**3. Exercise 7: Bayes Optimal Classifier, Naïve Bayes, Random Variables and Distributions, EM Clustering**

**4. Exercise 7-1 Bayes Optimal**

(1 point)

We have a classification problem with two classes " and " , three trained classifiers , , and , with the following probabilities of the classifiers, given the training data :

For the three test instances , the classifiers give the following class probabilities:

We combine the three classifiers to get a Bayes optimal classifier. Which class probabilities will we get from this Bayes optimal classifier for the three test instances?

**5. Suggested solution:**

The Bayes optimal classifier adds the conditional class probabilities given the classifier, weighted with the conditional classifier probabilities given the data:

The resulting probabilities are:

The predictions are therefore:

Exercise 7-2 Naive Bayes

(1 point)

The skiing season is open. To reliably decide when to go skiing and when not, you could use a classifier such as Naïve Bayes. The classifier will be trained with your observations from the last year. Your notes include the following attributes:

The weather: The attribute weather can have the following three values: sunny, rainy, and snow. The snow level: The attribute snow level can have the following two values: (There are at least of snow) and (There are less than of snow).

Assume you went skiing 8 times during the previous year. Here is the table with your decisions:

|  |  |  |
| --- | --- | --- |
| Weather | Snow level | Ski? |
| sunny |  | no |
| rainy |  | no |
| rainy |  | no |
| snow |  | yes |
| snow |  | no |
| sunny |  | yes |
| snow |  | yes |
| rainy |  | yes |

1. Compute the a priori probabilities for both classes ski yes and ski no (on the training set)!

Suggested solution:

1. Compute the distribution of the conditional probabilities for the two classes for each attribute.

Suggested solution:

1. Decide for the following weather and snow conditions, whether to go skiing or not! Use the Naïve Bayes classifier as trained in the previous steps for your decision.

|  |  |  |
| --- | --- | --- |
|  | Weather | Snow level |
| day A | sunny |  |
| day B | rainy |  |
| day C | snow |  |

**6. Suggested solution:**

**7. Day A:**

Day B:

Do not ski

Day C:

Do not ski Exercise 7-3 Random Variables and Probability Distributions

(1 point)

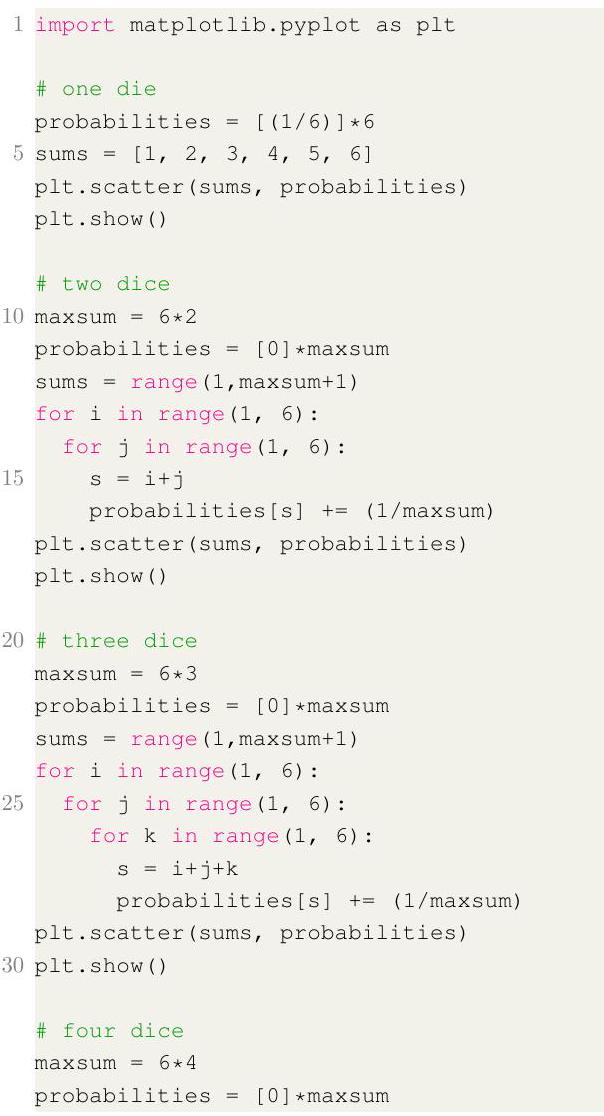
We played a lot with dice in the lecture. When we take the sum of dice (a random variable), we get a probability distribution over the possible values. For just one die, this distribution is discrete with equal probabilities over . For two dies, the probabilities are unequally distributed over .

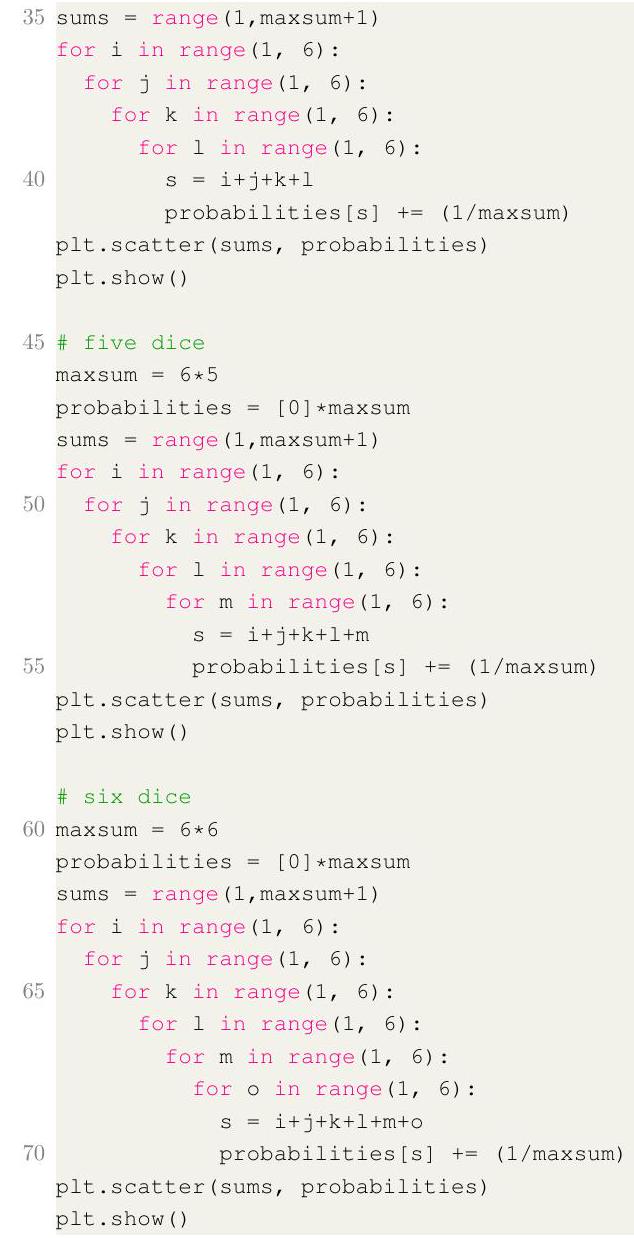
How does the shape of the probability distribution develop with increasing ?

Visualize the development of the shape with a little program or script.

Suggested solution:

Can be done with a simple python script, using matplotlib.





Exercise 7-4 Assignments in the EM-Algorithm

(1 point)

Given a data set with 100 points consisting of three Gaussian clusters and and the point

The cluster contains of all objects and is represented using the mean of all its points and the covariance matrix

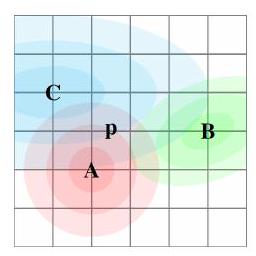
You will need the inverse .

The cluster contains of all objects and is represented using the mean of all its points and the covariance matrix

The cluster contains of all objects and is represented using the mean of all its points and the covariance matrix

The point is given by the coordinates .

The following sketch is not exact, and only gives a rough idea of the cluster locations:



Compute the three probabilities of belonging to the clusters , and . Suggested solution: Cluster A:

Cluster B:

Cluster C:

|  |  |  |  |
| --- | --- | --- | --- |
|  |  |  |  |
| Density |  |  |  |
| Size |  |  |  |
| Score |  |  |  |
| Sum |  |  |  |
| Weight |  |  |  |