Preface

The twenty-first century is seeing the emergence of the first truly quantum technologies; that is, technologies that rely on the counter-intuitive properties of individual quantum systems and can often outperform any conventional technology. Examples include quantum computing, which promises to be much faster than conventional computing for certain problems, and quantum metrology, which promises much more sensitive parameter estimation than that offered by conventional techniques. To realize these promises, it is necessary to understand the measurement and control of quantum systems. This book serves as an introduction to quantum measurement and control, including some of the latest developments in both theory and experiment.

Scope and aims

To begin, we should make clear that the title of this book is best taken as short-hand for 'Quantum measurements with applications, principally to quantum control'. That is, the reader should be aware that (i) a considerable part of the book concerns quantum measurement theory, and applications other than quantum control; and (ii) the sort of quantum control with which we are concerned is that in which measurement plays an essential role, namely feedback (or feedforward) control of quantum systems.[1]

Even with this somewhat restricted scope, our book cannot hope to be comprehensive. We aim to teach the reader the fundamental theory in quantum measurement and control, and to delve more deeply into some particular topics, in both theory and experiment.

Much of the material in this book is new, published in the last few years, with some material in Chapter 6 yet to be published elsewhere. Other material, such as the basic quantum mechanics, is old, dating back a lifetime or more. However, the way we present the material, being informed by new fields such as quantum information and quantum control, is often quite unlike that in older text-books. We have also ensured that our book is relevant to current developments by discussing in detail numerous experimental examples of quantum measurement and control.

[1] In using the term 'feedback' or 'feedforward' we are assuming that a measurement step intervenes – see Section 5.8.1 for further discussion.

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We have not attempted to give a full review of research in the field. The following section of this preface goes some way towards redressing this. The 'further reading' section which concludes each chapter also helps. Our selection of material is naturally biassed towards our own work, and we ask the forbearance of the many workers in the field, past or present, whom we have overlooked.

We have also not attempted to write an introduction to quantum mechanics suitable for those who have no previous knowledge in this area. We do cover all of the fundamentals in Chapter 1 and Appendix A, but formal knowledge is no substitute for the familiarity which comes with working through exercises and gradually coming to grips with new concepts through an introductory course or text-book.

Our book is therefore aimed at two groups wishing to do research in, or make practical use of, quantum measurement and control theory. The first is physicists, for whom we provide the necessary introduction to concepts in classical control theory. The second is control engineers who have already been introduced to quantum mechanics, or who are introducing themselves to it in parallel with reading our book.

In all but a few cases, the results we present are derived in the text, with small gaps to be filled in by the reader as exercises. The substantial appendices will help the reader less familiar with quantum mechanics (especially quantum mechanics in phase space) and stochastic calculus. However, we keep the level of mathematical sophistication to a minimum, with an emphasis on building intuition. This is necessarily done at the expense of rigour; ours is not a book that is likely to appeal to mathematicians.

Historical background

Quantum measurement theory provides the essential link between the quantum formalism and the familiar classical world of macroscopic apparatuses. Given that, it is surprising how much of quantum mechanics was developed in the absence of formal quantum measurement theory – the structure of atoms and molecules, scattering theory, quantized fields, spontaneous emission etc. Heisenberg [Hei30] introduced the 'reduction of the wavepacket', but it was Dirac [Dir30] who first set out quantum measurement theory in a reasonably rigorous and general fashion. Shortly afterwards von Neumann [vN32] added a mathematician's rigour to Dirac's idea. A minor correction of von Neumann's projection postulate by Lüders [Lüd51] gave the theory of projective measurements that is still used today.

After its formalization by von Neumann, quantum measurement theory ceased to be of interest to most quantum physicists, except perhaps in debates about the interpretation of quantum mechanics [Sch49]. In most experiments, measurements were either made on a large ensemble of quantum particles, or, if they were made on an individual particle, they effectively destroyed that particle by detecting it. Thus a theory of how the state of an individual quantum system changed upon measurement was unnecessary. However, some mathematical physicists concerned themselves with generalizing quantum measurement theory to describe non-ideal measurements, a programme that was completed in the 1970s by Davies [Dav76] and Kraus [Kra83]. Davies in particular showed how the new formalism could describe a continuously monitored quantum system, specifically for the case of quantum jumps [SD81].

By this time, experimental techniques had developed to the point where it was possible to make quantum-limited measurements on an individual quantum system. The prediction [CK85] and observation [NSD86, BHIW86] of quantum jumps in a single trapped ion was a watershed in making physicists (in quantum optics at least) realize that there was more to quantum measurement theory than was contained in von Neumann's formalization. This led to a second watershed in the early 1990s when it was realized that quantum jumps could be described by a stochastic dynamical equation for the quantum state, giving a new numerical simulation method for open quantum systems [DCM92, GPZ92, Car93]. Carmichael [Car93] coined the term 'quantum trajectory' to describe this stochastic evolution of the quantum state. He emphasized the relation of this work to the theory of photodetection, and generalized the equations to include quantum diffusion, relating to homodyne detection.

Curiously, quantum diffusion equations had independently, and somewhat earlier, been derived in other branches of physics [Bel02]. In the mathematical-physics literature, Belavkin [Bel88, BS92] had made use of quantum stochastic calculus to derive quantum diffusion equations, and Barchielli [Bar90, Bar93] had generalized this to include quantum-jump equations. Belavkin had drawn upon the classical control theory of how a probability distribution could be continuously (in time) conditioned upon noisy measurements, a process called filtering. He thus used the term quantum filtering equations for the quantum analogue. Meanwhile, in the quantum-foundations literature, several workers also derived these sorts of equations as attempts to solve the quantum-measurement problem by incorporating an objective collapse of the wavefunction [Gis89, Dió88, Pea89, GP92a, GP92b].

In this book we are not concerned with the quantum measurement problem. By contrast, Belavkin's idea of making an analogy with classical control theory is very important for this book. In particular, Belavkin showed how quantum filtering equations can be applied to the problem of feedback control of quantum systems [Bel83, Bel88, Bel99]. A simpler version of this basic idea was developed independently by the present authors [WM93c, Wis94]. Quantum feedback experiments (in quantum optics) actually date back to the mid 1980s [WJ85a, MY86]. However, only in recent years have sophisticated experiments been performed in which the quantum trajectory (quantum filtering equation) has been essential to the design of the quantum control algorithm [AAS$^+$02, SRO$^+$02].

At this point we should clarify exactly what we mean by 'quantum control'. Control is, very roughly, making a device work well under adverse conditions such as (i) uncertainties in parameters and/or initial conditions; (ii) complicated dynamics; (iii) noise in the dynamics; (iv) incomplete measurements; and (v) resource constraints. Quantum control is control for which the design requires knowledge of quantum mechanics. That is, it does not mean that the whole control process must be treated quantum mechanically. Typically only a small part (the 'system') is treated quantum mechanically, while the measurement device, amplifiers, collators, computers, signal generators and modulators are all treated classically.

As stated above, we are primarily concerned in this book with quantum feedback control. However, there are other sorts of quantum control in which measurement theory does not

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play a central role. Here we briefly discuss a few of these; see Ref. [MK05] for a fuller review of types of quantum control and Ref. [MMW05] for a recent sample of the field. First, *open-loop control* means applying control theory to manipulate the dynamics of systems in the absence of measurement [HTC83, D'A07]. The first models of quantum computing were all based upon open-loop control [Pre98]. It has been applied to good effect in finite quantum systems in which the Hamiltonian is known to great precision and real-time measurement is impossible, such as in nuclear magnetic resonance [KGB02, KLRG03]. Second, there is *learning control*, which applies to systems in which the Hamiltonian is not known well and real-time measurement is again impossible, such as chemical reactions [PDR88]. Here the idea is to try some control strategy with many free parameters, see what results, adjust these parameters, and try again. Over time, an automated learning procedure can lead to significant improvements in the performance of the control strategy [RdVRMK00]. Finally, general mathematical techniques developed by control theorists, such as semi-definite programming and model reduction, have found application in quantum information theory. Examples include distinguishing separable and entangled states [DPS02] and determining the performance of quantum codes [RDM02], respectively.

The structure of this book

The structure of this book is shown in Fig. 1. It is not a linear structure; for example, the reader interested in Chapter 7 could skip most of the material in Chapters 2, 3 and 6. Note that the reliance relation (indicated by a solid arrow) is meant to be transitive. That is, if Chapter C is indicated to rely upon Chapter B, and likewise Chapter B upon Chapter A, then Chapter C may also rely directly upon Chapter A. (This convention avoids a proliferation of arrows.) Not shown in the diagram are the two Appendices. Material in the first, an introduction to quantum mechanics and phase space, is used from Chapter 1 onwards. Material in the second, on stochastic differential equations, is used from Chapter 3 onwards.

For the benefit of readers who wish to skip chapters, we will explain the meaning of each of the dashed arrows. The dashed arrow from Chapter 2 to Chapter 7 is for Section 2.5 on adaptive measurements, which is used in Section 7.9. That from Chapter 3 to Chapter 4 is for Section 3.6, on the Lindblad form of the master equation, and Section 3.11, on the Heisenberg picture dynamics. That from Chapter 3 to Chapter 6 is for Section 3.8 on preferred ensembles. That from Chapter 5 to Chapter 6 is for Section 5.5 on homodyne-based Markovian feedback. Finally, that from Chapter 6 to Chapter 7 is for the concept of an optimal quantum filter, which is introduced in Section 6.5. Of course, there are other links between various sections of different chapters, but these are the most important.

Our book is probably too long to be covered in a single graduate course. However, selected chapters (or selected topics within chapters) could be used as the basis of such a course, and the above diagram should aid a course organizer in the selection of material. Here are some examples. Chapters 1, 3 and 4 could be the text for a course on open quantum

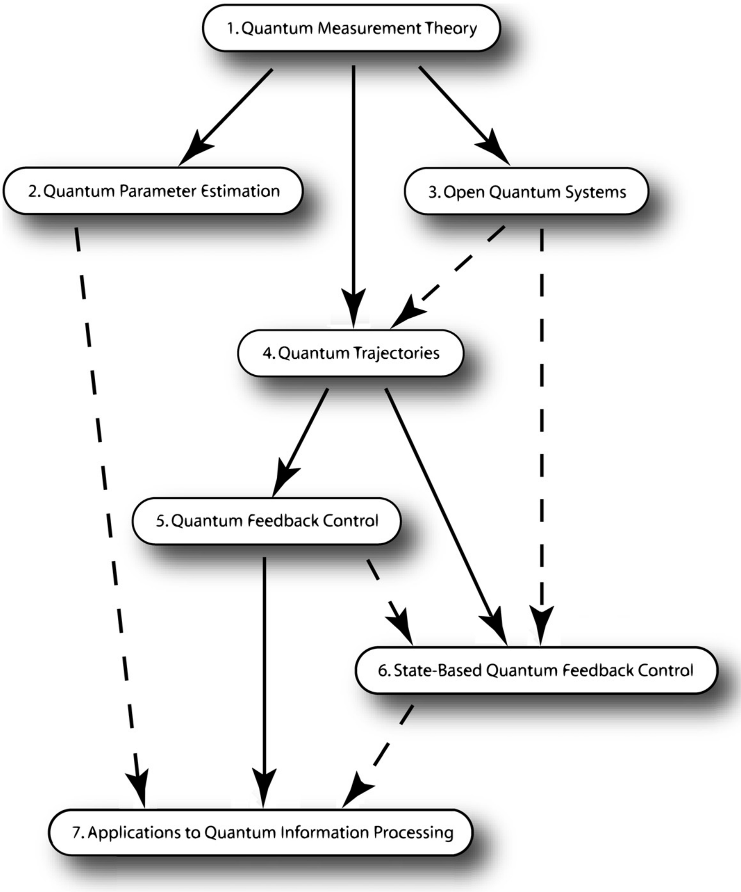

Fig. 1 The structure of this book. A solid arrow from one chapter to another indicates that the latter relies on the former. A dashed arrow indicates a partial reliance.

systems. Chapters 1, 4 and 6 (plus selected other sections) could be the text for a course on state-based quantum control. Chapters 1 and 2 could be the text for a course on quantum measurement theory.

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Chapter 6 requires a special discussion. Sections 6.3–6.6, while building on Ref. [WD05], contain a large number of hitherto unpublished results obtained by one of us (H.M.W) in collaboration with Andrew Doherty and (more recently) Andy Chia. This material has circulated in the community in draft form for several years. It is our intention that much of this material, together with further sections, will eventually be published as a review article by Wiseman, Doherty and Chia.

The contributions of others of course take away none of the responsibility of the authors for errors in the text. In a book of this size, there are bound to be very many. Readers are invited to post corrections and comments on the following website, which will also contain an official list of errata and supplementary material:

`www.quantum-measurement-and-control.org`