Olga Ibáñez Solé
MA in Bioinformatics and Modeling (ULB)

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2nd assignment: Evolutionary dynamics in a spatial context

Part I:

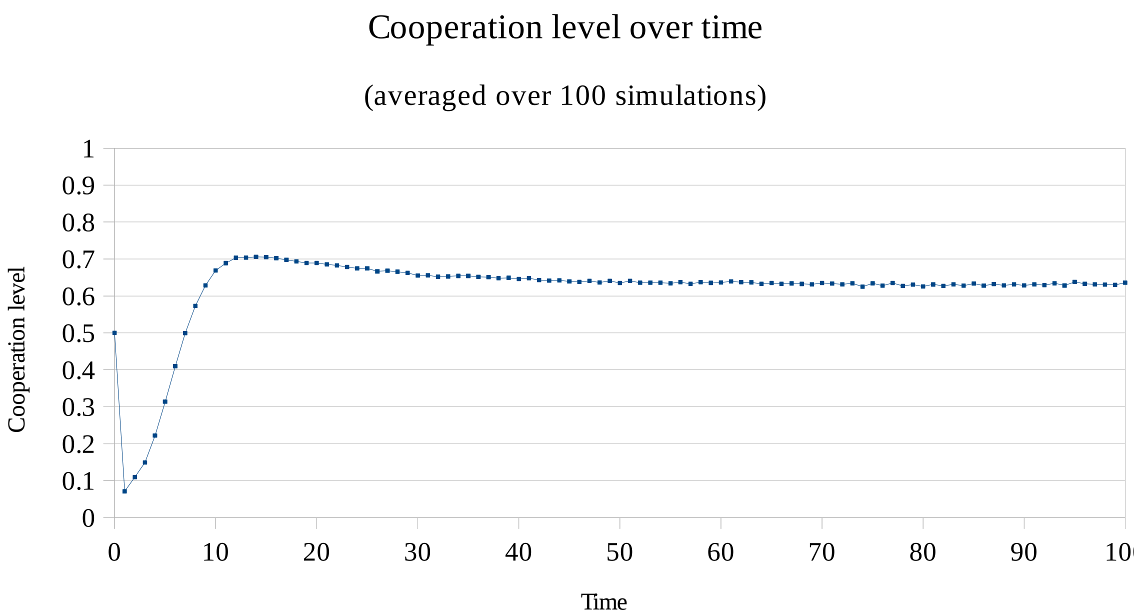
We wrote a program in Python that builds a lattice with the following features:

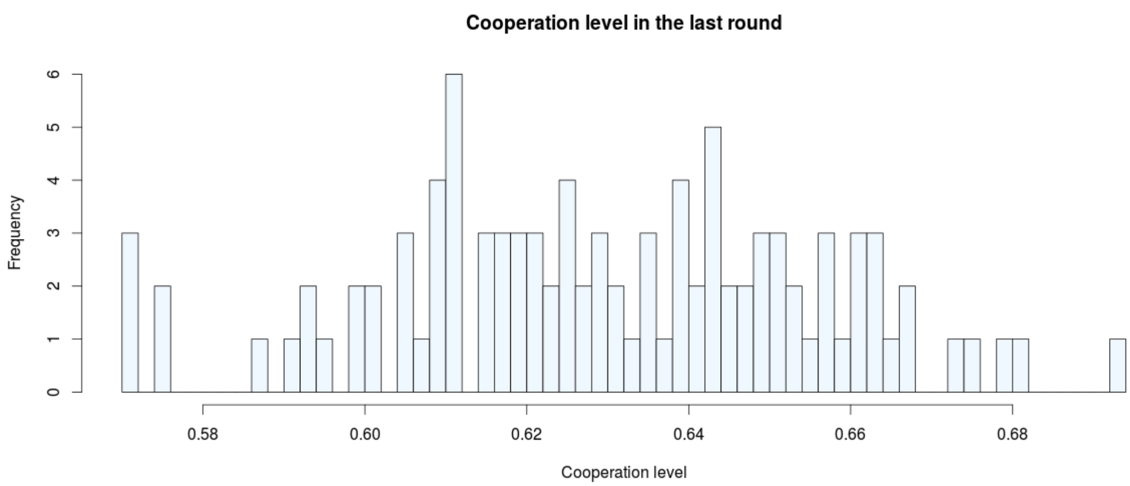
- The lattice is sized NxN.
- Each player can have either 8 (Moore's) or 4 neighbors (Von Neumann's neighborhood).
- It has periodic boundary conditions, that is, the above neighbors of the top row are the players of the bottom row, and the left neighbors of the first column are the players at the right edge column.
- A score matrix is built from the lattice, which contains the total score of the player at each position of the lattice, calculated as the sum of all the payoffs she would obtain when playing against all of her neighbors (but not against herself).
- Each agent plays a weak Prisoner's Dilemma game, in which T=10, R=7 and P=S=0. Each agent plays the same action (either Cooperate or Defect) against all of her neighbors in each round.
- The lattice is initialized by assigning one action (C or D) to each agent randomly, with 50% of probability.
- The lattice is updated in each round. Each agent looks for her best neighbor, that is to say, the one that obtained a highest score (sum of payoffs) in the previous round, and mimicks her action (unconditional imitation) in the next round.

Cooperation level over time

We performed 100 simulations in which 50x50 agents played 100 rounds and we kept track of the cooperation level (the proportion of agents in the lattice that were playing Cooperate) in each round. Below is shown the average cooperation level per round over 100 rounds, averaged over 100 simulations. We can notice that after 100 simulations, the steady state is reached and the cooperation level is stabilized.

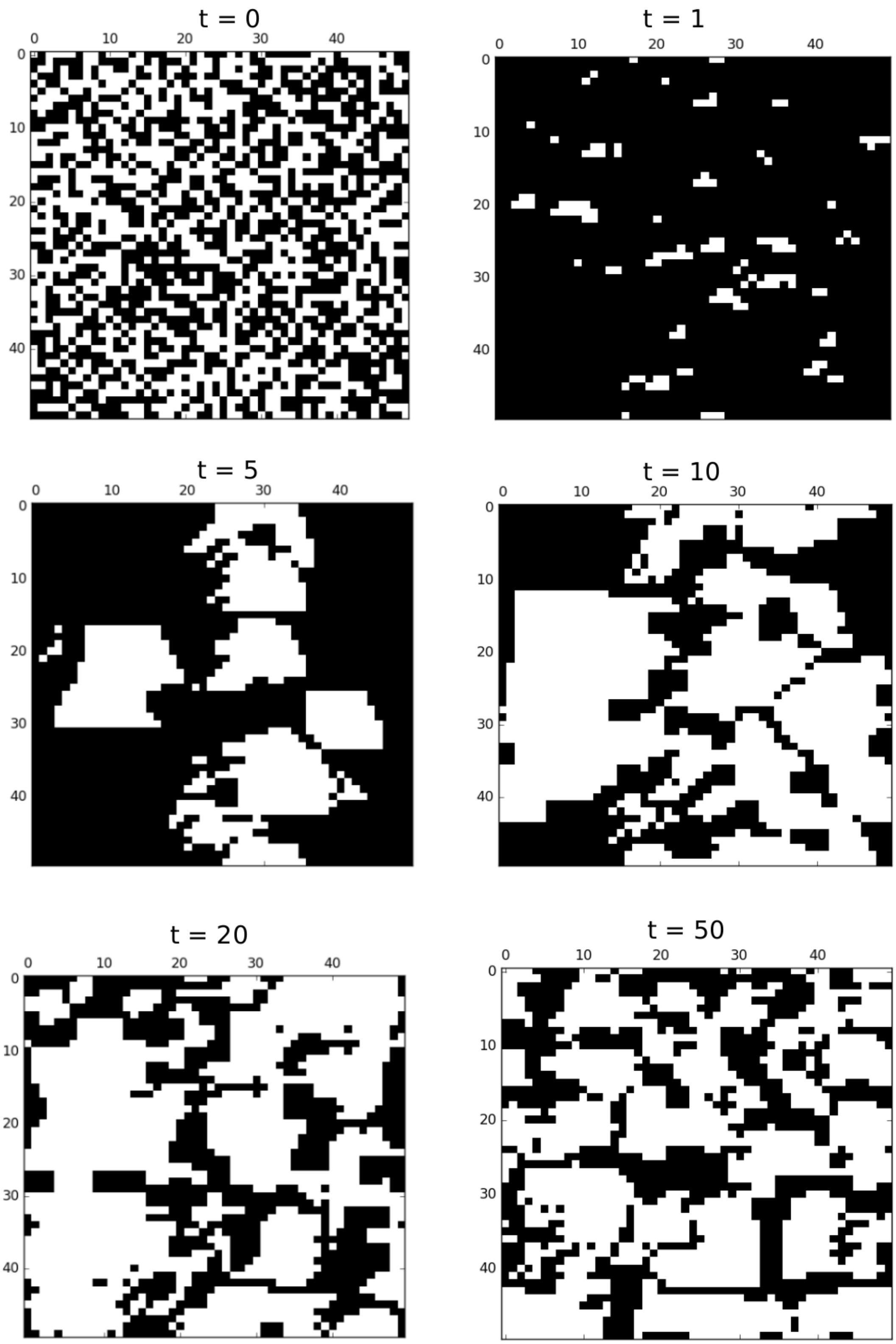
We plot a histogram of the final cooperation level values, shown under the cooperation time evolution plot. We can observe that the final cooperation level stays between quite a narrow range of values, the minimum being 0.57 and the maximum 0.6928. Thus, we can say that the cooperation level converges over time.





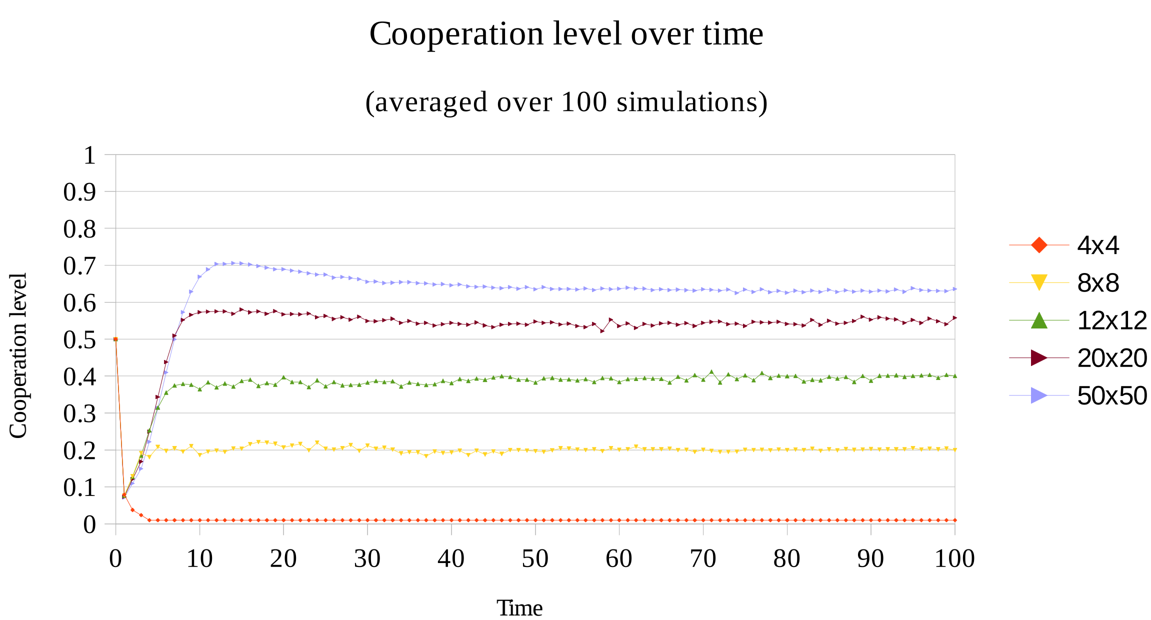
Visualization of the lattices

We visualized a run in which the cooperation level was 0.5 at the beginning. Below is shown the full matrix of cooperation for the rounds t=0, t=1, t=5, t=10, t=20 and t=50. Cooperating agents are shown in white, while defecting ones are shown in black. We can observe that half of the players were cooperating at the beginning of the simulation, as set in the initialization of the lattice. The consequence of this random assignment of actions was that most players had at least one very succesful neighbor that was defecting, so they decided to defect in the next round. The reason why this happens is that defecting is a good strategy only if you are surrounded by cooperators. This is also why after t=1, players that are located in the boundary between a group of cooperators and a group of defectors will choose to play cooperate in the next round. After some iterations, the group of cooperators will be growing and by t=50 the cooperators take the majority of the surface in the lattice.



Effect of the lattice size on the evolution of cooperation

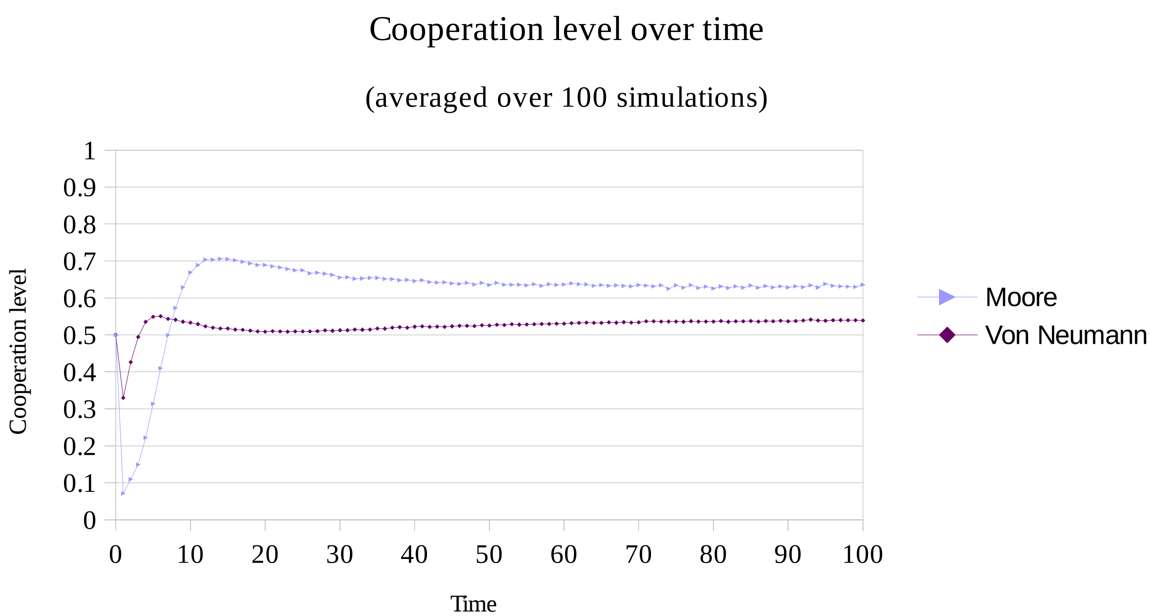
We performed four additional sets of 100 simulations using the lattice sizes 4x4, 8x8, 12x12 and 20x20 and added the results to the plot shown above. In the plot shown below we can see the evolution of the cooperation level over 100 rounds (averaged over 100 runs) for each lattice size. We can notice that 4x4 lattices never retain even a small level of cooperation, and stabilize as a pure defector lattice. However, 8X8 lattices retain a small percentage of cooperation (around 20%) that persists over time. The cooperation level in which the lattices reach the steady state increases with lattice size, and it seems that the time in which the steady state is reached is also dependent on lattice size. For example, 12x12 lattices seem to reach their steady state a little bit earlier than 50x50 matrices.



Effect of the type of neighborhood on the evolution of the cooperation

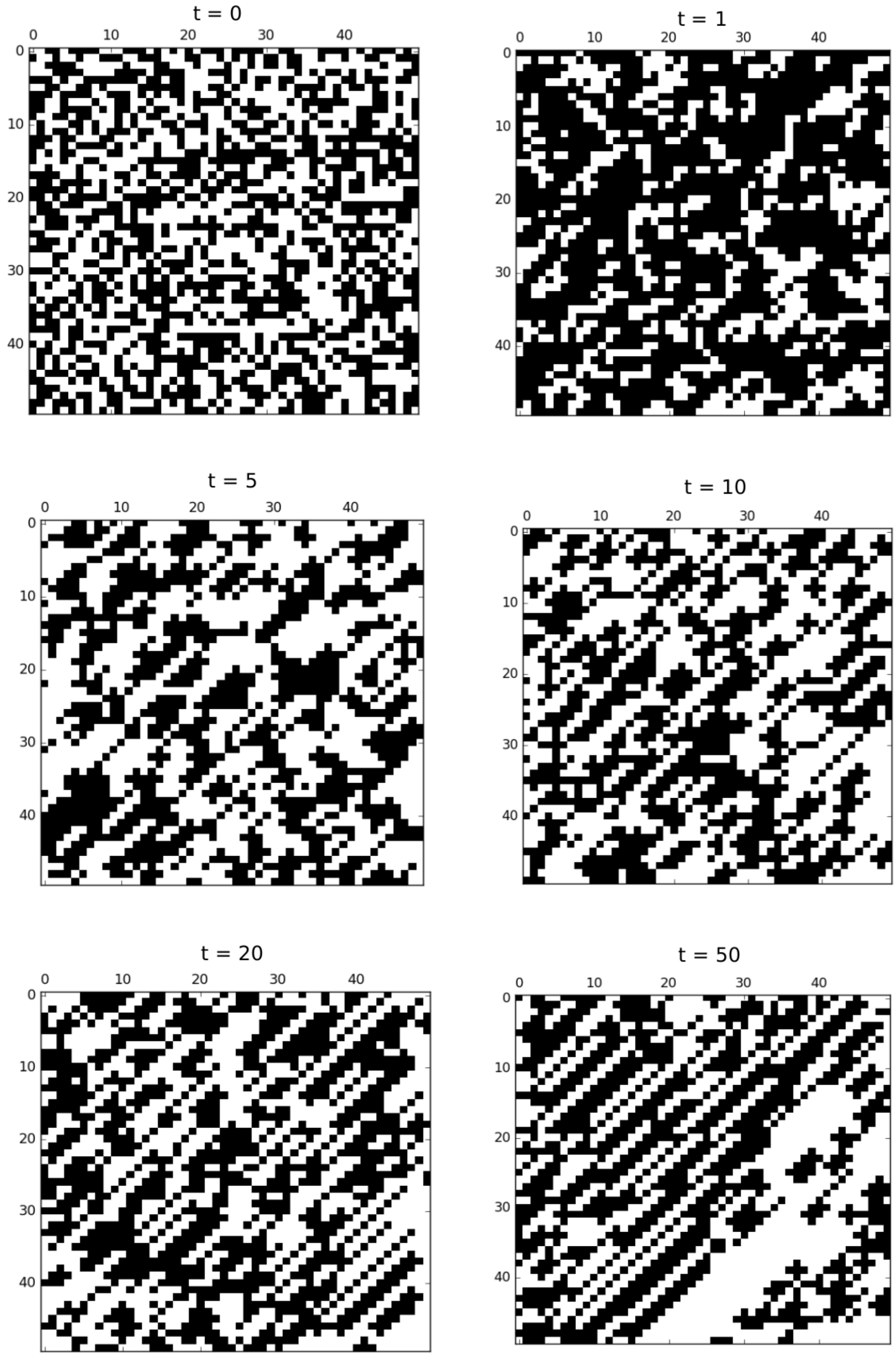
We changed the neighborhood type and considered Von Neumanns neighborhood, according to which every agent has only four neighbors. Below we compare the results for a 50x50 lattice using both types of neighborhoods, regarding their cooperation level over time. We can notice that the steady state is reached at different cooperation levels: the average cooperation level in the last round using Moore's neighborhood was 0.63594, while it was 0.53869 using Von Neumann's neighborhood. Therefore, we conclude that Moore's neighborhood allows for a greater cooperation level to persist than Von Neumann's neighborhood. The shape of the curves is also different depending on the neighborhood type: the former falls to very low levels of cooperation immediately after the first round, and then reaches much higher values and is kept at higher levels than the latter during the whole simulation. This makes sense since the probability that any given agent has a very succesful defector neighbor is much higher in Moore's neighborhood (8 neighbors) than in Von Neumann's neighborhood (4). Thus, for equal values of initial cooperation, many more agents will choose to play defect if they have to choose their best neighbor among eight players rather than just four. After the first round, the cooperation level increases at a higher speed in Moore's neighborhood because, again, if there is a small group of cooperators that have survived the first round, many more agents will "detect" them and imitate them than if we considered Von Neumann's neighborhood, and therefore the population of cooperators will increase more rapidly over time.

We could sum up this by stating that, thanks to Moore's neighborhood, the individual agents are more sensitive to their environment and change their actions in a more rapid way than with Von Neumann's neighborhood.



Visualization of the lattices

Below are shown the lattices (t=0, t=1, t=5, t=10, t=20 and t=50) corresponding to a simulation in which the Von Neumann neighborhood was considered. The lattice size was kept at 50x50 and the initial level of cooperation was 0.5. We can notice that the level of cooperation decreases dramatically in the first round, yet not as dramatically as in Moore's neighborhood simulation. We can also notice that some interesting geometrical patterns such as diagonal lines appear in the lattice, probably due to the fact that the groups of agents playing the same action are constrained in fewer directions than in Moore's neighborhood. In Moore's neighborhood, cooperators were affected from eight different directions and thus cooperator groups take rounder forms. Conversely, in Von Neumann's neighborhood cooperators are affected only from four directions and cooperator groups can more easily grow in diagonal, leading to thin diagonal groups. Although it might seem that there is a great difference between the matrix in t = 20 and t = 50, the matrix remains quite constant (in terms of cooperation level) from t=35 on (not shown).



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Part II

We changed the game to the Snowdrift game (T=10, R=7, S=3, P=0) and updated the mechanism by which the agents choose their action to the replicator rule. That is to say, every agent chooses her action by randomly picking a neighbor and then imitating her action with a probability p, defined as:

$$P_{ij} = (1 + [W_j - W_i]/[n^*(\max\{P,R,T,S\}-\min\{P,R,T,S\})]) / 2$$

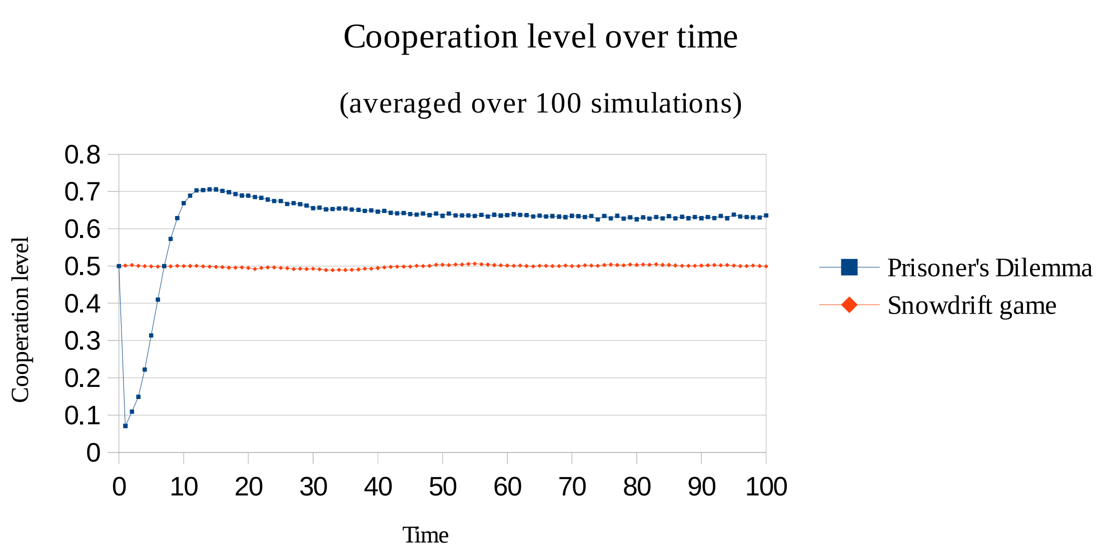
Where W_j and Wi are sum of payoffs of players i and j against all of their neighbors in the previous round, and n is the number of neighbors considered (eight if we take Moore's neighborhood, four if we take Von Neumann's).

This way we can ensure that, the better the strategy (the higher the score that the randomly picked neighbor obtained in the previous round), the greater the chances that her strategy will be imitated.

We performed a similar analysis as in Part I, and obtained the following results.

<u>Cooperation level over time</u>

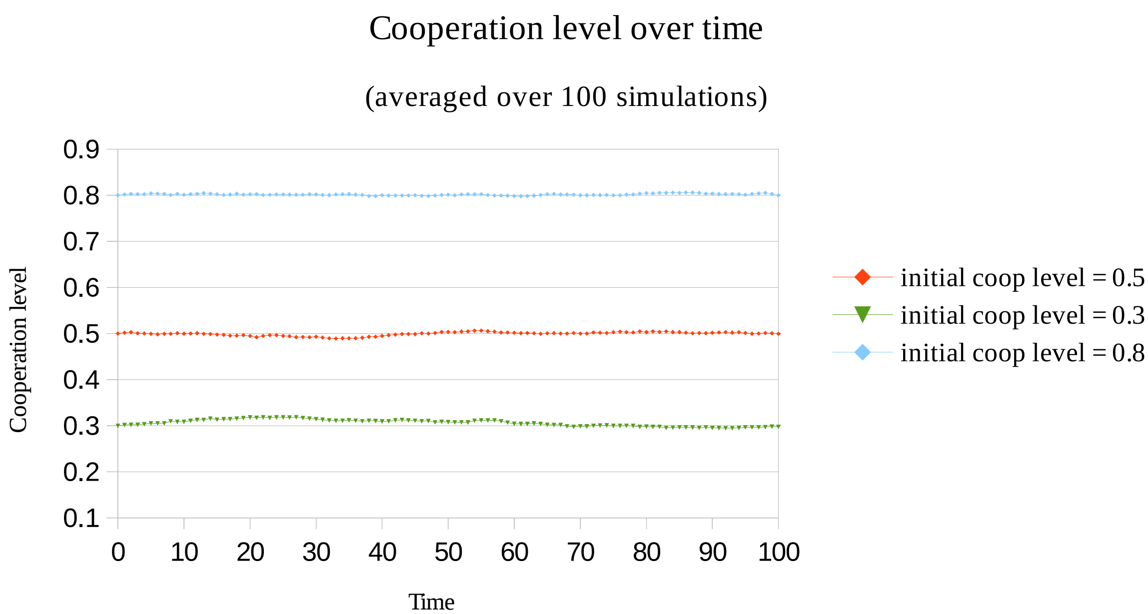
We run a set of 100 simulations using a 50x50 lattice and Moore's neighborhood and calculated the average cooperation level per round. Below is shown the plot corresponding to the averaged results of this set of simulations, together with the results we obtained in Part 1 for the prisonner's dilemma. Note that everything is kept constant except for the game (snowdrift game vs prisonner's dilemma) and the mechanism by which every agent chooses her next action (replicator rule vs unconditional imitation).



We can notice that the cooperation level remains almost constant, it lightly fluctuates around the initial cooperation value.

Effect of the initial cooperation level on the evolution of cooperation

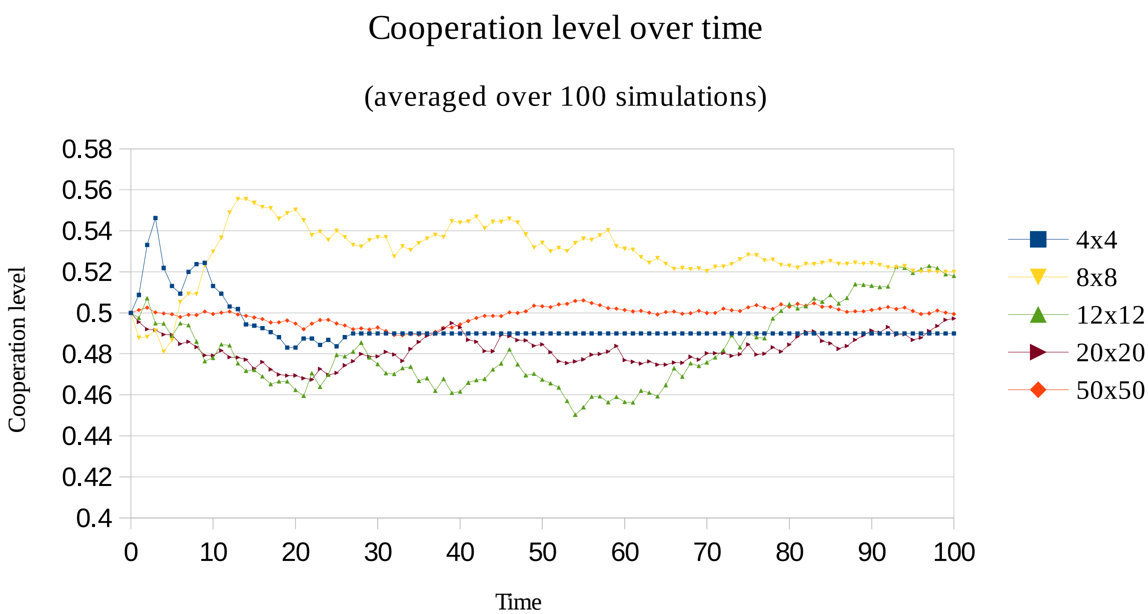
We performed three simulations using different initial cooperation values: 0.3, 0.5 and 0.8. The lattice size was 50x50 and we used Moore's neighborhood in all of them. Below is shown the time evolution plot of the cooperation level.



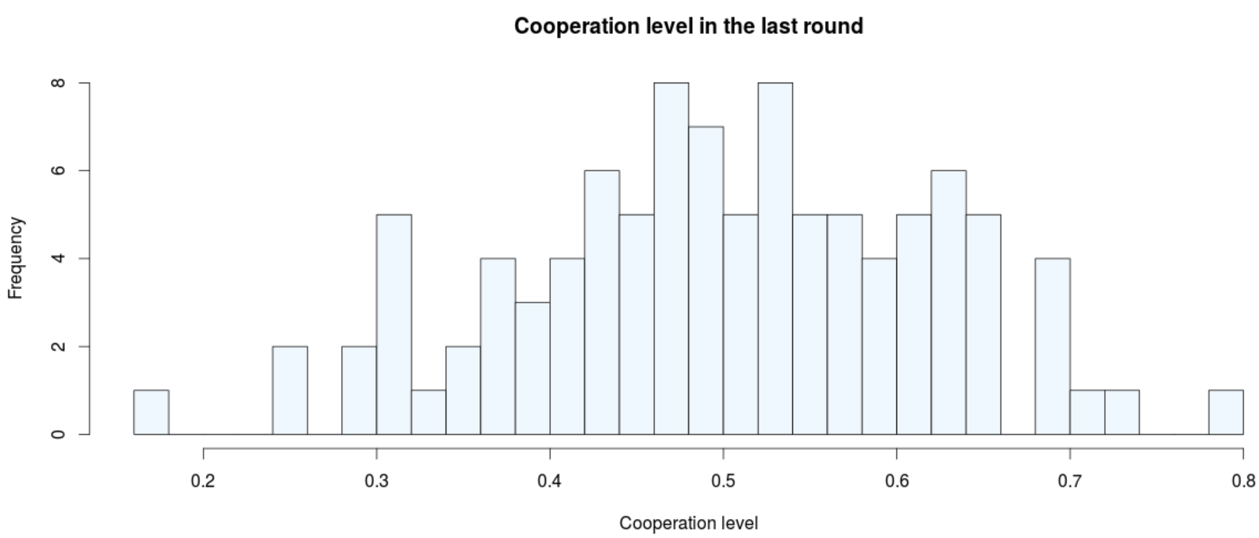
We can observe that the final cooperation level is mainly determined by the initial cooperation level and remains almost constant, with very little variations, over time.

Effect of lattice size on the evolution of cooperation

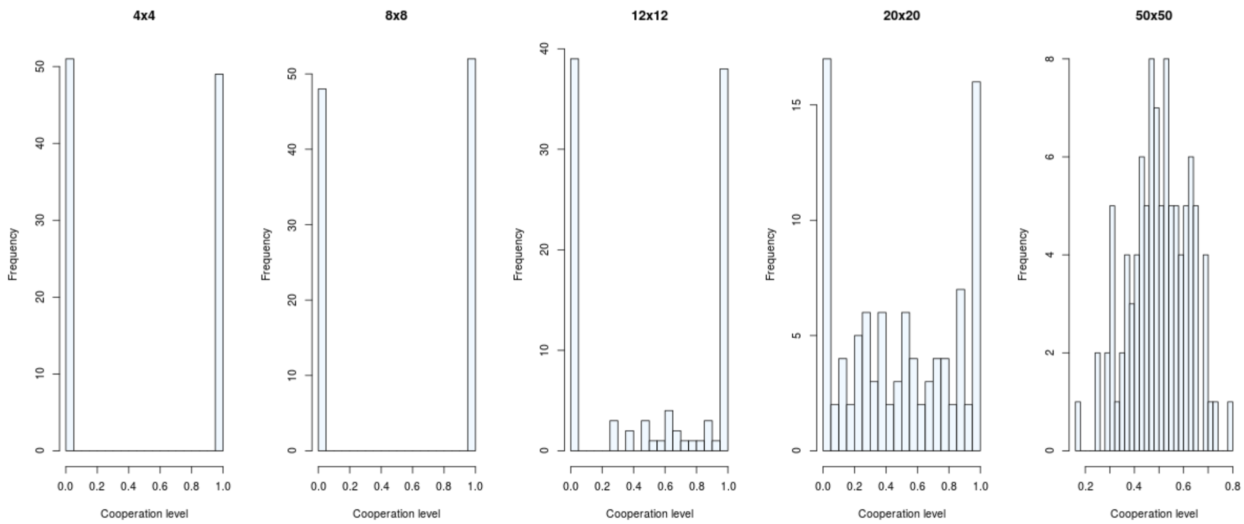
We performed five sets of 100 simulations of 100 rounds each using different lattice sized and computed the cooperation level over time. Below are shown the averaged cooperation levels per round associated to each lattice size (note that the scale is different in each plot).



We can see that the average cooperation level remains constant over time. However, if we look at the distribution of the cooperation levels in the last round (using a 50x50 lattice) shown in the histogram below, we can see that they range between a wide range of values, that almost go from 0 to 1.



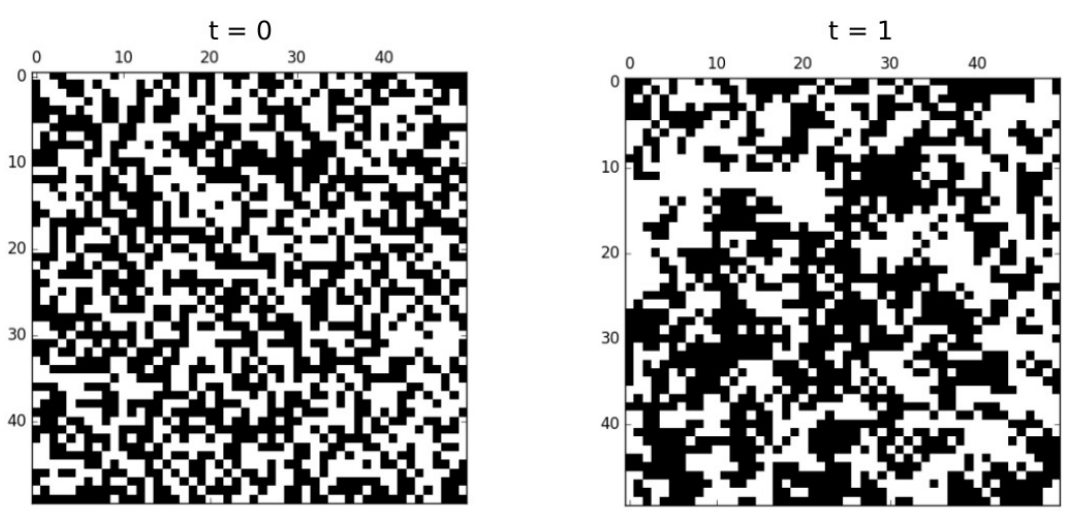
In fact, the minimum value for the cooperation level in the last round among the 100 simulations was 0.1684 and the maximum 0.7824. If we look at the histograms below, which show the distribution of the cooperation level in the last round for different lattice sizes, we can realise that the time evolution plot was a little bit misleading, because it averaged very distant values and gave similar results for the five tested lattice sizes even though their behavior was extremely different.

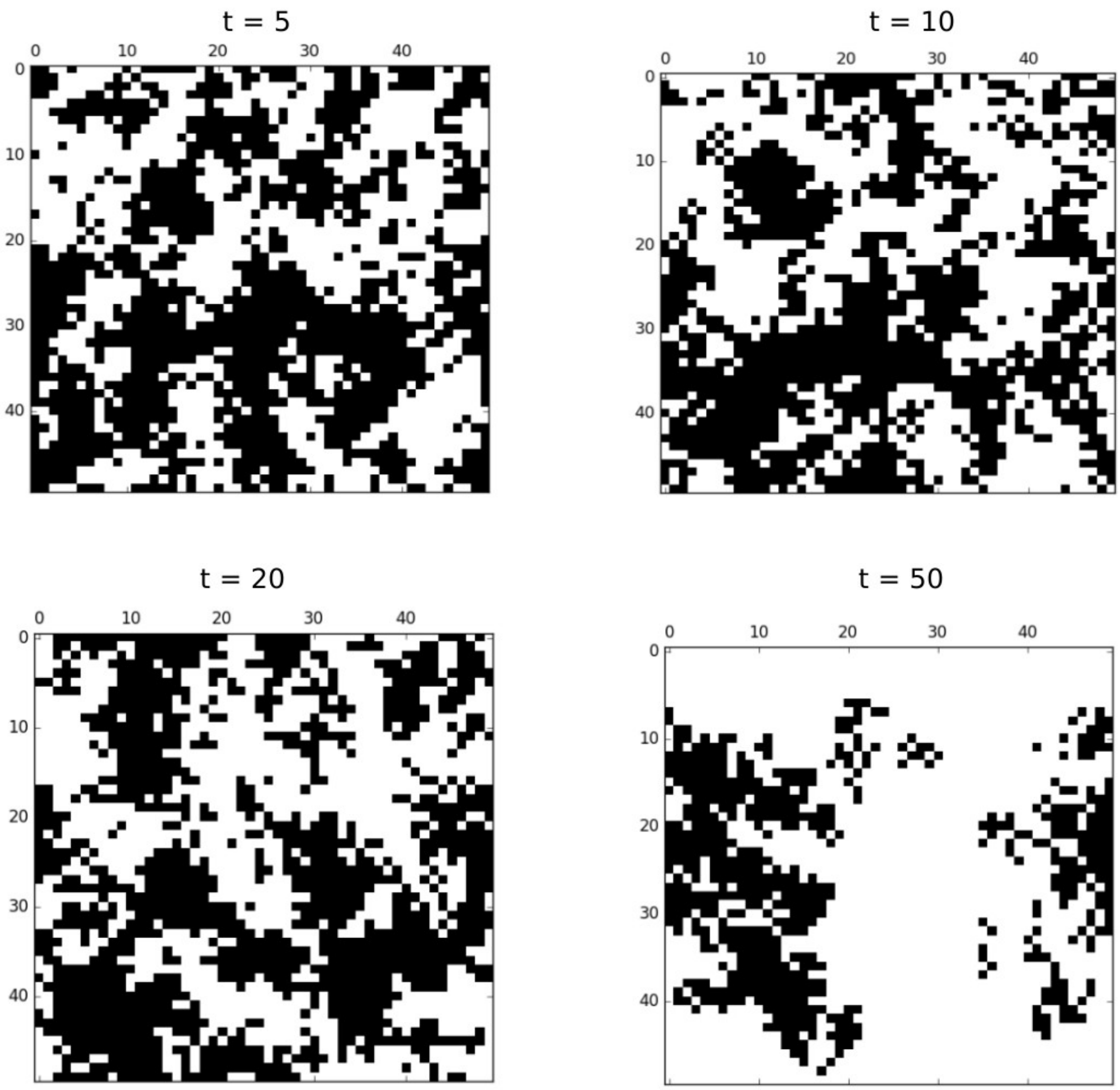


We can appreciate that for very small lattices (4x4 and 8x8), the final cooperation levels are aither 0 or 1, depending on the run. As the lattice size increases, intermediate levels of cooperation appear. This makes sense because for very small lattices, there is an all-or-none response: either the whole lattice becomes cooperative or defective.

<u>Visualization of the lattices</u>

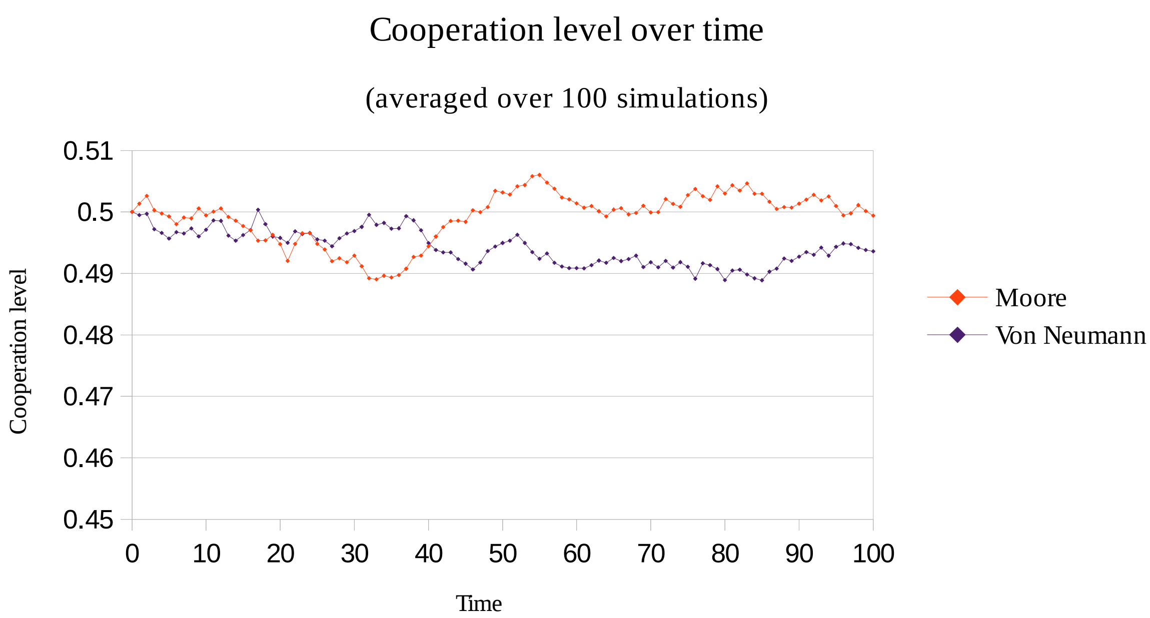
Below are shown the lattices at t=0, t=1, t=5, t=10, t=20 and t=50 from a simulation using 50x50 sized lattices, Moore's neighborhood and 50% initial cooperation. We can see that the geometrical patterns differ from those oberved in the Prisonner's Dilemma game.





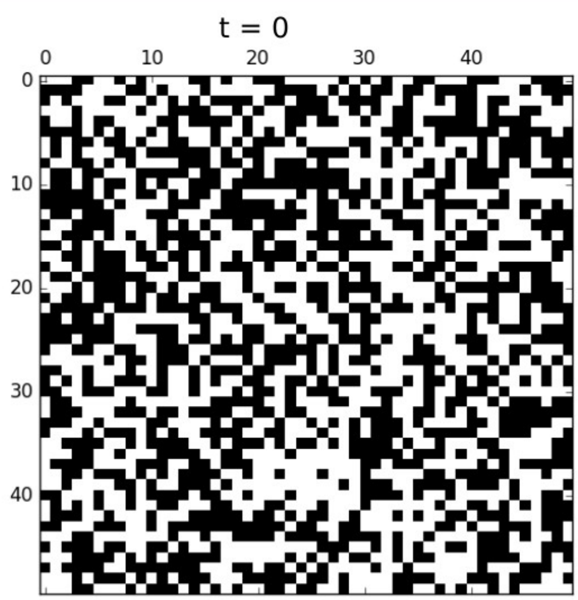
<u>Effect of the neighborhood type on the evolution of cooperation</u>

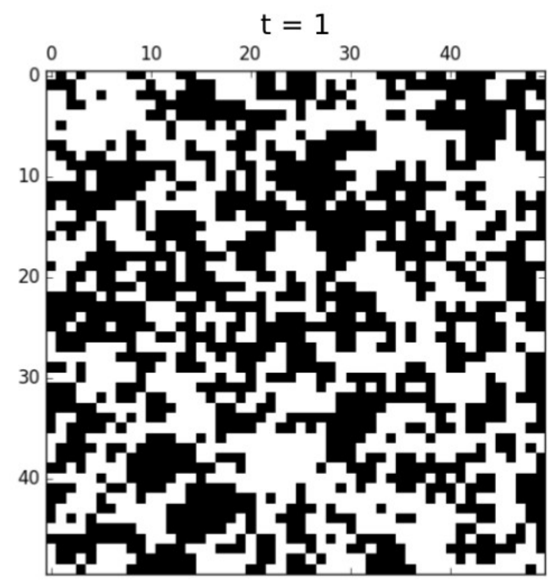
Two sets of 100 simulations using 50x50 lattices and only changing the neighborhood type gave very similar results. Notice that evolution of the cooperation level is more similar between Moore’s and Von Neumann’s neighborhood simulations than between 50x50 and 20x20 lattices. That is, cooperation seems to be more affected by lattice size than neighborhood type when it comes to the snowdrift game (using the replicator rule). Just like in Part I, it appears that Moore’s neighborhood can retain a slightly higher percentage of cooperants and somehow promotes cooperation, although it is not clear in this case that Moore’s neighborhood leads to cooperation levels that converge to values that are always higher than those of Von Neumann neighborhood simulations.

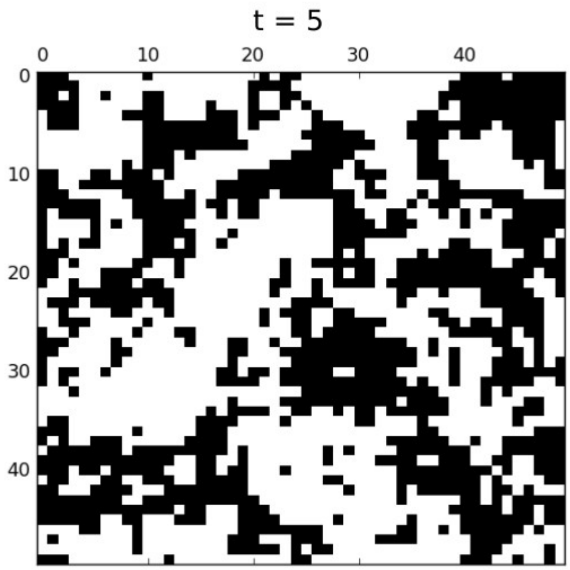


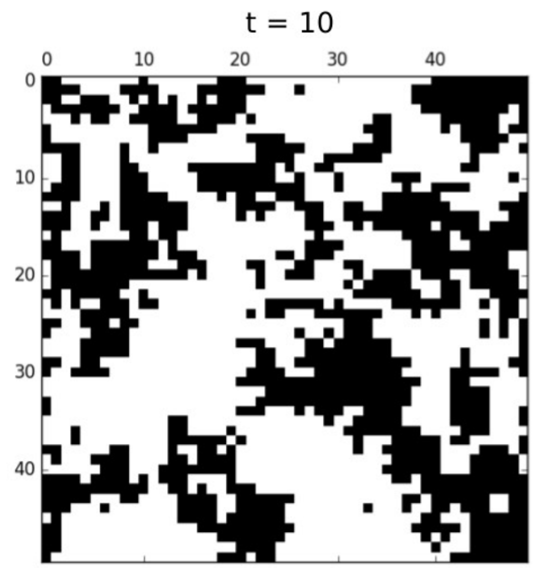
Visualization of the lattices

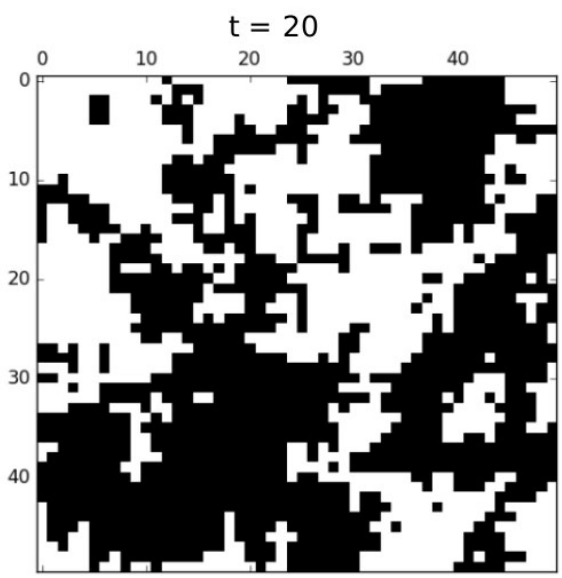
Below we can observe the lattices at t=0, t=1, t=5, t=10, t=20 from a simulation with lattice size 50x50, Von Neumann neighborhood and initial cooperation level of 0.5.

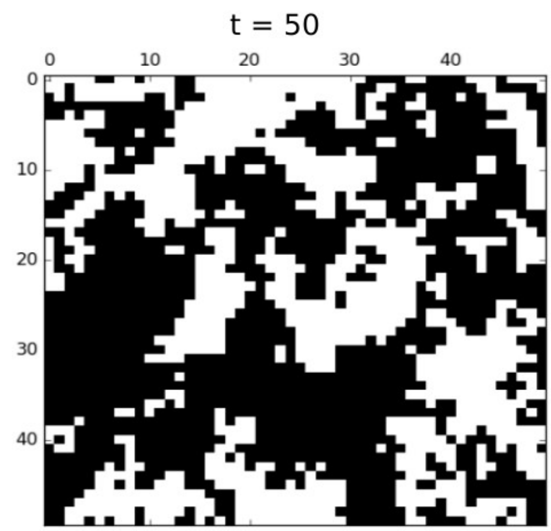












We observe some differences between the patterns observed in the Snowdrift game using Moore's neighborhood and Von Neumann's neighborhood, but we feel uncertain about the interpretation of such differences. Therefore, we cannot draw any certain conclusions on the dynamics of the system that might cause them.