Assignment 5

Date due: October 24 (in class)   See Assignment 2 for instructions for electronic submission.

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1. Let arrays $X[1:n]$ and $Y[1:n]$ describe a set of intervals (x_i, y_i), $x_i < y_i$, $i = 1, ..., n$. Two intervals (x_i, y_i) and (x_j, y_j) overlap if:

$$x_i \leq x_j \wedge x_j \leq y_i \vee x_j \leq x_i \wedge x_i \leq y_j.$$

   Two overlapping intervals (x_i, y_i) and (x_j, y_j) can be merged into one (x_k, y_k), where $x_k = \min(x_i, x_j)$ and $y_k = \max(y_i, y_j)$.

   Write an integer Fortran function $Merge(X, Y, n)$ which iteratively merges the overlapping intervals and returns the number of resulting, non-overlapping intervals (the merged intervals replace the original intervals in arrays X and Y).

   Also, write a Fortran program which reads the data, invokes $Merge$ and prints the results.

2. A block–diagonal matrix is a square matrix in which all non-zero elements are in the form of disjoint submatrices (called blocks) on the main diagonal, as shown below (where nonzero elements are represented by asterisks):

$$
\begin{array}{cccccccccc}
* & * & * & 0 & 0 & 0 & 0 & 0 & 0 \\
* & * & * & 0 & 0 & 0 & 0 & 0 & 0 \\
* & * & * & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & * & * & * & * & 0 & 0 \\
0 & 0 & 0 & * & * & * & * & 0 & 0 \\
0 & 0 & 0 & * & * & * & * & 0 & 0 \\
0 & 0 & 0 & * & * & * & * & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & * & * \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & * & * \\
\end{array}
$$

   Write an integer function $Blocks(A,n,B)$ which checks if a real matrix $A[1:n, 1:n]$ is block–diagonal and which returns the sizes of consecutive blocks in array $B[1:n]$ (for the example above the returned values of B are [3,4,2]). The value returned by the function is the number of blocks (i.e., 3 for the example above).

   Also, write a Fortran program which reads the data, invokes $Blocks$ and prints the results.

   Hint: Start with the element $A[1, 1]$ and, for consecutive values of $k = 1, 2, \cdots$ check if all elements $k + 1$ to n in the first k rows and the first k columns are equal to zero. If a nonzero element is found, increase k and repeat. When the first block is identified, repeat the same process starting from the next diagonal element, until the whole matrix A is checked.