Nonlinear Optimization

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Abstract

The course aims to present the main methods for non-linear optimization, both continuous and discrete. Topics include: optimality conditions for both unconstrained and constrained problems, Lagrangian functions, and duality; gradient-based methods, Newton's methods, and step-size reduction techniques; recursive quadratic programming; and methods using penalty functions.

Contents

CHAPTER 1

Introduction

1.1 Introduction

1.1.1 Localization and transportation

We want to locate the warehouses. Decision variables:

- w_{ij} is the amount of product transported from warehouse i to client j, $\forall i, j$.
- (x_i, y_i) the coordinates of the i-th warehouse, $i`i \leq n$.
- t_{ij} dinstance between warehouse i and client j, $\forall i, j$.

minimze $\sum_{i=1}^{m} \sum_{j=1}^{n} t_{ij} w_{ij}$ such that $\sum_{j=1}^{n} w_{ij} \leq p_i \qquad \forall i$ $\sum_{i=1}^{m} w_{ij} \geq pd_j \qquad \forall j$ $t_{ij} = \sqrt{(x_i - a_j)^2 + (y_i - b_j)^2}$ $\forall i, j$ $(x_i, y_i) \in A_i \forall i$ $w_{ij} \geq 0 t_{ij} \geq 0 \forall i, j$

t_{ij} can be directly inserted into the objective function simplifying the function description.

1.1.2 Image reconstruction

On slides

1.1.3 Combinatorial auctions

T is an objet. The assumption is that the amount for S and T is greater than S and T separately. Decision variable $x_S = \begin{cases} 1 & \text{if the highest bid on } S \text{ is accepted} \\ 0 & \text{otherwise} \end{cases} \qquad \forall S \subseteq M$

So the formulation is maximize $\sum_{S \subseteq M} b(S) x_S$ such that $\sum_{S \subseteq M : i \in S} x_S \leq 1 \qquad \forall i \in M$ $x_S \in \{0, 1\} \forall S \subseteq M$

SLIDEs