

Escape Velocity

We start with Newton's law of gravitation and derive the escape velocity of a planet.

Exercise 1 Let's find a function $f(x)$ so that $f(2) = 4$.

Solution

Hint: Try a linear function.

An example is $f(x) = x^2$.

That is a great example.

Observation 2 Newton's Law of Gravitation. The force of gravity on an object of mass m that is distance r from the center of the earth is given by

$$F(r) = \frac{GMm}{r^2}$$

where G is the universal gravitational constant and M is the mass of the Earth. In the metric system, the units of GM are $\frac{m^3}{s^2}$.

Question 3 Use the following two facts to find the value of GM in $\frac{m^3}{s^2}$ to three significant digits.

Observation 4 A 100-kg object weighs 981 N at the surface of the Earth.

Observation 5 The Earth's radius is 6380 km.

Solution

Hint: The two facts from this question can be plugged into the law of gravity.

Hint: There is a problem with the units of the Earth's radius. Make sure you convert it to meters!

Hint: Plugging in the values gives $981N = \frac{GM(100kg)}{(6380000m)^2}$, so solve for GM .

$$GM = 3.99 * 10^{14} \frac{m^3}{s^2}.$$

Question 6 The value of G is $6.67 \cdot 10^{-11} \frac{m^3}{kg \cdot s^2}$. Find the mass of the Earth in kilograms. Give 3 significant digits.

Learning outcomes: Use an improper integral in an application problem. Integrate a force function to compute work. Correctly handle units during calculus operations.

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Solution

Hint: You found the value of GM in the previous question, so you should now be able to find M by dividing.

Hint: Dividing GM by G and find 3 significant digits.

$$M = 5.99 * 10^{24} \text{ kg.}$$
