Escape Velocity

We start with Newton's law of gravitation and derive the escape velocity of a planet.

Observation 1 (Newton's Law of Gravitation) *The force of gravity on an object of mass m that is distance r from the center of the earth is given by*

$$F(r) = \frac{GMm}{r^2}$$

where G is the universal gravitational constant and M is the mass of the Earth. In the metric system, the units of GM are $\frac{m^3}{s^2}$.

Finding the value of GM

Question 2 *Use the following two facts to find the value of GM in $\frac{m^3}{s^2}$ to three significant digits.*

- *A 100-kg object weighs 981 N at the surface of the Earth.*
- *The Earth's radius is 6380 km.*

Solution

(a) $\mathrm{G\ M} = 3.99 \cdot 10^8 \frac{m^3}{s^2}$

(b) $\mathrm{G\ M} = 3.99 \cdot 10^{14} \frac{m^3}{s^2}$ ✓

(c) $\mathrm{G\ M} = 418 \frac{m^3}{s^2}$

(d) $\mathrm{G\ M} = 4.18 \cdot 10^8 \frac{m^3}{s^2}$

Hint: *The two facts from this question can be plugged into the law of gravity.*

Hint: *There is a problem with the units of the Earth's radius. Make sure you convert it to meters!*

Plugging in the values gives $981N = \frac{GM(100kg)}{(6380000m)^2}$, and so $GM = 3.99 \cdot 10^{14} \frac{m^3}{s^2}$.

Finding the value of M

Question 3 *The value of G is $6.67 \cdot 10^-11 \frac{m^3}{kg \cdot s^2}$. Find the mass of the Earth.*

Solution

Learning outcomes: Use an improper integral in an application problem. Integrate a force function to compute work. Correctly handle units during calculus operations.

(a) M $= 1.67 \cdot 10^{-25} kg$

(b) M $= 3.67 \cdot 10^{23} kg$

(c) M $= 5.99 \cdot 10^{24} kg$ ✓

(d) k $= 2.67 \cdot 10^{4} kg$

Hint: *You found the value of GM in the previous question, so you should now be able to find M by dividing.*

Dividing GM by G yields $5.99 \cdot 10^{24} kg$.