ETH zürich

DISS. ETH NO. 23220

Financial Market Risk of Speculative Bubbles

A thesis submitted to attain the degree of

DOCTOR OF SCIENCES of ETH ZURICH

(Dr. sc. ETH Zurich)

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2016

Abstract

Understanding the origins and characteristics of large stock price movements is key to the management of financial market risk. Traditionally it is assumed that large drawdowns are caused by unforeseeable external shocks adversely affecting financial markets. However, new research suggests that most crashes are the burst of a speculative bubble that endogenously builds up over a long time. This dissertation contributes in multiple ways to a better understanding of the intrinsic instability of financial markets. First, a theoretical model and computational simulations shed light on the dynamics of speculative bubbles. We observe asset prices that grow super-exponentially and derive analytical conditions defining the unstable regime. Second, an econometric analysis of derivative prices allows a quantitative characterization of the boom and bust cycle of the S&P 500 stock market throughout the decade around the Global Financial Crisis of 2008. In particular, we document investors' expectations of super-exponentially growing asset prices. And third, a recent risk measure is applied as a predictive tool for return downturns. The risk measure is found to add information beyond standard measures such as value at risk, expected shortfall and risk-neutral volatility. We conclude with an empirical study of a large hedge fund that investigates what communication structures can be associated with successful trading. We address these questions drawing from a rich and interdisciplinary set of methodologies such as agent-based modeling, risk-neutral density estimation, change point analysis, Granger-causality analysis, Monte Carlo methods, variance-ratio tests, network analysis and large scale text mining.

Zusammenfassung

Das Verständnis der Ursprünge und Charakteristika großer Aktienpreisänderungen ist von entscheidender Bedeutung für das Management von Finanzmarktrisiken. Eine traditionelle Annahme ist, dass erhebliche Preissprünge durch unvorhersehbare, externe, negativ wirkende Schocks verursacht werden. Im Gegensatz dazu sieht neuere Forschung einen Crash jedoch als die unvermeidliche Konsequenz einer spekulativen Blase, die sich über lange Zeit aufbaut und schließlich platzt, und damit als system-endogenes Phänomen. Diese Dissertation trägt in mehrfacher Hinsicht zu einem besseren Verständnis der intrinsischen Instabilität von Finanzmärkten bei. Zunächst wird die Dynamik spekulativer Blasen mit Hilfe eines theoretischen Modells und von Computersimulationen untersucht. Wir beobachten superexponentiell wachsende Vermögenspreise und leiten analytische Bedingungen für das instabile Regime her. Zweitens erlaubt eine ökonometrische Analyse von Derivatpreisen die quantitative Charakterisierung des Boom-Bust-Zyklus des S&P 500 Aktienmarktes über die Dekade um die globale Finanzkrise von 2008. Insbesondere dokumentieren wir die Erwartungen superexponentiell steigender Aktienpreise von Investoren. Schließlich wenden wir ein neues Risikomaß als Prediktor für Renditerückgänge an. Es wird gezeigt, dass das Risikomaß über einen Informationsgehalt verfügt, der über die Standardmaße Value at Risk, Expected Shortfall und risikoneutrale Volatilität hinaus geht. Außerdem untersuchen wir empirisch anhand eines großen Hedgefonds welche Kommunikationsstrukturen mit erfolgreichem Handel verknüpft sind. Wir adressieren diese Fragen mit einer Reihe verschiedener Methoden wie etwa agentenbasierten Modellen, risikoneutraler Dichteschätzung, Wechselpunktanalyse, Granger-Kausalitätsanalyse, Monte-Carlo-Simulationen, Varianz-Quotienten-Tests, sowie Netzwerk- und Textanalyse.

Acknowledgements

ETHZ has been an amazing place for doing research. First of all, it put me in the fortunate position of having not one, but *two* great advisors. I would like to thank Dirk Helbing and Didier Sornette for the academic freedom, intellectual stimulation, support and guidance throughout the years. Furthermore, I am very grateful to Heinrich H. Nax, who has accompanied my work during most of my Ph.D. and has given me innumerable times advice, inspiration, support and guidance.

Moreover, I would like to thank Abhinav Anand, Stefano Balietti, Peter Cauwels, Thomas Chadefaux, Karsten Donnay, Stephen Figlewski, Vladimir Filimonov, Zalán Forró, Dean Foster, Sergiu Hart, Thorsten Hens, Emőke-Ágnes Horvát, Jens Jackwerth, Caleb Koch, Ralf Kohrt, Terry Lyons, Michael Mäs, Yannick Malevergne, Tom Norman, Ole Peters, Daniel Philipp, Bary Pradelski, Lloyd Sanders, Christian Schulz, Olivia Woolley Meza, all members of COSS, ER and NICO, as well as the participants of the seminar series of the Oxford MAN Institute, the London Mathematical Laboratory and the 26th Stony Brook International Conference on Game Theory for helpful comments and suggestions to my research projects. Special gratitude goes to Brian Uzzi, who invited me for a research visit to the Kellogg School of Management, where I learned a lot.

I am thankful for the support by the ETH Risk Center. And last but not least, I would like to thank Nancy McLaughlin, Petra Monsch, Isabella Bieri and, in particular, Dietmar Huber for making everything happen.

Zurich, November 2015

Matthias Leiss

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Chapter 1

Introduction[1]

Understanding the nature, that is, the origins and characteristics, of large stock price movements is key to the management of financial market risk. The classical paradigm underlying the theory of financial market crashes relies on the efficient market hypothesis (EMH), stating that price movements are governed by unforeseeable external shocks. Today's terminology of market efficiency goes back to Eugene Fama who defined a market to be informationally efficient if prices *"fully reflect"* all information available to investors (Fama, 1970, p. 383). A price fully reflecting all information means the nonexistence of arbitrage – the possibility of gains at zero risk. Fama distinguished three forms of informational market efficiency. First, in its *"weak form"*, the set of available information involves only all historical prices. Then, in its *"semi-strong form"*, the efficient market hypothesis states that prices reflect all publicly available information up to that very moment. Finally, in the *"strong form"*, the information set includes all publicly and privately available information. Hence, even insider information regarding a company and its competitors would have been incorporated in the price.

There is a connection between market efficiency and random dynamics of asset prices. Assume the corresponding information set allowed the investor to guess the development of the price of an asset with a probability higher than chance. Then this would open an arbitrage possibility, a *free lunch*, and the market would cease to be efficient. Thus, in a truly efficient market prices should fluctuate randomly.

[1] **Parts of this chapter are based on Leiss (2015).**

Bachelier (1900) was the first to give a mathematical formulation to the random nature of stock market prices by employing a random walk model. His line of work was picked up by Samuelson (1965), first proving that properly anticipated prices must fluctuate randomly. Later, he extended his result by showing that stock prices based on a stochastic, but correctly anticipated dividend process also change in a random way (Samuelson, 1973). The two necessary assumptions for both proofs are a price formation mechanism as in the repeated general equilibrium model and complete rationality of all agents.[2]

The efficient market hypothesis has been challenged by questioning the assumptions of both equilibrium and rationality. Notably, Grossman and Stiglitz (1980) contended the assumption of a competitive market being always in general equilibrium. Since any acquisition of information is costly, but efficient markets exclude returns due to information gathering by arbitrage, they argued that individuals had no incentive to trade in an efficient market. But without, trading new information will not be incorporated into prices and the market ceases to be efficient. Instead of a general equilibrium they proposed *"an equilibrium degree of disequilibrium"* (Grossman and Stiglitz, 1980, p. 393), i.e., a price system that only imperfectly reflects publicly available information.

The theoretical argument by Grossman and Stiglitz was supported by empirical work of Bouchaud et al. (2009) who analyzed the characteristics of continuous double auction order books of the biggest financial stock exchanges. Regarding market efficiency they concluded that in orders of magnitude *"markets can only be informationally efficient at first order but must necessarily be inefficient at second order"* (Bouchaud et al., 2009, p. 67).[3]

Furthermore, a number of psychologists and behavioral economists argued for the implausibility of the assumption of completely rational individuals. For example Amos Tversky and Daniel Kahneman observed a number of heuristics, i.e., deviations from complete rationality, that people employ when making judgments

[2]Samuelson's proof also neglects the trade-off between risk and return in finance such that a positive expected price move just reflects the reward necessary to attract investors to hold an asset with a strongly fluctuating price (Lo and MacKinlay, 2011, p. 5).

[3] Bouchaud et al. (2009) argue for the EMH to be a good first approximation of real markets, but also an inherently insufficient one.

under uncertainty (Tversky and Kahneman, 1974). Although the heuristics are found to be effective in most cases, they also imply systematic errors. In a subsequent study, Kahneman and Tversky (1979) showed expected utility theory to be an unsuitable descriptive model for decisive behavior under risk. In particular, they described the well-known loss aversion – the tendency of individuals to give potential losses over-proportional importance compared to potential gains. De Bondt and Thaler (1985) investigated whether this type of "overreaction" of most people affected stock prices. By analyzing monthly return data they gave empirical evidence for substantial weak-form market inefficiencies.[4]

The insights from behavioral economics were incorporated into new models of markets where rational agents interact with less rational ones. The so-called *noise traders* were first introduced by Kyle (1985) and Black (1986), who characterized them as trading *"on noise as if it were information"* (Black, 1986, p. 531). According to Black (1986, p. 530), *"noise makes financial markets possible, but also makes them imperfect"* by introducing uncertainty to stock prices such that *"we might define an efficient market as one in which price is within a factor of 2 of [the fundamental] value"* (Black, 1986, p. 533). In Kyle's sequential equilibrium model a single risk neutral insider benefiting from private information acts in a market with risk neutral market makers and random noise traders. The noise traders camouflage the insider's trading from the market makers, breaking the no-trade argument by Grossman and Stiglitz with the consequence that all private information is dynamically incorporated into prices.

More recent critiques claim that the efficient market hypothesis is *impossible to test*. For example, Farmer and Lo (1999, p. 9992) explained this so-called *joint hypothesis problem*:

> "[...] the EMH, by itself, is not a well posed and empirically refutable hypothesis. To make it operational, one must specify additional structure: e.g., investors' preferences, information structure, etc. But then a test of the EMH becomes a test of several auxiliary hypotheses as well, and

[4] Akerlof and Shiller (2010) give an elegant review of how human psychology may play a role in economic contexts.

a rejection of such a joint hypothesis tells us little about which aspect of the joint hypothesis is inconsistent with the data."

The picture drawn by the efficient market hypothesis stands in contrast to the paradigm of speculative bubbles (Sornette, 2003), a concept that resonates with the understanding of financial markets as a complex systems (Helbing, 2013). Such a regime is different from the uncertainty in stock prices due to noise trading as described by Fischer Black, which corresponds to stochastic fluctuations around a long-term trend of exponential growth in the fundamental value. On the contrary, during a speculative bubble the price systematically detaches from the fundamentals over an extended period of time leading the market into an ever more unstable state (Sornette, 2003). Usually the bubble develops until a critical point is reached and the unsustainable dynamics can no longer be maintained. As a consequence, the market undergoes a change of regime, often characterized by a sudden price correction to the long-term trend (Sornette, 2003).[5] At this point it is worth noting that there are positive and negative bubbles associated with positive and negative deviations from the fundamental value, respectively. Intuitively one can observe both, as everything in finance is relative to the numeraire. For example, the trajectory of the Swiss franc in Euro in summer 2011 was a positive bubble, whereas the inverse, i.e. Euro in Swiss franc, was a negative bubble (Sornette and Cauwels, 2015). In the former case one would speak of a *crash*, and in the latter of a *rally* or *rebound*. In any case, the key insight is that the fundamental reason for the large stock market move was the endogenous instability and not some external shock.

In general, speculative bubbles are thought to be fueled by positive feedback mechanisms, which drive the market price away from equilibrium and fundamentals. Sornette and Cauwels (2015) classify them into two broad groups: technical positive feedback on the one hand and behavioral on the other hand.

Sircar and Papanicolaou (1998) discuss dynamic option hedging as an example of the first group, which is a special form of a portfolio insurance strategy often based on the model of Black and Scholes (1973). This model suggests that risk associated

[5] Sornette and Cauwels (2015) discuss problems with the notion of a fundamental value and explain on the basis of the dividend discount model by Gordon and Shapiro (1956) how small ambiguity in the discount rate may amplify to large uncertainty in the evaluation of a stock.

with selling, say, a call option can be eliminated by replicating each small change of the price of the underlying stock in one's portfolio. So if the underlying stock appreciates, the seller of the call option will buy more of the underlying – a clear case of positive feedback.

Examples of a behavioral positive feedback mechanism are social imitation and herding. They are not irrational per se, as in times of strong information asymmetries and uncertainty averaging over the actions of one's peers may represent a good estimate of the price determined by the overall average sentiment of the market (Sornette and Cauwels, 2015). However, Lorenz et al. (2011) show that social influence may undermine this "wisdom of crowd" effect. Imitation and herding can become particularly dangerous when combined with the typical mindset prevailing during a speculative bubble. Economic history suggests that, throughout time and across countries, speculative bubbles start with a new technology or business opportunity as well as experts chiming *"This Time Is Different"* (title of Reinhart and Rogoff 2009), claiming that the old rules of valuation no longer apply. Soon, the initial wave of funding and the extraordinary prospects encourage other investors to follow. The price increases, which in turn will attract even more investors.

Most of economics and finance is characterized by exponential growth. This reflects the multiplicative nature of growth processes such as compounding interests at a constant rate of return. However, both technical and behavioral positive feedback mechanisms imply a cycle where an increasing price leads to an increase in demand and vice versa. Consequently, the rate of return is no longer constant, but itself increases over time. The price then grows faster than exponentially, or *super-exponentially.* To see this, let us define the infinitesimal return of an asset with price $p(t)$ as is commonly done in mathematical finance (see e.g., Black and Scholes 1973; Fouque et al. 2000):

$$\frac{\mathrm{d}p(t)}{p(t)} = r \ \mathrm{d}t + \sigma \ \mathrm{d}W(t), \tag{1.1}$$

where r, σ and $\mathrm{d}W(t)$ are the rate of return, the volatility and infinitesimal increments of a Brownian motion, respectively. For simplicity we may consider only the deterministic equation, as everything carries over to the stochastic case. Thus

$$\mathbb{E}\left[\frac{\mathrm{d}p(t)}{p(t)}\right] = r\ \mathrm{d}t. \tag{1.2}$$

Usually, the rate of return r is left constant, but let us assume it grows linearly in time:

$$r(t) = r_0 + \gamma t, \tag{1.3}$$

with constants r_0 and γ. Then the solution of equation (1.2) is given by

$$\mathbb{E}\left[p(t)\right] = p_0\ \mathrm{e}^{r_0 t + \gamma t^2/2}\ . \tag{1.4}$$

For $\gamma = 0$ we recover the well-known standard exponential growth due to compounding interests, which is commonly found in economic processes and reflects Gibrat's law of proportional growth (Gibrat, 1931). However, for positive $\gamma > 0$ the rate of return itself grows in time such that the price increases much faster than an exponential, which we refer to as *super-exponential* growth. We can expect an even faster increase in the price of the asset for rates of return that exhibit stronger than linear transient growth dynamics. In general, such a growth path is not sustainable and therefore of a transient nature. Thus, it is usually associated with the build-up of instabilities, that in finance are often termed bubbles.

In fact, speculative bubbles may be defined as transient phases with super-exponential growth (Sornette, 2003). Investing in a stock with a super-exponentially increasing price can be embedded in a rational expectations model (Blanchard, 1979; Blanchard and Watson, 1982), as long as the expected return is proportional to the crash risk (Johansen et al., 2000).[6] There is, however, a distinct difference in the dynamics implied by increasing returns as compared to standard exponential growth: super-exponential dynamics can lead to a singularity in finite time, at which the model ceases to describe the underlying process. It would be a mistake, however, to conclude that the model is flawed. The finite time singularity rather reflects the fact that the current dynamics are unsustainable and that the system will undergo a change of regime – such as an end of imitation and herding – resulting in a price correction, i.e., in a burst of the bubble.

[6] Scheinkman and Xiong (2003) provide yet another model for speculative bubbles.

Speculative bubbles are ubiquitous in real financial markets. In general, it is difficult to decide whether a market was in a bubble or not (Camerer, 1989; Stiglitz, 1990; Bhattacharya and Yu, 2008). However, especially over the past few years, there has been a lot of progress in methodology (Jarrow et al., 2011; Evanoff et al., 2012; Lleo and Ziemba, 2012; Anderson et al., 2013; Sornette et al., 2013). Speculative bubbles have been empirically documented for international equity markets (Jiang et al., 2010; Phillips et al., 2011; Yan et al., 2012; Phillips et al., 2012), real estate (Zhou and Sornette, 2003, 2006), commodities and derivatives (Sornette and Woodard, 2010), as well as bonds, gold and foreign exchange markets (Johansen and Sornette, 2010). Furthermore, Hüsler et al. (2013) observed super-exponential growth dynamics in controlled experiments in the laboratory.

In the following, we will outline how this dissertation contributes to a better understanding of the intrinsic instability of financial markets.

Chapter contributions

All materials contained in this cumulative thesis represent sole or joint first author contributions. The main body of this dissertation consists of four chapters that are based on individual papers: two of these papers are published in peer-reviewed journals, one is a working paper currently under review, and one is based on work in progress. For the purpose of this thesis, the chapters have been extended in parts using content from a single-authored book chapter (Leiss, 2015) and results related to the two master theses which the author co-supervised (Philipp, 2015; Kohrt, 2015). Below, a brief outline of the individual chapters is given.

Chapter 2 is published in the Journal of Economic Behavior & Organization (Kaizoji, Leiss, Saichev, and Sornette, 2015). It builds on an agent-based model to theoretically assess the emergence of super-exponentially growing prices. Agent-based models (ABM) represent a natural way to study the aggregate outcome due to interactions among possibly heterogeneous individuals. In a first important contribution, De Long et al. (1990a,b) employed an ABM to quantitatively study a financial market populated by “fundamental” and “technical / chartist” traders. While fundamentalists base their investment decision on fundamental values, chartists exhibit erroneous stochastic beliefs leading to an unpredictable additional risk in the asset price. As a result, fundamentalist investors fail to exploit the noise traders' irrational behavior and market prices significantly deviate from fundamentals. A number of works have extended the set-up of noise traders to account for group psychological and sociological effects such as herding and trend-following (Kirman, 1993; Lux and Marchesi, 1999; Lux, 2009; Brock and Hommes, 1997, 1998; Chiarella and He, 2001; Chiarella et al., 2006, 2009).

Although successful in explaining many statistical regularities of financial markets such as a fat-tail distribution of returns and volatility clustering, to the best of our knowledge no ABM has studied the link between interactions among investors and the transiently super-exponential growth of asset prices yet. However, such an approach is crucial for a qualitative and quantitative micro-understanding of speculative bubbles. This gap in the literature has been closed by Kaizoji, Leiss, Saichev, and Sornette (2015), which we present in chapter 2. Starting from well-known set-ups of agent-based models of financial markets, we derive analytically and in simulations conditions for the occurrence of explosive price paths. Despite being the main driver in the bubble regime, technical trading strategies are shown to be transiently profitable, supporting these strategies as enhancing herding behavior. In section 2.A we will extend the model by including a third group of investors, who try to arbitrage the arising super-exponential price patterns based on the LPPLS methodology by Sornette and Johansen (1998); Sornette et al. (2013); Filimonov and Sornette (2013). Philipp (2015) quantifies the impact of LPPLS traders on the market as a whole and on the development of bubbles in particular. The presence of LPPLS investors is found to increase a bubble's peak in proportion to their market power, but not its duration.

Chapter 3 is published in the Journal of Economic Dynamics and Control (Leiss et al., 2015). It is dedicated to empirical observations of investors' return expectations. Following Leiss et al. (2015), we estimate risk-neutral probability distributions from financial option quotes on the S&P 500 stock index over the period 2003 to 2013. We employ the method by Figlewski (2010), which is essentially a model-free technique, allowing for nonstandard density features such as bimodality, fat tails and general asymmetry. It is therefore particularly suited to study the profound impacts of the Global Financial Crisis of 2008. Evaluating the resulting risk-neutral distributions in terms of their moments, tail characteristics and implied returns allows us to endogenously define three different regimes: a pre-crisis, crisis and post-crisis phase. Interestingly, the pre-crisis period is characterized by linearly rising returns as in equation (1.3), which translates into super-exponential growth expectations of the representative investor under risk neutrality. Granger-causality tests show that expected returns lead 3-month Treasury Bill yields prior to the crisis, while the inverse is true in the post-crisis period.

Chapter 4 is a working paper currently under review at Quantitative Economics (Leiss and Nax, 2015). Here, we address the question whether traditional and new risk measures would have captured the market risk posed by large price drawdowns, when evaluated on the risk-neutral densities of chapter 3. Foster and Hart (2009) intriguingly proposed a measure promising sustainable riskiness in the sense that it seeks to avoid bankruptcy at all times. Translating this idea from abstract gambles to applied finance, we determine the Foster-Hart (FH) bound. This measure indicates the maximal sustainable exposure to the S&P 500 and we compare it to value-at-risk (VaR), expected shortfall (ES) and risk-neutral volatility. It turns out, that the FH bound yields additional information compared to traditional risk measures and is a significant predictor of ahead-return downturns. We explain this by showing that the FH bound is able to capture more characteristics of the risk-neutral probability distributions than other measures.

Finally, chapter 5 is built on work in progress. Here, we study the digital communication network of a large hedge fund and relate it to its trading activity. We define random trading as those sequences of buys and sells that statistically cannot be distinguished from a random walk. It turns out that random trading significantly underperforms. Furthermore, we are able to associate meaningful decision making with two characteristics of the communication. Those are clustered and balanced (measured by entropy) internal communication on the one hand, and diverse communication networks in terms of external information sources on the other.

Methodology

Methodologically, this dissertation approaches the topic of market risk of speculative bubbles from two angles. First, we provide a theoretical model which we study with analytical derivations and computational simulations in chapter 2. Second, we analyze data using various econometric methods in chapters 3 and 4.

In chapter 2, guided by economic principles such as market clearing and expected utility, we set up a dynamic equilibrium model of a financial market. The model consists of two types of actors and is fully specified by a set of stochastic equations in closed form. Reducing the system to its deterministic version, we analytically derive the model dynamics for various parameter specifications via a fixed point analysis. Beyond that, however, computational simulations can still yield insights for two reasons. First, complex adaptive systems characterized by nonlinear interactions among *agents* exhibit emergent phenomena such as mutating collective behavior and self-organization (Miller and Page, 2009). Thus, an agent-based simulation as in chapter 2 is ideally suited for the study of how interactions at the micro-level, as for example social imitation between individual investors, may lead to certain outcomes at the macro-level such as speculative bubbles. Second, computational simulations allow a quantification of statistical regularities of the model, so-called *stylized facts*, that may be compared with those of real markets to provide empirical credibility. In the case of a financial market model, stylized facts involve the fat-tailed distribution of returns and volatility clustering (Lux, 2009), which may be tested with the toolkit provided by Clauset et al. (2009).

By contrast, in chapters 3 and 4, we let the data speak and make as few modeling assumptions as possible. This ensures that we do not impose certain results nor are blind to others. It is particularly important for studying nonstationary systems, as for example stock markets that changed throughout the Global Financial Crisis of 2008 (see chapter 3). Risk-neutral probability densities implied by financial options data underly this part. We employ the estimation technique by Figlewski (2010), which combines two very general approaches for the main body and tails of the distributions, respectively. The former is estimated using smoothed numerical derivatives

(Shimko et al., 1993), and the latter are characterized by the generic family of generalized extreme value distributions (Embrechts et al., 1997, 2005). Repeating the estimation multiple times with small noise in the input data informs about the robustness of results, a process very similar to Monte Carlo methods (Hammersley and Handscomb, 1964). In chapter 3, the properties of the resulting risk-neutral probability distributions are evaluated using standard methods of time series analysis. This involves a change point analysis to detect regime shifts throughout the Global Financial Crisis (Page, 1954; Scott and Knott, 1974) and a Granger-causality analysis relating market dynamics with monetary policy (Granger, 1969). In chapter 4, we compute various risk-measures with the estimated densities and analyze their predictive power via standard regressions on future returns.

The empirical study of a large hedge fund in chapter 5 required a whole set of new tools for data and network analysis as well as text mining. The network structures were analyzed for clustering using the clustering coefficient (Watts and Strogatz, 1998) and the pair-wise similarity measure by Adamic and Adar (2003), as well as for balance in terms of Shannon entropy (Shannon, 1948) of conversational turn-taking and (inverted) concentration index by Herfindahl (1950). We followed Lijffijt et al. (2011) to model the distribution of topics in the message full texts based on a glossary of financial terms by Harvey (2015). Finally, the sequences of buying and selling decisions were classified as random if the null hypothesis of variance-ratio tests could not be rejected (Charles and Darné, 2009; Lo and MacKinlay, 2011).

Chapter 2

Super-exponential endogenous bubbles in an equilibrium model of fundamentalist and chartist traders

This chapter is an edited version of Kaizoji et al. (2015), of which I am a joint first author. Section 2.A is an extension of the original paper based on two projects related to master theses that I co-supervised (Philipp, 2015; Kohrt, 2015).

Abstract

We introduce a model of super-exponential financial bubbles with two assets (risky and risk-free), in which fundamentalist and chartist traders co-exist. Fundamentalists form expectations on the return and risk of a risky asset and maximize their constant relative risk aversion expected utility with respect to their allocation on the risky asset versus the risk-free asset. Chartists are subjected to social imitation and follow momentum trading. Allowing for random time-varying herding propensity, we are able to reproduce several well-known stylized facts of financial markets such as a fat-tail distribution of returns and volatility clustering. In particular, we observe transient faster-than-exponential bubble growth with approximate log-periodic behavior and give analytical arguments why this follows from our framework. The model accounts well for the behavior of traders and for the price dynamics that developed during the dotcom bubble in 1995-2000. Momentum strategies are shown to be transiently profitable, supporting these strategies as enhancing herding behavior.

2.1 Introduction

The very existence of financial bubbles has been a controversial and elusive subject. Some have argued that financial bubbles play a huge role in the global economy, affecting hundreds of millions of people (Kindleberger and Aliber, 1978; Shiller, 2000; Sornette, 2003). Others have basically ignored or refuted their possibility (Fama, 1998). Moreover, until recently, the existence of such bubbles, much less their effects, have been ignored at the policy level. Finally, only after the most recent historical global financial crisis, officials at the highest level of government and academic finance have acknowledged the existence and importance of identifying and understanding bubbles. the President of the Federal Reserve Bank of New York, William C. Dudley, stated in April 2010 "what I am proposing is that we try—try to identify bubbles in real time, try to develop tools to address those bubbles, try to use those tools when appropriate to limit the size of those bubbles and, therefore, try to limit the damage when those bubbles burst." Such a statement from the New York Fed representing, essentially, the monetary policy of the United States governmental banking system would have been, and, in some circles, still is, unheard of. This, in short, is a bombshell and a wake-up call to academics and practitioners. Dudley exhorts to try to develop tools to address bubbles.

But before acting against bubbles, before even making progress in ex-ante diagnosing bubbles, one needs to define what is a bubble. The problem is that the "econometric detection of asset price bubbles cannot be achieved with a satisfactory degree of certainty. For each paper that finds evidence of bubbles, there is another one that fits the data equally well without allowing for a bubble. We are still unable to distinguish bubbles from time-varying or regime-switching fundamentals, while many small sample econometrics problems of bubble tests remain unresolved." summarizes Gürkaynak (2008) in his review paper.

Let us start with the rather generally accepted stylized fact that, in a period where a bubble is present, the stock return exhibits transient excess return above the long-term historical average, giving rise to what could be termed a "bubble risk premium puzzle". For instance, as we report in the empirical section, the valuation of the Internet stock index went from a reference value 100 in January 1998 to a peak of 1400.06 in March 9, 2000, corresponding to an annualized return of more than 350% ! A year and a half later, the Internet stock valuation was back at its pre-1998 level. Such explosive super-exponential growth has been documented extensively for bubbles in real markets (see for example Sornette et al. 2009; Jiang et al. 2010; Yan et al. 2012) and recently observed in lab experiments (Hüsler et al., 2013). Another stylized fact well represented during the dotcom bubble is the highly intermittent or punctuated growth of the stock prices, with super-exponential accelerations followed by transient corrections, themselves followed by further vigorous rebounds (Johansen and Sornette, 2010; Sornette and Woodard, 2010)

Bubbles are usually followed by crashes, in an often tautological logic resulting from the fact that the existence of a crash is usually taken as the ex-post signature of a bubble, as summarized by Greenspan (2002): "We, at the Federal Reserve... recognized that, despite our suspicions, it was very difficult to definitively identify a bubble until after the fact, that is, when its bursting confirmed its existence..." More optimistically but still controversial, recent systematic econometric studies have shown that it is possible to relate objectively an anomalous transient excess return and the subsequent crash (Sornette, 2003; Johansen and Sornette, 2010; Sornette et al., 2013). Furthermore, there is another relatively new stream of literature devoted to the early detection of bubbles, which also focuses on the often observed extreme growth of the price mentioned above. Phillips et al. (2011) have employed mildly explosive autoregressive processes of the log-price with an AR coefficient slightly larger than one decreasing towards one over time. This model results in super-exponential growth of the price and has led to bubble tests based on Markov-switching state-space models (Al-Anaswah and Wilfling, 2011; Lammerding et al., 2013), as well as sequential Chow-type and augmented Dickey-Fuller testing procedures for a structural breaks. Such a break could be either the start of a bubble, i.e. a transition from a random walk to a mildly explosive regime (Phillips et al., 2011; Homm and Breitung, 2012; Phillips et al., 2012) or vice versa its end (Breitung and Kruse, 2013). Both methods rely on the type of indirect stationarity tests initiated by Diba and Grossman (1988) and Hamilton and Whiteman (1985).

Going from econometrics to financial economics, there are several branches dedicated to modeling deviations from fundamental value. One important class of theories is related to *noise traders* (also referred to as positive-feedback investors), a term first introduced by Kyle (1985) and Black (1986) to describe irrational investors. Thereafter, many scholars exploited this concept to extend the standard models by introducing the simplest possible heterogeneity in terms of two interacting populations of rational and noise traders. One can say that the one-representative-agent theory is being progressively replaced by a two-representative-agents theory, analogously to the progress from the one-body to the two-body problems in physics. It has been often explained that markets bubble and crash in the absence of significant shifts in economic fundamentals when herders such as chartists deliberately act against their private information and follow the crowd.

De Long et al. (1990a,b) proposed the first model of market bubbles and crashes which exploits this idea of the possible role of noise traders following positive feedback or momentum investment strategies in the development of bubbles. They showed a possible mechanism for why asset prices may deviate from the fundamentals over long time periods. The key point is that trading between rational arbitrageurs and chartists gives rise to bubble-like price patterns. In their model, rational speculators destabilize prices because their trading triggers positive feedback trading by noise traders. This in turn leads to a positive auto-correlation of returns at short horizons. Eventually, arbitrage by rational speculators will pull the prices back to fundamentals. Their arbitrage trading leads to a negative autocorrelation of returns at longer horizons.

Their work was followed by a number of empirical studies on positive feedback trading. Influential empirical evidence on positive feedback trading came from the works of De Bondt and Thaler (1985), and Jegadeesh and Titman (1993, 2001), which established that stock returns exhibit momentum behavior at intermediate horizons, and reversals at long horizons. That is, strategies which buy stocks that have performed well in the past and sell stocks that have performed poorly in the past generate significant positive returns over 3- to 12- month holding periods. However, stocks that perform poorly in the past perform better over the next 3 to 5 years than stocks that perform well in the past. Behavioral models that explain the coexistence of intermediate horizon momentum and long horizon reversals in stock returns are proposed by Barberis et al. (1998), Daniel et al. (1998), and Hong and Stein (1997).

The behavior of investors who are driven by group psychology and the aggregate behavioral outcomes, have also been studied using frameworks suggested by Weidlich and Haag (1983); Blume (1993, 1995); Brock (1993); Arthur et al. (1997); Durlauf (1999); Kirman (1993); Brock and N Durlauf (2000); Aoki and Yoshikawa (2007); Chiarella et al. (2009); Hommes and Wagener (2009). Phani et al. (2004) summarize the formalism starting with different implementation of the agents' decision processes whose aggregation is inspired from statistical mechanics to account for social influence in individual decisions. Lux (1995); Lux and Marchesi (1999); Brock and Hommes (1998); Kaizoji (2000, 2010); Kirman and Teyssiere (2002) have developed related models in which agents' successful forecasts reinforce the forecasts. Such models have been found to generate swings in opinions, regime changes and long memory. An essential feature of these models is that agents are wrong for a fraction of the time but, whenever they are in the majority, they are essentially right by a kind of self-fulfilling prophecy. Thus, they are not systematically irrational (Kirman, 1997). Sornette and Zhou (2006) showed how irrational Bayesian learning added to the Ising model framework reproduces the stylized facts of financial markets. Harras and Sornette (2011) showed how over-learning from lucky runs of random news in the presence of social imitation may lead to endogenous bubbles and crashes.

Here, we follow this modeling path and develop a model of the pricing mechanism and resulting dynamics of two co-existing classes of assets, a risky asset representing for instance the Internet sector during the dotcom bubble and a risk-free asset, in the presence of two types of investors having different opinions concerning the risky asset (Harrison and Kreps, 1978; Scheinkman and Xiong, 2003). The first type of traders is a group of fundamentalists who maximize their expected utility. The second type of traders is a group of "chartists" who trade only the risky asset by using heuristics such as past momentum and social imitation. The chartist traders do not consider the fundamentals, while the fundamentalist investors allocate their wealth based on their expectation of the future returns and risks of the risky asset.

Our framework combines elements from various groundbreaking works. The setup of chartists follows closely Lux and Marchesi (1999), where an opinion index determined by past momentum and social imitation describes the prevailing investment behavior among this group. The description of fundamentalists is related to Brock and Hommes (1998) and to Chiarella et al. (2009). In particular, we employ a utility function with constant relative risk aversion, as this is a realistic choice in a growing economy.

One important ingredient that we introduce here is that we do not allow our agents to switch their investment behavior from rational to noise trading or vice versa. This reflects the empirical fact that many large institutional investors such as pension funds have to follow strict guidelines on how to split their portfolio on assets of different risk classes. In previous models, the occurrence of a bubble was related to a convergence of a large fraction of traders on noise trading, see for example Lux and Marchesi (1999). Instead of strategy switching, we account for the volatility of the imitation propensity of chartists by assuming that it fluctuates randomly around some anchoring value as in (Stauffer and Sornette, 1999; Harras et al., 2012). By keeping track of the agents' wealth levels, we are able to explain bubbles only with the transient increasing influence of chartists on the market price during an appreciation of the risky asset. While its price is rising, noise traders believing in momentum tend to invest more in the risky asset and thus become richer, thereby gaining more importance. The chartists' belief is further reinforced by social imitation, which becomes self-fulfilling. This, in turn, has destabilizing effects leading to an increase in the volatility and usually finishes in a crash when the prevailing opinion switches to pessimistic.

Within our simple setup without strategy switching we show theoretically and by simulations that bubbles start with a phase of transient super-exponential growth. As mentioned before, faster-than-exponential growth behavior has recently been picked up by the econometric literature, but to our knowledge it has been rarely discussed in the context of agent-based models. A first instantiation is found in (Corcos et al., 2002), in a much simplified model of imitative and contrarian agents.

The present model is one of the first in which we can provide a transparent analytical explanation for the existence of a transient faster-than-exponential growth. Moreover, we observe approximate log-periodic behavior during the rise of a bubble, that can result from the nature of the fluctuations of the opinion index. Furthermore, our model reproduces several stylized facts of financial markets. The distribution of returns is fat-tailed. Also, signed returns are characterized by a fast-decaying autocorrelation, while the autocorrelation function for absolute returns has a long memory (volatility clustering). While many of the ingredients and conditions used in our agent-based model may be found in various forms in some previous agent-based models, none have documented explicitly the important transient super-exponential behavior associated with bubbles, nor explained qualitatively or quantitatively the underlying mechanisms and the coexisting salient stylized facts.

The paper is organized as follows: the basic model is presented in Section 2 and Section 3 and analyzed theoretically in Section 4. Numerical simulations of the model are performed and the results are discussed in Section 5, together with a discussion of the price dynamics, its returns and momentum strategies during the dotcom bubble from 1998 to 2000. We conclude in Section 6.

2.2 Set-up of the model of an economy made of fundamentalists and chartists

We consider fixed numbers N_f of fundamentalist and N_c of chartist investors who trade the same risky asset, represented here for simplicity by a single representative risky asset fund. The former diversify between the risky asset and a risk-free asset on the basis of maximizing their constant relative risk aversion expected utility of returns and variance of the risky asset over the next period. The latter use technical and social indicators, such as price momentum and social imitation to allocate their wealth. A dynamically evolving fraction of them buys the risky asset while others stay out of the risky asset and have their wealth invested in the risk-free asset.

In the next subsection 2.2.1, we solve the standard allocation problem for the fundamentalists that determines their demand for the risky asset. Then, in subsection 2.2.2, the general ingredients controlling the dynamics of the demand of chartists are developed.

2.2.1 Allocation equation for the fundamentalists

The objective of the N_{f} fundamentalists is assumed to be the maximization at each time t of the expected utility of their expected wealth W^f_{t+1} at the next period, thus following Chiarella et al. (2009) and Hommes and Wagener (2009). To perform this optimization, they select at each time t a portfolio mix of the risky asset and of the risk-free asset that they hold over the period from t to $t+1$. Such one-period ahead optimization strategy can be reconciled with underlying expected utility maximizing stories as given for example in Brock and Hommes (1997, 1998); Chiarella et al. (2009); Boswijk et al. (2007); Hommes and Wagener (2009).

The fundamentalists are assumed to be identical, so that we can consider the behavior of one representative fundamental trader hereafter. We shall assume that fundamentalists are myopic mean-variance maximizers, which means that only the expected portfolio value and its variance impact their allocation. We denote P_t the price of the risky asset and x^f_t the number of risky assets that the representative fundamentalist holds at instant t. We also assume that the risky asset pays a dividend d_t at each period t. Similarly, P_{ft} and X_{ft} correspond to the price and number of a risk-free asset held by the fundamentalist. The risk-free asset is in perfectly elastic supply and pays a constant return R_f. Thus, at time t, the wealth of the fundamentalist is given by

$$W^f_t = P_t x^f_t + P_{ft} X_{ft} \ . \tag{2.1}$$

The wealth of the fundamentalist changes from time t to $t+1$ according to

$$W^f_{t+1} - W^f_t = (P_{t+1} - P_t) x^f_t + (P_{ft+1} - P_{ft}) X_{ft} + d_{t+1} x^f_t \ . \tag{2.2}$$

This expression takes into account that the wealth at time $t+1$ is determined by the allocation choice at time t and the new values of the risky and the risk-free asset at time $t+1$, which includes the payment of the dividend ($W^f_{t+1} = P_{t+1} x^f_t + P_{ft+1} X_{ft} +$

$d_{t+1}x_t^f$). Let us introduce the variables

$$x_t^f := \frac{P_t x_t^f}{W_t^f} \ , \qquad R_{t+1} := \frac{P_{t+1}}{P_t} - 1 \ , \qquad R_f := \frac{P_{ft+1}}{P_{ft}} - 1 \ . \tag{2.3}$$

They are respectively the fraction x_t^f of the fundamentalist's wealth invested in the risky asset at time t, the discrete time return R_{t+1} per stock of the risky asset from time t to $t+1$ and the risk-free rate of return R_f assumed constant. This allows us to rewrite (2.2) as giving the total relative wealth variation from t to $t+1$:

$$W_{t+1}^f - W_t^f = W_t^f \left[R_f + x_t^f \left(R_{t+1} - R_f + \frac{d_{t+1}}{P_t} \right) \right] \equiv W_t^f \left[R_f + x_t^f R_{\text{excess},t+1} \right] \ , \tag{2.4}$$

where we define

$$R_{\text{excess},t+1} = R_{t+1} - R_f + d_{t+1}/P_t \tag{2.5}$$

as the excess return of capital and dividend gains over the risk-free rate.

The problem of the fundamentalist at time t is to maximize the expected utility of his wealth for the next period by choosing the right proportion of wealth x_t^f to invest in the risky asset,

$$\max_{x_t^f} \ \mathrm{E}_t \left[U(W_{t+1}^f) \right] \ , \tag{2.6}$$

where $\mathrm{E}_t[.]$ means the expectation of the variable in the bracket performed at time t, i.e., under the knowledge of available information up to and including time t. If we assume the fundamentalist to have constant relative risk aversion, this proportion is constant in time and wealth. This can be shown by employing the explicit utility function $U(W)$ exhibiting constant relative risk aversion γ:

$$U(W) = \begin{cases} \log(W), & \text{for } \gamma = 1 \ , \\ \frac{W^{1-\gamma}}{1-\gamma}, & \text{for } \gamma \neq 1 \ . \end{cases} \tag{2.7}$$

Given this utility function and wealth evolution (2.4), it is easy to see that the maximization condition (2.6) is independent of W_t^f.

We may obtain an approximate solution for x_t^f in the special case where the wealth does not change much, i.e. in the case of small returns, so that the following expansion becomes approximately valid: $R_f, R_{\text{excess},t+1} \ll 1$.

$$\begin{aligned} \mathrm{E}_t[U(W_{t+1}^f)] &= U(W_t^f) + U'(W_t^f) W_t^f (R_f + x_t^f \mathrm{E}_t[R_{\text{excess},t+1}]) \\ &\quad + \frac{1}{2} U''(W_t^f) W_t^2 (x_t^f)^2 \mathrm{Var}_t[R_{\text{excess},t+1}] + \mathcal{O}(R_f^3, R_{\text{excess},t+1}^3) \ . \end{aligned} \tag{2.8}$$

Maximizing this expression with respect to x_t^f gives

$$x_t^f = \frac{1}{\gamma} \frac{\mathrm{E}_t[R_{\mathrm{excess},t+1}]}{\mathrm{Var}_t[R_{\mathrm{excess},t+1}]} \ , \tag{2.9}$$

where

$$\gamma \equiv -\frac{W_t^f U''(W_t^f)}{U'(W_t^f)} \ . \tag{2.10}$$

In expression (2.9), $\mathrm{E}_t[R_{\mathrm{excess},t+1}] \equiv \mathrm{E}_t[R_{t+1}] - R_f + \mathrm{E}_t[d_{t+1}]/P_t$ represents the total excess expected rate of return of the risky asset from time t to $t+1$ above the risk-free rate. In the following, we assume myopic fundamentalists who do not learn but invest according to fundamental valuation. They expect a steady relative growth rate embodied by a constant total excess rate of return R_{excess}, which is based on the behavior of stock markets in the long run:

$$R_{\mathrm{excess}} := E_t[R_{t+1}] - R_f + \frac{\mathrm{E}_t[d_{t+1}]}{P_t} = \ \text{constant} \ . \tag{2.11}$$

We will assume that $R_{\mathrm{excess}} > 0$, so that the risky asset is desirable. The variance $\mathrm{Var}_t[R_{\mathrm{excess},t+1}]$ will be denoted by $\tilde{\sigma}^2$ and is given by

$$\tilde{\sigma}^2 := \mathrm{Var}_t[R_{\mathrm{excess},t+1}] = \sigma^2 + \frac{\mathrm{Var}[d_{t+1}]}{P_t^2} \ , \qquad \sigma^2 := \mathrm{Var}[R_{t+1}] \ . \tag{2.12}$$

The expression for $\mathrm{Var}_t[R_{\mathrm{excess},t+1}]$ relies on the absence of correlation between R_{t+1} and d_{t+1}, because the dividend policy is assumed independent of the market price and vice-versa. Modigliani and Miller (1958, 1963) show that this holds true in the case of symmetric information and bounded rationality. Our fundamentalists believe to act in this world and take the quantities R_{t+1} and d_{t+1} as exogenous to the price dynamics developed below, because they reflect the information coming from a fundamental analysis.

In the sequel, we assume that $\tilde{\sigma}^2$ is independent of the price P_t and that $P_t \gg \sqrt{\mathrm{Var}[d_{t+1}]/\sigma^2}$. Thus, $\tilde{\sigma}^2 \simeq \sigma^2$ and $\tilde{\sigma}^2$ is approximately constant, as long as the fundamentalist investors form a non-varying expectation of the volatility of future prices of the risky asset. The assumption that $\tilde{\sigma}^2$ is constant is also made by Chiarella et al. (2009) and in the framework of Boswijk et al. (2007), if investors are assumed to be myopic, i.e. only look at the next period.

Expression (2.9) then becomes

$$x_t = x := \frac{R_{\text{excess}}}{\gamma \tilde{\sigma}^2} \ , \tag{2.13}$$

which is a constant. Note that this is not an ad hoc assumption, but a consequence of constant relative risk aversion and of the stationary nature of the dividend process. In particular, because of the constant relative risk aversion of the fundamentalists, as already mentioned, x is independent of the current wealth W_t^f of the agents. This allows us to treat all fundamentalists as one group with total wealth W_t^f irrespective of the distribution of the agents' individual wealth levels within the group. From here on, we will call W_t^f the wealth of *the fundamentalists*.

The assumption, that the variance $\tilde{\sigma}^2$ given by (2.12) is constant, implies

$$\text{Var}[d_t] = (\tilde{\sigma}^2 - \sigma^2) P_{t-1}^2. \tag{2.14}$$

Therefore, the flow of dividend d_t follows the stochastic process

$$d_t = P_{t-1} \left[r + \sigma_r u_t \right] \ , \tag{2.15}$$

where $r := R_{\text{excess}} - \text{E}_{t-1}[R_t] + R_f$, $\sigma_r = \sqrt{\tilde{\sigma}^2 - \sigma^2}$ and u_t forms a series of standard i.i.d. random variables with distribution $N(0,1)$.

Thus, under the above assumptions, the fundamentalist investors rebalance their portfolio so as to have a constant relative weight exposure to the risky asset. This is equivalent to the traditional portfolio allocation benchmark of 70% bonds and 30% stocks used by many mutual and pension funds. Rewriting expression (2.2) with the condition of a fixed fraction x invested in the risky asset, the wealth W_t^f at time t of the fundamentalists becomes at $t+1$

$$W_{t+1}^f = (P_{t+1} + d_{t+1}) x \frac{W_t^f}{P_t} + (1-x) W_t^f (1 + R_f) \ . \tag{2.16}$$

The excess demand of the risky asset from $t-1$ to t of the group of fundamentalists is defined by

$$\Delta D_t^{\text{f}} := P_t x_t^f - P_t x_{t-1}^f = P_t x_t^f - \frac{P_t}{P_{t-1}} P_{t-1} x_{t-1}^f = x W_t^f \left(1 - \frac{P_t}{P_{t-1}} \frac{W_{t-1}^f}{W_t^f} \right) \ . \tag{2.17}$$

Expression (2.2) with definitions (2.3) gives

$$\frac{P_t}{P_{t-1}} \frac{W_{t-1}^f}{W_t^f} = \frac{P_t}{(P_t + d_t) x + P_{t-1} (1-x)(1+R_f)} \ . \tag{2.18}$$

This allows us to rewrite the excess demand ΔD_t^{f} as

$$\Delta D_t^{\mathrm{f}} = xW_{t-1}^f \left[(1-x)\frac{P_{t-1}(1+R_f)-P_t}{P_{t-1}} + \frac{xd_t}{P_{t-1}}\right] , \qquad (2.19)$$

where x is given by expression (2.13). This last expression can be written, using (2.15), as

$$\Delta D_t^{\mathrm{f}} = xW_{t-1}^f \left[(1-x)\frac{P_{t-1}(1+R_f)-P_t}{P_{t-1}} + x(r+\sigma_r u_t)\right] . \qquad (2.20)$$

This corresponds to a kind of mean-reversing excess demand, where fundamentalists tend to buy the risky asset when its price is low and vice-versa. But this mean-reversing excess demand is adjusted by taking into account two factors that quantify an abnormal price increase (resp. decrease), which would justify unloading (resp. adding) the risky asset to the fundamentalists' portfolio. First, a price change is compared with the change that would occur if the corresponding wealth was instead invested in the risk-free asset. Second, even if its price decreases, the risky asset may still be attractive if it pays a sufficient dividend to compensate.

In absence of chartists, the market clearing condition $\Delta D_t^{\mathrm{f}} = 0$ leads to

$$P_t = (1+R_f)P_{t-1} + \frac{x}{1-x}d_t . \qquad (2.21)$$

In the simplified case where the dividends d_t are growing at a constant rate $g > 0$ such that $d_t = d_0(1+g)^t$, equation (2.21) solves into

$$P_t = (1+R_f)^t P_0 + \frac{x}{1-x}(1+R_f)^t \frac{d}{R_f - g} , \qquad (2.22)$$

for $g < R_f$, neglecting a term $[(1+g)/(1+R_f)]^t$ compared to 1. One recognizes the Gordon-Shapiro fundamental valuation, price = dividend/$(R_f - g)$, multiplied by a scaling factor taking into account the partitioning of the wealth of the fundamentalists with the condition that a constant fraction is invested in the risky asset.

2.2.2 Excess demand of the chartists

General framework

We assume that (a) the chartists are characterized by polarized decisions (in or out of the risky asset), (b) they tend to herd and (c) they are trend-followers.

A large body of literature indeed documents a lack-of-diversification puzzle (Kelly, 1995; Baxter and Jermann, 1995; Statman, 2004) as well as over-reactions De Bondt and Thaler (1985, 1987, 1990). There is strong evidence for imitation and herding, even among sophisticated mutual fund managers (Wermers, 1999), and technical analysis and chart trading is ubiquitous.

We account for the observations of lack-of-diversification by assuming that a chartist trader is fully invested either in the risky asset or in the risk-free asset. In contrast to the fundamentalist agents, our chartists have different opinions, which fluctuate stochastically according to laws given below. Due to the probabilistic setup the assumption of an "all or nothing" behavior at the individual level translates into a continuous investment weight of chartists at the group level and is given by the fraction of chartists invested in the risky asset varying smoothly between 0 and 1. The number of chartist investors invested in the risky asset (respectively invested in the risk-free asset) is N_t^+ (respectively N_t^-), and we have

$$N_t^+ + N_t^- \equiv N_\mathrm{c} \ . \tag{2.23}$$

We do not aim at describing the heterogeneity between chartists, which has been shown to lead to fat-tailed distribution of their wealth as a result of heterogenous investment decisions (Bouchaud and Mézard, 2000; Klass et al., 2007; Harras and Sornette, 2011). This is not a restriction in so far as we consider their aggregate impact.

Therefore, as for the fundamentalists, we treat the chartists as one group with total wealth W_t^c. The ratio of wealth of the group of chartists invested in the risky asset corresponds to the ratio of bullish investors among the population of chartists. Let us denote this quantity at time t by

$$x_t^c := \frac{N_t^+}{N_\mathrm{c}} \ . \tag{2.24}$$

Then, the wealth W_t^c of chartists at time t becomes at $t+1$

$$W_{t+1}^c = (P_{t+1} + d_{t+1}) x_t^c \frac{W_t^c}{P_t} + (1 - x_t^c) W_t^c (1 + R_f) \ . \tag{2.25}$$

The excess demand of the chartists over the time interval $(t-1, t)$ is equal to

$$\Delta D_t^c = x_t^c W_t^c - \frac{P_t}{P_{t-1}} x_{t-1}^c W_{t-1}^c = \tag{2.26}$$

$$W^c_{t-1}\left[x^c_t(1-x^c_{t-1})(1+R_f)-x^c_{t-1}(1-x^c_t)\frac{P_t}{P_{t-1}}+x^c_tx^c_{t-1}\frac{d_t}{P_{t-1}}\right]. \tag{2.27}$$

Let us introduce the opinion index (Lux and Marchesi, 1999)

$$s_t := \frac{N^+_t - N^-_t}{N_\mathrm{c}} \in [-1,1] \ , \tag{2.28}$$

which can be interpreted as the aggregate bullish ($s_t > 0$) versus bearish ($s_t < 0$) stance of the chartists with respect to the risky asset. With this definition (2.28) and with (2.23), we have

$$\frac{N^+_t}{N_\mathrm{c}} = \frac{1}{2}(1+s_t) = x^c_t, \quad \frac{N^-_t}{N_\mathrm{c}} = \frac{1}{2}(1-s_t) = 1-x^c_t \ . \tag{2.29}$$

Expression (2.27) with (2.29) yields

$$\begin{aligned}\Delta D^\mathrm{c}_t = \frac{W^c_{t-1}}{4P_{t-1}}&\left[(1+s_t)(1-s_{t-1})(1+R_f)P_{t-1}-\right.\\ &\left.(1-s_t)(1+s_{t-1})P_t+(1+s_t)(1+s_{t-1})d_t\right] \ .\end{aligned} \tag{2.30}$$

Master equation for the bullish/bearish chartist trader unbalance s_t

Let us now specify the dynamics of the opinion index s_t. We assume that, at each time step, each chartist trader may change her mind and either sell her risky portfolio if she was previously invested or buy the risky portfolio if she had only the risk-free asset. Again, we assume an all-or-nothing strategy for each chartist trader at each time step. Let p^+_{t-1} be the probability that any of the N^+_{t-1} chartists who is currently fully invested in the risky portfolio decides to remove her exposure during the time interval $(t-1,t)$. Analogously, let p^-_{t-1} be the probability that any of the N^-_{t-1} traders who are currently (at time $t-1$) out of the risky market decides to buy it. For a chartist k who owns the risky asset, her specific decision is represented by the random variable $\zeta_k(p^+)$, which takes the value 1 (sell) with probability p^+ and the value 0 (keep the position) with probability $1-p^+$. Similarly, for a chartist j who does not own the risky asset, her specific decision is represented by the random variable $\xi_j(p^-)$, which takes the value 1 (buy) with probability p^- and the value 0 (remain invested in the risk-free asset) with probability $1-p^-$. For given p^+ and p^-, the variables $\{\xi_j(p^+)\}$ and $\{\zeta_k(p^-)\}$ are i.i.d..

Aggregating these decisions over all chartists invested in the risky asset at time t, we have

$$N_t^+ = \sum_{k=1}^{N_{t-1}^+} [1 - \zeta_k(p_{t-1}^+)] + \sum_{j=1}^{N_{t-1}^-} \xi_j(p_{t-1}^-) \ . \tag{2.31}$$

The first term in the r.h.s. of (2.31) corresponds to all the traders who held the risky asset at $t-1$ and continue to hold it at t. The second term in the r.h.s. of (2.31) represents the chartists who were holding the risk-free asset at $t-1$ and sold it to buy the risky asset at time t. Similarly,

$$N_t^- = \sum_{k=1}^{N_{t-1}^+} \zeta_k(p_{t-1}^+) + \sum_{j=1}^{N_{t-1}^-} [1 - \xi_j(p_{t-1}^-)] \ . \tag{2.32}$$

The opinion index s_t (2.28) is thus given by

$$s_t = \frac{1}{N_{\rm c}} \left(\sum_{k=1}^{N_{t-1}^+} [1 - 2\zeta_k(p_{t-1}^+)] + \sum_{j=1}^{N_{t-1}^-} [2\xi_j(p_{t-1}^-) - 1] \right) \ . \tag{2.33}$$

Using the i.i.d. property of the $\{\xi_j(p)\}$ and $\{\zeta_k(p)\}$ variables allows us to obtain the following exact expression for the mean of s_t:

$$\mathrm{E}\left[s_t\right] = s_{t-1} + p_{t-1}^-(1 - s_{t-1}) - p_{t-1}^+(1 + s_{t-1}) \ . \tag{2.34}$$

Influence of herding and momentum on the behavior of chartists

As can be seen from (2.30) together with (2.33), the probabilities $p^\pm$ embody completely the behavior of the chartists. We assume that $p^\pm$ at time $t-1$ are both a function of s_{t-1} (social imitation effect) defined by (2.28) and of a measure H_t of the price momentum given by

$$H_t = \theta H_{t-1} + (1-\theta)\left(\frac{P_t}{P_{t-1}} - 1\right) \ , \tag{2.35}$$

which is nothing but the expression for an exponential moving average of the history of past returns. The parameter $0 \le \theta < 1$ controls the length of the memory that chartists keep of past returns, the closer to 1, the longer the memory $\sim 1/(1-\theta)$.

Considering that the probabilities $p^\pm$ are functions of s_{t-1} and H_{t-1},

$$p_{t-1}^\pm = p^\pm(s_{t-1}, H_{t-1}) \ , \tag{2.36}$$

means that the chartists make their decisions to buy or sell the risky Internet stock based on (i) the majority view held by their group and (ii) the recent capital gains that the risky asset has provided over a time frame $\sim 1/(1-\theta)$. We assume the chartists buy and sell symmetrically with no bias: a strong herding in favor of the risky asset or a strong positive momentum has the same relative effect on the drive to buy (or to sell) than a strong negative sentiment or strong negative momentum on the push to sell (or to buy). This is expressed by the following symmetry relation

$$p^-(s,H) = p^+(-s,-H) \ . \tag{2.37}$$

The simplest functions satisfying (2.37) are the linear expressions[1]

$$p^-(s,H) = \frac{1}{2}\left[p + \kappa \cdot (s+H)\right] \ , \quad p^+(s,H) = \frac{1}{2}\left[p - \kappa \cdot (s+H)\right] \ . \tag{2.38}$$

This defines two parameters p and κ, chosen sufficiently small such that $p^-(s,H)$ and $p^+(s,H)$ remain between 0 and 1. The positive parameter p controls the average holding time of the positions in the absence of any other influence. In other words, a position will last typically $\sim 2/p$ time steps in the absence of social imitation and momentum influence. The parameter κ quantifies the strength of social imitation and of momentum trading. Instead of κ, one could use two parameters for the opinion index and momentum, respectively. For the sake of parsimony we will only work with one parameter treating s and H symmetrically. For instance, for $\kappa > 0$, if there is already a majority of agents holding the risky asset and/or if its price has been increasing recently, then the probability for chartists holding the risk-free asset to shift to the risky asset is increased and the probability for the chartists who are already invested to sell their risky asset is decreased. The reverse holds for $\kappa < 0$, which describes "contrarian" traders. In the sequel, we will only consider the case $\kappa > 0$, which describes imitative and trend-following agents. Generalizations to allow for additional heterogeneous beliefs, involving mixtures as well as adaptive

[1] Another possibility for the transition probabilities which we have explored but do not elaborate on in this paper is the hyperbolic tangent: $p^{\pm}(s,H) = \frac{1}{2}\left[1 \mp \kappa/p \tanh(s+H)\right]$. This corresponds to the Glauber transition rates of an ensemble of spins on a fully connected graph with equal interaction strengths, see for example Harras et al. (2012). However, already the linear probabilities (2.38) translate into a very nonlinear S-like behavior at the aggregate level, which is quantitatively similar to the nonlinear case.

imitative and contrarian agents, is left for other communications. In this spirit, let us mention that Corcos et al. (2002) have introduced a simple model of imitative agents who turn contrarian when the proportion of herding agents is too large, which generates chaotic price dynamics.

Putting expressions (2.38) in (2.34) yields

$$\mathrm{E}\left[s_t\right] = (1+\kappa-p)s_{t-1} + \kappa H_{t-1}\ . \tag{2.39}$$

2.3 Dynamical market equations

2.3.1 Market clearing condition and price dynamics

The equation for the risky asset price dynamics is obtained from the condition that, in the absence of external supply, the total excess demand summed over the fundamentalist and chartist traders vanishes:

$$\Delta D_t^{\mathrm{f}} + \Delta D_t^{\mathrm{c}} = 0\ . \tag{2.40}$$

In other words, the net buy orders of chartists are satisfied by the net sell orders of the fundamentalists, and vice-versa. Substituting in (2.40) expression (2.20) for the excess demands ΔD_t^{f} of the fundamentalists and equation (2.30) for the excess demand ΔD_t^{c} of the chartists, we obtain the price equation

$$\begin{aligned}\frac{P_t}{P_{t-1}} = & \Big[(1+s_t)\left((1+R_f)(1-s_{t-1}) + (r+\sigma_r u_t)(1+s_{t-1})\right) W_{t-1}^c \\ & + 4x\left((1+R_f)(1-x) + (r+\sigma_r u_t)x\right) W_{t-1}^f\Big] / \\ & \Big[(1+s_{t-1})(1-s_t)W_{t-1}^c + 4W_{t-1}^f x(1-x)\Big]\ .\end{aligned} \tag{2.41}$$

Expression (2.41) shows that the price of the risk asset changes as a result of two stochastic driving forces: (i) the dividend-price ratio $(r+\sigma_r u_t)$ and (ii) the time increments of the bullish/bearish chartist unbalance $\{s_t\}$. The impact of $\{s_t\}$ is controlled by the wealth of the group of chartists W_{t-1}^c. As we shall demonstrate below, this becomes particularly important during a bubble where trend-following chartists tend to gain much more than fundamentalists. With the increasing influence of chartists, the market becomes much more prone to self-fulfilling prophecies.

Fundamentalist traders are less able to attenuate the irrational exuberance – they simply do not have enough wealth invested in the game.

2.3.2 Complete set of dynamical equations

Let us put all ingredients of our model together to state concisely all the equations controlling the price dynamics coupled with the opinion forming process of the chartists. As discussed above, the wealth levels of the fundamentalist and chartist traders are also time-dependent and influence the market dynamics. We thus arrive at the following equations.

Dynamics of the chartists opinion index:

$$s_t = \frac{1}{N_{\mathrm{c}}}\left(\sum_{k=1}^{N_{\mathrm{c}}(1+s_{t-1})/2}\left[1-2\zeta_k(p^+_{t-1})\right]+\sum_{j=1}^{N_{\mathrm{c}}(1-s_{t-1})/2}\left[2\xi_j(p^-_{t-1})-1\right]\right)\ , \tag{2.42}$$

where $\zeta_k(p^+_{t-1})$ takes the value 1 with probability p^+_{t-1} and the value 0 with probability $1-p^+_{t-1}$, $\xi_j(p^-_{t-1})$ takes the value 1 with probability p^-_{t-1} and the value 0 with probability $1-p^-_{t-1}$, and p^+_{t-1} and p^-_{t-1} are given by expressions (2.38):

$$\begin{aligned} p^-_{t-1}(s_{t-1},H_{t-1}) &= \frac{1}{2}\left[p+\kappa\cdot(s_{t-1}+H_{t-1})\right]\ , \\ p^+_{t-1}(s_{t-1},H_{t-1}) &= \frac{1}{2}\left[p-\kappa\cdot(s_{t-1}+H_{t-1})\right]\ . \end{aligned} \tag{2.43}$$

Thus, $\mathrm{E}\left[s_t\right]$ given by expression (2.39).

Dynamics of the risky asset price:

$$\begin{aligned} P_t/P_{t-1} \ = \ & \Big[(1+s_t)\left((1+R_f)(1-s_{t-1})+(r+\sigma_r u_t)(1+s_{t-1})\right)W^c_{t-1}+ \\ & +4x\left((1+R_f)(1-x)+(r+\sigma_r u_t)x\right)W^f_{t-1}\Big]\ / \\ & \Big[(1+s_{t-1})(1-s_t)W^c_{t-1}+4x(1-x)W^f_{t-1}\Big]\ . \end{aligned} \tag{2.44}$$

Wealth dynamics of fundamentalists:

$$W^f_t/W^f_{t-1} = x\left(\frac{P_t}{P_{t-1}}+(r+\sigma_r u_t)\right)+(1-x)(1+R_f)\ . \tag{2.45}$$

Wealth dynamics of chartists:

$$W^c_t/W^c_{t-1} = \frac{1+s_{t-1}}{2}\left(\frac{P_t}{P_{t-1}}+(r+\sigma_r u_t)\right)+\frac{1-s_{t-1}}{2}(1+R_f)\ . \tag{2.46}$$

Momentum of the risky asset price:

$$H_t = \theta H_{t-1} + (1-\theta)\left(\frac{P_t}{P_{t-1}} - 1\right) \ . \tag{2.47}$$

And u_t forms a series of standard i.i.d. random variables with distribution $N(0,1)$.

The set of equations (2.42) to (2.47) together with the realization of the stochastic dividend process u_t completely specify the model and its dynamics. Equation (2.42) describes how chartists form their opinion s_t based on the previous prevalent opinion s_{t-1} and the recent price trend H_t. Fundamentalist traders stick to their choice of investing x in the risky asset. Equation (2.44) gives the new market price P_t when excess demands of both groups are matched. Equations (2.45) and (2.46) describe the evolution of the wealth levels W_t^f and W_t^c for fundamentalist and chartist traders, respectively. There are capital gains and dividend gains from the risky asset, and interest payments by the risk-free asset. The new market price also feeds into the momentum of the risky asset described by equation (2.47).

We have the following flow of causal influences:

1. The recent price trend H_{t-1} and the prevailing opinion s_{t-1} among chartists determine the investment decision of chartists governed by s_t, while fundamentalists invest a constant fraction x of their wealth.

2. Market clearing determines the price P_t based on investment decisions x and s_t, and previous wealth levels W_{t-1}^f and W_{t-1}^c for fundamentalist and chartist traders, respectively.

3. The new wealth levels W_t^f and W_t^c are based on the market price P_t and investment decisions x and s_t.

2.3.3 Control parameters and their time-scale dependence

The set of equations (2.42) to (2.47) depends on the following parameters:

1. x quantifies the constant fraction of wealth that fundamentalists invest in the risky asset.

2. θ fixes the time scale over which chartists estimate price momentum. By construction, $0 \leq \theta < 1$.

3. N_{c} is the number of chartists that controls the fluctuations of the majority opinion of chartists.

4. p controls the average holding time of the positions of chartists in the absence of any other influence.

5. κ quantifies the strength of social imitation and of momentum trading by chartists.

6. R_f is the rate of return of the risk-free asset.

7. r and σ_r are the mean and standard deviation of the dividend-price ratio.

In order to have an intuitive understanding of the role and size of these parameters, it is useful to discuss how they depend on the time scale over which traders reassess their positions. Until now, we have expressed the time t in units of a unit step 1, which could be taken for instance to be associated with the circadian rhythm, i.e., one day. But there is no fundamental reason for this choice and our theory has the same formulation under a change of the time step. Let us call τ the time interval between successive reassessments of the fundamentalists, with τ being measured in a calendar time scale, for instance, in seconds, hours or days.

First, the parameters N_{c} and N_{f} are a priori independent of τ, while they may be a function of time t. We neglect this dependence as we are interested in the dynamics over time scales of a few years that are characteristic of bubble regimes. The parameter γ is also independent of τ.

In contrast, the parameters R_f, r and σ_r^2 are functions of τ, as the return of the risk-free asset, the average expected dividend return and its variance depend on the time scale. The simplest and standard dependence of Wiener processes or discrete random walks is $R_f \sim r \sim \sigma_r^2 \sim \tau$. Because of its definition, $x = R_{\text{excess}}/\gamma\tilde{\sigma}^2$, the fraction of wealth x fundamentalists hold is independent of time.

By construction, the parameter θ characterizing the memory of the price momentum influencing the decisions of chartists depends on τ. This can be seen by replacing $t-1$ by $t-\tau$ to make explicit the unit time scale in expression (2.35), giving

$$\frac{H_t - H_{t-\tau}}{\tau} = \frac{1-\theta}{\tau}\left(\frac{P_t}{P_{t-\tau}} - 1 - H_{t-\tau}\right) . \tag{2.48}$$

Requesting a bona-fide limit for small τ's leads to

$$\frac{1-\theta}{\tau} = \varrho = \text{const} , \tag{2.49}$$

where the time scale $\mathcal{T}_H := 1/\varrho$ is the true momentum memory. Thus, we have

$$1-\theta = \varrho \cdot \tau, \qquad \mathcal{T}_H := \frac{1}{\varrho} = \frac{\tau}{1-\theta} . \tag{2.50}$$

2.4 Theoretical analysis and super-exponential bubbles

2.4.1 Reduction to deterministic equations

It is possible to get an analytical understanding of the solutions of the set of equations (2.42) to (2.47) if we reduce them into their deterministic components. The full set including their stochastic contributions will be studied with the help of numerical simulations in the next section.

Taking $u_t \equiv 0$ and replacing s_t by its expectation $\mathrm{E}[s_t]$ given by (2.39), we obtain the following deterministic equations

Dynamics of the chartists opinion index:

$$s_t = (1+\kappa-p)s_{t-1} + \kappa H_{t-1} , \tag{2.51}$$

Dynamics of the risky asset price:

$$\begin{aligned} P_t/P_{t-1} \quad = \quad & \Big[(1+s_t)\left((1+R_f)(1-s_{t-1}) + r(1+s_{t-1})\right) W^c_{t-1} + \\ & + 4x\left((1+R_f)(1-x)+rx\right) W^f_{t-1}\Big] / \\ & \Big[(1+s_{t-1})(1-s_t)W^c_{t-1} + 4x(1-x)W^f_{t-1}\Big] . \end{aligned} \tag{2.52}$$

Wealth dynamics of fundamentalists:

$$W^f_t/W^f_{t-1} = x\left(\frac{P_t}{P_{t-1}} + r\right) + (1-x)(1+R_f) , \tag{2.53}$$

Wealth dynamics of chartists:

$$W_t^c / W_{t-1}^c = \frac{1 + s_{t-1}}{2} \left(\frac{P_t}{P_{t-1}} + r \right) + \frac{1 - s_{t-1}}{2} (1 + R_f) \ , \tag{2.54}$$

Momentum of the risky asset price:

$$H_t = \theta H_{t-1} + (1 - \theta) \left(\frac{P_t}{P_{t-1}} - 1 \right) \ . \tag{2.55}$$

This system of five coupled deterministic equations is non-linear and completely coupled, there is no autonomous subsystem. In particular, the multiplicative price equation is highly non-linear. The wealth equations describe the multiplicative process of capital accumulation depending on the choice of how to split the portfolio on the risky and risk-free asset yielding capital gains, given the dividend gains and the risk-free rate.

2.4.2 Fixed points and stability analysis

To gain insights into the system of coupled equations, we will consider the stationary case where the wealth levels of fundamentalist and chartist traders only change slowly and remain of roughly the same order of magnitude.[2] This happens when both groups keep their portfolio allocation approximately fixed and their endowments mainly grow due to dividends and risk-free returns. According to (2.51), we are in the regime $\kappa < p$ and may treat the ratio of wealth levels ν as approximately constant,

$$\nu := \frac{W_t^c}{W_t^f} \simeq \text{const} \sim \mathcal{O}(1) \ . \tag{2.56}$$

This allows us to decouple the equations for H_t, s_t and P_t from the wealth equations. The fixed points $\{(H^*, s^*)\}$ are determined by the system:

$$H^* = R_f + r \frac{\nu(1 + s^*)^2 + 4x^2}{\nu(1 + s^*)(1 - s^*) + 4x(1 - x)} \ , \tag{2.57}$$

$$s^* = \frac{\kappa}{p - \kappa} H^* \ , \tag{2.58}$$

[2]This last condition is necessary for nontrivial dynamics, as both populations remain relevant to the economy.

Since this system is essentially one third-order equation, it can be solved analytically yielding three fixed points. As we will see later, for typical parameter values, there is one solution $s^*, H^* \ll 1$, while the other two lie outside the restricted domain of $[-1, 1]$ for s. An expansion in the small parameters $r, R_f \ll 1$ permits the approximation:

$$H^* = R_f + \frac{\nu + 4x^2}{\nu + 4x - 4x^2} r + \mathcal{O}(r^2, R_f^2) \ , \tag{2.59}$$

$$s^* = \frac{\kappa}{p - \kappa} \left[R_f + \frac{\nu + 4x}{\nu + 4x - 4x^2} r + \mathcal{O}(r^2, R_f^2) \right] \ . \tag{2.60}$$

This fixed point is stable for $\kappa < p$ over a range of the other parameter values and unstable for $\kappa > p$. A deviation from the fixed point due to stochastic fluctuations in the opinion index leads to a price change in the same direction. According to (2.51), for $\kappa > p$, the opinion index grows transiently exponentially (until its saturation). Since the stability is mainly governed by the relative value of the two parameters κ and p characterizing chartist behavior, we conclude that there is an inherent instability caused by herding and trend following, which is independent of the stochastic dividend process.

2.4.3 Super-exponential bubbles

It is well-known that many bubbles in financial markets start with a phase of super-exponential growth, see for example Sornette et al. (2009) for oil prices, Jiang et al. (2010) for the Chinese stock market and Yan et al. (2012) for major equity markets. Furthermore, Sornette et al. (2013) discuss various theoretical and empirical questions related to faster-than-exponential growth of asset prices, while Hüsler et al. (2013) document super-exponential bubbles in a controlled experiment in the laboratory.

One of the main findings of this paper is that phases with faster-than-exponential growth of the price are inherent also in the present model. If a bubble is essentially driven by herding and trend following, we may neglect the dividend process and expand the pricing formula (2.52) in terms of r and R_f:

$$\frac{P_t}{P_{t-1}} = \frac{(1 + s_t)(1 - s_{t-1}) + 4x(1 - x) W_{t-1}^f / W_{t-1}^c}{(1 - s_t)(1 + s_{t-1}) + 4x(1 - x) W_{t-1}^f / W_{t-1}^c} + \mathcal{O}(r, R_f) \ . \tag{2.61}$$

Again, we focus on the scenario that the ratio of the wealth levels W^f_{t-1} and W^c_{t-1} of the fundamentalist and chartist traders remains approximately constant,

$$W^c_{t-1}/W^f_{t-1} = W^c_{t_0}/W^f_{t_0} = \nu \simeq \text{const} \ . \tag{2.62}$$

This is the case if the endowments of both groups grow at the same constant exponential growth rate or, more accurate here, if both grow in the same super-exponential way. Starting with an opinion index s_0 at time $t = t_0$, we can further simplify the price equation to:

$$\frac{P_t}{P_{t-1}} = 1 + b(s_t - s_{t-1}) + \mathcal{O}(r, R_f, (s - s_0)^2) \ , \tag{2.63}$$

where the constant quantity b is of order 1 provided the initial levels of wealth were of the same order of magnitude:

$$b = \frac{2}{1 + 4x(1-x)\nu - s_0^2} \sim \mathcal{O}(1) \ . \tag{2.64}$$

Therefore, up to terms of order $\mathcal{O}(r, R_f, (s - s_0)^2)$, the price evolves as

$$\frac{P_t}{P_0} = \prod_{j=1}^{t} [1 + b(s_j - s_{j-1})] \simeq \prod_{j=1}^{t} \mathrm{e}^{b(s_j - s_{j-1})} = \mathrm{e}^{b(s_t - s_0)} \ . \tag{2.65}$$

Since s_t grows exponentially with time according to expression (2.51) for $\kappa > p$, the price P_t grows as an exponential of an exponential of time. In other words, for the regimes when the opinion index grows exponentially ($\kappa > p$), we expect super-exponential bubbles in the price time series. Since our equations are symmetric in the sign of the opinion index s_t, the same mechanism leads also to "negative bubbles" for a negative herding associated with a transition from bullish to bearish behavior for which the price drops also super-exponentially in some cases.

2.4.4 Time-dependent social impact and bubble dynamics

The strength of herding is arguably regime dependent. In some phases, chartists are prone to herding, while at other times, they are more incoherently disorganized "noise" traders. This captures in our dynamical framework the phenomenon of regime switching (Hamilton, 1989; Lux, 1995; Hamilton and Raj, 2002; Yukalov et al., 2009; Binder and Gross, 2013; Fischer and Seidl, 2013; Kadilli, 2013), where successive phases are characterized by changing values of the herding propensity. In this respect, we follow the model approach of Harras et al. (2012) developed in a similar context and assume that the strength κ of social imitation and momentum influence slowly varies in time. In this way, we incorporate the effects of a changing world on financial markets such as a varying economic and geopolitical climate into the model. More generally, we allow for varying uncertainties influencing the behavior of chartists. As we shall show, this roots the existence of the bubbles documented below in the mechanism of "sweeping of an instability" (Sornette, 1994; Stauffer and Sornette, 1999).

More specifically, we propose that κ undergoes a discretized Ornstein-Uhlenbeck process:[3]

$$\kappa_t - \kappa_{t-1} = \eta(\mu_\kappa - \kappa_{t-1}) + \sigma_\kappa \upsilon_t \ . \tag{2.66}$$

Here $\eta > 0$ is the mean reversion rate, μ_κ is the mean reversion level and $\sigma_\kappa > 0$ is the step size of the Wiener process realized by the series υ_t of standard i.i.d. random variables with distribution $N(0, 1)$.

Our approach is related to how Lux (1995) describes switching between bear and bull markets. While we propose a stochastic process for the strength of social imitation κ, Lux adds a new deterministic term proportional to $\mathrm{d}\log P_t/\mathrm{d}t$ to the transition probabilities, which corresponds to a direct positive feedback.

The interesting case is $\mu_\kappa \lesssim p$, where κ is on average below the critical value p but, due to stochastic fluctuations, may occasionally enter the regime with faster-than-exponential growth $\kappa > p$ described in the previous subsection. Since an Ornstein-Uhlenbeck process with deterministic initial value is a Gaussian process, its distribution is fully determined by the first and second moments. Starting from an initial value κ_0, the non-stationary mean and covariance are given by:

$$\mathrm{E}[\kappa_t] = \kappa_0 \mathrm{e}^{-\eta t} + \mu_\kappa \left(1 - \mathrm{e}^{-\eta t}\right) \ , \tag{2.67}$$

$$\mathrm{Cov}[\kappa_s, \kappa_t] = \frac{\sigma_\kappa^2}{2\eta} \left(\mathrm{e}^{-\eta(t-s)} + \mathrm{e}^{-\eta(t+s)}\right) \ , \quad s < t \ . \tag{2.68}$$

[3] Choosing a confined random walk yields similar results, but the mean reversion is then effectively nonlinear (or threshold based), which is less standard.

Both moments converge such that in the long run κ_t admits the following stationary distribution:

$$\kappa_t \sim N\left(\mu, \frac{\sigma_\kappa}{\sqrt{2\eta}}\right) \ . \tag{2.69}$$

If, at some time t, the social imitation strength is above the critical value $\kappa_t \equiv \kappa_0 > p$, the time ΔT needed for κ_t to revert to the subcritical regime $\kappa_t < p$ can be estimated from equation (2.67):

$$\Delta T = \frac{1}{\eta} \log\left(\frac{\kappa_0 - \mu_\kappa}{p - \mu_\kappa}\right) \ . \tag{2.70}$$

Expressions (2.69) and (2.70) will allow us to estimate how often the group of chartists will interact in the supercritical regime of the opinion index related to transient faster-than-exponential growth in the price and how long a typical bubble will last.

2.5 Numerical simulations and qualitative comparison with the dotcom bubble

2.5.1 Estimation of parameter values

Let us take $\tau = 1$ day and assume a typical memory used by chartists for the estimation of price momentum equal to about one month. This amounts approximately to 20 trading days, hence $\mathcal{T}_H \simeq \frac{\tau}{1-\theta} = 20$, leading to $\theta = 0.95$.

We calibrate the average dividend-price ratio r and its standard deviation σ_r to the values given by Engsted and Pedersen (2010), which are quite similar for various countries. We set the mean daily dividend-price ratio to $r = 1.6 \cdot 10^{-4}$ and the daily standard deviation to $\sigma_r = 9.5 \cdot 10^{-4}$. Furthermore, we assume a constant return of the risk-free asset of annualized 2%, i.e. a daily value of $R_f = 8 \cdot 10^{-5}$.

Fundamentalists keep 30% of their wealth in the risky asset, that is, $x = 0.3$. The wealth levels W_t^f and W_t^c of fundamentalist and chartist traders evolve dynamically and determine the relative influence of the two groups. We analyze the importance of the initial endowments W_0^f and W_0^c on the stability of the market. We capture this by the parameter $\nu = W_0^c / W_0^f$ and set ν to 1, 2 or 0.5 in three different sets of simulations.[4]

For the parameter p entering in expressions (2.38), recall that it is equal to twice the probability that during a given day some chartist will buy (or sell) the risky asset. We posit $p = 0.2$, which means that the natural trading frequency of traders in absence of social influence is about two weeks. For the parameter κ in (2.38) describing the strength of social imitation and of momentum trading, we assume that it is close to the parameter p. Specifically, for the Ornstein-Uhlenbeck process given in expression (2.66), we choose $\mu_\kappa = 0.98p = 0.196$. We set the mean reversion speed η and the step size σ_κ such that (i) the Ornstein-Uhlenbeck process has a standard deviation of $0.1p$ and (ii) a deviation of κ_t two standard deviations above μ_κ in the supercritical regime will revert within $\Delta T = \mathcal{T}_H = 20$:

$$\eta = \frac{1}{\Delta T} \log\left(\frac{\mu_\kappa + 2 \cdot 0.1p - \mu_\kappa}{p - \mu_\kappa}\right) = \log(10)/20 \simeq 0.11 \ , \tag{2.71}$$

$$\sigma_\kappa = 0.1p\sqrt{2\eta} \simeq 0.001 \ . \tag{2.72}$$

Summarizing, the numerical simulations presented in the figures correspond to

$$\theta = 0.95, \quad r = 1.6 \cdot 10^{-4}, \quad \sigma_r = 9.5 \cdot 10^{-4}, \quad R_f = 8 \cdot 10^{-5}, \quad x = 0.3 \ , \tag{2.73}$$

$$p = 0.2, \quad \mu_\kappa = 0.196, \quad \sigma_\kappa = 0.001, \quad \eta = 0.11 \ , \tag{2.74}$$

and ν will be varied as $\nu = 0.5, 1, 2$. Furthermore, we run the simulations over 20 trading years, i.e. $T = 5000$.

We can now test our claims from the fixed points analysis in section 2.4.2 numerically. Assuming that κ_t will not deviate further than five standard deviations from its mean μ_κ, we find that one fixed point for the opinion index is indeed close to zero, $s^* \sim \mathcal{O}(10^{-3})$, while the other two lie well outside of the domain of definition $[-1, 1]$.

[4]Note that this is equivalent to setting the ratio of group sizes $\nu = N_\mathrm{c}/N_\mathrm{f}$ with the assumption that both groups consist of representative agents with equal initial wealth. In our formulation, N_c has no further importance than controlling the smoothness of the opinion index. Thus it disappears from the deterministic equations (2.51) to (2.55). The simulations are run with $N_\mathrm{c} = 1000$.

2.5.2 Results and interpretation

Figures 2.1, 2.4 and 2.5 show the time dependence of the variables P_t, s_t, κ_t, H_t, W_t^f, W_t^c and the time series of returns that are generated by numerical solutions of the set (2.42) to (2.47) for three different parameter values for $\nu = 1, 2$ and 0.5 respectively, of the relative important of chartists compared with fundamentalists in their price impact.

Figure 2.1 corresponds to the situation where both groups have equal initial endowments ($\nu = 1$). One can observe a general positive log-price trend biasing upward a fluctuating random walk-like trajectory. The upward drift reflects a combination of the dividend gains, of the rate of return paid by the risk-free asset as well as a component resulting from the herding behavior of chartists who tend intermittently to push prices in a kind of self-fulfilling prophecy or convention à la Boyer and Orléan (1993); Orléan (2004); Eymard-Duvernay et al. (2005).

But the most striking aspect of the price dynamics is the occurrence of four clearly identifiable bubbles occurring within the chosen time interval, defined by the transient explosive growth of the price P_t followed by sharp crashes bringing the prices back approximately to pre-bubble levels. As seen from the second panel of Figure 2.1 showing the opinion index dynamics of the chartists, the bubbles are essentially driven by the chartist traders. As described in section 2.4.3, the start of the growth of herding among chartists feeds the price dynamics, resulting in a larger price momentum (fourth panel), which amplifies herding, enhancing further the bubble growth and so on. One can observe in each bubble that the growth of the opinion index (or equivalently the fraction of wealth invested in the risky asset) precedes and then accompanies the explosive price growth, as predicted by expression (2.65). The transient bubbles and their subsequent crashes are associated with clustered volatility and the existence of outliers in the price momentum. During the bubbles, the wealth levels of chartists and of fundamentalists diverge. In the long run, chartists outperform fundamentalists because they tend to invest more in the risky asset, which exhibits higher average returns.

Figure 2.2 presents a more detailed analysis of a typical bubble from the time series shown in Figure 2.1, demonstrating the characteristic transient faster-than-exponential growth behavior predicted theoretically in section 2.4.3. For periods when $\kappa_t > p$, we may approximate the opinion index as exponentially growing:

$$s_t = s_1(\alpha_1^t - 1) \ , \tag{2.75}$$

where t runs over the growth period $[t_1, t_1 + \Delta T]$, with initial value $s_1 \equiv s_{t_1}$ and where $\alpha_1 > 1$ is an empirical effective multiplicative factor, $\log \alpha_1$ being the effective growth rate of s_t. One can verify that the length ΔT of such a period is compatible with our theoretical prediction (2.70), which for our chosen parameters gives $\Delta T = 20$. Bubbles with longer lifetimes are easily engineered in our framework by allowing κ to remain close and higher than p for longer times. Our model supports therefore the view that long-lived bubbles may be associated with excess positive sentiments catalyzing a herding propensity that is sustained and self-reinforcing (via the momentum mechanism) over long periods.

Furthermore, the exponential growth in the opinion index results in a faster-than-exponential growth of the price, as can be seen in the log-linear plot of P_t. From expression (2.65), we deduce

$$\log(P_t) = b_1 s_1(\alpha_1^t - 1) + \log(P_0) \ , \tag{2.76}$$

where $b_1 = b_{t_1}$, which fits well the transient super-exponential price dynamics. These observations presented in Figure 2.2 are in agreement with the theoretical derivation of section 2.4.3. It is interesting to note also that the dynamics of κ_t, with its tendency to present a transient oscillatory behavior due to the interplay between rare large excursions with the mean reversal of the constrained random walk associated with the discrete Ornstein-Uhlenbeck process, leads to an approximate log-periodic behavior[5] of the price during its ascendency, which is similar to many observations reported empirically (Sornette, 2003; Johansen and Sornette, 2010; Jiang et al., 2010; Yan et al., 2012; Sornette et al., 2013).

Figure 2.3 presents three statistical properties of our generated price time series. Various well-known stylized facts are matched by our model. First, we show the distribution of absolute values of the returns, which has a fat-tail $p(x) \sim x^{-1-\alpha}$ with

[5]Log-periodicity here refers to transient oscillations with increasing local frequency. Formal mathematical definitions and illustrations can be found in Sornette and Johansen (1998).

exponent $\alpha = 3.0$, which is in the range of accepted values in the empirical literature (Vries, 1994; Pagan, 1996; Guillaume et al., 1997; Gopikrishnan et al., 1998; Jondeau and Rockinger, 1999). Furthermore, signed returns R_t are characterized by a fast-decaying autocorrelation function, which is consistent with an almost absence of arbitrage opportunities in the presence of transaction costs. In contrast, the absolute values $|R_t|$ of returns have an autocorrelation function with longer memory (Ding et al., 1993; Cont, 2007).

Figures 2.4 and 2.5 present the same panels as in Figure 2.1 but with $\nu = 2$ and $\nu = 0.5$, respectively. Due to their larger relative weight compared to the case shown in Figure 2.1, one can observe in Figure 2.4 bubbles with stronger "explosive" trajectories. The wealth of chartists fluctuates widely, but amplifies to values that are many times larger than that of fundamentalists. This is due to the self-fulfilling nature of the chartist strategies that impact the price dynamics. In contrast, Figure 2.5 with $\nu = 0.5$ shows that the wealth of the fundamentalists remains high for a long transient, even if in the long term the chartists end up dominating the price dynamics. The chartists also transiently over-perform dramatically the fundamentalists during the bubbles. It is informative to observe that, even a minority of chartists ($\nu = 0.5$ shown in Figure 2.5) ends up creating bubbles and crashes. Their influence progressively increases and their transient herding behavior becomes intermittently destabilizing.

2.5.3 Comparison with the dotcom bubble

This section compares the insights obtained from the above theoretical and numerical analyses to empirical evidence on momenta and reversals in the period when the dotcom bubble developed.[6] We study the characteristics of the share prices of Internet-related companies over the period from January 1, 1998 to December 31, 2002, which covers the period of the development of the dotcom bubble and its collapse. We use the list of 400 companies belonging to the Internet-related sector that has been published by Morgan Stanley and has already been investigated by Ofek

[6] The dotcom bubble (followed by its subsequent crash) is widely believed to be a speculative bubble, as documented by Ofek and Richardson (2003); Brunnermeier and Nagel (2004); Battalio and Schultz (2006).

and Richardson (2003). The criteria for a company to be included in that list is that it must be considered a "pure" internet company, i.e., whose commercial goals are associated exclusively to the Internet. This implies that technology companies such as Cisco, Microsoft, and telecommunication firms, notwithstanding their extensive Internet-related businesses, are excluded.

2.6 graphs the index of an equally weighted portfolio of the Internet stocks over the sample period of January 1998 to December 2002. The time evolution of the equally weighted portfolio of the Internet stocks is strikingly different from that shown in 2.7 for the index of an equally weighted portfolio of non-Internet stocks over this same period. The two indexes are scaled to be 100 on January 2, 1998. The two figures illustrate clearly the widely held view that a divergence developed over this period between the relative pricing of Internet stocks and the broad market as a whole. In the two year period from early 1998 through February 2000, the internet related sector earned over 1300 percent returns on its public equity while the price index of the non-internet sectors rose by only 40 percent. However, these astronomical returns of the Internet stocks had completely evaporated by March 2001. Note how 2.6 is strikingly similar to the dynamics generated by the theoretical model in the bubble regime shown at the end of the top panel of Figure 2.5 ($\nu = 0.5$).

Table 2.1: Annual Returns for Internet and non-Internet stock indices.

Year	1998	1999	2000	2001	2002
Internet stock index	116.8%	815.6%	−875.9%	−62%	−48.8%
(per month)	(9.7%)	(68%)	(−73%)	(−5.2%)	(−4.1%)
Non-internet stock index	6.5%	17%	−9%	3.6%	−9%
(per month)	(0.5%)	(1.4%)	(−0.8%)	(0.3%)	(−0.7%)

We now focus our attention on the profitability of the momentum strategies studied by Jegadeesh and Titman (1993, 2001) and others. Table 2.1 provides some descriptive statistics about annual returns of the Internet-stock index versus of the non-Internet stock index from the beginning of 1998 to the end of 2002. In the 12 months of 1998, the annual cumulative return of the Internet stock index was 117 percent, while that of the non-Internet stock index was 6.5 percent. In the 12 months of 1999, the annual cumulative return of the Internet stock index surged to 816 percent, and that of the non-Internet stock index increased to 16.6 percent. The Internet stock index clearly outperformed the non-Internet stock index by 800 percent in 1999. This implies a strong profitability of momentum strategies applied to the Internet stocks over the period of the dotcom bubble. However, after its burst in March 2000, the return of the Internet stocks sharply declined, from 2000 to 2002. In the 12 months of 2000, the annual return of the internet-stock index fell to - 876 percent, followed by - 62 percent and - 49 percent in 2001 and in 2002, respectively. On the other hand, the annual returns of the non-Internet stock index in the period from 2000 to 2002 remain modest in amplitude at - 9 percent, 3.6 percent and - 9 percent, respectively. After the bust of the dotcom bubble, the Internet stocks continued to underperform the non-Internet stocks.

Table 2.2 shows the cumulative returns for the Internet stock index and for the non-Internet stock index in the five years from the beginning of 1998 to the end of 2002. The cumulative return of the Internet stock index in the first 24 months of the holding period is 932.5 percent, but the cumulative returns ends at the net loss of - 54.2 percent over the five year holding period. In contrast, the cumulative returns of the non-Internet stock index over the same five year holding period is 8.6 percent.

These figures can be reproduced by our simulations, and are visualized by the extremely good performance of our chartists during the bubble phases, as shown in the fifth panels (from the top) of Figures 2.1, 2.4 and 2.5.

Table 2.2: Cumulative Returns for Internet and non-Internet stock indices.

Year	1998	1999	2000	2001	2002
Internet stock index	116.8%	932.5%	56.6%	−5.4%	−54.2%
(per month)	(9.7%)	(38.9%)	(1.6%)	(−0.1%)	(−0.9%)
Non-internet stock index	6.5%	23.1%	−14%	17.6%	8.6%
(per month)	(0.5%)	(1.0%)	(0.4%)	(0.4%)	(0.1%)

In summary, these empirical facts constitute strong evidence for the Internet stock for momentum profit at intermediate time scales of about two years and reversals at longer time scales of about 5 years. These empirical facts confirm for this specific bubble and crash period the general evidence documented by many researchers (e.g. Jegadeesh and Titman 1993, 2001). They are consistent with the stylized facts described by the model that predict that the momentum profits will eventually reverse in cycle bubbles and crashes as illustrated above. The qualitative comparison between the empirical data and our simulations suggest that chartists do not need to be a majority, as their superior performance during the bubble make them dominate eventually utterly the investment ecology.

2.6 Conclusions

We have introduced a model of financial bubbles with two assets (risky and risk-free), in which fundamentalists and chartists co-exist. Fundamentalists form expectations on the return and risk of a risky asset and maximize their constant relative risk aversion expected utility with respect to their portfolio allocation. Chartists are subjected to social imitation and follow momentum trading.

In contrast to various previous models, agents do not switch between investment strategies. By keeping track of their wealth levels, we still observe the formation of endogenous bubbles and match several stylized facts of financial markets such as a fat-tail distribution of returns and volatility clustering. In particular, we observe transient faster-than-exponential bubble growth with approximate log-periodic behavior. Although faster-than-exponential growth at the beginning of a bubble has been found in many econometric studies of bubbles in real markets and recent lab experiments, it has been hardly discussed in the context of agent-based models. Our model is one of the first offering a transparent analytical explanation for this stylized fact.

To the important question of whether and when fundamentalist investors are able to stabilize financial markets by arbitraging chartists, our analysis suggests that chartists may eventually always lead to the creation of bubbles, given sufficient time, if a mechanism exists or some sentiment develops that increase their propensity for herding. Momentum strategies have been shown to be transiently profitable, supporting the hypothesis that these strategies enhance herding behavior.

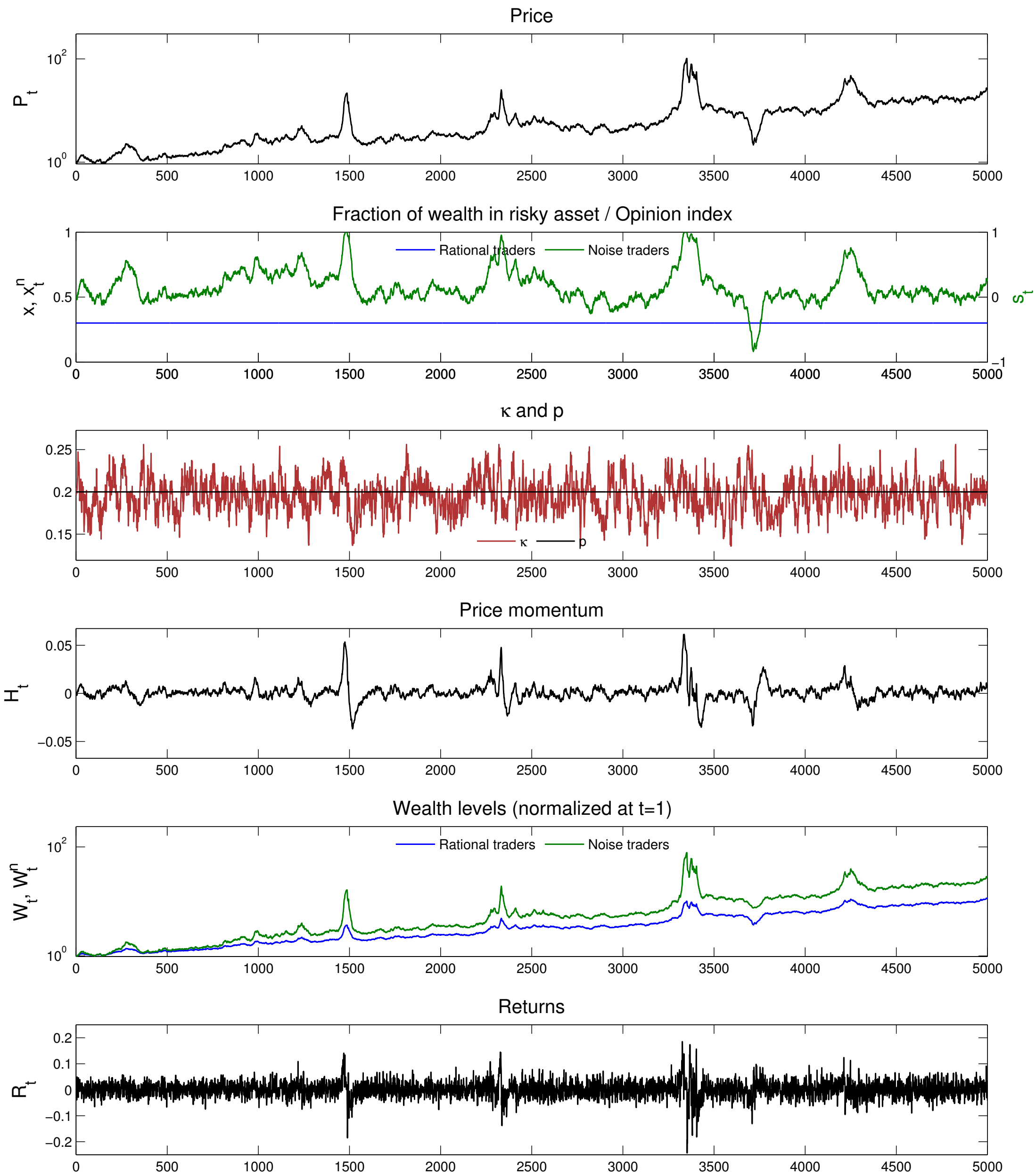


Figure 2.1: Time dependence of the variables P_t (log-scale), s_t, κ_t, H_t, W_t^f, W_t^c (both in log-scale) and the time series of returns that are generated by numerical solutions of the set (2.42) to (2.47) for the value $\nu = 1$ of the relative important of chartists compared with fundamentalists in their price impact at the origin of time.

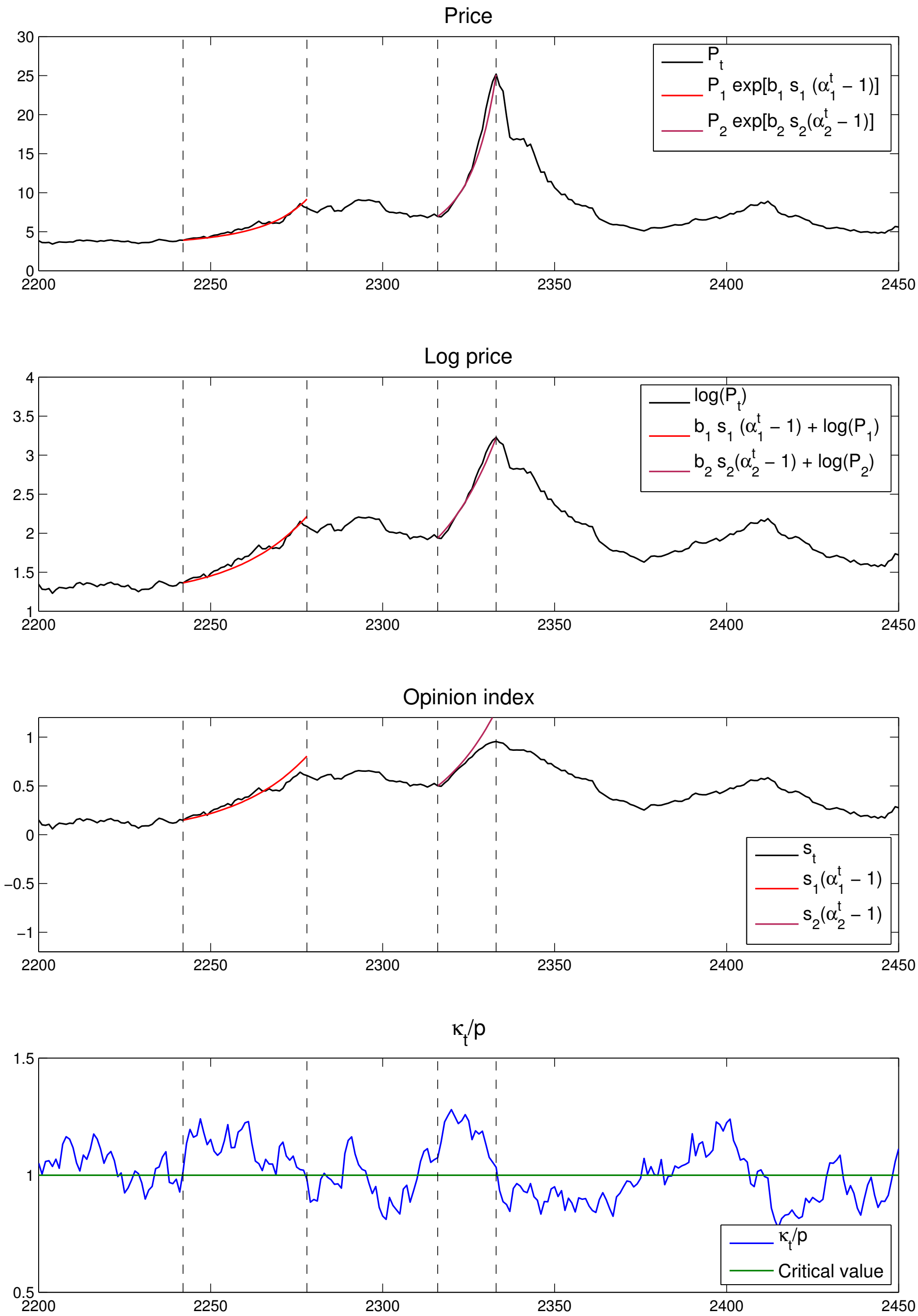


Figure 2.2: Zoom of Figure 2.1 for the price P_t, log-price $\ln P_t$, opinion index s_t and imitation parameter κ_t in units of the random component strength p as a function of time around bubbles. The panels show the behavior of these variables for typical bubbles, demonstrating the characteristic transient faster-than-exponential growth behavior.

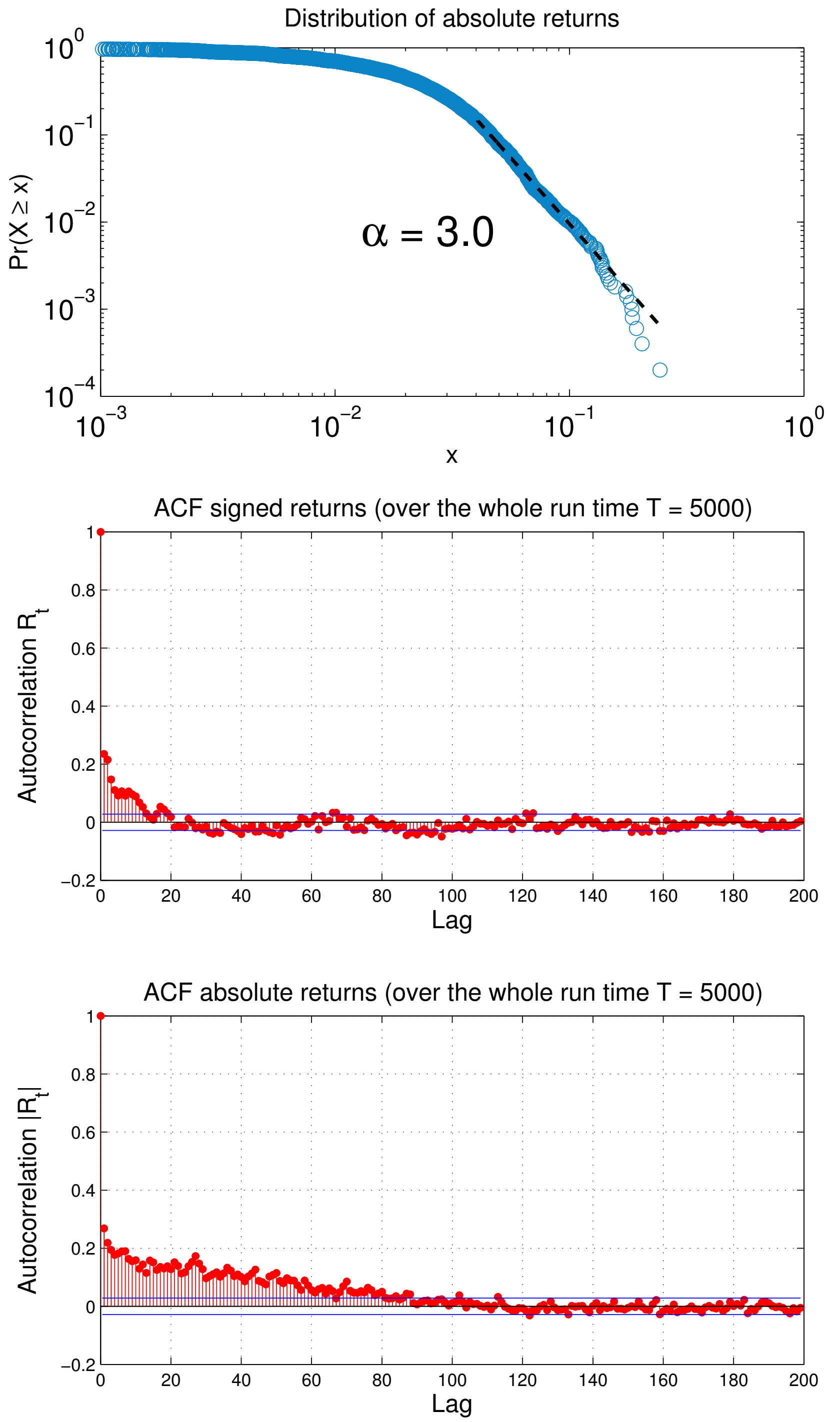


Figure 2.3: Top panel: complementary cumulative distribution function of absolute values of the returns in log-log, where the straight dashed line qualifies a fat-tail $p(x) \sim x^{-1-\alpha}$ with exponent $\alpha = 3.0$; Middle panel: auto-correlation function of the signed returns R_t; Lower panel: autocorrelation function of the absolute values $|R_t|$ of returns. The parameters are the same as in Figure 2.1.

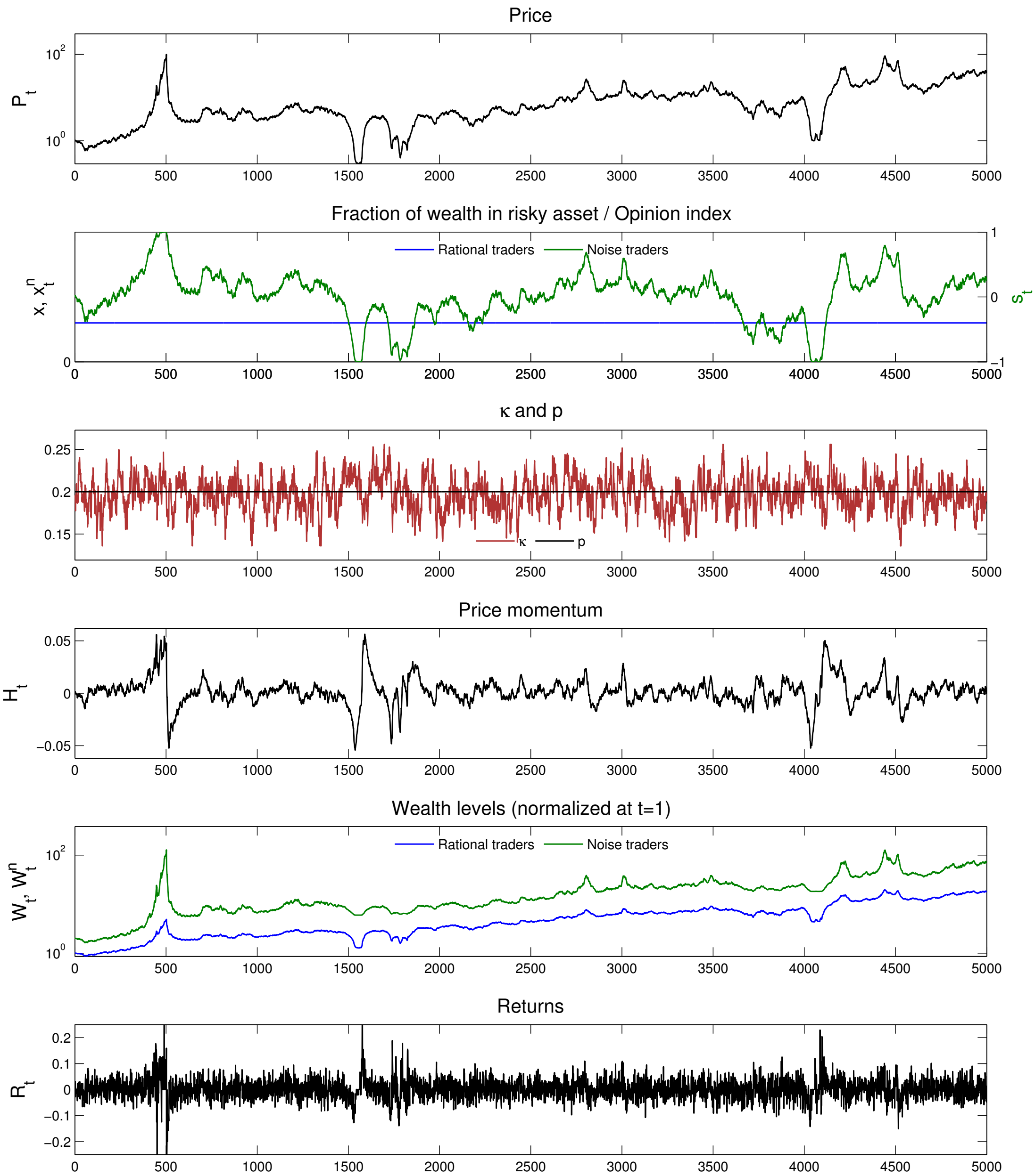


Figure 2.4: Same as Figure 2.1 for $\nu = 2$, i.e. chartists than fundamentalists.

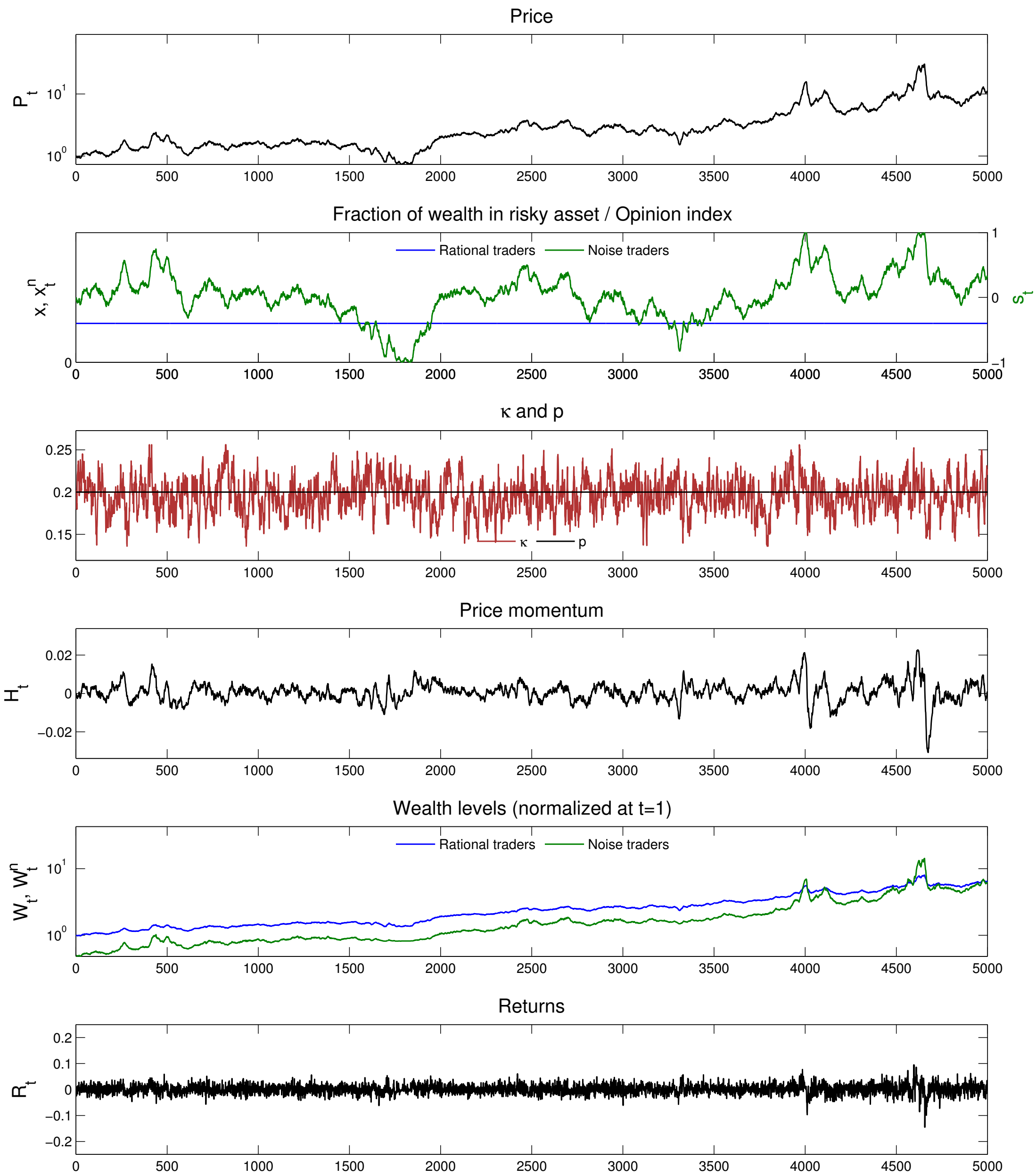


Figure 2.5: Same as Figure 2.1 for $\nu = 0.5$, i.e. more fundamentalists than chartists.

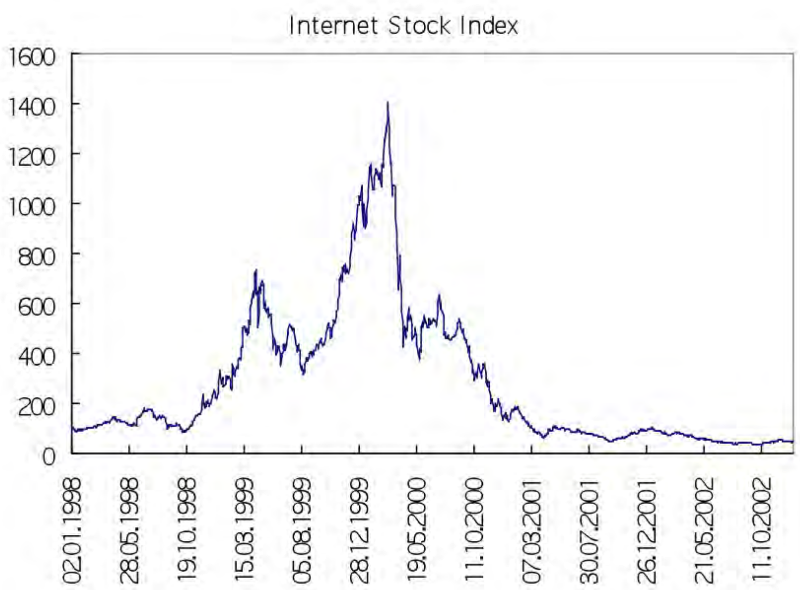


Figure 2.6: The equally weighted Internet stock index for the period 1/2/1998-12/31/2002. The index is scaled to be 100 on 1/2/1998.

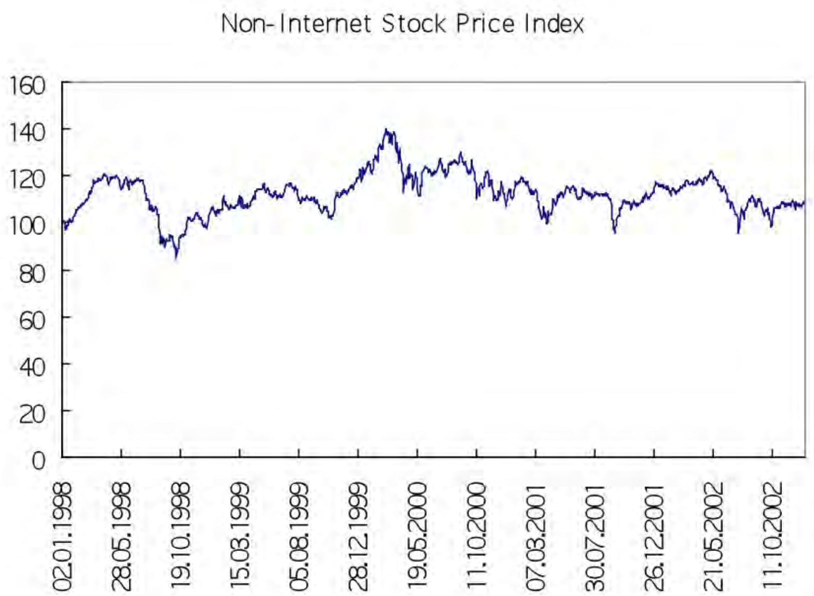


Figure 2.7: The equally weighted non-Internet stock index for the period 1/2/1998-12/31/2002. The index is scaled to be 100 on 1/2/1998.

Appendix

2.A The effect of LPPLS traders on the market

Since the bubbles generated by the model defined in this chapter exhibit super-exponential growth behavior modulated by approximate log-periodic oscillations, it is intriguing to enrich the model with a third group of investors who try to detect and exploit them based on the LPPLS methodology (Sornette and Johansen, 1998; Sornette et al., 2013). How will they impact the market price in general and especially during bubbles? The efficient market hypothesis claims that investors with superior information of temporary deviations from fundamentals will use it for arbitrage and push prices back to fundamentals, leading to full efficiency. Already Friedman (1953) argued that this mechanism would be even more effective in the long run, as agents with persistently superior knowledge would survive, while the others would lose their capital and eventually be driven out of the market.

Reflexivity, however, can lead to market outcomes that differ strongly from what the efficient market hypothesis predicts. In his conventionalist approach Orléan (2004) argued that prices were essentially the result of interactions driven by the various beliefs of market participants. Then it could be possible that prices stabilize due to self-referential interactions at levels different from the fundamental values as predicted by the EMH – this is what Orléan calls a *"convention"*. However, as a consequence the dominance of investors with superior information of fundamentals will no longer be guaranteed because *"markets can remain irrational a lot longer than you and I can remain solvent."* (attributed to John Maynard Keynes, see Shilling 1993).

Following the conventionalist idea, Wyart and Bouchaud (2007) developed a quantitative model where agents define trading strategies using correlations between certain past information and prices. The impact of these strategies on the market price creates a feedback loop, which can lead to the emergence of conventions in the sense of Orléan – substantial and long-lived deviations from market efficiency. One could also interpret their result as a transformation of correlation into causation. There is also empirical support for the existence of conventions. Lorenz et al. (2011) show that social influence can undermine the wisdom of the crowd effect in simple estimation tasks. In particular, information about estimates of others led to a convergence of the group's average estimate that often is further away from the true value than when no information is given.

Philipp (2015) presents a first study of a financial market as in Kaizoji, Leiss, Saichev, and Sornette (2015) with additional LPPLS traders. Those traders use the methodological framework by Sornette and Johansen (1998); Sornette et al. (2013); Filimonov and Sornette (2013) to detect bubble signals on various time scales. As long as they do not find evidence of a bubble building up, they invest similarly to fundamentalist traders. However, once they find a significant LPPLS signature in the price time series, they try to ride the bubble by fully investing until shortly before the anticipated crash. Figure 2.A.1 presents the average and median effect of LPPLS traders on the price during a bubble. Philipp (2015) finds the presence of LPPLS investors to increase a bubble's peak proportional to their market power, but not its duration.

Kohrt (2015) extends the analysis in two dimensions (among others). First, fundamentalists adjust their exposure to the risky asset based on the time-varying dividend price ratio. Second, short selling is allowed and is actively included into the investment strategy of LPPLS traders. Figure 2.A.2 shows the price impact and cumulative performance of LPPLS traders during a bubble relative to their initial market share. As in Philipp (2015), LPPLS traders magnify a bubble's peak relative to their market share. However, since LPPLS traders successfully start short-selling at the peak of the bubble, they also exacerbate the subsequent crash. On average they succeed in recognizing the peak such they make money.

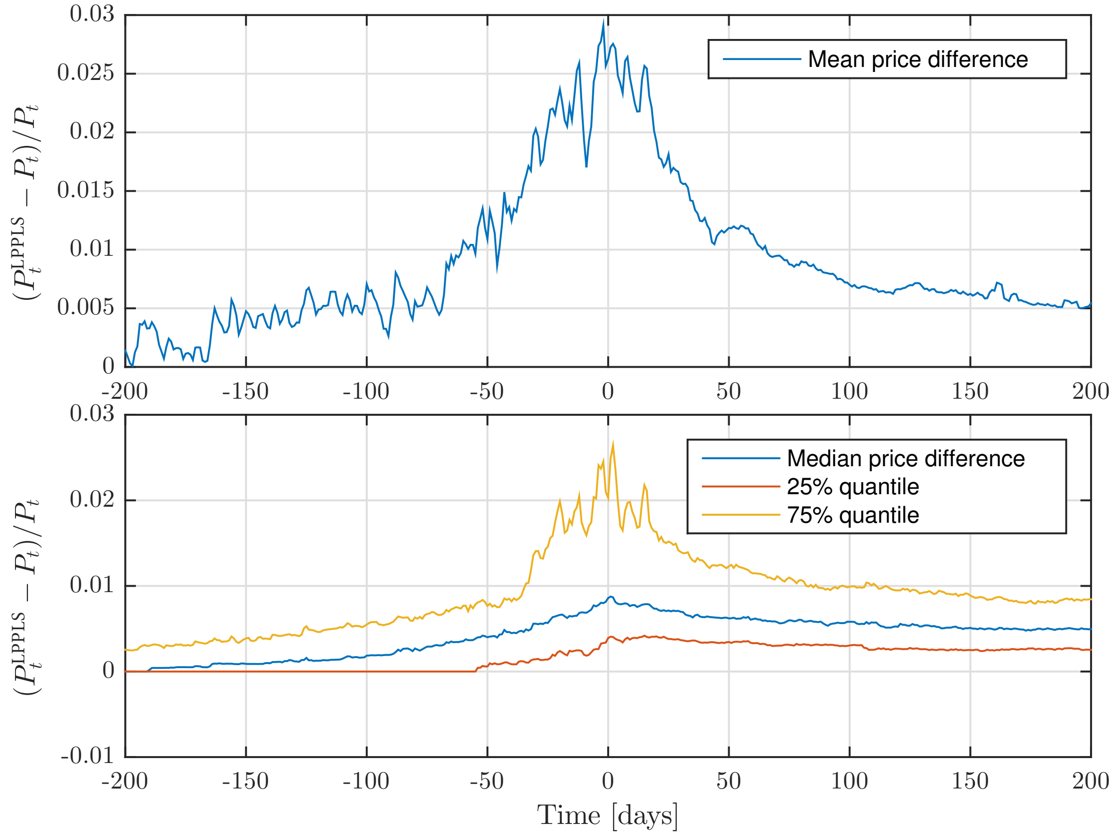
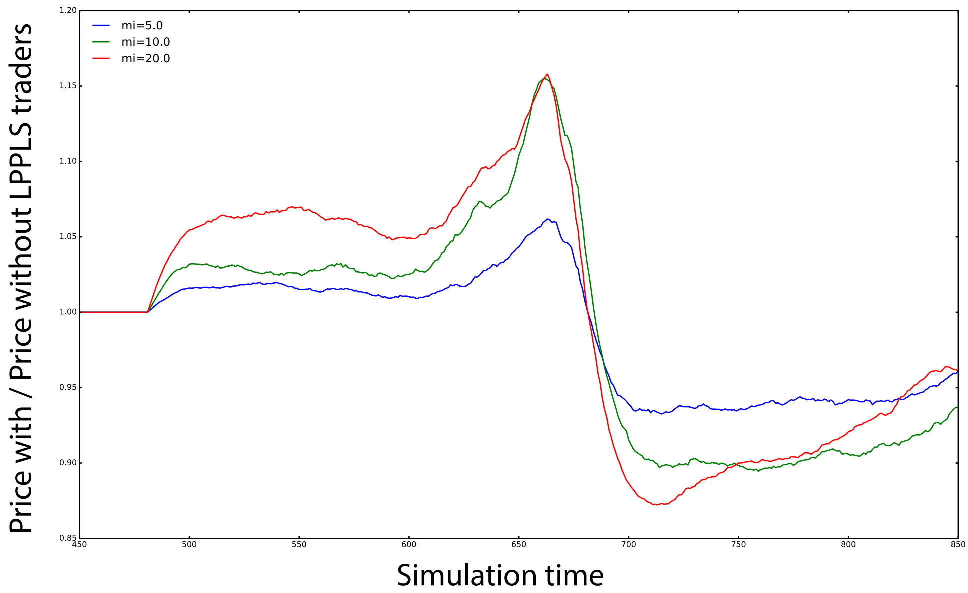
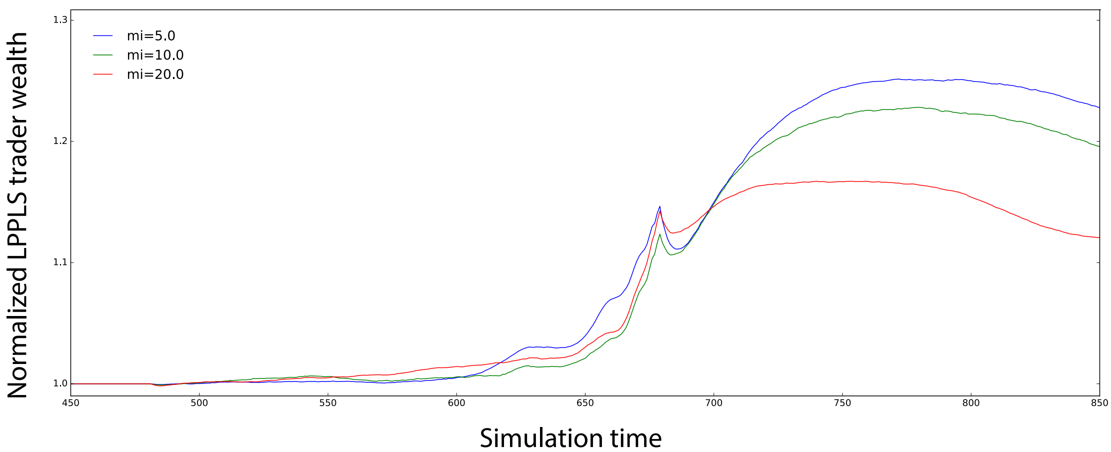


Figure 2.A.1: The mean and median effect of LPPLS traders with 3% wealth share on the risky asset price during a bubble as compared to a market without LPPLS traders. Reprinted with permission from Philipp (2015).



(a) Price impact of LPPLS traders during a bubble.



(b) Change in LPPLS trader wealth during a bubble.

Figure 2.A.2: The mean effect over 3,500 simulations of LPPLS traders with 5% (blue), 10% (green) and 20% (red) market share on the risky asset price during a bubble as compared to a market without LPPLS traders (upper panel) and their cumulated performance (lower panel). Reprinted with permission from Kohrt (2015).

Chapter 3

Super-Exponential Growth Expectations and the Global Financial Crisis

This chapter is an edited version of Leiss et al. (2015). It has been extended by section 3.C containing robustness tests based on Monte Carlo simulations.

Abstract

We construct risk-neutral return probability distributions from S&P 500 options data over the decade 2003 to 2013, separable into pre-crisis, crisis and post-crisis regimes. The pre-crisis period is characterized by increasing realized and, especially, option-implied returns. This translates into transient unsustainable price growth that may be identified as a bubble. Granger tests detect causality running from option-implied returns to Treasury Bill yields in the pre-crisis regime with a lag of a few days, and the other way round during the post-crisis regime with much longer lags (50 to 200 days). This suggests a transition from an abnormal regime preceding the crisis to a "new normal" post-crisis. The difference between realized and option-implied returns remains roughly constant prior to the crisis but diverges in the post-crisis phase, which may be interpreted as an increase of the representative investor's risk aversion.

3.1 Introduction

The Global Financial Crisis of 2008 brought a sudden end to a widespread market exuberance in investors' expectations. A number of scholars and pundits had warned ex ante of the non-sustainability of certain pre-crisis economic developments, as documented by Bezemer (2011). Those who warned of the crisis identified as the common elements in their thinking the destabilizing role of uncontrolled expansion of financial assets and debt, the flow of funds, and the impact of behaviors resulting from uncertainty and bounded rationality. However, these analyses were strongly at variance with the widespread belief in the "Great Moderation" (Stock and Watson, 2003) and in the beneficial and stabilizing properties of financial derivatives markets by their supposed virtue of dispersing risk globally (Summers et al., 1999; Greenspan, 2005). In hindsight, it became clear to everyone that it was a grave mistake to ignore issues related to systemic coupling and resulting cascade risks (Bartram et al., 2009; Hellwig, 2009). But could we do better in the future and identify unsustainable market exuberance ex ante, to diagnose stress in the system in real time before a crisis starts?

The present article offers a new perspective on identifying growing risk by focussing on growth expectations embodied in financial option markets. We analyze data from the decade around the Global Financial Crisis of 2008 over the period from 2003 to 2013.[1] We retrieve the full risk-neutral probability measure of implied returns and analyze its characteristics over the course of the last decade. Applying a change point detection method (Killick et al., 2012), we endogenously identify the beginning and end of the Global Financial Crisis as indicated by the options data. We consistently identify the beginning and end of the Crisis to be June 2007 and May 2009, which is in agreement with the timeline given by the Federal Reserve Bank of St. Louis (2009).[2]

The resulting pre-crisis, crisis and post-crisis regimes differ from each other in several important aspects. First, during the pre-crisis period, but not in the cri-

[1]Related existing work has considered data from pre-crisis (Figlewski, 2010) and crisis (Birru and Figlewski, 2012).

[2]See section 3.3.2 for more details on market and policy events marking the Global Financial Crisis of 2008.

sis and post-crisis periods, we identify a continuing increase of S&P 500 expected returns. This corresponds to super-exponential growth expectations of the price. By contrast, regular expectation regimes prevail in the crisis and post-crisis periods. Second, the difference between realized and option-implied returns remains roughly constant prior to the crisis but diverges in the post-crisis phase. This phenomenon may be interpreted as an increase of the representative investor's risk aversion. Third, Granger-causality tests show that changes of option-implied returns Granger-cause changes of Treasury Bill yields with a lag of few days in the pre-crisis period, while the reverse is true at lags of 50 to 200 days in the post-crisis period. This role reversal suggests that Fed policy was responding to, rather than leading, the financial market development during the pre-crisis period, but that the economy returned to a "new normal" regime post-crisis.

The majority of related option market studies have used option data for the evaluation of risk. An early contribution to this strand of work is Aït-Sahalia and Lo (2000) who proposed a nonparametric risk management approach based on a value at risk computation with option-implied state-price densities. Another popular measure of option-implied volatility is the Volatility Index (VIX), which is constructed out of options on the S&P 500 stock index and is meant to represent the market's expectation of stock market volatility over the next 30 days (Chicago Board Options Exchange, 2009). Bollerslev and Todorov (2011) extended the VIX framework to an "investor fears index" by estimating jump tail risk for the left and right tail separately. Bali et al. (2011) define a general option-implied measure of riskiness taking into account an investor's utility and wealth leading to asset allocation implications. What sets our work apart is the focus on identifying the long and often slow build-up of risk during an irrationally exuberant market that typically precedes a crisis.

Inverting the same logic, scholars have used option price data to estimate the risk attitude of the representative investor as well as its changes. These studies, however, typically impose stationarity in one way or another. Jackwerth (2000), for example, empirically derives risk aversion functions from option prices and realized returns on the S&P 500 index around the crash of 1987 by assuming a constant return probability distribution. In a similar way, Rosenberg and Engle (2002) analyze the

S&P 500 over four years in the early 1990s by fitting a stochastic volatility model with constant parameters. Bliss and Panigirtzoglou (2004), working with data for the FTSE 100 and S&P 500, propose another approach that assumes stationarity in the risk aversion functions. Whereas imposing stationarity is already questionable in "normal" times, it is certainly hard to justify for a time period covering markedly different regimes as around the Global Financial Crisis of 2008. We therefore proceed differently and merely relate return expectations implicit in option prices to market developments, in particular to the S&P 500 stock index and yields on Treasury Bills. We use the resulting data trends explicitly to identify the pre-crisis exuberance in the trends of market expectations and to make comparative statements about changing risk attitudes in the market.

The importance of market expectation trends has not escaped the attention of many researchers who focus on 'bubbles' (Galbraith, 2009; Sornette, 2003; Shiller, 2000; Soros, 2009; Kindleberger and Aliber, 1978). One of us summarizes their role as follows: "In a given financial bubble, it is the expectation of future earnings rather than present economic reality that motivates the average investor. History provides many examples of bubbles driven by unrealistic expectations of future earnings followed by crashes" (Sornette, 2014). While there is an enormous econometric literature on attempts to test whether a market is in a bubble or not, to our knowledge our approach is the first trying to do so by measuring and evaluating the market's expectations directly.[3]

This paper is structured as follows. Section 2 details the estimation of the risk-neutral return probability distributions, the identification of regime change points, and the causality tests regarding market returns and expectations. Section 3 summarizes our findings, in particular the evidence concerning pre-crisis growth of expected returns resulting in super-exponential price growth. Section 4 concludes with a discussion of our findings.

[3]For the econometric literature regarding assessments as to whether a market is in a bubble or not see Stiglitz (1990) (and the corresponding special issue of the Journal of Economic Perspectives), Bhattacharya and Yu (2008) (and the corresponding special issue of the Review of Financial Studies), as well as Camerer (1989); Scheinkman and Xiong (2003); Jarrow et al. (2011); Evanoff et al. (2012); Lleo and Ziemba (2012); Anderson et al. (2013); Phillips et al. (2012); Hüsler et al. (2013).

3.2 Materials and Methods

3.2.1 Estimating risk-neutral densities

Inferring information from option exchanges is guided by the fundamental theorem of asset pricing stating that, in a complete market, an asset price is the discounted expected value of future payoffs under the unique risk-neutral measure (see e.g. Delbaen and Schachermayer, 1994). Denoting that measure by $\mathbb{Q}$ and the risk-neutral density by f, respectively, the current price C_0 of a standard European call option on a stock with price at maturity S_T and strike K can therefore be expressed as

$$C_0(K) = \mathrm{e}^{-r_f T}\, \mathbb{E}_0^{\mathbb{Q}}\left[\max(S_T - K, 0)\right] = \mathrm{e}^{-r_f T} \int_K^{\infty} (S_T - K) f(S_T) \mathrm{d}S_T, \qquad (3.1)$$

where r_f is the risk-free rate and T the time to maturity. From this equation, we would like to extract the density $f(S_T)$, as it reflects the representative investor's expectation of the future price under risk-neutrality. Since all quantities but the density are observable, inverting equation (3.1) for $f(S_T)$ becomes a numerical task.

Several methods for inverting have been proposed, of which Jackwerth (2004) provides an excellent review. In this study, we employ a method by Figlewski (2010) that is essentially model-free and combines standard smoothing techniques in implied-volatility space and a new method of completing the density with appropriate tails. Tails are added using the theory of Generalized Extreme Value distributions, which are capable of characterizing very different behaviors of extreme events.[4] This method cleverly combines mid-prices of call and put options by only taking into account data from at-the-money and out-of-the-money regions, thus recovering non-standard features of risk-neutral densities such as bimodality, fat tails, and general asymmetry.

Our analysis covers fundamentally different market regimes around the Global Financial Crisis. A largely nonparametric approach, rather than a parametric one, seems therefore appropriate, because an important question that we shall ask is

[4] As Birru and Figlewski (2012) note, the theoretically correct extreme value distribution class is the Generalized Pareto Distribution (GPD) because estimating beyond the range of observable strikes corresponds to the peak-over-threshold method. For our purposes, both approaches are known to lead to equivalent results.

whether and how distributions actually changed from one regime to the next. We follow Figlewski's method in most steps, and additionally weight points by open interest when interpolating in implied-volatility space – a proxy of the information content of individual sampling points permitted by our data. We give a more detailed review of the method in appendix 3.A.

3.2.2 Data

We use end-of-day data for standard European call and put options on the S&P 500 stock index provided by Stricknet[5] for a period from January 1st, 2003 to October 23rd, 2013. The raw data includes bid and ask quotes as well as open interest across various maturities. For this study, we focus on option contracts with quarterly expiration dates, which usually fall on the Saturday following the third Friday in March, June, September and December, respectively. Closing prices of the index, dividend yields and interest rates of the 3-month Treasury Bill as a proxy of the risk-free rate are extracted from Thomson Reuters Datastream.

We apply the following filter criteria as in Figlewski (2010). We ignore quotes with bids below \$0.50 and those that are larger than \$20.00 in the money, as such bids exhibit very large spreads. Data points for which the midprice violates no-arbitrage conditions are also excluded. Options with time to maturity of less than 14 calendar days are discarded, as the relevant strike ranges shrink to smaller and smaller lengths resulting in a strong peaking of the density.[6] We are thus left with data for 2,311 observations over the whole time period and estimate risk-neutral densities and implied quantities for each of these days.

3.2.3 Subperiod classification

As the Global Financial Crisis had a profound and lasting impact on option-implied quantities, it is informative for the sake of comparison to perform analyses to subperiods associated with regimes classifiable as pre-crisis, crisis and post-crisis.

[5]The data is accessible via stricknet.com, where it can be purchased retrospectively.

[6]Figlewski (2010) points out that rollovers of hedge positions into later maturities around contract expirations may lead to badly behaved risk-neutral density estimates.

Rather than defining the relevant subperiods with historical dates, we follow an endogenous segmentation approach for identifying changes in the statistical properties of the risk-neutral densities. Let us assume we have an ordered sequence of data $x_{1:n} = (x_1, x_2, ..., x_n)$ of length n, e.g. daily values of a moment or tail shape parameter of the risk-neutral densities over n days. A change point occurs if there exists a time $1 \leq k < n$ such that the mean of set $\{x_1, ..., x_k\}$ is statistically different from the mean of set $\{x_{k+1}, ..., x_n\}$ (Killick et al., 2012). As a sequence of data may also have multiple change points, various frameworks to search for them have been developed. The binary segmentation algorithm by Scott and Knott (1974) is arguably the most established detection method of this kind. It starts by identifying a single change point in a data sequence, proceeds iteratively on the two segments before and after the detected change and stops if no further change point is found.

As in the case of estimating risk-neutral densities, we refrain from making assumptions regarding the underlying process that generates the densities and choose a nonparametric approach. We employ the numerical implementation of the binary segmentation algorithm by Killick et al. (2012) with the cumulative sum test statistic (CUSUM) proposed by Page (1954) to search for at most two change points. The idea is that the cumulative sum, $S(t) := \sum_{i=1}^{t} x_i$, $1 \leq t < n$, will have different slopes before and after the change point. As opposed to moving averages, using cumulative sums allows rapid detection of both small and large changes. We state the mathematical formulation of the test statistic in appendix 3.B.[7]

3.2.4 Determining lag-lead structures

Option-implied quantities may be seen as expectations of the (representative) investor under $\mathbb{Q}$. A popular question in the context of self-referential financial markets is whether expectations drive prices or vice versa. To get a feeling of the causality, we analyze the lag-lead structure between the time series based on the classical method due to Granger (1969). Informally, 'Granger causality' means that the knowledge of one quantity is useful in forecasting another. Formally, given two

[7] Interested readers may consult Brodsky and Darkhovsky (1993) as well as Csörgö and Horváth (1997) for a deeper discussion of theory, applications, and potential pitfalls of these methods.

time series X_t and Y_t, we test whether Y_t Granger-causes X_t at lag m as follows. We first estimate the univariate autoregression

$$X_t = \sum_{j=1}^{m} a_j X_{t-j} + \varepsilon_t, \tag{3.2}$$

where ε_t is an uncorrelated white-noise series. We then estimate the augmented model with lagged variables

$$X_t = \sum_{j=1}^{m} b_j X_{t-j} + \sum_{j=1}^{m} c_j Y_{t-j} + \nu_t, \tag{3.3}$$

where ν_t is another uncorrelated white-noise series. An F-test shows if the lagged variables collectively add explanatory power. The null hypothesis "Y_t does not Granger cause X_t" is that the unrestricted model (3.3) does not provide a significantly better fit than the restricted model (3.2). It is rejected if the coefficients $\{c_j, j = 1...m\}$ are statistically different from zero as a group. Since the model is only defined for stationary time series, we will test for Granger causality with standardized incremental time series in identified subperiods as described in section 3.2.3.

3.3 Results

3.3.1 First-to-fourth return moment analyses

We start by analyzing the moments and tail shape parameters of the option-implied risk-neutral densities over the whole period (see Figure 3.1). For comparability, we rescale the price densities by the S&P 500 index level S_t, i.e. assess $f(S_T/S_t)$ instead of $f(S_T)$.[8] In general, we recover similar values to the ones found by Figlewski (2010) over the period 1996 to 2008. The annualized option-implied log-returns of the S&P 500 stock index excluding dividends are defined as

$$r_t = \frac{1}{T-t} \int_0^\infty \log\left(\frac{S_T}{S_t}\right) f(S_T) \mathrm{d}S_T. \tag{3.4}$$

[8] We do not go into the analysis of the first moment, which, in line with efficient markets, is equal to 1 by construction of $f(S_T/S_t)$ (up to discounting).

They are on average negative with a mean value of -3%, and exhibit strong fluctuations with a standard deviation of 4%. This surprising finding may be explained by the impact of the Global Financial Crisis and by risk aversion of investors as explained below. The annualized second moment, also called risk-neutral volatility, is on average 20% (standard deviation of 8%). During the crisis from June 22nd, 2007 to May 4th, 2009, we observe an increase in risk-neutral volatility to $29 \pm 12\%$.

A skewness of -1.5 ± 0.9 and excess kurtosis of 10 ± 12 indicate strong deviations from log-normality, albeit subject to large fluctuations.[9] During the crisis, we measure a third (-0.9 ± 0.3) and fourth moment (4.4 ± 1.6) of the risk-neutral densities closer to those of a log-normal distribution than before or after the crisis. Birru and Figlewski (2012) find a similar dynamic using intraday prices for S&P 500 Index options. For the period from September 2006 until October 2007, they report an average skewness of -1.9 and excess kurtosis of 11.9, whereas from September to November 2008 these quantities change to -0.7 and 3.5, respectively.

As the fourth moment is difficult to interpret for a strongly skewed density, one must be careful with the implication of these findings. One interpretation is that, during crisis, investors put less emphasis on rare extreme events or potential losses, that is, on fat tails or leptokurtosis, while immediate exposure through a high standard deviation (realized risk) gains importance.[10] Another interpretation of the low kurtosis and large volatility observed during the crisis regime would be in terms of the mechanical consequences of conditional estimations. The following simple example illustrates this. Suppose that the distribution of daily returns is the sum of two Normal laws with standard deviations 3% and 20% and weights 99% and 1% respectively. This means that 99% of the returns are normally distributed with a standard deviation of 3%, and that 1% of the returns are drawn from a Gaussian distribution with a standard deviation of 20%. By construction, the unconditional excess kurtosis is non zero (27 for the above numerical example). Suppose that one

[9] For the sake of comparison, note that a log-normal distribution with standard deviation 20% has skewness of 0.6 and excess kurtosis of 0.7. In particular, skewness is always positive.

[10] In other words, this interpretation indicates that investors, during crisis, focus on the unfolding risk, while, during non-crisis regimes, investors worry more about possible/unlikely worst case scenarios. Related to this interpretation are hypothesis regarding human behavioral traits according to which risk-aversion versus risk-taking behaviors are modulated by levels of available attention (Gifford, 2010).

observes a rare spell of large negative returns in the range of -20%. Conditional on these realizations, the estimated volatility is large, roughly 20%, while the excess kurtosis close to 0 a consequence of sampling the second Gaussian law (and Gaussian distributions have by construction zero excess kurtosis).

It is interesting to note that Jackwerth and Rubinstein (1996) reported opposite behaviors in an early derivation of the risk-neutral probability distributions of European options on the S&P 500 for the period before and after the crash of October 1987. They observed that the risk-neutral probability of a one-standard deviation loss is larger after the crash than before, while the reverse is true for higher-level standard deviation losses. The explanation is that, after the 1987 crash, option traders realized that large tail risks were incorrectly priced, and that the volatility smile was born as a result thereafter (Mackenzie, 2008).

The left tail shape parameter ξ with values of 0.03 ± 0.23 is surprisingly small: a value around zero implies that losses are distributed according to a thin tail.[11] Moreover, with -0.19 ± 0.07, the shape parameter ξ for the right tail is consistently negative indicating a distribution with compact support, that is, a finite tail for expected gains.

3.3.2 Regime change points

A striking feature of the time series of the moments and shape parameters is a change of regime related to the Global Financial Crisis, which is the basis of our subperiod classification. A change point analysis of the left tail shape parameter identifies the crisis period as starting from June 22nd, 2007 and ending in May 4th, 2009. As we obtain similar dates up to a few months for the change points in risk-neutral volatility, skewness and kurtosis, this identification is robust and reliable (see Table 3.1 for details). Indeed, the determination of the beginning of the crisis as June 2007 is in agreement with the timeline of the build-up of the financial crisis[12] (Federal Reserve Bank of St. Louis, 2009), opening the gates of

[11] When positive, the tail shape parameter ξ is related to the exponent α of the asymptotic power law tail by $\alpha = 1/\xi$.

[12] (i) S&P's and Moody's Investor Services downgraded over 100 bonds backed by second-lien subprime mortgages on June 1, 2007, (ii) Bear Stearns suspended redemption of its credit strategy

loss and bankruptcy announcements. Interestingly, when applying the analysis to option-implied returns instead, we detect the onset of the crisis only on September 5th, 2008, more than a year later. This reflects a time lag of the market to fully endogenize the consequences and implication of the crisis. This is in line with the fact that most authorities (Federal Reserve, US Treasury, etc.) were downplaying the nature and severity of the crisis, whose full blown amplitude became apparent to all only with the Lehmann Brother bankruptcy.

The identification of the end of the crisis in May 2009 is confirmed by the timing of the surge of actions from the Federal Reserve and the US Treasury Department to salvage the banks and boost the economy via "quantitative easing", first implemented in the first quarter of 2009.[13] Another sign of a change of regime, which can be interpreted as the end of the crisis per se, is the strong rebound of the US stock market that started in March 2009, thus ending a strongly bearish regime characterized by a cumulative loss of more than 60% since its peak in October 2007.

Finally, note that the higher moments and tail shape parameters of the risk-neutral return densities in the post-crisis period from May 4th, 2009 to October 23, 2013 progressively recovered their pre-crisis levels.

3.3.3 Super-exponential return: bubble behavior before the crash

Apart from the market free fall, which was at its worst in September 2008, the second most remarkable feature of the time series of option-implied stock returns shown in Figures 3.1a and 3.2a is its regular *rise* in the years prior to the crisis. For the pre-crisis period from January 2003 to June 2007, a linear model estimates an average increase in the option-implied return of about 0.01% per trading day (p-value < 0.001, $R^2 = 0.82$, more details can be found in Table 3.2). As a matter

funds on June 7, 2007, (iii) S&P put 612 securities backed by subprime residential mortgages on credit watch, (iv) Countrywide Financial warned of "difficult conditions" on July 24, 2007, (v) American Home Mortgage Investment Corporation filed for Chapter 11 bankruptcy protection on July 31, 2007 and (vi) BNP Paribas, France's largest bank, halted redemptions on three investment funds on Aug. 9, 2007 and so on.

[13] On March 18, 2009 the Federal Reserve announced to purchase $750 billion of mortgage-backed securities and up to $300 billion of longer-term Treasury securities within the subsequent year, with other central banks such as the Bank of England taking similar measures.

of fact, this increase is also present in the realized returns, from January 2003 until October 2007, i.e. over a slightly longer period, as shown in Figure 3.2a. Note, however, that realized returns have a less regular behavior than the ones implied by options since the former are realized whereas the latter are expected under $\mathbb{Q}$. An appropriate smoothing such as the exponentially weighted moving average is required to reveal the trend, see Figure 3.2a for more details.

In the post-crisis period, in contrast, the option-implied returns exhibit less regularity, with smaller upward trends punctuated by abrupt drops. We find that option-implied returns rise on average 0.003% per trading day from May 2009 to October 2013 (p-value < 0.001). However, a coefficient of determination of $R^2 = 0.20$ suggests that this period is in fact not well-described by a linear model.

To the best of our knowledge, super-exponential price growth expectations have not previously been identified as implied by options data. This finding has several important implications that we shall now detail.

The upward trends of both option-implied and realized returns pre-crisis signal a transient "super-exponential" behavior of the market price, here of the S&P500 index. To see this, if the average return $r(t) := \ln[p(t)/p(t-1)]$ grows, say, linearly according to $r(t) \approx r_0 + \gamma t$ as can be approximately observed in Figure 3.2a from 2003 to 2007, this implies $p(t) = p(t-1)e^{r_0+\gamma t}$, whose solution is $p(t) = p(0)e^{r_0 t + \gamma t^2/2}$. In absence of the rise of return ($\gamma = 0$), this recovers the standard exponential growth associated with the usual compounding of interests. However, as soon as $\gamma > 0$, the price is growing much faster, in this case as $\sim e^{t^2}$. Any price growth of the form $\sim e^{t^\beta}$ with $\beta > 1$ is faster than exponential and is thus referred to as "super-exponential." Consequently, if the rise of returns is faster than linear, the super-exponential acceleration of the price is even more pronounced. For instance, Hüsler et al. (2013) reported empirical evidence of the super-exponential behaviour $p(t) \sim e^{e^t}$ in controlled lab experiments (which corresponds formally to the limit $\beta \to \infty$). Corsi and Sornette (2014) presented a simple model of positive feedback between the growth of the financial sector and that of the real economy, which predicts even faster super-exponential behaviour, termed transient finite-time singularity (FTS). This dynamics can be captured approximately by the novel FTS-

GARCH, which is found to achieve good fit for bubble regimes (Corsi and Sornette, 2014). The phenomenon of super-exponential price growth during a bubble can be accommodated within the framework of a rational expectation bubble (Blanchard, 1979; Blanchard and Watson, 1982), using for instance the approach of Johansen et al. (1999, 2000) (JLS model).[14] In a nutshell, these models represent crashes by jumps, whose expectations yield the crash hazard rate. Consequently, the condition of no-arbitrage translates into a proportionality between the crash hazard rate and the instantaneous conditional return: as the return increases, the crash hazard rate grows and a crash eventually breaks the price unsustainable ascension. See Sornette et al. (2013) for a recent review of many of these models.

Because super-exponential price growth constitutes a deviation from a long-term trend[15] that can only be transient, it provides a clear signature of a non-sustainable regime whose growing return at the same time embodies and feeds over-optimism and herding through various positive feedback loops. This feature is precisely what allows the association of these transient super-exponential regimes with what is usually called a "bubble" (Kaizoji and Sornette, 2009), an approach that has allowed bubble diagnostics ex-post and ex-ante (see e.g. Johansen et al., 1999; Sornette, 2003; Lin and Sornette, 2013; Sornette and Cauwels, 2014, 2015).

3.3.4 Dynamics of realized and option-implied returns

Realized S&P 500 and option-implied S&P 500 returns exhibit different behaviors over time (Figure 3.2a). Note that this difference persists even after filtering out short-term fluctuations in the realized returns.[16] During the pre-crisis period (from January 2003 to June 2007), the two grow at roughly the same rate, but the realized

[14] Alternative rational expectations frameworks include Sornette and Andersen (2002); Lin and Sornette (2013); Lin et al. (2014). Also related is the literature on mildly explosive bubbles (Phillips et al., 2011, 2012).

[15] Long-term exponential growth is the norm in economics, finance and demographics. This simply reflects the Gibrat law of proportional growth (Gibrat, 1931), which has an extremely broad domain of application (Yule, 1925; Simon, 1955; Saichev et al., 2009).

[16] Realized S&P 500 returns show more rapid fluctuations than option-implied ones, which is not surprising given that the former are realized whereas the latter are expected (under $\mathbb{Q}$). In this section we only focus on dynamics on a longer timescale, thus Figure 3.2a presents realized returns smoothed by an exponential weighted moving average (EWMA) of daily returns over 750 trading days. Different values or smoothing methods lead to similar outcomes.

returns grow are approximately 8% larger than the option-implied returns. This difference can be ascribed to the "risk premium" that investors require to invest in the stock market, given their aggregate risk aversion.[17] This interpretation of the difference between the two return quantities as a risk premium, which one may literally term "realized-minus-implied risk premium", is based on the fact that the option-implied return is determined under the risk-neutral probability measure while the realized return is, by construction, unfolding under the real-world probability measure.[18] In other words, the risk-neutral world is characterized by the assumption that all investors agree on asset prices just on the basis of fair valuation. In contrast, real-world investors are in general risk-adverse and require an additional premium to accept the risks associated with their investments. During the crisis, realized returns plunged faster and deeper in negative territory than the option-implied returns, then recovered faster into positive and *growing* regimes post-crisis. Indeed, during the crisis, the realized-minus-implied risk premium surprisingly became negative.

While the option-implied returns exhibit a stable behavior punctuated by two sharp drops in 2010 and 2011 (associated with two episodes of the European sovereign debt crisis), one can observe that the realized returns have been increasing since 2009, with sharp drop interruptions, suggesting bubbly regimes diagnosed by transient super-exponential dynamics (Sornette and Cauwels, 2015). Furthermore, the realized-minus-implied risk premium has steadily grown since 2009, reaching approximately 16% at the end of the analyzed period (October 2013), i.e. twice its pre-crisis value. This is qualitatively in agreement with other analyses (Graham and Harvey, 2013) and can be rationalized by the need for investors to be remunerated against growing uncertainties of novel kinds, such as created by unconventional policies and sluggish economic recovery.[19]

[17]To understand variations in the risk premium in relation to the identification of different price regimes, we cannot rely on many of the important more sophisticated quantitative methods for derivation of the the risk premium, but refer to the literature discussed in the introduction. There are many avenues for promising future research to develop hybrid approaches between these more sophisticated approaches and ours which a priori allows the premium to vary freely over time.

[18]The standard definition, which usually takes the expected 10-year S&P 500 return relative to a 10-year U.S. Treasury bond yield (Fernandez, 2009; Duarte and Rosa, 2013) captures different information.

[19]An incomplete list of growing uncertainties at that time is: instabilities in the middle-East, concerns about sustainability of China's growth and issues of its on-going transitions, and many

3.3.5 Granger causality between option-implied returns and the 3-month Treasury Bill

We now examine possible Granger-causality relationships between option-implied returns and 3-month Treasury Bill yields. First note that option-implied returns and the 3-month Treasury Bill yields reveal a much weaker correlation than between realized returns and option-implied returns. A casual glance at Figure 3.2b suggests that their pre-crisis behaviors are similar, up to a vertical translation of approximately 3%. To see if the Fed rate policy might have been one of the drivers of the pre-crisis stock market dynamics, we perform a Granger causality test in both directions. Since a Granger test is only defined for stationary time series, we consider first differences in option-implied S&P 500 returns and 3-month Treasury Bill yields, respectively. Precisely, we define

$$SP_t = r_t - r_{t-1}, \quad TB_t = y_t - y_{t-1}. \tag{3.5}$$

where r_t is the option-implied return (3.4) and y_t is the Bill yield at trading day t. Before testing, we standardize both SP_t and TB_t, i.e. we subtract the mean and divide by the standard deviation, respectively.

There is no evidence that Federal Reserve policy has influenced risk-neutral option-implied returns over this period, as a Granger causality test fails to reject the relevant null at any lag (see Table 3.3 and Figure 3.3a). The other direction of Granger causality is more interesting, revealing Granger-causal influence of the option-implied returns on the 3-month Treasury Bill. A Granger causality test for SP_t on TB_t rejects the null for a lag of $m = 5$ trading days. This suggests that the Fed policy has been responding to, rather than leading, the development of the market expectations during the pre-crisis period. Previous works using a time-adaptive lead-lag technique had only documented that stock markets led Treasury Bills yields as well as longer term bonds yields during bubble periods (Zhou and Sornette, 2004; Guo et al., 2011). It is particularly interesting to find a Granger causality of the

other uncertainties involving other major economic players, such as Japan, India and Brazil, quantitative easing operations in the US, political will from European leaders and actions of the ECB to hold the eurozone together.

forward-looking expected returns, as extracted from option data, onto a backward-looking Treasury Bill yield in the pre-crisis period and the reverse thereafter. Thus, expectations were dominant in the pre-crisis period as is usually the case in efficient markets, while realized monetary policy was (and still is in significant parts) shaping expectations post-crisis (as shown in Table 3.3 and Figure 3.3b). The null of no influence is rejected for Treasury Bill yields Granger causing option-implied returns lagged by 50 to 200 days. This is coherent with the view that the Fed monetary policy, developed to catalyze economic recovery via monetary interventionism, has been the key variable influencing investors and thus options/stock markets.

Analyses of Granger causality with respect to realized returns yield no comparable results. Indeed, mutual influences with respect to Bivariate Granger tests involving the first difference time series of realized returns (with both option-implied returns and Treasury Bill yields) confirm the results that would have been expected. Both prior to and after the crisis, Treasury Bill yields Granger-cause realized returns over long time periods ($p < 0.1$ for lags of 150 and 200 trading days, respectively), whereas option-implied returns Granger-cause realized ones over short time periods ($p < 0.01$ for a lag of 5 trading days).

3.4 Conclusion

We have extracted risk-neutral return probability distributions from S&P 500 stock index options from 2003 to 2013. Change point analysis identifies the crisis as taking place from mid-2007 to mid-2009. The evolution of risk-neutral return probability distributions characterizing the pre-crisis, crisis and post-crisis regimes reveal a number of remarkable properties. Indeed paradoxically at first sight, the distributions of expected returns became very close to a normal distribution during the crisis period, while exhibiting strongly negative skewness and especially large kurtosis in the two other periods. This reflects that investors may care more about the risks being realized (volatility) during the crisis, while they focus on potential losses (fat left tails, negative skewness and large kurtosis) in quieter periods.

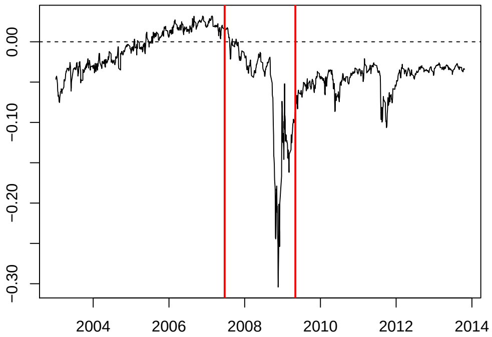
Our most noteworthy finding is the continuing increase of the option-implied average returns during the pre-crisis (from January 2003 to mid-2007), which more than parallels a corresponding increase in realized returns. While a constant average return implies standard exponential price growth, an increase of average returns translates into super-exponential price growth, which is unsustainable and therefore transient. This finding corroborates previous reports on increasing realized returns and accelerated super-exponential price trajectories, which previously have been found to be hallmarks of exuberance and bubbles preceding crashes.

Moreover, the comparison between realized and option-implied expected returns sheds new light on the development of the pre-crisis, crisis and post-crisis periods. A general feature is that realized returns adapt much faster to changes of regimes, indeed often overshooting. Interpreted as a risk premium, literally the "realized-minus-implied risk premium", these overshoots can be interpreted as transient changes in the risk perceptions of investors. We find that the realized-minus-implied risk premium was approximately 8% in the pre-crisis, and has doubled to 16% in the post-crisis period (from mid-2009 to October 2013). This increase is likely to be associated with growing uncertainties and concern with uncertainties, fostered possibly by unconventional financial and monetary policy and unexpectedly sluggish economic recovery.

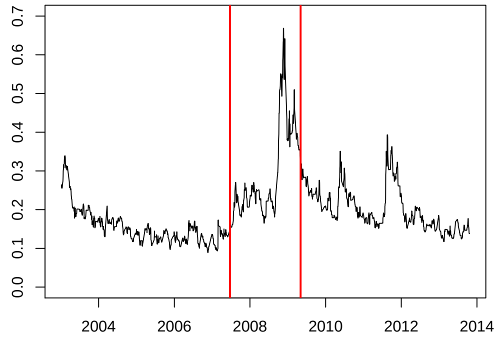
Finally, our Granger causality tests demonstrate that, in the pre-crisis period, changes of option-implied returns lead changes of Treasury Bill yields with a short lag, while the reverse is true with longer lags post-crisis. In a way, the post-crisis period can thus be seen as a return to a "normal" regime in the sense of standard economic theory, according to which interest rate policy determines the price of money/borrowing, which then spills over to the real economy and the stock market. What makes it a "new normal" (El-Erian, 2011) is that zero-interest rate policies in combination with other unconventional policy actions actually dominate and bias investment opportunities. The pre-crisis reveals the opposite phenomenon in the sense that expected (and realized returns) lead the interest rate, thus in a sense "slaving" the Fed policy to the markets. It is therefore less surprising that such an abnormal period, previously referred to as the "Great Moderation" and hailed as the successful taming of recessions, was bound to end in disappointments as a bubble was built up (Sornette and Woodard, 2010; Sornette and Cauwels, 2014).

These results make clear the existence of important time-varying dynamics in both equity and variance risk premia, as exemplified by the difference between the pre- and post-crisis periods in terms of the Granger causalities. The option-implied returns show that expectations have been changed by the 2008 crisis, and this confirms another massive change of expectations following the crash of October 1987, embodied in the appearance of the volatility smile (Mackenzie, 2008). We believe that extending our analysis to more crises will confirm the importance of accounting for changes of expectations and time-varying premia, and we will address these issues in future research.

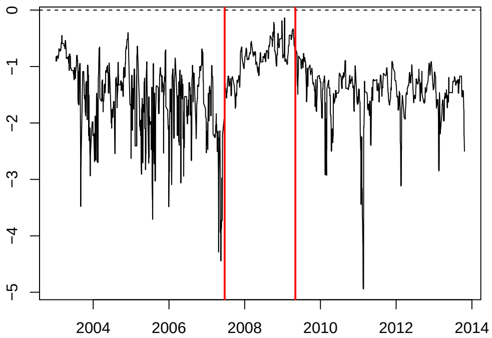
Returns and distributional moments implied by S&P 500 options



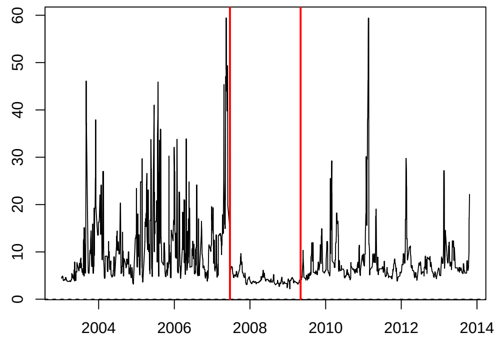
(a) Ann. expected returns.



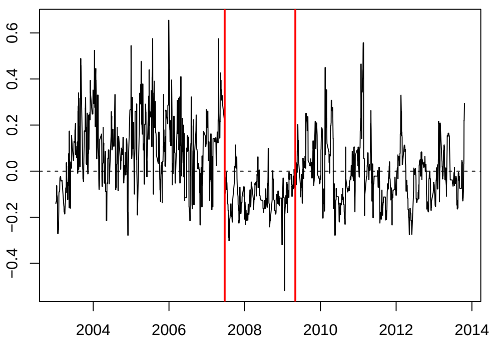
(b) Ann. risk-neutral volatility.



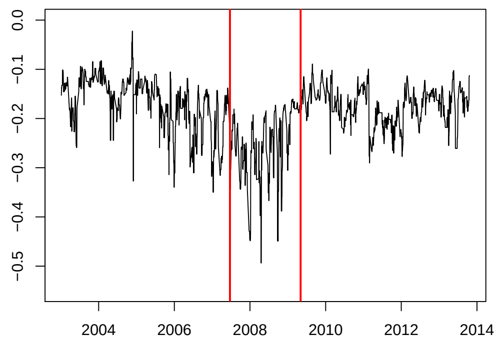
(c) Skewness.



(d) Excess kurtosis.



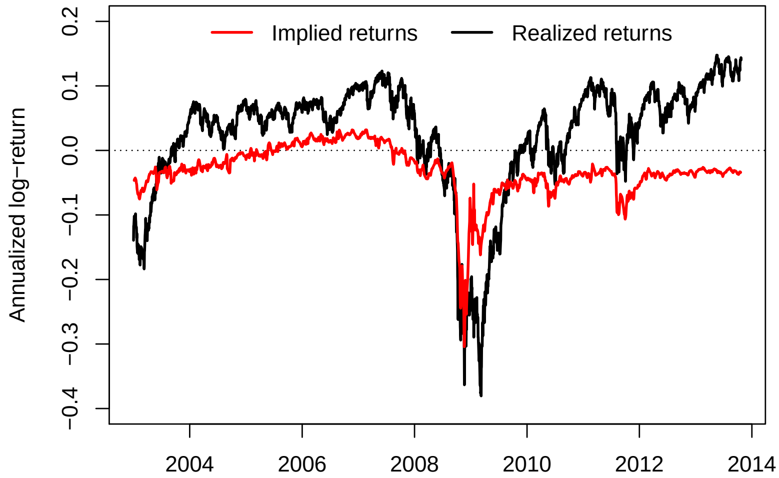
(e) Left tail shape parameter.



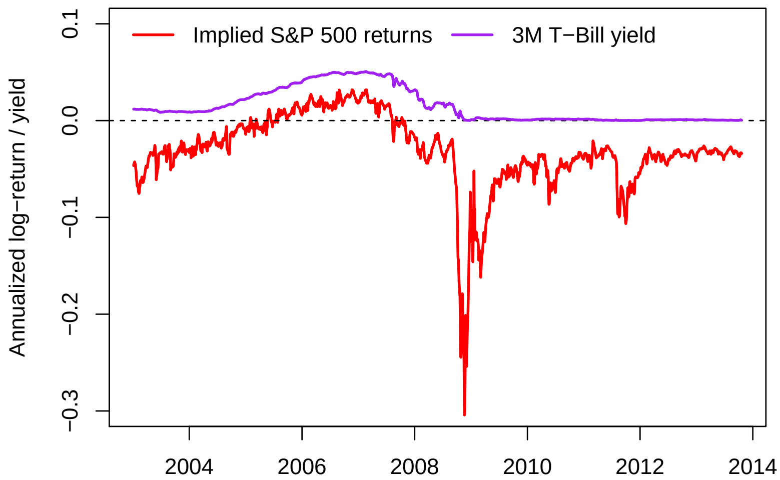
(f) Right tail shape parameter.

Figure 3.1: This figure presents returns and distributional moments implied by S&P 500 options. Structural changes around the financial crisis are identified consistently with a change point analysis of the means of the higher moments and tail shape parameters (vertical lines).

Option-implied returns vs realized returns and Treasury Bill yields

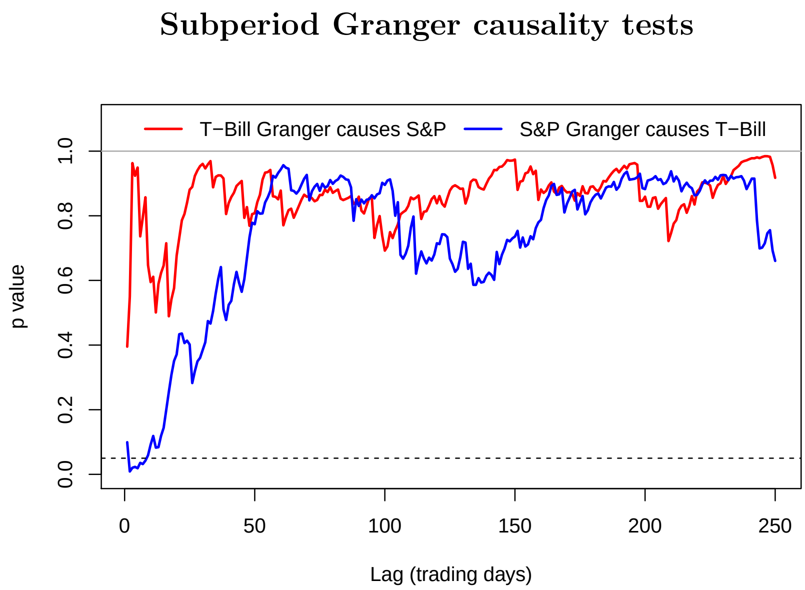


(a) Annualized realized returns and option-implied S&P 500 returns. Realized returns are calculated by exponential weighted moving average (EWMA) smoothing of daily returns over 750 trading days.

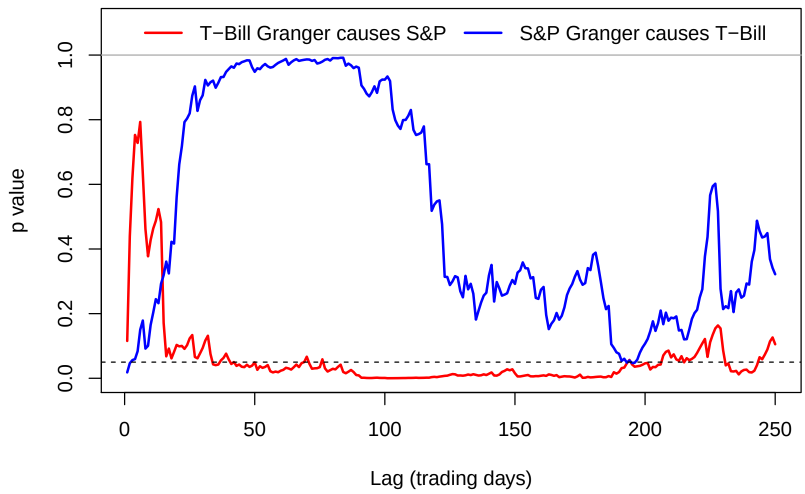


(b) 3-month Treasury Bill yields and annualized option-implied S&P 500 returns (5-day moving averages).

Figure 3.2: This figure presents time series of option-implied S&P 500 returns, realized returns and Treasury Bill yields over the time period 2003–2013.



(a) Pre-crisis: January 1st, 2003 to June 22nd, 2007. No evidence for Treasury Bill yields Granger causing option-implied S&P 500 returns at any lag, but rather that option-implied S&P 500 returns Granger cause Treasury Bill yields at lags of a few trading days.



(b) Post-crisis: May 4th, 2009 to October 23rd, 2013. Treasury Bill yields Granger cause option-implied S&P 500 returns over a large range of lags.

Figure 3.3: Subperiod Granger causality tests on incremental changes in annualized option-implied S&P 500 returns and 3-month Treasury Bill yields. The $p = 0.05$ line is plotted as dashed black.

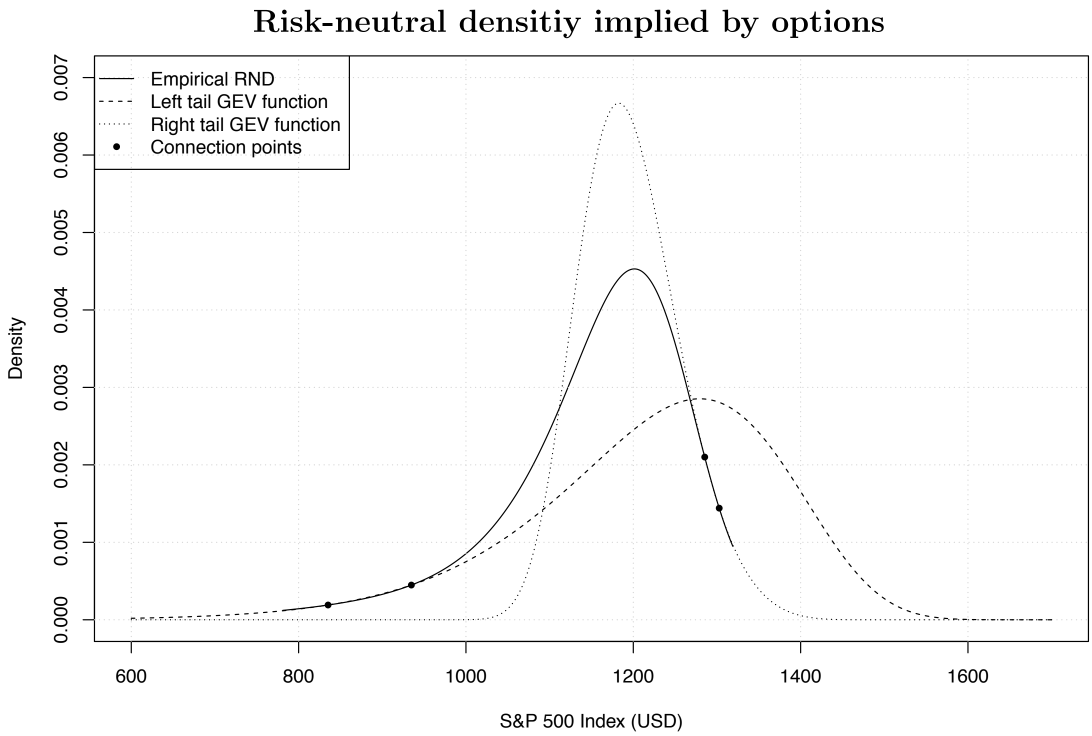


Figure 3.4: Risk-neutral density implied by S&P 500 options from 2010-10-06 for the index level on 2010-12-18. The empirical part is directly inferred from option quotes, whereas tails must be estimated to account for the range beyond observable strike prices. Together, they give the full risk-neutral density. The method is reviewed in section 3.2.1 and appendix 3.A.

Table 3.1: Start and end dates of the Global Financial Crisis as identified by a change point analysis of statistical properties of option-implied risk-neutral densities. The dates found in the left tail shape parameter and higher moments identify consistently the crisis period as ca. June 2007 to ca. October 2009. Interestingly, the return time series signals the beginning only more than a year later, as September 2008. See section 3.2.3 for a review of the method, and 3.3.2 for a more detailed discussion of the results.

Variable	*Crisis start date*	*Crisis end date*
Left tail shape parameter	2007-06-22***	2009-05-04***
Right tail shape parameter	2005-08-08***	2009-01-22***
Risk-neutral volatility	2007-07-30***	2009-11-12***
Skewness	2007-06-22***	2009-10-19***
Kurtosis	2007-06-19***	*NA*[a]
Option-implied returns	2008-09-05***	2009-07-17***

Note: *p<0.1; **p<0.01; ***p<0.001

[a] No change point indicating a crisis end date found.

Table 3.2: Results of a linear regression of option-implied returns of the S&P 500 index on time (trading days) by sub-period. In particular, a linear model fits well the pre-crisis, indicating the regular rise of expected returns, but not the post-crisis. This translates into super-exponential price growth expectations in the pre-crisis period. Standard deviations are in parentheses.

	Option-implied returns (in percent):		
	Pre-crisis	Crisis	Post-crisis
linear coefficient	0.009^{***}	-0.043^{***}	0.003^{***}
per trading day	(0.0001)	(0.002)	(0.0002)
Constant	-4.747^{***}	2.836^{***}	-5.775^{***}
	(0.072)	(0.485)	(0.097)
Observations	942	411	958
R^2	0.820	0.520	0.196
Note:			*p<0.1; **p<0.01; ***p<0.001

Table 3.3: This table reports the results of a Granger-causality test of option-implied S&P 500 returns and Treasure Bill yields by sub-period. While we do not find evidence that Treasury Bill yields may have Granger-caused implied returns pre-crisis, there is Granger-influence in the other direction at a lag of 5 trading days both pre- and especially post-crisis. Notably, our test strongly suggests that post-crisis Treasury Bill yields have Granger-causal influence on option-implied returns at lags of 50 to 200 trading days.

Pre-crisis				
	S&P Granger-causes T-Bill		*T-Bill Granger-causes S&P*	
Lag	F-ratio[a]	Degrees of freedom	F-ratio[a]	Degrees of freedom
5	2.72*	5, 926	0.23	5, 926
50	0.84	50, 791	0.82	50, 791
100	0.82	100, 641	0.92	100, 641
150	0.92	150, 491	0.77	150, 491
200	0.86	200, 341	0.87	200, 341
250	0.95	250, 191	0.83	250, 191
Post-crisis				
	S&P Granger-causes T-Bill		*T-Bill Granger-causes S&P*	
Lag	F-ratio[a]	Degrees of freedom	F-ratio[a]	Degrees of freedom
5	1.95*	5, 942	0.56	5, 942
50	0.69	50, 807	1.37*	50, 807
100	0.79	100, 657	1.55**	100, 657
150	1.07	150, 507	1.32*	150, 507
200	1.16	200, 357	1.23*	200, 357
250	1.06	250, 207	1.18	250, 207

Note: *p<0.1; **p<0.01; ***p<0.001

[a] Refers to the F-test for joint significance of the lagged variables.

Appendix

3.A Estimating the risk-neutral density from option quotes

In this study, we estimate the option-implied risk-neutral density with a method developed by Figlewski (2010), which is based on equation (3.1). For completeness, we shall briefly review the method as employed in this paper, but refer the interested reader to the original document for more detail. The raw data are end-of-day bid and ask quotes of European call and put options on the S&P 500 stock market index with a chosen maturity. Very deep out of the money options exhibit spreads that are large relative to the bid, i.e. carry large noise. Due to the redundancy of calls and puts, we may discard quotes with bid prices smaller than \$0.50. In this paper, we perform the calculation with mid-prices, which by inverting the Black-Scholes model translate into implied volatilities.

In a window of ±\$20.00 around the at-the-money level, the implied volatilities of put and call options are combined as weighted averages. The weights are chosen in order to ensure a smooth transition from puts to calls by gradually blending calls into puts when going to higher strikes. Below and above that window, we only use call and put data, respectively. We then fit a fourth order polynomial in implied volatility space. Here, we deviate slightly from Figlewski (2010) because we use open interest as fitting weights. By doing so, we give more weight to data points carrying more market information. The Black-Scholes model transforms the fit in implied volatility space back to price space. The resulting density bulk is called "empirical density".

To obtain a density estimate beyond the range of observable strike prices, we must append tails to the empirical part. Figlewski (2010) proposes to add tails of the family of generalized extreme value (GEV[20]) distributions with connection conditions: a) matching value at the 2%, 5%, 92% and 95% quantile points, and b) matching probability mass in the estimated tail and empirical density. An example can be seen in Figure 3.4. The empirical density together with the tails give the complete risk-neutral density.

3.B Change point detection

The following framework is used for significance testing in section 3.3.2 and Table 3.1. For more details, see Csörgö and Horváth (1997). Let $x_1, x_2, ..., x_n$ be independent, real-valued observations. We test the "no change point" null hypothesis,

$$H_0 : \mathbb{E}(x_1) = \mathbb{E}(x_2) = ... = \mathbb{E}(x_n), \tag{3.6}$$

against the "one change in mean" hypothesis,

$$H_1 : \text{there is a } k, 1 \leq k < n, \text{ such that } \mathbb{E}(x_1) = ... = \mathbb{E}(x_k) \neq \mathbb{E}(x_{k+1}) = ... = \mathbb{E}(x_n), \tag{3.7}$$

using the auxiliary functions

$$A(x) := \sqrt{2 \log \log x} \ , \quad D(x) := 2 \log \log x + \frac{1}{2} \log \log \log x - \frac{1}{2} \log \pi \ . \tag{3.8}$$

Then, following corollary 2.1.2 and in light of remark 2.1.2. (Csörgö and Horváth, 1997, pp. 67-68), under mild regularity conditions, H_0 and for large sample sizes, one has

$$P\left(A(n) \max_k \frac{1}{\hat{\sigma}_n} \left(\frac{n}{k(n-k)} \right)^{1/2} \left| S(k) - \frac{k}{n} S(n) \right| - D(n) \leq t \right) = \exp\left(-2\mathrm{e}^{-t}\right), \tag{3.9}$$

where $\hat{\sigma}_n$ is the sample standard deviation and $S(t) := \sum_{i=1}^{t} x_i$ the cumulative sum of observations.

[20]See Embrechts et al. (1997) for a detailed theoretical discussion of GEV distributions and modeling extreme events.

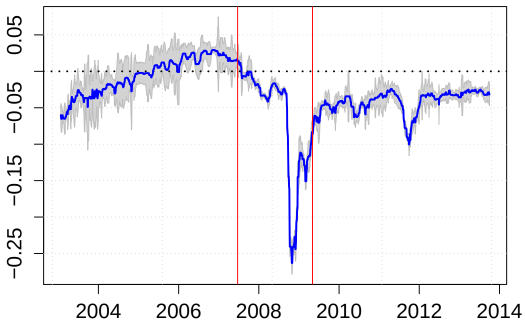
3.C Robustness tests based on Monte Carlo simulations

The density estimation method by Figlewski (2010) outlined in section 3.A proposes to fit a function to the implied volatilities of mid prices, i.e. the averaged bid and ask prices. In this section, we analyze the robustness of our results by using interpolation points that are chosen uniformly at random from the bid-ask spread instead. This allows us to study the implications of both the somewhat arbitrary choice of fitting mid-price volatilities and the inevitable microstructure noise for the estimated risk-neutral densities. Or, put differently, we quantify to what extent uncertainty in the input data leads to uncertainty in the outputs. Thus, we estimate every risk-neutral density 500 times with prices drawn i.i.d. uniformly at random from the bid-ask spread for each strike and estimation. This procedure based on repeated random sampling is also known as Monte Carlo method (Hammersley and Handscomb, 1964). Ye (1998) proposes a similar idea to estimate model sensitivity by applying perturbations to observed data.

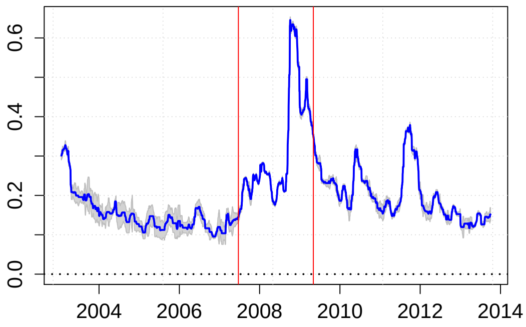
Figures 3.C.1 and 3.C.2 show the returns and moments of mid-price data with one-standard-deviation bands computed from the Monte Carlo simulations implied by monthly and quarterly S&P 500 options, respectively. Figure 3.C.3 presents the information of those graphs plotted together. In general, the uncertainty in quarterly options (as used in the analysis of this chapter) is smaller than in monthly options. This is probably due to the higher liquidity of quarterly options that reduces both bid-ask spreads and microstructure noise. As the traded volume of options increased over our data period, we also observe shrinking standard-deviation bands over time.

Overall, the main findings of the chapter hold despite the intrinsic and inevitable uncertainty in the input data. Those are a rate of return that rises in the pre-crisis period and remains flat after the crisis, as well as markedly different higher moments and tail shape parameters during the crisis as to compared to the non-crisis periods.

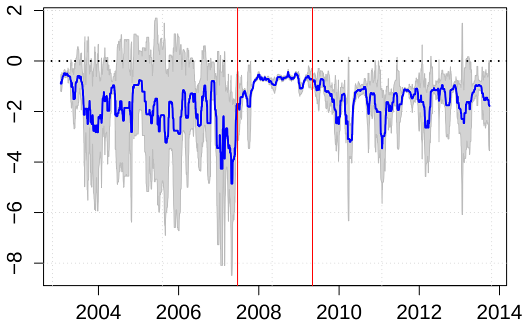
Returns and moments implied by monthly S&P 500 options



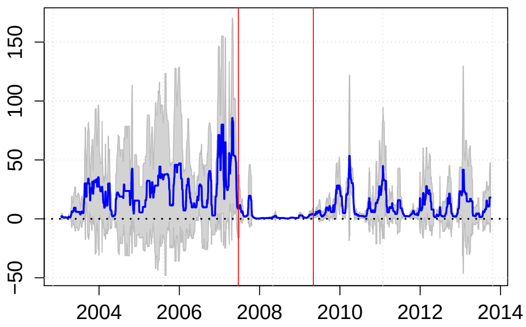
(a) Ann. expected returns.



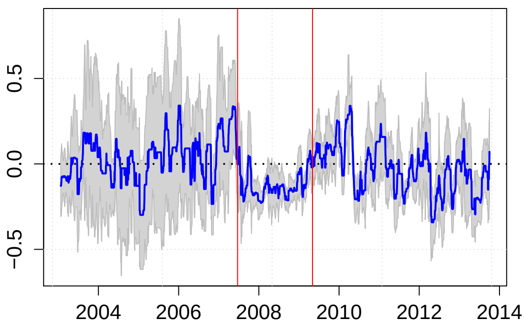
(b) Ann. risk-neutral volatility.



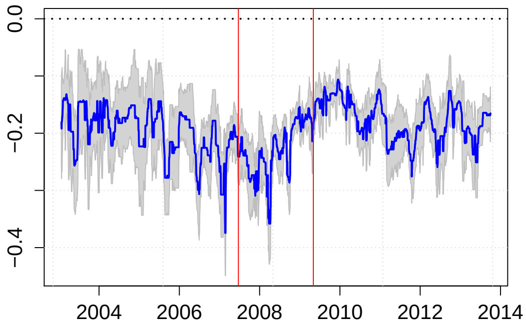
(c) Skewness.



(d) Excess kurtosis.



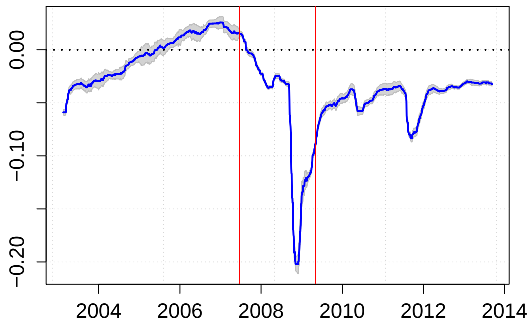
(e) Left tail shape parameter.



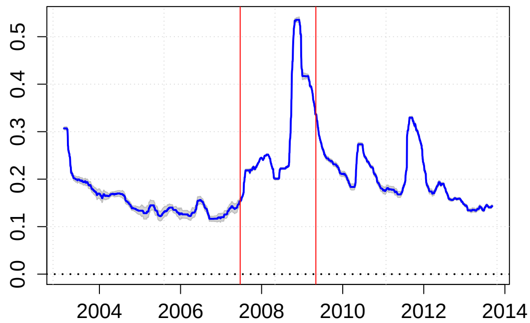
(f) Right tail shape parameter.

Figure 3.C.1: This figure presents returns and distributional moments implied by S&P 500 options with time to maturity of one month. Structural changes around the financial crisis are identified consistently with a change point analysis of the means of the higher moments and tail shape parameters (vertical lines). The grey area marks the one-standard-deviation band based on 500 Monte Carlo simulations of density estimation. The time series have been smoothed by a 1-month rolling median.

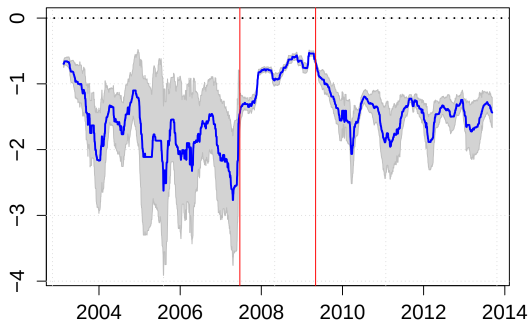
Returns and moments implied by quarterly S&P 500 options



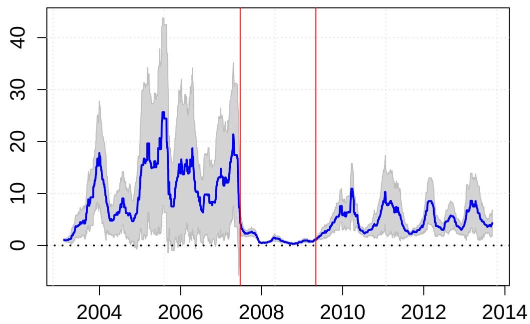
(a) Ann. expected returns.



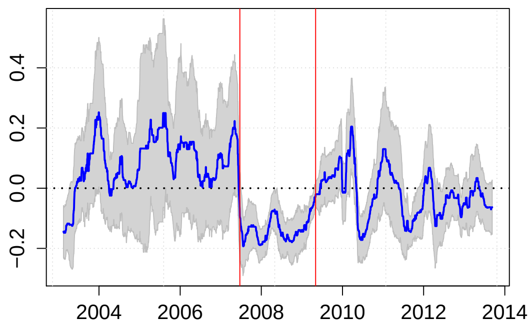
(b) Ann. risk-neutral volatility.



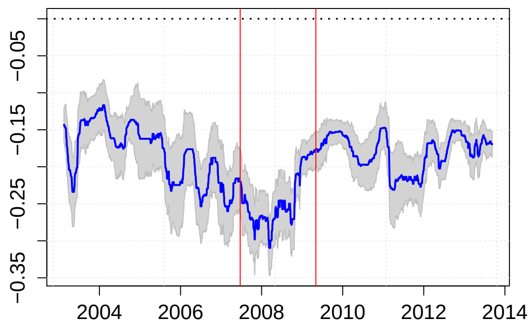
(c) Skewness.



(d) Excess kurtosis.



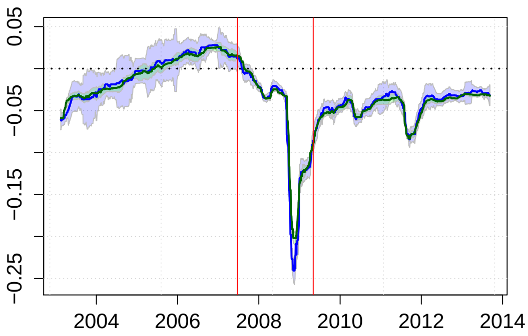
(e) Left tail shape parameter.



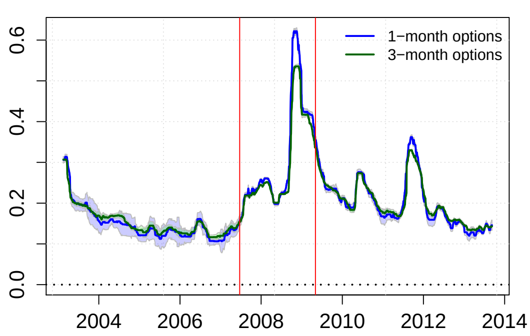
(f) Right tail shape parameter.

Figure 3.C.2: This figure presents returns and distributional moments implied by S&P 500 options with time to maturity of up to three months. Structural changes around the financial crisis are identified consistently with a change point analysis of the means of the higher moments and tail shape parameters (vertical lines). The grey area marks the one-standard-deviation band based on 500 Monte Carlo simulations of density estimation. The time series have been smoothed by a 3-months rolling median.

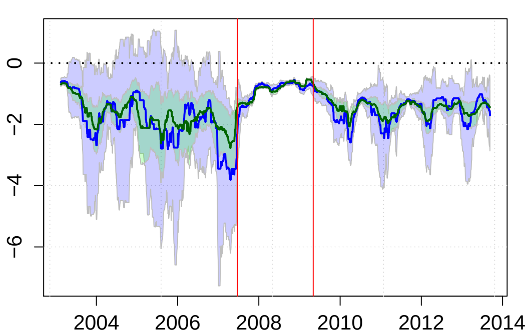
Returns and moments implied by monthly and quarterly S&P 500 options



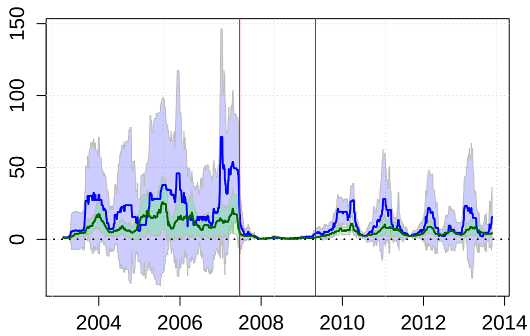
(a) Ann. expected returns.



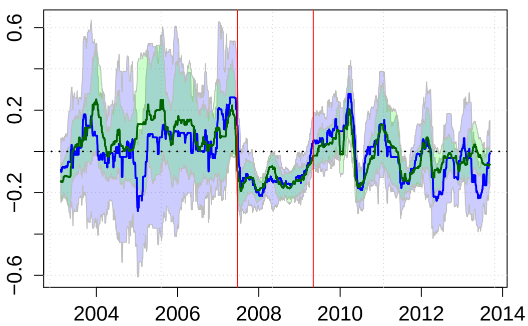
(b) Ann. risk-neutral volatility.



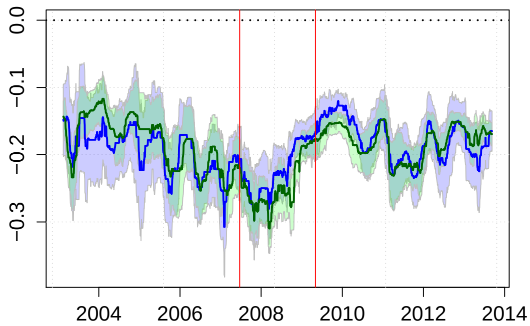
(c) Skewness.



(d) Excess kurtosis.



(e) Left tail shape parameter.



(f) Right tail shape parameter.

Figure 3.C.3: This figure compares returns and distributional moments implied by S&P 500 options with time to maturity of up to one month (blue) and three months (green), respectively. Structural changes around the financial crisis are identified consistently with a change point analysis of the means of the higher moments and tail shape parameters (vertical lines). The shaded areas marks the one-standard-deviation band based on 500 Monte Carlo simulations of density estimation. The time series have been smoothed by a 3-months rolling median.

Chapter 4

Option-Implied Objective Measures of Market Risk

This chapter is an edited version of Leiss and Nax (2015).

Abstract

Foster and Hart (2009) introduce an *objective* measure of the riskiness of an asset that implies a bound on how much of one's wealth is 'safe' to invest in the asset while (a.s.) guaranteeing no-bankruptcy in the long run. In this study, we translate the Foster-Hart measure from static and abstract gambles to dynamic and applied finance using nonparametric estimation of risk-neutral densities from S&P 500 call and put option prices covering 2003 to 2013. This exercise results in an option-implied market view of objective riskiness. The dynamics of the resulting 'option-implied Foster-Hart bound' are analyzed and assessed in light of well-known risk measures including value at risk, expected shortfall and risk-neutral volatility. The new measure is shown to be a significant predictor of ahead-return downturns. Furthermore, it is able to grasp more characteristics of the risk-neutral probability distributions than other measures, furthermore exhibiting predictive consistency. The robustness of the risk-neutral density estimation method is analyzed via Monte Carlo methods.

The price which a man whose available fund is n pounds may prudently pay for a share in a speculation... (Whitworth 1870, p.217)

4.1 Introduction

Foster and Hart (2009) introduce a concept that relates the riskiness of a given gamble to a critical wealth level above which it is 'safe' to enter that gamble. Entering gambles below the critical wealth level is not safe in the sense that it results in risk exposures that exhibit a positive probability of bankruptcy in finite time. Conversely, safe gambles (a.s.) guarantee no-bankruptcy. Importantly, the Foster-Hart risk measure is law-invariant; i.e. it depends only on the underlying distribution and not on the risk attitude of the investor. In this sense Foster and Hart (2009) refer to it as 'objective'.

Thus far, despite its interesting theoretical properties, the Foster-Hart criterion for no-bankruptcy has not been applied much in finance.[1] In this paper, we set out to propose a novel application of the measure using an option-implied (hence forward-looking) perspective on the stock market, and and to evaluate the resulting option-implied risk measure as regards its predictive significance and consistency.

Before we proceed to elaborate on our application in detail, it is worth placing the Foster-Hart criterion amongst the most closely related risk measures, namely the 'economic index of riskiness' (Aumann and Serrano, 2008) and the 'Kelly criterion' (Kelly, 1956; Samuelson, 1979). The 'economic index of riskiness' measures a gamble's risk instead of individuals' risk perceptions (Aumann and Serrano, 2008). The Aumann-Serrano index constitutes the motivation for the formulation of the Foster-Hart criterion as an 'objective' (also 'operational') interpretation as a wealth level/fraction. The resulting Foster-Hart criterion resembles the Kelly criterion formally with the difference that Kelly maximizes growth rates over sequences of gambles, while Foster-Hart guarantees no-bankruptcy.[2] Both, Kelly and Foster-Hart criteria are deeply rooted in the very origins of mathematical risk analyses, and in-

[1] Exceptions include Anand et al. (2015); Bali et al. (2011, 2012).

[2] We shall make the mathematical connections clear when we formally introduce Foster-Hart riskiness in the next section.

deed they are both first expressed in Whitworth's seminal book *Choice and Chance* in the year 1870.[3] Importantly, these measures have important decision-theoretic properties that are interesting for finance applications, and, moreover, thet have been shown to matter crucially from the perspective of evolutionary stability of stock markets (Evstigneev et al., 2006).

In this note, we translate FH from abstract gambles to applied option-market dynamics using nonparametric estimation of risk-neutral densities from S&P 500 call and put option prices covering 2003 to 2013. As the underlying decisions are purchases of stocks, we are looking at scalable gambles, which means that the Foster-Hart measure of riskiness as a 'critical wealth level' can be re-interpreted as a 'bound' (between zero and one) that defines the share of one's wealth that is safe to invest. Henceforth, we shall write 'FH' as shorthand for the Foster-Hart measure of riskiness in the bounds/shares interpretation.[4]

We shall address the question of *how much of one's wealth can one, in the sense of Foster and Hart (2009), safely invest in the S&P 500 stock index.*

Obviously, the answer is not straightforward, because the pig in the poke regarding such real-world investment decisions is the underlying probability distribution of the stock market, which typically is unknown not only to the decision-maker but also to us as scientists. One may use the word 'speculation' instead of 'gamble' to stress this feature (as in the above citation by Whitworth 1870).[5] Fundamental to our analysis is therefore a formulation of probability distributions for developments of the S&P 500 stock index over some finite horizon. One way to approach this would be to employ historical return distributions, possibly in combination with a dynamic model as, for example, in Anand et al. (2015). We, by contrast, extract risk-neutral densities from the information contained in call and put option prices written on the S&P 500 stock index.

The economic rationale for choosing the option-implied approach over other alternatives is that the option-implied information is inherently forward-looking, hence

[3]See pp. 216-219 for the no-bankruptcy proofs.

[4]The crucial (a.s.) no-bankruptcy guarantee is preserved by FH.

[5]Indeed, many of the rare real-world situations where the return distributions are known to the decision-maker (and to the scientist) in a strict sense are gambling or casino situations, where FH would typically commend not to invest at all.

the information used to quantify the ‘gamble’ regarding future realizations of the S&P 500 stock index is both available when the investment decision made and directly targeted at the consequences of that decision. We rely on efficient markets or, perhaps somewhat weaker, a “wisdom of the market” argument as regards why this information should be informative, instead of making specific modeling assumptions. In other words, we assume that the options are priced, on average, accurately to reflect underlying market risks. The resulting ‘option-implied Foster-Hart bound’ is an aggregation of the market information to a fraction of one’s wealth that is ‘safe’ to invest in the S&P 500 stock index while guaranteeing no-bankruptcy.

The motivating assumption is that the investor considers how much of his (unleveraged) wealth to invest in one single index stock, in our case one that is tied to the S&P 500, based on information about the associated options market. We analyze and assess option-implied FH in light of well-known risk measures including value at risk (VaR), expected shortfall (ES), risk-neutral volatility (RNV) and the volatility index (VIX), as well as the spread (TED) between the London Interbank Offered Rate (LIBOR) and Treasury bills (T-Bill) as a measure of credit risk. This approach turns out to be predictively fruitful, especially in predicting large market downturns.

There are, however, some technical issues we have to address to make the exercise work, because, in contrast to the original formulation of FH, the gamble of investing in a stock based on option-implied information is both *continuous* and *dynamic*. The original operationalization by Foster and Hart (2009) has recently been generalized to our setting by Riedel and Hellmann (2015) and Hellmann and Riedel (2015). To get most information out of the options data, our estimation of the underlying risk-neutral densities (RNDs) is done nonparametrically from S&P 500 call and put options prices using a variant of the method by Figlewski (2010) introduced in (Leiss et al., 2015).

The empirical literature that is directly related to ours, to the best of our knowledge, only consists of two excellent, recent papers; Bali et al. (2011, 2012). Bali et al. (2011) propose a generalized measure of riskiness nesting those of Aumann and Serrano (2008) and Foster and Hart (2009). Their measure is shown to significantly predict risk-adjusted market returns, and in some cases even outperforms standard risk measures as evaluated historically. Bali et al. (2012) build on Bali et al. (2011), but focus on economic downturns, finding empirical support for both Foster-Hart and Aumann-Serrano. Compared with our approach, there are three key differences between ours and the studies by Bali et al. (2011) and Bali et al. (2012), and while we advance beyond them in some dimensions, we also take steps backward in others.

First, quite differently to the risk measure introduced by Bali et al. (2011), which is a generalization of Foster and Hart (2009), ours is an attempt to return to the objectivity feature of Foster-Hart in terms of its independence from risk aversion. We achieve this by making a somewhat 'brutal' (because direct) move from the physical probability measure ($\mathbb{P}$) to the option-implied risk-neutral measure ($\mathbb{Q}$). While this is inappropriate for most financial analyses, where alternative approaches are preferred (e.g., Bliss and Panigirtzoglou 2004; Kostakis et al. 2011), the direct $\mathbb{P}$-$\mathbb{Q}$ move is both theoretically and empirically validated in our setting as it results in a 'bound for the Foster-Hart bound'; i.e. taking into account agents' risk-aversion would only lead to higher and thus riskier values for FH. It is in this sense that our approach is closer in spirit to the original 'satisficing' approach of (Foster and Hart, 2009, p. 802) (as opposed to an 'optimizing' approach as in, for example, Kelly 1956). Furthermore, our approach thus also remains independent of risk aversion, making it 'objective' as in Foster and Hart (2009) (as opposed to Aumann and Serrano 2008 and Bali et al. 2011).

Second, in contrast to Bali et al. (2011), and more in the spirit of Bali et al. (2012), we compare FH with other option-implied risk measures rather than with historical measures. We believe this establishes a somewhat level playing field, as then all measures are forward-looking. Our estimation of full RNDs allows us to calculate virtually any option-implied risk measure.

Finally, and perhaps most importantly, we address the dynamic feature of option-implied information as the time to maturity diminishes. By contrast, Bali et al. (2011, 2012) use the smoothed volatility surface by OptionMetrics, which interpolates the raw options data so that the windows of forward-looking remain of constant lengths. While this smoothed surface is preferable for most scientific enquiries (hence the popularity of that data in the literature), we are particularly interested in the dynamic component of FH, to which the theoretical work by Hellmann and Riedel (2015) recently opened the door. We therefore use the non-smoothed dynamic 'raw' options data (provided by Stricknet).

Our main findings summarize as follows. First, a linear correlation analysis suggests that FH provides an investor with additional information beyond standard risk measures. Second, FH is shown to be a significant predictor of large return downturns. Third, by contrast to standard risk measures, FH captures a large number of characteristics of the risk-neutral probability distributions. And finally, we evaluate a form of time-consistency of the risk measures and find FH to be predictively consistent.

The remainder of this document is structured as follows. Next, we formally introduce and discuss FH in section two, and turn to the estimation of RNDs in section three. Section four contains the analysis. Finally, section five concludes.

4.2 Foster-Hart riskiness

4.2.1 No-bankruptcy

When applying Foster and Hart (2009) finance, it will prove useful to work within the setup where the decision maker is allowed to take any proportion of the offered gamble. In our case the gamble g consists of buying some multiple of the risky asset at price S_0, holding it over a period $T > 0$ and finally selling it at price S_T. Including dividends, we may define g as the absolute return $g := S_T + Y - S_0$, where Y is the monetary amount of dividends being paid over the period. This allows us to define the Foster-Hart bound $\mathrm{FH} \in (0, 1)$ for a gamble with positive expectation as the zero of the equation

$$\mathrm{E}\left[\log\left(1 + r\,\mathrm{FH}\right)\right] = 0, \tag{4.1}$$

with $r := g/S_0 = (S_T + Y - S_0)/S_0$ being the relative return. Since in reality any risky asset might default, FH is bounded from above by 1. Riedel and Hellmann (2015) show that there exist gambles for which equation (4.1) has no solution $\mathrm{FH} \in (0,1)$, even if the expected return is positive. In this case we may consistently set FH to one, $\mathrm{FH} = 1$.

FH connects to the original definition of the Foster-Hart objective measure of riskiness as a wealth level R simply as $\mathrm{FH} = S_0/R$ (Foster and Hart, 2009, p. 791). Varying between 0 and 1, one may interpret it as the fraction of wealth at which it becomes risky to invest in the asset. Formally, this may be expressed via a no-bankruptcy criterion. Following Foster and Hart (2009), we define no-bankruptcy as a vanishing probability for ending up with zero wealth when confronted with a sequence of gambles

$$\mathrm{P}\left[\lim_{t\to\infty} W_t = 0\right] = 0. \tag{4.2}$$

Foster and Hart (2009) (Theorem 2) show that no-bankruptcy is guaranteed if, and only if, the fraction of wealth invested in the risky asset is always smaller than FH. In this case, wealth actually diverges; $\lim_{t\to\infty} W_t \to \infty \quad \text{(a.s.)}$.

4.2.2 Growth rates

FH can be interpreted as the limit between the positive and negative geometric means of the gamble outcomes. A simple example may provide some intuition. Assume that a risky asset at price $S_0 = \$300$ will, with equal probability of one half, increase to $S_T = \$420$ or decrease to $S_T = \$200$. Solving equation (4.1) reads as $\left(1 + \frac{2}{5}\mathrm{FH}\right)\left(1 - \frac{1}{3}\mathrm{FH}\right) = 1$. The solution $\mathrm{FH} = 0.5$ is exactly that quantity balancing the potential gain and loss to an expected geometric mean of 1. By contrast, investing a higher (lower) fraction of wealth will result in a negative (positive) expected geometric mean. Thus FH separates the regimes of expected negative and positive growth rates of wealth. For an infinite sequence of gambles only investments in the latter avoid bankruptcy.

A natural question is why FH (equation 4.1) sets the expected growth rate to zero instead of maximizing it. Indeed, there is an extensive literature on the corresponding maximal growth rate, which is often referred to as the ‘Kelly criterion’ (Kelly, 1956; Samuelson, 1979). (Foster and Hart, 2009, p. 802) succinctly comment on this relation as follows:

> *While the log function appears there too, our approach is different. We do not ask who will win and get more than everyone else [...], but rather who will not go bankrupt and will get good returns. It is like the difference between 'optimizing' and 'satisficing'.*

In our eyes, and more importantly for our purposes, the main difference between Kelly and FH lies in their respective applications. While the first is an investment strategy explicitly stating how to allocate one's portfolio in order to maximize wealth growth, the latter is a risk measure indicating the set of mathematically problematic portfolio allocations in the sense of incurring bankruptcy risks. For us, the goal is to identify risky investment decisions, which is why we prefer the latter interpretation.

4.2.3 A more conservative bound

While FH (equation 4.1) is defined under the physical probability measure $\mathbb{P}$, we will evaluate it under the option-implied risk-neutral measure $\mathbb{Q}$. Although Cox et al. (1985) argue from a theoretical standpoint that the RND will converge to the physical probability density as the aggregate wealth of an economy rises, more recent econometric work questions this hypothesis (e.g. Brown and Jackwerth 2001). Since studies such as Bliss and Panigirtzoglou (2004) find remarkable consistencies in the deviations of the two measures across markets, utility functions and time horizons, we shall address in this section what our direct move between $\mathbb{P}$ and $\mathbb{Q}$ means for the validity of option-implied FH in our analysis.

Intuitively, given a risk-averse representative investor, FH will be lower under $\mathbb{Q}$ than under $\mathbb{P}$. Hence, our FH thus evaluated under $\mathbb{Q}$, even though defined under $\mathbb{P}$ is justified as a 'bound on the bound'. To make this statement more formal, we follow Bliss and Panigirtzoglou (2004) to reconstruct the subjective density function p from the RND q assuming, as an example, a power utility function;

$$p(S_T) = \frac{q(S_T)/U'(S_T)}{\int q(x)/U'(x)\mathrm{d}x} = \frac{q(S_T)S_T^{\gamma}}{\int q(x)x^{\gamma}\mathrm{d}x}, \tag{4.3}$$

where S_T is the price of the underlying at maturity and $U(S_T) = (S_T^{1-\gamma} - 1)/(1 - \gamma)$. For a positive relative risk aversion coefficient $\gamma > 0$, it is clear that this transformation shifts probability mass from lower towards higher prices.[6] Be $S_1 > S_0 > 0$, then

$$\frac{p(S_1)/p(S_0)}{q(S_1)/q(S_0)} = \left(\frac{S_1}{S_0}\right)^{\gamma} > 1. \tag{4.4}$$

Technically, the above argument shows that p first-order stochastically dominates q, hence FH increases – as the gamble becomes more 'attractive' (Foster and Hart, 2009). This means that FH under will be a more conservative risk measure under $\mathbb{Q}$ than under $\mathbb{P}$, such that the bankruptcy property persists. Throughout the literature one finds positive coefficients of relative risk aversion for the representative agent, albeit of various magnitude (e.g. Arrow 1971; Friend and Blume 1975; Hansen and Singleton 1982, 1984; Epstein and Zin 1991; Normandin and St-Amour 1998). In the spirit of Foster and Hart (2009), and in the light of recent findings (Leiss et al., 2015) that indicate changing risk attitudes over time as a result of events such as the Global Financial Crisis, for example, we restrain from making somewhat arbitrary assumptions on the utility of a representative agent and pursue directly with option-implied quantities instead.

4.3 Risk-neutral densities

4.3.1 Theory

Several methods for estimating RNDs from options data as underlying various risk assessment studies exist (e.g., Aït-Sahalia and Lo 2000; Aït-Sahalia et al. 2001; Panigirtzoglou and Skiadopoulos 2004; Figlewski 2010, to name just some of the most popular). Jackwerth (2004) provides an excellent review. All these methods share the fundamental 'inversion' logic, which we shall now proceed to sketch out.[7]

The fundamental theorem of asset pricing, stating that, in a complete market, the current price of a derivative may be determined as the discounted expected value of the future payoff under the unique risk-neutral measure (e.g., Delbaen and

[6] Note that the same argument applies to exponential utilities with $U(S_T) = -(\mathrm{e}^{-\gamma S_t})/\gamma$, i.e. to the other type of utility function discussed by Bliss and Panigirtzoglou (2004).

[7] Skipping the risk-neutral density estimation, the spanning formula by Bakshi and Madan (2000) poses a way of directly estimating the option-implied FH bound, as well as risk-neutral volatility, skewness and kurtosis (Bakshi et al., 2003).

Schachermayer, 1994), guides the way of inferring information from financial options. The price C_t of a standard European call option at time t with exercise price K and exercise time T on a stock with price S is thus given as

$$C_t(K) = \mathrm{e}^{-r_f(T-t)}\ \mathbb{E}_t^{\mathbb{Q}}\left[\max(S_T - K, 0)\right] = \mathrm{e}^{-r_f(T-t)} \int_K^{\infty} (S_T - K) f_t(S_T) \mathrm{d}S_T, \quad (4.5)$$

where $\mathbb{Q}$ and f_t are the risk-neutral measure and the corresponding RND, respectively. Since option prices as well as the risk-free rate, r_f, and time to maturity, $T-t$, are observable, we may hope to invert equation (4.5) for the RND.[8]

Several inversion methods have been proposed (Jackwerth, 2004). Besides parametric approaches, where one assumes a specific form for the RND with parameters that minimize the pricing error, a 'trick' by Breeden and Litzenberger (1978) opens another route: if strikes were distributed continuously on the positive real line, we could simply differentiate equation (4.5) with respect to K to obtain the RN-distribution F_t and RND f_t as

$$F_t(S_T) = \mathrm{e}^{r_f(T-t)} \frac{\partial}{\partial K} C_t(K) + 1, \quad f_t(S_T) = \mathrm{e}^{r_f(T-t)} \frac{\partial^2}{\partial K^2} C_t(K). \quad (4.6)$$

Again various methods exist to overcome the numerical problems associated with the fact that options are only traded at discrete and unevenly spaced strikes (Rubinstein, 1994; Aït-Sahalia and Lo, 2000; Shimko et al., 1993).

4.3.2 A nonparametric approach

For our purposes, the relatively new approach by Figlewski (2010), as adapted in the recent study (Leiss et al., 2015), turns out to be most suited in order to get as much information out of the data as regard extreme events. It combines a 4^{th}-order polynomial interpolation of data points in implied volatility space with appended generalized extreme value (GEV) tails beyond the range of observed strikes. We shall briefly present this method here.

We start from bid and ask quotes for puts and calls with a given maturity and transform the mid-prices to implied-volatility space via the Black-Scholes equation (Black and Scholes, 1973). Note that we do not assume the Black-Scholes model

[8] One may at least proxy the *true* risk-free rate with, say, yields on 13-week T-Bills or LIBOR.

to price options correctly, but only use the equation as a mathematical tool. The implied volatilities of puts and calls are blended together such that only the more liquid, and thus informative, out-of-the-money and at-the-money data points are considered while ensuring a smooth transition from puts to calls. The resulting famous 'volatility smirk' is interpolated with a 4^{th}-order polynomial weighted by open interest, thus, giving higher importance to data points which contain more market information.[9] After a retransformation of the fit values to price space, we numerically evaluate the empirical part of the RND according to equation (4.6).

As the range of strikes is finite, we have to choose a functional form of the tails. Instead of the often-used log-normal function, Figlewski (2010) employs the family of generalized extreme value (GEV) distributions (Embrechts et al., 2005, p. 265). The Fisher-Tippett theorem supports this choice, stating that, under weak regularity conditions and after rescaling, the maximum of any i.i.d. random variable sample converges in distribution to a GEV distribution (Embrechts et al., 2005, p. 266). The GEV family contains many relevant distributions, in particular also those with heavy tails.[10] A distribution of GEV-type is characterized by three parameters: location, scale and shape. We determine them by imposing the following three connection conditions for the left and right tails separately: the GEV density should match the empirical one at two specified quantile points and conserve the probability mass in the tail.

Joining the empirical part with the tails eventually gives the full option-implied RND. While there exist many approaches to estimate RNDs, we argue that Figlewski's method, as a combination of a model-free empirical part and flexible extreme value tails, belongs to the most unbiased ones. Allowing for non-standard features such as bimodality and fat tails will be of advantage for analyzing the highly different regimes around the Global Financial Crisis of 2008. We refer the interested reader to

[9] Since our data set admits open interest weighting, we deviate as in Leiss et al. (2015) here from the original approach by Figlewski (2010), who weighs such that fits outside the bid-ask spread are penalized instead.

[10] The conceptually correct choice of extreme value family is the generalized Pareto distribution (GPD), since the risk-neutral tails correspond rather to the peaks-over-threshold method than the block-maxima method (Embrechts et al., 2005, pp. 264-291). Mathematically this translates into applying the Pickands-Balkema-de Haan theorem instead of Fisher-Tippett. However, because of their asymptotic equivalence and quantitative similarity we may use either and refer to Embrechts et al. (1997, 2005); Birru and Figlewski (2012) for a more detailed discussion.

Leiss et al. (2015), who discuss in detail the properties of RNDs during and around the Financial Crisis (see also Figlewski 2010; Birru and Figlewski 2012).

In order to assess the robustness of our analysis regarding fitting assumptions, we also fitted risk-neutral densities by mix of two log-normal distributions to equation (4.5) as, for example, in Bahra (1996); Melick and Thomas (1997); Söderlind and Svensson (1997); Jondeau et al. (2007). We found qualitatively and quantitatively similar results, but the log-normal approach to be less stable. Furthermore, we repeated the nonparametric technique, while slightly perturbing the input data as follows. For every iteration, instead of using mid-prices, we choose points in the bid-ask spread of every option uniformly at random and proceed as described above, thus obtaining a different risk-neutral density. Iterating 500 times per business day gives us statistical significance of option-implied quantities in a way that is known as a Monte Carlo method (Hammersley and Handscomb, 1964), see also section 3.C for more details.

4.3.3 Data

In this study, we employ end-of-day data for standard European call and put options on the Standard & Poor's 500 stock market index (SPX) covering the period from January 1st, 2003, to October 23rd, 2013. During this decade the market for SPX options grew substantially from about 150K to 890K contracts in average daily volume and from 3,840K to 11,883K in open interest at the end of period, respectively.[11] Our data provided by Stricknet consists of bid and ask quotes as well as open interest across various maturities, but we focus on the very liquid monthly options expiring on the third Friday every month. Daily values for index level, its dividend yield and the yield of the 3-Month Treasury bill as a proxy of the risk-free rate are taken from the Thomson Reuters Datastream.

We follow Figlewski (2010) in filtering the raw data, ignoring quotes with bids below \$0.50 and those that are more than \$20.00 in the money, as such bids come with high ambiguity due to large spreads. Moreover, we also discard data points with

[11] See http://www.cboe.com/SPX for a detailed description of the options contracts and recent market data.

midprices violating static no-arbitrage conditions. Finally, to ensure well-behaved densities, we restrict our analysis to dates with time to expiration of at least one week.[12]

In the following section, we will derive the option-implied risk measures that we shall consider in our analysis. These are the Foster-Hart bound (FH), value at risk (VaR), expected shortfall (ES) and risk-neutral volatility (RNV). Moreover, we control for other risk measures popular in the industry such as the Chicago Board Options Exchange Market Volatility Index (VIX), also known as the "fear index" (Chicago Board Options Exchange, 2009), which we obtain from the Thomson Reuters Datastream.[13] Furthermore, as a measure of perceived credit risk, we include the TED spread, which is the difference between the 3-Month London Interbank Offered Rate (LIBOR) and the interest rate on 3-Month Treasury bills (T-Bill).[14]

4.4 Empirical results

4.4.1 Option-implied risk indicators

Aït-Sahalia and Lo (2000) is the pioneering work on option-implied measures of risk. Their study suggests that VaR under the risk-neutral measure $\mathbb{Q}$ may capture aspects of market risk that VaR under the physical measure $\mathbb{P}$ does not. Aït-Sahalia and Lo (2000) argue that "risk management is a complex process that is unlikely to be driven by any single risk measure", and conclude that the option-implied measure should rather be seen as a compliment than substitute. In a similar fashion, Bali et al. (2011) set out to assess the added value of their option-implied 'generalized risk measure' against traditional ones such as historical VaR, historical ES and an option-implied measure of skewness (QSKEW; Xing et al. 2010). A Fama and MacBeth (1973)-type of regression shows that their option-implied measure

[12](i) As the range of relevant strikes shrinks on the way towards maturity, RNDs show a strong peaking. (ii) Figlewski (2010) also notes that another reason may be price effects from rollovers of hedge positions into later maturities around contract expirations.

[13]It is calculated as the 30-day expected variance of the S&P 500 Index and represents volatility risk.

[14]LIBOR data is available at https://research.stlouisfed.org/fred2/series/USD3MTD156N.

successfully explains the cross section of 1-, 3-, 6- and 12-month-ahead risk-adjusted stock returns.[15] Furthermore, Bali et al. (2012) find strong predictive power of the riskiness measures of both Aumann and Serrano (2008) and Foster and Hart (2009) evaluated under option-implied the risk-neutral measure for economic downturns as measured by the Chicago Fed National Activity Index.[16]

In this work, we combine the previous approaches by Bali et al. (2011) and Bali et al. (2012) and analyze if option-implied FH may help predict large downturns in stock returns when controlling for other quantities evaluated under the risk-neutral measure.[17] For that, we calculate VaR and ES for option-implied log-returns at the $\alpha = 5\%$ level at time t,

$$\mathrm{VaR}_t^{\mathbb{Q}} = -\inf\{x \in \mathbb{R} : F_t^r(x) \leq \alpha\}, \quad \mathrm{ES}_t^{\mathbb{Q}} = -\mathbb{E}_t^{\mathbb{Q}}\left[x \in \mathbb{R} : x \leq -\mathrm{VaR}_t\right], \qquad (4.7)$$

where F_t^r is the risk-neutral distribution of log-returns estimated at time t.[18] In this definition VaR and ES are expressed in losses such that higher values indicate higher risk. RNV is defined as the standardized second moment of the risk-neutral density $f(S_T)$,

$$\mathrm{RNV}_t^2 = \frac{1}{(T-t)S_t^2} \int_0^\infty (S_T - \mu_t)^2 \, f_t(S_T) \mathrm{d}S_T, \qquad (4.8)$$

where $T-t$ is the time to maturity, S_t the price of the underlying and μ_t the mean of the density f_t extracted at time t, respectively. Figure 4.1 displays and compares the resulting quantities. All measures exhibit signatures of the Global Financial Crisis of 2008 as well as the Greek and European sovereign debt crises in 2010 and late 2011, respectively. Yet, it appears from Figure 4.1 that the behavior of FH is distinctly different from VaR, ES and RNV. The corresponding one-standard-deviation bands are comparatively small, suggesting that this observation is robust with respect to

[15]Note, however, that the asset allocation implications of Bali et al. (2011)'s result are limited: across all investment horizons the time-varying investment choice of an investor with a relative risk aversion of three over the whole sample period of 1996–2008 ranges only over a few percentage points.

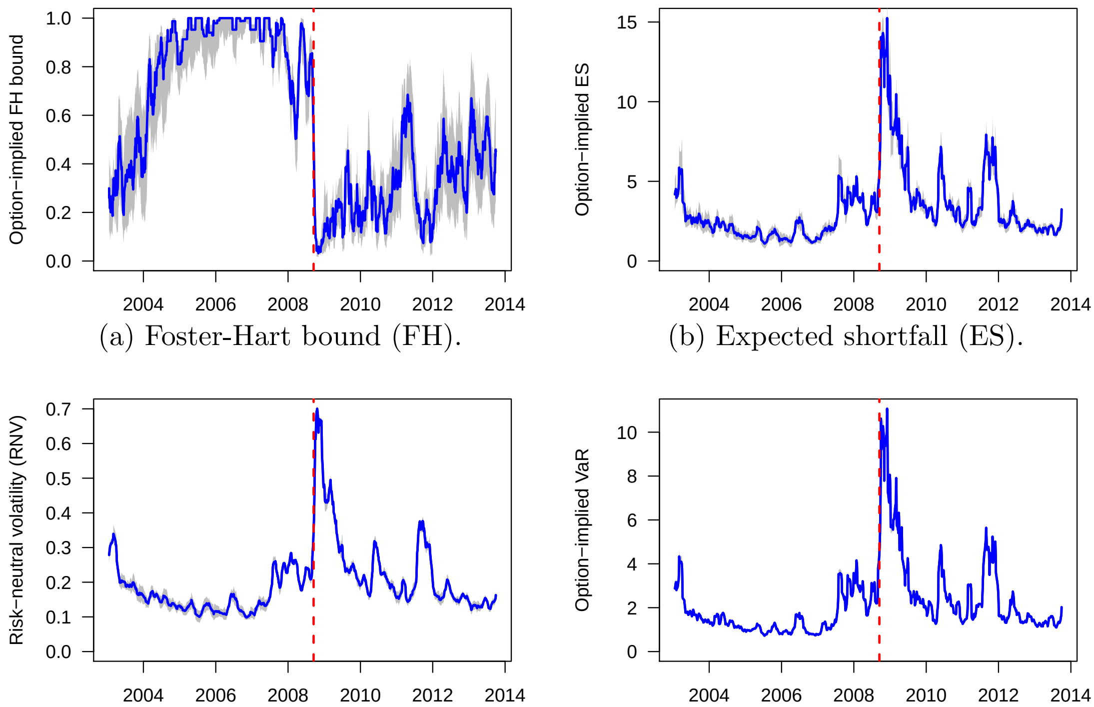
[16]The Chicago Fed National Activity Index is an aggregate measure for overall economic activity and inflationary pressure. https://www.chicagofed.org/research/data/cfnai/historical-data

[17]Indeed, Bali et al. (2011) compare the option-implied Bali measure to historical VaR and ES. Evaluating all risk measures under the same information set represents a somewhat level playing field.

[18]One can easily go from annualized log-returns to prices as $r = \log(S_T/S_0)/T$. The RND expressed in log-returns is $f^r(r) = TS_T f^S(S)$.

noise in the options price data and not an artefact of our estimation technique. In particular, the respective changes in the risk measures due to the crisis are by far larger than the estimated variance.

Figure 4.1: Option-implied measures of market risk.



(a) Foster-Hart bound (FH).

(b) Expected shortfall (ES).

(c) Risk-neutral volatility (RNV).

(d) Value at risk (VaR).

Various risk measures evaluated under the option-implied risk-neutral measure over time (21-day moving average). The dashed red line marks the bankruptcy filing of Lehman Brothers on September 15, 2008. FH (a) clearly shows a different behavior from ES (b), RNV (c) and VaR (d), although all measures are determined on the same information set. The grey area marks the one-standard-deviation band based on 500 Monte Carlo iterations of density estimation.

A correlation table provides some first quantification of the relation between the various option-implied risk measures as well as with VIX and TED spread (see Table 4.1). While the tail measures VaR and ES, as well as RNV, are highly correlated amongst each other (with 98% and 81%), they correlate with FH to only 20% - 46%. Indeed, it seems that FH captures different information than VaR, ES or RNV. Furthermore, VIX and RNV exhibit a linear correlation of 94%. This was to be expected as both are meant to capture option-implied ahead-volatility. To avoid multicollinearity problems, we will use in the following analyses only ES instead of

VaR, and the VIX instead of RNV.

Table 4.1: Correlations between risk measures.

	-FH$^{\mathbb{Q}}$	VaR$^{\mathbb{Q}}$	ES$^{\mathbb{Q}}$	RNV	VIX	TED	Skew	Kurt
-FH$^{\mathbb{Q}}$								
VaR$^{\mathbb{Q}}$	0.20							
ES$^{\mathbb{Q}}$	0.26	0.98						
RNV	0.46	0.81	0.82					
VIX	0.48	0.64	0.64	0.94				
TED	-0.02	0.43	0.41	0.60	0.56			
Skew	-0.23	0.16	0.03	0.10	0.23	0.16		
Kurt	0.18	-0.09	0.02	-0.05	-0.19	-0.11	-0.96	
Left Tail	0.29	-0.18	-0.05	-0.06	-0.17	-0.12	-0.80	0.73

Correlations between various measures of risk. The option-implied risk measures are Foster-Hart bound (FH$^{\mathbb{Q}}$), value at risk (VaR$^{\mathbb{Q}}$), expected shortfall (ES$^{\mathbb{Q}}$) and risk-neutral volatility (RNV). Furthermore, the volatility index (VIX) and the TED spread between interbank loans (Libor) and T-Bills are included.

4.4.2 Downturns

A glance at the FH definition (4.1) suggests that, due to the strongly concave logarithm, FH may be particularly sensitive to left-tail risks, i.e. extreme losses. We test this hypothesis using a dummy variable Δr_t^ρ that is one whenever the S&P 500 ahead-return until maturity of the option, $r_{t\to T} := \log(S_T/S_t)$, is lower than a threshold value ρ and zero otherwise:

$$\Delta r_t^\rho = \begin{cases} 1, & \text{if } r_{t\to T} = \log\left(S_T/S_t\right) < \rho, \\ 0, & \text{if } r_{t\to T} \geq \rho, \end{cases} \tag{4.9}$$

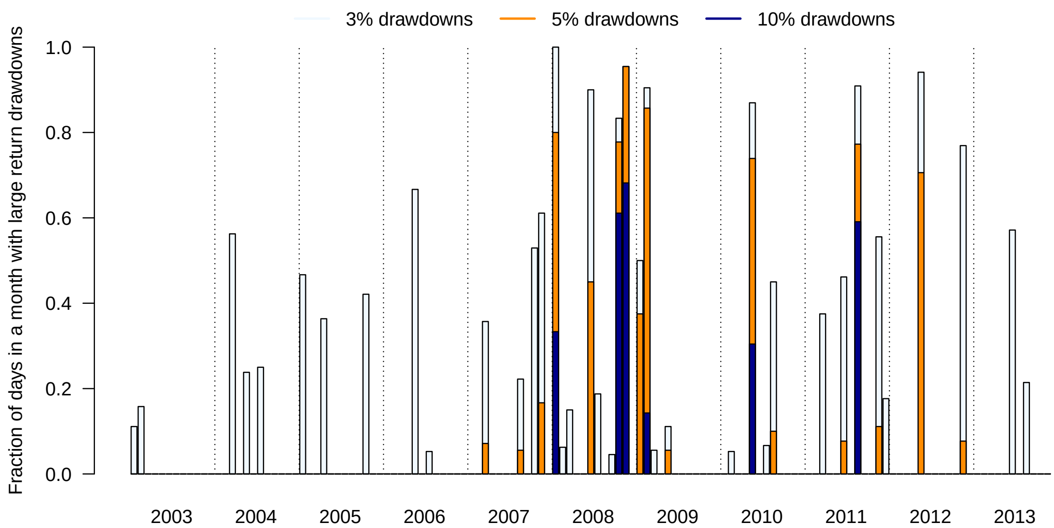
where S_T is the realized price at maturity. Figure 4.2 represents when those return drawdowns occur over our data period. We run the logistic regressions,

$$\Delta r_t^\rho = a_{0,t} + a_{1,t}\ R_t, \tag{4.10}$$

where $R \in \mathcal{R} \equiv \left\{\text{FH}^{\mathbb{Q}},\ \text{ES}^{\mathbb{Q}},\ \text{TED},\ \text{VIX}\right\}$. Furthermore, we control for skewness and excess kurtosis of the RND,

$$\text{Skew}_t = \mathbb{E}_t^{\mathbb{Q}}\left[\left(\frac{S-\mu_t}{\sigma_t}\right)^3\right],\ \text{Kurt}_t = \mathbb{E}_t^{\mathbb{Q}}\left[\left(\frac{S-\mu_t}{\sigma_t}\right)^4\right] - 3, \tag{4.11}$$

Figure 4.2: Distribution of return downturns over time.



Moments when the return decreases by more than a certain value (3, 5 or 10%). Most of the larger drawdowns occur during the Global Financial Crisis (late 2007 to early 2009) as well as the Greek and European sovereign debt crisis in 2010 and 2011, respectively.

where μ_t and σ_t are the first two moments of the RND estimated at time t. Finally, we also control for the left tail shape parameter, which we denote by Left Tail$_t$. Thus, we also run the logistic regression

$$\Delta r_t^{\rho} = a_{0,t} + a_{1,t}\text{Skew}_t + a_{2,t}\text{Kurt}_t + a_{3,t}\text{Left Tail}_t + \sum_{R \in \mathcal{R}} a_{R,t} R_t. \tag{4.12}$$

Results for $\rho = -5\%$ and $\rho = -10\%$, with 138 and 54 downturns respectively, are reported in Table 4.2. As expected from theory, lower FH and higher ES, VIX and TED spread, individually, all indicate a higher probability for a return drawdown. However, in the full regression (4.12), the VIX looses significance and ES surprisingly changes sign. Hence, only FH and TED spread preserve their predictive characteristics in the joint regression.

The regression results are robust with respect to choice of threshold, ρ, over a wide range of numerical values. Furthermore, stepwise backward model selection (Venables and Ripley, 2002, p. 175) based on the Akaike (1974) information criterion shows that the full model (4.12) is preferred over all submodels. Finally, a leave-one-out cross-validation (Kohavi et al., 1995) yields a classification rate of 94%.

4.4.3 The impact of RNDs on risk measures

How do characteristics of the RND shape the various risk measures? To answer that question we regress the risk measures $R \in \mathcal{R}$ on the (standardized) second, third and fourth moment as well as the left tail shape parameter of the RNDs:

$$R_t = a_{0,t} + a_{1,t}\mathrm{RNV}_t + a_{2,t}\mathrm{Skew}_t + a_{3,t}\mathrm{Kurt}_t + a_{4,t}\mathrm{Left\ Tail}_t. \tag{4.13}$$

Results are presented in Table 4.3. FH bound captures all variations in the properties of the RND. In particular, a higher RNV and fatter left tail, as well as more negative skewness lead to lower FH.[19] By contrast, the other risk measures do not grasp all the RND characteristics. For instance, ES and VIX seem to react mainly to the second moment of the density (RNV), whereas a thinner left tail actually makes them take on higher values. This is particularly surprising in the case of ES, which is formulated specifically to capture tail risks, but a phenomenon that is not an artefact of our estimation technique. Moreover, TED spread is significantly explained by the risk-neutral volatility, but not by the tail shape parameter.

4.4.4 Time-consistency

Hellmann and Riedel (2015) point out that FH lacks time-consistency, similarly to VaR and ES. Somewhat loosely speaking, they define a risk measure to be time-consistent, if the knowledge of gamble X_t^1 being riskier than X_t^2 in any state of the world tomorrow should imply X_t^1 to be considered riskier than X_t^2 already today. Hellmann and Riedel (2015) construct an example showing that, in general, FH is not time-consistent.

Due to the fixed expiration dates of options, our option-implied risk measures also exhibit a naturally dynamic structure, thus raising the issue of time-consistency in the above sense for the special case of risk-neutral densities approaching maturity. Tests thereof, in the sense of Hellmann and Riedel (2015), however, are not possible as the structure of the dynamic gamble is *a priori* not known to the representative investor. Instead, we may, however, get at the issue of predictive time-consistency

[19]Note, that throughout our data period the RNDs consistently exhibit a *negative* skewness of -1.5 ± 0.9, such that a positive slope coefficient means a reversal to (log-)normality.

by comparing how the informativeness of predicting return downturns behaves for the various measures of riskiness depending on the time to maturity. Table 4.4 summarizes the predictive power of FH, ES, VIX and TED spread when evaluated 2, 3, 4 or 5 weeks before the exercise date of the option. While the slope coefficient of FH is consistently positive, it is significant only 4 and 5 weeks before maturity. Similarly, the TED spread has explanatory power 3 to 5 weeks ahead, whereas the option-implied ES does not show significance at any time window. Surprisingly, the VIX significantly changes sign when derived at different times relative to the exercise date. Hence, only FH and TED spread are predictively time-consistent.[20]

4.5 Conclusion

The main contribution of this paper has been the translation of the objective risk measure by Foster and Hart (2009) (using Riedel and Hellmann 2015's and Hellmann and Riedel 2015's generalizations) to a typical decision context in finance. This was done by extracting the underlying risk-neutral densities from option prices and deriving the corresponding option-implied risk measure. Rather than optimal estimates, we chose an approach which could be described as deriving a conservative bound on these. Our resulting measure was shown to have additional information compared to standard risk measures. In fact, it outperformed the standard measures including value at risk, expected shortfall, risk-neutral volatility and the volatility index.[21] Indeed, not only had our option-implied Foster-Hart measure of riskiness interesting macroscopic patterns, in that it indicated rather extreme regime shifts in the dawn of the financial crisis, but also was it shown to be of microscopic interest, in that it turned out a significant predictor of large return downturns. Finally, in future work, we would like to study option-implied Foster-Hart measures in richer investment settings, for example, when investment into more than one asset and/or leverage with hedging is allowed.

[20] Recall that it was also FH and TED who preserved their predictive characteristics in the joint estimation of regression (4.12).

[21] Solely the TED spread (between Libor and Treasury bills), as a measure of credit risk, turned out similarly predictive and consistent, despite the known reliability issues associated with fraudulent Libor setting.

Table 4.2: Regression of return downturns on risk measures.

Regression of -5% downturns (138 such events) on risk measures and RND characteristics								
(Intercept)	-2.01^{***}	-2.92^{***}	-3.58^{***}	-4.64^{***}	-2.35^{***}	-2.39^{***}	-2.64^{***}	-6.04^{***}
	(0.12)	(0.11)	(0.14)	(0.22)	(0.14)	(0.11)	(0.09)	(0.59)
$-\mathrm{FH}^{\mathbb{Q}}$	1.41^{***}							0.74^{*}
	(0.23)							(0.36)
$\mathrm{ES}^{\mathbb{Q}}$		0.07^{***}						-0.08^{**}
		(0.02)						(0.03)
TED			1.30^{***}					1.18^{***}
			(0.12)					(0.18)
VIX				0.08^{***}				0.06^{***}
				(0.01)				(0.01)
Skewness					0.20^{*}			-2.36^{***}
					(0.09)			(0.39)
Kurtosis						-0.02^{**}		-0.14^{***}
						(0.01)		(0.03)
Left Tail							-0.82^{*}	-1.11
							(0.33)	(0.63)
AIC	973.25	991.60	860.96	870.30	1006.00	999.11	1006.78	769.06
Log Likelihood	-484.62	-493.80	-428.48	-433.15	-501.00	-497.56	-501.39	-376.53
Num. obs.	2037	2037	1989	2037	2037	2037	2037	1989
Regression of -10% downturns (54 such events) on risk measures and RND characteristics								
(Intercept)	-2.69^{***}	-3.80^{***}	-4.87^{***}	-5.66^{***}	-3.47^{***}	-3.46^{***}	-3.60^{***}	-6.70^{***}
	(0.17)	(0.16)	(0.24)	(0.32)	(0.20)	(0.16)	(0.14)	(0.96)
$-\mathrm{FH}^{\mathbb{Q}}$	2.64^{***}							3.41^{***}
	(0.49)							(0.70)
$\mathrm{ES}^{\mathbb{Q}}$		0.05^{**}						-0.48^{***}
		(0.02)						(0.13)
TED			1.42^{***}					2.44^{***}
			(0.15)					(0.30)
VIX				0.08^{***}				0.04
				(0.01)				(0.03)
Skewness					0.09			-3.50^{***}
					(0.11)			(0.74)
Kurtosis						-0.01		-0.20^{***}
						(0.01)		(0.05)
Left Tail							-0.14	-1.95
							(0.47)	(1.15)
AIC	458.59	497.84	391.65	431.28	501.77	499.83	502.54	305.41
Log Likelihood	-227.30	-246.92	-193.82	-213.64	-248.88	-247.92	-249.27	-144.71
Num. obs.	2037	2037	1989	2037	2037	2037	2037	1989

$^{***}p < 0.001$, $^{**}p < 0.01$, $^{*}p < 0.05$

This table reports the intercept and slope coefficients of the regression of downturns of ahead-returns until maturity of the underlying option of more than 5% (upper part) or 10% (lower part) on the option-implied Foster-Hart bound $\mathrm{FH}^{\mathbb{Q}}$, option-implied expected shortfall $\mathrm{ES}^{\mathbb{Q}}$, the spread between interbank loans and T-Bills TED, and the VIX, as well as skewness, excess kurtosis and the left tail shape parameter of the risk-neutral density. Standard deviations are given in brackets, significance according to p-values is indicated by stars.

Table 4.3: Regression of risk measures on RND characteristics.

	$-\text{FH}^{\mathbb{Q}}$	$\text{ES}^{\mathbb{Q}}$	VIX	TED
(Intercept)	-1.08^{***}	-2.46^{***}	5.51^{***}	0.05
	(0.03)	(0.16)	(0.23)	(0.03)
RNV	1.83^{***}	26.78^{***}	79.48^{***}	2.72^{***}
	(0.07)	(0.42)	(0.59)	(0.09)
Skewness	-0.19^{***}	-0.02	0.30	0.10^{***}
	(0.02)	(0.12)	(0.18)	(0.03)
Kurtosis	-0.01^{***}	0.02^{**}	-0.02^{**}	0.00^{**}
	(0.00)	(0.01)	(0.01)	(0.00)
Left Tail	0.27^{***}	-1.30^{***}	-0.79^{*}	-0.02
	(0.04)	(0.27)	(0.38)	(0.06)
R^2	0.34	0.68	0.91	0.37
Adj. R^2	0.34	0.68	0.91	0.37
Num. obs.	2037	2037	2037	1989
RMSE	0.34	2.14	3.05	0.44

$^{***}p < 0.001$, $^{**}p < 0.01$, $^{*}p < 0.05$

This table reports the intercept and slope coefficients of the regression of various risk measures on characteristics of the density. The risk measures are the option-implied FH bound, the option-implied expected shortfall, the volatility index and the TED spread.

Table 4.4: Predictive consistency of risk measures.

	All	2 weeks	3 weeks	4 weeks	5 weeks
(Intercept)	-3.38^{***}	-9.89^{*}	-6.02^{***}	-2.56^{**}	-1.11
	(0.48)	(4.11)	(1.40)	(0.80)	(0.86)
$-\mathrm{FH}^{\mathbb{Q}}$	3.31^{***}	24.60	1.01	2.96^{**}	5.35^{**}
	(0.64)	(24.69)	(1.42)	(1.03)	(1.77)
$\mathrm{ES}^{\mathbb{Q}}$	-0.54^{***}	-1.62	0.17	1.45	0.60
	(0.13)	(0.84)	(0.63)	(0.75)	(1.01)
TED	2.02^{***}	-0.47	2.07^{***}	2.78^{***}	2.30^{*}
	(0.28)	(1.46)	(0.53)	(0.57)	(0.95)
VIX	0.04	0.39^{*}	0.02	-0.25^{*}	-0.16
	(0.02)	(0.17)	(0.10)	(0.10)	(0.14)
AIC	327.73	27.40	85.05	117.33	80.62
Log Likelihood	-157.86	-7.70	-36.52	-52.67	-34.31
Num. obs.	1989	498	578	582	214

$^{***}p < 0.001$, $^{**}p < 0.01$, $^{*}p < 0.05$

This table reports the intercept and slope coefficients of the regression of downturns of ahead-returns until maturity of the underlying option of more than 10% on the option-implied Foster-Hart bound $\mathrm{FH}^{\mathbb{Q}}$, option-implied expected shortfall $\mathrm{ES}^{\mathbb{Q}}$, the spread between interbank loans and T-Bills TED, the VIX and realized volatility by time to maturity of the underlying option. Standard deviations are given in brackets, significance according to p-values is indicated by stars.

Chapter 5

How communication networks help achieve collective goals

This chapter is based on work in progress in collaboration with Christian Schulz[a], Emőke-Ágnes Horvát[b], Dirk Helbing[a] and Brian Uzzi[bc]. Intermediate results were presented at the 2015 International School and Conference in Network Science[d] and the 2015 Network Frontier Workshop[e].

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Abstract

Knowledge about how groups of individuals self-organize to achieve a common goal in complex environments is needed to advance our understanding of collective behavior and network performance. Here, we study how structure and temporal dynamics of the communication network of a large hedge fund influence trading activity of the fund. To isolate the effects of internal communication we control for information inflow due to business news and direct email contact with other institutional investors and investment banks. Setting up natural experiments, we analyze transactions in US equities executed by human traders and supervised by portfolio managers. Our findings indicate that a large fraction (33%) of the sequences of buys and sells is statistically indistinguishable from a random walk. We find that the random trades significantly underperform. Most interestingly, they can be associated with certain characteristics of the communication network inferred from emails and instant messages. Specifically, a more clustered and a more balanced (as measured by entropy) internal communication, as well as a more diverse external network of information sources are closely associated with nonrandom and thus more profitable trading. Our results indicate how individual decision-makers can structure their interactions in complex environments to achieve individual and collective goals.

5.1 Introduction

Since the first empirical analysis of the *wisdom of crowds effect* (Galton, 1907), it is often believed that the aggregate judgement of a group of individuals can be more accurate than that of a single expert. Indeed, the efficient market hypothesis formalizes that markets constitute an example of this phenomenon (Fama, 1970). It states that market prices fully and correctly reflect all information available to investors. Building on this idea, prediction markets have been shown to efficiently aggregate dispersed information, generating forecasts that typically outperform most moderately sophisticated benchmarks (Wolfers and Zitzewitz, 2006). At the same time, much is already known about the limits of collective intelligence. For example, social influence increases the unpredictability of success (Salganik et al., 2006) and decreases group diversity without improvements to collective judgement (Lorenz et al., 2011). In fact, the existence of a suitable aggregation mechanism that turns private evaluations into collective decisions has been identified as a necessary criterion for the wisdom of crowds effect to work (Surowiecki, 2005). Otherwise, informational cascades may develop (Easley and Kleinberg, 2010) or, more generally, rational individuals may form an irrational group (Mayo-Wilson et al., 2013).

It remains an open research question, however, how people self-organize in groups to achieve optimal collective decision-making. This is especially important because collective performance depends on the diversity and social sensitivity of group members as well as equality in distribution of conversational turn-taking (Page, 2008; Woolley et al., 2010). Furthermore, implications may be counterintuitive; for example, a team comprised of individual best-performers may be outperformed by a randomly formed team (Hong and Page, 2004). A first step was taken by Guimera et al. (2005), who propose a dynamic model of group assembly mechanisms involving team size, diversity and collaboration networks. They conclude that assembly mechanisms determine the performance of teams within both artistic and scientific fields. Goldstone and Janssen (2005) provide a computational model of plausible mechanisms from individual decisions to the emergence of different group-level organization, including group pattern formation, contagion and cooperation. Yet this work is still in its infancy. Couzin (2007) argues, concerning collective goals, we are only "beginning to comprehend more fully how individuals in groups can gain access to higher-order collective computational capabilities such as the simultaneous acquisition and processing of information from widely distributed sources."

In particular when it comes to risky decisions in complex systems, successful self-organization of a team crucially depends on the communication flows between its members. Eckmann et al. (2004) observe in a dynamic network of e-mail traffic the development of self-organized structures that turn out to be functional and goal-oriented. Saavedra et al. (2011) show how day traders use their instant messaging network to synchronize behavior, thereby boosting individual and collective performance. In terms of the structure of the communication network, neither dispersed nor concentrated networks respond most effectively to informational uncertainty (Wu et al., 2004; Easley and Kleinberg, 2010). Generally, the optimal network structure depends on the problem space being explored (Mason et al., 2008). Networks that incorporate spatially based cliques are advantageous when problems benefit from broad exploration. On the other hand, networks with greater long-range connectivity have an advantage when problems require less exploration. Helbing et al. (2014) highlight that suitable system design and management can help to stop undesirable cascade effects and to enable favorable kinds of self-organization in the system. In the best case, *group cognition* emerges, enabling organization-dependent cognitive capacities that go beyond simple aggregation of the cognitive capacities of individuals (Goldstone and Gureckis, 2009; Theiner et al., 2010).

In this paper, we set up 35,000 natural experiments to study how the structure of the communication network of a large hedge fund relates to its trading activity. We control for the inflow of information through business news and direct email contact between the fund and other institutional investors. Our analysis of 1.2 million transactions in US equities with an overall volume of 900 billion USD executed by 97 human traders, and supervised by 55 portfolio managers, reveals that a surprisingly large fraction (33%) of their trading is statistically indistinguishable from a random walk. These random trades significantly underperform nonrandom ones. Most interestingly, they can be associated with certain characteristics of the digital communication network consisting of 2 million emails and 6 million instant messages.

Our analysis yields two main findings. First, a more clustered and a more balanced (as measured by entropy) internal communication network can be related to less random and thus, more successful, behavior. This is in line with previous experimental results suggesting that groups where a few individuals dominate communication were less collectively intelligent and performant than groups with a more equal distribution of conversational turn-taking (Woolley et al., 2010). Also, our work resonates with the "causal entropic principle" (Mann and Garnett, 2015), whereby agents follow behavioral rules that maximize their entropy, thus finding to collectively intelligent outcomes. Second, we can associate networks that are diverse in external information sources to success. We find that more business news and a more diverse incoming communication with investment banks significantly reduce the likelihood of random trading. This is in accordance with previous empirical results at the level of companies, which suggest that firm embeddedness has positive effects on financing conditions and survival (Uzzi, 1996, 1997, 1999).

The remainder of this paper is structured as follows. Section 5.2 summarizes the empirical setting and data, section 5.3 discusses the methods used and section 5.4 concludes.

5.2 Empirical setting and data

We observed the hedge fund's communication and trading activity from January 2008 to December 2012. During this time period, the firm employed 55 portfolio managers, 97 traders and 60 analysts. We discard data from other employees such as IT and administrative staff. The financial employees often work in teams, where an analyst delivers intelligence with respect to specific stocks and the overall market to a portfolio manager. The latter carries the operative responsibility and takes buying and selling decisions, which are then executed by a trader. However, some portfolio managers also do the trading themselves. A large part of the internal coordination takes place via two channels. Emails are used for the exchange of

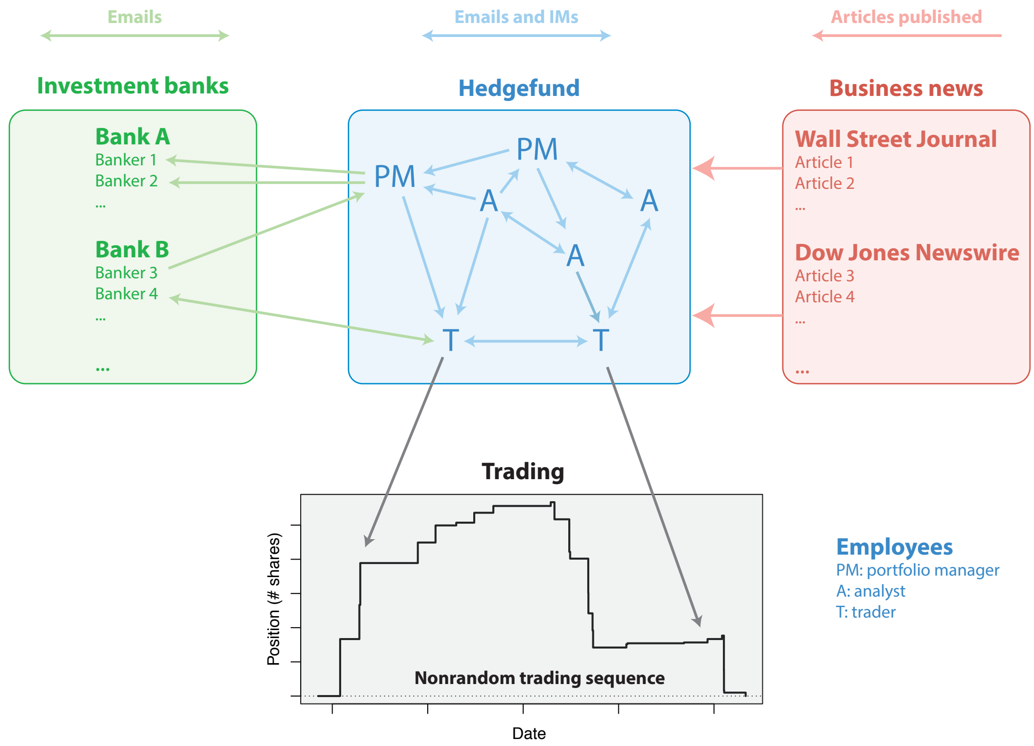


Figure 5.1: Graphical representation of the empirical setting and data. Portfolio managers (PM), analysts (A) and traders (T) employed at the hedge fund internally communicate via emails and instant messages (blue lines). They are in exchange with investment bankers at various banks via email (green lines) and are exposed to business news published online (red lines). Portfolio managers make decisions to buy or sell a stock, which are then executed by traders (grey lines). A variance-ratio test classifies the resulting sequence of buys and sells as random or nonrandom.

detailed information, whereas instant messaging serves to share information quickly and to align immediate actions. For this study we analyzed the full texts of 2 million emails and 6 million instant messages. As with emails, instant messages may have multiple recipients.

Furthermore, the hedge fund was in brisk contact with investment banks, which often take the "sell side" in market making and promote securities. Thus, we also evaluated the full texts of 12 million anonymized emails sent to the fund from 170 investment banks. Finally, financial professionals regularly consult business news to keep themselves informed about recent developments. To control for those effects, we use the Ravenpack Analytics dataset[1], which continuously collects and consolidates

[1]The dataset is available at `http://www.ravenpack.com` and is widely used for systematic

information from sources such as the Dow Jones Newswires and the Wall Street Journal. The studied time period covers 80 million news articles that may be linked to the stocks traded by the fund.

The hedge fund protocoled 1.2 million and 3.4 million transactions in US equities by human and algorithmic trading with an overall volume of 900 billion and 400 billion USD, respectively. The fund's internal structure is given by 186 trading accounts, each of which is either administered by one or several portfolio managers or run by algorithmic trading. Usually the trading accounts operate on different sectors such as technology, financial or consumer goods, but may sometimes overlap. Managers at the firm explained that the buying and selling of a stock over time without a full closeout within such a trading account measure a portfolio manager's conviction or prediction for that stock. Thus, we focus on those sequences of buys and sells of an asset in between subsequent full closeouts, which we refer to as blocks. Over the time period of our study we observed 89,000 and 268,000 blocks on human and computerized trading accounts, respectively. On average a block associated with human trading lasts 25 days and involves 13 trades, whereas the numbers are 22 days and 8 trades for an algorithmically executed block, respectively. Overall, short selling accounts for slightly less than half of the cases. Figure 5.1 shows a schematic representation of the empirical setting and section 5.A gives two activity examples.

5.3 Methods

5.3.1 Testing for randomness

The Random Walk Hypothesis (RWH) was conceived as early as in the 16th century as a model of games of chance, and has been studied by many famous scientists such as Bachelier, Einstein, Lévy, Kolmogorov and Wiener (Lo and MacKinlay, 2011). Given a time series $\{X_t\}_{t=1}^{T}$, it can be formulated recursively as

$$X_t = \mu + X_{t-1} + \varepsilon_t, \tag{5.1}$$

analysis of unstructured data for finance.

where μ is an arbitrary drift parameter and ε_t is the random term. A simple form of the null hypothesis of a random walk assumes identically and independently drawn Normal random numbers,

$$\text{RWH}: \ \varepsilon_t \text{ i.i.d. } N(0, \sigma^2), \tag{5.2}$$

but this may be generalized to heteroskedastic innovations with vanishing mean and serial correlation (Lo and MacKinlay, 2011). The most popular tests of the RWH (5.2) are based on the variance-ratio methodology, of which Charles and Darné (2009) give an excellent review. The main idea is to use the fact that the variance of random walk increments is linear in the observation interval (Lo and MacKinlay, 2011). It can be shown that under the RWH the ratio of variances of increments is asymptotically Normally distributed for all sampling intervals $k = 2, 3, ...$

$$\sqrt{T} J_r \equiv \sqrt{T} \left(\frac{\hat{\sigma}^2(k)}{\hat{\sigma}^2(1)} - 1 \right) \sim N(0, 1), \tag{5.3}$$

where $\hat{\sigma}^2(k)$ is the unbiased estimator of the k-period variance of increments (Charles and Darné, 2009; Lo and MacKinlay, 2011). While the idea behind variance-ratio tests is intuitive, many sophisticated test statistics have been developed to deal with overlapping data, small sample sizes and joint tests for various sampling intervals. Given that the blocks involve only a small number of trades, we will use the joint Wright sign test by Belaire-Franch and Contreras (2004), who extended the individual variance-ratio test by Wright (2000) to a joint one for multiple k-periods. A recent application can be found in Kim and Shamsuddin (2008), who investigate time-varying return predictability of the Dow Jones Industrial Average index from 1900 to 2009. This type of test has special advantages when the sample size is small, notably because the sampling distribution is exact (Charles and Darné, 2009). Figure 5.2 presents examples of trading sequences that are classified as nonrandom and random, respectively.

5.3.2 Network structures

We relate the trading and communication activity on a stock-by-stock basis. For each business day we narrow the flow of emails and instant messages down to

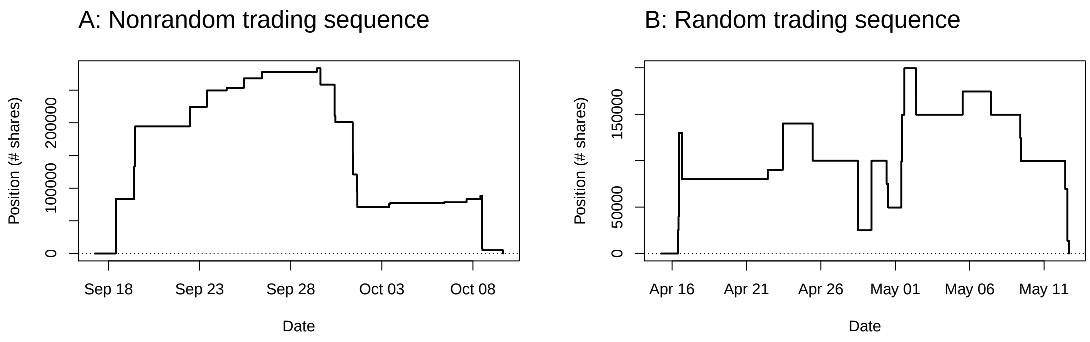


Figure 5.2: Illustrative examples for trading sequences classified as blocks for the case of nonrandom (A) and random trading (B).

that subset of messages that are specific to a certain stock. For example, if we are interested in the communication network on a specific day around the stock of Apple Inc., we proceed as follows: we include anonymized senders and receivers of messages that contain the company's ticker symbol ("AAPL") or words that frequently co-occur with it (e.g. "iPhone") as well as those messages that are exchanged between the same communication partners within a 5-minute interval before or after.[2] This gives us the networks of email and instant message communication corresponding to a certain stock on a certain business day, respectively.

Subsequently, we analyze the resulting stock-day-communication networks along various structural dimensions for internal and external communication partners separately. Regarding the internal structure, we subset the network to employees of the hedge fund to compute both the clustering coefficient (Watts and Strogatz, 1998) and the balance, measured as the Shannon entropy (Shannon, 1948), of communication participants. The clustering coefficient of an employee i is defined as

$$C_i = \frac{2\,|\{e_{jk} : \nu_j, \nu_k \in N_i\}|}{k_i(k_i - 1)} \ , \tag{5.4}$$

where e_{jk} is a communication event between employees j and k that are both in the immediate network neighborhood N_i consisting of k_i employees. Thus, C_i is the fraction of all possible edges in the undirected neighborhood network. The global clustering coefficient of a communication network is then computed as the mean of clustering coefficients of all its participating employees. Similar to the Shannon

[2]We find that our results are robust when choosing a different interval length such as 1, 2 or 3 minutes.

entropy we define communication balance as

$$B = -\sum_i P_i \ \log_2 (P_i) \tag{5.5}$$

where P_i is the fraction of the total communication of an employee i. The reported results are robust with respect to the specific measures and also hold if we quantify clustering by the pair-wise similarity measure by Adamic and Adar (2003) or balance by the (inverted) concentration index by Herfindahl (1950). Furthermore, we measure the diversity of communication with investment banks by the number of distinct banks to which hedge fund employees send emails to or receive emails from, respectively.

5.3.3 Communication complexity

The full texts of messages allow us to go beyond structure and analyze the content of communication. This, however, is difficult, as employees often use a large amount of nonstandard language. We thus employ the following working hypothesis: the complexity of trading stocks increases with the number of distinct topics in the internal communication. We define the list of topics as the entries in Campbell Harvey's finance glossary (Harvey, 2015), a standard glossary of about 8,000 financial terms such as "illiquid" and "sell out". We quantify complexity for every business day and sector. Following Lijffijt et al. (2011), we identify a topic q as being discussed in sector s, if it appears more often in the sector-specific communication R_t^s on day t than expected by chance from the overall communication full texts $T_t = \{R_1^s, R_2^s, ..., R_T^s\} \setminus \{R_t^s\}$ regarding that sector on other days, i.e. if the one-tailed empirical p-value is lower than 0.01

$$\hat{p}\left(q, R_t^s, T_t\right) = \frac{1 + \sum_{k \neq t} I\left(freq(q, R_t^s) \leq freq(q, R_k^s)\right)}{T}, \tag{5.6}$$

where I is the indicator function. Thus, the sector-day-complexity is given by the number of topics discussed with respect to internal communication regarding a certain sector on a certain day. Figure 5.3 shows the communication complexity over time averaged over all the sectors. Section 5.B discusses the robustness of our definition of communication complexity.

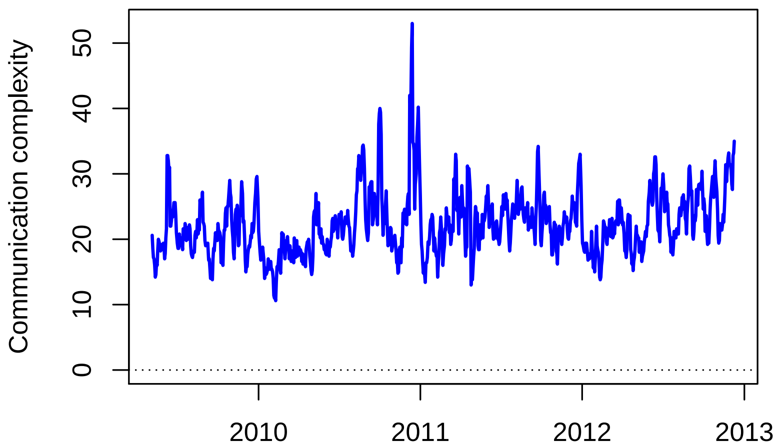


Figure 5.3: Communication complexity measured as number of distinct topics being discussed over time, averaged over all sectors.

5.3.4 Experimental setup

Now we can set up equivalents to natural experiments in order to assess the effects on randomness in human trading of the contextual variables internal clustering, internal balance, informational diversity, content complexity and business news volume. First, to avoid events attributable to the Global Financial Crisis of 2008, we limit our time period to the "post-crisis" case separated starting on May 4th, 2009. This date has been found in the econometric study of Leiss et al. (2015) via a change point analysis of financial market characteristics and is in line with macroeconomic and political events such as the first implementation of "quantitative easing" by the Federal Reserve as well as the Bank of England in the first quarter of 2009 (see also chapter 3). Second, we compute for each stock the means of the contextual variables for the three cases of random trading, nonrandom trading and no trading. The latter serves as a control group and is subtracted from the resulting values.

5.4 Results

5.4.1 Prevalence and profitability of random trading

In order for the variance-ratio test to remain meaningful we restrict our analysis to blocks that involve at least 10 trades. This leaves us with 35,000 and 41,000

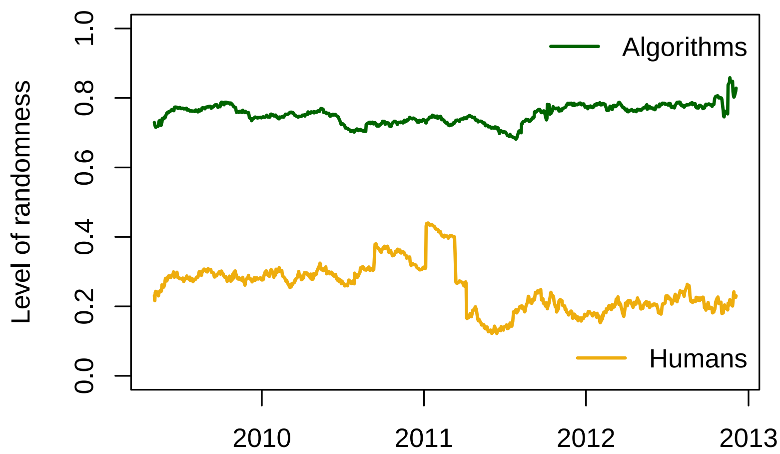
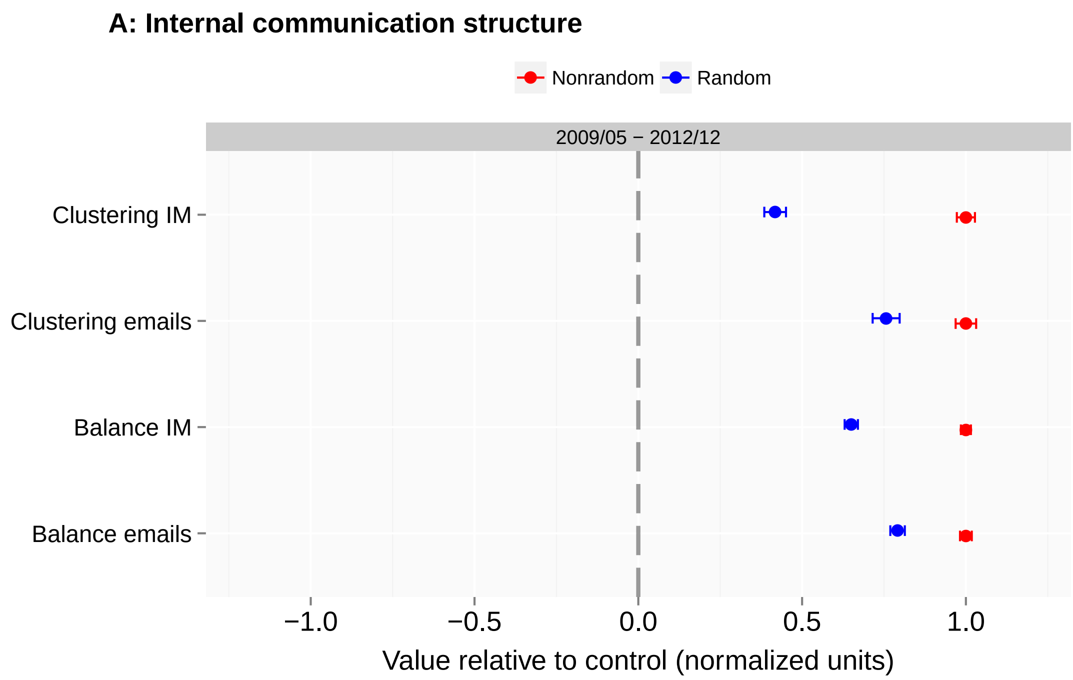


Figure 5.4: Level of randomness for human trading (yellow) and algorithmic trading (green) over time.

sequences of buys and sells executed by humans and algorithms, respectively. The average block has a relative return of 0.25% at a turnover of 18 million USD for humans, whereas machines achieve 0.04% at a turnover of 3.3 million USD. We find a surprisingly large prevalence of randomness in both human and algorithmic trading, with 33% and 81% of those sequences, respectively, confirming the random walk hypothesis. While Daniels et al. (2003) explain many statistical regularities of financial markets with a model based on Poisson random order arrival and cancellation processes, to our knowledge we present the first empirical analysis of individually random trading sequences. Figure 5.4 shows how the levels of randomness change over time. While there is some temporal variation, the prevalence of randomness remains distinctively different between human and algorithmic trading. Sequences of buys and sells classified as nonrandom can be associated with a significantly larger return ($p < 0.001$ in a linear regression). The effect is particularly strong in human trading, where it holds true both for long blocks of 20 trades and more ($p < 0.001$) and short blocks ($p < 0.05$), whereas for algorithmic trading we find only long random blocks significantly less profitable than nonrandom ones ($p < 0.05$).

Network structure and contextual variables associated with human random trading



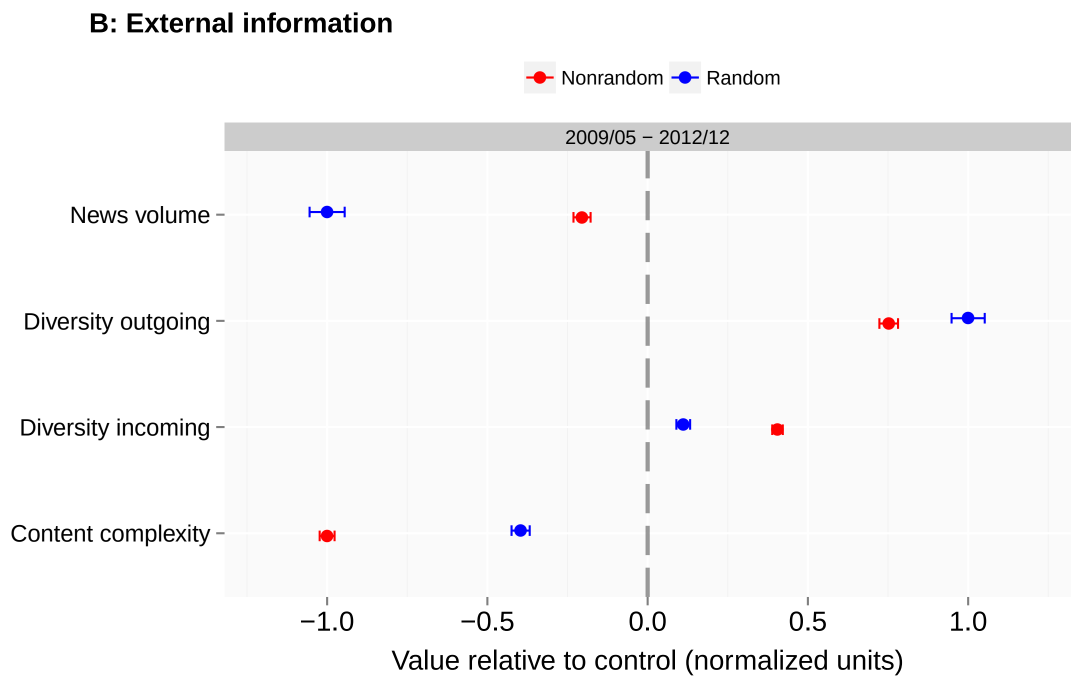


Figure 5.5: Contextual variables associated with random trading by humans relative to non-trading (control). (A) Especially after the Global Financial Crisis random trading is associated with less clustered and less balanced email and instant messaging (IM) internal communication networks. (B) A stock is more likely to be traded randomly when there is less business news reported, when the incoming information is less diverse and when there is more sector-specific complexity in the market. By contrast, a more diverse outgoing communication pattern does not lead to a reduced likelihood of trading random.

5.4.2 Network structure and contextual variables associated with human random trading

We now assess the relationship of random trading with the contextual variables. Random trading can be associated with internal communication networks that are less balanced and less clustered (see Figure 5.5). This is in accordance with previous findings from laboratory experiments, where equality in distribution of conversational turn-taking increases collective intelligence and group performance (Woolley et al., 2010). In addition to that, balanced networks bring to mind the "causal entropic principle" (Mann and Garnett, 2015), whereby agents follow behavioral rules that maximize their entropy, which leads to collectively intelligent outcomes.

As for the information exchange with outside the hedge fund, we find that more business news and a more diverse incoming communication with investment banks significantly reduce the likelihood of random trading (Figure 5.5). The same is not true, however, for outgoing communication from the fund to investment banks. This result emphasizes the importance of diversity for collective performance Woolley et al. (2010); Hong and Page (2004); Page (2008). It also resonates with the positive effects of the social embeddedness of firms on financing conditions and survival (Uzzi, 1996, 1997, 1999).

5.5 Conclusion

We have studied the structure and node dynamics of the digital communication network of a large hedge fund and related it to its trading activity. Based on variance-ratio tests, we defined nonrandom behavior as sequences of buys and sells that cannot be explained by a random walk. Both human and algorithmic random trades were found to significantly underperform nonrandom ones. This allowed us to identify two characteristics of communication networks associated with collectively successful decision-making. On the one hand, internal networks must be clustered and balanced. On the other hand, external networks should be diverse in terms of information sources. Our research may help create conditions that enable successful collective performance.

Appendix

5.A Activity examples

We present two activity examples to illustrate the use of instant messaging within the hedge fund. First, in April 2010, a portfolio manager (P) and a trader (T) and second, in August 2010, two traders (T1 and T2) communicate, respectively. Both communications end with a trade of a mentioned stock. Those are buying GOOG stock for 2.8 million USD and selling AAPL stock for 0.6 million USD, respectively.

```
2010 April
IM    11:24:30 P->T: buy 5k goog no hurry
IM    11:24:36 T->P: b 5 goog
IM    11:24:40 T->P: no rush
IM    11:26:46 T->P: ACN just guided to fiscal 11 revs up 7-10%.
IM    11:28:05 T->P: EPS also better
IM    11:30:03 T->P: NOK - hearing chattter only ... -
                     talking down numbers as they might be seeing a
                     squeeze on both ends. Getting hit on the Low end
                     by Samsung and LG?on the High end buy AAPL,
                     GOOG and RIMM
TRADE 11:30:42 GOOG USD 2.828 M
```

```
2010 August
IM    15:27:48 T1->T2: sorry, we are working the goog (1k to short)
                       into the bell?
IM    15:29:09 T1->T2: ty -
IM    15:29:52 T1->T2: cover another 5k ca
IM    15:30:48 T1->T2: thx
IM    15:31:16 T1->T2: i think they always say "buyer out of europe"
                       to make us believe its "dumb money"
IM    15:32:21 T1->T2: lol
IM    15:32:47 T1->T2: sell another 2.5k aapl
IM    15:33:44 T1->T2: cover 5k more wdc
TRADE 15:33:44 AAPL USD -0.649 M
IM    15:34:00 T1->T2: make it 10k more wdc - gets me to 40k short
```

5.B Robustness tests of content complexity

Based on a method by Lijffijt et al. (2011), we defined communication complexity in equation (5.6) by the number of topics with a one-tailed empirical p-value smaller than 0.005. To verify the robustness of our definition, we vary this threshold parameter over the range $\hat{p} < 0.001, ..., 0.02$. Table 5.B.1 presents the correlation between the resulting complexity scores averaged over all sectors. Since the correlations across thresholds are quite high, we conclude that our definition of communication complexity is fairly robust.

	$\hat{p} < 0.001$	$\hat{p} < 0.002$	$\hat{p} < 0.005$	$\hat{p} < 0.010$	$\hat{p} < 0.020$
$\hat{p} < 0.001$					
$\hat{p} < 0.002$	0.83				
$\hat{p} < 0.005$	0.72	0.89			
$\hat{p} < 0.010$	0.62	0.79	0.91		
$\hat{p} < 0.020$	0.52	0.68	0.82	0.93	
$\hat{p} < 0.050$	0.33	0.47	0.62	0.75	0.88

Table 5.B.1: Dependence of communication complexity averaged over all sectors on the threshold parameter for the one-tailed empirical p-value.

Chapter 6

Conclusion

In this dissertation, we studied the origins and characteristics of large stock price movements. Our approach was contrary to the standard paradigm of efficient markets that explains price movements by the arrival of new information. Instead, we have worked with the notion of speculative bubbles. There, the price of an asset systematically detaches from the fundamental value over an extended period of time. It is generally thought that speculative bubbles are fueled by positive feedback mechanisms, which can be of a both technical and behavioral nature. In any case, they imprint statistical patterns on the price trajectory, in particular a price that grows faster than an exponential, or super-exponentially.

In chapter 2, we defined a theoretical model of a financial market with a risky asset that is traded by agents of two types. First, fundamentalists myopically optimize their expected mean-variance utility based on the asset's dividend process. Second, chartists employ a mixture of technical trading and social imitation of peers. We showed transient super-exponential price growth to be an inherent phenomenon of such a model driven by the herding of chartist traders. The model successfully explained empirically observed statistical regularities such as a fat-tailed return distribution and volatility clustering. Furthermore, we extended the original model by introducing a third type of traders, who use statistical techniques for the detection of speculative bubbles. Instead of eliminating the latter, they actually increased their magnitude.

In chapters 3 and 4, we estimated risk-neutral densities for the dynamics of the

S&P 500 index price based on financial options. Our observation period ranged from 2003 to 2013, thus spanning the Global Financial Crisis of 2008. Chapter 3 was dedicated to studying the structural changes induced by the crisis. We applied a change point analysis to density characteristics in order to endogenously identify the pre-crisis, crisis and post-crisis sub-periods. Interestingly, while the higher moments and tail parameters indicated an early beginning of the crisis around mid-2007, the option-implied returns only changed significantly in September 2008. Moreover, we observed super-exponential growth expectations prior to the crisis. During the post-crisis period, however, we found evidence for monetary policy to Granger-cause option-implied returns at time lags of 50 to 200 days.

In chapter 4, we used the estimated risk-neutral densities to analyze the predictive power of various risk-measures regarding large stock price movements. We included standard measures such as risk-neutral volatility as well as the tail measures value at risk and expected shortfall. In addition, we studied the Foster-Hart bound, a measure that promises no-bankruptcy in the long run. The Foster-Hart bound was shown to be a significant predictor of large future return drawdowns, as it was able to capture more characteristics of the risk-neutral probability distributions than other measures.

Finally, in chapter 5, we empirically studied at the example of a large hedge fund how communication networks may help achieve collective goals. Variance-ratio tests determined whether sequences of buys and sells were statistically different from a random walk. While nonrandom trading was found to significantly outperform, we were able to relate it to two characteristics of the accompanying communication. Those were more clustered and balanced internal communication networks on the one hand, and more diversity in terms of information sources, on the other hand.

What follows from this research? On the one hand, the implications for individual investors, risk managers and financial players seem clear. Given a certain risk budget, one may allocate a portfolio to maximize returns conditional on the amount of risk one is willing to bear. This analysis should include insights on large stock market moves, which were key to this dissertation, and limit the corresponding exposure.

On the other hand, also policy making could benefit from a better understanding of the nature of large stock price movements. It is an open question to what extent regulators should try smoothening out the boom and bust cycles characterizing capitalist economies. While intuitively it appears plausible that in the short term society would be better off without a stock market crash, the combined long term consequences of the bust *together* with the bullish period preceding it might as well raise total welfare. For example, would we observe a Silicon Valley as innovative and profitable as it is today without the overly optimistic funding of Internet companies throughout the dot-com bubble during the late 1990s? Would Google and Amazon have been able to grow to be the global players they are now? Would there be a Facebook at all? Indeed, there is research suggesting that most boom and bust cycles may have a positive net contribution to total welfare (Gisler and Sornette, 2009; Gisler et al., 2011). Therefore, the policy implications of regulating bubbles and crashes may involve a tradeoff between short term stability and long term growth, which is a question that each society may have to answer for itself.

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Computational Social Science D-GESS, Entrepreneurial Risks D-MTEC and ETH Risk Center

Ph.D. thesis "Financial Market Risk of Speculative Bubbles"

2009-2012 MSc in Physics - Ludwig-Maximilians-Universität München

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Publications

2015 Kaizoji, T., M. Leiss[1], A. Saichev, and D. Sornette.
Super-exponential endogenous bubbles in an equilibrium model of fundamentalist and chartist traders.
Journal of Economic Behavior & Organization 112, 289 – 310.

2015 Leiss, M., H. H. Nax, and D. Sornette.
Super-exponential growth expectations and the global financial crisis.
Journal of Economic Dynamics and Control 55, 1 – 13.

2015 Leiss, M.
Super-exponential bubbles and expectations: theory, simulations and empirics.
Finance, mathematics & philosophy. In E. Ippoliti and P. Chen (Eds.).

Publications under review and working papers

2015 Leiss, M. and H. H. Nax.
Option-implied objective measures of market risk.
Social Science Research Network Working Paper Series, 2690476, submitted to Quantitative Economics.

Conferences and invited talks

2015, December Network Frontier Workshop.
Poster "How communication networks help achieve collective goals".
Northwestern University, Evanston, United States.

2015, November System Social and Financial Risk:
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Talk "Option-implied indicators of market risk".
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2015, July	The 26th International Conference on Game Theory. *Talk "The option-implied Foster-Hart riskiness".* Stony Brook University, Stony Brook, United States.
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2014, June	Finance, Mathematics and Philosophy. *Talk "Super-exponential bubbles in an equilibrium model of rational and noise traders".* Sapienza, Rome, Italy.
2014, May	Oxford-Man Institute ECR Lecture Series. *Talk "The option-implied Foster-Hart riskiness".* Oxford-Man Institute, Oxford, United Kingdom.
2014, May	London Mathematical Laboratory Lecture Series. *Talk "The option-implied Foster-Hart riskiness".* London Mathematical Laboratory, London, United Kingdom.
2014, March	DPG-Frühjahrstagung. *Talk "Super-exponential bubbles in an equilibrium model of rational and noise traders".* TU Dresden, Dresden, Germany.
2013, October	Supervising Banks in Complex Financial Systems. Deutsche Bundesbank, Frankfurt am Main, Germany.
2013, June	18th Annual Workshop on the Economic Science with Heterogeneous Interacting Agents. Reykjavik University, Reykjavik, Iceland.

Awards and grants

2015	Travel grant of CHF 1000 by the Swiss Academy of Humanities and Social Sciences
2015	St. Gallen Wings of Excellence Award
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Technical skills

Programming	R, Matlab, Python, Java, SQL, distributed computing, text mining.
Statistical methods	Linear and nonlinear regression, extreme value theory, bootstrapping, flexible classification (random forests, support vector machines, etc.).
Econometrics	Time series analysis, risk-neutral density estimation, change point analysis, Granger causality analysis.
Numerical methods	Monte Carlo simulations, agent-based modeling.

Languages

German	Native
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