

Contents

<i>Preface and Acknowledgments</i>	xv
I PROBABILITY	1
1 Probability	3
1.1 Trials, Sample Spaces, and Events	3
1.2 Probability Axioms and Probability Space	9
1.3 Conditional Probability	12
1.4 Independent Events	15
1.5 Law of Total Probability	18
1.6 Bayes' Rule	20
1.7 Exercises	21
2 Combinatorics—The Art of Counting	25
2.1 Permutations	25
2.2 Permutations with Replacements	26
2.3 Permutations without Replacement	27
2.4 Combinations without Replacement	29
2.5 Combinations with Replacements	31
2.6 Bernoulli (Independent) Trials	33
2.7 Exercises	36
3 Random Variables and Distribution Functions	40
3.1 Discrete and Continuous Random Variables	40
3.2 The Probability Mass Function for a Discrete Random Variable	43
3.3 The Cumulative Distribution Function	46
3.4 The Probability Density Function for a Continuous Random Variable	51
3.5 Functions of a Random Variable	53
3.6 Conditioned Random Variables	58
3.7 Exercises	60
4 Joint and Conditional Distributions	64
4.1 Joint Distributions	64
4.2 Joint Cumulative Distribution Functions	64
4.3 Joint Probability Mass Functions	68
4.4 Joint Probability Density Functions	71
4.5 Conditional Distributions	77
4.6 Convolutions and the Sum of Two Random Variables	80
4.7 Exercises	82
5 Expectations and More	87
5.1 Definitions	87
5.2 Expectation of Functions and Joint Random Variables	92
5.3 Probability Generating Functions for Discrete Random Variables	100

5.4	Moment Generating Functions	103
5.5	Maxima and Minima of Independent Random Variables	108
5.6	Exercises	110
6	Discrete Distribution Functions	115
6.1	The Discrete Uniform Distribution	115
6.2	The Bernoulli Distribution	116
6.3	The Binomial Distribution	117
6.4	Geometric and Negative Binomial Distributions	120
6.5	The Poisson Distribution	124
6.6	The Hypergeometric Distribution	127
6.7	The Multinomial Distribution	128
6.8	Exercises	130
7	Continuous Distribution Functions	134
7.1	The Uniform Distribution	134
7.2	The Exponential Distribution	136
7.3	The Normal or Gaussian Distribution	141
7.4	The Gamma Distribution	145
7.5	Reliability Modeling and the Weibull Distribution	149
7.6	Phase-Type Distributions	155
7.6.1	The Erlang-2 Distribution	155
7.6.2	The Erlang- r Distribution	158
7.6.3	The Hypoexponential Distribution	162
7.6.4	The Hyperexponential Distribution	164
7.6.5	The Coxian Distribution	166
7.6.6	General Phase-Type Distributions	168
7.6.7	Fitting Phase-Type Distributions to Means and Variances	171
7.7	Exercises	176
8	Bounds and Limit Theorems	180
8.1	The Markov Inequality	180
8.2	The Chebychev Inequality	181
8.3	The Chernoff Bound	182
8.4	The Laws of Large Numbers	182
8.5	The Central Limit Theorem	184
8.6	Exercises	187
II	MARKOV CHAINS	191
9	Discrete- and Continuous-Time Markov Chains	193
9.1	Stochastic Processes and Markov Chains	193
9.2	Discrete-Time Markov Chains: Definitions	195
9.3	The Chapman-Kolmogorov Equations	202
9.4	Classification of States	206
9.5	Irreducibility	214
9.6	The Potential, Fundamental, and Reachability Matrices	218
9.6.1	Potential and Fundamental Matrices and Mean Time to Absorption	219
9.6.2	The Reachability Matrix and Absorption Probabilities	223

9.7	Random Walk Problems	228
9.8	Probability Distributions	235
9.9	Reversibility	248
9.10	Continuous-Time Markov Chains	253
9.10.1	Transition Probabilities and Transition Rates	254
9.10.2	The Chapman-Kolmogorov Equations	257
9.10.3	The Embedded Markov Chain and State Properties	259
9.10.4	Probability Distributions	262
9.10.5	Reversibility	265
9.11	Semi-Markov Processes	265
9.12	Renewal Processes	267
9.13	Exercises	275
10	Numerical Solution of Markov Chains	285
10.1	Introduction	285
10.1.1	Setting the Stage	285
10.1.2	Stochastic Matrices	287
10.1.3	The Effect of Discretization	289
10.2	Direct Methods for Stationary Distributions	290
10.2.1	Iterative versus Direct Solution Methods	290
10.2.2	Gaussian Elimination and <i>LU</i> Factorizations	291
10.3	Basic Iterative Methods for Stationary Distributions	301
10.3.1	The Power Method	301
10.3.2	The Iterative Methods of Jacobi and Gauss–Seidel	305
10.3.3	The Method of Successive Overrelaxation	311
10.3.4	Data Structures for Large Sparse Matrices	313
10.3.5	Initial Approximations, Normalization, and Convergence	316
10.4	Block Iterative Methods	319
10.5	Decomposition and Aggregation Methods	324
10.6	The Matrix Geometric/Analytic Methods for Structured Markov Chains	332
10.6.1	The Quasi-Birth-Death Case	333
10.6.2	Block Lower Hessenberg Markov Chains	340
10.6.3	Block Upper Hessenberg Markov Chains	345
10.7	Transient Distributions	354
10.7.1	Matrix Scaling and Powering Methods for Small State Spaces	357
10.7.2	The Uniformization Method for Large State Spaces	361
10.7.3	Ordinary Differential Equation Solvers	365
10.8	Exercises	375
III	QUEUEING MODELS	383
11	Elementary Queueing Theory	385
11.1	Introduction and Basic Definitions	385
11.1.1	Arrivals and Service	386
11.1.2	Scheduling Disciplines	395
11.1.3	Kendall’s Notation	396
11.1.4	Graphical Representations of Queues	397
11.1.5	Performance Measures—Measures of Effectiveness	398
11.1.6	Little’s Law	400

11.2	Birth-Death Processes: The $M/M/1$ Queue	402
11.2.1	Description and Steady-State Solution	402
11.2.2	Performance Measures	406
11.2.3	Transient Behavior	412
11.3	General Birth-Death Processes	413
11.3.1	Derivation of the State Equations	413
11.3.2	Steady-State Solution	415
11.4	Multiserver Systems	419
11.4.1	The $M/M/c$ Queue	419
11.4.2	The $M/M/\infty$ Queue	425
11.5	Finite-Capacity Systems—The $M/M/1/K$ Queue	425
11.6	Multiserver, Finite-Capacity Systems—The $M/M/c/K$ Queue	432
11.7	Finite-Source Systems—The $M/M/c//M$ Queue	434
11.8	State-Dependent Service	437
11.9	Exercises	438
12	Queues with Phase-Type Laws: Neuts' Matrix-Geometric Method	444
12.1	The Erlang- r Service Model—The $M/E_r/1$ Queue	444
12.2	The Erlang- r Arrival Model—The $E_r/M/1$ Queue	450
12.3	The $M/H_2/1$ and $H_2/M/1$ Queues	454
12.4	Automating the Analysis of Single-Server Phase-Type Queues	458
12.5	The $H_2/E_3/1$ Queue and General $Ph/Ph/1$ Queues	460
12.6	Stability Results for $Ph/Ph/1$ Queues	466
12.7	Performance Measures for $Ph/Ph/1$ Queues	468
12.8	Matlab code for $Ph/Ph/1$ Queues	469
12.9	Exercises	471
13	The z-Transform Approach to Solving Markovian Queues	475
13.1	The z -Transform	475
13.2	The Inversion Process	478
13.3	Solving Markovian Queues using z -Transforms	484
13.3.1	The z -Transform Procedure	484
13.3.2	The $M/M/1$ Queue Solved using z -Transforms	484
13.3.3	The $M/M/1$ Queue with Arrivals in Pairs	486
13.3.4	The $M/E_r/1$ Queue Solved using z -Transforms	488
13.3.5	The $E_r/M/1$ Queue Solved using z -Transforms	496
13.3.6	Bulk Queueing Systems	503
13.4	Exercises	506
14	The $M/G/1$ and $G/M/1$ Queues	509
14.1	Introduction to the $M/G/1$ Queue	509
14.2	Solution via an Embedded Markov Chain	510
14.3	Performance Measures for the $M/G/1$ Queue	515
14.3.1	The Pollaczek-Khintchine Mean Value Formula	515
14.3.2	The Pollaczek-Khintchine Transform Equations	518
14.4	The $M/G/1$ Residual Time: Remaining Service Time	523
14.5	The $M/G/1$ Busy Period	526
14.6	Priority Scheduling	531
14.6.1	$M/M/1$: Priority Queue with Two Customer Classes	531
14.6.2	$M/G/1$: Nonpreemptive Priority Scheduling	533

14.6.3	$M/G/1$: Preempt-Resume Priority Scheduling	536
14.6.4	A Conservation Law and SPTF Scheduling	538
14.7	The $M/G/1/K$ Queue	542
14.8	The $G/M/1$ Queue	546
14.9	The $G/M/1/K$ Queue	551
14.10	Exercises	553
15	Queueing Networks	559
15.1	Introduction	559
15.1.1	Basic Definitions	559
15.1.2	The Departure Process—Burke’s Theorem	560
15.1.3	Two $M/M/1$ Queues in Tandem	562
15.2	Open Queueing Networks	563
15.2.1	Feedforward Networks	563
15.2.2	Jackson Networks	563
15.2.3	Performance Measures for Jackson Networks	567
15.3	Closed Queueing Networks	568
15.3.1	Definitions	568
15.3.2	Computation of the Normalization Constant: Buzen’s Algorithm	570
15.3.3	Performance Measures	577
15.4	Mean Value Analysis for Closed Queueing Networks	582
15.5	The Flow-Equivalent Server Method	591
15.6	Multiclass Queueing Networks and the BCMP Theorem	594
15.6.1	Product-Form Queueing Networks	595
15.6.2	The BCMP Theorem for Open, Closed, and Mixed Queueing Networks	598
15.7	Java Code	602
15.8	Exercises	607
IV	SIMULATION	611
16	Some Probabilistic and Deterministic Applications of Random Numbers	613
16.1	Simulating Basic Probability Scenarios	613
16.2	Simulating Conditional Probabilities, Means, and Variances	618
16.3	The Computation of Definite Integrals	620
16.4	Exercises	623
17	Uniformly Distributed “Random” Numbers	625
17.1	Linear Recurrence Methods	626
17.2	Validating Sequences of Random Numbers	630
17.2.1	The Chi-Square “Goodness-of-Fit” Test	630
17.2.2	The Kolmogorov-Smirnov Test	633
17.2.3	“Run” Tests	634
17.2.4	The “Gap” Test	640
17.2.5	The “Poker” Test	641
17.2.6	Statistical Test Suites	644
17.3	Exercises	644

18 Nonuniformly Distributed “Random” Numbers	647
18.1 The Inverse Transformation Method	647
18.1.1 The Continuous Uniform Distribution	649
18.1.2 “Wedge-Shaped” Density Functions	649
18.1.3 “Triangular” Density Functions	650
18.1.4 The Exponential Distribution	652
18.1.5 The Bernoulli Distribution	653
18.1.6 An Arbitrary Discrete Distribution	653
18.2 Discrete Random Variates by Mimicry	654
18.2.1 The Binomial Distribution	654
18.2.2 The Geometric Distribution	655
18.2.3 The Poisson Distribution	656
18.3 The Accept-Reject Method	657
18.3.1 The Lognormal Distribution	660
18.4 The Composition Method	662
18.4.1 The Erlang- r Distribution	662
18.4.2 The Hyperexponential Distribution	663
18.4.3 Partitioning of the Density Function	664
18.5 Normally Distributed Random Numbers	670
18.5.1 Normal Variates via the Central Limit Theorem	670
18.5.2 Normal Variates via Accept-Reject and Exponential Bounding Function	670
18.5.3 Normal Variates via Polar Coordinates	672
18.5.4 Normal Variates via Partitioning of the Density Function	673
18.6 The Ziggurat Method	673
18.7 Exercises	676
19 Implementing Discrete-Event Simulations	680
19.1 The Structure of a Simulation Model	680
19.2 Some Common Simulation Examples	682
19.2.1 Simulating the $M/M/1$ Queue and Some Extensions	682
19.2.2 Simulating Closed Networks of Queues	686
19.2.3 The Machine Repairman Problem	689
19.2.4 Simulating an Inventory Problem	692
19.3 Programming Projects	695
20 Simulation Measurements and Accuracy	697
20.1 Sampling	697
20.1.1 Point Estimators	698
20.1.2 Interval Estimators/Confidence Intervals	704
20.2 Simulation and the Independence Criteria	707
20.3 Variance Reduction Methods	711
20.3.1 Antithetic Variables	711
20.3.2 Control Variables	713
20.4 Exercises	716
Appendix A: The Greek Alphabet	719
Appendix B: Elements of Linear Algebra	721
B.1 Vectors and Matrices	721
B.2 Arithmetic on Matrices	721

B.3	Vector and Matrix Norms	723
B.4	Vector Spaces	724
B.5	Determinants	726
B.6	Systems of Linear Equations	728
	B.6.1 Gaussian Elimination and LU Decompositions	730
B.7	Eigenvalues and Eigenvectors	734
B.8	Eigenproperties of Decomposable, Nearly Decomposable, and Cyclic Stochastic Matrices	738
	B.8.1 Normal Form	738
	B.8.2 Eigenvalues of Decomposable Stochastic Matrices	739
	B.8.3 Eigenvectors of Decomposable Stochastic Matrices	741
	B.8.4 Nearly Decomposable Stochastic Matrices	743
	B.8.5 Cyclic Stochastic Matrices	744
Bibliography		745
Index		749

This page intentionally left blank