Expectation

This activity deals with the expected value *of an experiment.*

This activity deals only with experiments that have *numerical outcomes*.

Example 1. *The following experiments have numerical outcomes.*

- *The number of minutes a randomly selected customer waits in line at the grocery store*
- *The number of olives in a randomly selected jar*
- *The lifetime in hours of randomly selected light bulb*

Example 2. *The following experiments* do not *have numerical outcomes*

- *The result (heads or tails) of flipping a coin*
- *The color of the shirt a randomly selected student is wearing*

Remark 1. *The outcomes of some experiments, while ostensibly numerical, might fail to have significance as numbers. For example, you could use a die to randomly select one of six roommates to take the trash out. In this case the outcomes* $1, 2, 3, 4, 5, 6$ *correspond with people, so the experiment is not considered to have numerical outcomes. In contrast, in Monopoly the outcome of rolling two dice determines the* number *of spaces a player advances. In this situation the roll* is *considered to have a numerical outcome.*

Exercise 1 *Do the following experiments have numerical outcomes?*

- *The number of books on a randomly selected shelf at the library*

  Multiple Choice:

  (a) *Yes* ✓
  (b) *No*

- *The first letter of the title of a randomly selected book at the library*

  Multiple Choice:

  (a) *Yes*

Learning outcomes: Students will be able to calculate and understand the meaning of the expected value of an experiment.

(b) *No* ✓

- *The time at which the next book will be checked out at the library*

  Multiple Choice:

  (a) *Yes*
  (b) *No*
  (c) *Possibly, depending on how the information will be used* ✓

Now suppose that an experiment has several outcomes $E_1, E_2, \ldots, E_n$. These should all be numbers. Also, suppose that p_1 is the probability of E_1, p_2 is the probability of E_2, and so on. Note that this means $1 = p_1 + p_2 + \cdots + p_n$.

Definition 1. *The* expectation *of the experiment described above is defined to be*

$$p_1E_1 + p_2E_2 + \cdots + p_nE_n.$$

Remark 2. *The reason we require the outcomes $E_1, E_2, \ldots, E_n$ to be numbers is because we add and multiply them in* **??**. *This only makes sense when $E_1, E_2, \ldots, E_n$ are numbers.*

Example 3. *Suppose you're taking an exam, but you don't know the answer to one of the multiple choice questions. You'll just have to guess or else skip the question! The question has five answer choices. You'll receive 2 points if you answer the question correctly, but you'll* lose *a half point for an incorrect response! However, if you skip the question, you won't be penalized. In other words, if you don't answer the question, your total score on the exam will neither increase nor decrease. Should you guess or skip the question?*

We'll list all the outcomes with their probabilities, assuming that you decide to guess.

- 2 *occurs with probability* $\dfrac{1}{5}$ *when a correct answer is chosen*
- $-\dfrac{1}{2}$ *occurs with probability* $\dfrac{4}{5}$ *when an incorrect answer is chosen*

The expectation on the question is therefore

$$2 \cdot \frac{1}{5} + \left(-\frac{1}{2}\right)\frac{4}{5} = \frac{2}{5} - \frac{4}{10} = \frac{4}{10} - \frac{4}{10} = 0.$$

So it doesn't hurt to guess! However, see **??** *for a slight variation on the scoring rules with a very different conclusion.*

Remark 3. *Expectation gives the expected value of an experiment* in long run. *It doesn't tell us the exact value we expect the next time the experiment repeated, because that information would generally be impossible to know. Expectation is used in the same way that averages are used.*

Exercise 2 *This exercise is a modification of 3. Now suppose that you receive only 1 point for a correct response but you lose 1/2 point for an incorrect response. Now should you guess or skip the question?*

Begin by calculating the probabilities of each of the possible outcomes.

- 1 *occurs with probability* $\boxed{1/5}$
- $-1/2$ *occurs with probability* $\boxed{4/5}$

Now multiply each probability by its outcome and add them up. The expectation is $\boxed{-1/5}$.

You conclude that

Multiple Choice:

(a) *There is no harm in guessing.* ✓

(b) *You shouldn't guess.*

(c) *We can't conclude anything.*

Exercise 3 *Marcus and Dave play following game. Marcus randomly selects card from a standard deck. If he selects ♣ then Dave gives Marcus \$4. Otherwise Marcus gives Dave \$2. The expected proceeds for Marcus are* $\boxed{-1/2}$.

Exercise 4 *The expected proceeds for Dave are* $\boxed{1/2}$.

Exercise 5 *You conclude that the game is unfair for*

Multiple Choice:

(a) *Marcus* ✓

(b) *Dave*

Exercise 6 *At charity event can purchase one of* 100 *tickets for* $\$2$ *The prize is* $\$50$. *Calculate expected proceeds of a participant who buys one ticket.*

- *The outcome* $48 = 50 - 2$ *occurs with probability* $\boxed{1/100}$ *if the player wins.*
- *The outcome* -2 *occurs with probability* $\boxed{99/100}$ *if the player loses.*
- *So the expected proceeds are* $\$\boxed{-3/2}$.

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for i in range(5):
  for j in range(5):
    print i+j,i,j
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