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Homework problems Classical Field Theory – SoSe 2022 – Set 9

due June 28 in lecture

Problem 30:

We consider again the example of a conducting sphere that we discussed in the lecture. The sphere is on fixed potential φ_0.

(a) Assume that the charge density ϱ for this problem vanishes. Furthermore, the potential is supposed to vanish at infinity. Use the Green's function that we derived in the lecture for this problem to compute the potential $\varphi(\vec{x}\,)$ everywhere in the exterior of the sphere.

(b) Now assume that the charge density *inside* the sphere is given. Find the Green's function for this case and give an expression for the potential everywhere inside the sphere.

(c) Suppose that $\varrho = 0$ in the interior of the sphere. Use the Green's function from **(b)** to compute the potential in the interior.

Problem 31:

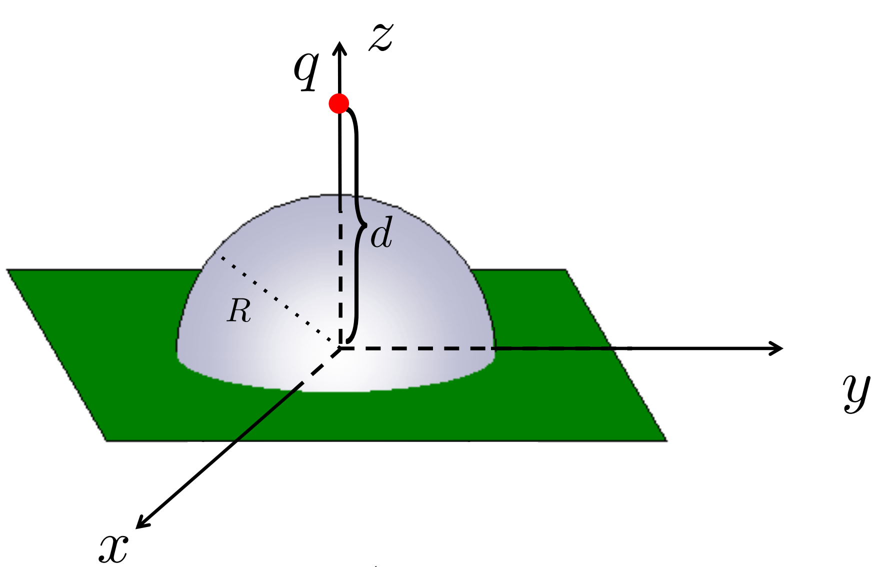
A plane has a bulge that has the shape of a half sphere with radius R (see figure below; the center of the sphere is in the coordinate origin). Plane and half sphere are conducting and grounded, so that they are on potential zero. The potential also vanishes at $|\vec{x}\,| \to \infty$. On the positive z axis at $z = d > R$ there is a point charge q.

(a) Find the potential of the system everywhere in $|\vec{x}\,| \geq R, z \geq 0$. Also, present the Green's function for this problem.

Help: Recall the example of the sphere considered in the lecture or in problem **30**.

(b) What charge Q is induced on the plane (without the bulge)? What charge Q' do you expect on the half sphere?

(c) Compute the force that acts on the point charge q.



Problem 32:

We consider the two charged conducting spheres shown in the figure below. They touch in one point. They both have the same radius R, and their surfaces are both on the same potential φ_0. The potential $\varphi(\vec{x}\,)$ vanishes at $|\vec{x}\,| \to \infty$. Compute the total charge Q that the spheres have to carry.

Help:

(1) It is simpler to first assume that (for example) the sphere on the right is on potential zero;

(2)

$$1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} \pm \ldots = \log(2)\,.$$

$\varphi = \varphi_0$