Homework 7 (39 Points)

SUGGESTED SOLUTIONS

Problem 1 (6 Points)

There are two firms in the market, company A and company B. Without taxation, both are currently emitting 100 metric tons of greenhouse gas (GHG) each year. To reduce q metric tons of emission, the marginal abatement cost for A is $mc = 5q$. The marginal abatement cost for B is $mc = 10q$.

The government is considering charging a price for emission in the form a carbon tax.

1. Derive the market demand for GHG emission as a function of emission price. (2 Points)

   Given an emission price p, A will reduce $q_A = \frac{p}{5}$ metric tons of GHG. B will reduce $q_B = \frac{p}{10}$ metric tons of GHG. Thus the total pollution abatement $Q^{Abatement} = \left(\frac{1}{10} + \frac{1}{5}\right) p = \frac{3}{10}p$. Since A and B together emit 200 metric tons of GHG without tax, the market demand for GHG emission is

   $$Q^D = 200 - \frac{3}{10}p$$

2. It is determined that each metric ton of GHG emission will cause \$100 equivalent of damage to the environment. What is the marginal benefit of emission reduction? What is the optimal level of tax the government should impose? (2 Points)

   Marginal benefit of abatement: \$100
   Optimal level of tax: \$100

3. If, instead of taxation, the government implements a cap-and-trade system, what is the level of cap the government should set? (2 Points)

   Cap level = emission level under optimal tax:

   $$Q = 200 - \frac{3}{10} \times 100 = 170$$

Problem 2 (4 points)

There are three groups in a community. Their demand curves for public television in hours of programming, T, are given respectively by

$$W_1 = \$200 - T$$
$$W_2 = \$240 - T$$
$$W_3 = \$320 - 2T$$

Suppose public television is a public good that can be produced at a cost of \$200 per hour.

1. What is the efficient number of hours of public television? (2 points)

   `Total demand for public TV:`

   $$W = 760 - 4T$$

   `Social optimum:`

   $$760 - 4T = 200 \Rightarrow T^* = 140$$

2. If the government charges each group for watching public TV at a price of \$200/hour, then it becomes excludable. In such a case, how many hours of programming would the three groups consume, respectively (2 points)

   $T_1 = 0, T_2 = 40, T_3 = 60.$

Problem 3 (0 Points)

Consider the following tragedy of the commons problem. There are N fishermen who go out fishing each day. Let t_i be the number of hours fisherman i spends fishing each day. The cost of fishing *per hour* is c. The amount of fish that *each* fisherman can catch *per hour* is

$$q = a - bT^2 \tag{1}$$

, where $T = \sum_{i=1}^{N} t_i$.

(1) says that the more fishing is done each day (in terms of the total number of hours spent by all fishermen), the less fish that *each* person can catch per hour. Let p be the price of fish.

1. How many hours of fishing will each fisherman do per day?[1] (2 Points)

   Each individual fishermen solves[2]:

$$\max_{t_i} \{pt_i q - ct_i\} = \max_{t_i} \left\{pt_i \left(a - bT^2\right) - ct_i\right\}$$

   $\Rightarrow$

$$t_i^* = \frac{a - \frac{c}{p} - bT^2}{2bT} \tag{2}$$

   Since $\sum_{i=1}^{N} t_i = T$, (2)$\Rightarrow$

$$N\left(\frac{a - \frac{c}{p} - bT^2}{2bT}\right) = T$$

$$\Rightarrow T_{Private}^* = \sqrt{\frac{a - \frac{c}{p}}{\left(1 + \frac{2}{N}\right) b}} \tag{3}$$

   , where we use $T_{Private}^*$ to denote the total amount of fishing that will be done by fishermen.
   (2) and (3) $\Rightarrow$

$$t_i^* = \sqrt{\frac{a - \frac{c}{p}}{bN\left(N + 2\right)}}$$

[1] Hint: If fisherman i spends t_i hours fishing, then she will be able to catch qt_i amount of fish, which will generate pqt_i in revenue, while paying a cost of ct_i.

[2] Note: T is a function of t_i.

2. What is the socially optimal level of fishing per day? i.e. what is the total number of hours of fishing that is optimal for all the fishermen collectively?[3] (2 Points)

   `The society solves:`

$$\max_{t_i} \{pTq - cT\} = \max_{t_i} \left\{pT\left(a - bT^2\right) - cT\right\}$$

   $\Rightarrow$

$$T^*_{Society} = \sqrt{\frac{a - \frac{c}{p}}{3b}} \tag{4}$$

3. Is the socially optimal amount of fishing smaller than, equal to, or larger than the actual amount of fishing that will be done by all the fishermen? (2 Points)

$$T^*_{Society} = \sqrt{\frac{a - \frac{c}{p}}{3b}} < \sqrt{\frac{a - \frac{c}{p}}{\left(1 + \frac{2}{N}\right) b}} = T^*_{Private}$$

[3] Hint: For the society as a whole, if T hours is spent fishing, then qT amount of fish will be caught, which will generate pqT in income, while the total cost is cT.

Problem 4 (14 points)

The Georges Bank, a highly productive fishing area off New England, can be divided into two zones in terms of fish population. The total daily fish catch (in tons) in Zone 1 is

$$F_1 = 200X_1 - 2\left(X_1\right)^2 \tag{5}$$

, where X_1 is the number of boats fishing there. Each boat then equally share the total daily fish catch.

The daily fish catch of Zone 2 is

$$F_2 = 100X_2 - \left(X_2\right)^2 \tag{6}$$

, where X_2 is the number of boats fishing in Zone 2. Each boat then equally share the total daily fish catch.

There are 100 boats now licensed by the government to fish in these two zones. The fish are sold at \$100 per ton. Total cost (capital and operating) per boat is constant at \$1,000 per day. Answer the following questions about this situation:

1. If the boats are allowed to fish where they want, with no government restriction, how many will fish in each zone (note: the number of fish need NOT be an integer)? What will be the total revenue and total profit? (6 points)

   People will fish until the catch in each zone is the same:

   $$\frac{F_1}{X_1} = \frac{F_2}{X_2} \Rightarrow 200 - 2X_1 = 100 - X_2 \Rightarrow X_1 = \frac{200}{3}, X_2 = \frac{100}{3}$$

   , where $X_1 + X_2 = 100$.

   Total catch:

   $$F_1 + F_2 = 200 \times \frac{200}{3} - 2 \times \left(\frac{200}{3}\right)^2 + 100 \times \frac{100}{3} - \left(\frac{100}{3}\right)^2 = 6666$$

   Total revenue: \$666,600

   Total profit:\$666,600 - \$1000×100 = \$566,600

2. Suppose the government wants to maximize the total profit from fishing. If the government can restrict in which zone the boats are allowed to fish, how many boats should be allocated to each zone? What will be the total revenue? What will be the total profit? Assume the total number of boats remains at 100. (6 points)

   The government maximizes total profit: $100 \times (F_1 + F_2) - 1000 \times 100$, which is equivalent to maximizing the total catch:

$$\begin{aligned} F_1 + F_2 &= 200X_1 - 2(X_1)^2 + 100X_2 - (X_2)^2 \\ &= 200X_1 - 2(X_1)^2 + 100 \times (100 - X_1) - (100 - X_1)^2 \end{aligned} \tag{7}$$

   Maximizing (7) w.r.t. $X_1 \Rightarrow X_1^* = 50, X_2^* = 50$

   Total catch: $F_1 + F_2 = 7500$

   Total revenue: \$750,000

   Total profit: \$650,000[4]

3. Suppose there is no limit on the number of licensed boats, how many boats should fish in each zone in order to maximize the total profit? (2 points)

   Profit in Zone 1:

$$\pi_1 = 100 \times \left(200X_1 - 2(X_1)^2\right) - 1000X_1 \tag{8}$$

   Maximizing (8) w.r.t. $X_1 \Rightarrow X_1^* = 47.5$

   Profit in Zone 2:

$$\pi_2 = 100 \times \left(100X_2 - (X_2)^2\right) - 1000X_2 \tag{9}$$

   Maximizing (9) w.r.t. $X_2 \Rightarrow X_2^* = 45$

[4] Notice that the profits are not evenly divided between boats in the two zones. The average catch in Zone 1 is 100 tons per boat, while the average catch in Zone 2 is 50 tons per boat. Therefore, fishing in Zone 1 yields a higher profit for the individual owner of the boat.

Problem 5 (5 Points)

Read this article on the proposed U.S. tax reform plans. Summarize the article. In particular, what do the bills do to corporate income tax rates? What incentives do they offer to U.S. multinationals to (1) move their overseas profits back to the U.S.; (2) move their intellectual property back to U.S. parent firms? What is a "global minimum tax" and what are its intended purposes? (5 Points)

`See the article.`

Problem 6 (5 Points)

Read this article and summarize its criticism of the proposed U.S. tax reform plans. For each criticism, briefly state the article's reasoning.

`See the article.`

Problem 7 (5 Points)

Read this article on Zuckerberg's pledge. According to this article, what could be some negative effects of tax-subsidized charity?

`If income and wealth inequality in an economy is high, then charity donation could be dominated by a few very wealthy individuals (say, billionaires). In this case what charities do could mainly reflect the preferences of these individuals, rather than the preferences of the public. When this is the case, using tax deductions to encourage charity donation could run counter to its goal, which is to allow the public to decide where the money is spent (on which social/externality problems).`