Exercise 9 (online: 26.06.2023. Return by: **Mo 03.07.2023 10:00**) **21P**

1. **NIS & SINIS junctions** 8P

   Consider a NIS junction (N is a normal metal, I is an insulator, S is a superconductor) and a SINIS chain, in which both S are made of the same material, *i.e.*, they are characterized by the same energy gap Δ for quasiparticle excitations.

   (a) Sketch the density of states (DoS) plot (see, *e.g.*, Fig. 4.4 in the lecture notes) and indicate the occupied states in the N and S parts of the NIS junction for the two values of the applied voltage $V = 0$ and $V = \Delta/e$ and $T = 0$. Compare it with that of a SINIS chain for $V = 0$ and $V = 2\Delta/e$. Here e is the single electron charge.(2P)

   (b) From DoS plots at different V, sketch qualitatively the $I(V)$ curve of a NIS junction and a SINIS chain at $T = 0$. Hint: the tunnel current I is proportional to the number of occupied states in the first electrode (N, in case of the NIS junction) times the number of unoccupied states in the second electrode (S, in the case of the NIS junction).(2P)

   (c) Sketch the DoS plot for the NIS junction at $V = \Delta/e$ for three different (electron subsystem) temperatures in the normal metal: $T_{N,1} = 0$, $T_{N,2} > T_{N,1}$ and $T_{N,3} > T_{N,2}$.(1P)

   (d) Assume now that the N-electrode is thermalized at ’high’ temperature $T_{N,3}$. How does (qualitatively) the temperature T_N of the electrons in N change when we voltage-bias the NIS junction to $V_b \lesssim \Delta/e$? Hint: Assume that the S electodes are large enough to be considered as electron-reservoirs (=there is a thermal equilibrium there). How does (qualitatively) T_N change when one uses a SINIS chain?(3P)

2. **Underdamped Josephson junction** 13P

   Within the RCSJ model, the dynamics of the phase difference $\delta(t)$ across a Josephson junction can be described in analogy with the motion of a particle in a tilted washboard potential.

   (a) Start with the equation of motion for δ (see lecture notes, Eq. (4.32)) and derive from this the normalized equation of motion, Eq. (4.40), and hence the definition of the Stewart-McCumber parameter β_C. Hint: Normalize currents to I_0, *i.e.*, introduce normalized currents $i \equiv I/I_0$ and $i_N \equiv I_N/I_0$. Further, normalize time to the inverse characteristic frequency $1/\omega_c$ ($\omega_c = 2\pi I_0 R/\Phi_0$) *i.e.*, introduce normalized time $\tau \equiv t\omega_c$.(2P)

   (b) The obtained normalized equation of motion can be rewritten as

   $$\beta_C \ddot{\delta} \equiv -\frac{\partial u_J}{\partial \delta} - \dot{\delta}, \tag{1}$$

   where $\dot{\delta} \equiv \partial\delta/\partial\tau$ is the derivative relative to the *normalized* time τ, $u_J(\delta)$ is the (normalized) potential. Find the expression for $u_J(\delta)$ by choosing the integration constant so that $u_J(0) = 0$. What is the physical meaning of the choice $i_N = 0$? (1P)

   (!) **For the rest of the exercise assume $i_N \equiv 0$!**

   (c) Eq. (1) is analogous to the equation of motion for a particle with coordinate δ and mass β_C, moving with friction ($\dot{\delta}$) in the potential $u_J(\delta)$. Using this analogy, explain

      i. Why does the Josephson junction, upon increasing the bias current I from 0 upwards, switches to the non-zero voltage state at $I = I_0$?

      ii. What happens if you decrease the current starting from some $I > I_0$ back to zero, for the case $\beta_C < 1$ and for the case $\beta_C > 1$? What is the major difference between the I–V curves in these two cases?(2P)

(d) Sketch the normalized potential $u_J(\delta)$ for $0 \leq \delta/(2\pi) \leq 2$ and for a fixed value of $i = 0.1$. Why is $u_J(\delta)$ called tilted washboard potential?

Assume that the particle is placed at the left (higher) maxiumum (at δ_{m1}) with velocity $\dot{\delta} = 0$ and that it then rolls down to the right. What will be the velocity of the particle when it reaches the next maximum at δ_{m2} in the case $i = i_r$ (i_r is the normalized retrapping current for an overdamped Josephson junction)? Explain your answer!

What is the relation between the potential energy difference $\Delta u_J = u_J(\delta_{m1}) - u_J(\delta_{m2})$ of the particle at the two maxima and the dissipated energy ΔE_{diss} during the travel of the particle from one maximum to the next one, for the case $i = i_r$.(3P)

(e) Calculate Δu_J for the case $i = i_r$.(1P)

(f) Calculate

$$\Delta E_{\text{diss}} \equiv \int_{\delta_{m1}}^{\delta_{m2}} \dot{\delta}\, d\delta, \tag{2}$$

by considering $i = i_r$ and the damping as a small perturbation, *i.e.*, assume in the 0-th approximation that the particle moves from δ_{m1} to δ_{m2} without friction and in an untilted potential. Hint: in this limit the system is conservative and $E_{tot} = E_{kin} + E_{pot}$ during the motion. Looking at the Eq. (1), what are the expressions for E_{kin} and E_{pot}? Choose a suitable value of E_{tot} and arrive to the expression for

$$\dot{\delta} = \sqrt{\frac{2}{\beta_C}} \cdot \sqrt{1 + \cos\delta}, , \tag{3}$$

and use it in Eq. (2) to find ΔE_{diss}.(3P)

(g) Finally, use the relation between Δu_J and ΔE_{diss} from your answer to the question (d) to calculate $i_r(\beta_C)$, using the explicit results for Δu_J and E_{diss} obtained above.(1P)