q_2

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Ex. 4 Q2

probability.

4.3. Let $\mathbf{X}$ be $N_3(\boldsymbol{\mu}, \boldsymbol{\Sigma})$ with $\boldsymbol{\mu}' = [-3, 1, 4]$ and

$$\boldsymbol{\Sigma} = \begin{bmatrix} 1 & -2 & 0 \\ -2 & 5 & 0 \\ 0 & 0 & 2 \end{bmatrix}$$

Which of the following random variables are independent? Explain.

(a) X_1 and X_2

(b) X_2 and X_3

(c) (X_1, X_2) and X_3

(d) $\dfrac{X_1 + X_2}{2}$ and X_3

(e) X_2 and $X_2 - \frac{5}{2}X_1 - X_3$

Let us first define the variables:

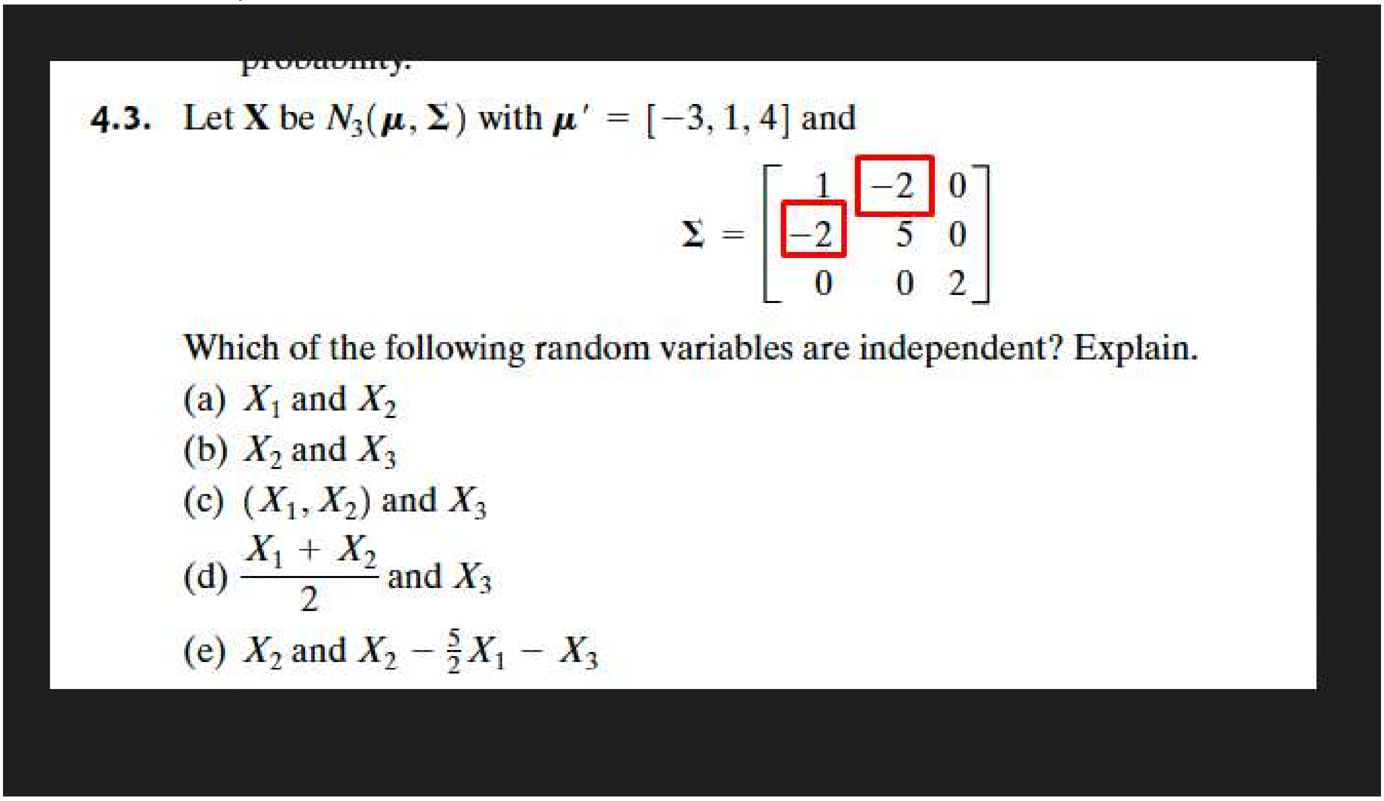
```
mu <- c(-3, 1, 4)

x_1 <- c(1, -2, 0)
x_2 <- c(-2, 5, 0)
x_3 <- c(0, 0, 2)
sigma <- data.frame(
  x_1,
  x_2,
  x_3
)
sigma <- data.matrix(sigma)
```

a

X_1 and X_2:

These are not independent since, their covariances are not zero:



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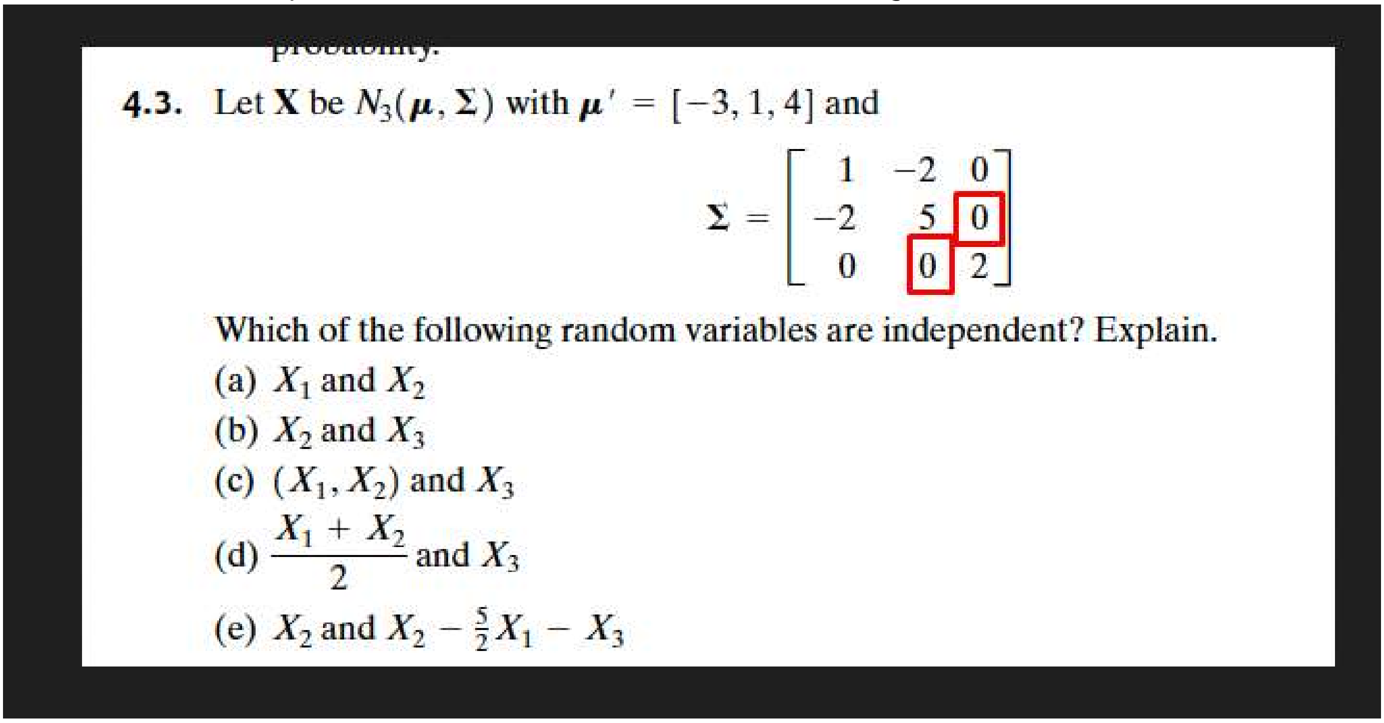
b

X_2 and X_3:

These seem to be independent since their co-variance is 0, but it is not given



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We can test this by getting the correlation ,matrix using the formula on p. 72 / 110 $P = (v^{1/2})^{-1}\Sigma(v^{1/2})^{-1}$
Here $(v^{1/2})$ is the inverse of the population standard deviation matrix, which is given by taking the square root of the diagonal for σ and letting the rest be 0:

```
# Get standard deviation matrix
std_dev <- sigma
std_dev[,] <- 0
diag(std_dev) <- sqrt(diag(sigma))
std_dev
```

```
##      x_1      x_2      x_3
## [1,]   1 0.000000 0.000000
## [2,]   0 2.236068 0.000000
## [3,]   0 0.000000 1.414214
```

```
# calculate correlation matrix
cor_mat <- solve(std_dev) %*% sigma %*% solve(std_dev)
cor_mat
```

```
##             [,1]       [,2] [,3]
## x_1  1.0000000 -0.8944272    0
## x_2 -0.8944272  1.0000000    0
## x_3  0.0000000  0.0000000    1
```

```
#The above was just for my own sake to make it easier just use R built in function
cov2cor(sigma)
```

```
##             x_1        x_2 x_3
## [1,]  1.0000000 -0.8944272   0
## [2,] -0.8944272  1.0000000   0
## [3,]  0.0000000  0.0000000   1
```

As we can see X_2 and X_3 has a correlation of zero, so I will conclude that they are most likely independent:

```
            x_1        x_2 x_3
[1,]  1.0000000 -0.8944272   0
[2,] -0.8944272  1.0000000   0
[3,]  0.0000000  0.0000000   1
```

c

(X_1, X_2) and X_3. Yes since both X_1 and X_2 are independent from X_3.

```
            x_1        x_2 x_3
[1,]  1.0000000 -0.8944272   0
[2,] -0.8944272  1.0000000   0
[3,]  0.0000000  0.0000000   1
```

d

((X_1 + X_2) / 2) and X_3
Yes again X_1 and X_2 might be dependent, but both variables are still independent from X_3

e

X_2 and X_2 - $5/2X_1$ - X_3
No Since x_1 and X_2 have a strong negative correlation and a shared co-variance they cannot be independent.