**3. Exercise 6**

Exercise 6-1 Conditional Probability

(1 point)

Suppose that of all individuals buying a certain digital camera, include an optional memory card in their purchase, include an extra battery, and include both a card and a battery. Consider randomly selecting a buyer and let memory card purchased and battery purchased . Then , and .

1. Given that the selected individual purchased an extra battery, what is the probability that an optional card was also purchased?

**4. Suggested solution:**

1. Given that the selected individual purchased a memory card, what is the probability that an optional extra battery was also purchased?

**5. Suggested solution:**

Exercise 6-2 Bayes' Theorem

(1 point)

Only 1 in 1000 adults is afflicted with a rare disease for which a diagnostic test has been developed. The test is such that when an individual actually has the disease, a positive result will occur of the time, whereas an individual without the disease will show a positive test result only of the time.

If a randomly selected individual is tested and the result is positive, what is the probability that the individual has the disease?

Suggested solution:

Bayes' Theorem:

We have the following events:

- Actual positive / has disease

- Actual negative / healthy

Test is positive

- Test is negative

We are given the following probabilities:

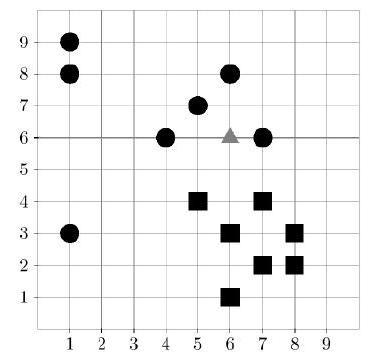
Total probability of positive test:

We want to calculate :

Exercise 6-3 Nearest Neighbour Classification

(1 point)

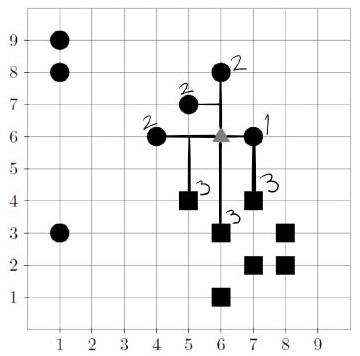
The feature vectors in the figure below belong to two different classes (circles and rectangles). Classify the object at - in the image represented using a triangle - using nearest neighbor classification. Use Manhattan distance norm as distance function, and use the non-weighted class counts in the -nearest-neighbor set, i.e. the object is assigned to the majority class within the nearest neighbors. Perform classification and compare the results with your own "intuitive" result for the following values.



**6. Suggested solution:**

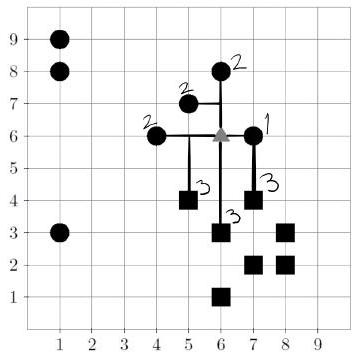
For :

Looking at the closest neighbours, we see that all 4 nearest neighbours are circles. The object at would be classified as a circle.



For

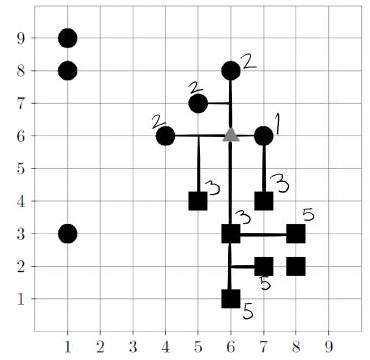
Looking at the closest neighbours, we see that of the nearest neighbours are circles. The object at would be classified as a circle.



For :

Looking at the closest neighbours, we see that of the nearest neighbours are circles. We also see that of the nearest neighbours are squares.

The object at would be classified as a square.



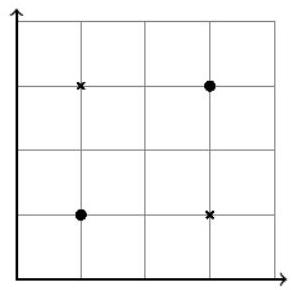
Exercise 6-4 Nearest Neighbour Classification

(1 point)

Find a scenario where we have a set of at least four points in 2 dimensions, such that the Nearest Neighbor classification only gives incorrect classification results when using any of these points as query point and the rest as training examples. Use Euclidean distance as distance function.

**7. Suggested solution:**

Various solutions are possible, e.g.:



Exercise 6-5 Linearity of Expectation and Variance

(1 point)

Suppose that the two variables and are statistically independent. Show that the mean and variance of their sum satisfies:

**8. Suggested solution:**

Proof for Linearity of Expectation:

values of .

Probability of sum of variables:

We never assume variables are independent, so this proof also works for dependent variables. Note: For continuous random variables, the proof is the same, but with integrals rather than sums. The proof can be extended to an arbitrary number of variables by induction.

Variance follows a similar proof. Recall definition of variance over a random variable:

We use the linearity of expectation for this proof: