Learning Dynamics (INFO-F409)
Assignment 1: Game Theory

Charlotte Nachtegael - ULB

November 1, 2016

1 The Hawk-Dove game

We observe here a competitive environment where the resource (V) can be either obtained by different ways : either by using the Hawk strategy, which is immediately taking the resources by fighting, or the Dove strategy, which is displaying themselves. Three different outcomes can be observed :

- **Hawk vs Hawk** : they have 50% chance of being injured (D) and 50% chance of winning;
- **Hawk vs Dove** : while the dove is displaying, the hawk attacks and takes everything;
- **Dove vs Dove** : both doves lose energy while displaying (T) and have 50% chance of winning the resources.

We want to know what would be the best logical strategy in function on the amount of resources (V), the amount of injuries after a fight between hawks (D) and the energy lost while displaying between doves (T).

If V > D, the Nash equilibrium is the pure strategy {Hawk;Hawk}
If V < D, we need to look for mixed strategy. First, we calculated the expected pay-offs for both players.
Expected pay-off for the player 1 :

- For a pure strategy "Hawk" (p) :
  $q * \left(\frac{V-D}{2}\right) + (1-q) * V = V - q * \left(\frac{V+D}{2}\right)$
- For a pure strategy "Dove" ($1-p$) :
  $q * 0 + (1-q) * \left(\frac{V}{2} - T\right) = \frac{V}{2} - T - q * \left(\frac{V}{2} + T\right)$

The player 1 will choose a pure "Hawk" strategy when :

$$V - q * \left(\frac{V+D}{2}\right) > \frac{V}{2} - T - q * \left(\frac{V}{2} + T\right)$$
$$\frac{V+2T}{D+2T} < q$$

So when $q > \dfrac{V+2T}{D+2T}$, player 1 will choose to play {Hawk}, so the best response is {Dove} or $p = 0$.
When $q < \dfrac{V+2T}{D+2T}$, player 1 will choose to play {Dove}, so the best response is {Hawk} or $p = 1$.

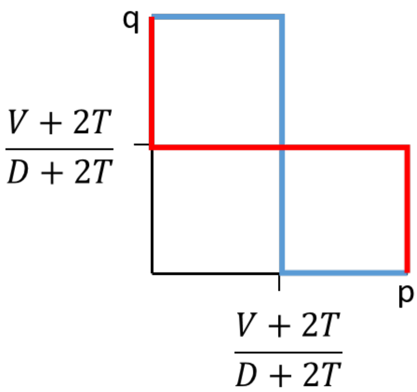
Expected pay-off for the player 2:

- For a pure strategy "Hawk" (q) :
  $p * \left(\frac{V-D}{2}\right) + (1-p) * V = V - p * \left(\frac{V+D}{2}\right)$
- For a pure strategy "Dove" $(1-q)$:
  $p * 0 + (1-p) * \left(\frac{V}{2} - T\right) = \frac{V}{2} - T - p * \left(\frac{V}{2} + T\right)$

The player 2 will choose a pure "Hawk" strategy when :

$$V - p * \left(\frac{V+D}{2}\right) > \frac{V}{2} - T - p * \left(\frac{V}{2} + T\right)$$
$$\frac{V+2T}{D+2T} < p$$

So when $p > \dfrac{V+2T}{D+2T}$, player 2 will choose to play {Hawk}, so the best response is {Dove} or $q = 0$.
When $p < \dfrac{V+2T}{D+2T}$, player 2 will choose to play {Dove}, so the best response is {Hawk} or $q = 1$.
We observe the subsequent graph :



So we have three different Nash equilibria when V $< D$:

- Two pure strategies : {Hawk;Dove} and {Dove;Hawk}
- One mixed strategy : $\{(\frac{V+2T}{D+2T}, 1-\frac{V+2T}{D+2T});(1-\frac{V+2T}{D+2T}, \frac{V+2T}{D+2T})\}$

Displaying is advantageous when we are in the case where the danger is greater than the resources ($V < D$), so it is better displaying and losing the sources to the adversary than fighting and ending with only injuries and not enough resources for it to be worthwhile.

2 Which social dilemna ?

First, we have to calculate the pay-off of the player A for every possible strategy of the player B. Then, you have to choose the best response of A to each strategy of B (highlighted in red in the following figure).

	C,C,C	C,C,D	C,D,C	D,C,C	D,D,C	D,C,D	C,D,D	D,D,D
C	3	8/3	4/3	7/3	2/3	2	1	1/3
D	4	7/3	11/3	8/3	7/3	1	2	2/3

Figure 1: Pay-off values for each action of A (row player) for each strategy of B (column player). In red are the best response to the strategy of B.

We then choose the best response of B for each action of A (Fig 2).

	C	D
C	2,2	0,5
D	5,0	1,1

Prisonners dilemma

	C	D
C	5,5	0,2
D	2,0	1,1

Stag Hunt game

	C	D
C	2,2	1,5
D	5,1	0,0

Snowdrift game

Figure 2: In green are the best response of B in response to each possible action of A for every game.

The Nash equilibria are found when the best-response found for A and B (Fig 1 and 2) match. We found two Nash equilibria :

- A choosing to play C and B following the strategy D,C,D (Fig 3)
- A choosing to play D and B following the strategy D,D,C (Fig 4)

The strategy for B is for the prisoners dilemma, stag hunt game and snowdrift game respectfully.

	C,C,C	C,C,D	C,D,C	D,C,C	D,D,C	D,C,D	C,D,D	D,D,D
C	3	8/3	4/3	7/3	2/3	2	1	1/3
D	4	7/3	11/3	8/3	7/3	1	2	2/3

	C	D
C	2,2	0,5
D	5,0	1,1

Prisonners dilemma

	C	D
C	5,5	0,2
D	2,0	1,1

Stag Hunt game

	C	D
C	2,2	1,5
D	5,1	0,0

Snowdrift game

Figure 3: Proof of the matching of the best response for A playing C and B playing D,C,D

	C,C,C	C,C,D	C,D,C	D,C,C	D,D,C	D,C,D	C,D,D	D,D,D
C	3	8/3	4/3	7/3	2/3	2	1	1/3
D	4	7/3	11/3	8/3	7/3	1	2	2/3

	C	D
C	2,2	0,5
D	5,0	1,1

Prisonners dilemma

	C	D
C	5,5	0,2
D	2,0	1,1

Stag Hunt game

	C	D
C	2,2	1,5
D	5,1	0,0

Snowdrift game

Figure 4: Proof of the matching of the best response for A playing D and B playing D,D,C

3 Sequential truel

Considering p_i as the probability of the player i successfully hitting his target, C is better off if he is a worst shooter than B, if $p_C < p_B$. Indeed, when A has to choose shooting C or B, they have to take into account the fact that the remaining player will target them. So it is more advantageous for A to shoot the better shooter between B and C, so that

their chances of survival when the remaining player shoots at them is higher. This is why in this case ”weakness is strength”.
The subgame perfect equilibria only happens when C can make their choice, which only happens when C was either never targeted or C was targeted but the shooter failed their shot. C will either target A or B in function of the danger they represent. They will target the most dangerous of the two.
On the same trend, B can only make their choice if they were not targeted by A or A missed their shot when targeting them. B will always target C because C is the only one with a bullet left who could target them.
A will target the best shooter, as explained above.
The subgame perfect equilibria would be for everyone to survive, which only happen by a probability of $(1 - p_A) * (1 - p_B) * (1 - p_C)$.

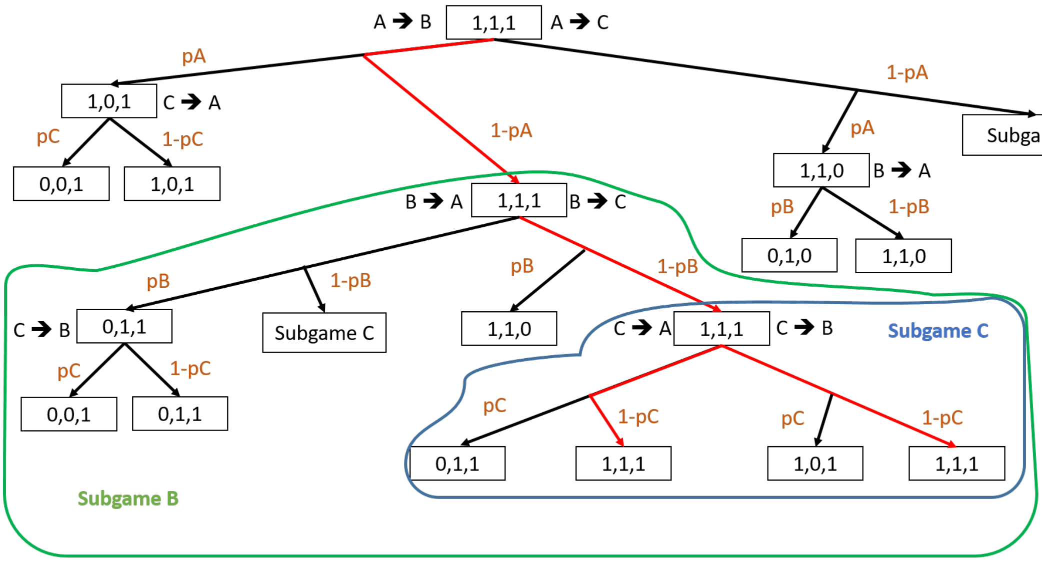


Figure 5: Sequential truel representation. The subgame perfect equilibria is highlighted in red.