Assignment 2: Evolutionary dynamics in a spatial context

General remarks:

_ Deadline 5-11-2015
_ Mail your results to Luis Martínez <l.martinez.vaquero@gmail.com>.
_ Provide a single (self-contained) *.PDF file.
_ Put your name and your affiliation (VUB/ULB) both on the document and in the file name.

Problem description

In this new assignment you will examine the evolutionary dynamics of cooperation and competition in a spatial lattice. Assume a population of 400 individuals that are distributed on a two-dimensional square lattice (20x20). Each individual can choose between two actions -- cooperation (C) or defection (D).

Given this populated lattice, carry out the following simulation:

- First ($t=0$), set up the initial conditions (initial actions for the individuals); see below

- In each round t (equivalent to a generation in this scenario):

  1st – Each individual interacts with its 4 closest neighbors (the one on her left, right, up and down) playing her action and accumulating the payoff W_k corresponding to the sum of the payoffs of the four interactions (according to a proposed game, see below).

  2nd – Each individual i chooses randomly one of her closest neighbors j and with a probability P_{ij} she changes her strategy (if both are different), where P_{ij} is defined as:

  $$P_{ij} = (1 + [W_j\text{-}W_i]/[4*(max\{0,1,T,S\}\text{-}min\{0,1,T,S\})]) / 2$$

  where T and S is defined by the game.

- This is repeated until the proportion of cooperators converges (or $t=2000$ rounds if the process does not converge).

The whole process is repeated for 100 realizations. And the results, i.e. the fraction of cooperators in the population for each round, are averaged over that number of realizations.

The payoff matrix of the games the individuals play is:

	C	D
C	1	S
D	T	0

a) Explain why P_{ij} was defined as it was proposed

b) Assume that the lattice has boundaries (for example, the individual in the position {1,1} only has two closest neighbors: {1,2} and {2,1}).

Plot the average fraction of cooperation in the population $f_c(t)$ for the different initial conditions and the games showed below (16 different functions in total). Comment the results.

Initial conditions:

- $f_c(0) = 0.3$, randomly distributed
- $f_c(0) = 0.5$, randomly distributed
- $f_c(0) = 0.7$, randomly distributed
- Only one defector in the position {10,10} of the lattice

Games:

- Prisoner Dilemma: $T=1.5$, $S=-0.5$
- Harmony: $T=0.5$, $S=0.5$
- Snowdrift: $T=1.5$, $S=0.5$
- Stug hunt: $T=0.5$, $S=-0.5$

c) Does anything change if there are no boundaries in the lattice (an individual in the border of the lattice can interact with the individual in the opposite border, and then every individual has 4 closest neighbours; for example, individual {1,1} has {1,2}, {2,1}, {1,20}, {20,1} as closest neighbours)? Repeat (b) for the Prisoner Dilemma game to check it.

*This assignment is related to the article published by Martin Nowak and Robert May in Nature; Nowak, M. & May, R. M. Evolutionary games and spatial chaos. Nature **359,** 826–829 (2002).*