

Escape Velocity

We start with Newton's law of gravitation and derive the escape velocity of a planet.

Observation 1 Newton's Law of Gravitation. The force of gravity on an object of mass m that is distance r from the center of the earth is given by

$$F(r) = \frac{GMm}{r^2}$$

where G is the universal gravitational constant and M is the mass of the Earth. In the metric system, the units of GM are $\frac{m^3}{s^2}$.

Finding the value of GM

Question 2 Use the following two facts to find the value of GM in $\frac{m^3}{s^2}$ to three significant digits.

- A 100-kg object weighs 981 N at the surface of the Earth.
- The Earth's radius is 6380 km.

Solution

- (a) $GM = 3.99 \cdot 10^8 \frac{m^3}{s^2}$
- (b) $GM = 3.99 \cdot 10^{14} \frac{m^3}{s^2}$ ✓
- (c) $GM = 418 \frac{m^3}{s^2}$
- (d) $GM = 4.18 \cdot 10^8 \frac{m^3}{s^2}$

Hint: The two facts from this question can be plugged into the law of gravity.

Hint: There is a problem with the units of the Earth's radius. Make sure you convert it to meters!

Plugging in the values gives $981N = \frac{GM(100kg)}{(6380000m)^2}$, and so $GM = 3.99 \cdot 10^{14} \frac{m^3}{s^2}$.

Finding the value of M

Question 3 The value of G is $6.67 \cdot 10^{-11} \frac{m^3}{kg \cdot s^2}$. Find the mass of the Earth.

Solution

Learning outcomes: Use an improper integral in an application problem. Integrate a force function to compute work. Correctly handle units during calculus operations.

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- (a) $M = 1.67 \cdot 10^{-25} kg$
- (b) $M = 3.67 \cdot 10^{23} kg$
- (c) $M = 5.99 \cdot 10^{24} kg$ ✓
- (d) $k = 2.67 \cdot 10^4 kg$

Hint: You found the value of GM in the previous question, so you should now be able to find M by dividing.

Dividing GM by G yields $5.99 \cdot 10^{24} kg$.
