Escape Velocity

We start with Newton's law of gravitation and derive the escape velocity of a planet.

Exercise 1 *Let's find a function $f(x)$ so that $f(2) = 4$.*

Solution

Hint: *Try a linear function.*

An example is $f(x) =$.

That is a great example.

Observation 2 ***Newton's Law of Gravitation.*** *The force of gravity on an object of mass m that is distance r from the center of the earth is given by*

$$F(r) = \frac{GMm}{r^2}$$

where G is the universal gravitational constant and M is the mass of the Earth. In the metric system, the units of GM are $\frac{m^3}{s^2}$.

Question 3 *Use the following two facts to find the value of GM in $\frac{m^3}{s^2}$ to three significant digits.*

Observation 4 *A 100-kg object weighs 981 N at the surface of the Earth.*

Observation 5 *The Earth's radius is 6380 km.*

Solution

Hint: *The two facts from this question can be plugged into the law of gravity.*

Hint: *There is a problem with the units of the Earth's radius. Make sure you convert it to meters!*

Hint: *Plugging in the values gives $981N = \frac{GM(100kg)}{(6380000m)^2}$, so solve for GM.*

$GM = 3.99 * 10^{14} \ \frac{m^3}{s^2}$.

Question 6 *The value of G is $6.67 \cdot 10^{-11} \frac{m^3}{kg \cdot s^2}$. Find the mass of the Earth in kilograms. Give 3 significant digits.*

Learning outcomes: Use an improper integral in an application problem. Integrate a force function to compute work. Correctly handle units during calculus operations.

Solution

Hint: *You found the value of GM in the previous question, so you should now be able to find M by dividing.*

Hint: *Dividing GM by G and find 3 significant digits.*

$M = 5.99 * 10^{24}\ kg$.