cameron-christopher-homework4.Rmd

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1. (10 points, 5 points each) Hoff, 3.10 (Change of variables).

$$p_\psi(\psi) = p_\theta(h(\psi))\left|\frac{dh}{d\psi}\right|$$

a) $\theta \sim beta(a, b)$ & $\psi = log[\theta/(1-\theta)]$. Obtain the form of p_ψ and plot it for the case that $a = b = 1$ where $\theta = h(\psi)$.

$$\psi = log\left[\frac{\theta}{1-\theta}\right]$$
$$e^\psi = \frac{\theta}{1-\theta}$$
$$\theta = e^\psi(1-\theta)$$
$$\theta = e^\psi - \theta e^\psi$$
$$e^\psi = \theta + \theta e^\psi$$
$$e^\psi = \theta(1+e^\psi)$$
$$\theta = \frac{e^\psi}{1+e^\psi}$$

$$\theta =: h(\psi) = \frac{e^\psi}{1+e^\psi}$$

$$\left|\frac{dh}{d\psi}\right| = \frac{e^\psi}{(1+e^\psi)^2}$$

$$p_\theta(h(\psi)) = p_\theta(\theta) = \frac{1}{B(a,b)}[h(\psi)]^{a-1}[1-h(\psi)]^{b-1}$$

$$p_\psi(\psi) = p_\theta(h(\psi))\left|\frac{dh}{d\psi}\right| = \frac{1}{B(a,b)}[h(\psi)]^{a-1}[1-h(\psi)]^{b-1}\frac{e^\psi}{(1+e^\psi)^2}$$

$$= \frac{1}{B(a,b)}\left[\frac{e^{\psi}}{1+e^{\psi}}\right]^{a-1}\left[\frac{1}{1+e^{\psi}}\right]^{b-1}\frac{e^{\psi}}{(1+e^{\psi})^2}$$

$$= \frac{1}{B(a,b)}\left[\frac{e^{\psi}}{1+e^{\psi}}\right]^{a}\left[\frac{1}{1+e^{\psi}}\right]^{b}\frac{e^{\psi}}{(1+e^{\psi})^2}\left(\frac{e^{\psi}}{1+e^{\psi}}\right)\left(\frac{1}{1+e^{\psi}}\right)$$

$$p_{\psi}(\psi) = \frac{1}{B(a,b)}\left[\frac{e^{\psi}}{1+e^{\psi}}\right]^{a}\left[\frac{1}{1+e^{\psi}}\right]^{b}$$

```
theta.sim <- seq(from = -2, to = 2, length.out = 1000)
a <- 1
b <- 1

y <- (gamma(a+b)/gamma(a)*gamma(b))*(((exp(log((theta.sim)/(1-theta.sim))))/(1+exp(log((thet
```

```
## Warning in log((theta.sim)/(1 - theta.sim)): NaNs produced

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```
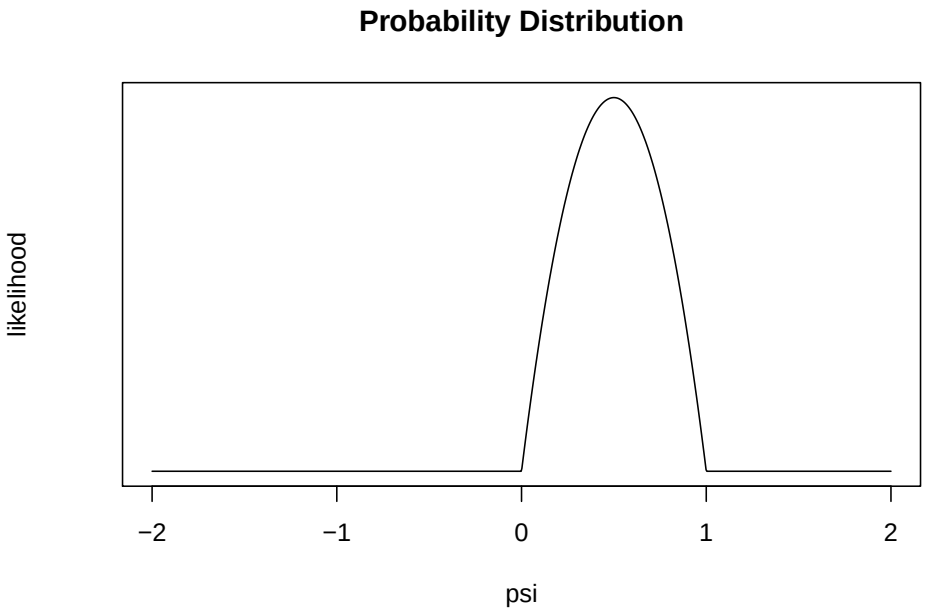

```
y[is.nan(y)] <- 0

plot(theta.sim, y, type = "l", main = "Probability Distribution",
     xlab = "psi", yaxt = "n", ylab = "likelihood")
```



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(25 points total) Please refer to lab 4 and complete tasks 4—5.