Exercise

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1. What is the multivariate normal density $\mathcal{N}(x|\mu, C)$?

2. Suppose X, Y are two real-valued random variables.

   (a) What is the formula for the covariance $\mathrm{Cov}(X, Y)$?

   (b) What is the formula for the correlation $\rho(X, Y)$?

3. Suppose you can generate $\mathcal{N}(0, 1)$ random variables. Give an explicit formula for generating a sample from $\mathcal{N}(\mu, C)$ where $C = AA^{\mathsf{T}}$.

Solution

1.
$$\frac{1}{(2\pi)^{d/2}|C|^{1/2}} \exp\big(-\tfrac{1}{2}(x-\mu)^{\mathtt{T}} C^{-1}(x-\mu)\big)$$

2. (a) $\mathrm{Cov}(X,Y) = \mathbb{E}(XY) - \mathbb{E}(X)\mathbb{E}(Y) = \mathbb{E}\big((X-\mathbb{E}X)(Y-\mathbb{E}Y)\big)$
   (b) $\rho(X,Y) = \mathrm{Cov}(X,Y)/(\sigma(X)\sigma(Y))$

3. If $Z_1, \ldots, Z_d \overset{\text{iid}}{\sim} \mathcal{N}(0,1)$ and $Z = (Z_1, \ldots, Z_d)^{\mathtt{T}}$, then $AZ + \mu \sim \mathcal{N}(\mu, AA^{\mathtt{T}})$.