## Probability of correct estimation

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## Question

Consider a Normal random variable  $\mathbf{z}$  with mean 1 and variance  $\sigma^2 = 4$ . Let  $\hat{\boldsymbol{\mu}}$  denote the sample average estimator. How many i.i.d. samples should I observe to guarantee that the following relation holds?

$$\operatorname{Prob}\{0.9 \leq \hat{\boldsymbol{\mu}} \leq 1.1\} \geq 0.85$$

## Solution

Since **z** is Normal, the distribution of  $\hat{\boldsymbol{\mu}}$  is Normal as well, with mean 1 and variance  $\sigma^2/N$ .

```
varz=4
muhat=1

for (N in 1:200){
   varhat=varz/N
   P=pnorm(1.1,muhat,varhat)-pnorm(0.9,muhat,varhat)
   ##cat("N=",N,"P=",P,"\n")
   if (P>=0.85)
        break;
}
```

The required number of samples is 58