ex 4

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ex. 4 - Q1

3.30. You are given the random vector $\mathbf{X}' = [X_1, X_2, X_3, X_4]$ with mean vector $\boldsymbol{\mu}'_{\mathbf{X}} = [4, 3, 2, 1]$ and variance–covariance matrix

$$\boldsymbol{\Sigma}_{\mathbf{X}} = \begin{bmatrix} 3 & 0 & 2 & 2 \\ 0 & 1 & 1 & 0 \\ 2 & 1 & 9 & -2 \\ 2 & 0 & -2 & 4 \end{bmatrix}$$

Partition $\mathbf{X}$ as

$$\mathbf{X} = \begin{bmatrix} X_1 \\ X_2 \\ \hdashline X_3 \\ X_4 \end{bmatrix} = \begin{bmatrix} \mathbf{X}^{(1)} \\ \hdashline \mathbf{X}^{(2)} \end{bmatrix}$$

Let

$$\mathbf{A} = [1 \quad 2] \quad \text{and} \quad \mathbf{B} = \begin{bmatrix} 1 & -2 \\ 2 & -1 \end{bmatrix}$$

and consider the linear combinations $\mathbf{AX}^{(1)}$ and $\mathbf{BX}^{(2)}$. Find

(a) $E(\mathbf{X}^{(1)})$
(b) $E(\mathbf{AX}^{(1)})$
(c) $\text{Cov}(\mathbf{X}^{(1)})$
(d) $\text{Cov}(\mathbf{AX}^{(1)})$
(e) $E(\mathbf{X}^{(2)})$
(f) $E(\mathbf{BX}^{(2)})$
(g) $\text{Cov}(\mathbf{X}^{(2)})$
(h) $\text{Cov}(\mathbf{BX}^{(2)})$
(i) $\text{Cov}(\mathbf{X}^{(1)}, \mathbf{X}^{(2)})$
(j) $\text{Cov}(\mathbf{AX}^{(1)}, \mathbf{BX}^{(2)})$

p. 78 / 116

Let us start by defining them in the code:

```
x_1 <- c(3, 0, 2, 2)
x_2 <- c(0, 1, 1, 0)
x_3 <- c(2, 1, 9, -2)
x_4 <- c(2, 0, -2, 4)
sigma_x = data.frame(
  x_1,
  x_2,
  x_3,
  x_4
)
sigma_x <- data.matrix(sigma_x)


mu <- c(4, 3, 2, 1)

A <- c(1, 2)

col_1 <- c(1, 2)
col_2 <- c( -2, -1)
B <- data.frame(
  col_1,
  col_2
)
B <- data.matrix(B)

sigma_x
```

```
##      x_1 x_2 x_3 x_4
## [1,]   3   0   2   2
## [2,]   0   1   1   0
## [3,]   2   1   9  -2
## [4,]   2   0  -2   4
```

```
mu
```

```
## [1] 4 3 2 1
```

```
A
```

```
## [1] 1 2
```

```
B
```

```
##      col_1 col_2
## [1,]     1    -2
## [2,]     2    -1
```

(a)

E($X^{(1)}$)

Since the expected value is the same as the mean or μ we just need to take the partioned μ, which are the two first elements of the list [4, 3, 2, 1]

```
mu[1:2]
```

```
## [1] 4 3
```

(b)

E(A$X^{(1)}$)

Here we just need to multiply the partioned mu with A:

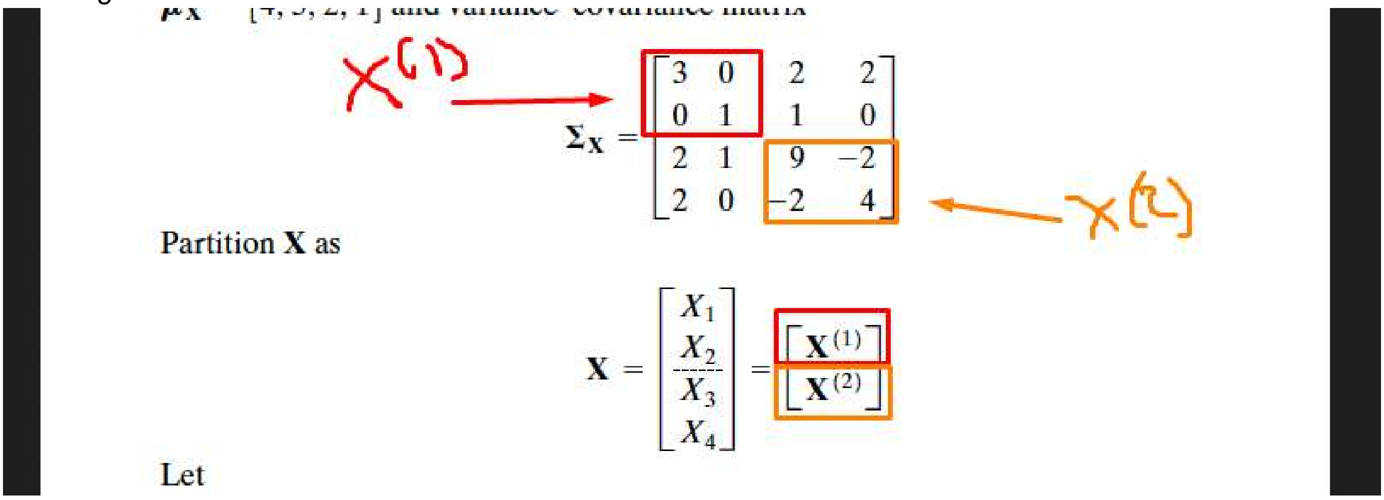
```
A %*% mu[1:2]
```

```
##      [,1]
## [1,]   10
```

(c)

Cov($X^{(1)}$)

To get the partioned variance-covariance matrix we need to look p.78 / 116, but the general case is the following:




```
sigma_x[1:2,1:2]
```

```
##      x_1 x_2
## [1,]   3   0
## [2,]   0   1
```

(d)

Cov(A$X^{(1)}$), now this is more tricky since we are dealing with the covariance. When scaling a covariance matrix, we need to use the formula on p. 76 (2-45):

```
A %*% sigma_x[1:2,1:2] %*% matrix(A) # matrix(A) is a way to transpose a vector
```

```
##      [,1]
## [1,]    7
```

(e)

E($X^{(2)}$)
Same as in a, we just partion μ

```
mu[3:4]
```

```
## [1] 2 1
```

(f)

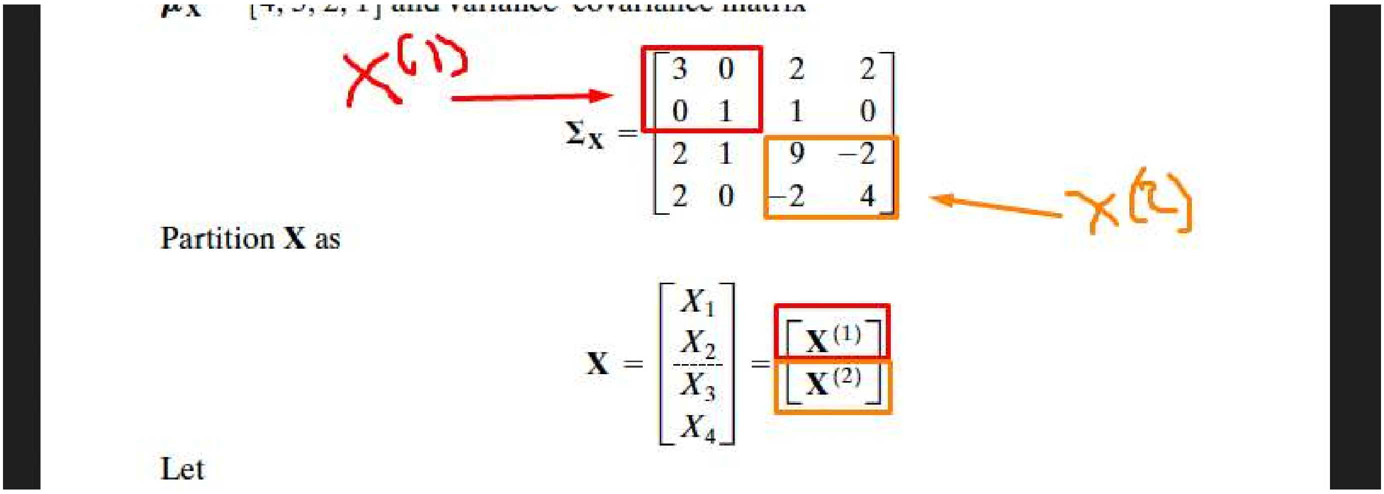
E(B$X^{(2)}$) Same as in b, we just multiply B by the partion from previous

```
B %*% mu[3:4]
```

```
##      [,1]
## [1,]    0
## [2,]    3
```

(g)

Cov($X^{(2)}$) here we just partion it as before:



$X^{(1)}$

$$\Sigma_{\mathbf{X}} = \begin{bmatrix} 3 & 0 & 2 & 2 \\ 0 & 1 & 1 & 0 \\ 2 & 1 & 9 & -2 \\ 2 & 0 & -2 & 4 \end{bmatrix}$$

$X^{(2)}$

Partition $\mathbf{X}$ as

$$\mathbf{X} = \begin{bmatrix} X_1 \\ X_2 \\ \hdashline X_3 \\ X_4 \end{bmatrix} = \begin{bmatrix} \boxed{\mathbf{X}^{(1)}} \\ \boxed{\mathbf{X}^{(2)}} \end{bmatrix}$$

Let

```
sigma_x[3:4, 3:4]
```

```
##      x_3 x_4
## [1,]   9  -2
## [2,]  -2   4
```

h

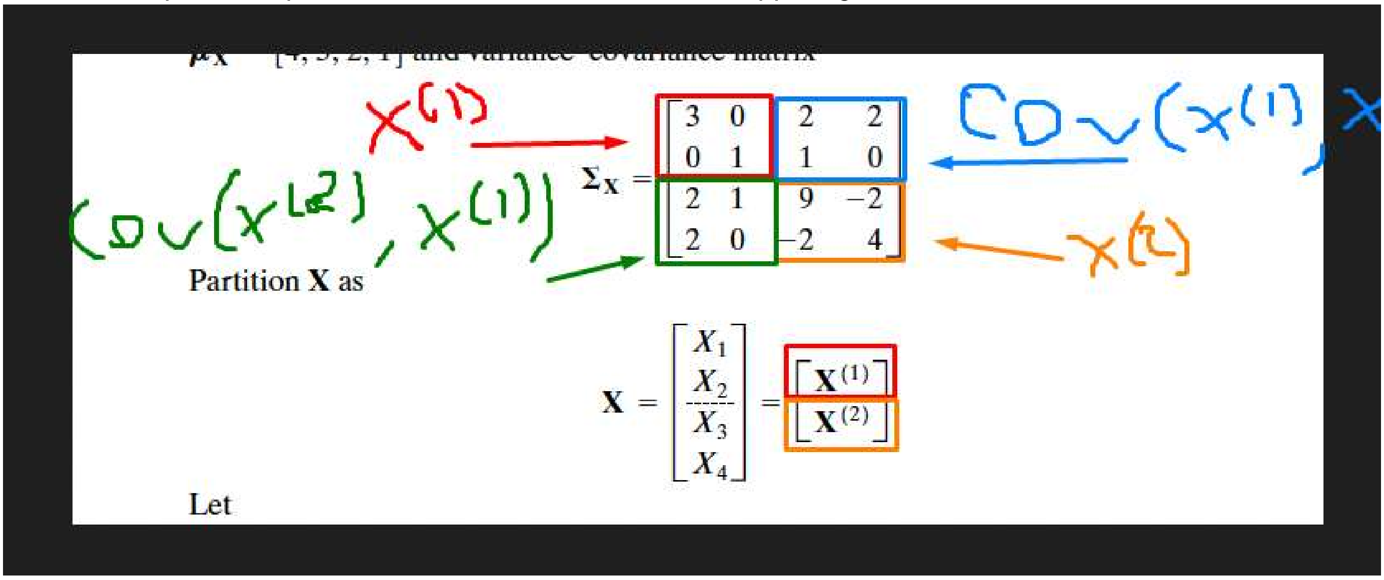
Cov(B$X^{(2)}$) we do as in d

```
B %*% sigma_x[3:4, 3:4] %*% t(B) # t(B) is used instead of matrix(B), since it is matrix not a vector, we are transposing
```

```
##      [,1] [,2]
## [1,]   33   36
## [2,]   36   48
```

i

The upper right and the lower left are transposed versions of each other and comprises this the covariances between the partitions p. 78 / 116. In this case we need the upper right matrix




```
sigma_x[1:2, 3:4]
```

```
##      x_3 x_4
## [1,]   2   2
## [2,]   1   0
```

j

Here we still use the formula from p. 76, but we are using the formula : Cov(AX1,BX2)=ACov(X1,X2)t(B) (exercise solutions):

```
A %*% sigma_x[1:2, 3:4] %*% t(B)
```

```
##      [,1] [,2]
## [1,]    0    6
```