The Neutral Decay Modes of the Eta-Meson*

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Abstract

The neutral decay modes of the η meson are reviewed. The most recent results obtained with the Crystal Ball multiphoton detector at BNL are incorporated. This includes a new, precise result for the slope parameter α of the Dalitz plot in $\eta \to 3\pi^0$ decay and a new, lower branching ratio for $\eta \to \pi^0 \gamma \gamma$ which is consistent with chiral perturbation theory. Recently-obtained limits are given for novel tests of CP and C invariance based on several rare η decays.

1 Introduction

The η meson has the interesting feature that all its possible strong decays are forbidden in lowest order: $\eta \not\to 2\pi$ and $\eta \not\to 4\pi^0$ by P and CP invariance, $\eta \not\to 3\pi$ because of G-parity conservation as well as isospin and charge symmetry invariance. First order electromagnetic η decays are forbidden as well: $\eta \to \pi^0 \gamma$ by conservation of angular momentum, $\eta \to 2\pi^0 \gamma$ and $\eta \to 3\pi^0 \gamma$ by C invariance. Only $\eta \to \pi^+\pi^-\gamma$ occurs, but at a suppressed rate because it involves the anomaly. The first allowed decay is the second-order electromagnetic transition $\eta \to 2\gamma$.

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The width of the η is about 1.3 keV; this is 5 orders of magnitude smaller than a typical strong decay, such as the ρ meson. This feature makes η decays 10^5 times more sensitive than ρ or ω decays at a comparable branching ratio for testing invariances.

The physical η is a mixture of the pseudoscalar SU(3) octet (η_8) and singlet (η_0) parametrized by the mixing angle θ :

$$|\eta\rangle = \cos\theta |\eta_8\rangle - \sin\theta |\eta_0\rangle$$

$$= \frac{1}{6}\sqrt{6}\cos\theta |\overline{u}u + \overline{d}d - 2\overline{s}s\rangle - \frac{1}{3}\sqrt{3}\sin\theta |\overline{u}u + \overline{d}d + \overline{s}s\rangle$$

The value for θ is $(20 \pm 2)^{\circ}$. If we choose $\theta = 19.5^{\circ}$, we have the following remarkable makeup of the physical η :

$$|\eta\rangle = \frac{1}{3}\sqrt{3}|\overline{u}u + \overline{d}d - \overline{s}s\rangle,$$

which means that the η is an eigenstate of the I, U, and V operators of SU(3). We also have $|\eta'\rangle = \sin\theta |\eta_8\rangle + \cos\theta |\eta_0\rangle = \frac{1}{6}\sqrt{6}|\overline{u}u + \overline{d}d + 2\overline{s}s\rangle$. In the limit $u = d \equiv q$, we have $\eta' = \frac{1}{3}\sqrt{6}|\overline{q}q + \overline{s}s\rangle$. The η' is an eigenstate of the operator that interchanges the s and the q quarks.

There are more than 14 different neutral η decays (see Table 6 at the end). They make up $(71.6 \pm 0.4)\%$ of all η decays [1].

$$2 \quad \eta \to 2\gamma$$

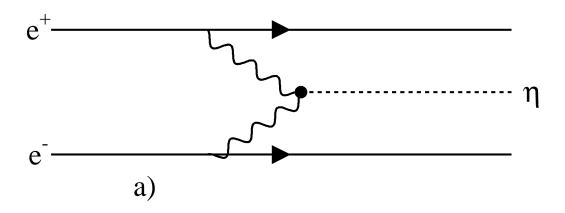
The largest branching ratio of the η is into two photons:

$$BR(\eta \to 2\gamma) = (39.5 \pm 0.2 \pm 0.3)\%.$$
 (1)

It was measured in a dedicated experiment [2] carried out at SACLAY with the SPES II η "factory" using the $pd \rightarrow ^3 \text{He} \eta$ production reaction.

The decay rate $\Gamma(\eta \to 2\gamma)$ has been determined by two means. The first one is the QED process $e^+e^- \to e^+e^-\gamma^*\gamma^* \to e^+e^-\eta \to e^+e^-2\gamma$ (see Fig. 1a). The calculation of the rate is believed to be well-understood and the uncertainty due to the virtual photon form factor is small [3]. The results of four experiments [4, 5, 6, 7] are given in Table 1. They are mutually consistent; the average value is

$$\Gamma(\eta \to 2\gamma) = 0.510 \pm 0.026 \text{ keV}.$$
 (2)



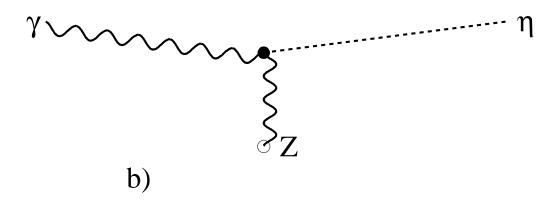


Figure 1: Feynman diagrams of η production for the determination of $\Gamma(\eta \to 2\gamma)$. a) η production in QED. b) η production by the Primakoff effect.

Table 1: Experimental results for $\Gamma(\eta \to 2\gamma)$ measured in $e^+e^- \to e^+e^-\eta$.

Experiment	Reference	$\Gamma(\eta \to 2\gamma)$
Bartel et al.	[4]	$0.53 \pm 0.04 \pm 0.04$
Williams et al.	[5]	$0.514 \pm 0.017 \pm 0.035$
Roe et al.	[6]	$0.490 \pm 0.010 \pm 0.048$
Baru et al.	[7]	$0.51 \pm 0.12 \pm 0.05$

There are 2 older measurements of very limited statistics [8, 9] which we are not using. Finally, there is a different type of measurement based on the Primakoff effect [10], $\gamma A \rightarrow \gamma \gamma A$ (see Fig. 1b), which gives

$$\Gamma(\eta \to 2\gamma) = 0.324 \pm 0.046 \text{ keV}.$$
 (3)

This experiment suffers from uncertainties in the dependence of the Primakoff form factor on the momentum transfer and production angle, and on the systematic error in the phase of the interference term. Because the difference between the Primakoff and the QED process is 4σ and the four QED measurements are consistent with one another and have fewer theoretical uncertainties, we recommend using the value of Eq. 2 rather than the average of both methods as done by the Particle Data Group [1]. Combining Eqs. 1 and 2, we obtain for the total η decay rate

$$\Gamma(\eta \to all) = 1.29 \pm 0.07 \text{ keV}, \tag{4}$$

which is 11% larger than the PDG value.

 $\Gamma(\eta \to 2\gamma)$ can be calculated in different ways. The order of magnitude is readily obtained by scaling the positronium decay rate treating $\eta \to 2\gamma$ as the electromagnetic annihilation of a constituent quark and its antiquark (see Fig. 2a), thus

$$\Gamma(\eta \to 2\gamma) = \left(\frac{m(\eta)}{m(e^+e^-)}\right)^3 \Gamma(positr. \to 2\gamma) = 0.81 \text{ keV},$$
 (5)

which agrees to a factor of 1.6 with the measured value. For comparison, note that this scaling law gives $\Gamma(\pi^0 \to 2\gamma) = 12$ eV, which is 1.5 times the measured result. This calculation gives $\Gamma(\eta' \to 2\gamma) = 4.3$ keV, also in good agreement with the measurement.

It was a stunning surprise when Sutherland [11] and Veltman [12] showed that in the limit of massless quarks, the 2γ decay of π^0 and η is forbidden. This embarrassing situation — after all, the dominant decay of both the π^0 and the η is into 2 photons — turned into a triumph when Adler [13] and Bell and Jackiw [14] figured out how all is saved by the QCD anomaly. The triangle graph (see Fig. 2b) results in the prediction

$$\Gamma(\eta \to 2\gamma) = \Gamma(\pi^0 \to 2\gamma) \left(\frac{m_\eta}{m_\pi}\right)^3 \times \phi,$$
 (6)

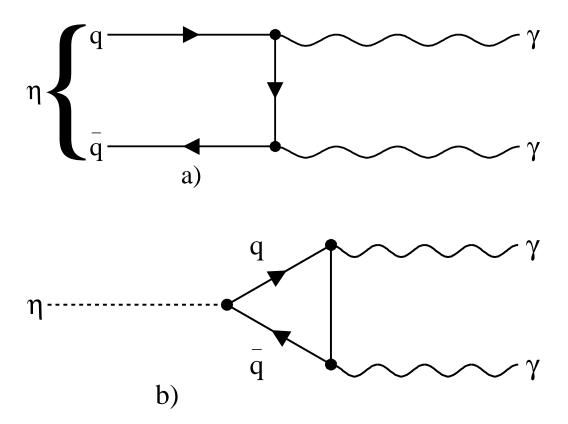


Figure 2: Feynman diagrams for the decay $\eta \to \gamma \gamma$. a) Electromagnetic annihilation of a constituent quark with its antiquark. b) Triangle graph incorporating the QCD anomaly.

where

$$\phi = \sqrt{18} \left[\frac{F_{\pi}}{F_8} \cos \theta (Q_u^2 + Q_d^2 - 2Q_s^2) - \sqrt{2} \frac{F_{\pi}}{F_0} \sin \theta (Q_u^2 + Q_d^2 + Q_s^2) \right]^2,$$

$$\Gamma(\pi^0 \to \gamma \gamma) = \alpha^2 m_{\pi}^3 N_c^2 (Q_u^2 - Q_d^2)^2 / 32 \pi^3 F_{\pi}^2.$$

The last equation gives a value of 7.6 eV for $\Gamma(\pi^0 \to \gamma\gamma)$, in agreement with the measurement of (7.9 ± 0.7) eV. Using $F_8 = 1.3 F_\pi$, $F_0 = (1.05 \pm 0.04) F_\pi$, and $\theta = 19.5^\circ$, we obtain $\phi = 1.3 \pm 0.2$ and $\Gamma(\eta \to 2\gamma) = 0.66$ keV, which is close to the experimental value (Eq. 2). A more sophisticated handling of the octet-singlet mixing that involves two mixing angles is available in the literature [15].

These results have been hailed as experimental verification that the number of colors, N_c , is 3. On detailed inspection this is not quite so as discussed in Ref. [16]. The branching ratio for $\eta \to 2\gamma$ plays a pivotal role in experimental eta physics, as it is used in the determination of the η flux in many experiments. It appears prudent therefore to have a new measurement of $BR(\eta \to 2\gamma)$, preferably with a 4π acceptance photon detector.

$$3 \quad \eta \to 3\pi^0$$

The second largest η decay mode is $\eta \to 3\pi^0$. The PDG recommends the value $BR(\eta \to 3\pi^0) = (32.24 \pm 0.29)\%$. This value was determined principally by GAMS-2000 [17] who reported that $\Gamma(2\gamma)/\Gamma(neutrals) = 0.549 \pm 0.004$ and $\Gamma(3\pi^0)/\Gamma(neutrals) = 0.450 \pm 0.004$, resulting in $\Gamma(3\pi^0)/\Gamma(2\gamma) = 0.820 \pm 0.009$. This agrees with the average of 3 direct measurements; the PDG quotes 0.825 ± 0.011 for the ratio.

The decay mode of the η into 3 pions violates isospin invariance. This η decay occurs because of the u-d quark mass difference, which can be seen as follows. The Lagrangian of QCD, \mathcal{L}_{QCD} , can be divided into two parts:

$$\mathcal{L}_{QCD} = \mathcal{L}_0 + \mathcal{L}_m,$$

where

$$\mathcal{L}_{0} = -\frac{1}{4} F_{\mu\nu}^{(a)} F^{(a)\mu\nu} + i \sum_{q} \overline{\psi}_{q}^{i} \gamma^{\mu} (D_{\mu})_{ij} \psi_{q}^{j},$$
$$F_{\mu\nu}^{(a)} = \partial_{\mu} A_{\nu}^{a} - \partial_{\nu} A_{\mu}^{a} + g_{s} f_{abc} A_{\mu}^{b} A_{\nu}^{c},$$

$$(D_{\mu})_{ij} = \delta_{ij}\partial_{\mu} - ig_s \sum_{a} \frac{\lambda_{i,j}^a}{2} A_{\mu}^a,$$

A is the gluon field, ψ_q is the quark field, g_s is the strong coupling factor, and f_{abc} is the SU(3) structure constant. Thus, \mathcal{L}_0 depends only on the quark and gluon fields and their derivatives, and is the same for all quarks. This is called the flavor symmetry of (massless) QCD, and isospin symmetry is flavor symmetry applied to the u and d quarks. The flavor symmetry is broken by the quark-mass term,

$$\mathcal{L}_m = -\sum_q m_q \overline{\psi}_q^i \psi_{q_i}.$$

Thus we obtain the following expression for π^0 - η mixing:

$$\langle \pi | H_m | \eta \rangle = \left\langle \frac{1}{2} \sqrt{2} (\bar{u}u - \bar{d}d) | \bar{u}m_u u + \bar{d}m_d d | \frac{1}{3} \sqrt{3} (\bar{u}u + \bar{d}d - \bar{s}s) \right\rangle$$
(7)
$$= \frac{1}{6} \sqrt{6} (m_u - m_d),$$

where we have used the value $\theta = -19.5^{\circ}$ for the SU(3) octet-singlet mixing angle. $\eta \to 3\pi$ is not an electromagnetic decay as is sometimes stated in the older literature; it is a limited strong decay, which depends only on \mathcal{L}_m and not on \mathcal{L}_0 . We also have $\Gamma(\eta \to 3\pi) \sim (m_u - m_d)^2$; it is one of the best ways to determine the u-d quark mass difference. A related way is via the ratio $r = \Gamma(\eta \to 3\pi^0)/\Gamma(\eta \to \pi^+\pi^-\pi^0)$ [18].

An interesting parameter to investigate is the slope in the Dalitz plot for $\eta \to 3\pi^0$. In lowest order the Dalitz plot should be uniform because of the identical final state particles. However, the $\pi - \pi$ interaction is strong and strongly energy dependent, which results in a tiny nonuniformity of the Dalitz plot. To incorporate the fact that there are 3 identical π^0 's in the final state, we use a symmetrized Dalitz plot as illustrated in Fig. 3. The deviation of the shape from a circle is the result of the use of relativistic kinematics. The experimental Dalitz plot density should be uniform in concentric circles around the center. A suitable variable to transform from the two-dimensional to a one-dimensional distribution is z:

$$z = 6\sum_{i=1}^{3} (E_i - m_{\eta}/3)^2 / (m_{\eta} - m_{\pi^0})^2 = \rho^2 / \rho_{max}^2,$$

where E_i is the energy of the *i*th pion in the η rest frame and ρ is the distance from the center of the Dalitz plot. The variable z varies from z = 0, when all three π^0 's have the same energy of $m_{\eta}/3$, to z = 1, when one π^0 is at rest.

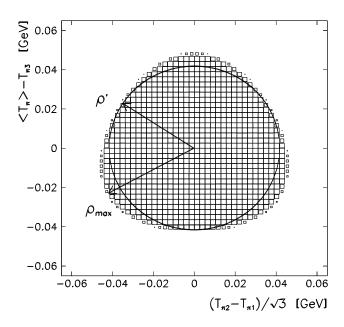


Figure 3: Symmetrized Dalitz plot for the decay $\eta \to 3\pi^0$. The plot is uniform along concentric circles. The deviation from a circular shape comes from using relativistic kinematics.

Table 2: Experimental measurements of the slope parameter α of the decay $\eta \to \pi^0 \pi^0 \pi^0$.

Experiment	Reference	α
Baglin et al.	[23]	-0.32 ± 0.37
GAMS-2000	[17]	-0.022 ± 0.023
Crystal Barrel	[24]	$-0.052 \pm 0.017 \pm 0.010$
SND	[25]	$-0.010 \pm 0.021 \pm 0.010$
Crystal Ball	[19]	-0.031 ± 0.004

The most accurate result comes from the very recent measurement by the Crystal Ball Collaboration [19] yielding $\alpha = -0.031(4)$. The world data is summarized in Table 2. Predictions based on chiral perturbation theory made by Kambor *et al.* [20], who used a dispersion calculation in which rescattering effects are treated to all orders, give values for α in the range -(0.014-0.007). There is agreement with experiment in the sign but not in the magnitude. B. Holstein [21] has suggested the addition of a new dynamical input.

For comparison note that the slope parameter in K_L decay is $2\alpha = (-6.1 \pm 0.9 \pm 0.5) \times 10^{-3}$ [22]; it agrees in sign as expected but it is much smaller, presumably because there is $\frac{1}{3}$ less energy released than in $\eta \to 3\pi^0$ decay. It will be of interest to make a slope measurement of $\eta' \to 3\pi^0$, where the energy release is sufficiently large to see the effects of a major $\pi - \pi$ resonance.

4
$$\eta \to \pi^0 \gamma \gamma$$

The smallest measured branching ratio of the neutral η decays is of the doubly radiative transition $\eta \to \pi^0 \gamma \gamma$. At first glance one might expect the BR to be just slightly smaller than $\eta \to 2\gamma$ because of the reduced phase space. In reality the decay rate is suppressed because of the chiral symmetry of the \mathcal{L}_0 term of \mathcal{L}_{QCD} . In terms of the momentum expansion used in chiral perturbation theory (χ PTh), the leading $O(p^2)$ term is absent for massless quarks; the $O(p^4)$ term is very small because there is no direct photon coupling to the π^0 and η . Thus, the decay $\eta \to \pi^0 \gamma \gamma$ is a (unique) test of the $O(p^6)$ terms of χ PTh. Various theoretical estimates are given in Table 3. The only sufficiently sensitive published result is by GAMS-

Table 3: Theoretical predictions of the decay $\eta \to \pi^0 \gamma \gamma$.

Prediction	Reference	$\Gamma(\eta \to \pi^0 \gamma \gamma) \text{ (eV)}$
$\operatorname{Ko}\left(O(p^4)\right)$	[26]	0.004
Ko	[27]	0.47 ± 0.20
Ametller et al.	[28]	0.42 ± 0.20
Nemoto et al.	[29]	0.92
Bellucci and Bruno	[30]	0.58 ± 0.3
Ng and Peters (VMD)	[31]	$0.30^{+0.16}_{-0.13}$
Ng and Peters (Box)	[32]	0.70

2000 [17]. Their result, $BR(\eta \to \pi^0 \gamma \gamma) = 7.1 \pm 1.7 \times 10^{-4}$, is based on a signal of 38 events. It is larger than every prediction based on χ PTh. The preliminary result of the CB experiment [33, 34] based on 500 events is $BR(\eta \to \pi^0 \gamma \gamma) = (3.2 \pm 0.9) \times 10^{-4}$. Together with Eq. 4, this yields $\Gamma(\eta \to \pi^0 \gamma \gamma) = 0.42 \pm 0.14$ eV, which is well within the range of the theoretical predictions. It will be of considerable interest to measure the Dalitz plot of this decay. Again, a comparison with $K_L \to \pi^0 \gamma \gamma$ is interesting.

5
$$\eta \to \pi^0 \pi^0 \gamma \gamma$$

The two pion, doubly radiative decay, $\eta \to \pi^0 \pi^0 \gamma \gamma$, is driven by the Wess-Zumino anomaly, and therefore is expected [35] to have a very small branching ratio. It provides another, though difficult case for testing the higher order terms in χ PTh. The original interest in $\eta \to \pi^0 \pi^0 \gamma \gamma$ was focused on providing important limits on the coupling of new, relatively light neutral gauge bosons, useful already at the level of $BR \lesssim 10^{-5}$, as discussed e.g. by D. Wyler [36]. The decay amplitude can be divided into two components:

$$A(\eta \to \pi^0 \pi^0 \gamma \gamma) = A_R \pm A_{NR}. \tag{8}$$

The first component, A_R , is of the resonance-type. It is proportional to the on-shell amplitudes for $\eta \to 3\pi^0$ and $\pi^0 \to 2\gamma$. Besides being maximized at the kinematics for $\eta \to 3\pi^0$, it has a limited value over the entire phase space. The non-resonant amplitude, A_{NR} , can be evaluated in χ PTh [37, 38]. For

 $s_{\gamma\gamma} > 0.23 m_{\eta}^2$, where $s_{\gamma\gamma}$ is the square of the $\gamma\gamma$ invariant mass, the non-resonant contribution is larger than the resonance by more than a factor of two [37, 38]. For an energy cut δm of 20 MeV, a tree-level analysis [39] gives

$$BR(\eta \to \pi^0 \pi^0 \gamma \gamma) \simeq 6 \times 10^{-7}.$$
 (9)

Extending the calculation to one loop [37], one obtains

$$BR(\eta \to \pi^0 \pi^0 \gamma \gamma) \simeq 8 \times 10^{-8}.$$
 (10)

Experimentally it is very difficult to measure the decay $\eta \to \pi^0 \pi^0 \gamma \gamma$ because of the background coming from $\eta \to \pi^0 \pi^0 \pi^0$. The first, very modest, upper limit has been obtained very recently by the Crystal Ball Collaboration [40]:

$$BR(\eta \to \pi^0 \pi^0 \gamma \gamma) < 3.1 \times 10^{-3} \tag{11}$$

with a CL of 90%.

6
$$\eta \rightarrow 4\gamma$$

The four photon decay of the neutral pseudoscalar mesons is allowed, and like the two photon decay, is driven by the anomaly described by the Wess-Zumino-Witten term [41]. To this should be added the $O(p^2)$ terms in the non-anomalous sector [42]. Based on the coupling strength alone, one has $\Gamma(\eta \to 4\gamma)/\Gamma(\eta \to 2\gamma) < (\alpha/\pi)^2 \simeq 10^{-5}$. In realistic models the decay $\eta \to 4\gamma$ is going to be much smaller because the lowest orbital angular momentum states are forbidden by the requirements of gauge invariance and PCAC [43]. The centrifugal barriers of the higher states inhibit the $\eta \to 4\gamma$ decay by extra powers of (kR) where k is the photon momentum and R is the interaction radius. Thus far no theoretical estimate or experimental search has been reported.

For the related case of $\pi^0 \to 4\gamma$ there are some estimates. A Vector-Meson-Dominance-with-PCAC calculation [43] gives $BR(\pi^0 \to 4\gamma) \sim 10^{-16}$, while a χ PTh evaluation [42] yields 7.1×10^{-18} . The purely electromagnetic photon-splitting term alone yields $BR(\pi^0 \to 4\gamma) \simeq 3 \times 10^{-11}$ (see Ref. [44]). The experimental upper limit [45] is $BR(\pi^0 \to 4\gamma) < 2 \times 10^{-8}$ at 90% CL.

Table 4: The four classes of C, P, and T violations assuming CPT invariance.

Class	Violated	Valid
1	C, P, CT, PT	T, CP
2	C, P, T, CP, CT, PT	
3	P, T, CP, CT	C, PT
4	C, T, CP, PT	P, CT

7 Search for New Physics

The Standard Model (SM) of the electroweak interactions has been phenomenally successful in giving a quantitative account of the various electroweak interactions. There is no evidence thus far for any failure. Yet the SM is not considered to be a theory. It needs 17 input parameters, not counting the neutrino sector. It does not explain such basic features as the existence of three families of fundamental fermions, the absence of the charge conjugates of the left-handed neutrinos and right-handed antineutrinos, the generation of widely different masses of the charged leptons, the origin of CP violation, etc. It is generally expected that the SM breaks down somewhere. A good place to look for New Physics appears to be the limit of validity of the basic symmetries of charge conjugation (C), parity (P), and time reversal (T), as well as CP and CPT in the different interactions.

If the CPT theorem holds, there are four distinct classes of the violations of C, P, and T (see Table 4). A particularly stringent limit on simultaneous P and T violation, class 3 of table 4, is obtained from the smallness of the limit on the electric dipole moment of the neutron. Yet, all classes, even #3, need better limits, in particular of electrostrong interactions of the quarks. If CPT is not valid there are 7 main classes of C, P, T, and CPT violations (see Table 5).

CPT invariance is a fundamental theorem in quantum field theory, which results from locality and Lorentz invariance. Its validity in the context of quantum gravity is questionable [46]. String theory is intrinsically non-local, which could lead to a violation of CPT. Instantons give rise to charge-non-conserving transitions on the world sheet, and hence to CPT violations. Thus, the extent of the validity of CPT invariance must rest on experimental evidence which is either dynamic, such as in K^0 and η decay, or static, such

Table 5: The seven classes of C, P, and T violations when CPT is not valid.

Class	Violated	Valid
1	C,CP,CT,CPT	P, T, PT
2	P,CP,PT,CPT	C, T, CT
3	T,CT,PT,CPT	C, P, CP
4	C, P, CP, CT, PT, CPT	T
5	C, T, CP, CT, PT, CPT	P
6	P, T, CP, CT, PT, CPT	C
7	C, P, T, CPT	

as the equality of masses, half lives, magnetic moments, etc., of particles and their antiparticles. The most precise static test is $|m_{K^0} - m_{\overline{K^0}}|/m$ (ave.) $< 10^{-18}$ [1]. Testing CPT is extremely difficult in dynamic cases. The decay $\eta \to \pi^0 \mu^+ \mu^-$ provides a special opportunity. The decay spectrum must be even in $\cos \phi$, where ϕ is the angle between the π^0 and the μ^+ in the rest frame of the muon pair [47, 48].

8 CP Violation

CP symmetry means that the interaction of a set of left-handed particles is identical to the interaction of the complimentary set of right-handed antiparticles. The discovery of a 0.2% CP violation in 1964 came as a great surprise. At the time of the discovery there were no theoretical models for CP violations, and the experimental upper limit for $K_L \to 2\pi$ was 0.3%! The origin of CP violation is still a mystery. There is widespread anticipation that detailed studies of CP violation may lead us to "New Physics" that goes beyond the Standard Model (SM). In the context of the SM, CP violation is described by the phase in the Cabibbo-Kobayashi-Maskawa quark-mixing matrix that is related to the existence of six quark flavors grouped into 3 families. CP violation shows up in family-changing interactions, while in family-conserving cases CP violation is unobservably small. The last consequence needs experimental verification which is lacking thus far. Of the 30 tests of CP listed in the Review of Particle Physics [1] only four are in this category: the 2π decays of the η and η' which at the present level are tests

of P as well as CP.

Doable tests of CP invariance are hard to find for lack of particles or states which are eigenstates of the CP operator and have family-conserving interactions. There are several speculative ideas about unconventional CP violation such as spontaneous CP violation in the extended Higgs sector but they are hardly compelling.

9
$$\eta \to \pi\pi$$

The strong decay $\eta \to 2\pi$ is forbidden by CP and P invariance. An η can decay via the weak interaction; at the level of 10^{-7} , parity is not conserved any longer and for $BR \lesssim 10^{-7}$, $\eta \to 2\pi$ becomes a real test of CP invariance. There is considerable evidence that parity is conserved in purely strong and electromagnetic interactions. Strong-electroweak interference can produce a violation of P in carefully chosen processes, usually at a small level which is of order 10^{-6} .

The current experimental limits on the 2π decay of the η are:

$$BR(\eta \to \pi^0 \pi^0) < 4.3 \times 10^{-4} [1].$$
 (12)

$$BR(\eta \to \pi^+ \pi^-) < 3.3 \times 10^{-4} [1].$$
 (13)

Both limits have been obtained at a ϕ factory where η 's are produced in the decay $\phi \to \eta \gamma$ (BR = 1.2%).

There are no η beams, as the η lifetime is too short. The η 's come from baryon decays, in particular from the $N(1535)\frac{1}{2}^-$ and $\Lambda(1670)\frac{1}{2}^-$ resonances, and from meson decays, observed e.g. in ϕ decay or in $\overline{p}p$ annihilation. In all cases there is plenty of 2π production; it is comparable to η production. Thus, it is a real experimental challenge to push the $\eta \to 2\pi$ limit down below the 10^{-7} level.

As indicated earlier, since $\eta \to 2\pi$ is a flavor-conserving interaction the expected BR in the SM is small. A recent calculation yielded $BR(\eta \to 2\pi) < 3 \times 10^{-17}$ [49]. The discovery of a much larger decay rate would be a sign for the existence of a nonconventional CP violating mechanism.

10
$$\eta \to 4\pi^0$$
, a new test of CP

The upper limit for the CP test $\eta \to 2\pi$ is hard to improve appreciably with currently available setups because of the sizeable 2π background in every

 η production reaction. It is thus of interest to find another test. A novel possibility is $\eta \to 4\pi^0$ which is forbidden by CP and P. For η 's produced in the reaction $\pi^- \ p \to \eta n$ near threshold there is no known background to $\eta \to 4\pi^0$. The chief drawback is the smallness of the final state phase space for $\eta \to 4\pi^0$ compared to $\eta \to 2\pi^0$.

The Crystal Ball Collaboration has recently produced the first upper limit [50]:

$$BR(\eta \to 4\pi^0) < 6.9 \times 10^{-7}.$$
 (14)

Together with Eq. 4, this gives $\Gamma(\eta \to 4\pi^0) < 8.9 \times 10^{-4}$ eV. No events were found in a sample of 3×10^7 η decays produced near threshold in $\pi^- p \to \eta n$ close to threshold. To evaluate the sensitivity of this test, note that the η meson is an eigenstate of the CP operator. This allows for a comparison with a related but CP-allowed decay. The decay of a hypothetical η meson, the η_{hyp} , with $J^{PC}=0^{++}$ into $4\pi^0$ is allowed. As η_{hyp} does not exist, we can instead use $f_0(1500) \to 4\pi^0$. The f_0 has the same quantum numbers as the η except positive parity. The experimental value for the partial width is $\Gamma(f_0 \to 4\pi^0) = 33$ MeV. The ratio of the phase space compared to $\eta \to 4\pi^0$ is $[51] \Phi(\eta \to 4\pi^0)/\Phi(f_0 \to 4\pi^0) = 4.7 \times 10^{-8}$, so we might expect $\Gamma(\eta_{hyp} \to 4\pi^0) \simeq 1.6$ eV. Thus, the CP-violating amplitude for $\eta \to 4\pi^0$ compared to a comparable, allowed decay is

$$A_{\overline{cp}}/A_{cp} < \left[\frac{8.9 \times 10^{-4} \,\mathrm{eV}}{1.6 \,\mathrm{eV}}\right]^{\frac{1}{2}} = 2.3 \times 10^{-2}$$
 (15)

at 90% CL.

11 Charge Conjugation

C invariance, or charge conjugation symmetry, is the invariance of a system to the interchange of the colored quarks with their antiquarks of anticolor, the charged leptons with their antileptons, the left(right)-handed neutrinos with the left(right)-handed antineutrinos, and vice versa. According to QED and QCD, C invariance holds for all purely electromagnetic and all strong interactions, but the experimental limits are not impressive. The Review of Particle Physics [1] lists "all weak and electromagnetic decays whose observation would violate conservation laws." Seventeen tests of C invariance are listed: eight involve decays of the η , six of the η' , two of the ω and one of

the π^0 . None has yielded a significant limit thus far [52]. Neither has the Pais test [53] which is the equality of any pair of C-symmetric reactions. Presented in ratio form, there is $R_1 = \sigma(\overline{p}p \to K^+X^-)/\sigma(\overline{p}p \to K^-X^+)$ and $R_2 = \sigma(\overline{p}p \to \pi^+Y^-)/\sigma(\overline{p}p \to \pi^-Y^+)$. The experimental data are $(R_1 - 1) < 2 \times 10^{-2}$ and $(R_2 - 1) < 1 \times 10^{-2}$ [54], which are not sufficiently precise for a useful analysis. The paucity of tests of C-invariance is related to the small number of eigenstates of the C-operator: only flavorless mesons such as η , η' , and self-conjugate systems like $(p\overline{p})$, (e^+e^-) , and $(K^0\overline{K^0})$ qualify.

According to Stuckelberg and Feynman, an antiparticle may be viewed as an ordinary particle that goes backward in time. This shows that the C, P, and T symmetries are interwoven. C reverses the sign of all additive quantum numbers of a particle but leaves its spin unaffected. Thus the C operator turns a left-handed neutrino into a left-handed antineutrino. The neutrinos studied in the lab all turn out to be left-handed and the antineutrinos right-handed, which means that there is full C violation of the weak interactions. The SM does not explain this; it merely is part of the input of the SM, namely it is assumed that all basic fermions come as left-handed doublets and right-handed singlets. This blatant asymmetry provides a strong impetus for making better experimental tests of the validity of C invariance.

Another argument which has kindled the interest in C is the experimental observations of the abundance of matter over antimatter in the universe, as well as photons over baryons: $n_B/n_{\gamma} < 10^{-10}$ [55]. In big-bang models of cosmology one naïvely expects the same abundance of matter and antimatter. The known CP violation is insufficient for explaining the experimental baryon/antibaryon asymmetry.

Finally, recent neutrino experiments have provided tantalizing hints of possible neutrino mixing and of a finite neutrino masses [56, 57].

12 $\eta \to \pi^0 \pi^0 \gamma$, a new Test of C Invariance

The eta meson has the charge conjugation eigenvalue C=+1, and the $\pi^0\pi^0\gamma$ system with $J^P=0^-$ has C=-1. Thus, the decay $\eta\to\pi^0\pi^0\gamma$ is strictly forbidden by C invariance. This decay would be an isoscalar electromagnetic interaction of hadrons. It has been suggested that there may exist an isotensor electromagnetic interaction with a C-violating component [58, 59]. The decay $\eta\to\pi^0\pi^0\gamma$ provides an opportunity to search for such an exotic

interaction; it would be a clear signal for New Physics.

No searches for $\eta \to \pi^0 \pi^0 \gamma$ have been reported in the literature. A preliminary upper limit has been obtained using the Crystal Ball detector [60] from a sample of $1.9 \times 10^7 \ \eta$'s. Candidate events in the signal region are predominantly ($\sim 85\%$) due to $\eta \to 3\pi^0$ decay with overlapping photon showers; the rest are due to $2\pi^0$ production with a split-off photon. The net yield is no events resulting in

$$BR(\eta \to \pi^0 \pi^0 \gamma) < 5 \times 10^{-4} \text{ at the } 90\% \text{ C.L.}$$
 (16)

This corresponds to $\Gamma(\eta \to \pi^0 \pi^0 \gamma) < 0.6 \,\mathrm{eV}$. To evaluate the sensitivity of this new result, we compare the upper limit with that of a C-allowed decay. In the absence of information on $f_0 \to \pi^0 \pi^0 \gamma$ — the f_0 is the preferred comparison state since it has $I^G(J^{PC}) = 0^+(0^{++})$ — we use $\rho \to \pi^+\pi^-\gamma$, with $BR = 1.0 \times 10^{-2}$, $\Gamma_\rho = 151$ MeV. In $\eta \to \pi^0 \pi^0 \gamma$ decay the $2\pi^0$ must be in a relative d-state. This implies that the decay goes by a magnetic quadrupole transition rather than a dipole as $\rho \to \pi^+\pi^-\gamma$ does, and we must include a reduction factor of order $(kR)^4$ (see Section 6); we estimate this factor to be $(\frac{1}{2})^4 = 0.06$ at worst. After small adjustments for the difference in phase space and the Clebsch-Gordan factor, we find that a C-allowed decay has an expected decay width of 2×10^3 eV. This value is in reasonable agreement with two other estimates; the first is based on the allowed decay $\rho \to 2\pi$ and the other on the suppressed decay $\phi \to \pi^0 \pi^0 \gamma$. The sensitivity of the CB upper limit for $\eta \to \pi^0 \pi^0 \gamma$ is

$$A_{\mathcal{C}}/A_{C} < \left[\frac{0.6 \,\text{eV}}{2 \times 10^{3} \,\text{eV}}\right]^{\frac{1}{2}} = 1.7 \times 10^{-2}.$$
 (17)

We are not aware of a more precise test of C invariance of an isoscalar interaction.

13 $\eta \to \pi^0 \pi^0 \pi^0 \gamma$, a test of C Invariance

The radiative decay $\eta \to \pi^0 \pi^0 \pi^0 \gamma$, is strictly forbidden by charge-conjugation invariance. No search for it has been published thus far. There are seven photons in the final state, which explains the need for a 4π acceptance detector. The background is mainly from $\eta \to 3\pi^0$ with either a split-off or an old photon shower from a previous interaction.

Recently a preliminary result, an upper limit, has been obtained using the Crystal Ball detector in an AGS experiment [40]:

$$BR(\eta \to \pi^0 \pi^0 \pi^0 \gamma) < 7 \times 10^{-5},$$
 (18)

which corresponds to

$$\Gamma(\eta \to \pi^0 \pi^0 \pi^0 \gamma) < 9.0 \times 10^{-2} \,\text{eV}, \text{ at the } 90\% \text{ C.L.}$$
 (19)

This is a test of an isovector electromagnetic interaction of hadrons. The sensitivity of this test has been evaluated using a similar approach as was used in the previous section. Starting from the strong decay of the ω -meson, $J^P=1^-$, $\Gamma(\omega\to\pi^+\pi^-\pi^0)=7.5$ MeV. We estimate the unknown radiative decay to be α times the strong decay width. Including an adjustment factor for the difference in phase space and the spin average weight factor [61] we obtain for an allowed $3\pi^0\gamma$ decay rate (if C invariance did not exist) 6.8×10^3 eV. The upper limit for a C-violating amplitude is thus

$$A_{\mathcal{C}}/A_{C} \le \left[\frac{9 \times 10^{-2} \,\text{eV}}{6.8 \times 10^{3} \,\text{eV}}\right]^{\frac{1}{2}} = 3.6 \times 10^{-3}.$$
 (20)

This is the best upper limit for an isovector electromagnetic transition.

14
$$\eta \rightarrow 3\gamma$$

The decay of a neutral, flavorless, C=+1, pseudoscalar meson into three photons is forbidden rigorously by C-invariance. The 3γ decay should be small as it is a third order electromagnetic interaction and $\alpha^3=4\times 10^{-7}$. The rate is further suppressed by substantial factors coming from phase space and angular momentum barrier considerations [62]. The decay $\eta \to 3\gamma$ can be isoscalar or isovector and even the hypothetical isotensor interaction. The Particle Data Group [1] lists the upper limit for the $\eta \to 3\gamma$ branching ratio as 5×10^{-4} .

The Crystal Ball experiment at the AGS has produced a new, still preliminary result which is [40, 63]

$$BR(\eta \to 3\gamma) < 1.8 \times 10^{-5} \tag{21}$$

at the 90% C.L. Using Eq. 4, this corresponds to

$$\Gamma(\eta \to 3\gamma) < 2.3 \times 10^{-2} \,\text{eV}. \tag{22}$$

The largest background in this experiment is from $\eta \to 3\pi^0$ decay when photon showers overlap in the detector. The background from $\eta \to \pi^0 \gamma \gamma$ with two overlapping photon showers is at the percent level, because $BR(\eta \to \pi^0 \gamma \gamma)$ is only 3×10^{-4} (see Section 4). The background from $\eta \to 2\gamma$ with a split-off is totally suppressed in our analysis.

There is no straightforward way to assess the sensitivity of the $\eta \to 3\gamma$ process analogous to the one used in the preceding two sections. The triplet positronium state decays into 3γ but it has $J^{PC}=1^{--}$, which is not quite suitable for the task at hand.

The decay $\eta \to 3\gamma$ can take place by the allowed, C-violating, CP conserving weak interaction of the SM denoted by $BR(\eta \to 3\gamma)_w$. Using a quark-loop model Dicus [64] has obtained

$$BR(\pi^0 \to 3\gamma)_w = (1.2 \times 10^{-5}) \frac{\alpha}{(2\pi)^5} G^2 m_\pi^4 \left(\frac{m_\pi}{m}\right)^8,$$
 (23)

where G is the Fermi constant and m is an effective quark mass. Choosing $m > (1/7)m_N$ this yields

$$BR(\pi^0 \to 3\gamma) < 6 \times 10^{-19}.$$
 (24)

A similar expression for the allowed weak decay of the η yields for $m > (1/5)m_N$ the limit [65]

$$BR(\eta \to 3\gamma)_{\omega} < 3 \times 10^{-12}. \tag{25}$$

 $\eta \to 3\gamma$ decay due to a CP-violating new interaction is not likely. P. Herczeg [66] has shown that in renormalizable gauge models with elementary quarks the flavor conserving nonleptonic interactions of the quarks do not contain in first order a P-conserving CP-violating component. The CP-violating contributions to $BR(\eta \to 3\gamma)$ in such models are therefore negliable relative to $BR(\eta \to 3\gamma)_{\omega}$. P. Herczeg [65] has considered the existence of a flavor conserving C- and CP-violating interaction (\bar{H}) . Using the stringent limits imposed by the upper limit of an electric dipole moment of the neutron, he obtains

$$B(\eta \to 3\gamma)_{\bar{H}} < 10^{-19}.$$
 (26)

It is of interest to compare the relative sensitivity of the decay rates of the three lightest pseudoscalar mesons into 3γ . The simplest effective

Hamiltonian for a 0^- meson decaying into 3γ contains seven derivatives [65], consequently,

$$BR \sim m_{0^-}^{12} \cdot \Gamma(0^- \to all).$$
 (27)

This results in the following sensitivity comparison

$$BR(\eta \to 3\gamma) : BR(\pi^0 \to 3\gamma) : BR(\eta' \to 3\gamma) = 1 : 10^{-5} : 5.$$
 (28)

Thus, even though the experimental limit for $BR(\pi^0 \to 3\gamma) < 3 \times 10^{-8}$ is small, it is 100 times less sensitive than the experimental upper limit for $\eta \to 3\gamma$ given in Eq. 21. On the other hand, $BR(\eta' \to 3\gamma) < 1 \times 10^{-4}$ quoted in Ref. [1] is a comparable test of C.

In recent years there has been a growing interest in quantum field theory over noncommutative spaces in part because of the connection to string theories. Noncommutative CP-violating effects are now being estimated. They may actually dominate over the SM contribution [67].

Grosse and Liao [68] have shown that a generalization of the anomalous $\pi^0 \to 2\gamma$ interaction can induce a *C*-violating $\pi^0 \to 3\gamma$ amplitude in non-commutative quantum electrodynamics. The prediction for $BR(\pi^0 \to 3\gamma) = 6 \times 10^{-21}$ is still far from the experimentally reachable level but it shows that there are new options. It will be interesting to see what noncommutative field theory will predict for $\eta \to 3\gamma$.

15 Weak Neutral η Decays

Most of η physics is focused on the investigations of the strong and electromagnetic interaction of confined $q\bar{q}$'s. It is appropriate to note that weak η decay could occur already at a BR level of 10^{-7} . The opportunity to actually observe a "standard" weak η decay is rather slim because of various suppression factors and the fact that the η is a neutral meson which eliminates charged weak leptonic decays. As a result the permitted weak η decays in the standard model are expected to occur at the level 10^{-13} and below [49]. This situation makes rare η decays a fair hunting ground for searching for new interactions which are mediated by new particles such as leptoquarks which are not subject to the constraints of the ordinary weak interactions.

Recent new experimental data from neutrino detectors, in particular Super-Kamiokande [56] and SNO [57] are being interpreted as evidence for neutrino oscillations. One or more neutrinos are expected to have a mass.

These new developments are enhancing the interest in very rare η decays which involve neutrinos. Weak semileptonic η decays are forbidden by G-parity conservation. They can occur by second class weak currents and are much suppressed [49]. This leaves us with double neutrino decays which experimentally look like $\eta \to nothing$. They can only be investigated indirectly. The present upper limit for $\eta \to nothing$ is a paltry 2.8% at the 90% C.L. [2] The experiment used an enhanced η source based on the Jacobian-peak method and clean tagged η 's as well as the determination of all η decay modes. The limit deserves to be improved using a good 4π -acceptance detector.

There are three categories of η decay into two neutrinos. The first category consists of decays which are allowed by lepton number conservation, $\nu_e \overline{\nu_e}$, $\nu_\mu \overline{\nu_\mu}$ and $\nu_\tau \overline{\nu_\tau}$. They can only occur if neutrino states of both chiralities exist, and imply New Physics which goes beyond the minimal SM. When neutrinos have mass the branching ratio is [65]

$$\Gamma(\eta \to \nu \bar{\nu}) \simeq 4 \times 10^{-9} (m_{\nu}/m_{\eta})^2. \tag{29}$$

Even the τ - neutrino which has the poorest limit on its mass, $m_{\nu_{\tau}} < 18$ MeV, still gives a discouragingly small η decay branching ratio, $< 4 \times 10^{-12}$.

The second category contains the lepton-family violating but lepton-number conserving $\nu \overline{\nu}$ final states such as $\nu_e \overline{\nu_\mu}$, $\overline{\nu_e} \nu_\mu$, $\nu_\mu \overline{\nu_\tau}$, etc. This is of special interest in view of the new data on neutrino oscillations.

In the third category are the decays which violate lepton number by two, such as η decay into $\nu_e\nu_e$, $\nu_\mu\nu_e$, etc. They may be generated by certain classes of leptoquarks. D. Wyler [36] has discussed bounds on the masses of various types of leptoquarks, B- photons, and other exotics, based on the upper limits for different leptonic and semileptonic pseudoscalar meson decays. He finds

$$BR(\eta \to \nu \nu) \simeq 10^{-6} (G_{la}^2 / G_F^2)$$
 (30)

where G_{lq} is the coupling strength of the leptoquark and G_F the weak coupling strength. With $G_F \simeq (300 \, \text{GeV})^{-2}$, we can relate the upper limit of rare eta decays to the mass M_{lq} and relevant coupling constant G_{lq} for instance

$$M_{lq} > G_{lq}(300 \,\text{GeV})/33[BR(\eta \to \nu \bar{\nu})]^{1/4}.$$
 (31)

The helicity suppression which hampers $\eta \to \nu \overline{\nu}$ decays by the factor $(m_{\nu}/m_{\eta})^2$ does not apply to $\eta \to \nu \overline{\nu} \gamma$ because the spin of the photon allows

the $\nu\overline{\nu}$ pair to have angular momentum J=1. The expected BR is still very low. Arnellos *et al.* [69] find using the standard quark model

$$BR(\eta \to \nu \overline{\nu} \gamma) \simeq 2 \times 10^{-15}.$$
 (32)

The weak decay $\eta \to \pi^0 \nu \overline{\nu}$ has been discussed briefly by P. Herczeg [65]. In the SM it is a second class weak interaction with $BR \simeq 10^{-13}$. Beyond the SM $\eta \to \pi^0 \nu \overline{\nu}$ could proceed via an interaction which couples neutrinos to a scalar quark current. The branching ratio is not expected to be larger than 10^{-9} [65].

The largest predicted weak decay of the η' is $\eta' \to K\pi$, which could be in the range $BR \simeq 10^{-9}$ [70].

16 Summary

The different neutral η decay modes are listed in Table 6 together with the experimental branching ratio or upper limit as well as the chief physics interest. The list shows tests of C and CP invariance and chiral perturbation theory. The rare and forbidden η decays are useful for placing limits on several proposed modifications of the Standard Model and on manifestations of New Physics.

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Table 6: The Neutral η Decays.

Decay Mode	Branching Ratio	Physics Highlight
All Neutrals	$(71.6 \pm 0.4)\%$	
2γ	$(39.3 \pm 0.3)\%$	SU(3) octet-singlet mixing
$3\pi^0$	$(32.2 \pm 0.3)\%$	$\chi PTh; m_u - m_d$
$\pi^0\gamma\gamma$	$(3.2 \pm 0.9) \times 10^{-4}$	$\chi PTh, O(p^6)$
$2\pi^0$	$< 4.3 \times 10^{-4}$	P and CP
$4\pi^0$	$< 6.9 \times 10^{-7}$	P and CP
$\pi^0\pi^0\gamma$	$< 5 \times 10^{-4}$	C (isoscalar)
$\pi^0\pi^0\pi^0\gamma$	$< 4.7 \times 10^{-5}$	C (isovector)
3γ	$<4.5\times10^{-5}$	C (isovector, isoscalar)
4γ	< 2.8%	
$\pi^0\pi^0\gamma\gamma$	$< 3.1 \times 10^{-3}$	χPTh , New Physics
$ u_e \overline{ u_e}$	< 2.8%	New Physics, leptoquarks
$ u_e ar{ u_\mu}$	< 2.8%	New Physics, leptoquarks
$ u_e \overline{ u_e}$	< 2.8%	New Physics, leptoquarks
$\gamma u \overline{ u}$	< 2.8%	New Physics, leptoquarks
$\pi^0 \nu \overline{\nu}$	< 2.8%	New Physics, leptoquarks

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