Learning Dynamics (INFO-F409)
Assignment 2: Evolutionary dynamics in a spatial context

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1 Part I : Prisoner's Dilemma

For the first part of the simulations, we play a weak Prisoner's dilemma game, which means that the payoff for the punishment is not higher than the sucker's payoff. We used the following parameters for the simulations : $R = 7, S = 0, T = 10, P = 0$.

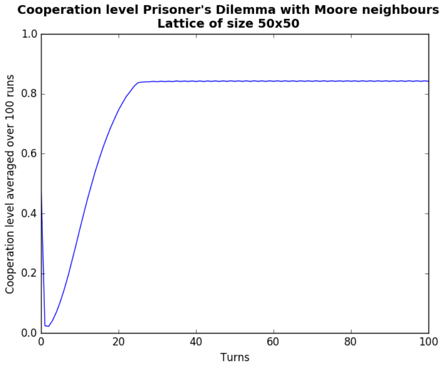
At each turn, all the players play with their assigned players then update their strategy, either cooperating or defecting, by following the unconditional imitation rule. This rule edicts that the player will follow the strategy of the player with the highest payoff among his neighbours, himself included.

We observed the evolution of the strategies used by the players over time by visualizing the results at time step $t = 0, 1, 5, 10, 20, 50$. We also observed the cooperation level over time averaged over 100 simulations and the distribution of the final cooperation level of these simulations.

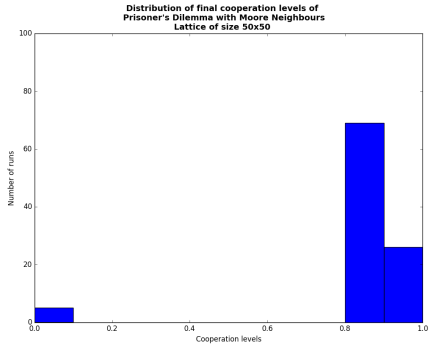
1.1 Moore neighbours

The Moore neighbours correspond to the eight adjacent tiles around the player.

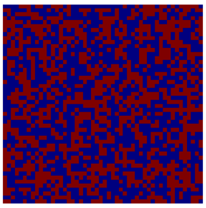
We first simulated with a lattice of size $50x50$. The cooperation level over time revealed that the first few turns the cooperation level drastically dropped then gradually increased until it stabilized around 90%. This result was confirmed by the visualization of a run with the cooperators in blue and the defectors in red. We could see the defectors almost taking over after one turn, but then over the following turns, clusters of cooperators would grow until only fine trails of defectors are left between the clusters (Fig 1).



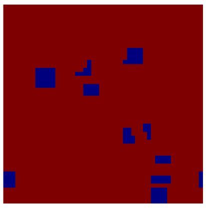
(a) Cooperation level over time



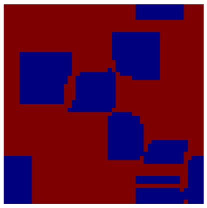
(b) Distribution of final cooperation levels



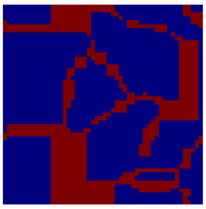
(c) $t = 0$



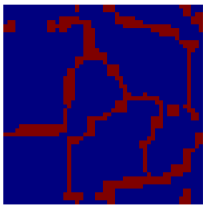
(d) $t = 1$



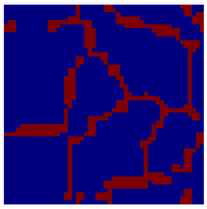
(e) $t = 5$



(f) $t = 10$



(g) $t = 20$



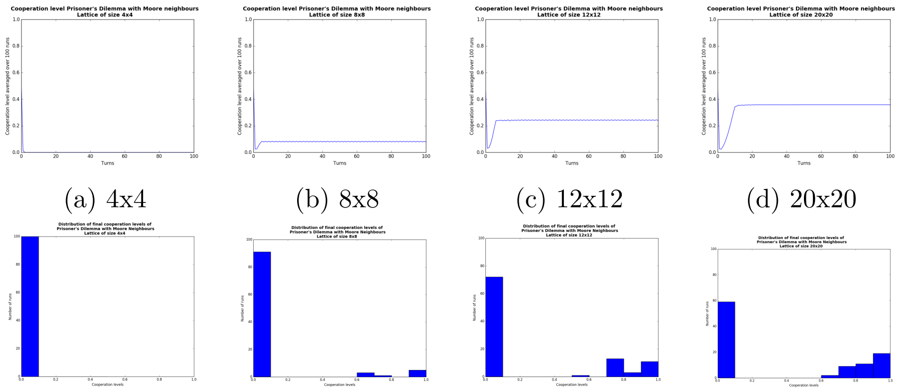
(h) $t = 50$

Figure 1: Prisoner's Dilemma with a lattice size 50x50

This phenomena can be witnessed only because defectors together earn less than cooperators together (P ¡ R). So at each turn, all defectors next to a cooperator will change to cooperator, and this is why the clusters of cooperators grow. The only defectors left are exploiting the fact that they are surrounded by cooperators and few defectors earning them enough to stay defectors but not to change their strategy to cooperators.

Then, we ran simulations with different lattice sizes.

When the network is too small, everybody will change to defectors as this is the strategy with the highest payoff as the repartition is 50% of each strategy, and they all stays defectors as they follow the unconditional imitation and they have no cooperators left to imitate. However, as the size of the lattice grows, the opportunity to have a cluster of cooperator left after the first turn also increases, which is translated in the distribution as the runs where you obtain a high final cooperation level as we can observe the same phenomena as with the lattice of 50x50. Thusly, as the size increase and the chance to have a cluster of cooperators left increase, so does the averaged level of cooperation over time (Fig 2).



(a) 4x4 (b) 8x8 (c) 12x12 (d) 20x20

(e) 4x4 (f) 8x8 (g) 12x12 (h) 20x20

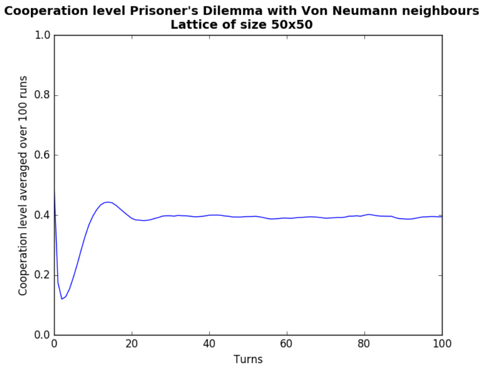
Figure 2: Prisoner's Dilemma with different lattice sizes. From a to d are the graphs of the cooperation level over time and from e to h are the graphs of the distribution of the final cooperations level.

1.2 Von Neumann neighbours

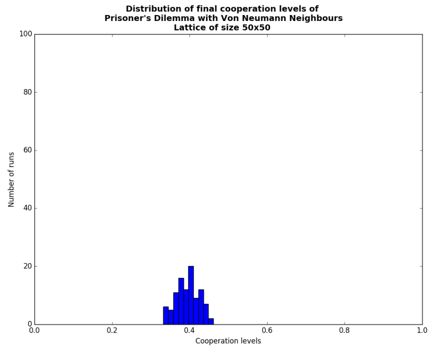
The Von Neumann neighbours correspond to the tiles left, right, above and beneath the player.

When we simulated the prisoner's dilemma with the Von Neumann neighbours, we did not observe the same cooperation level as with the Moore neighbours. Even though we observed the same phenomena where the level of cooperation drastically dropped after the first turn and then gradually increased, we did not reach the 90% of final cooperation level as in the Moore neighbours, but only 40%. The distribution of the final cooperation level are closely centered around 40% (Fig 3).

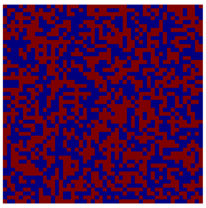
This difference can be observed because the defectors are not are not as penalized as previously with the Moore neighbours. This means that even though the clusters are able to grow, the two strategies reach an equilibria where the defectors are more predominant. Another explanation would be also the fact that when you play with Moore neighbours, two players may have up to four common neighbours, whereas with Von Neumann neighbours you could have no common neighbours even though you are in the surrounding of the other player.



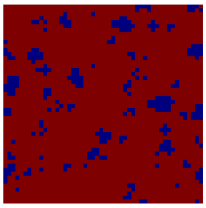
(a) Cooperation level over time



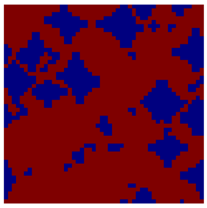
(b) Distribution of final cooperation levels



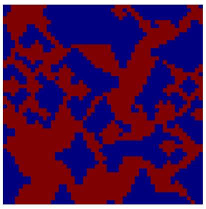
(c) $t = 0$



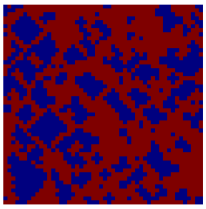
(d) $t = 1$



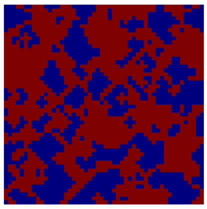
(e) $t = 5$



(f) $t = 10$



(g) $t = 20$



(h) $t = 50$

Figure 3: Prisoner's Dilemma with Von Neumann neighbours with a lattice size 50x50

2 PART II : Snowdrift game

The snowdrift game is different from the Prisoner's dilemma in the sense that the Surcker's payoff is larger than the punishment. This means that the game should promote the cooperation, as nobody wins when both player are defectors. We simulated with the following parameters : $R = 7, S = 3, T = 10, P = 0$.

The update rule for this simulation is the replicator rule, where the player i chooses randomly one of its neighbour j and adopts its strategy with a probability P_{ij} according to the formula :

$$P_{ij} = \frac{\left(1 + \dfrac{W_j - W_i}{N * (max\{P, R, S, T\} - min\{P, R, S, T\})}\right)}{2}$$

Where W_n is the payoff of the player n and N is the number of neighbours of the player.

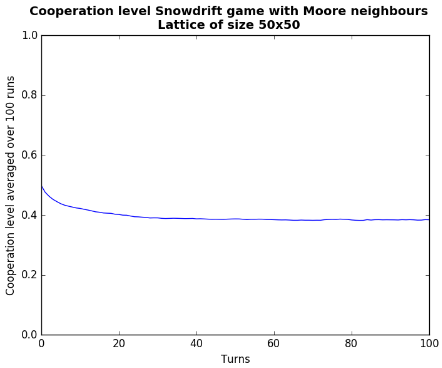
We want a probability that will reflect the fact that if the other player earns more, the current player will have a higher probability of choosing the strategy of the other player, and on the other hand if the other player has a smaller payoff, the probability should also be small.

The difference between W_i and W_j allows to mark the difference between the current and the other player. The division by $N * max\{P, R, S, T\} - N * min\{P, R, S, T\}$ will divide according to the worst case scenario, which is that one of the player has the best payoff possible and the other one the worst one, and have the result of this division situated between 1 and -1, where 1 means that the other player has the best payoff possible and the current player the worst, and -1 the opposite situation. The addition of 1 and the division of the sum by 2 will result in a probability comprised between 0 and 1.

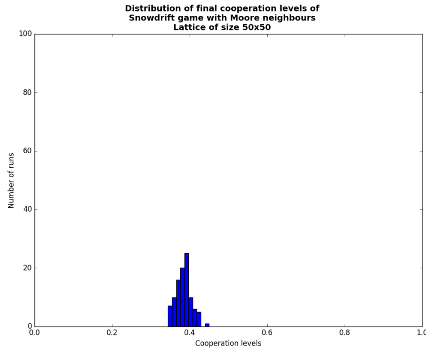
We repeated the same analysis that we did in the part I.

2.1 Moore neighbours

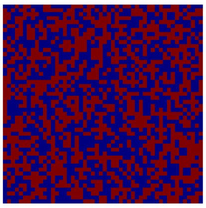
Surprisingly, although the game seems to promote cooperation, we did not observe a distinctive promotion of cooperation. We only observe a mean level of cooperation around 40%, with a distribution of the final cooperation level also closely centered around 40% (Fig 4). This difference in the result compared to the previous part with the prisoner's dilemma is due to the update rule. The replicator rule enters a stochastic component whereas the unconditional imitation rule is deterministic. Thusly, cooperation is not promoted.



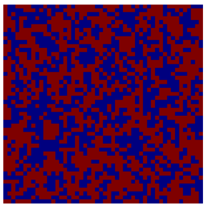
(a) Cooperation level over time



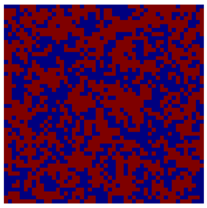
(b) Distribution of final cooperation levels



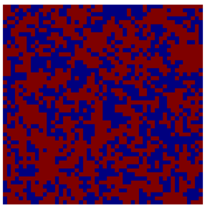
(c) $t = 0$



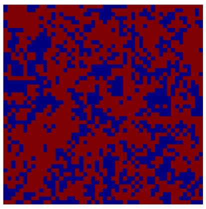
(d) $t = 1$



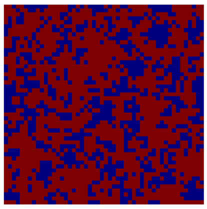
(e) $t = 5$



(f) $t = 10$



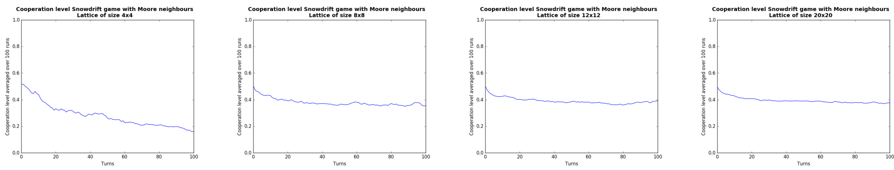
(g) $t = 20$



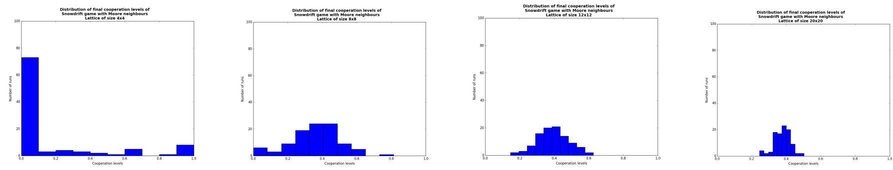
(h) $t = 50$

Figure 4: Snowdrift with Moore neighbours with a lattice size 50x50

The different sizes of the lattice revealed that the main difference is the dispersion around the final cooperation level of 40%. The bigger the lattice the more centered around 40% is the distribution of the final cooperation levels. This means that despite the stochastic element, the equilibria seems to be situated around 40% of cooperation level.



(a) 4x4 (b) 8x8 (c) 12x12 (d) 20x20

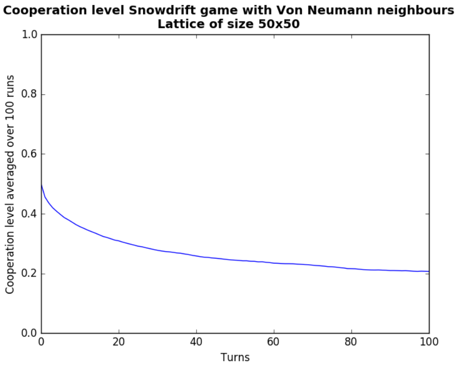


(e) 4x4 (f) 8x8 (g) 12x12 (h) 20x20

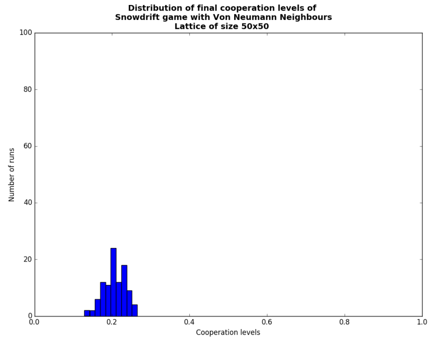
Figure 5: Snowdrift with different lattice sizes. From a to d are the graphs of the cooperation level over time and from e to h are the graphs of the distribution of the final cooperation level.

2.2 Von Neumann neighbours

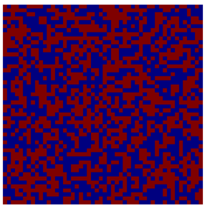
The diminution of neighbours seemed to be translated in a decrease of the cooperation level, around 20% (Fig 6). This difference with the Moore neighbours is due to the fact that, lacking comparison, the players tends to change in a slower pace than with the Moore neighbours and limits also the exposition to different strategies.



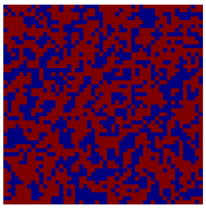
(a) Cooperation level over time



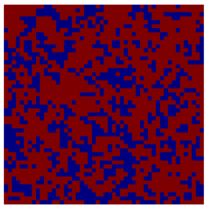
(b) Distribution of final cooperation levels



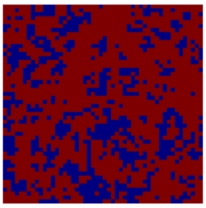
(c) $t = 0$



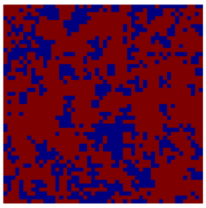
(d) $t = 1$



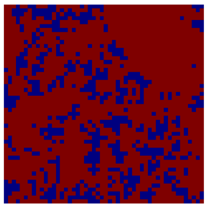
(e) $t = 5$



(f) $t = 10$



(g) $t = 20$



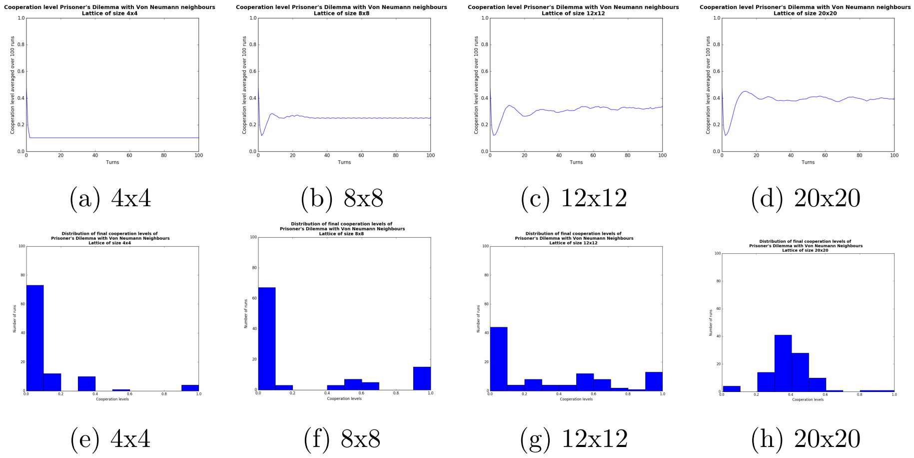
(h) $t = 50$

Figure 6: Snowdrift with Von Neumann neighbours with a lattice size 50x50

SUPPLEMENTARY PART

2.3 Prisoner's dilemma with Von Neumann neighbours with different lattice sizes

We simulated the Prisoner's dilemma with different sizes with the Von Neumann neighbours (Fig 7). We observed the same phenomena as with the Snowdrift game, where the bigger the lattice, the more centered around the final cooperation level, here 40%, the distribution is.



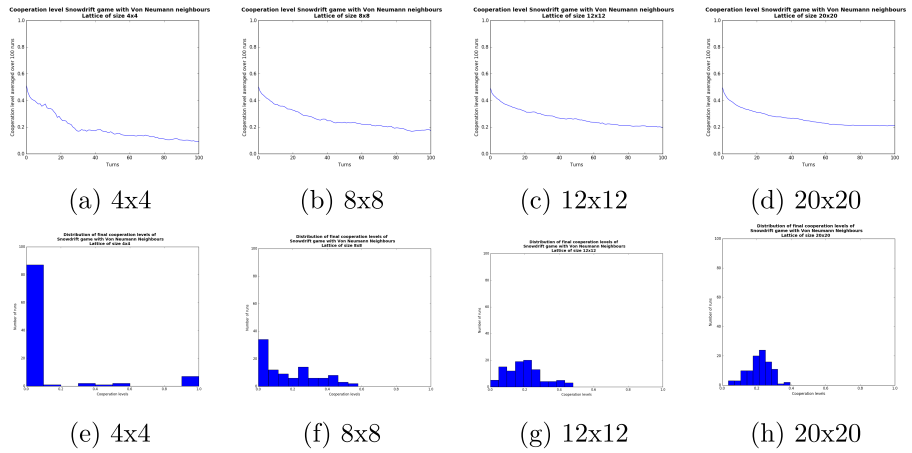
(a) 4x4 (b) 8x8 (c) 12x12 (d) 20x20

(e) 4x4 (f) 8x8 (g) 12x12 (h) 20x20

Figure 7: Prisoner's Dilemma with Von Neumann neighbours with different lattice sizes. From a to d are the graphs of the cooperation level over time and from e to h are the graphs of the distribution of the final cooperations level.

2.4 Snowdrift game with Von Neumann neighbours with different lattice sizes

As with the Moore neighbours, we observed that the bigger the size of the lattice the more centered around the final cooperation level, here 20%, the distribution is (Fig 8). This is because the more players you have, the less noise you have from one simulation to another.



(a) 4x4 (b) 8x8 (c) 12x12 (d) 20x20

(e) 4x4 (f) 8x8 (g) 12x12 (h) 20x20

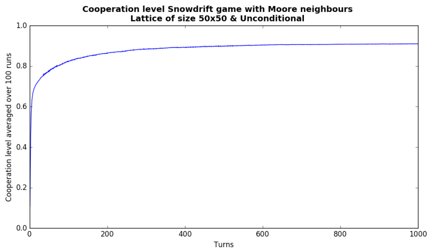
Figure 8: Snowdrift with different lattice sizes. From a to d are the graphs of the cooperation level over time and from e to h are the graphs of the distribution of the final cooperations level.

2.5 Snowdrift game with unconditional imitation rule

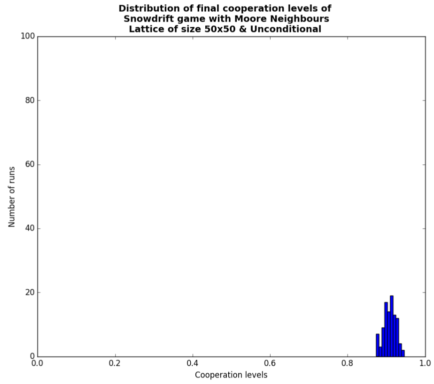
We wanted to know if we would observe something different than what we saw with the replicator rule. We simulated the Snowdrift game with the unconditional imitation rule with both Moore neighbours and with Von Neumann ones.

The simulation with Moore neighbours showed a higher cooperation level than with the replicator rule, centered on the distribution of the final level around 90%(Fig 4). This can be explained by the fact that nobody wins when the both defect, but even if one defects and the other cooperates there is some positive payoffs. This means that the cooperation strategy can have higher payoff than defection strategy when surrounded by mixed strategies. As the player will adopt automatically the strategy of the player with the highest payoff, the cooperators can invade the defectors population more easily.

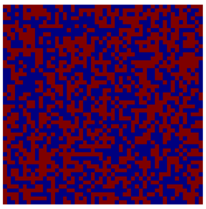
With the Von Neumann neighbours, we did observe a higher cooperation level, but not at the level observed with the Moore neighbours(Fig 10). This difference was also observed in the Prisoner's dilemma. The player is only compared to a limited number of its surrounding, limiting the changes brought to the spatial configuration.



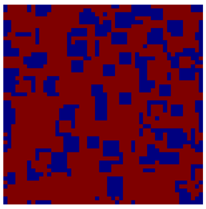
(a) Cooperation level over time



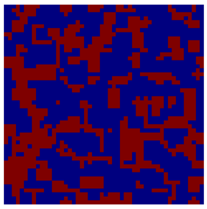
(b) Distribution of final cooperation levels



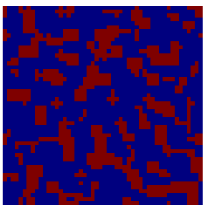
(c) $t = 0$



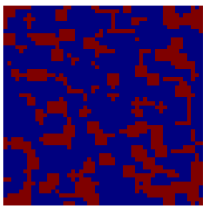
(d) $t = 1$



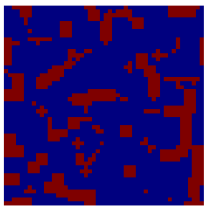
(e) $t = 5$



(f) $t = 10$

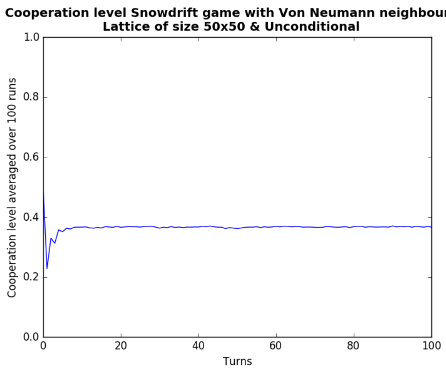


(g) $t = 20$

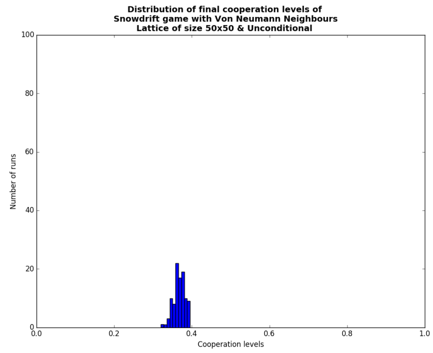


(h) $t = 50$

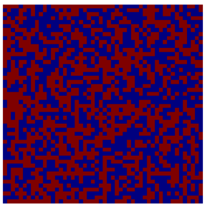
Figure 9: Snowdrift with Moore neighbours with a lattice size 50x50



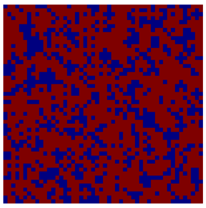
(a) Cooperation level over time



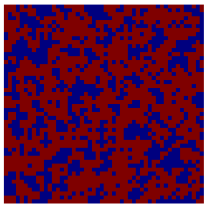
(b) Distribution of final cooperation levels



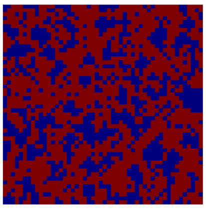
(c) $t = 0$



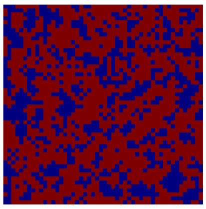
(d) $t = 1$



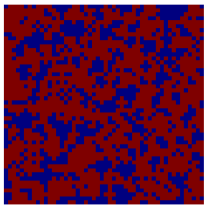
(e) $t = 5$



(f) $t = 10$



(g) $t = 20$



(h) $t = 50$

Figure 10: Snowdrift with Von Neumann neighbours with a lattice size 50x50