Lab 1 Report Chris Lipscomb

I. Introduction

In this report I will explain some of the physics and math behind my calculations, as well as an explanation of my plots and where they come from. I used the programming language Python and a few libraries to aid in calculations and plotting: matplotlib, numpy, and scipy. Using these tools I was able to create models of the Earth and Moon's gravitational force interactions, as well as estimate the burn time of the Saturn V Stage 1 rocket.

II. The gravitational potential of the Earth-Moon System

Gravitational potential is given by the formula $\Phi(r) = -\frac{GM}{r}$ , where G is the gravitational constant, M is the mass of the planetary body, and r is the distance from the surface of the body. I used this formula in python to plot this function as a meshplot and a contour plot. To create these 2-D plots I had to create a function that would vectorize r and make sure the program would not be able to accidentally divide by zero. I also put the plots on a logarithmic scale, for better visualization of this potential. The Earth is plotted to the origin of the graph, and the moon was set at its real-life orbit distance. As you can see, the attraction to  Earth is much more impactful than the moon's, because gravity is based on the mass of the object.

III. The gravitational force of the Earth-Moon system

Gravitational Force is $\vec{F}_{21} = -G\frac{M_1 m_2}{|\vec{r}_{21}|^2}\hat{r}_{21}$ , with G being the gravitational constant, M1 representing the mass of  one planetary body of the system, m2 being the mass of another body that M1 is applying force to. Again I had to account for potential errors with dividing by zero. This time I made a 2-D streamplot to help visualize the gravitational force near the Earth-Moon system. This plot displays the magnitude and direction of the force that will act on the Saturn V due to Earth and the Moon's gravity. As you can see from the color bar, gravity's force gets more and more weak the further you move from the object. This is due to a physics concept known as the inverse square law, where the effect of this force decreases exponentially, by the square of your distance to that object. This is important to note because this means as the rocket pushes farther away from Earth's surface, it will require less force, therefore less fuel, to escape its gravitational pull.

IV. Projected performance of the Saturn V Stage 1

The final calculation I ran was for the change in velocity, or Delta V, of the Saturn V Stage 1 rocket. This is calculated using the Tsiolkovsky rocket equation,

$\Delta v(t) = v_e \ln\left(\frac{m_0}{m(t)}\right) - gt$ . Ve is the fuel exhaust velocity. M0 is the "wet" mass of the rocket, which means the mass of the rocket itself and the payload as well as the fuel. M(t) is the mass of the rocket as a function of time. This is important because the mass of the rocket does change over time due to the burning of fuel. g is the acceleration due to the force of gravity, and t represents the time measured in seconds. We used this calculation to find the altitude that the first rocket can reach, by using the formula

$h = \int_0^T \Delta v(t)dt$ , which you can think of as taking little sections of the velocity over the time it takes the rocket to run out of fuel, and adding them all up to get the total distance it will travel in that time. Also using the equation $T = \frac{m_0 - m_f}{\dot{m}}$, with Mdot being the burn rate of S-1C, we found the total burn time of the first stage's fuel. Using these calculations I found that the rocket will be able to burn for 157.7 seconds, reaching an altitude of 74094 meters.

## V. Discussion and Future Work

I hope my diagrams and calculations were informative. However it is important for me to point out some assumptions made to simplify these calculations, which make them only approximate. First off the gravitational force generated by the Earth is not completely uniform. Since mass is not evenly distributed in the Earth's mantle and core, gravity is not the exact same strength on Earth. However, for most purposes, it is negligent enough to ignore. Something else that could be taken into consideration for more accurate results is the rate of fuel consumption, which is also not the same through time, and is effected by many different factors. Another example was realized last week, when we received the test results of the first prototype. Although my predictions were in the correct general range, they did overestimate the altitude that the rocket could go. My program predicted higher maximum height, because it did not take into account the resistance due to air, or drag. In the future, My program would be more accurate if I included variables like air resistance.