Problem Chosen	2024 MCM/ICM Summary Sheet	Team Control Number
A		2403982

Uncovering the secrets of the lamprey population

Summary

Adaptive sex ratio variation is a special phenomenon in the ecosystem, where the sex ratio of animals with this type of trait changes according to the external environment. This paper focuses on the ability of a typical example, the sea lamprey, to adapt to its environment by adjusting its sex ratio in the face of changes in resource availability and the effects of this trait on the ecosystem.

First, in order to analyze the impacts on the larger ecosystem when the sea lamprey changes its sex ratio, we developed an Ecosystem model of Population with Variable Sex ratio (**EPVS**), and for this large model, we modeled the dynamics of the sea lamprey population using the **equations of constant differential dynamics of ecosystems**, as well as **Lotka-Volterra** variants of the equations.We analyzed the effects of five factors: food chain, ecological balance and natural resources, reproduction rate, population size change, and the relationship between ecological niche and sex ratio.

Secondly, we analyzed the advantages and disadvantages by using the **EPVS model** in two directions: to itself and to the outside world. In terms of advantages, we found that **the self-regulatory mechanism** of the sea lamprey provides a unique advantage for the long-term reproduction and adaptive evolution of the population. In terms of disadvantages, there may be important implications for the sea lamprey itself but also for the ecosystem.

Next, we established an ecosystem stability evaluation model to quantitatively assess ecosystem stability based on 2 major indicators and 5 minor indicators by using the **EWM-Topsis method**. After modeling, we conducted **100** sets of simulation experiments **200** mathematical simulations to reveal the stability state of the ecosystem under different sex ratios in the sea lamprey population.

Finally, in order to analyze the dominant role of sex ratio changes on other species in the ecosystem, such as parasites, we developed a **model of the dynamics of the sea lamprey-parasite ecosystem** based on the first question, and further analyzed the relationship between the number of natural resources, the number of sea lamprey, and the number of the three types of parasites over time and time, when there are parasites that have direct and indirect competitive relationships with the lamprey, symbiotic relationships, and predator-prey relationships with the lamprey. parasites over time and the sex ratio of the sea lamprey population.

Keywords: Sea lamprey; Adaptive sex ratio variation; Dynamic modeling; Lotka-Volterra equation; EWM-Topsis

Contents

1 Introduction

1.1 Background

Sex plays a crucial role in the animal kingdom, with the vast majority of animal species having a 1:1 sex ratio, but there are always special species that have a disproportionate sex ratio, such as the American alligator and the lampreys, as a result of adaptive sex ratio variation. One notable example is the lamprey, an ancient lineage of jawless fish of the order Petromyzontiformes that lives in coastal and fresh waters habitats. The ecological role of lampreys is complex, it is not only a source of food in some areas, but also a parasite with significant ecological impacts.

Studies have shown that the sex ratio of the sevengill is influenced by growth rate, which is influenced by the environment in which it grows. In environments with low food availability, due to slow growth rates, most eels will develop into males, with the proportion of males reaching nearly 78%; while in an environment with more food supply, the male ratio will slightly decrease to about 56%. We used the example of the sea lampreys to explore the ability or pros and cons of species to alter their sex ratio based on resource availability. Thus, our task is to develop and test a model that assesses the impact of this variation in sex on ecosystem interactions.

Figure 1: The Sea Lamprey

1.2 Restatement of the Problem

Establish and examine a mathematical model to provide insights into the resulting interactions in an ecosystem：

- When a population of sea lampreys can change its sex ratio, what are the implications for the larger dimension of the ecosystem?
- What are the strengths and weaknesses of sea lampreys populations?
- Given changes in the sex ratio of sea lampreys, what are the implications for ecosystem stability?
- Can ecosystems with changing sex ratios of sea lampreys populations provide advantages to other species in the ecosystem (e.g.parasites)?

1.3 Our Work

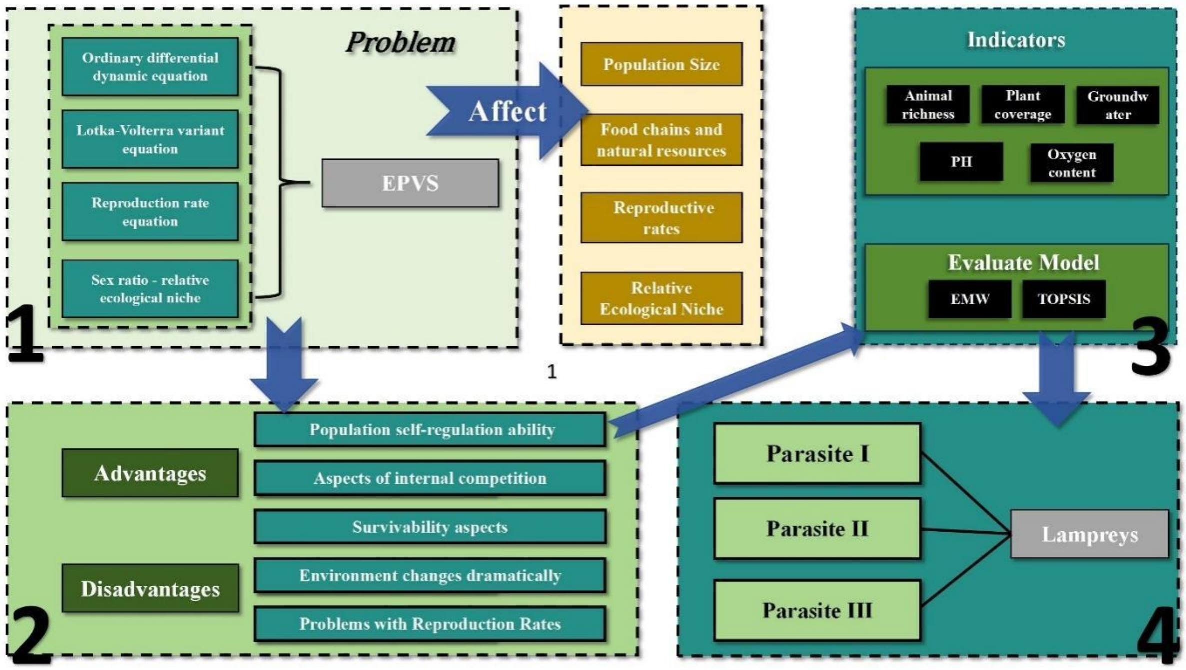


Figure 2: The flow chart of our work

2 Assumptions and Justifications

To simplify the problems, we propose the following foundational assumptions, each supported by appropriate justification:

- **Assumption 1:** Ecosystems are not disturbed by external factors. Ecosystems often do not exist independently and are always subject to external natural and human factors that can interfere and cause the ecosystem to be transformed in one way or another. In order to minimize unnecessary influencing factors, we consider that the outside world of an ecosystem has little or no influence on it.
- **Assumption2:** The food chain relationship between species in an ecosystem remains unchanged. The interactions between species in an ecosystem are extremely complex and can change over time, but to simplify the model, we assume that they remain constant over shorter time scales.
- **Assumption3:** It is assumed that adaptive sex ratio adjustments in sea lamprey populations may be passed on and remain relatively stable in the population over long time scales.
- **Assumption4:** It is hypothesized that adjustments in the sex ratio of sea lampreys may affect parasite survival and reproduction, providing an advantage or disadvantage to the parasite.

3 Notations

The primary notations used in this paper are listed in Table 1.

Table 1: Notations

Symbol	Description
θ_{male}	Male growth rate relative to resources
μ_{male}	Mortality rate of males relative to their own numbers
θ_{female}	Female growth rate relative to resources
μ_{female}	Mortality rate of females relative to their own numbers
R	Number of natural resources available to all sea lampreys
P	The size of the sea lamprey population
H	The number of predators
α	The natural growth rate of the sea lamprey
ρ	Sex ratio of the sea lamprey, i.e. male rate
k	Consumption rate of natural resources by sea lampreys
S	Reproductive success of the sea lamprey
L	Ecotope width of the sea lamprey
F_i	Population size of parasite i
δ_i	Natural growth rate of parasite i
γ_i	Natural mortality of parasite i

4 Problem 1: Ecosystem model of Population with Variable Sex ratio (EPVS)

The role of organisms in nature is complex and requires consideration of many factors. Although we studied the effect of sex ratio of a single species on the ecosystem, there are many species in nature that interact and associate with the sea lamprey, perhaps in a competitive relationship, a predator-prey relationship, a mutually beneficial relationship, etc. Due to the differences in habitat selection, living habits, and food intake between females and males, changes in the sex ratio of sea lampreys will inevitably affect natural resources, reproduction rates, ecological niche, and so on, as well as the size of the population of sea lampreys. Since these connections are interlocked, if the sex ratio of sea lampreys changes, it will affect the ecosystem in a larger dimension like a butterfly effect. Therefore, in order to investigate the effect of sex ratio on the larger ecosystem, we need to first analyze the dynamics of the sea lamprey population, and then study its effect on other factors on this basis.

In this section, we constructed an ecosystem model of Population with Variable Sex ratio (EPVS model). For this model, we first established the Ordinary Differential Dynamics Equation for Ecosystems, determined the parameters by fitting, so as to describe the dynamic changes of the population of the sea lamprey, and then used the Lotka-Volterra variant equation, and considered the relationship between the food chain, the ecological balance and the natural resources, the reproduction rate, the change in population size, and the relationship between the ecological niches and the sex ratio in order to complete the construction of the model.

4.1 Ordinary Differential Dynamics Equation for Ecosystems

4.1.1 Model construction

In order to simulate the dynamics of the ecosystem over time in terms of the number of females and males in the sea lamprey population and natural resources, we constructed the following model:

Male population dynamics change:

$$\frac{dm}{dt} = \theta_{male} \cdot R - \mu_{male} \cdot m$$

Female population dynamics change:

$$\frac{df}{dt} = \theta_{female} \cdot R - \mu_{female} \cdot f$$

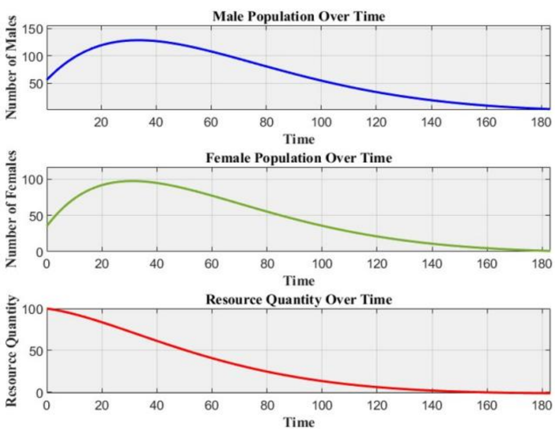
Natural resource population dynamics change:

$$\frac{dR}{dt} = - x \cdot (m + f)$$

where m is the number of males, f is the number of is the number of natural resources (all sources of nutrients that can be ingested and utilized by the sea lamprey), θ_{male} is the natural rate of growth of males, μ_{male} is the mortality rate of males relative to their own number, θ_{female} is the natural rate of growth of females, μ_{female} is the mortality rate of females relative to their own number, and x is a parameter for the rate of resource consumption.

4.1.2 Result

According to the background information, we know that according to the amount of available food resources, the proportion of lamprey males in the population varies roughly between 56% and 78%, and here we define a certain time span, which represents the end of the cycle when a certain amount of natural resources is consumed. According to the results of the simulation experiment, the fluctuation of the proportion of male sea lampreys in the population meets the above requirements, and then according to the above information, using Matlab to fit the parameter, and to make a visualization as shown in Fig. 3 and Fig. 4.



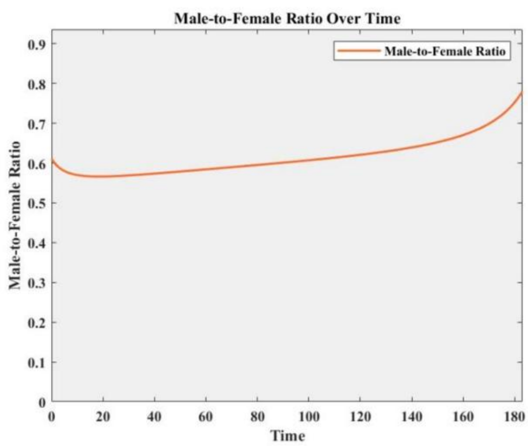


Figure 3:Population and resource over time Figure 4: Male-to-Female ratio over time

The parameters of the Ordinary Differential Power Equation obtained by fitting are as follows:

$$\theta_{female} = 0.075 \qquad \mu_{female} = 0.040$$

$$\theta_{male} = 0.068 \qquad \mu_{male} = 0.050 \qquad x = 0.005$$

From Figure 4, we can see that the proportion of male lampreys at this time with the passage of time and the reduction of natural resources roughly showed a gradual upward trend, in general, to meet the 56% to 78% interval of change, the determination of these parameters for the subsequent study as a basis.

4.2 Food chains and natural resources: the Lotka-Volterra variant model

4.2.1 Lotka-Volterra variant modelling

To mathematically model Problem 1, this paper utilizes an enhanced Lotka-Volterra equation with additional parameters to describe a more complex ecosystem. In contrast to the ordinary Lotka-Volterra equation, we account for the effect of sex ratio on the rate of natural resource consumption by lampreys and incorporate the sex ratio of sea lampreys into the equation of natural resource dynamics. [1] This model focuses on the dynamics of natural resources, lampreys, and predators, which use lampreys as a source of nutrients. The model is presented below:

Dynamic equations for natural resources:

$$\frac{dR}{dt} = \mathrm{Y} - k\rho\mathrm{P}R$$

Dynamic equations of sea lamprey:

$$\frac{dP}{dt} = \alpha(0.5 + 0.5\sin(\pi\rho))P - \beta(0.5 + 0.5\sin(\pi\rho))PH$$

Dynamic equations of predator:

$$\frac{dH}{dt} = \delta(0.5 + 0.5\sin(\pi\rho))PH - \gamma H$$

We define:

$\boldsymbol{R}$ is the number of natural resources
$\boldsymbol{Y}$ is the natural resource recovery rate
P is the quantity of sea lamprey
$\boldsymbol{H}$ is the number of predators
$\boldsymbol{\alpha}$ is the natural growth rate of sea lamprey
$\boldsymbol{\beta}$ is the rate at which the predator grows by taking sea lamprey
$\boldsymbol{\delta}$ is the natural mortality rate of the predator
$\boldsymbol{\gamma}$ is the rate at which predators grow by feeding on sea lamprey
$\boldsymbol{k}$ is the rate of consumption of natural resources by the sea lamprey
$\boldsymbol{\rho}$ is the male ratio of the sea lamprey

In the model we constructed, the male ratio will affect the rate of consumption of natural resources by sea lamprey because male sea lampreys have higher resource utilization, so the higher the male ratio, the slower the rate of consumption, and the lower the male ratio, the faster the rate of consumption. And, the closer the male-to-female ratio is to 1:1, the faster the growth rate of the sea lampreys. We will further use this model for simulation analysis later.

4.2.2 Result

According to Lotka-Volterra variant equation Model, we assume that the initial number of natural resources is 1000, the initial number of sea lamprey is 500, the initial number of predators is 1, and the time span is 200(All data are relative rather than real). Trends in predators, sea lampreys, and natural resources were plotted over time to reflect the effects of changing sex ratios on natural resources, the food chain, and the population of sea lampreys themselves. In order to more visually compare the effects of different sex ratios on the ecosystem, here we plotted the dynamics of each parameter for a higher proportion of males (80%) and a lower proportion of males (40%), as shown in Figure 5:

Translated with www.DeepL.com/Translator (free version)

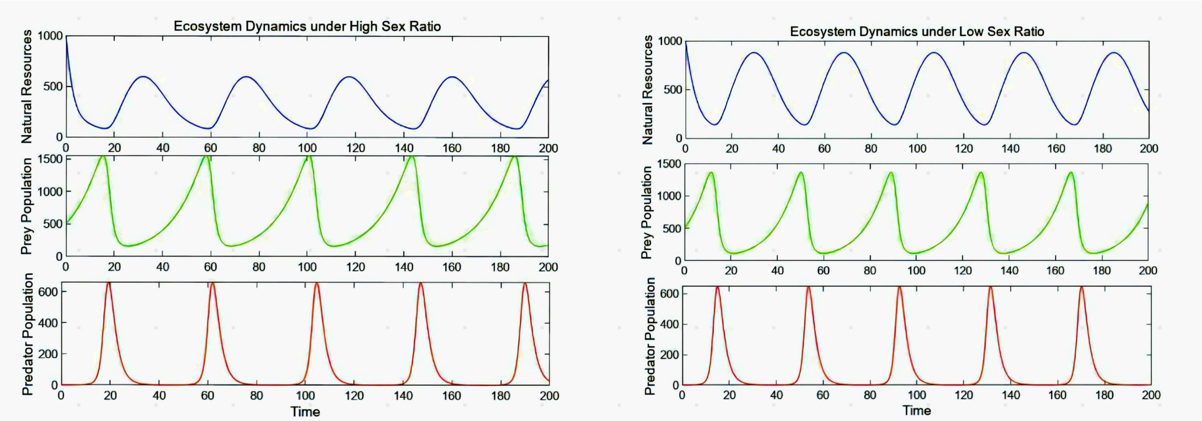


Figure 5:Time variation of predator population sea lamprey population natural resource under high and low male sex ratio

As can be seen in Figure 5, the size of the sex ratio affects the population size of the sea lamprey, for example, when the ratio of males to females is closer, reproductive behaviors are more likely to occur among the sea lamprey population, the reproduction rate will be much higher, and the rate of increase of the population size will be faster. When the proportion of males is higher, the peak in the population size of sea lamprey will be higher due to the higher utilization of natural resources by male sea lampreys. Natural resources and sea lampreys interact with each other; when there are fewer natural resources, less nutrients are consumed by sea lampreys, which will cause the proportion of males in the population to rise; whereas, when the proportion of males in sea lampreys is higher, it means that the population as a whole utilizes natural resources more, and fewer natural resources may be needed instead. And the number of predators rises and falls as the number of sea lampreys rises and falls, with an overall lag, which is in line with the objective law of the earth's ecosystem. Overall, the three populations in our constructed ecosystem promote and restrain each other, and the population numbers all show periodic fluctuation changes.

4.3 Reproductive rates in sea lamprey populations

The sex ratio of lamprey population will inevitably affect its reproductive rate. The reproductive rate is not only related to the ratio of male to female, but also to the quality of nutrients that the species can absorb, that is, the amount of available food resources.

Firstly, from a biological point of view, when the ratio of the number of females to males tends to be close to 1:1, the proportion of pairs rises considerably, and so does the reproductive rate.

Secondly, the abundance or otherwise of food resources directly affects the nutritional status of individuals. Good nutritional status is essential for reproduction as it affects the fertility of females and the reproductive input of males. Adequate food resources help to ensure that pregnant females are able to provide sufficient energy and nutrients to support the birth and development of the young. At the same time, abundant food resources may lead to seasonal peaks in fecundity, while scarce resources may lead to reduced or stagnant reproductive activity.

Overall, accessible food resources and sex ratio are among the key factors in the reproductive rates of animal populations. They are intertwined and together affect species survival, reproduction and adaptation. Adaptive sex ratio variation is a biological adaptation to adjust reproductive strategies under different environmental conditions, but its effects also depend on the complex interactions of other environmental factors. Summarizing the above analysis and considering only the effects of gender ratio and natural resources, we develop the following model.

$$S(\hbar, R) = \alpha_f \hbar(1 - \hbar)R$$

where S denotes reproductive success, R denotes the amount of resources, α_f is the coefficient, $\hbar$ is the sex ratio, and the model assumes that reproductive success is proportional to the product of the amount of resources and sex ratio.

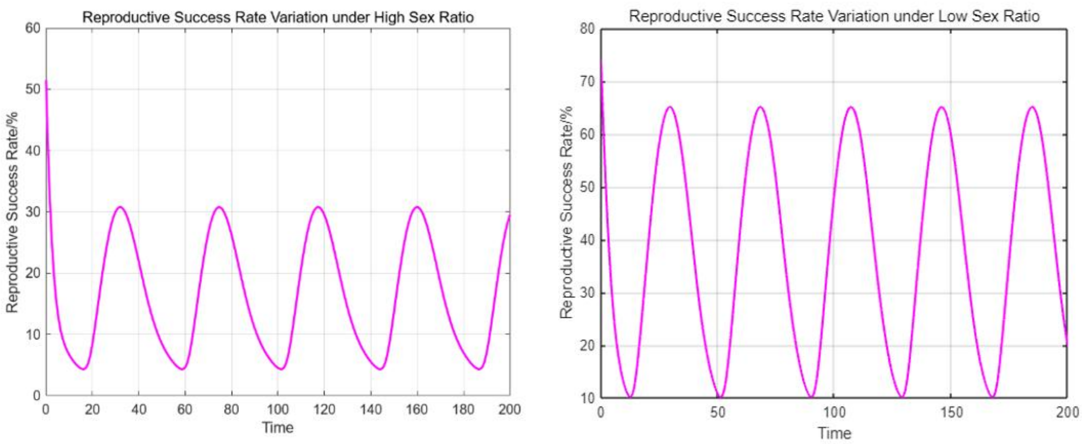


Figure 6:Time variation of reproductive success rate under high and low male sex ratio

From the graph, we can see that the reproduction rate will be lower when the proportion of males is relatively high (away from 50%) and higher when the proportion of males is low (close to 50%), in line with our logical analysis.

4.4 Effects of sex ratio on ecological niches

4.4.1 Model construction

With limited resources, we set the width of relative ecological niche of the sea lamprey to be L and the sex ratio to be $\hbar$. The more we reproduce, the more we increase the relative ecological niche.

$$\mathrm{L}(\hbar+\Delta\hbar)=L(\hbar)+b(\hbar)-d(\hbar)+\sigma$$

We define:

$\boldsymbol{L(\hbar)}$ is the width of relative ecological niche when the sex ratio is $\hbar$

$\boldsymbol{b(\hbar)}$ is the width of relative ecological niche increased by reproduction when the sex ratio is$\hbar$

$\boldsymbol{d(\hbar)}$ is the width of relative ecological niche reduced by mortality when the sex ratio is $\hbar$

$\boldsymbol{\sigma}$ is the random disturbance factor used to model the effects of anthropogenic or natural factors

The greater the relative ecological niche width of the current species or the greater the environmental capacity,the more relative ecological niche increases, and vice versa. According to Beverton-Holt and Lotka-Volterra equation to get:

$$b(\hbar)=g_r\cdot L(\hbar)\cdot\left(1-\frac{L\left(\hbar+\sum c_i\cdot L(\hbar)\right)}{s}\right)$$

Where:

$\boldsymbol{g_r}$ is the growth rate of the species

$\boldsymbol{c_i}$ is the relative ecological niche relationship between the species and other related species

$\boldsymbol{s}$ is the ecosystem area

$$\begin{cases} c_i=1, & \text{symbiosis} \\ c_i=0, & \text{neutrality} \\ c_i=-1, & \text{competition} \end{cases}$$

$$d(\hbar)=f(de)\cdot\alpha\cdot L(\hbar)$$

where $f(de)$ is the response function to the adverse effects of weather on the species community, the exact form of which can be obtained from the specific environmental situation.

Associating the above equations:

$$L(\hbar + \Delta\hbar) = L(\hbar) + b(\hbar) - f(de)\cdot\alpha\cdot L(\hbar) + \sigma$$

Differential form:

$$\frac{dl}{d\hbar} = g_r\cdot L(\hbar) - g_r\cdot L(\hbar)\cdot\frac{L\left(\hbar + \sum c_i\cdot L(\hbar)\right)}{s} - f(de)\cdot\alpha\cdot L(\hbar) + \sigma$$

4.4.2 Result

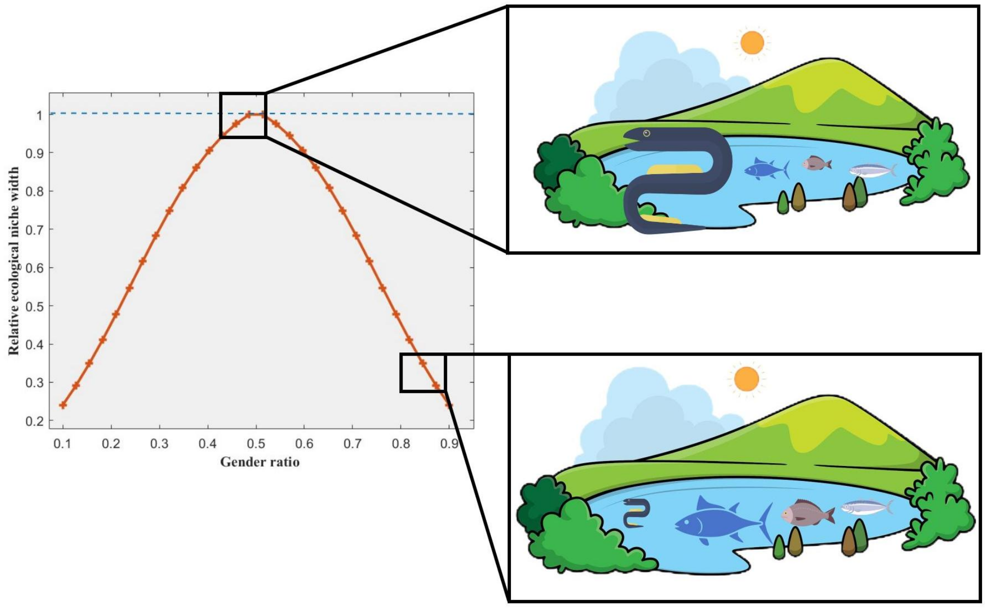


Figure 7:Variation of relative niche width with sex ratio (The size of the graph represents the relative niche width of species but does not represent the true scale)

5 Problem 2: Strengths and weaknesses of the sea lamprey stock

5.1 Advantages

5.1.1 Population self-regulation ability

The sea lamprey population has a certain self-regulation ability, and can adapt to different living environments by changing the sex ratio of its own population. This self-regulation ability enables the sea lamprey to maintain the stability of its population under different ecological conditions, and it is not easy to have explosive growth or sudden extinction. When the demand for a particular sex in the environment increases or decreases, the sea lamprey population may adjust its sex ratio to adapt to the changes in the environment. If food resources are abundant in the environment, the number of females may increase to increase the reproduction rate of the population, and if there are more predators in the environment, the number of males may increase to enhance the population's defence and competition.

This ability to regulate sex ratio allows the sea lamprey to better adapt to changing ecological environments, contributing to the survival of its own population, essentially for the sake of its offspring to be more adapted to the current environment, and is a kind of flexible self-regulation of the species. At the same time, through the change of sex ratio, the population can increase the communication and mating opportunities between individuals, and promote the exchange and flow of genes, which helps to reduce the accumulation of genetic defects and maintain the health and stability of the population.

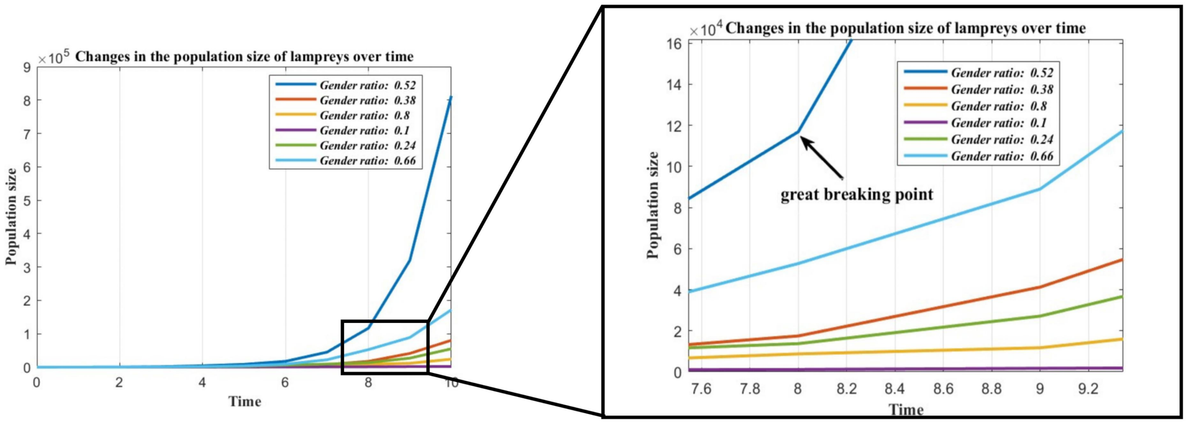


Figure 8:Growth of lamprey population with time under different sex ratios

5.1.2 Aspects of internal competition

An appropriate sex ratio can reduce the competitive pressure between individuals of the same sex. With a relatively balanced sex ratio, competition between individuals for resources and territories may be reduced, thus lowering the incidence of internal conflicts, which helps to maintain relative harmony and balance within the population.

5.1.3 Survivability aspects

In order to reproduce and grow, sea lamprey populations will live in living or marine habitats and migrate upstream to spawn and reproduce their offspring, and are able to adapt to different conditions of water temperature and water quality, with a strong ability to adapt to the waters. At the same time, the likelihood of extinction is very low compared to other species, not only because it is a parasitic fish with a wide range of food sources, but also because it has a strong adaptive and regulatory ability due to the sex reversal from female to male. At the same time, adaptive sex ratio variation may help increase the species' resilience. When environmental conditions are changing, sex ratio adjustments can make populations more resilient and able to adapt to new ecological challenges.[2]

5.2 Disadvantages

5.2.1 Problems when the environment changes dramatically

Sea lamprey populations rely on sex ratio adjustments to adapt to their environment, but this adaptation may make the population vulnerable to rapid changes in the external environment. If the region environment changes drastically, the population may need time to adapt, and this process may not always be rapid or successful.[3]

5.2.2 Problems with Reproduction Rates

Moderate variation in sex ratio may be beneficial to a species, but excessive or frequent variation may lead to a series of problems that threaten the survival and reproduction of the species. In some cases, excessive sex-ratio adjustments may lead to reduced reproductive rates, and when sex ratios are out of balance, effective reproductive opportunities may be reduced, thus affecting population growth and maintenance. For example, if the proportion of females is chronically too low, populations may face reproductive difficulties, as an insufficient number of females may lead to reduced reproductive opportunities and may even threaten the survival of the species.

6 Problem 3: Evaluation of Ecosystem Stability

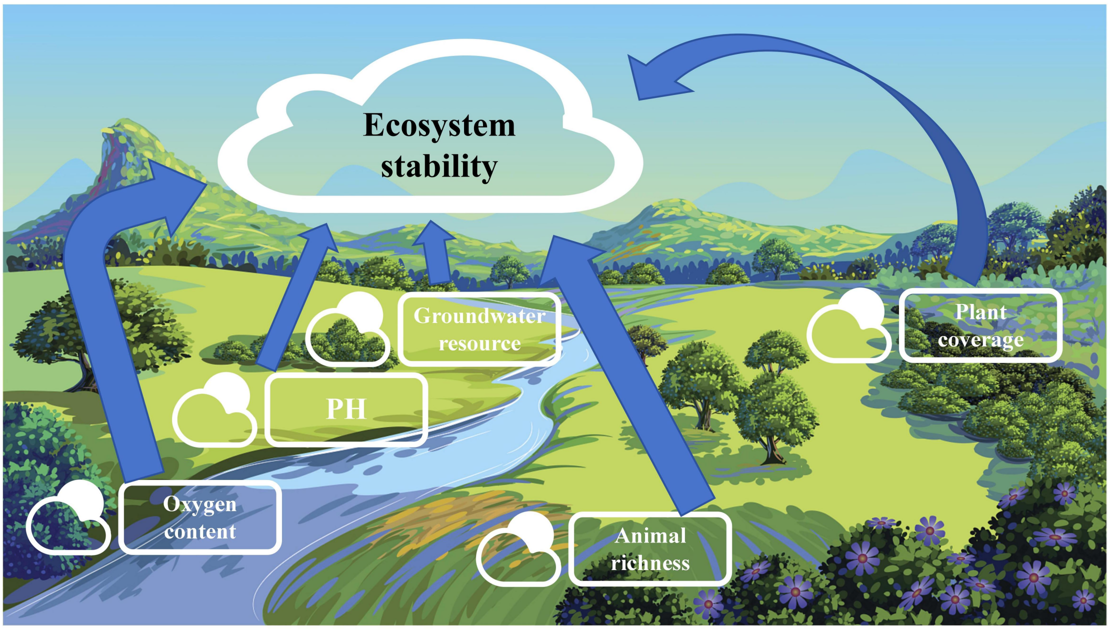


Figure 9:Schematic diagram of ecosystem stability assessment

Ecosystem stability has always been the key to ecological research[4], for the definition of ecosystem stability, different scholars have different definitions, and due to the complexity and multidimensionality of ecosystems, the stability of the definition of the concept of evaluation does not have a unified concept, for this, we establishes the ecosystem stability evaluation model, by two large indicators and five small indicators of ecosystem stability to assess.

6.1 Explanation of indicators:

Animal richness:

Higher animal species richness is usually associated with more stable ecosystems. As different species play different roles in ecosystems and form complex interrelationships between them, including predation, competition and symbiosis, such diversity can provide more functional groups and ecological niches for ecosystems to adapt to external changes and disturbances, and when one species is threatened or becomes extinct, others can fill its ecological role, thus maintaining the ecosystem stability. are able to fill their ecological roles, thus maintaining the stability of the system. In this paper, we use animal species abundance as a quantitative indicator of animal species richness, taking sea lamprey and lake trout (or other prey objects of sea lamprey) as representatives, and its calculation formula is as follows.

$$\begin{cases} SLT = \dfrac{N_{lam}}{100} \\ LTA = \dfrac{N_{tro}}{100} \\ AR = LTA - SLT \end{cases}$$

We define：

$\boldsymbol{SLT}$ is sea lamprey abundance

$\boldsymbol{LTA}$ is lake trout abundance

$\boldsymbol{N_{lam}}$ is the number of sea lamprey species

$\boldsymbol{N_{tro}}$ is the number of lake trout species

$\boldsymbol{AR}$ is animal species richness vegetation cover

Plant coverage：

Adequate vegetation cover provides protection and habitat that helps prevent soil erosion, soil decay and natural disasters such as droughts and floods, and vegetation maintains ecosystem stability by fixing soil, absorbing water and reducing climate change. In addition, vegetation plays a key role in ecosystem health and stability by providing important ecological services such as oxygen, absorbing carbon dioxide, regulating climate and providing food. Therefore, increasing vegetation cover contributes to the stability of ecosystems, reduces the risk of natural disasters, and maintains biodiversity and ecological balance, as quantified by the following formula.

$$PR = \frac{Pla}{S}$$

Where Pla is the area covered by vegetation and $\boldsymbol{S}$ is the total land area.

Water Resources pH:

The pH of water resources has a direct impact on ecosystem stability. Changes in the pH of a water body can lead to changes in the community of organisms in the water, thus affecting the stability of the entire ecosystem. For example, when the pH value of the water body is too high or too low, it will adversely affect the health of the organisms in the water, leading to the death or migration of the organisms, and destroying the ecological balance. We use the method of calculating the Euclidean distance between the local water resource PH and the optimal SPH of the place for the quantification of the indicator, and the quantification formula is as follow:

$$\sqrt{\frac{(PH-SPH)^2}{SPH}}$$

The SPH is determined by the environment of the local water resource.

Groundwater resource reserves :

Groundwater is an important source of water in many ecosystems, supporting the survival and functioning of ecosystems such as wetlands, rivers, forests and grasslands. Adequate groundwater resources can maintain wetlands in a moist state, keep rivers flowing and flowing, nourish the growth of vegetation, and sustain animals and microorganisms, as quantified by the following formula.

$$Gr = Gr$$

Oxygen content:

Dissolved oxygen in water is a key element necessary for aquatic organisms to sustain life, especially for fish and other aquatic organisms. Adequate oxygen content helps to support the metabolic and respiratory processes of aquatic organisms and maintain their normal physiological functions, which is quantified by the following formula.

$$Oc = Oc$$

6.2 Ecosystem Stability Modelling Based on EWM-Topsis

6.2.1 Entropy Weight Method

Step1: Normalisation (min-max)

$$\widetilde{Z_{ij}} = \frac{x_{ij} - min\{x_{1j}, \ x_{2j}, \ \cdots x_{nj}\}}{max\{x_{1j}, \ x_{2j}, \ \cdots x_{nj}\} - min\{x_{1j}, \ x_{2j}, \ \cdots x_{nj}\}}$$

Step2: Calculate the probability

$$P_{ij} = \frac{\widetilde{Z_{ij}}}{\sum_{i=1}^{n} \widetilde{Z_{ij}}}$$

$$\widetilde{Z} = \begin{pmatrix} \widetilde{Z_{1j}} & \cdots & \widetilde{Z_{1m}} \\ \vdots & \ddots & \vdots \\ \widetilde{Z_{n1}} & \cdots & \widetilde{Z_{nm}} \end{pmatrix}$$

Step3:Calculate the information entropy of each indicator

For the j indicator, information entropy:

$$e_j = -\frac{1}{ln(n)} \sum_{i=1}^{n} P_{ij} \, ln(P_{ij}) \ (j = 1,2, \ \cdots\cdots, \ m)$$

Step4: Calculate the weights

$$d_j = 1 - e_j$$

$$w_j = \frac{d_j}{\sum_{j=1}^{n} d_j} (j = 1,2, \ \cdots\cdots, \ m)$$

6.2.2 Topsis method

Step1:Normalisation

Step2: De-measurement of the forward matrix

$$\widetilde{Z_{ij}} = \frac{x_{ij}}{\sqrt{\sum_{i=1}^{n} x_{ij}{}^2}}$$

Step3: Calculate the score and normalise the matrix

$$\begin{cases} D_i^+ = \sqrt{\sum_{j=1}^{n} w_j \left(\widetilde{Z_j}^+ - \widetilde{Z_{ij}}\right)^2} \\ D_j^- = \sqrt{\sum_{j=1}^{n} w_j \left(\widetilde{Z_j}^- - \widetilde{Z_{ij}}\right)^2} \end{cases}$$

Where, w_j is its weight, determined by entropy weight method.

6.3 Result

6.3.1 Experimental results

In order to facilitate the unified quantitative analysis, we weighted the scores of the five indicators and mapped them proportionally to the interval of 0 to 5, and obtained the stability coefficient K after summing, according to the range of K to determine the stability of the ecosystem, when the scores are in the interval of 0 to 10, we believe that the ecosystem is in an unstable state, facing the risk of collapse, when the scores are in the interval of 10 to 15, we believe that the ecosystem is in a more stable state, and there is a greater risk of collapse. When the score is in the range of 10 to 15, we consider that the ecosystem is in a stable state and at risk of collapse; when the score is in the range of 10 to 15, we consider that the ecosystem is in a stable state and at low risk of instability.

Table 2 Ecosystem stability assessment classification table

Score	Riskiness	Stability
0<K<10	**High risk**	**Unstable**
10<K<15	**Risky**	**More stable**
15<K<25	**Low risk**	**Very Stable**

6.3.2 Outcome simulation

We set up 100 sets of experiments using the EPVS model for a population containing adjustable sex ratios of sea lampreys in the ecosystem in which it is embedded, with two simulations for each set of experiments, and in the two simulations, randomly set combinations of sex ratios of the sea lampreys, and then evaluated them using the Ecosystem Stability Evaluation Model after obtaining the results, which were designed to analyse the effects of changes in sex ratios on ecosystem stability in different sex ratios, using the method of controlled analysis. This was done in order to analyse the effect of changing the sex ratio of the lamprey population on ecosystem stability under different sex ratios using a controlled analysis. Due to space constraints, we will only show two sets of experiments that are more meaningful for the analysis, and the sex ratio combinations of the two sets of experiments were set as (0.17, 0.52) and (0.85, 0.66), respectively, and the results of the evaluation are as follows:

Table 3 Scores of each part

Gender ratio	AR	PR	PH	Gr	OC	Score
0.17	**3**	**3.3**	**2.66**	**4.1**	**2.54**	**15.6**
0.52	**0.8**	**3**	**2.6**	**4**	**1.1**	**11.5**
0.85	**3.4**	**3.2**	**2.54**	**4.3**	**2.34**	**15.78**
0.66	**0.6**	**3.5**	**2.55**	**4.3**	**1.01**	**11.96**

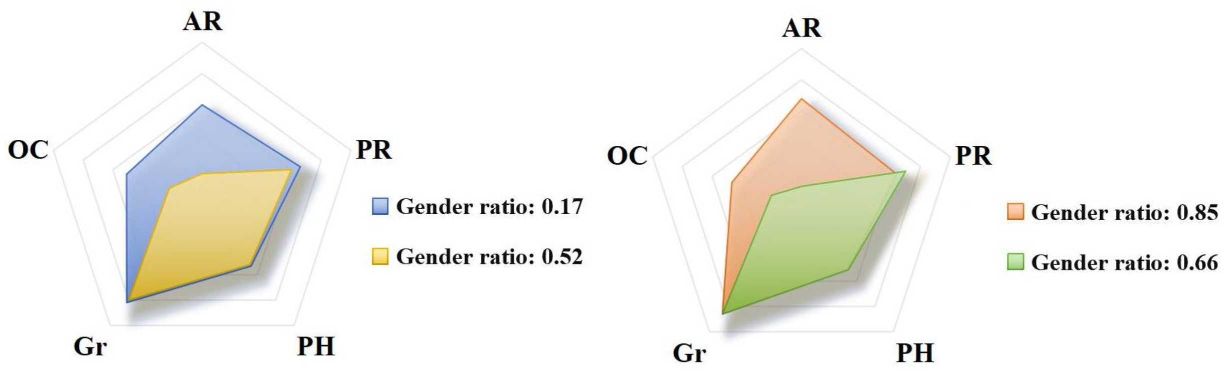


Figure 10: Rating radar chart of each part

Through the experimental simulation, there were 24 groups of experiments in which the sex ratio of the sea lamprey population was between 0.40 and 0.65, and among these 24 groups of experiments the results were judged to be ecosystem instability in 16 groups (66.7%), the results were judged to be ecosystems that are more stable in 5 groups (20.8%), and the results were judged to be ecosystems that are unstable in 3 groups (12.5%). .

From this, we judge that when the sex ratio of the sea lamprey population is close to 0.5, it has a large impact on the stability of the ecosystem, with a high risk and a high correlation with the instability of the ecosystem. And its impact on the organic aspect of the ecosystem is large, and its impact on the inorganic environment of the ecosystem is small, and its impact on the organic aspect is mainly reflected in the abundance of species, which has a greater relationship with their sex ratio, when the sex ratio of the population is close to 0.5, the reproduction rate of the population is high, and the relative ecological niche is close to 1, which will deal a great blow to its host populations due to the characteristics of its parasitic lethality[5], thus causing food web rupture and greatly reducing ecosystem stability.

7 Problem 4: Beneficial effects of changes in the sex ratio of sea lampreys on other organisms in the ecosystem

7.1 Model establishment

The relationship between lampreys and parasites is diverse and complex. We mainly divide them into the following three categories :

- Parasite I : Parasites parasitizing lamprey food (natural resources), direct competitive relationship
- Parasite II : Parasites that parasitize other organisms in the ecosystem, indirect competitive relationships
- Parasite III : Parasites parasitizing lampreys, symbiotic relationship

To mathematically model problem four, we upgrade the Lotka-Volterra variant of the equation model developed in the first question with a more sophisticated optimization that includes more parameters to describe the more complex sea lamprey-parasite ecosystems.[6]

7.1.1 Model of Changes in Natural Resource Quantity

We need to establish an equation for the variation of natural resource quantity over time. According to our definition, parasite I competes with lampreys for natural resources, so the consumption of natural resources is related to the consumption rates of lampreys and parasite I Based on this[5], we establish the following equation:

$$\frac{dR}{dt} = Re - k\rho PR - k_1F_1R$$

R is the number of natural resources, Re is the recovery rate of natural resources without any external interference, k is the consumption rate of lamprey resources, ρ is the sex ratio of lamprey, P is the number of lamprey population, F_1 is the number of parasite 1 population, and k_1 is the consumption rate of parasite I to natural resources.

7.1.2 Model of population change of parasite III

Parasite III is a parasite that parasitizes on lampreys. When lampreys are abundant, it is equivalent to more food resources (or habitat resources) of parasite III. Therefore, the population size of parasite III is also related to the population size of lampreys. Based on this, we establish the following equation :

$$\frac{dF_3}{dt} = \delta_3(0.5 + 0.5\,sin\,(\pi\rho))PF_3 - \gamma_3F_3$$

F_3 is the number of parasite III population, δ_3 is the rate parameter of parasite III by parasitic lamprey growth, γ_3 is the natural mortality of parasite III, ρ is the sex ratio of lamprey, and P is the number of lamprey population.

7.1.3 Model of population change of parasite I

When analyzing the changes of parasite I population, we should consider that it is competitive with lampreys. When the number of lampreys is large, the competitive pressure is large, and the population will decrease due to lack of resources. Based on this, the following equation is established :

$$\frac{dF_1}{dt} = \delta_1 F_1 - \gamma_1 F_1 - X_2 P F_1$$

F_1 is the population size of parasite I, δ_1 is the natural growth rate of parasite I, γ_1 is the natural mortality of parasite I, X_2 is the influence rate of lamprey on parasite I, P is the population size of lamprey.

7.1.4 Model of population change of parasite II

The same as the analysis of parasite I :

$$\frac{dF_2}{dt} = \delta_2 F_2 - \gamma_2 F_2 - X_3 P F_2$$

F_2 is the number of parasite II population, δ_2 is the natural growth rate of parasite II, γ_2 is the natural mortality of parasite II, X_3 is the influence rate of lamprey on parasite II, and P is the number of lamprey population.

$$\frac{dP}{dt} = \alpha(0.5 + 0.5\,sin\,(\pi\rho))PR - \beta(0.5 + 0.5\,sin\,(\pi\rho))PF_3 - X_1 P F_1 - X_4 P F_2$$

β is the effect rate of parasite III on the death of lamprey, X_1 is the effect rate of parasite I on lamprey, and X_4 is the effect rate of parasite II on lamprey.

7.2 Result

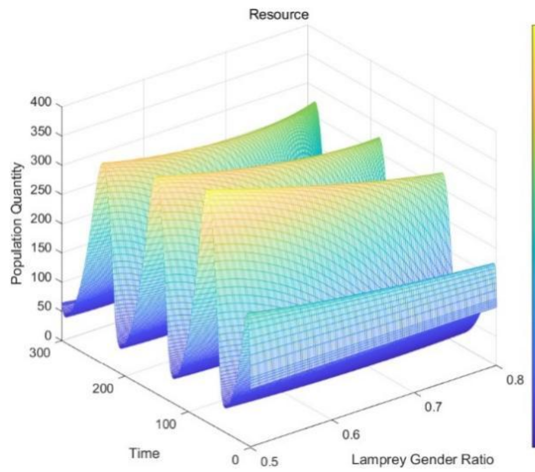
After many times of fitting and parameter adjustment, we finally determined that the relevant parameters are:

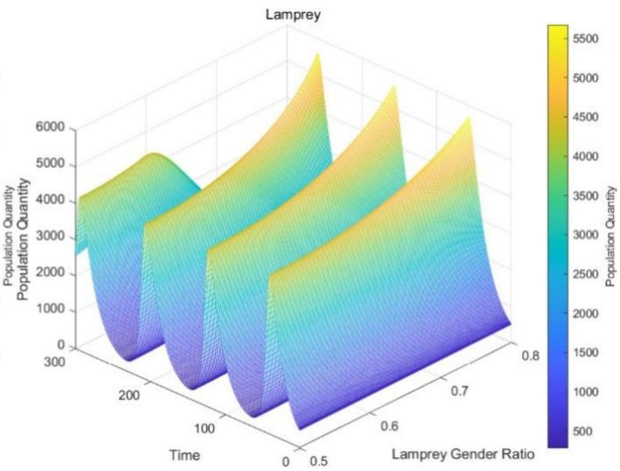
$$\alpha = 0.08;\ \beta = 0.01;\ \delta_3 = 0.0001;\ \gamma_3 = 0.15;\ \delta_1 = 0.05;$$

$$\gamma_1 = 0.04;\ X_1 = 0.0002;\ \delta_2 = 0.02;\ \gamma_2 = 0.015;\ X_3 = 0.000002$$

$$X_2 = 0.000002;\ k = 0.001;\ \rho = 0.78;\ X_4 = 0.001$$

Based on the above parameters, in order to analyze the impact more intuitively, we have drawn a three-dimensional map of the number of natural resources, the number of lampreys, and the number of three types of parasites over time and the sex ratio of lamprey population, as shown below :





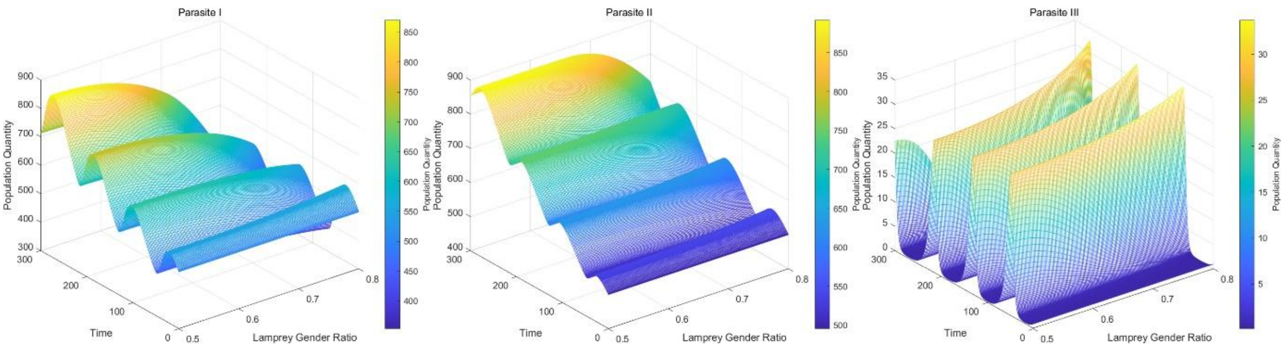


Figure 11:The above figure from left to right is the stereogram of the number of natural resources, the number of lampreys over time and the sex ratio of lamprey population. The following figure from left to right is the stereogram of the number of three types of parasites over time and the sex ratio of lamprey population.

The model is in line with our expectations for changes in each population. Lamprey is in a dominant position in the competition of natural resources. Natural resources and parasite III show periodic fluctuation changes with the change of lamprey number. Parasite II has unlimited living space due to the ideal situation of the problem. In the early stage, the pressure of insufficient natural resources is small, and the population number shows a fluctuating upward trend. Parasite I is in a weak position in the competition with lamprey resources. The population number is also affected by natural resources and lamprey population, but the peak value and change rate of population number are less than lamprey.

8 Sensitivity Analysis

8.1 On the sensitivity analysis of natural resource changes

Due to seasonal changes, special natural disasters, regional differences and other reasons, it may cause differences in the amount of natural resources (and all sources of nutrition that can be used for their own growth) in ecosystems at different times and places. According to the difference in the amount of natural resources, we divide different environmental differences into slight differences, large differences and huge differences.

We use the natural resources used in the model in Problem 4 as a baseline value to see the effect of changing the amount of natural resources on the stability of the ecosystem model we constructed. We compare the differences in the peak value, trough value and average value of wave distance of population change by experiment under different natural resources, after enough time until the population change of each species tends to be stable.

Table4：Sensitivity analyses of natural resources and peaks in population size changes

Changes in natural resources	+10%	+100%	+500%	-10%	-50%	-99%
Resource（%）	0.01	ε	0.02	0.02	0.01	0.01
Lamprey（%）	ε	ε	ε	ε	ε	ε

Parasite 1（%）	ε	ε	ε	ε	ε	ε
Parasite 2（%）	ε	ε	ε	ε	ε	ε
Parasite 3（%）	0.01	ε	ε	0.01	0.01	ε

Table5： Sensitivity analysis of natural resources to the trough of population size changes

Changes in natural resources	+10%	+100%	+500%	-10%	-50%	-99%
Resource（%）	0.01	ε	0.03	0.01	ε	0.01
Lamprey（%）	0.03	ε	0.01	0.02	0.02	0.01
Parasite 1（%）	ε	ε	ε	ε	ε	ε
Parasite 2（%）	ε	ε	ε	ε	ε	ε
Parasite 3（%）	ε	ε	0.07	ε	0.07	ε

Table6： Sensitivity analysis of natural resources and population size change wave pace

Changes in natural resources	+10%	+100%	+500%	-10%	-50%	-99%
Resource（%）	ε	0.55	2.06	0.14	0.27	0.41
Lamprey（%）	ε	ε	0.20	ε	0.20	0.20
Parasite 1（%）	ε	ε	0.20	ε	0.20	ε
Parasite 2（%）	ε	ε	0.20	0.20	0.20	0.20
Parasite 3（%）	0.20	ε	0.40	0.20	0.40	ε

Note: where ε is for data less than 0.01%.

Observing the data, we can see that our ecosystem model can recover to a stable ecological balance quickly and automatically after a certain period of development in the case of slight differences in natural resources, large differences and huge differences. It shows that our ecosystem model conforms to the objective law of the earth 's ecosystem. After a certain degree of external interference, it can be restored to a stable balance after sufficient time.This shows that our model is robust.

8.2 Sensitivity analysis of population variation of lamprey

In life, people regularly catch and remove some lampreys according to the number of lampreys, so we observe whether the ecosystem can be restored to stability or destroyed in a stable cycle if the lampreys are removed in large numbers.

We set the ratio of male and female lampreys to an ideal 1 : 1, and in the time range of 120-180, fished and removed lampreys to stabilize their number at the initial number and observe the overall changes of the ecosystem.

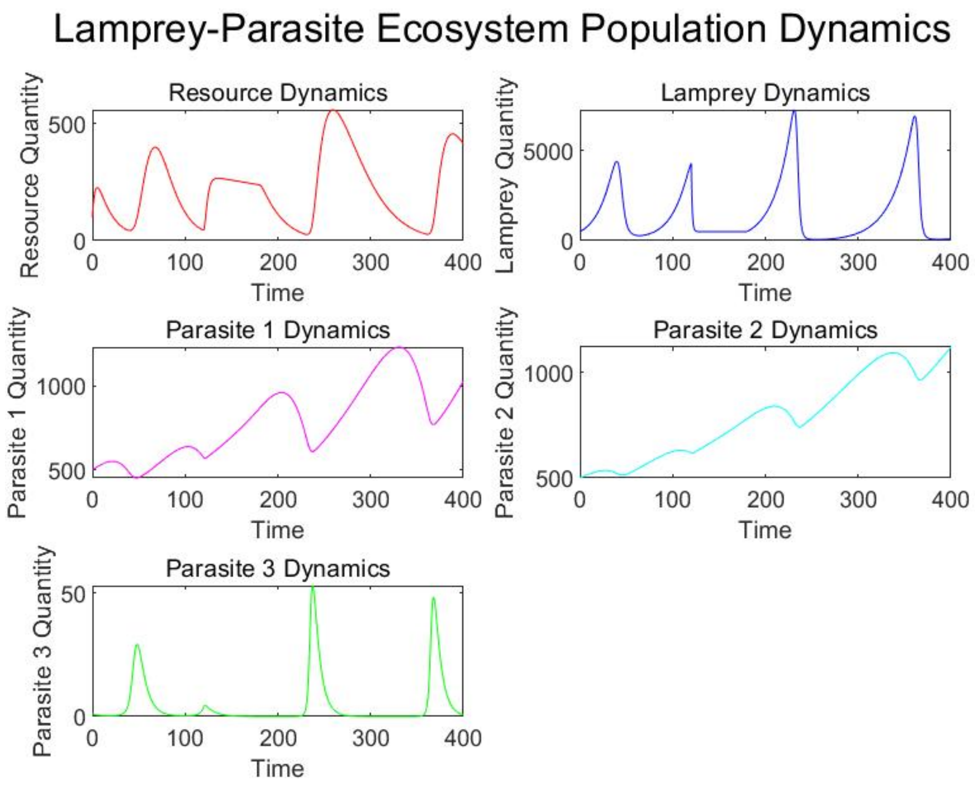


Figure 12: Lamprey-Parasite Ecosystem Population Dynamics

It can be seen from the figure that when the number of lampreys remains low due to human fishing, the number of competitors has achieved rapid growth, resulting in a decline in natural resources, and the number of parasites associated with them has decreased due to the decline in the number of hosts. However, after a period of recovery, we found that our ecosystem can still evolve into a stable state, which is consistent with the objective law of the earth 's ecosystem, indicating that our model is robust.

9 Strengths and Weaknesses

9.1 Strengths

- **The improved Lotka-Volterra model we used was able to dynamically describe the interactions and competitive relationships between species, which is more in line with the dynamic evolution of real ecosystems.** This was extremely helpful for us to qualitatively analyze the dynamics of sea lamprey populations and the effects of the sex ratio of sea lampreys on the ecosystems in which they are found, taking into account direct competitive relationships between species, indirect competitive relationships, symbiotic relationships, predator-prey relationships, and simplifying the complex food web.
- **For the assessment of ecosystem stability, we consider multiple indicators.** The stability of the ecosystem is not determined by a single factor, but the result of multiple factors in nature. Therefore, we establish an ecosystem stability evaluation model, considering two large indicators and five small indicators, and analyze their respective

weights. This makes our model more comprehensive and more convincing.

- **Classification analysis of parasites.** Our model not only considers the parasites that have direct or indirect competitive relationships with lampreys, but also considers the parasites that have symbiotic relationships with lampreys, and analyzes each parasite in a targeted manner.

9.2 Weaknesses

- **The model 's assumptions on the environment are relatively simplified, ignoring some complex influencing factors in reality, which may lead to deviations between the results and the real situation.** The real environment is extremely complex, which will inevitably affect the survival and development of the population, thus causing a certain impact on the problems studied in this paper. Our results may have certain limitations and cannot cover all situations.
- **Some parameters are difficult to calculate accurately.** In the determination of the parameters of the lamprey population dynamics model, we are based on the proportion of lamprey males in the population between 56 % and 78 %. Although we have flexibility in determining the parameters of the model, this estimation still lacks accuracy.

10 References

[1]Huang Guangqiu, Zhao Weijuan, Lu Qiuqin. Optimization algorithm for population kinetic function based on 3-population Lotka-Volterra model[J]. Computer Sci.,2013,40(08):214-219.

[2] R.J. Young, G.C. Christie, R.B. McDonald, D.W. Cuddy, T.J. Morse, and N.R. PayneCan. J. Fish. Aquat. Sci. Vol. 53(Suppl. 1), 1996

[3] J. Great Lakes Res. 29 (Supplement 1):1–14 Internat. Assoc. Great Lakes Res., 2003

[4]Li Jiaying, Hu Guanzheng, Zeng Weihua. Stability evaluation of grassland ecosystems based on system dynamics--a case study of Mado County, Yellow River Source, Sanjiangyuan National Park[J/OL]. China Environmental Science:1-15[2024-02-03]

[5]J. Great Lakes Res. 33 (Special Issue 2):35－47Internat. Assoc. Great Lakes Res., 2007

[6]Niu Hong, Wang Yidan, Wang He. Complexity analysis of Lotka-Volterra differential-algebraic models with competing populations[J]. Journal of Liaoning University of Petrochemical Technology,2018,38(02):90-93.