ex 2b

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Ex 2b

2.23 p. 143

Verify the relationships $V^{1/2} \quad p \quad V^{1/2} = \Sigma$ and $p = (V^{1/2})^{-1} \quad \Sigma \quad (V^{1/2})^{-1}$ So here is the deal

p = population correlation matrix
$V^{1/2}$ = Population standard deviation matrix
Σ = Population co-variance matrix

This basically shows that if we know V and P we can obtain Σ, and if we know Σ we can obtain p.
How to prove it:
$V^{1/2}pV^{1/2} = \Sigma$
$pV^{1/2} = (V^{1/2})^{-1}\Sigma$
$p = (V^{1/2})^{-1}\Sigma(V^{1/2})^{-1}$

2.41 p. 147

You are given the random vector $[X_1, X_2, X_3, X_4]$ with mean vector $\mu' x$ equal to:

```
mu <- c(3, 2, -2, 0)
print(mu)
```

```
## [1]  3  2 -2  0
```

and variance-covariance matrix:

```
x_1 <- c(3, 0, 0, 0)
x_2 <- c(0, 3, 0, 0)
x_3 <- c(0, 0, 3, 0)
x_4 <- c(0, 0, 0, 3)
co_var_matrix <- data.frame(x_1,
                            x_2,
                            x_3,
                            x_4)
print(co_var_matrix)
```

```
##   x_1 x_2 x_3 x_4
## 1   3   0   0   0
## 2   0   3   0   0
## 3   0   0   3   0
## 4   0   0   0   3
```

Let *A* equal

```
col_1 <- c(1, 1, 1)
col_2 <- c(-1, 1, 1)
col_3 <- c(0, -2, 1)
col_4 <- c(0, 0, -3)
A <- data.frame(col_1,
                col_2,
                col_3,
                col_4)

print(A)
```

```
##   col_1 col_2 col_3 col_4
## 1     1    -1     0     0
## 2     1     1    -2     0
## 3     1     1     1    -3
```

2.41 a)

Find E(AX), the mean of AX. Since we already have the mean vector mu, we can just multiply A with the mean vector to get the expected outcome of A times our data frame which gives the expected value of AX:

```
print(data.matrix(A) %*% mu)
```

```
##      [,1]
## [1,]    1
## [2,]    9
## [3,]    3
```

2.41 b)

Find cov(AX), the variances and covariances of AX.

This is essentially just scaling the co variance matrix, which has the following formula: $Z = A\Sigma A^T$ found here https://math.stackexchange.com/questions/1184523/scaling-a-covariance-matrix (https://math.stackexchange.com/questions/1184523/scaling-a-covariance-matrix) or at page 76 upper page number / 114 lower page number

```
print(data.matrix(A) %*% data.matrix(co_var_matrix) %*% t(data.matrix(A)))
```

```
##      [,1] [,2] [,3]
## [1,]    6    0    0
## [2,]    0   18    0
## [3,]    0    0   36
```

2.41 c)

Which pairs of linear combinations have zero covariances? 1, 2
2, 1
1, 3
3, 1
2, 3

3, 2

All of the above have zero covariances since there is zero at. said another way, since it is only the diagonal, which is not zero, then they all have zero covariance, ie. they have variance individually, but they do not display any related pattern in their variance.