1 Preliminaries

- Abstract from labor income (in the end, everything conditional on y)
- We fix a grid $\mathcal{G} = \{\mathcal{B}_i, \mathcal{K}_j\}_{i,j}$ for liquid and illiquid assets
- We have policy functions $b_a^*(b,k), k_a^*(b,k), b_n^*(b,k)$ known on a grid for $\mathcal{G}$
- In the following, we describe the baseline algorithm and **Alternative B**, where the distribution over the wealth grid is *additional* part of the iteration

2 First steps

- Calculate total wealth for each gridpoint combination b, k $w = b + qk$
- Use the policy functions $b_a^*(b,k), k_a^*(b,k), b_n^*(b,k)$ to construct for each tupel in $\mathcal{G}: w_a^* = b_a^*(\mathcal{B}_i, \mathcal{K}_j) + q' k_a^*(\mathcal{B}_i, \mathcal{K}_j), w_n^* = b_n(\mathcal{B}_i, \mathcal{K}_j) + \mathcal{K}_j$.

3 Iteration

Given are: F_k, marginal cumulative distribution function of illiquid asset k, and $F_{b|k}$, marginal cumulative distribution function of liquid asset b *conditional* on $k \in \mathcal{K}$.

Alternative B: Additionally have marginal cumulative distribution function of available resources / wealth, F_w, as object that is iterated over.

We are looking for same objects at end of period, denoted by $\tilde{F}$.

3.1 Adjustment and non-adjustment: Plan

There is exogenous, time-constant share of adjusters λ, determined at beginning of period. Hence, distribution $\tilde{F}$ is a convex combination of distributions *conditional* on (non-)adjustment. This holds for some of the objects we are looking for: $\tilde{F}_k(k') = \lambda \tilde{F}_k(k' \mid adj) + (1-\lambda) F_k(k')$, where we use that $\tilde{F}_k(k' \mid nadj) = F_k(k')$, and (only **Alternative B**) $\tilde{F}_w(w') = \lambda \tilde{F}_w(w' \mid adj) + (1-\lambda)\tilde{F}_w(w' \mid nadj)$.

We cannot directly build end-of-period *conditional* distributions, since for the integration over adjustment and non-adjustment, one has to account for the fact that the marginal distribution $\tilde{F}_k$ also depends on adjustment vs non-adjustment. Therefore, we first build the *joint distribution* end-of-period:

$$\tilde{F}_{b,k}(b',k') = \lambda\left(\tilde{F}_{b|k}(b' \mid 0, adj)\tilde{F}_k(0 \mid adj) + \underbrace{\int_0^{k'} \tilde{F}_{b|k}(b' \mid k, adj)\tilde{f}_k(k \mid adj)dk}_{(I)}\right)$$

$$+(1-\lambda)\left(\tilde{F}_{b|k}(b' \mid 0, nadj)\tilde{F}_k(0 \mid nadj) + \underbrace{\int_0^{k'} \tilde{F}_{b|k}(b' \mid k, nadj)\tilde{f}_k(k \mid nadj)dk}_{(II)}\right)$$

We discuss below how to compute the two integrals (I) and (II).

Next, we approximate the end-of-period distributions of b conditional on $k \in \mathcal{K}$ (on-grid) with differential coefficients:

$$\tilde{F}_{b|k}(b' \mid 0) = \frac{\tilde{F}_{b,k}(b',0)}{\tilde{F}_k(0)}, \tag{1}$$

$$\tilde{F}_{b|k}(b' \mid \mathcal{K}_i) = \frac{\tilde{F}_{b,k}(b',\mathcal{K}_i) - \tilde{F}_{b,k}(b',\mathcal{K}_{i-1})}{\tilde{F}_k(\mathcal{K}_i) - \tilde{F}_k(\mathcal{K}_{i-1})}, i > 1 \tag{2}$$

3.2 Adjustment transitions

Baseline: Construct F_w. Use

$$F_w(w) = F_{b|k}(w/R^b \mid k=0)F_k(0) + \int_0^\infty F_{b|k}\left(\frac{w-(q+r)k}{R^b} \mid k\right) f_k(k)dk \tag{3}$$

See below for a derivation and how to compute the integral numerically. This formulation should already allow for mass point at $w = 0$.

Alternative B: Start with F_w.

Leverage that k_a^* is monotonically increasing in w. Use DEGM to get $\tilde{F}_k(\cdot \mid adj)$ with interpolation nodes $(k_a^*(b_i,k_j), F_w(w(b_i,k_j))_{i,j}$. Allow for mass point at $k = 0$ by setting $\tilde{F}_k(0 \mid adj) = F_w(\bar{w})$, where $\bar{w}$ is defined as largest w such that $k_a^*(w) = 0$ (if existant; see below).

Alternative B: Given w_a^* (as defined above), which is mon. increasing in w, use DEGM to get $\tilde{F}_w(\cdot \mid adj)$ with interpolation nodes $(w_a^*(b_i,k_j), F_w(w(b_i,k_j))_{i,j}$.

To compute $\tilde{F}_{b|k}(b' \mid k', adj)$, need to use some results from optimal portfolio choice.

1. $k_a^* > 0$ implies that household is not financially constrained. Then, b_a^* is *determined* by k_a^* through equilibrium conditions. It follows that, if $k' > 0$, we have

$$\begin{aligned}\tilde{F}_{b|k}(b' \mid k', adj) &= P(b_a^*(k') \leq b') \qquad (4)\\ &= \mathcal{I}_{\{b_a^*(k') \leq b'\}} \qquad (5)\end{aligned}$$

$b_a^*(k')$ is taken directly from EGM algorithm, i.e. household optimization, where it is called "m_a_aux".

2. If $k_a^* = 0$, household financially constrained (other than in zero-probability case where 0 is actually optimal), so that b_a^* depends on available resources w. We have

$$\tilde{F}_{b|k}(b' \mid k' = 0, adj) = P(b_a^*(w) \leq b' \mid k_a^*(w) = 0) \tag{6}$$

There exists a $\bar{w}$ such that $k_a^*(w) = 0 \; \forall w \leq \bar{w}$, unless $k_a^* > 0$ always (for given y). Assume it exists (o.w., probability is irrelevant since it conditions on zero probability event). This leads to

$$\begin{aligned}
\tilde{F}_{b|k}(b_a^*(\hat{w}) \mid k' = 0, adj) &= P(b_a^*(w) \leq b_a^*(\hat{w}) \mid k_a^*(w) = 0) && (7)\\
&= P(w \leq \hat{w} \mid w \leq \bar{w}) && (8)\\
&= \frac{F_w(\min(\hat{w}, \bar{w}))}{F_w(\bar{w})} && (9)
\end{aligned}$$

This gives the interpolation nodes

$$\left(b_a^*(b_i, k_j), \frac{F_w(\min(w(b_i, k_j), \bar{w}))}{F_w(\bar{w})}\right)_{i,j} \tag{10}$$

to interpolate $\tilde{F}_{b|k}(b' \mid k' = 0, adj)$. To get $\bar{w}$, easiest to take largest w s.t. $k_a^*(w) = 0$. For more precision, go to optimality conditions in EGM. Assume there exists $\bar{w}_m$ as largest w such that $b_a^*(\bar{w}_m) = \underline{m}$. Then we allow for mass point there by setting $\tilde{F}_{b|k}(\underline{m} \mid k' = 0, adj) = \frac{F_w(\bar{w}_m)}{F_w(\bar{w})}$.

3.3 Non-Adjustment transitions

Need to compute

$$\tilde{F}_{b|k}(b' \mid k, nadj) = P(b_n^*(b \mid k) \leq b' \mid k) \tag{11}$$

Using monotonicity:

$$\begin{aligned}
\tilde{F}_{b|k}(b_n^*(\hat{b} \mid k) \mid k, nadj) &= P(b_n^*(b \mid k) \leq b_n^*(\hat{b} \mid k) \mid k) && (12)\\
&= P(b \leq \hat{b} \mid k) && (13)\\
&= F_{b|k}(\hat{b} \mid k) && (14)
\end{aligned}$$

Interpolation nodes for $\tilde{F}_{b|k}(b' \mid \mathcal{K}, nadj)$:

$$\left(b_n^*(b \mid k), F_{b|k}(b \mid k)\right)_{(b,k) \in \mathcal{G}} \tag{15}$$

Use DEGM to compute $\tilde{F}_{b|k}(\cdot \mid k, nadj)$ for any $k \in \mathcal{K}$. Allow for mass point at $\underline{m}$ by finding $\bar{b}(k)$ as largest b such that $b_n^*(\bar{b}(k) \mid k) = \underline{m}$. If existent, set $\tilde{F}_{b|k}(\underline{m} \mid k, nadj) = F_{b|k}(\bar{b}(k) \mid k)$.

3.3.1 Alternative B: update w-marginal when non-adjusting

Use

$$
\begin{aligned}
&\tilde{F}_w(w' \mid nadj) = \tilde{F}_w(w' \mid k=0, nadj)F_k(0) + \int_0^\infty \tilde{F}_w(w' \mid k, nadj) f_k(k) dk \\
&= \cdots + \int_0^\infty P((q+r)k + R^b b_n^*(b \mid k) \leq w' \mid k) f_k(k) dk \\
&= \tilde{F}_{b|k}(w'/R^b \mid k=0, nadj)F_k(0) + \int_0^\infty \tilde{F}_{b|k}\left(\frac{w' - (q+r)k}{R^b} \mid k, nadj\right) f_k(k) dk
\end{aligned}
$$

Several approaches to compute integral. One would be with some quadrature procedure, where pdf f_k stems from numerical derivative of spline-interpolated cdf F_k.

A different one is with average of lower and upper sum of Riemann integral. Upper sum:

$$
\tilde{F}_w(w' \mid k > 0, nadj)^U := \sum_j \tilde{F}_{b|k}\left(\frac{w' - (q+r)\mathcal{K}_j}{R^b} \mid \mathcal{K}_j, nadj\right)[F_k(\mathcal{K}_j) - F_k(\mathcal{K}_{j-1})]
$$

Lower sum:

$$
\tilde{F}_w(w' \mid k > 0, nadj)^L := \sum_j \tilde{F}_{b|k}\left(\frac{w' - (q+r)\mathcal{K}_{j-1}}{R^b} \mid \mathcal{K}_{j-1}, nadj\right)[F_k(\mathcal{K}_j) - F_k(\mathcal{K}_{j-1})]
$$

Then:

$$
\begin{aligned}
\tilde{F}_w(w' \mid nadj) \approx &\tilde{F}_{b|k}(w'/R^b \mid k=0, nadj)F_k(0) \\
&+ .5\left(\tilde{F}_w(w' \mid k > 0, nadj)^U + \tilde{F}_w(w' \mid k > 0, nadj)^L\right)
\end{aligned} \tag{16}
$$