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LEARNING DYNAMICS

Assignment 2

ULB student

Learning dynamics (VUB)

Academic year: 2015-2016

A) Explain Pij

$$P_{ij} = (1 + [W_j - W_i] / [4*(\max\{0,1,T,S\} - \min\{0,1,T,S\})]) / 2$$

Wi = sum of the payoffs of the four interactions of someone
Wj = sum of the payoffs of the four interactions of his neighbor
Wj-Wi > 0 when the payoff of the neighbor is higher

1) 4*(max{0,1,T,S} - min{0,1,T,S}) → 4 times the best payoff for a duel – 4 times the worst payoff for a duel.

2) So:

- If Wj has the 4 best payoffs in duel with his neighbors and Wi has the 4 worst payoffs:

[Wj - Wi] / [4*(max{0,1,T,S} - min{0,1,T,S})] = 1

-And if Wj has the 4 worst payoffs in duel with his neighbors and Wi has the 4 best payoffs:

[Wj - Wi] / [4*(max{0,1,T,S} - min{0,1,T,S})] = -1

-And if Wj = Wi :

[Wj - Wi] / [4*(max{0,1,T,S} - min{0,1,T,S})] = 0

3) A probability must be comprised between 0 and 1. And [Wj - Wi] / [4*(max{0,1,T,S} - min{0,1,T,S})] is comprised between 1 and -1. To have a probability, we have to add 1 and divide by 2. So:

- Pij= 1 if Wj = 4 best payoffs and Wi= 4 worst payoffs. Player i will change if he has done another choice (C or D) that J because he notices that his neighbor j earns far more than him.

- Pij = 1/2 if Wi = Wj. Player i will change half the time if he has done another choice (C or D) that J because he notices that his neighbor j earns the same payoff.

- Pij = 0 if Wj = 4 worst payoffs and Wi= 4 best payoffs. Player i will not change because he notices that his neighbor j earns far less than him.

B) Lattice with boundaries

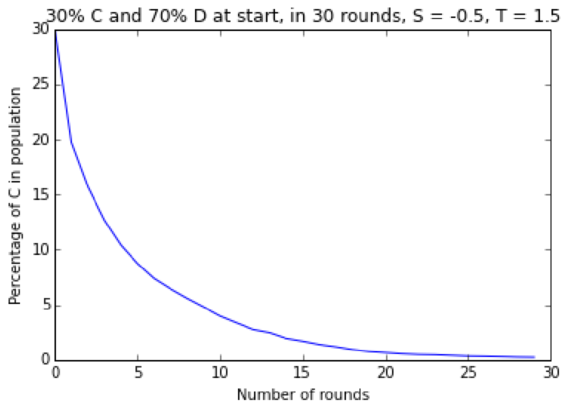
(See conclusion for more explanations about the graphs in each game)

C = cooperators
D = defectors
(B) = with boundaries
(No B) = without boundaries
R = reward, S = sucker's payoff, T = temptation to defect and P = punishment

Prisoner Dilemma: T= 1.5 , S= -0.5 (B)

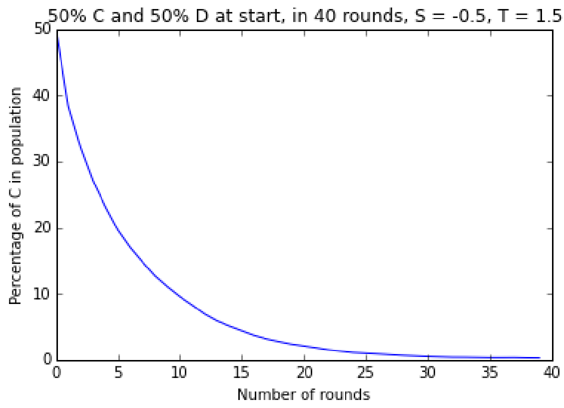
There are greed (T > R) and fear (P > S)

30% C and 70% D at start



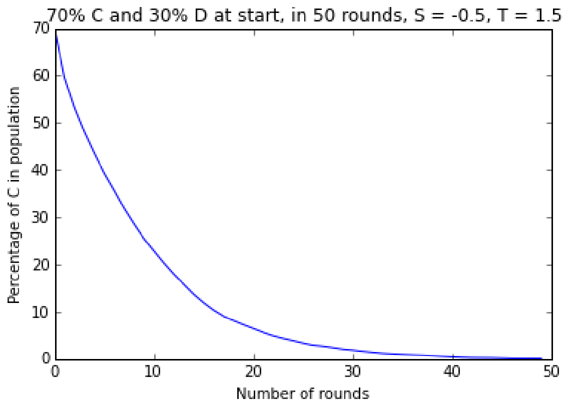
It tends to 0 cooperator and 100% defectors after about 30 rounds.

50% C and 50% D at start



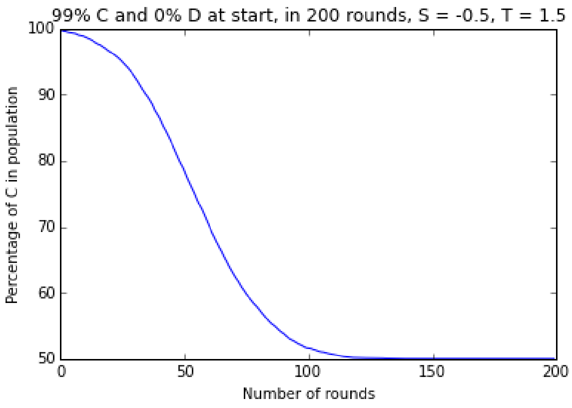
It tends to 0 cooperator and 100% defectors after about 40 rounds.

70% C and 30% D at start



It tends to 0 cooperators and 100% defectors after about 45 rounds.

One defector in the position {10,10} at start



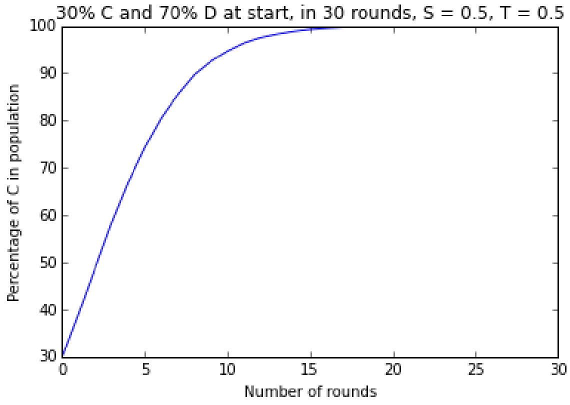
The mean tends to 45-50% of cooperators after about 120 rounds. After that, it stays at the same level round after round.

If we look at the lattices individually, we see that they are full C or full D at the end. It's why the mean is about 50% cooperators. It depends of the first few defectors at the first round and the few rounds after.

Harmony: T= 0.5 , S= 0.5 (B)

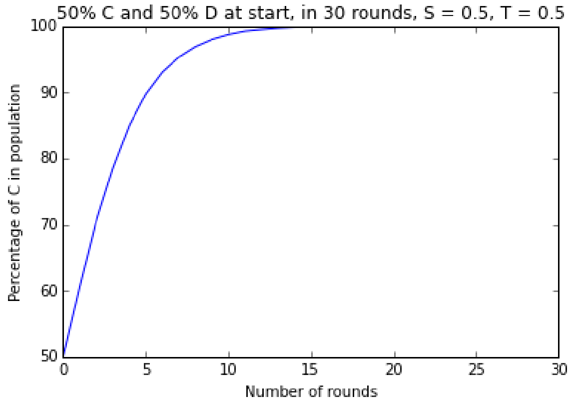
There is neither greed (T < R) and nor fear (P < S)

30% C and 70% D at start



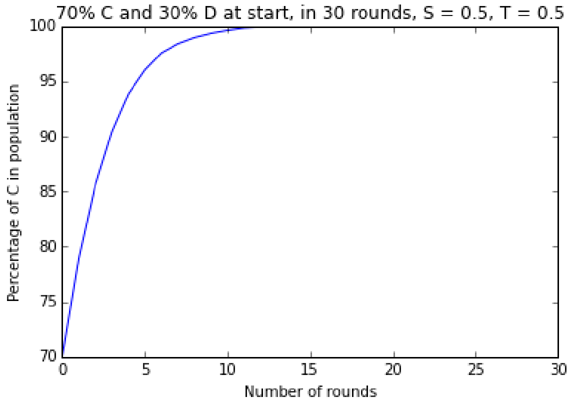
It tends to 100% cooperators and 0% defector after about 17 rounds.

50% C and 50% D at start



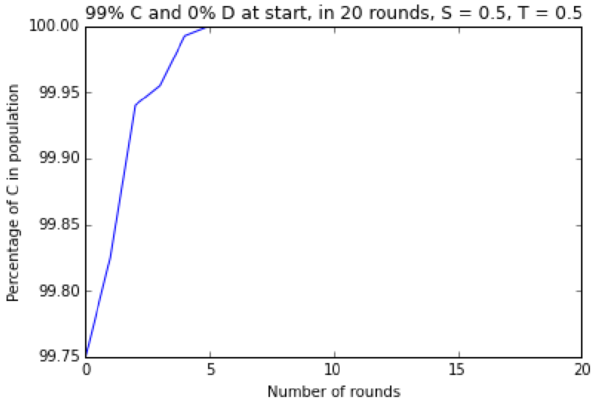
It tends to 100% cooperators and 0% defector after about 15 rounds.

70% C and 30% D at start



It tends to 100% cooperators and 0% defector after about 12 rounds.

One defector in the position {10,10} at start

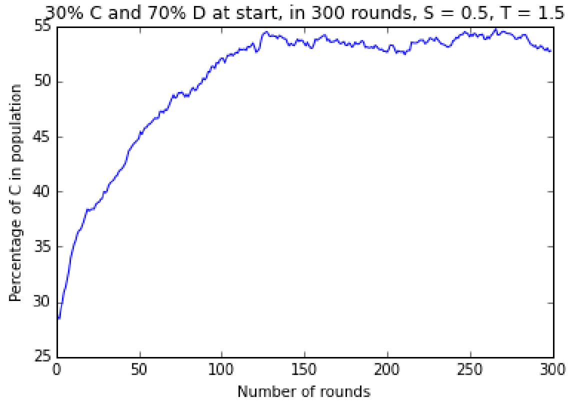


It tends to 100% cooperators and 0% defector after about 5 rounds. Some lattices take more time to convert the defector because of random changes.

Snowdrift: T= 1.5 , S= 0.5 (B)

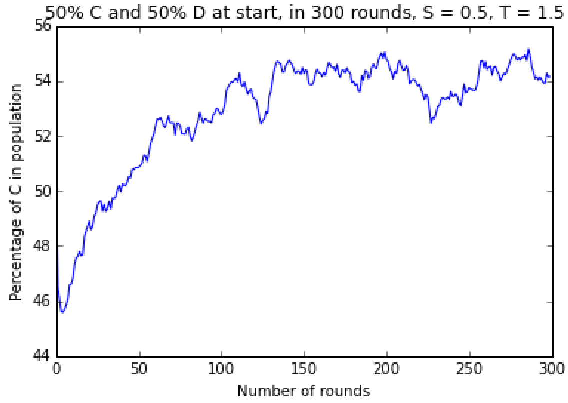
There is greed (T > R) and no fear (P < S)

30% C and 70% D at start



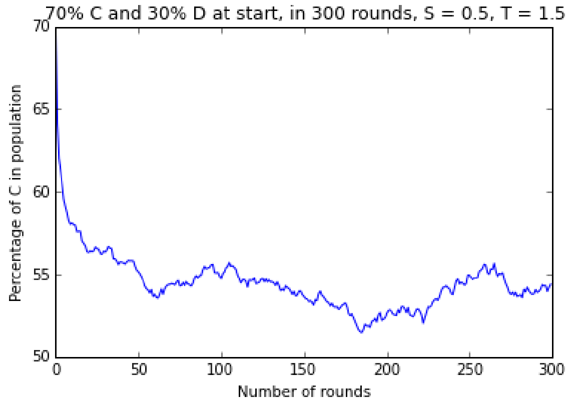
Every value of cooperators is near 50-55% of population.
It goes quickly to the 50% C and 50% D.

50% C and 50% D at start



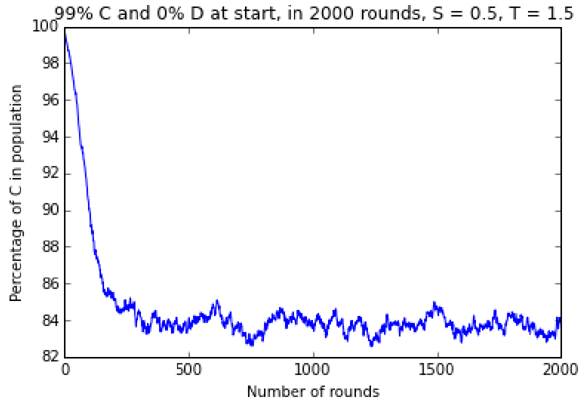
Every value of cooperators is near 50-55% of population.

70% C and 30% D at start



Every value of cooperators is near 50-55% of population.
It goes quickly to the 50% C and 50% D.

One defector in the position {10,10} at start



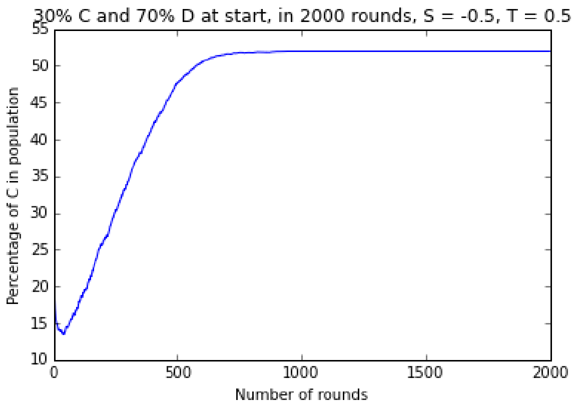
If we look at each turn we can notice that some lattices end with 100% cooperators and the other with around 50% cooperators.

If the few first defectors aren't converted in cooperators, the proportion of C will tend to 50%, just like in the other cases where we start with 30%, 50% or 70% of C.

Stag hunt: T= 0.5 , S= -0.5 (B)

There is no greed (T < R) and fear (P > S)

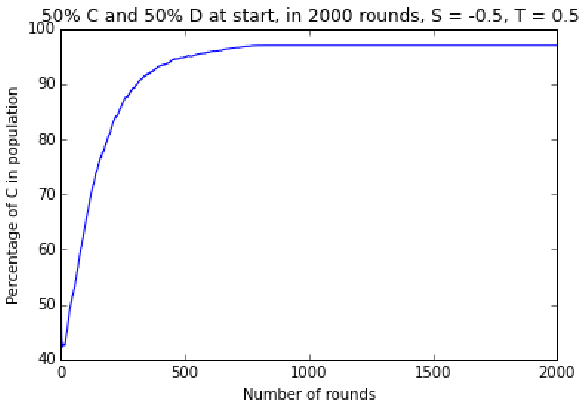
30% C and 70% D at start



0 or 100% cooperators at the end of the 2000 rounds. Around 50% fall of to 0 after around 50 rounds. If cooperators can group together, they will win.

See conclusion for more explanations.

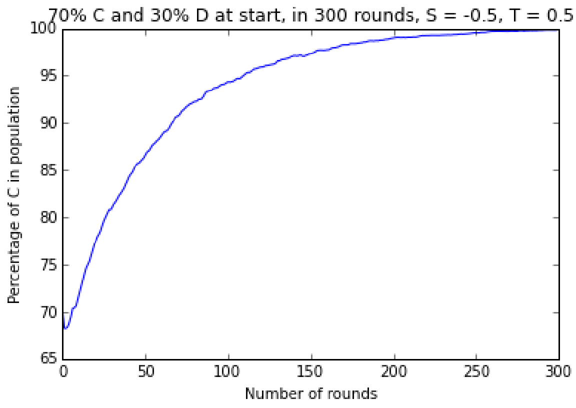
50% C and 50% D at start



0 or 100% cooperators at the end of the 2000 rounds. Around 5% fall of to 0 after around 50 rounds. If cooperators can group together they will win.

See conclusion for more explanations.

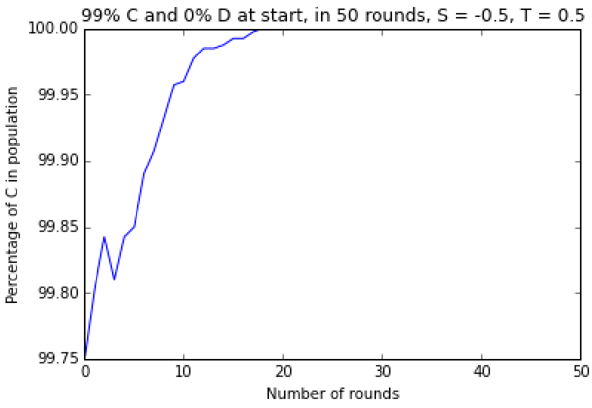
70% C and 30% D at start



There are more cooperators initially (70%) so they will easily group together. If they are grouped, they will convert the defectors and we will have 100% cooperators.

See conclusion for more explanations.

One defector in the position {10,10} at start



Cooperators are grouped at the start of the game so the only defector will be converted quickly.

See conclusion for more explanations.

Conclusion

Prisoner Dilemma

	C	**D**
C	R = 1	S = -0.5
D	T = 1.5	P = 0

Defectors will win because players will always have a better payoff if they defect rather than cooperate. There are greed (T > R) and fear (P > S) because the temptation to defect (1.5) is too high and the sucker's payoff (-0.5) is too low. Defect gives always the best payoff in a duel in this game. The defectors can invade since R < T. The temptation to defect is higher than the reward of cooperation.

With one defector we obtain around half lattices with 100% C and half with 0% C. It depends highly of the first defector to be converted. We can calculate the payoff of the 1st D:

Wi = 4 * 1.5 , Wj = 2.5 , max{0,1,T,S} = 1.5 , min{0,1,T,S} = - 0.5

Pij = (1 + (2.5-6)/(4*1.5 – 4 * (-0.5))) / 2 =(1- 7/16)/2 = 0.28 → Pij = 30%.

The first D has a probability of 0.30 to be converted and in this case we reach 100% of cooperators. The neighbor (C) has a probability of 0.70 to be converted in D and converge to 0% C. If some neighbors C are converted in D, there is still a probability of being "reconverted" in C. That's why, at the end, we reach approximately half lattices full of C and half lattices full of D.

- ➔ With greed and fear, D dominates C.

Harmony

	C	**D**
C	R = 1	S = 0.5
D	T = 0.5	P = 0

Cooperators will win because the best payoff is when both cooperate. There is neither greed (T < R) and nor fear (P < S). The temptation to defect is lower than the reward if both cooperate and the sucker's payoff is higher than the punishment. Cooperate gives always the best payoff in a duel in this game. The cooperators can invade since P < S.

So with only 1 defector he will quickly be converted in C and we will reach the 100% cooperators. Defectors can't invade because R > T, the reward of cooperation is higher than the temptation to defect.

- ➔ With neither greed nor fear, C dominates D.

Snowdrift

	C	D
C	R = 1	S = 0.5
D	T = 1.5	P = 0

It will tend to 50% of cooperators that look randomly distributed. There is greed (T > R) but no fear (P < S). C can invade because P < S but D can also invade because R < T. When your opponent cooperate it's better to defect and when opponent defects it's better to cooperate. So if the proportion of cooperators increases, players will tend to defect to earn more. And when there are too many defectors, players will tend to cooperate to increase their payoff. So neither C nor D can reach the 100% because if one increases, players will tend to choose the other to increase their payoff. It's why there is a "steady state" around the 50%.

If we have only 1 defector, the result depends highly of the first interaction. We can easily calculate his probability to change in C:

Wi = 4 * 1.5, Wj = 3 .5 , max{0,1,T,S} = 1.5, min{0,1,T,S} = 0

Pij = (1 + (3.5-6)/(4*1.5)) / 2 = (1- 5/12) /2 = 0.29 → Pij = 29%, so approximately 30%

The first D has 30% to be converted and in this case we reach 100% of cooperators. The neighbor (C) has 70% to be converted in D and converge to 50% C and 50% D.

So at this state, we should have 30 % of lattices full cooperators and 70% of lattices converging to 50% of cooperators. We should have a mean of 0.3 * 100% + 0.7 * 50% = 65%. Here we see that the mean after 2000 rounds is about 84%. It's because even if some cooperators has been converted in defector they still have a probability to be "reconverted" in C and reach the 100% cooperators. When more cooperators are converted in defectors, the probability to reach the 100% C is decreasing.

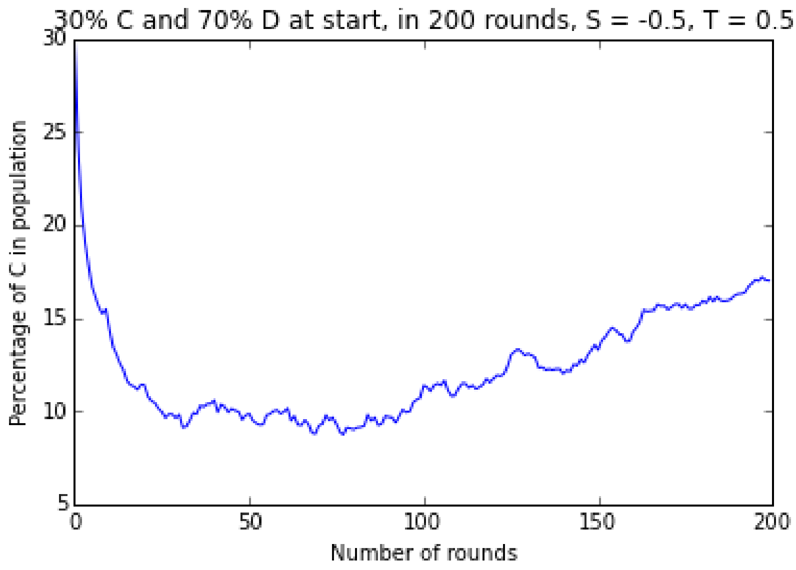
After 500 rounds (and after 2000 rounds) we have approximately a mean of 84% which represents: 69% of all lattices giving 100% C and 31% of all lattices giving 50% C and 50% D.

➔ With only greed, C and D coexist (half C and half D).

Stag hunt

	C	D
C	R = 1	S = -0.5
D	T = 0.5	P = 0

There is no greed (T < R) but there is fear (P > S) because the sucker's payoff is lower than the punishment. Neither cooperators (P > S) nor defectors (R > T) can invade.



Here is the graph of the 200 first rounds in the stag hunt game with initially 30% C. We can notice that there is a large decrease of cooperators at the beginning until the mean reach a point where cooperators begin to increase.

To explain this peculiar curve we have to look the lattice at each round (with 30% of C at the start for example). 1 = cooperator and 0 = defector.

1st lattice *(30% C)*

0	0	0	1	0	0	0	1	0	0	0	0	1	1	1	0	0	1	1	0
0	0	0	1	1	1	0	1	1	1	0	0	1	0	0	0	0	0	1	1
0	0	0	1	0	0	0	1	0	0	0	0	0	0	0	1	0	1	1	0
0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	1	1	0	0	1
0	1	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	1
0	1	1	1	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0
0	0	0	1	0	0	0	1	0	0	0	0	1	0	1	1	0	0	0	0
0	0	1	0	1	1	0	0	0	0	0	0	0	0	1	0	0	0	0	1
0	0	1	0	0	0	0	0	0	0	1	1	0	0	1	0	0	0	1	0
0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	1
0	0	1	0	0	0	0	0	0	0	1	1	0	0	0	0	0	0	0	1
0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1
0	0	0	1	0	1	1	0	1	0	0	0	0	0	0	0	0	0	0	0
0	0	0	1	1	1	1	1	1	1	1	0	0	0	0	0	0	0	1	0
0	0	0	1	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	1	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	1
0	0	0	0	1	1	0	0	0	1	0	0	0	0	0	0	0	0	0	1
1	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0
0	1	0	1	0	1	0	0	0	1	1	0	0	0	0	0	0	0	0	1

We can see here the first lattice where 30 % cooperators are randomly distributed.

<u>13th lattice</u> *(45/400 → 13.5% C)*

0	0	0	0	0	0	1	1	1	1	1	1	1	0	0	1	1	1	0	0
0	0	0	0	0	0	0	0	1	1	1	1	1	0	1	1	1	1	1	0
0	0	0	0	0	0	0	0	0	1	1	1	1	1	1	1	1	1	1	1
0	0	0	0	0	0	0	0	0	0	0	1	1	1	1	0	1	1	1	1
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0	0	0	0	0	0	0	0	0	0	0	0	1	1	1	1	0	0	0	0
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0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
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0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

Here I choose a lattice that end with 100% of cooperators. At the 13th lattice, we notice that the percentage of C is decreasing but they are grouping. There is still a small group of 3 cooperators.

<u>20th lattice</u> *(66/400 → 16.5% C)*

0	0	0	0	0	0	0	0	1	1	1	1	1	1	0	1	1	1	1	1
0	0	0	0	0	0	0	0	0	1	1	1	1	1	1	1	1	1	1	1
0	0	0	0	0	0	0	0	0	1	1	1	1	1	1	1	1	1	0	0
0	0	0	0	0	0	0	0	0	0	1	1	1	1	1	1	1	1	0	0
0	0	0	0	0	0	0	0	0	0	1	1	1	1	1	0	1	0	0	0
0	0	0	0	0	0	0	0	0	0	0	1	1	1	1	1	1	0	0	0
0	0	0	0	0	0	0	0	0	0	0	1	1	1	1	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	1	1	1	1	1	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	1	1	1	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	1	1	1	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

After 20 rounds we can see that the isolated cooperators have disappeared. Only the big group of cooperators stays and grows up around. Then the cooperators will convert slowly the defectors around the group of C and they will reach the 100% C.

If the cooperators can't group together, the defectors will convert all the cooperators around and then we will reach the 0% of cooperators.

Considering a defector surrounded by 2 C and 2 D (in blue) facing a cooperator (in red). How many cooperators does this C need as neighbors to have a better payoff? X represents the number of C needed.

/	C	?	/
D	D	C	?
/	D	?	/

W(defector) = 2 * 0.5 + 2 * 0 = 1

W(cooperator) = 1 * X + (4-X) * (-0.5)

Wd = Wc when X = 2. So if the cooperator is surrounded by 2 cooperators, the defector has a probability of 0.5 to be converted if he is surrounded by 2 C and 2 D. If the cooperator is surrounded by 3 other C, he will increase his payoff and so the defector has more chance to be converted. If the cooperator is surrounded by 3 C, probability that D is converted in C increases:

Wd = 1

Wc = 3 - 0.5 = 2.5

Pij = (1 + (2.5 – 1)/ 6)/ 2 = 15/24 = 62.5 %

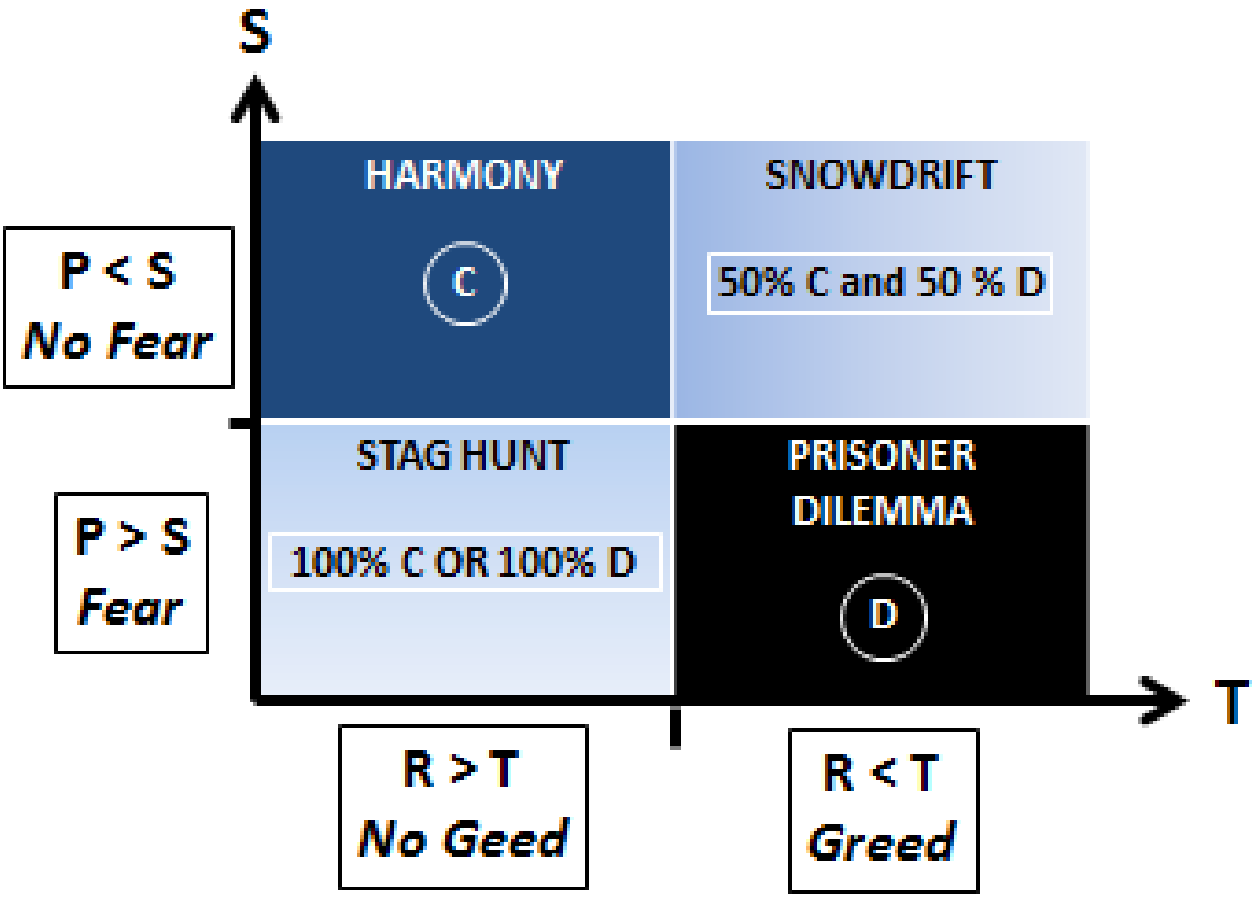
So when the cooperators are grouped, the defectors facing them have better probability to be converted.

So at the end of the 2000 rounds, we have lattices with 100% C when cooperators can group and we have lattices with 0% C we they can't group. The more C we have initially, the more easily they group together and so reach the 100% cooperators. With 70% C initially there is a little decrease at the start where defectors convert some isolated C. But all lattices reach the 100% C because the cooperators are easily grouping.

With only 1 defector, this one will be easily converted in cooperator because all C are grouped at the start.

➔ With only fear, D and C are “bistable” (it's converging to full C or full D)

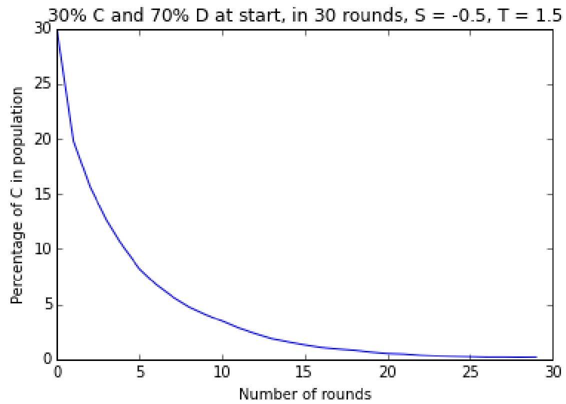
Summary of the 4 games



C) Lattices without boundaries:

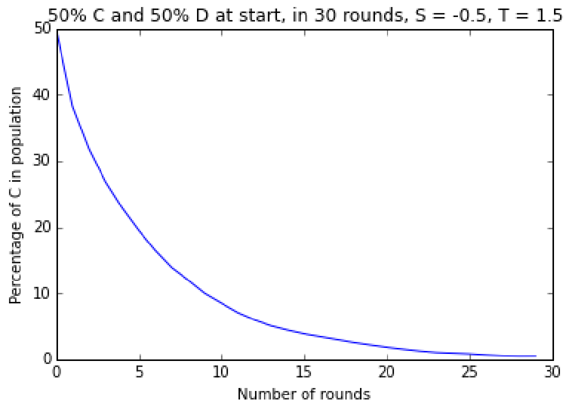
Prisoner Dilemma: T= 1.5 , S= -0.5 (No B)

30% C and 70% D at start



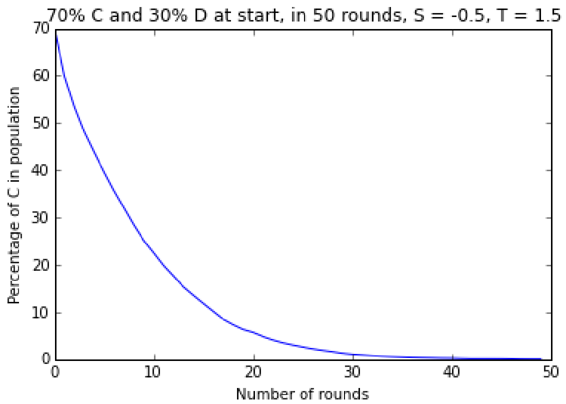
No change when there are no boundaries.

50% C and 50% D at start



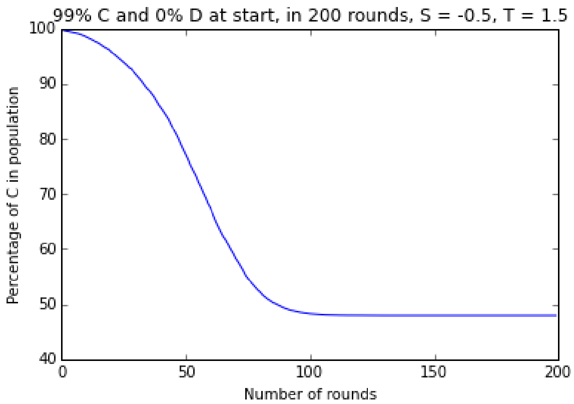
No change when there are no boundaries.

70% C and 30% D at start



No change when there are no boundaries.

One defector in the position {10,10} at start



Little change when there are no boundaries

There is no real change between with or without boundaries with 30%, 50% and 70% of cooperators at the start.

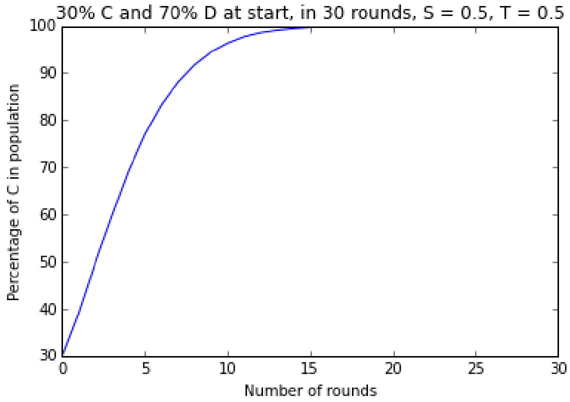
For the lattice with only one defector, it seems to need a little bit less round to reach the mean 50% (50% full C and 50% full D). Maybe the defectors can easily convert the cooperators without boundaries to stop them. The difference is small.

In annexes we can find the other games without boundaries. We can't notice any change.

Annexes:

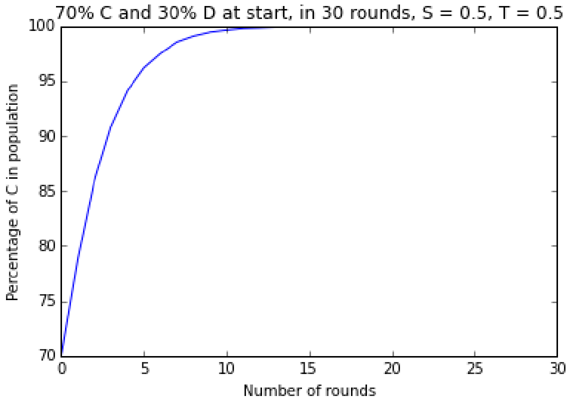
Harmony: T= 0.5 , S= 0.5 (No B)

30% C and 70% D at start



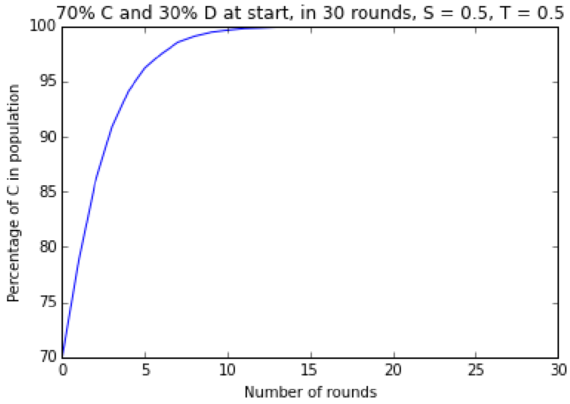
No change when there are no boundaries.

50% C and 50% D at start



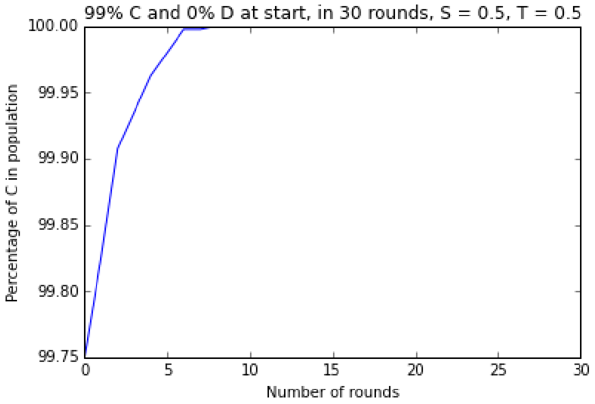
No change when there are no boundaries.

70% C and 30% D at start



No change when there are no boundaries.

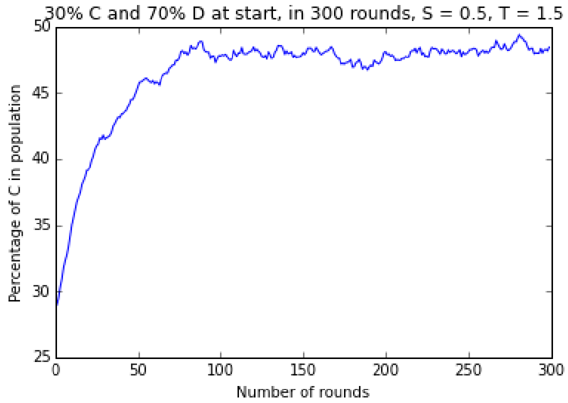
One defector in the position {10,10} at start



No change when there are no boundaries.

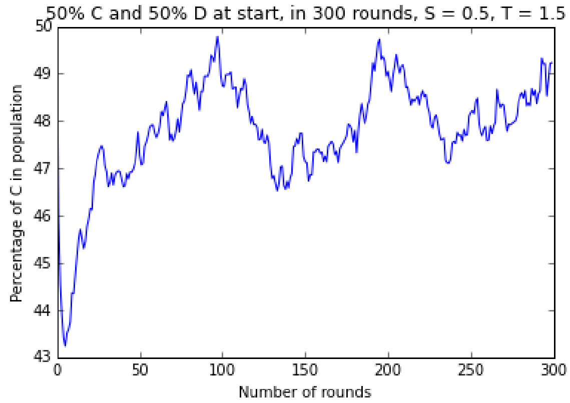
Snowdrift: T= 1.5 , S= 0.5 (No B)

30% C and 70% D at start



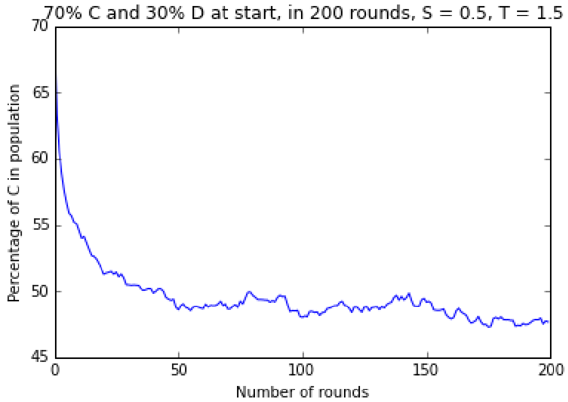
No change when there are no boundaries.

50% C and 50% D at start



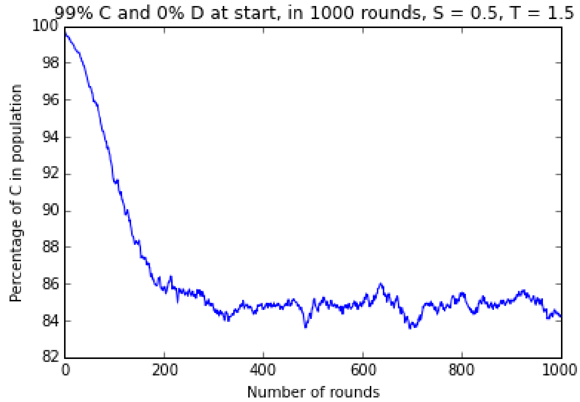
No change when there are no boundaries.

70% C and 30% D at start



No change when there are no boundaries.

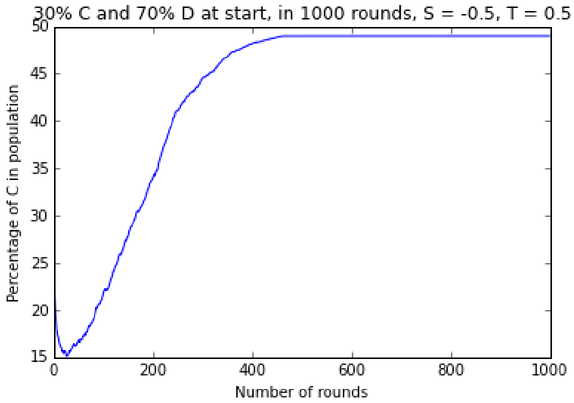
One defector in the position {10,10} at start



No change when there are no boundaries.

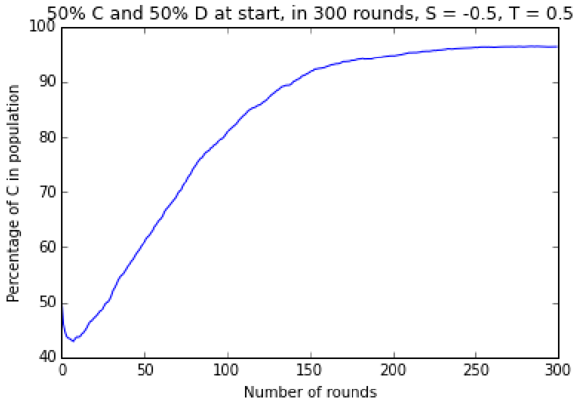
Stag hunt: T= 0.5 , S= -0.5 (No B)

30% C and 70% D at start



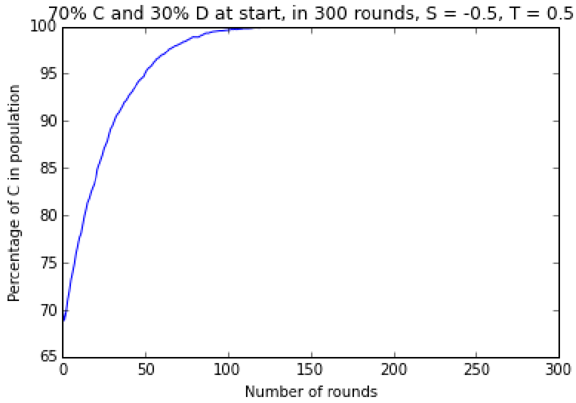
No change when there are no boundaries.

50% C and 50% D at start



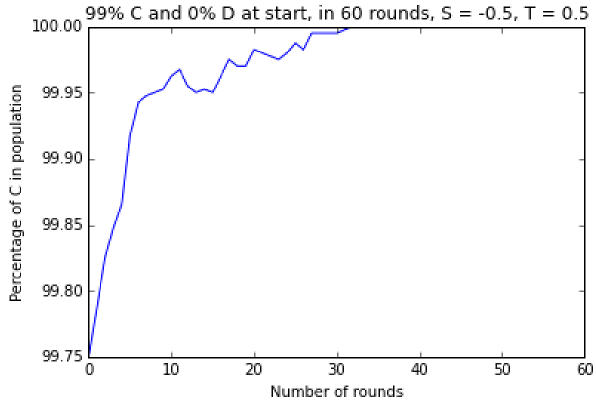
No change when there are no boundaries.

70% C and 30% D at start



No change when there are no boundaries.

One defector in the position {10,10} at start



No change when there are no boundaries.