

# Escape Velocity

We start with Newton's law of gravitation and derive the escape velocity of a planet.

**Axiom 1 (Newton's Law of Gravitation)** The force of gravity on an object of mass  $m$  that is distance  $r$  from the center of the earth is given by

$$F(r) = \frac{GMm}{r^2}$$

where  $G$  is the universal gravitational constant and  $M$  is the mass of the Earth. In the metric system, the units of  $GM$  are  $\frac{m^3}{s^2}$ .

## Finding the value of $GM$

**Question 2** Use the following two facts to find the value of  $GM$  in  $\frac{m^3}{s^2}$  to three significant digits.

- A 100-kg object weighs 981 N at the surface of the Earth.
- The Earth's radius is 6380 km.

**Solution**

- (a)  $GM = 3.99 \cdot 10^8 \frac{m^3}{s^2}$
- (b)  $GM = 3.99 \cdot 10^{14} \frac{m^3}{s^2}$  ✓
- (c)  $GM = 418 \frac{m^3}{s^2}$
- (d)  $GM = 4.18 \cdot 10^8 \frac{m^3}{s^2}$

**Hint:** The two facts from this question can be plugged into the law of gravity.

**Hint:** There is a problem with the units of the Earth's radius. Make sure you convert it to meters!

Plugging in the values gives  $981N = \frac{GM(100kg)}{(6380000m)^2}$ , and so  $GM = 3.99 \cdot 10^{14} \frac{m^3}{s^2}$ .

## Finding the value of $M$

**Question 3** The value of  $G$  is  $6.67 \cdot 10^{-11} \frac{m^3}{kg \cdot s^2}$ . Find the mass of the Earth.

**Solution**

Learning outcomes: Use an improper integral in an application problem. Integrate a force function to compute work. Correctly handle units during calculus operations.

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- (a)  $M = 1.67 \cdot 10^{-25} kg$
- (b)  $M = 3.67 \cdot 10^{23} kg$
- (c)  $M = 5.99 \cdot 10^{24} kg$  ✓
- (d)  $k = 2.67 \cdot 10^4 kg$

**Hint:** You found the value of  $GM$  in the previous question, so you should now be able to find  $M$  by dividing.

Dividing  $GM$  by  $G$  yields  $5.99 \cdot 10^{24} kg$ .

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