Pierard
Florian
Bioinformatics Master ULB

15/10/2015

LEARNING DYNAMICS

Assignment 1

1) A Coordination game

For player 1 :

- Expected payoff for the Player 1 for a pure No effort strategy (p) :

q * 0 + (1-q) * 0 = 0

- Expected payoff for the Player 1 for a pure Effort strategy (1-p) :

q * - C + (1-q)*(1-C) = q * -C + (1 – C) + q * C = 1 – C – q

➔ Player 1 will chose a pure No effort strategy if :

0 > 1 – C – q → So if q is higher than 1 – C → **q > 1 – C**.

For player 2 :

- Expected payoff for the Player 2 for a pure No effort strategy (q) :

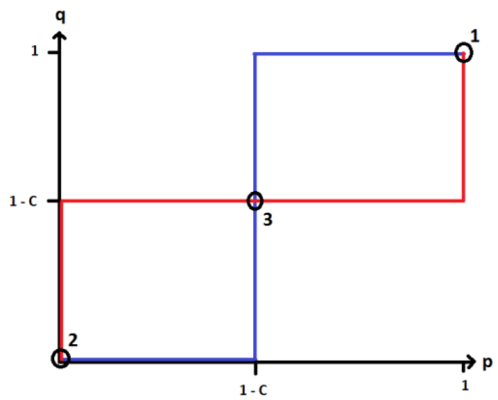
p * 0 + (1-p) * 0 = 0

- Expected payoff for the Player 2 for a pure Effort strategy (1-q) :

p * -C + (1-p)*(1-C) = p * -C + (1 – C) + p * C = 1 – C – p

➔ Player 2 will chose a pure No effort strategy if :

0 > 1 – C – p → So if p is higher than 1 – C → **p > 1 – C**.



So we have 3 possible mixed nash equilibria with one depending on C:

1) {(1,0);(1,0)} → Both choose « No effort »
2) {(0,1),(0,1)} → Both choose « Effort »
3) {(1-C , C);(1-C , C)} → The mixed nash equilibirum depending on C.

As C increases, the mixed nash equilibrium is moving near (0 ; 0). So the probability of doing effort is decreasing and the probability of doing “No effort” is increasing. It seems logic because the cost of doing effort while the other player chose “No effort” increases (- C) and the reward of doing “Effort” together decreases (1 – C). So players are more likely to choose “No effort”.

We also calculated that player 1 (or 2) will chose pure No effort strategy if p > 1- C. So if C increases, 1-C decreases and so, the player will chose easily “No effort”.

2) Which social dilemma?

First we group the values and we do the mean of every value from each game: Prisonners dilemma, Stag-Hunt game and Snowdrift game. We do this for every choices of player A (Cooperate = C or Defect = D). We obtain this table:

	C,C,C	C,C,D	C,D,C	C,D,D	D,C,C	D,C,D	D,D,C	D,D,D
C	9/3	**8/3**	4/3	3/3	7/3	**6/3**	2/3	1/3
D	**12/3**	7/3	**11/3**	**6/3**	**8/3**	3/3	**7/3**	**2/3**

The best choice for player A for each combination is in red.

Than we look to the best choice of player B in each game (also in red) in this table:

Prisonners dilemma		
	C	D
C	2	**5**
D	0	**1**

Stag-Hunt Game		
	C	D
C	**5**	2
D	0	**1**

Snowdrift game		
	C	D
C	2	**5**
D	**1**	0

Finally we compare where best choice for A is also best choice for B. We obtain 2 pure Nash Equilibria:

1) C for player A and D,C,D for player B. **{C, (D,C,D)}**

	C,C,C	C,C,D	C,D,C	C,D,D	D,C,C	D,C,D	D,D,C	D,D,D
C	9/3	**8/3**	4/3	3/3	7/3	**6/3**	2/3	1/3
D	**12/3**	7/3	**11/3**	**6/3**	**8/3**	3/3	**7/3**	**2/3**

Prisonners dilemma		
	C	D
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Stag-Hunt Game		
	C	D
C	**5**	2
D	0	**1**

Snowdrift game		
	C	D
C	2	**5**
D	**1**	0

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2) D for player A and D,D,C for player B. **{D, (D,D,C)}**

	C,C,C	C,C,D	C,D,C	C,D,D	D,C,C	D,C,D	D,D,C	D,D,D
C	9/3	8/3	4/3	3/3	7/3	6/3	2/3	1/3
D	12/3	7/3	11/3	6/3	8/3	3/3	7/3	2/3

Prisonners dilemma		
	C	D
C	2	5
D	0	1

Stag-Hunt Game		
	C	D
C	5	2
D	0	1

Snowdrift game		
	C	D
C	2	5
D	1	0

3) Sequential truel

Considering:
pA= probability that player A hits his intended target
pB= probability that player B hits his intended target
pC= probability that player C hits his intended target

We have 3 types of sub-games (only one of each group is shown on the diagram):
-Sub-game A: it's all the game, regrouping every choice
-Sub-game B: when B and C still has to choose their target
-Sub-game C: when C has to choose his target

We can affirm that B will always target C because C is the only one who still has a bullet to kill B. But who will A target:

"Weakness is strength" for C because A will always try to shoot the one who has the highest chance to shoot him. Indeed:

When A shoots:

- If he misses (1 – pA) his intended target, it doesn't matter if he targets B or C, it gives the same result.
- If he hits his target (pA), B or C will die depending on who is targeted.
  - **If A kills B:** C will target A on the second round. So, in this case, the probability of A to live is equal to probability A hits B multiplied by probability C misses his shot on A => pA * (1-pC)
  - **If A kills C:** B will target A on the second round. So, in this case, the probability of A to live is equal to probability A hits C multiplied by probability C misses his shot on A => pA * (1-pB)

➔ **So A will target B if pA * (1-pC) > pA* (1-pB)**

→ **1-pC > 1-pB**

→ **pC < pB**

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So, A will always try to kill the one who has the best chance to kill him, so he will kill the one with the best probability to hit his target.

For example, if B has a better probability to hit than C, A will target B. And after that, if B is alive, he will target C. And C can target whomever he wants; it doesn't matter for his own survivability.

Considering that, the red arrows show the best choice for everybody if pC < pB. So the sub-game perfect equilibria are when A targets B (if pB > pC) and when B targets C. After that, the payoff of each one depends only on their probability to hit the target. The best end is when all miss his his target, so they are all alive (1,1,1). It happens with a probability of (1-pA) * (1-pB) * (1-pC).

DIAGRAM