Expectation

This activity deals with the expected value *of an experiment.*

This activity deals only with experiments that have *numerical outcomes*.

Example 1. *The following experiments have numerical outcomes.*

- *The number of minutes a randomly selected customer waits in line at the grocery store*
- *The number of olives in a randomly selected jar*
- *The lifetime in hours of randomly selected light bulb*

Example 2. *The following experiments* do not *have numerical outcomes*

- *The result (heads or tails) of flipping a coin*
- *The color of the shirt a randomly selected student is wearing*

Remark 1. *The outcomes of some experiments, while ostensibly numerical, might fail to have significance as numbers. For example, you could use a die to randomly select one of six roommates to take the trash out. In this case the outcomes* $1, 2, 3, 4, 5, 6$ *correspond with people, so the experiment is not considered to have numerical outcomes. In contrast, in Monopoly the outcome of rolling two dice determines the* number *of spaces a player advances. In this situation the roll* is *considered to have a numerical outcome.*

Exercise 1 *Do the following experiments have numerical outcomes?*

- *The number of books on a randomly selected shelf at the library*

  Multiple Choice:

  (a) *Yes* ✓
  (b) *No*

- *The first letter of the title of a randomly selected book at the library*

  Multiple Choice:

  (a) *Yes*

Learning outcomes: Students will be able to calculate and understand the meaning of the expected value of an experiment.

(b) *No* ✓

- *The time at which the next book will be checked out at the library*

  Multiple Choice:

  (a) *Yes*
  (b) *No*
  (c) *Possibly, depending on how the information will be used* ✓

Now suppose that an experiment has several outcomes $E_1, E_2, \ldots, E_n$. These should all be numbers. Also, suppose that p_1 is the probability of E_1, p_2 is the probability of E_2, and so on. Note that this means $1 = p_1 + p_2 + \cdots + p_n$.

Definition 1. *The* expectation *of the experiment described above is defined to be*

$$p_1E_1 + p_2E_2 + \cdots + p_nE_n. \tag{1}$$

Remark 2. *The reason that we require the outcomes $E_1, E_2, \ldots, E_n$ to be numbers is because we add and multiply them in Equation 1. This only makes sense when $E_1, E_2, \ldots, E_n$ are numbers.*

Example 3. *Suppose you're taking an exam, but you don't know the answer to a multiple choice question. You'll just have to guess or else skip the quesgion! The question has five answer choices. You'll get 2 points for a correct response but you'll* lose $-1/2$ *point for an incorrect response! However, if you skip the question, you won't be penalized. Should you guess or skip the question.*

To answer this, we list all the outcomes and their probabilities in the situation that you decide to guess.

- 2 *occurs with probability* $1/5$ *when correct answer chosen*
- $-1/2$ *occurs with probability* $4/5$ *when incorrect answer chosen*

The expectation on the question is therefore

$$2 \cdot \frac{1}{5} + \left(-\frac{1}{2}\right)\frac{4}{5} = \frac{2}{5} - \frac{4}{10} = \frac{4}{10} - \frac{4}{10} = 0$$

So it doesn't hurt to guess!

Remark 3. *The expectation gives the expected value of an experiment* in long run*. It doesn't tell us the exact value we expect the next time the experiment repeated, because that information would generally be impossible to know. Expectation is used in the same way that averages are used.*