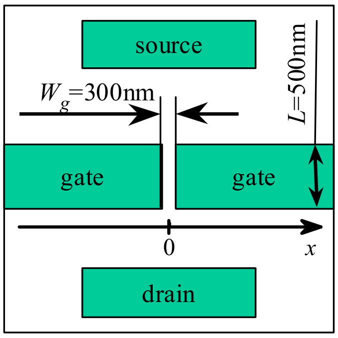
Exercise 4 (online: 15.05.2023. Return by: **Mo 22.05.2023 10:00**) 8P

1. **Quantum Point-Contacts as Quantum Well** 8P



So-called “split-gates” with a channel length $L = 500\,\mathrm{nm}$ and a width $W_g = 300\,\mathrm{nm}$ (the **g**ap between the two gate electrodes) can be fabricated on the surface of a heterostructure with 2DEG using e-beam lithography. Low temperature Hall measurements are then used to determine the electronic density of the two-dimensional system: $N_s = 3.1 \cdot 10^{15}\,\mathrm{m}^{-2}$. At a gate voltage $V_g = -0.5\,\mathrm{V}$ (applied to both gates) the quantum point-contact (QPC) is defined. At a gate voltage of $V_g = -3.5\,\mathrm{V}$ the QPC is driven into “Pinch-Off” ($G = 0$). In total $N = 13$ quantised steps in the conductance with $G = (2e^2/h) \cdot n$ and $n = N \ldots 1$ are observed between $V_g = -0.5 \ldots -3.5\,\mathrm{V}$. An idealised 1D model can be used to simulate such a QPC: a quantum well with infinitely high side-walls. For such a well the confining potential is given by:

$$V(x) = \begin{cases} 0, & \text{for } |x| \leq W/2 \\ \infty, & \text{otherwise,} \end{cases} \tag{1}$$

where a gate-voltage dependent *electrostatic* well width W (different from the *geometrical* width W_g) is assumed to change as

$$W(V_g) = W_o - \beta(-0.5\,\mathrm{V} - V_g). \tag{2}$$

(a) Sketch the gate characteristics (conductance) $G(V_g)$ of the QPC for V_g between $-3.5\,\mathrm{V}$ and $0\,\mathrm{V}$.(2P)

(b) Calculate the Fermi energy E_F (relative to the lowest energy state) and the Fermi wavelength λ_F of the 2D system. For GaAs the effective mass is $m = 0.067\,m_e$, where $m_e \approx 9.1 \cdot 10^{-31}\,\mathrm{kg}$ is the free electron mass. Estimate the number of conducting channels/modes from the relation $N \simeq W_g/(\lambda_F/2)$ and compare the result with the observed 13 conductance steps.(2P)

(c) Write down the (well-known expression for the) energy spectrum ϵ_n of an infinite quantum well (1) with the *electrostatic* width W. Using the fact that $\epsilon_n = \epsilon_F$ at step n, obtain $W(n)$ – the *electrostatic* width at step n (use the known explicit expression for ϵ_F in 2D system). Further, use the energy ϵ_N of the highest occupied state at the QPC definition voltage to calculate W_o.(2P)

(d) Calculate the value of β from the gate voltage at “Pinch-Off”.(1P)

(e) Hence calculate the linear trend in the gate-voltage dependent conductance for the onsets of the steps (the gate voltage from $V_g = -0.5\,\mathrm{V}$ downwards at which the new step starts) for this model. Note: Experimentally a roughly linear $G(V_g)$ dependence of the number of occupied 1D modes is observed. (1P)