INTRODUCTION TO MICROELECTRONIC DEVICES AND CIRCUITS

Review of

SEMICONDUCTOR DEVICES

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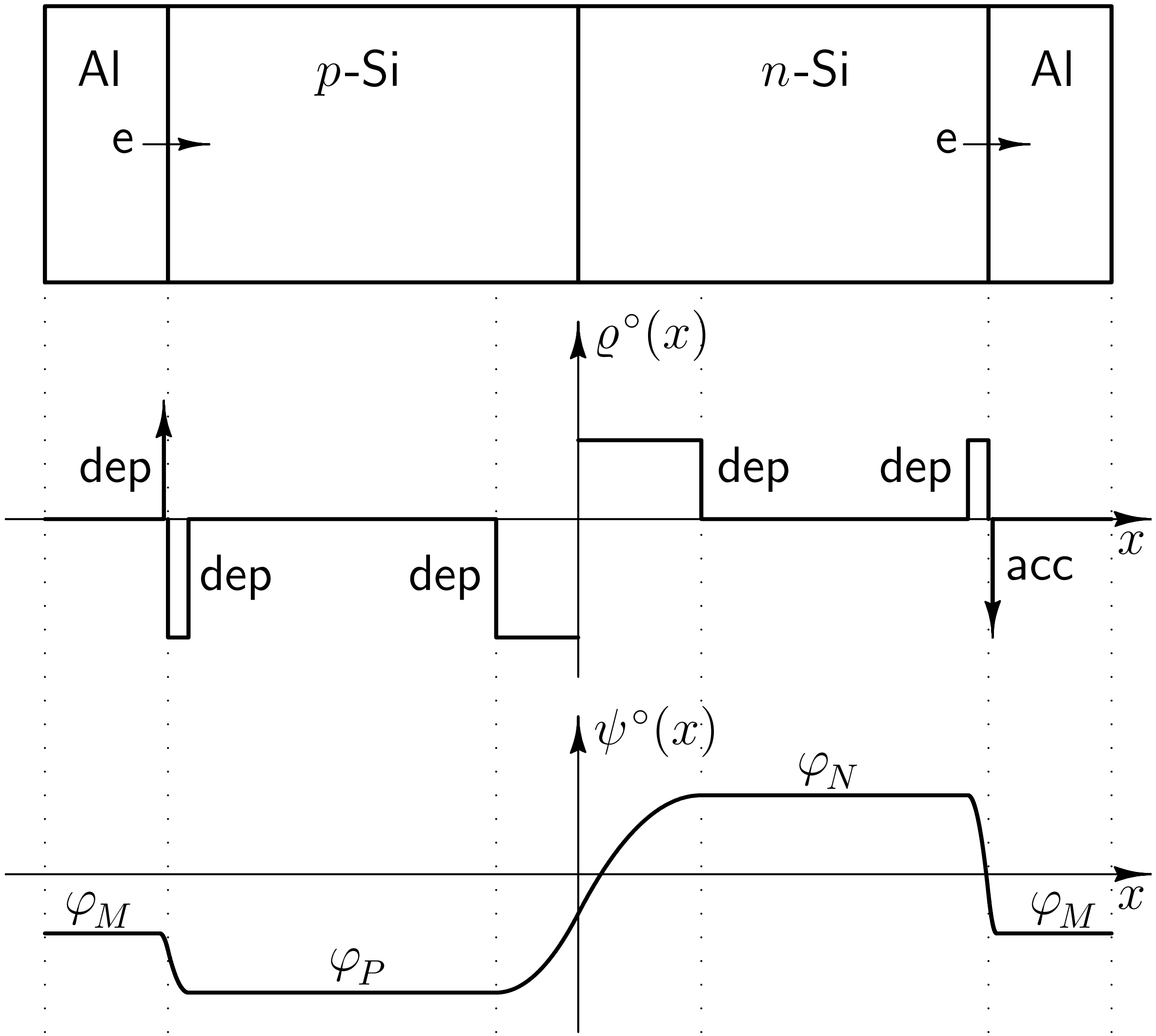
FORMATION OF THE DEPLETION REGION IN PN-JUNCTION DIODES

- The depletion region is formed because holes from the p-side diffuse to the n-side where they recombine with electrons, and electrons from the n-side diffuse to the p-side where they recombine with holes.
- The concentrations of electrons and holes in the depletion region are much smaller than N_a in the p-side and N_d in the n-side.
- In the depletion region, there is a built-in electric field and non-zero carrier-concentration gradients:
  - Hole diffusion $\rightarrow$ is cancelled by hole drift $\leftarrow$, and
  - Electron diffusion $\leftarrow$ is cancelled by electron drift $\rightarrow$.
- The net recombination or generation rate distributions are zero the depletion region and the quasi neutral regions.
- Charge conservation requires that

$$N_a^- W_{d,P}^\circ = N_d^+ W_{d,N}^\circ .$$

PN-JUNCTION DIODE AT THERMAL EQUILIBRIUM



PN-JUNCTION DIODE AT THERMAL EQUILIBRIUM

- The built-in potential V_{bi} is given by

$$V_{bi} = \frac{k_B T}{q} \ln \left(\frac{N_a^- \, N_d^+}{n_i^2} \right) .$$

- The depletion-region width W_d° at thermal equilibrium is given by

$$W_d^\circ = \sqrt{\frac{2\,\epsilon_{Si}}{q} \cdot \frac{\left(N_a^- + N_d^+\right)}{N_a^- \, N_d^+} \, V_{bi}} .$$

PN-JUNCTION DIODE AT THERMAL EQUILIBRIUM

- The p-side depletion-region width $W^{\circ}_{d,P}$ at thermal equilibrium is

$$W^{\circ}_{d,P} = \frac{N_d^+}{\left(N_a^- + N_d^+\right)} W^{\circ}_d = \sqrt{\frac{2\,\epsilon_{Si}}{q} \cdot \frac{N_d^+}{N_a^- \left(N_a^- + N_d^+\right)} V_{bi}} \; .$$

- The n-side depletion-region width $W^{\circ}_{d,N}$ at thermal equilibrium is

$$W^{\circ}_{d,N} = \frac{N_a^-}{\left(N_a^- + N_d^+\right)} W^{\circ}_d = \sqrt{\frac{2\,\epsilon_{Si}}{q} \cdot \frac{N_a^-}{N_d^+ \left(N_a^- + N_d^+\right)} V_{bi}} \; .$$

BIAS-VOLTAGE EFFECTS IN A PN-JUNCTION DIODE

- For $V_{PN} > 0$:
  - $W_d(V_{PN}) \downarrow$,
  - The barrier becomes $(V_{bi} - V_{PN}) \downarrow$,
  - The magnitude of the field in the depletion region $\downarrow$, and
  - The depletion-region carrier gradients $dp/dx \uparrow$ and $dn/dx \uparrow$.
- For $V_{PN} < 0$:
  - $W_d(V_{PN}) \uparrow$,
  - The barrier becomes $(V_{bi} - V_{PN}) \uparrow$,
  - The magnitude of the field in the depletion region $\uparrow$, and
  - The depletion-region carrier gradients $dp/dx \downarrow$ and $dn/dx \downarrow$.

BIAS-VOLTAGE EFFECTS IN A PN-JUNCTION DIODE

- The width of the depletion region $W_d\,(V_{PN})$ is given by

$$W_d\,(V_{PN}) = \sqrt{\frac{2\,\epsilon_{Si}}{q} \cdot \frac{\left(N_a^- + N_d^+\right)}{N_a^-\,N_d^+}\left(V_{bi} - V_{PN}\right)}\ .$$

- The p-side depletion-region width is

$$W_{d,P}(V_{PN}) = \sqrt{\frac{2\,\epsilon_{Si}}{q} \cdot \frac{N_d^+}{N_a^-\,\left(N_a^- + N_d^+\right)}\left(V_{bi} - V_{PN}\right)}\ .$$

- The n-side depletion-region width is

$$W_{d,N}(V_{PN}) = \sqrt{\frac{2\,\epsilon_{Si}}{q} \cdot \frac{N_a^-}{N_d^+\,\left(N_a^- + N_d^+\right)}\left(V_{bi} - V_{PN}\right)}\ .$$

- The field at $x = 0$ is given by

$$
\begin{aligned}
\mathcal{E}_{x \cdot max}(V_{PN}) &\equiv \mathcal{E}_x(0, V_{PN}) \,, \\
&= - \, \frac{q \, N_a^- \, W_{d,P}(V_{PN})}{\epsilon_{Si}} = - \, \frac{q \, N_d^+ \, W_{d,N}(V_{PN})}{\epsilon_{Si}} \,, \\
&= - \sqrt{\frac{2\,q}{\epsilon_{Si}} \cdot \frac{N_a^- \, N_d^+}{\left(N_a^- + N_d^+\right)} \left(V_{bi} - V_{PN}\right)} \,.
\end{aligned}
$$

BIAS-VOLTAGE EFFECTS IN A PN-JUNCTION DIODE

- The minority-carrier concentrations at the depletion-region edges under bias conditions are given by

$$p_N(W_{d,N}, V_{PN}) = p_N^\circ \exp\left(\frac{q\,V_{PN}}{k_B T}\right) ,$$

and

$$n_P(-W_{d,P}, V_{PN}) = n_P^\circ \exp\left(\frac{q\,V_{PN}}{k_B T}\right) .$$

These important relationships are called the **law of the junction**.

BIAS-VOLTAGE EFFECTS IN A PN-JUNCTION DIODE

- Under forward-bias static conditions, holes are transported in the positive x direction by drift and diffusion in the p-side quasi-neutral region where they also recombine with electrons injected from the n-side of the junction.
- Holes and electrons recombine throughout the depletion region. Under the assumption of a uniform net recombination rate throughout this region, the hole current density decreases linearly from the p-side depletion-region edge to the n-side depletion-region edge.
- Holes are transported in the positive x direction mainly by diffusion in the n-side quasi-neutral region where they also recombine with the majority electrons.

STATIC $I_D(V_{PN})$ CHARACTERISTICS

- Under forward-bias static conditions, electrons are transported in the negative x direction by drift and diffusion in the n-side quasi-neutral region where they also recombine with holes injected from the p-side of the junction.
- Electrons and holes recombine throughout the depletion region. Under the assumption of a uniform net recombination rate throughout this region, the electron current density decreases linearly from the n-side depletion-region edge to the p-side depletion-region edge.
- Electrons are transported in the negative x direction mainly by diffusion in the p-side quasi-neutral region where they also recombine with the majority holes.

- The PN-junction diode current density is then given by

$$J_D(V_{PN}) = J_{S\cdot diff}\left[\exp\left(\frac{q\,V_{PN}}{k_B T}\right) - 1\right] + J_{S\cdot scr}(V_{PN})\left[\exp\left(\frac{q\,V_{PN}}{2\,k_B T}\right) - 1\right],$$

where

$$J_{S\cdot diff} = \frac{q\,D_{n,P}\,n_i^2}{L_{n,P}\,N_a^-} + \frac{q\,D_{p,N}\,n_i^2}{L_{p,N}\,N_d^+},$$

$$J_{S\cdot scr}(V_{PN}) = \frac{q\,n_i\,W_d\,(V_{PN})}{2\,\tau_{rec,SCR}},$$

and

$$\tau_{rec,SCR} \approx \frac{\tau_{rec,P} + \tau_{rec,N}}{2}.$$

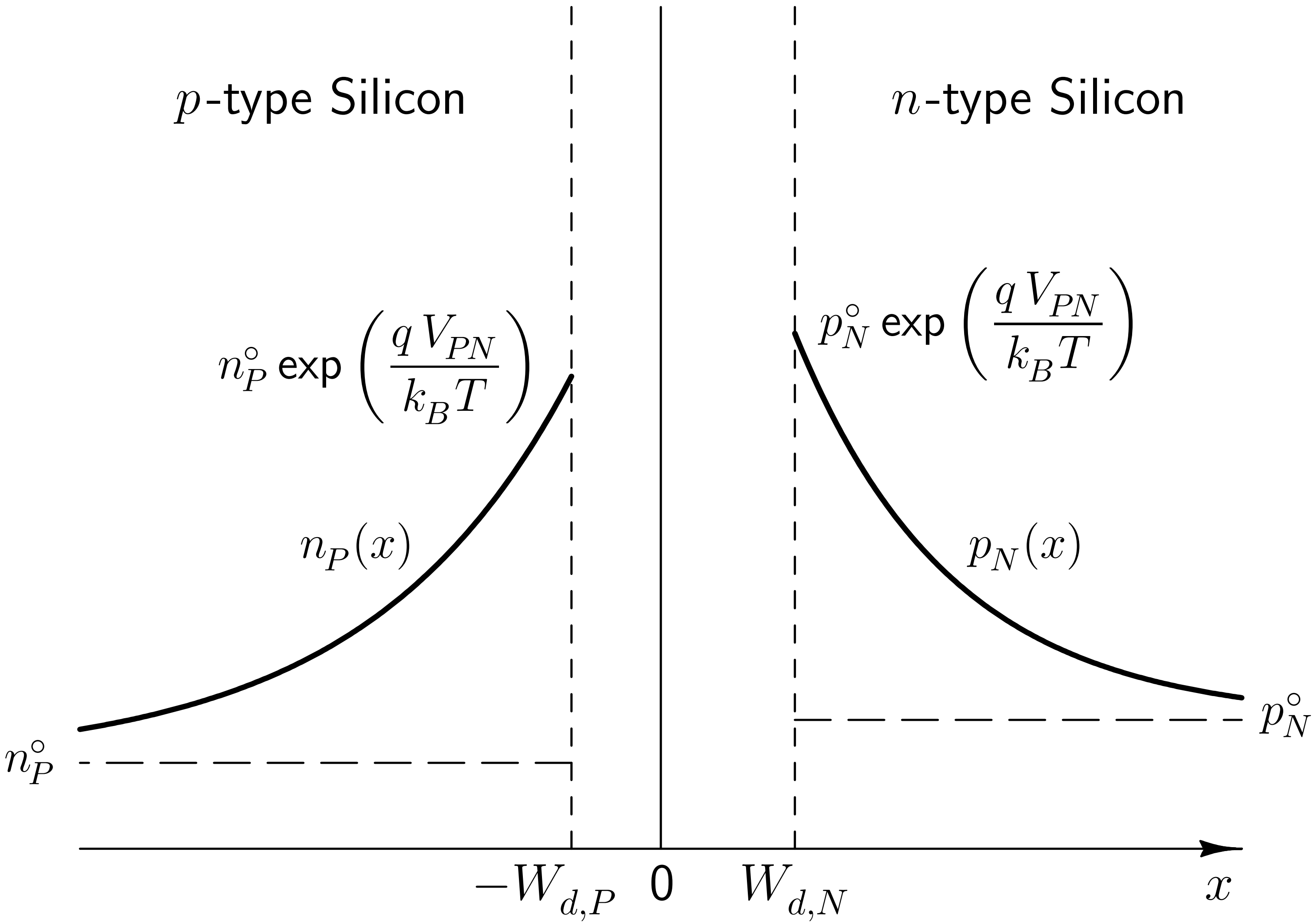
STATIC $I_D(V_{PN})$ CHARACTERISTICS

- The PN-junction diode current $I_D(V_{PN})$ is then given by

$$
\begin{aligned}
I_D(V_{PN}) &= A_d\, J_D(V_{PN})\,, \\
&= I_{S\cdot diff}\left[\exp\left(\frac{q\,V_{PN}}{k_B T}\right) - 1\right] + I_{S\cdot scr}(V_{PN})\left[\exp\left(\frac{q\,V_{PN}}{2\,k_B T}\right) - 1\right],
\end{aligned}
$$

where A_d is the diode area.

STATIC $I_D(V_{PN})$ CHARACTERISTICS



STATIC $I_D(V_{PN})$ CHARACTERISTICS

- When the applied bias voltage V_{PN} is negative, the PN-junction diode is reverse-biased. Under such conditions
  - $W_d(V_{PN}) \uparrow$,
  - $|\mathcal{E}_x| \uparrow$ in the depletion region,
  - $(V_{bi} - V_{PN}) \uparrow$ across the junction because $V_{PN} < 0$,
  - $p_N(W_{d,N}) \downarrow < p_N^\circ$ because $V_{PN} < 0$ and $\exp(qV_{PN}/k_BT) < 1$,
  - Holes in the n-side quasi-neutral region diffuse towards the junction,
  - $n_P(-W_{d,P}) \downarrow < n_P^\circ$ because $V_{PN} < 0$ and $\exp(qV_{PN}/k_BT) < 1$,
  - Electrons in the p-side quasi-neutral region diffuse towards the junction, and
  - There is net electron-hole-pair generation throughout the diode.

STATIC $I_D(V_{PN})$ CHARACTERISTICS

- The minority-carrier lifetimes and diffusion lengths used in calculating the carrier-concentration distributions are minority-carrier generation lifetimes and diffusion lengths. In other words :

$$\begin{aligned}
\tau_{rec\cdot p,N} \quad &\longrightarrow \quad \tau_{gen\cdot p,N} \,, \\
L_{p,N} = \sqrt{\tau_{rec\cdot p,N}\, D_{p,N}} \quad &\longrightarrow \quad L_{p,N} = \sqrt{\tau_{gen\cdot p,N}\, D_{p,N}} \,, \\
\tau_{rec\cdot n,P} \quad &\longrightarrow \quad \tau_{gen\cdot n,P} \,, \\
L_{n,P} = \sqrt{\tau_{rec\cdot n,P}\, D_{n,P}} \quad &\longrightarrow \quad L_{n,P} = \sqrt{\tau_{gen\cdot n,P}\, D_{n,P}} \,, \\
\tau_{rec,SCR} \quad &\longrightarrow \quad \tau_{gen,SCR} \,.
\end{aligned}$$

STATIC $I_D(V_{PN})$ CHARACTERISTICS

- Holes generated in the n-side quasi-neutral region diffuse towards the depletion region where they join holes generated there. These holes are swept by the electric field in the depletion region towards the p-side depletion-region edge. More holes are generated in the p-side quasi-neutral region, and all holes are transported there by drift and diffusion towards the metal contact at $x = -W_P$, and

- Electrons generated in the p-side quasi-neutral region diffuse towards the depletion region where they join electrons generated there. These electrons are swept by the electric field in the depletion region towards the n-side depletion-region edge. More electrons are generated in the n-side quasi-neutral region, and all electrons are transported there by drift and diffusion towards the metal contact at $x = W_N$.

STATIC $I_D(V_{PN})$ CHARACTERISTICS

- Under reverse-bias conditions, the space-charge-generation current density is given by

$$J_{D\cdot scg}(V_{PN}) = \frac{q\, n_i\, W_d\,(V_{PN})}{2\,\tau_{gen,SCR}} \left[\mathrm{exp}\left(\frac{q\, V_{PN}}{2\, k_B T} \right) - 1 \right] ,$$

$$\approx -\, \frac{q\, n_i\, W_d\,(V_{PN})}{2\,\tau_{gen,SCR}} \qquad \text{for } V_{PN} \leq -\,6\, k_B T/q \ .$$

It is evident that the reverse-bias space-charge-generation current density has a weak bias-voltage dependence.

REVERSE-BIAS BREAKDOWN IN A PN-JUNCTION DIODE

- At large enough reverse-bias voltages, the reverse-bias diode current I_D increases substantially due to **avalanche multiplication**, a process that results directly from carrier impact ionization.
- Breakdown can also occur due to carrier tunneling at low bias voltages in heavily doped PN-junction diodes. As the dopant concentrations in both sides of a PN-junction diode are increased, the depletion width decreases.
- For depletion widths near 50 Å, carriers can tunnel through the potential barrier between valence and conduction bands. This lower voltage breakdown in heavily doped PN-junction diodes is called **tunneling breakdown** or **Zener breakdown**.

SMALL-SIGNAL $C_{pn}(V_{PN})$ CHARACTERISTICS

- The small-signal capacitance per unit area $C_{pn}(V_{PN})$ of a PN-junction diode is given by :

$$
\begin{aligned}
C_{pn}(V_{PN}) &\equiv \left|\frac{dQ_{pn}(V_{PN})}{dV_{PN}}\right| , \\
&= \left|\frac{dQ_{pn\cdot dep}(V_{PN})}{dV_{PN}}\right| + \left|\frac{dQ_{pn\cdot diff,P}(V_{PN})}{dV_{PN}}\right| + \left|\frac{dQ_{pn\cdot diff,N}(V_{PN})}{dV_{PN}}\right| , \\
&= \quad C_{pn\cdot dep}(V_{PN}) \quad + \quad C_{pn\cdot diff,P}(V_{PN}) \quad + \quad C_{pn\cdot diff,N}(V_{PN}) \, .
\end{aligned}
$$

SMALL-SIGNAL $C_{pn}(V_{PN})$ CHARACTERISTICS

- The small-signal PN-junction diode depletion capacitance $C_{pn \cdot dep}(V_{PN})$ per unit area is given by

$$C_{pn \cdot dep}(V_{PN}) = \sqrt{\frac{q\,\epsilon_{Si}}{2} \cdot \frac{N_a^- \, N_d^+}{\left(N_a^- + N_d^+\right)} \cdot \frac{1}{\left(V_{bi} - V_{PN}\right)}} \; .$$

SMALL-SIGNAL $C_{pn}(V_{PN})$ CHARACTERISTICS

- The small-signal diffusion capacitance $C_{pn\cdot diff,N}(V_{PN})$ per unit area on the n-side of the junction for a long-base n-side quasi-neutral region is given by

$$C_{pn\cdot diff,N}(V_{PN}) = \frac{q^2\, n_i^2\, L_{p,N}}{k_B T\, N_d^+} \exp\left(\frac{q\, V_{PN}}{k_B T}\right) .$$

- The small-signal diffusion capacitance $C_{pn\cdot diff,P}(V_{PN})$ per unit area on the p-side of the junction for a long-base p-side quasi-neutral region is given by

$$C_{pn\cdot diff,P}(V_{PN}) = \frac{q^2\, n_i^2\, L_{n,P}}{k_B T\, N_a^-} \exp\left(\frac{q\, V_{PN}}{k_B T}\right) .$$

SMALL-SIGNAL $C_{pn}(V_{PN})$ CHARACTERISTICS

- The PN-junction diffusion capacitance per unit area can be expressed as

$$C_{pn\cdot diff}(V_{PN}) = \Big[\, C_{pn\cdot diff,P}(0) + C_{pn\cdot diff,N}(0) \,\Big] \exp\left(\frac{q\,V_{PN}}{k_B T}\right) ,$$

$$= C_{pn\cdot diff}(0) \exp\left(\frac{q\,V_{PN}}{k_B T}\right) ,$$

where

$$C_{pn\cdot diff}(0) = C_{pn\cdot diff,P}(0) + C_{pn\cdot diff,N}(0) \, .$$

SMALL-SIGNAL $C_{pn}(V_{PN})$ CHARACTERISTICS

- The small-signal PN-junction diode capacitance per unit area is given by

$$C_{pn}(V_{PN}) = C_{pn\cdot dep}(0)\,\sqrt{\frac{V_{bi}}{(V_{bi} - V_{PN})}} + C_{pn\cdot diff}(0)\,\exp\left(\frac{q\,V_{PN}}{k_B T}\right)\,,$$

where

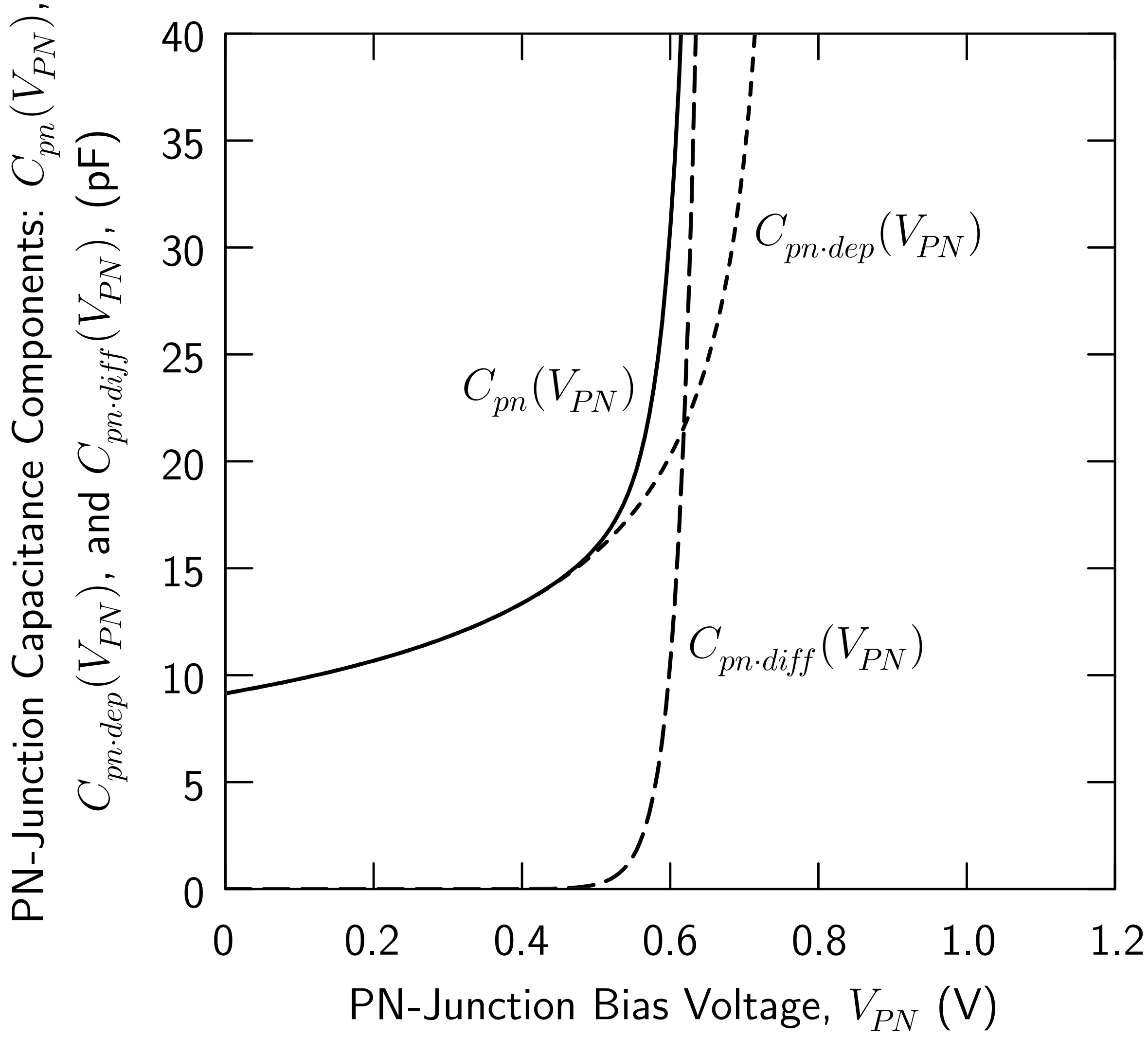
$$C_{pn\cdot dep}(0) = \sqrt{\frac{q\,\epsilon_{Si}}{2}\cdot\frac{N_a^-\,N_d^+}{(N_a^- + N_d^+)}\cdot\frac{1}{V_{bi}}}\,,$$

$$C_{pn\cdot diff,P}(0) = \frac{q^2\,n_i^2\,L_{n,P}}{k_B T\,N_a^-}\,,$$

and

$$C_{pn\cdot diff,N}(0) = \frac{q^2\,n_i^2\,L_{p,N}}{k_B T\,N_d^+}\,.$$

SMALL-SIGNAL $C_{pn}(V_{PN})$ CHARACTERISTICS



MOSCAP PARAMETERS, VARIABLES, AND DISTRIBUTIONS

- $N_{a,B}^{-}$ Ionized-acceptor concentration in the substrate region,
- X_{ox} Gate-oxide thickness,
- Q_f Fixed-oxide charge density ($Q_f = q\, N_f$),
- $Q_{it}(V_{GB})$ Interface-trap charge density $\left(Q_{it}(V_{GB}) = q\, N_{it}(V_{GB})\right)$, and
- φ_{PM} Contact-potential difference between the p-type substrate and the backside metal contact.
- $Q_G(V_{GB})$ Charge density in the metal-gate region,
- $\mathcal{E}_{x,OX}(V_{GB})$ Electric field in the x-direction in a charge-free gate-oxide layer,
- $V_{OX}(V_{GB})$ Voltage across the gate-oxide layer,
- $Q_{SC,B}(V_{GB})$ Charge density in the substrate space-charge region, and
- $V_{SC,B}(V_{GB})$ Voltage across the substrate space-charge region.

- The charge density $Q_{SC,B}(V_{GB})$ (C/cm^2) in the substrate space-charge region is defined as

$$Q_{SC,B}(V_{GB}) \equiv \int_{0^+}^{\text{neutral bulk}} \varrho_B(x, V_{GB})\, dx\,.$$

BASIC RELATIONSHIPS

- The electrostatic-potential-distribution relationship

$$V_{GB} = V_{OX}(V_{GB}) + V_{SC,B}(V_{GB}) + \varphi_{PM} \,,$$

- The charge-density-conservation relationship

$$Q_G(V_{GB}) + Q_f + Q_{it}(V_{GB}) + Q_{SC,B}(V_{GB}) = 0 \,.$$

- The electric-displacement-vector continuity at the metal/gate-oxide interface

$$\begin{aligned} \epsilon_{ox}\, \mathcal{E}_{x,OX}(V_{GB}) + Q_f + Q_{it}(V_{GB}) &= \epsilon_{Si}\, \mathcal{E}_{x,B}(0^+, V_{GB}) \,, \\ &= -\, Q_{SC,B}(V_{GB}) \,. \end{aligned}$$

- The electric-displacement-vector continuity at the gate-oxide/substrate interface

$$Q_G(V_{GB}) = \epsilon_{ox}\, \mathcal{E}_{x,OX}(V_{GB}) \,.$$

BASIC RELATIONSHIPS

- The voltage $V_{OX}(V_{GB})$ across a charge-free gate-oxide layer and the electric field $\mathcal{E}_{x,OX}(V_{GB})$ in it are related by :

$$V_{OX}(V_{GB}) = X_{ox}\, \mathcal{E}_{x,OX}(V_{GB})\,, \quad \text{or} \quad \mathcal{E}_{x,OX}(V_{GB}) = \frac{V_{OX}(V_{GB})}{X_{ox}}\,.$$

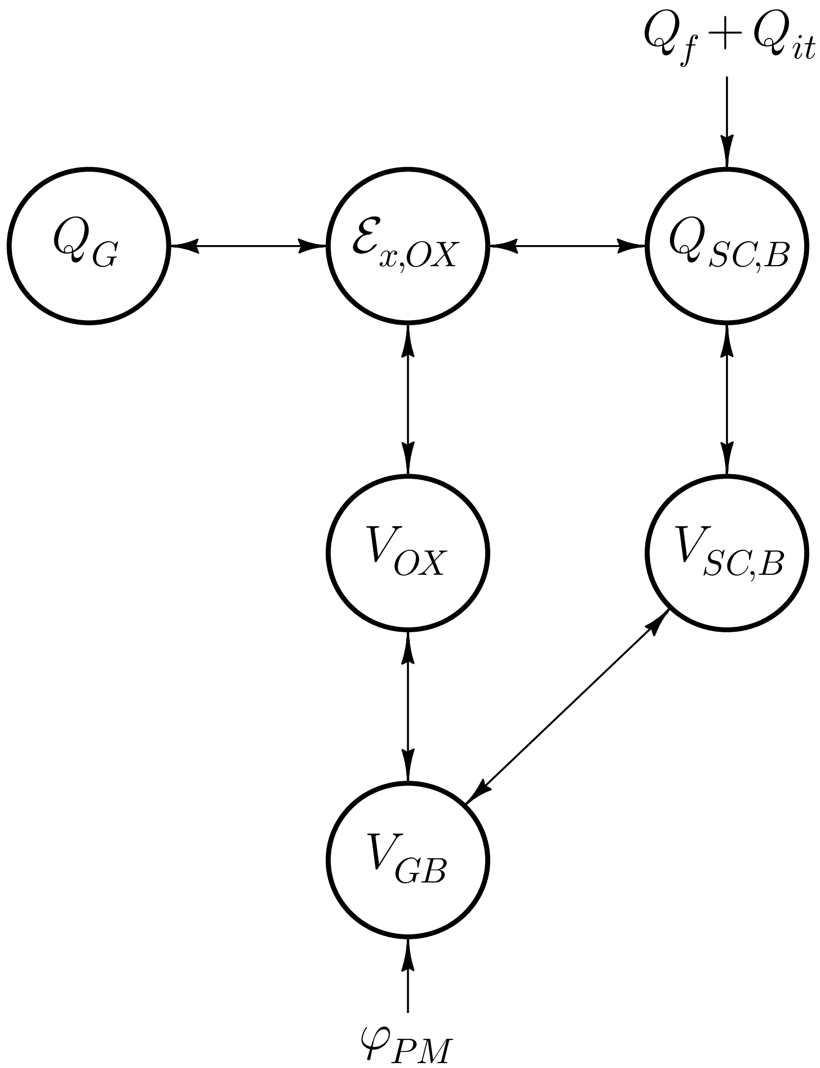
- The electron and hole concentration distributions are related to the electrostatic-potential distribution by

$$n_B(x, V_{GB}) = n_B^{\circ} \exp\left(\frac{q\left[\,\psi_B(x, V_{GB}) - \psi_B(\text{neutral bulk})\,\right]}{k_B T}\right)\,,$$

and

$$p_B(x, V_{GB}) = p_B^{\circ} \exp\left(-\frac{q\left[\,\psi_B(x, V_{GB}) - \psi_B(\text{neutral bulk})\,\right]}{k_B T}\right)\,.$$

BASIC RELATIONSHIPS



MOSCAP AT THERMAL EQUILIBRIUM

- A metal-gate MOS capacitor is at thermal equilibrium at $V_{GB} = 0$. At thermal equilibrium,

$$V^{\circ}_{OX} + V^{\circ}_{SC,B} + \varphi_{PM} = 0\,.$$

- At thermal equilibrium, the charge-density-conservation relationship at thermal equilibrium is expressed as

$$Q^{\circ}_{G} + Q_{f} + Q^{\circ}_{it} + Q^{\circ}_{SC,B} = 0\,.$$

- The values of the flat-band and threshold voltages will determine the condition of the substrate space-charge region at thermal equilibrium. The approximate analysis can be then applied to find $V^{\circ}_{SC,B}$ and $Q^{\circ}_{SC,B}$.

MOSCAP AT THERMAL EQUILIBRIUM

- The oxide electric field $\mathcal{E}^\circ_{x,OX}$ at thermal equilibrium is given by

$$\mathcal{E}^\circ_{x,OX} = \frac{Q^\circ_G}{\epsilon_{ox}} = -\,\frac{Q_f + Q^\circ_{it} + Q^\circ_{SC,B}}{\epsilon_{ox}}\,,$$

and the thermal-equilibrium oxide voltage V°_{OX} is given by

$$\begin{aligned} V^\circ_{OX} &= X_{ox}\,\mathcal{E}^\circ_{x,OX}\,, \\ &= -\,X_{ox}\,\frac{Q_f + Q^\circ_{it} + Q^\circ_{SC,B}}{\epsilon_{ox}}\,, \\ &= -\,\frac{Q_f + Q^\circ_{it} + Q^\circ_{SC,B}}{C_{ox}} = -\,\left(\,V^\circ_{SC,B} + \varphi_{PM}\,\right)\,. \end{aligned}$$

which can be rearranged to read

$$Q^\circ_{SC,B}(V^\circ_{SC,B}) = C_{ox}\,V^\circ_{SC,B} + C_{ox}\,\varphi_{PM} - \left(\,Q_f + Q^\circ_{it}\,\right)\,.$$

The Accumulation Bias Range:

- $V_{GB} \leq V_{FB \cdot N}$.
- $p_B(0, V_{GB}) \geq p_B^\circ$.
- $n_B(0, V_{GB}) \leq n_B^\circ$.

In the approximate analysis:

- $V_{SC,B}^{acc}(V_{GB}) \ll V_{OX}^{acc}(V_{GB}) + \varphi_{PM}$,
- $V_{GB} \approx V_{OX}^{acc}(V_{GB}) + \varphi_{PM}$.
- $V_{OX}^{acc}(V_{GB}) \approx V_{GB} - \varphi_{PM}$.
- $\mathcal{E}_{x,OX}^{acc}(V_{GB}) \approx \dfrac{V_{GB} - \varphi_{PM}}{X_{ox}}$.
- $Q_G^{acc}(V_{GB}) \approx C_{ox}\left(V_{GB} - \varphi_{PM}\right)$.
- $Q_{SC,B}^{acc}(V_{GB}) \approx -C_{ox}\left(V_{GB} - V_{FB \cdot N}\right)$.

The Flat-Band Voltage:

- $Q_{SC,B}(V_{FB\cdot N}) \equiv 0\,.$
- $V_{SC,B}(V_{FB\cdot N}) \equiv 0\,.$
- $V_{FB\cdot N} = V_{OX}(V_{FB\cdot N}) + \varphi_{PM}\,.$
- $Q_G(V_{FB\cdot N}) = -\left(Q_f + Q_{it}(V_{FB\cdot N})\right)\,.$
- $\mathcal{E}_{x,OX}(V_{FB\cdot N}) = \dfrac{Q_G(V_{FB\cdot N})}{\epsilon_{ox}} = -\dfrac{\left(Q_f + Q_{it}(V_{FB\cdot N})\right)}{\epsilon_{ox}}\,.$
- $V_{OX}(V_{FB\cdot N}) = X_{ox}\,\mathcal{E}_{x,OX}(V_{FB\cdot N}) = -\dfrac{\left(Q_f + Q_{it}(V_{FB\cdot N})\right)}{C_{ox}}\,.$
- $V_{FB\cdot N} = \varphi_{PM} - \dfrac{\left(Q_f + Q_{it}(V_{FB\cdot N})\right)}{C_{ox}}\,.$

The Flat-Band Voltage:

- The flat-band voltage $V_{FB \cdot N}$ can be negative, zero, or positive.
- At the flat-band voltage :

$$n_B(x, V_{FB \cdot N}) = n_B^\circ = \frac{n_i^2}{N_{a,B}^-} ,$$

$$p_B(x, V_{FB \cdot N}) = p_B^\circ = N_{a,B}^- ,$$

$$\varrho_B(x, V_{FB \cdot N}) = 0 ,$$

and

$$\mathcal{E}_{x,B}(x, V_{FB \cdot N}) = 0 .$$

The Depletion Bias Range:

- $V_{T \cdot N} \geq V_{GB} \geq V_{FB \cdot N}$.
- $p_B^\circ \geq p_B(0, V_{GB}) \geq n_B^\circ$.
- $n_B^\circ \leq n_B(0, V_{GB}) \leq p_B^\circ$.
- $W_{d,B}^{dep}(V_{GB})$ increases from 0 to $W_{d,B}^{dep}(V_{T \cdot N})$.
- $V_{SC,B}^{dep}(V_{GB})$ increases from 0 to $2\,\varphi_{F,B}$.

In the approximate analysis:

- $W_{d,B}^{dep}(V_{GB}) \approx \sqrt{\dfrac{2\,\epsilon_{Si}}{q\,N_{a,B}^{-}}\,V_{SC,B}^{dep}(V_{GB})}$.
- $Q_{SC,B}^{dep}(V_{GB}) \approx -\sqrt{2\,q\,\epsilon_{Si}\,N_{a,B}^{-}\,V_{SC,B}^{dep}(V_{GB})}$.

The Depletion Bias Range:

- $Q_G^{dep}(V_{GB}) \approx \sqrt{2\, q\, \epsilon_{Si}\, N_{a,B}^{-}\, V_{SC,B}^{dep}(V_{GB})} - \left[Q_f + Q_{it}^{dep}(V_{GB}) \right] .$
- $\mathcal{E}_{x,OX}^{dep}(V_{GB}) \approx \dfrac{\sqrt{2\, q\, \epsilon_{Si}\, N_{a,B}^{-}\, V_{SC,B}^{dep}(V_{GB})}}{\epsilon_{ox}} - \dfrac{\left[Q_f + Q_{it}^{dep}(V_{GB}) \right]}{\epsilon_{ox}} .$
- $V_{OX}^{dep}(V_{GB}) \approx \dfrac{\sqrt{2\, q\, \epsilon_{Si}\, N_{a,B}^{-}\, V_{SC,B}^{dep}(V_{GB})}}{C_{ox}} - \dfrac{\left[Q_f + Q_{it}^{dep}(V_{GB}) \right]}{C_{ox}} .$
- $V_{GB} \approx \dfrac{\sqrt{2\, q\, \epsilon_{Si}\, N_{a,B}^{-}\, V_{SC,B}^{dep}(V_{GB})}}{C_{ox}} + V_{SC,B}^{dep}(V_{GB}) + V_{FB \cdot N} .$

The Depletion Bias Range:

- $$V_{SC,B}^{dep}(V_{GB}) \approx V_{GB} - V_{FB \cdot N} + \frac{q\,\epsilon_{Si}\,N_{a,B}^{-}}{C_{ox}^{2}} \left[1 - \sqrt{1 + \frac{2\,C_{ox}^{2}}{q\,\epsilon_{Si}\,N_{a,B}^{-}} \left(V_{GB} - V_{FB \cdot N} \right)} \, \right] .$$

- From this approximate relationship, we can calculate $V_{SC,B}^{dep}(V_{GB})$ for a certain value of V_{GB}. It is then possible to calculate all other MOS variables which depend only on $V_{SC,B}^{dep}(V_{GB})$.

The Threshold Voltage:

- $n_B(0, V_{T\cdot N}) = p_B^\circ$,
- $p_B(0, V_{T\cdot N}) = n_B^\circ$,
- $V_{T\cdot N} = V_{OX}(V_{T\cdot N}) + V_{SC,B}(V_{T\cdot N}) + \varphi_{PM}$,
- $Q_G(V_{T\cdot N}) + Q_f + Q_{it}(V_{T\cdot N}) + Q_{SC,B}(V_{T\cdot N}) = 0$.
- $$V_{SC,B}(V_{T\cdot N}) = \frac{2\,k_B T}{q} \ln\left(\frac{N_{a,B}^-}{n_i}\right) = 2\,\varphi_{F,B}\ ,$$
- $$\varphi_{F,B} = \frac{k_B T}{q} \ln\left(\frac{N_{a,B}^-}{n_i}\right)\ .$$

The Threshold Voltage:

- $$W_{d,B}(V_{T\cdot N}) = \sqrt{\frac{2\,\epsilon_{Si}}{q\,N_{a,B}^{-}}\,V_{SC,B}(V_{T\cdot N})} = \sqrt{\frac{4\,\epsilon_{Si}\,\varphi_{F,B}}{q\,N_{a,B}^{-}}}\,.$$

- $$Q_{SC,B}(V_{T\cdot N}) = -\,q\,N_{a,B}^{-}\,W_{d,B}(V_{T\cdot N}) = -\sqrt{4\,q\,\epsilon_{Si}\,N_{a,B}^{-}\,\varphi_{F,B}}\,.$$

- $$Q_G(V_{T\cdot N}) = \sqrt{4\,q\,\epsilon_{Si}\,N_{a,B}^{-}\,\varphi_{F,B}} - \left[\,Q_f + Q_{it}(V_{T\cdot N})\,\right].$$

- $$\mathcal{E}_{x,OX}(V_{T\cdot N}) = \frac{Q_G(V_{T\cdot N})}{\epsilon_{ox}} = \frac{\sqrt{4\,q\,\epsilon_{Si}\,N_{a,B}^{-}\,\varphi_{F,B}} - \left[\,Q_f + Q_{it}(V_{T\cdot N})\,\right]}{\epsilon_{ox}}\,.$$

- $$V_{OX}(V_{T\cdot N}) = \frac{\sqrt{4\,q\,\epsilon_{Si}\,N_{a,B}^{-}\,\varphi_{F,B}}}{C_{ox}} - \frac{Q_f + Q_{it}(V_{T\cdot N})}{C_{ox}}\,.$$

The Threshold Voltage:

- $$V_{T \cdot N} = V_{FB \cdot N} + 2\,\varphi_{F,B} + \frac{\sqrt{4\,q\,\epsilon_{Si}\,N^{-}_{a,B}\,\varphi_{F,B}}}{C_{ox}}\,.$$
- $$\varphi_{PM} = -\,0.51165 - \frac{k_B T}{q} \ln\left(\frac{N^{-}_{a,B}}{n_i}\right)\,.$$

The Inversion Bias Range:

- $V_{GB} \geq V_{T \cdot N}$,
- $n_B(0, V_{GB}) \geq p_B^{\circ}$,
- $p_B(0, V_{GB}) \leq n_B^{\circ}$.

In the approximate analysis:

- $Q_{SC,B}^{inv}(V_{GB}) = Q_{DEP,B}^{inv}(V_{GB}) + Q_{INV,B}^{inv}(V_{GB})$.
- $W_{d,B}^{inv}(V_{GB}) = W_{d,B}^{max} \approx W_{d,B}^{dep}(V_{T \cdot N}) \approx \sqrt{\dfrac{4\,\epsilon_{Si}\,\varphi_{F,B}}{q\,N_{a,B}^{-}}}$.
- $V_{SC,B}^{inv}(V_{GB}) \approx V_{SC,B}^{inv}(V_{T \cdot N}) \approx 2\,\varphi_{F,B}$.
- The inversion-layer charge is assumed to be a charge sheet located at the gate-oxide/substrate interface.

The Inversion Bias Range:

- $Q^{inv}_{DEP,B}(V_{GB}) \approx -q\, N^{-}_{a,B}\, W^{max}_{d,B} \approx -\sqrt{4\, q\, \epsilon_{Si}\, N^{-}_{a,B}\, \varphi_{F,B}}\,.$
- $Q^{inv}_{INV,B}(V_{GB}) \approx Q^{inv}_{SC,B}(V_{GB}) + \sqrt{4\, q\, \epsilon_{Si}\, N^{-}_{a,B}\, \varphi_{F,B}}\,.$
- $V_{GB} \approx V^{inv}_{OX}(V_{GB}) + 2\,\varphi_{F,B} + \varphi_{PM}\,.$
- $V^{inv}_{OX}(V_{GB}) \approx V_{GB} - 2\,\varphi_{F,B} - \varphi_{PM}\,.$
- $\mathcal{E}^{inv}_{x,OX}(V_{GB}) \approx \dfrac{V_{GB} - 2\,\varphi_{F,B} - \varphi_{PM}}{X_{ox}}\,.$
- $Q^{inv}_{G}(V_{GB}) \approx C_{ox}\,\left(V_{GB} - 2\,\varphi_{F,B} - \varphi_{PM}\right)\,.$
- $Q^{inv}_{SC,B}(V_{GB}) \approx -C_{ox}\,\left(V_{GB} - 2\,\varphi_{F,B} - V_{FB\cdot N}\right)\,.$
- $Q^{inv}_{INV,B}(V_{GB}) \approx -C_{ox}\,\left(V_{GB} - V_{T\cdot N}\right)\,.$

SMALL-SIGNAL CAPACITANCE-VOLTAGE CHARACTERISTICS

- $$C_{gb}(V_{GB}) = \frac{C_{ox}\, C_{sc,B}(V_{GB})}{C_{ox} + C_{sc,B}(V_{GB})}\,.$$
- $$C_{gb}^{acc}(V_{GB}) \approx C_{ox}\,.$$
- $$C_{sc,B}^{dep}(V_{GB}) \approx \frac{\epsilon_{Si}}{W_{d,B}^{dep}(V_{GB})} \approx \frac{\epsilon_{Si}}{\sqrt{2\,\epsilon_{Si}\, V_{SC,B}^{dep}(V_{GB}) \,/\, q\, N_{a,B}^{-}}}\,.$$
- $$W_{d,B}^{inv}(V_{GB}) = W_{d,B}^{dep}(V_{T\cdot N}) = W_{d,B}^{max} = \sqrt{\frac{4\,\epsilon_{Si}\,\varphi_{F,B}}{q\, N_{a,B}^{-}}}\,.$$
- $$C_{sc,B\cdot HF}^{inv}(V_{GB}) = \frac{\epsilon_{Si}}{W_{d,B}^{max}} = \sqrt{\frac{q\,\epsilon_{Si}\, N_{a,B}^{-}}{4\,\varphi_{F,B}}}\,.$$

SMALL-SIGNAL CAPACITANCE-VOLTAGE CHARACTERISTICS

- $C^{inv}_{gb \cdot HF}(V_{GB}) = \dfrac{C_{ox}\, C^{inv}_{sc,B \cdot HF}(V_{GB})}{C_{ox} + C^{inv}_{sc,B \cdot HF}(V_{GB})} = \dfrac{1}{\left(\dfrac{X_{ox}}{\epsilon_{ox}} + \dfrac{W^{max}_{d,B}}{\epsilon_{Si}}\right)} \neq f(V_{GB})\,.$

- $C^{inv}_{gb \cdot LF}(V_{GB}) \approx C_{ox}\,.$

MOSFET TYPES

- N-Channel and P-Channel MOSFETs
- Enhancement-Mode and Depletion-Mode MOSFETs
- Surface-channel and Buried-channel MOSFETs
- Bulk and Silicon-on-Insulator (SOI) MOSFETs

- $V_{T \cdot N}(V_{BS}) = V_{FB \cdot N0} + 2\,\varphi_{F,B} + \dfrac{1}{C_{ox}} \sqrt{2\,q\,\epsilon_{Si}\,N_{a,B}^{-}\left(2\,\varphi_{F,B} - V_{BS}\right)}\,.$
- $V_{T \cdot N0} \equiv V_{T \cdot N}(V_{BS} = 0) = V_{FB \cdot N0} + 2\,\varphi_{F,B} + \gamma_{B \cdot N}\sqrt{2\,\varphi_{F,B}}\,.$
- $\gamma_{B \cdot N} \equiv \dfrac{\sqrt{2\,q\,\epsilon_{Si}\,N_{a,B}^{-}}}{C_{ox}}\,.$
- $V_{DSat} = V_{GS} - V_{T \cdot N}(V_{BS}).$
- $K_N = \mu_{n \cdot ch}\,C_{ox}\,\dfrac{W_{ch}}{L_{ch}}\,.$
- For $V_{GS} \leq V_{T \cdot N}(V_{BS})$, the NMOSFET is off and

$$I_D^{off} = 0\,.$$

STATIC DRAIN-CURRENT CHARACTERISTICS

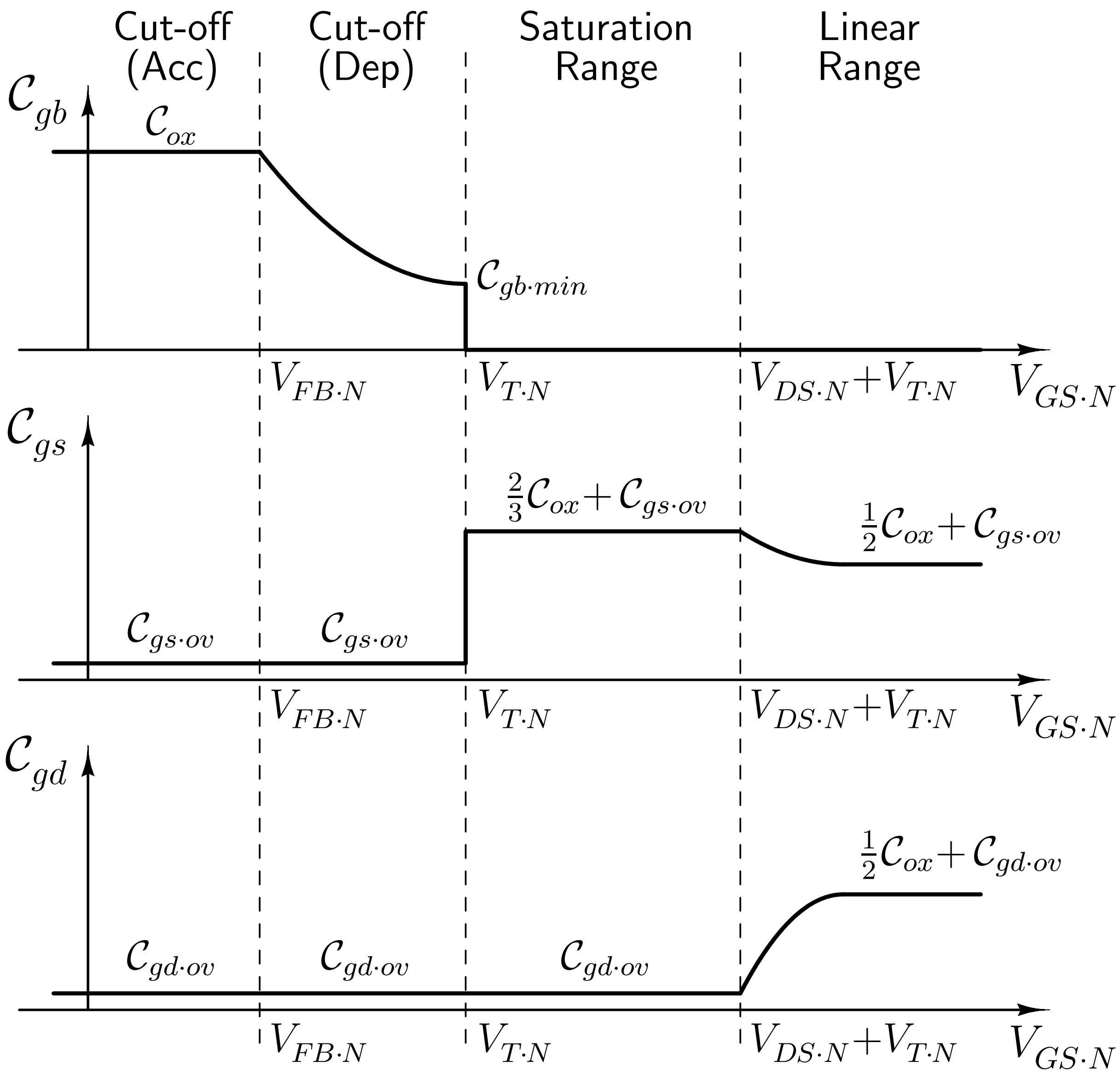
- For $V_{GS} \geq V_{T \cdot N}(V_{BS})$ and $V_{DS} \leq V_{DSat}$, the NMOSFET is in the linear bias range and

$$I_D^{lin} = K_N \left[\Big(V_{GS} - V_{T \cdot N}(V_{BS}) \Big) V_{DS} - \frac{1}{2} V_{DS}^2 \right] .$$

- For $V_{GS} \geq V_{T \cdot N}(V_{BS})$ and $V_{DS} \geq V_{DSat}$, the NMOSFET is in the saturation bias range and

$$I_D^{sat} = \frac{K_N}{2} \Big(V_{GS} - V_{T \cdot N}(V_{BS}) \Big)^2 \left[1 + \lambda_N \left(V_{DS} - V_{DSat} \right) \right] .$$

MOSFET CAPACITANCES



SMALL-SIGNAL MOSFET MODEL

- $i_d = g_m\, v_{gs} + g_{mb}\, v_{bs} + g_o\, v_{ds}\,.$
- $g_m^{sat} = K_N \left(V_{GS} - V_{T \cdot N}(V_{BS}) \right) \left[1 + \lambda_N \left(V_{DS} - V_{DSat} \right) \right].$
- $g_{mb}^{sat} = \dfrac{\gamma_{B \cdot N}}{2\sqrt{2\,\varphi_{F,B} - V_{BS}}}\, g_m^{sat}\,.$
- $g_o^{sat} = \dfrac{\lambda_N\, K_N}{2} \left(V_{GS} - V_{T \cdot N}(V_{BS}) \right)^2 = \dfrac{1}{r_o^{sat}}\,,$
- $f_T = \dfrac{g_m^{sat}}{2\,\pi \left(\mathcal{C}_{gs} + \mathcal{C}_{gd} \right)} = \dfrac{3\,\mu_{n \cdot ch}}{4\,\pi\, L_{ch}^2} \left(V_{GS} - V_{T \cdot N}(V_{BS}) \right).$