

# Practice Exercise 2023

## Exam I Practice Problem

We write  $X \sim \text{Poisson}(\theta)$  if  $X$  has the Poisson distribution with rate  $\theta > 0$ , that is, its p.m.f. is

$$p(x|\theta) = \text{Poisson}(x|\theta) = e^{-\theta} \theta^x / x!$$

for  $x \in \{0, 1, 2, \dots\}$  (and is 0 otherwise). Suppose  $X_1, \dots, X_n \stackrel{\text{iid}}{\sim} \text{Poisson}(\theta)$  given  $\theta$ , and your prior is

$$p(\theta) = \text{Gamma}(\theta|a, b) = \frac{b^a}{\Gamma(a)} \theta^{a-1} e^{-b\theta} \mathbf{1}(\theta > 0).$$

1. **Derive** (step by step)  $p(x_{1:n} | \theta)$  where  $x_{1:n} = (X_1, \dots, X_n)$ .
2. Assume the squared error loss function. **Provide** the Bayes rule (under the model defined above). (You do not have to prove the result).
3. **Derive** (step by step) the posterior predictive distribution of  $p(\tilde{x} | x_{1:n})$ , for a new observation  $\tilde{x}$ . Identify the distribution of the posterior predictive and its parameters for full credit.