Bayesian optimization for truss structures

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Abstract

This project investigates the application of Bayesian optimization to identify an optimal truss design that minimizes weight while meeting structural performance constraints, particularly on natural frequencies. The design space consists of nodal coordinates and cross-sectional properties of truss members, with the goal of minimizing weight while ensuring that the first three natural frequencies satisfy specified thresholds. A Python-based finite element solver is integrated into the Bayesian optimization process, employing a probabilistic surrogate to efficiently guide the search for the global optimum. The framework will be tested on problems of increasing complexity, beginning with simpler 1D and 2D cases and progressing to a 19-dimensional design space. The results include the optimal truss design and an evaluation of the effectiveness of Bayesian optimization for different dimensional problems.

Introduction

Truss optimization is important because it enables the design of lightweight, cost-effective, and environmentally sustainable structures. However, evaluating truss structures, which are typically multi-dimensional optimization problems, can be computationally expensive. Compared to meta-heuristic optimization algorithms, the Bayesian optimizer is more data-efficient, as it can incorporate prior knowledge about the function being optimized. In this project, the performance of the Bayesian optimizer is investigated for a truss structure, where the dimensionality is increased step by step. [1, 2]

Problem formulation

Objective function:

$$\min R(x) = \sum_{q=1}^{37} A_q \cdot \rho_q \cdot L_q$$

such that:

$$g_1(x) = 20 - \omega_1 \leq 0$$
$$g_2(x) = 40 - \omega_2 \leq 0$$
$$g_3(x) = 60 - \omega_3 \leq 0$$

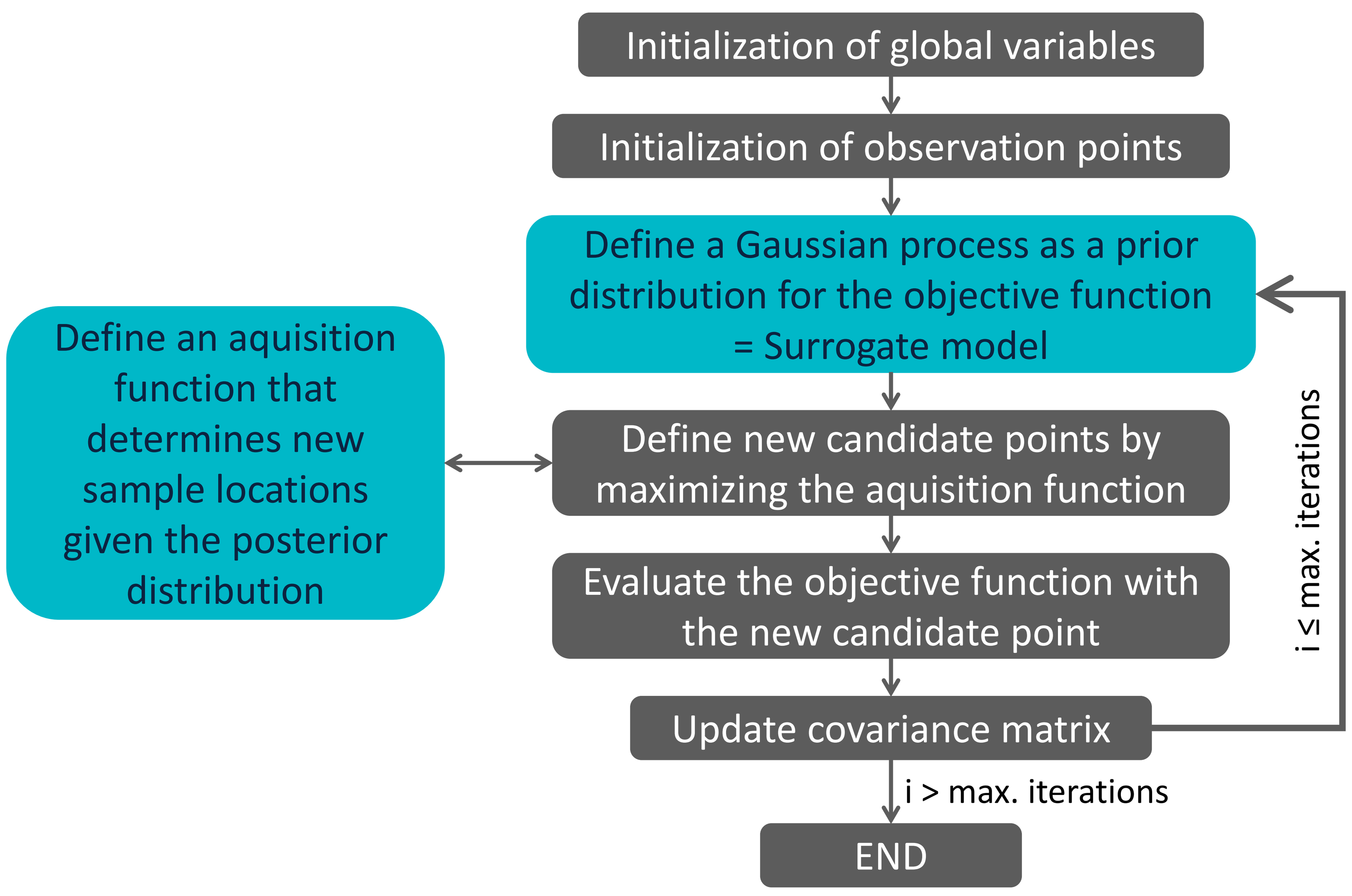
Where:

R(x):	weight of the truss structure
A_q:	cross section area
ρ_q:	density
L_q:	length of the qth truss member

To evaluate the given optimization problem using a Bayesian optimizer, the objective of weight minimization is combined with the constraints on the lowest natural frequencies. For this purpose, the following objective function is formulated based on the augmented Lagrange function with the penalty weight p:

$$L(x,p) = \sum_{q=1}^{37} A_q \cdot \rho_q \cdot L_q + p \cdot \sum_{j=1}^{3} \left[\max\left(0, g_j(x)\right)\right]^2$$

Methodology



Conclusion

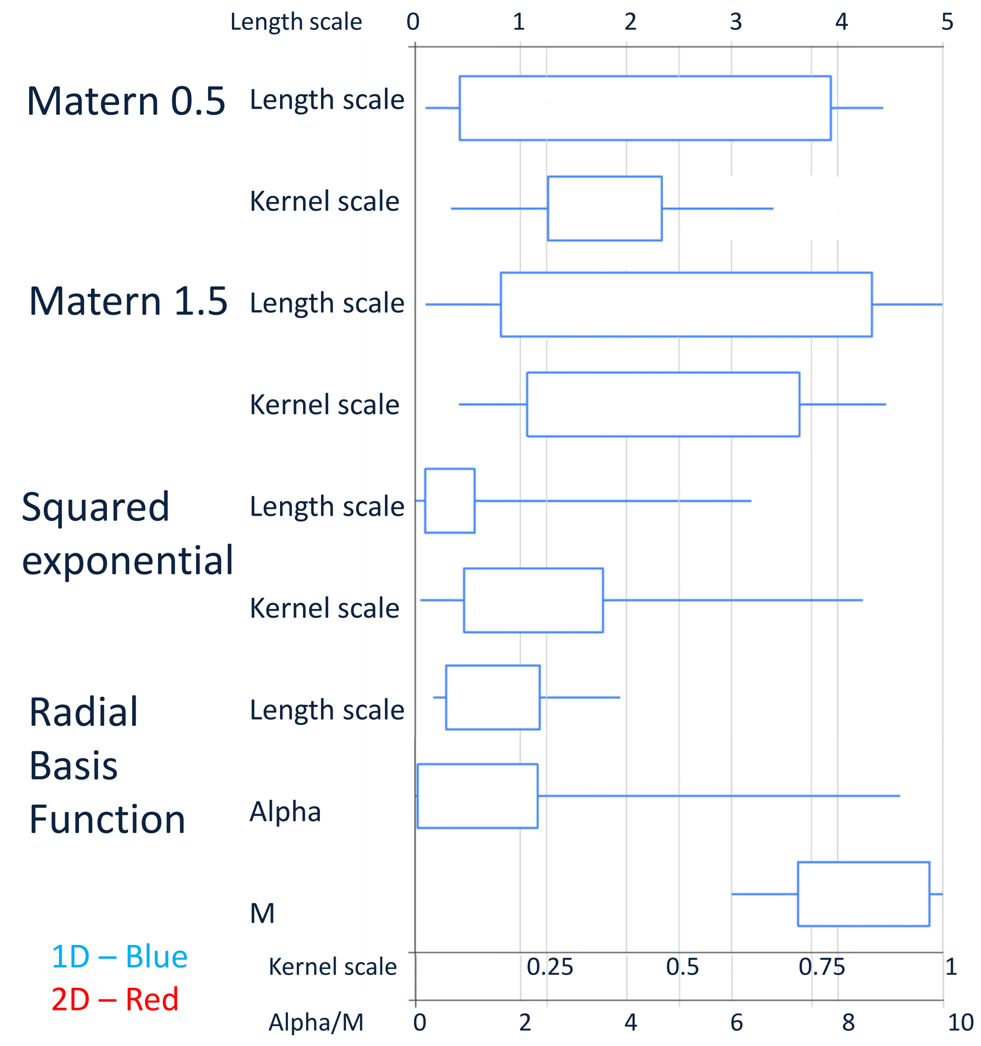
In this project, the application of Bayesian optimization to a truss structure was demonstrated in order to achieve a minimum structural weight while meeting the natural frequency constraints. In the 1D and 2D cases, the Bayesian optimizer was able to find the optimal solution within a few iterations, showing that this method is extremely effective for low-dimensional optimization problems. However, in the higher-dimensional cases (6D and 19D), the Bayesian optimizer required significantly more initial points to find a near-optimal solution, and the optimal solution reported in [1] was never achieved. The difficulties encountered in the 6D and 19D cases are consistent with the statement in [2] that extending the Bayesian optimization to high-dimensional problems remains a critical open challenge.

References:

[1] Kanarachos, S., Griffin, J., Fitzpatrick, M. E. (2017). Efficient truss optimization using the contrast-based fruit fly optimization algorithm. Computers and Structures, 182, 137-148
[2] Wang, X., Jin, Y., Schmitt, S., Olhofer, M. (2023). Recent Advances in Bayesian Optimization. ACM Computing Surveys, 55(13), 1-36. https://doi.org/10.1145/3582078
[3] Vink, R. (2019, August 25). Algorithm Breakdown: Bayesian Optimization. Ritchie Vink. https://www.ritchievink.com/blog/2019/08/25/algorithm-breakdown-bayesian-optimization/

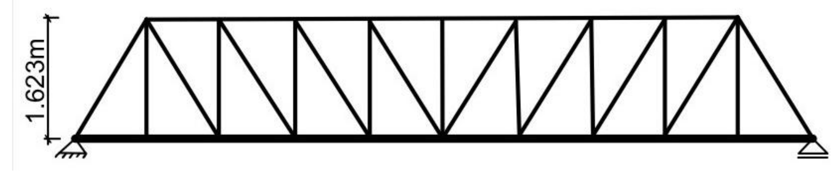
Hyperparameters

The hyperparameters were tuned for 1D and 2D problems using the Bayesian optimizer. Based on the results for different kernels, it can be concluded that the hyperparameters function has many local minima. Due to the very random nature and different starting points of the Bayesian optimizer, the optimizer rarely converges to the same local minima, regardless of the kernel. The squared exponential kernel has shown the least variety in the values it converged to and required fewer iterations than other kernels. Therefore, that kernel was chosen to proceed.

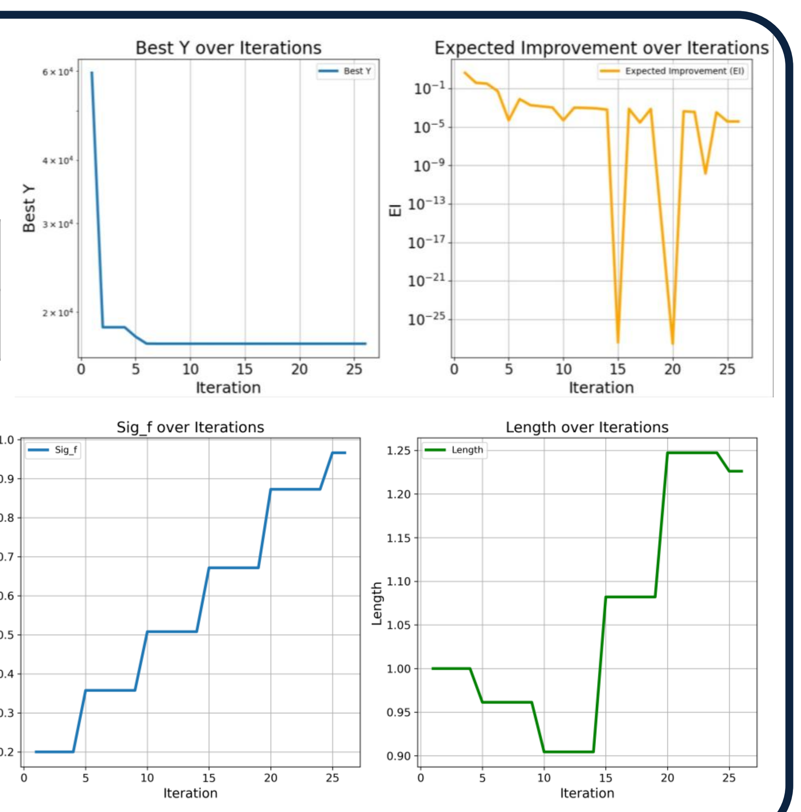


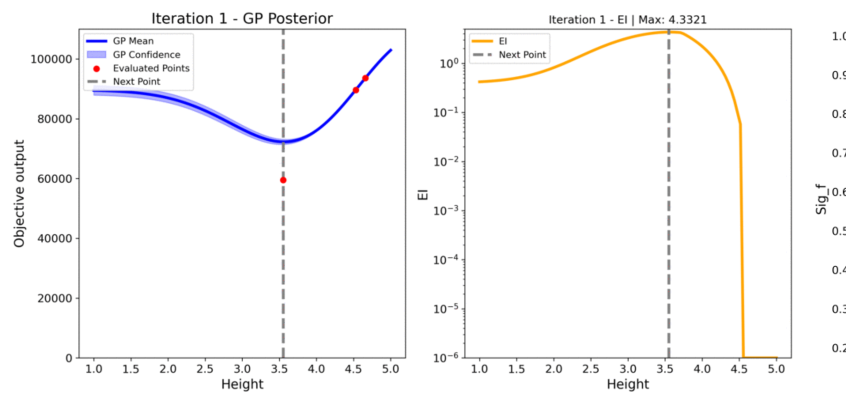
Results

1D

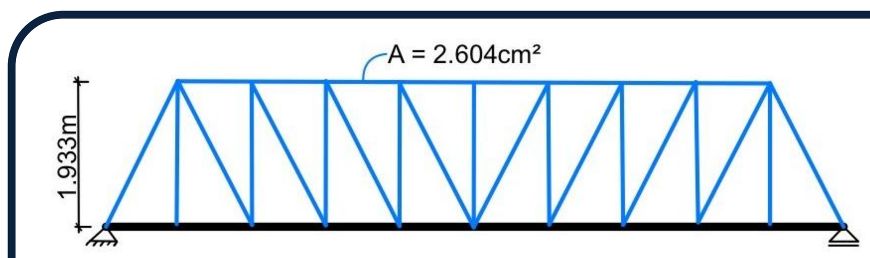


ω1 [Hz]	ω2 [Hz]	ω3 [Hz]	Weight [kg]
12.373	33.434	51.724	344.506

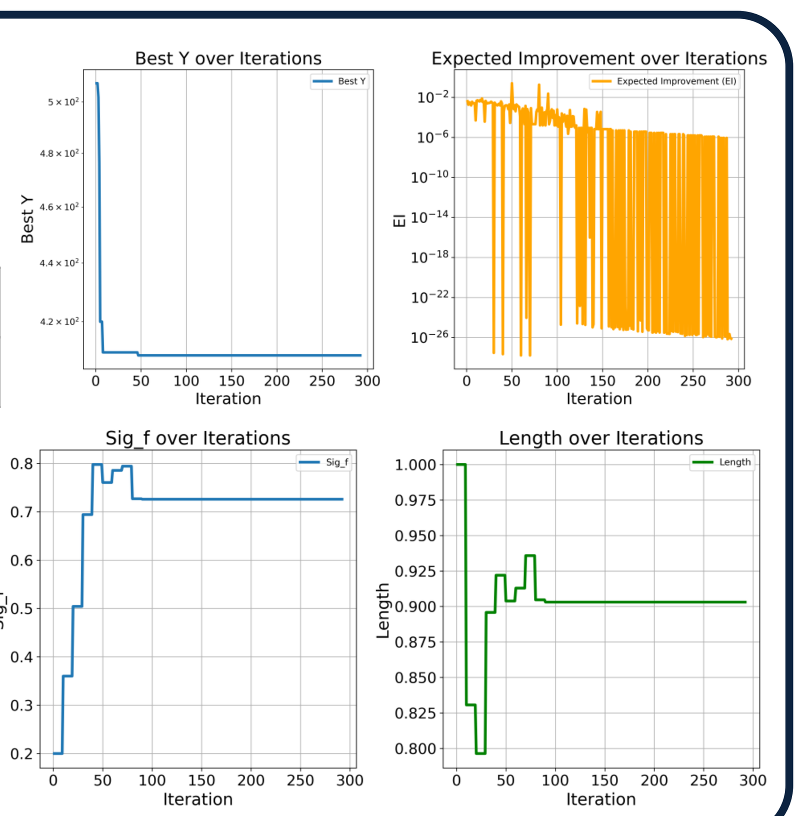


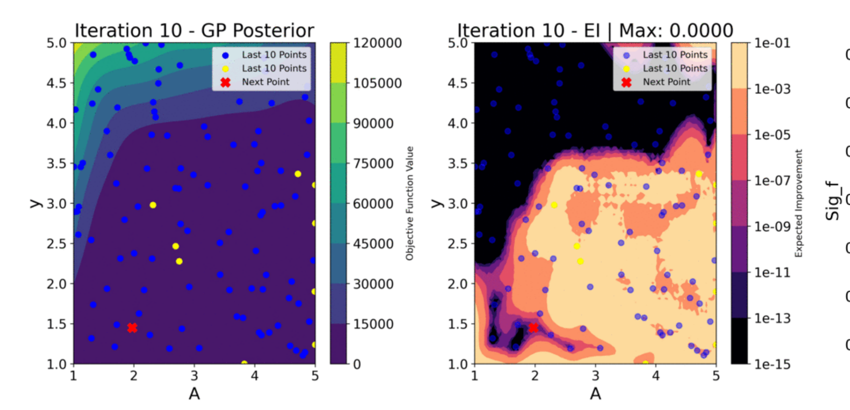


2D

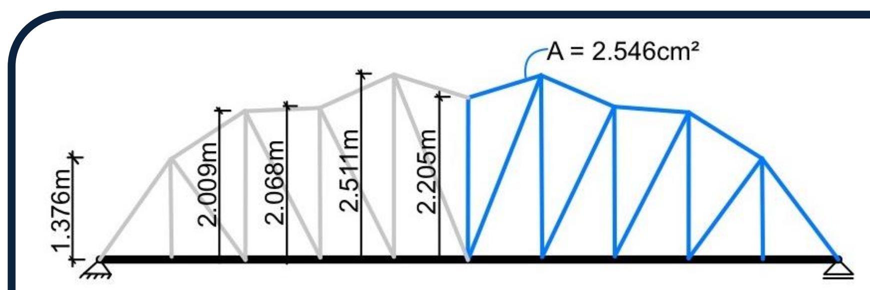


ω1 [Hz]	ω2 [Hz]	ω3 [Hz]	Weight [kg]
19.895	50.193	75.971	407.806

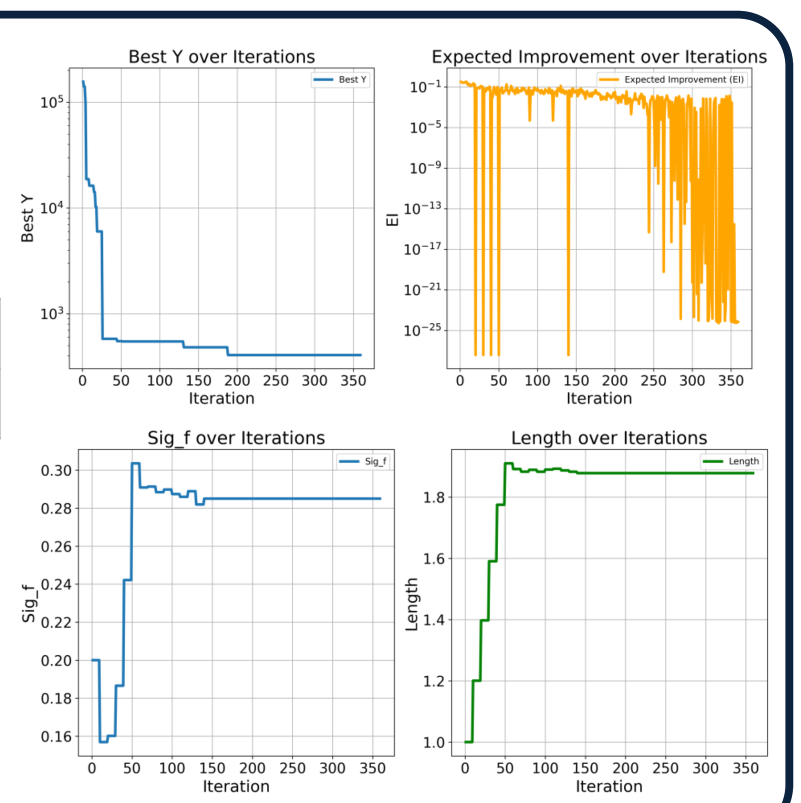




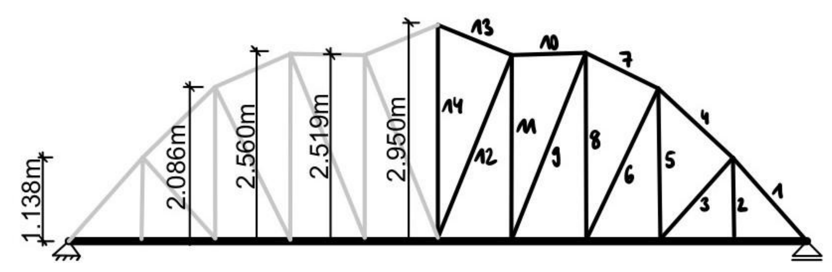
6D



ω1 [Hz]	ω2 [Hz]	ω3 [Hz]	Weight [kg]
21.114	45.587	70.500	407.475



19D



ω1 [Hz]	ω2 [Hz]	ω3 [Hz]	Weight [kg]
20.508	40.202	63.945	406.965

Optimal cross sections [cm²]					
A1	1.852	A6	3.693	A11	2.667
A2	2.313	A7	1.734	A12	1.499
A3	2.588	A8	3.453	A13	2.915
A4	2.903	A9	2.093	A14	1.235
A5	2.192	A10	2.109