

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Abstract

The course topics are:

- Hardware infrastructure of datacenters:
  - Basic components, rack structure, cooling.
  - Hard Disk Drive and Solid State Disks.
  - RAID architectures.
  - Hardware accelerators.
- Software infrastructure of datacenters:
  - Virtualization: basic concepts, technologies, hypervisors and containers.
  - Computing Architecture: Cloud, Edge and Fog Computing.
  - Infrastructure, platform and software-as-a-service.
- Methods:
  - Scalability and performance of datacenters: definitions, fundamental laws, queuing network theory basics.
  - Reliability and availability of datacenters: definitions, fundamental laws, reliability block diagrams.

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CHAPTER 1

Exercise session I

1.1 Exercise one

A pacemaker for the heart has a failure rate of $\lambda = 0.25 \cdot 10^{-8}$ per hour.

1. Compute the mean time to failure.
2. Compute the probability that it fails during the first five years of operation.

Solution

1. The average time to failure is calculated as:

$$\text{MTTF} = \frac{1}{\lambda} = \frac{1}{0.25 \cdot 10^{-8}} = 4 \cdot 10^{8} \text{ hours} \approx 45.662 \text{ years}$$

   Hence, the average time to failure of the heart pacemaker is approximately 45.6 years.

2. The reliability of the system can be expressed as:

$$R(t) = e^{-\lambda t}$$

   Given λ as specified and a duration of five years, we have:

$$R(5) = e^{-5\lambda} = e^{-1.25 \cdot 10^{-8}} = 0.9999999875$$

   Subsequently, the probability of failure during the initial five years of operation is calculated as:

$$F(5) = 1 - R(5) = 1 - 0.9999999875 = 1.25 \cdot 10^{-8}$$

1.2 Exercise two

Let's consider a generic component D. We are tasked with determining the minimum integer value for the mean time to failure of D to ensure that $R_{\text{D}}(t) \geq 0.96$ at five days.

Solution

The reliability is defined as:

$$R(t) = e^{-\lambda t} = e^{-\frac{t}{\text{MTTF}}}$$

We need to satisfy:

$$R_{\text{D}}(5) \geq 0.96$$

Which is equivalent to:

$$e^{-\frac{5\text{days}}{\text{MTTF}}} \geq 0.96$$

Thus, we have:

$$\rightarrow \text{MTTF} \geq -\frac{5\text{days}}{\ln(0.96)} \rightarrow \text{MTTF} \geq 122.5 \text{ days}$$

Therefore, the minimum mean time to failure required to meet the given conditions is 122.5 days.

1.3 Exercise three

A smartphone manufacturer determines that their products have a mean time to failure of 59 years in normal use. They want to estimate how long a warranty should be set if no more than 5% of the items are to be returned for repair.

Solution

We seek a time t_w such that:

$$1 - R(t_w) = 0.05 \rightarrow R(t_w) = 0.95$$

Given the mean time to failure, we can express the reliability as:

$$R(t) = e^{-\lambda t} \rightarrow e^{-\frac{t}{\text{MTTF}}}$$

Substituting, we get:

$$R(t_w) = e^{-\frac{t_w}{\text{MTTF}}} = 0.95 \rightarrow t_w = -59 \ln(0.95) = 3.026 \text{ years}$$

Hence, the warranty should be set for approximately 3 years to ensure that no more than 5% of the items are returned for repair.

1.4 Exercise four

A complex system has a failure rate of $\lambda = 0.25 \cdot 10^{-4}$ per hour and a mean time to repair of 72 hours in normal use.

1. Compute the steady-state availability.

2. If mean time to repair is increased to 120 hours, compute the failure rate that can be tolerated without decreasing the availability of the system.

Solution

1. To calculate the steady-state availability, we first find the mean time to failure:
$$\text{MTTF} = \frac{1}{\lambda} = \frac{1}{0.25 \cdot 10^{-4}} = 40000 \text{ hours}$$
Then, the availability is computed as:
$$A = \frac{\text{MTTF}}{\text{MTTF} + \text{MTTR}} = \frac{40000}{40000 + 72} = 0.9982$$

2. If we increase the mean time to repair while maintaining the same availability of 0.9982, we first find the new minimum mean time to failure:
$$A = \frac{\text{MTTF}_{new}}{\text{MTTF}_{new} + \text{MTTR}} \rightarrow \text{MTTF}_{new} = -\frac{A \cdot \text{MTTR}}{A - 1} \rightarrow \text{MTTF}_{new} = 66666.66 \text{ hours}$$
From this, we derive the new failure rate:
$$\lambda_{new} = \frac{1}{\text{MTTF}_{new}} = \frac{1}{66666.66} = 1.5 \cdot 10^{-5}$$

1.5 Exercise five

Consider a server architecture composed of three main components: CPU, memory, and hard drive. Each component has a constant failure rate of $\frac{1}{64}$, $\frac{1}{58}$ and $\frac{1}{28}$ per year, respectively, and failures are assumed to be independent events.

1. Visualize the reliability block diagram of the server architecture.
2. Calculate the mean time to failure for the server.
3. Determine the reliability of the server over a three-year mission.

Solution

1. The reliability block diagram of the server architecture is depicted below:

Given that the components are in series, the total reliability is the sum of the reliabilities of each component:
$$R(t) = R_{CPU}(t) + R_{memory}(t) + R_{hard\,disk}(t) = e^{-\lambda_{CPU}t} + e^{-\lambda_{memory}t} + e^{-\lambda_{hard\,disk}t}$$

2. As the components are in series, the total failure rate is the sum of individual failure rates:
$$\lambda_{tot} = \lambda_{CPU} + \lambda_{memory} + \lambda_{hard\,disk} = \frac{1}{64} + \frac{1}{58} + \frac{1}{28} = \frac{891}{12992}$$
The mean time to failure is the inverse of the total failure rate:
$$\text{MTTF}_{tot} = \frac{1}{\lambda_{tot}} = \frac{1}{\frac{891}{12992}} = 14.58 \text{ years}$$

3. The reliability for a three-year mission is:
$$R_{tot}(3 \text{ years}) = e^{-3\lambda_{tot}} = 0.814$$

CHAPTER 2

Exercise session II

2.1 Exercise one

A computer system is engineered with a failure rate of one fault every five years under typical usage conditions. This system lacks fault tolerance capabilities, meaning it ceases functioning upon encountering its initial fault.

1. Compute the mean time to failure of the system.
2. Compute the probability that the system will fail during its first year of operation.
3. The standard warranty for the system covers 2 years of operation. However, the vendor aims to provide extended insurance against failures for the initial 5 years of operation, for which they plan to charge an additional fee. The vendor intends to charge 20$ for every 1% decrease in reliability to offer this insurance. Compute how much should the vendor charge for such an insurance.

Solution

1. The mean time to failure for the system is calculated as the inverse of the failure rate:

$$\text{MTTF} = \frac{1}{\lambda} = \frac{1}{\frac{1}{5}} = 5 \text{ years}$$

2. The failure probability within a given time frame is given by:

$$F(t) = 1 - R(t)$$

For the first year:

$$F(1) = 1 - R(1) = 1 - e^{-\lambda} = 1 - 0.818 = 0.18$$

3. We compute the reliability at 2 and 5 years:

$$R(2 \text{ year}) = e^{-2\lambda} = 0.67$$

$$R(5\text{ year}) = e^{-2\lambda} = 0.37$$

This yields a reliability drop of 0.3. Since the vendor wants to charge 20$ for each 1% drop in reliability, the cost is:

$$\text{cost} = \frac{0.3}{0.01} = 30 \cdot 20\$ = 600\$$$

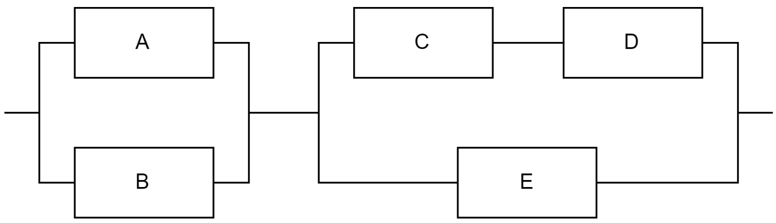
2.2 Exercise two

A system comprising five modules (A, B, C, D, and E) is designed to function properly under the following conditions: either modules A or B operate correctly, and concurrently modules C and D operate correctly, or alternatively, module E operates correctly.

1. Draw the reliability block diagram of the system.
2. Find the expression for the reliability of the system.
3. Given that the mean time to failure for modules A and B is 3412 hours, and for modules C, D, and E is 1245 hours, we aim to calculate the reliability of the system after 1 month.

Solution

1. The reliability block diagram of the system is depicted below:



2. The blocks A and B are in parallel, hence the total reliability is computed as:

$$R_{\text{AB}} = 1 - (1 - R_{\text{A}})(1 - R_{\text{B}})$$

The blocks C and D are in series, therefore the total reliability is:

$$R_{\text{CD}} = R_{\text{C}} R_{\text{D}}$$

Now, with the new block CD in parallel with E, the reliability becomes:

$$R_{\text{CDE}} = 1 - (1 - R_{\text{CD}})(1 - R_{\text{E}})$$

Finally, the entire system's reliability is the product of the reliabilities of blocks AB and CDE:

$$R_s = [1 - (1 - R_{\text{A}})(1 - R_{\text{B}})][1 - (1 - R_{\text{CD}})(1 - R_{\text{E}})]$$

3. We can compute the monthly mean time to failures:

$$\text{MTTF}_{\text{A}} = \text{MTTF}_{\text{B}} = 4,74 \text{ months}$$

$$\mathrm{MTTF_C} = \mathrm{MTTF_D} = \mathrm{MTTF_E} = 1,73 \text{ months}$$

The monthly failure rates are the inverses of the mean time to failures:

$$\lambda_\mathrm{A} = \lambda_\mathrm{B} = \frac{1}{4,74 \text{ months}} = 0.21$$

$$\lambda_\mathrm{C} = \lambda_\mathrm{D} = \lambda_\mathrm{E} = \frac{1}{1,73 \text{ months}} = 0.578$$

We can now compute the reliability of each block:

$$R_\mathrm{A}(1) = R_\mathrm{B}(1) = e^{-\lambda t} = e^{-1 \cdot 0.21} = 0.81$$

$$R_\mathrm{B}(1) = R_\mathrm{C}(1) = R_\mathrm{D}(1) = e^{-\lambda t} = e^{-1 \cdot 0.578} = 0.56$$

From the previously derived formula, we find:

$$R_s(1) = [1 - (1 - R_\mathrm{A}(1))\,(1 - R_\mathrm{B}(1))]\,[1 - (1 - R_\mathrm{CD}(1))\,(1 - R_\mathrm{E}(1))] = 0.67$$

Specifically, we have:

- $R_\mathrm{AB} = 1 - (1 - R_\mathrm{A})\,(1 - R_\mathrm{B}) = 0.96$.
- $R_\mathrm{CD} = R_\mathrm{C} R_\mathrm{D} = 0.31$.
- $R_\mathrm{CDE} = 1 - (1 - R_\mathrm{CD})\,(1 - R_\mathrm{E}) = 0.70$.

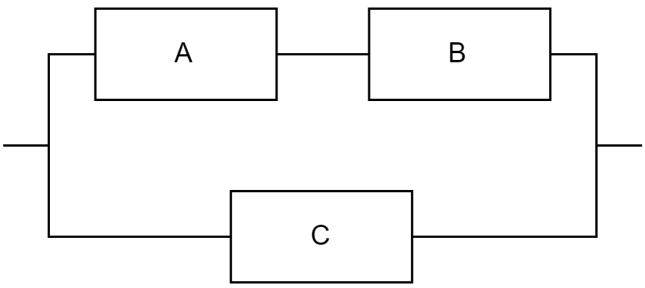
2.3 Exercise three

The system comprises three modules (A, B, and C) and is designed to function correctly if both A and B operate correctly or if C operates correctly. The availabilities of the modules are as follows: $A_\mathrm{A} = 0.97$, $A_\mathrm{B} = 0.92$, and $A_\mathrm{D} = 0.95$.

1. Illustrate the reliability block diagram of the system.
2. Compute the system's availability.

Solution

1. The reliability block diagram of the system is depicted below:



2. To calculate the availability of the system, we first determine the availability of blocks A and B in series:

$$A_\mathrm{AB} = A_\mathrm{A} A_\mathrm{B} = 0.97 \cdot 0.92 = 0.89$$

Subsequently, with block AB in parallel with C, the overall availability becomes:

$$A_s = 1 - (1 - A_\mathrm{AB})\,(1 - A_\mathrm{C}) = 1 - (0.11)(0.04) = 0.9956$$

2.4 Exercise four

A system comprises four non-redundant components, with a 1-year reliability of 0.92. A new version of the system incorporates a novel feature and utilizes six non-redundant components. Compute the 1-year reliability of the new system, assuming all components have the same mean time to failure.

Solution

The reliability of the initial system is:

$$R_{old}(1 \text{ year}) = \left(e^{-\lambda}\right)^4 = e^{-4\lambda}$$

Given this expression equals 0.92, we can determine the value of the failure rate:

$$e^{-4\lambda} = 0.92 \rightarrow \lambda = \frac{\ln(0.92)}{-4} = 0.02$$

From the failure rate, we can derive the mean time to failure:

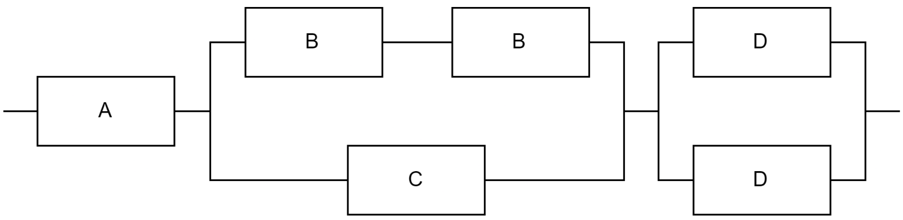
$$\text{MTTF} = \frac{1}{\lambda} = \frac{1}{0.02} = 47.97 \text{ years}$$

In the new scenario, the reliability becomes:

$$R_{old}(1 \text{ year}) = \left(e^{-\lambda}\right)^6 = e^{-6\lambda} = 0.88$$

2.5 Exercise five

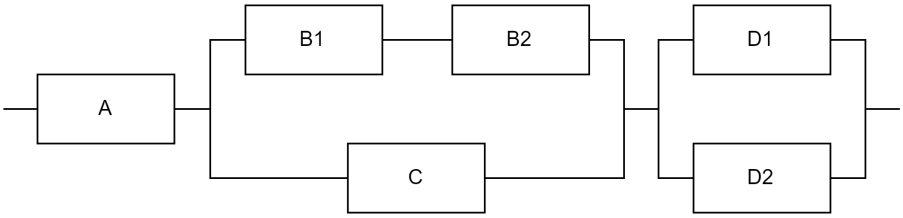
A system is composed of components arranged in the following reliability block diagram:



1. Identify all possible configurations of components that may fail without causing the entire system to fail.
2. Calculate MTTF_B knowing that $R(t) = 83\%$ at two years.
3. Compute the availability of the entire system given that $\text{MTTF}_\text{A} = \text{MTTF}_\text{D} = 1$ year, $\text{MTTF}_\text{C} = 14$ months, and $\text{MTTR} = 21$ days for all components.

Solution

1. Renaming the blocks in the reliability block diagram as:



   The pairs of blocks that may fail without causing the entire system to fail are:

   - B1 and D1.
   - B1 and D2.
   - B2 and D1.
   - B2 and D2.
   - C and D1.
   - C and D2.

2. The reliability of each block B is calculated as:

$$R_B(2 \text{ years}) = e^{-\frac{2}{\mathrm{MTTF_B}}}$$

   Given this expression equals 0.83, we solve for MTTF:

$$e^{-\frac{2}{\mathrm{MTTF_B}}} = 0.83 \rightarrow \mathrm{MTTF} = -\frac{2}{\ln(0.83)} = 10.73 \text{ years}$$

3. The mean time to failure of block C in years is:

$$\mathrm{MTTF_C} = \frac{14 \text{ months}}{12 \text{ months}} = 1.17 \text{ years}$$

   Similarly, the mean time to recovery is:

$$\mathrm{MTTR} = \frac{21 \text{ days}}{365 \text{ days}} = 0.058 \text{ years}$$

   We can compute the availability for each block:

   - $A_\mathrm{A} = A_\mathrm{D} = \frac{\mathrm{MTTF}}{\mathrm{MTTF+MTTR}} = \frac{1}{1+0.058} = 0.95$
   - $A_\mathrm{C} = \frac{\mathrm{MTTF}}{\mathrm{MTTF+MTTR}} = \frac{1.17}{1.17+0.058} = 0.95$
   - $A_\mathrm{B} = \frac{\mathrm{MTTF}}{\mathrm{MTTF+MTTR}} = \frac{10.73}{10.73+0.058} = 0.99$

   Now, we combine the two blocks B into a single block:

$$A_\mathrm{BB} = A_\mathrm{B} A_\mathrm{B} = 0.99 \cdot 0.99 = 0.98$$

Then, we have the parallel connection between the new block and C:

$$A_{\text{BBC}} = 1 - (1 - A_{\text{BB}})\,(1 - A_{\text{C}}) = 1 - (0.02)(0.05) = 0.999$$

Similarly, for the parallel connection between the two blocks D:

$$A_{\text{DD}} = 1 - (1 - A_{\text{D}})\,(1 - A_{\text{D}}) = 1 - (0.05)(0.05) = 0.997$$

Finally, the total availability of the system can be computed as the series of A, BBC, and DD:

$$A_s = 1 - (1 - A_{\text{A}})\,(1 - A_{\text{BBC}})\,(1 - A_{\text{DD}}) = 1 - (0.05)(0.001)(0.003) = 0.946$$

2.6 Exercise six

In the C-5 aircraft, there are 12 identical AC generators, and at least 9 of them must be operational for the aircraft to complete its mission. Failures follow an exponential distribution with a failure rate of 0.01 failure per hour. Compute the reliability of the generator system over a ten-hour mission in case the switch is perfect.

Solution

For a system composed of n identical replicas where at least r replicas must function for the entire system to operate correctly, the system reliability is given by:

$$R_s(t) = R_V \sum_{i=r}^{n} R_c^i (1 - R_c)^{n-1} \binom{n}{i}$$

Here:

- R_s is the system reliability.
- R_c is the component reliability.
- R_v is the voter reliability.
- n is the number of components.
- r is the minimum number of components that must function.

For each generator over a ten-hour mission, the reliability is:

$$R_m(10) = e^{-\lambda t} = e^{-0.01 \cdot 10} = 0.9048374$$

Substituting into the formula, we get:

$$R_s = \sum_{i=9}^{1} 2R_m^i\,(1 - R_m)^{12-i} \binom{12}{i} = 165R_m^{12} + 540R_m^{11} + 549R_m^1 0 + 220R_m^9 = 0.9782773$$