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Exercise sheet 9
WS 2022/23
January 10, 2023

Quantum Field Theory

Exercise 34:

We start from the S-matrix expansion

$$S = \sum_{n=0}^{\infty} S^{(n)} = \mathcal{T} exp\left(-i \int d^4x \mathcal{H}_{int}(x)\right)$$

involving the QED interaction Hamiltonian density

$$\mathcal{H}_{int}(x) = -e : \bar{\psi}(x)\gamma^{\mu} A_{\mu}(x)\psi(x) : \ .$$

In the following we consider the process (Bhabha scattering)

$$e^+(\vec{p}_1) + e^-(\vec{p}_2) \to e^+(\vec{p}_1{}') + e^-(\vec{p}_2{}') \ ,$$

where spin indices are suppressed:

(a) Derive the lowest-order non-vanishing S-matrix operator $S^{(2)}(e^+e^- \to e^+e^-) = S_a + S_b$ with (4P)

$$S_a = -e^2 \int d^4x_1 d^4x_2 : \left[(\bar{\psi}^- \gamma^{\alpha}\psi^+)_{x_1}(\bar{\psi}^+\gamma^{\beta}\psi^-)_{x_2}\right] : iD_{F\alpha\beta}(x_1 - x_2)$$

and

$$S_b = -e^2 \int d^4x_1 d^4x_2 : \left[(\bar{\psi}^- \gamma^{\alpha}\psi^-)_{x_1}(\bar{\psi}^+\gamma^{\beta}\psi^+)_{x_2}\right] : iD_{F\alpha\beta}(x_1 - x_2) \ .$$

(b) Further show that the corresponding Feynman-amplitude $\mathcal{M} = \mathcal{M}_a + \mathcal{M}_b$ is given by (4P)

$$\mathcal{M}_a = -ie^2 \left[\bar{u}(\vec{p}_2{}')\gamma_{\alpha} u(\vec{p}_2)\right] \frac{1}{(p_1 - p_1{}')^2} \left[\bar{v}(\vec{p}_1)\gamma^{\alpha} v(\vec{p}_1{}')\right]$$

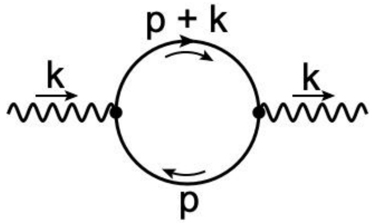
and

$$\mathcal{M}_b = ie^2 \left[\bar{u}(\vec{p}_2{}')\gamma_{\alpha} u(\vec{p}_1{}')\right] \frac{1}{(p_1 + p_2)^2} \left[\bar{v}(\vec{p}_1)\gamma^{\alpha} u(\vec{p}_2)\right] \ .$$

(c) Draw and label the respective Feynman-diagrams for both amplitudes. (2P)

Exercise 35:

Starting from the S-matrix expansion of Exercise 34 show that the Feynman-amplitude for the photon self-energy with the diagram



is given by

$$\mathcal{M} = \frac{-e^2}{(2\pi)^4} \int d^4p \; Tr \left[\not{\epsilon}_r(\vec{k}) S_F(p+k) \; \not{\epsilon}_r(\vec{k}) S_F(p) \right]$$

where $\vec{k}$ und $\epsilon_r(\vec{k})$ are the momentum and polarization vectors of the photon. Tr refers to the trace of a matrix. (4P)

Exercise 36:

The Lagrangian density for pseudo-scalar mesons (ϕ) of mass m, for fermions (ψ) of mass M and their Yukawa-interaction is defined by

$$\mathcal{L} \; = \; \frac{1}{2}\partial_\mu \phi \partial^\mu \phi \; - \; \frac{1}{2} m^2 \phi^2 + \bar{\psi}(i\gamma^\mu \partial_\mu - M)\psi - ig\bar{\psi}\gamma_5 \psi \phi \; .$$

The interaction part of the Lagrangian density is similar to that of QED, except that $e\gamma^\alpha$ is replaced by $(-ig\gamma_5)$, and the photon field by the meson field ϕ.

(a) Exploit this similarity to write down the Feynman rules for pseudo-scalar meson theory. (2P)

(b) Use these Feynman rules to determine the transition amplitude to order g^2 for the scattering processes $\psi\psi \to \psi\psi$ and $\psi\bar{\psi} \to \psi\bar{\psi}$. (4P)

Worked-out solutions to the homework problems should be handed in latest at the beginning of the lecture of January 17, 2023.