Homework 6 (52 Points)

SUGGESTED SOLUTIONS

Problem 1 (10 Points)

In a village, each person has the following willingness to pay for beer:

1^{st} bottle	\$5
2^{nd}	4
3^{rd}	3
4^{th}	2
5^{th}	1
Further bottles	0

1. The cost of producing beer is \$1.50, and the competitive suppliers sell it at this price. (The supply curve is horizontal.) How many bottles will each villager consume? What is each person's consumer surplus? (4 Points)

   At a price of \$1.50, each person will consume 4 bottles of beer. Each consumer's total willingness to pay is \$14 (= \$5 + \$4 + \$3 + \$2). The total spent by each person on beer is \$6 (= \$1.50 × 4). Therefore, each consumer receives \$8 in consumer surplus (=\$14 - \$6).

2. Suppose producing beer creates pollution. Each bottle has an external cost of $1. Taking this additional cost into account, what is total surplus per person? (2 Points)

   Each consumer receives consumer surplus = $8 and generates $4 in external cost. Total producer surplus = 0, since producers are selling at marginal cost and make 0 profit. Total surplus $TS = CS + PS -$ total external cost $= 8N + 0 - 4N$, where N is the number of villagers. Therefore total surplus per person = $4.

3. The mayor of the village imposes a $1 tax on beer. What is consumption per person now? Calculate each person's consumer surplus and total surplus per person. Based on your calculations, would you support the mayor's policy? (6 Points)

   The $1 tax raises the price of a bottle of beer to $2.50. (The entire tax will be borne by consumers because supply is perfectly elastic.) Each villager will purchase only 3 bottles at the higher price and each consumer's total willingness to pay is now $12 (= $5 + $4 + $3). Each villager pays $7.50 (= $2.50 × 3) in total. Therefore, each villager receives $4.50 ($12 - $7.50) in consumer surplus and generates $3 in external cost ($1 per bottle x 3 bottles). The government collects $3 per villager in tax revenue. Total surplus per person is therefore equal to $4.50 - $3.00 + $3.00 = $4.50. Since this is higher than $4, we should recommend the mayor's policy.

Problem 2 (4 Points)

There are three industrial firms in Happy Valley.

Firm	Initial Pollution	Cost of Reducing Pollution by 1 Unit
A	70 units	\$20
B	80 units	\$25
C	50 units	\$10

The government wants to reduce pollution to 120 units, so it gives each firm 40 tradable pollution permits.

1. Who sells permits and how many do they sell? Who buys permits and how many do they buy? (2 Points)

   B buys 40 permits and C sells 40 permits.

   Reasoning: If B buys permits from C at a price $\in (\$10, \$25)$, then both B and C would benefit from the trade. If A buys permits from C at a price $\in (\$10, \$20)$, then both A and C would benefit from the trade. Who gets to buy from C? Since B is willing to pay a price $\in (\$20, \$25)$, A would not be able to compete with B, therefore, B would be able to buy all 40 permits from C. After buying 40 permits from C, B no longer needs permits and C no longer has permits to sell, therefore, A will not participate in any trade and will keep its 40 permits[1].

[1] What if B buys 40 permits from A instead? It could also happen, but in this case, after selling 40 permits to B at a price $\in (\$20, \$25)$, A can then buy 40 permits from C at a price $\in (\$10, \$20)$. So in the end, B will get 80 permits, A will have 40, and C will have none.

2. What is the total cost of pollution abatement in this situation? How much higher would the costs of pollution reduction be if the permits could not be traded? (2 Points)

C sells all 40 of its permits and needs to reduce its pollution by 50 units at a cost of $10 × 50 = $500. A still has 40 permits and needs to reduce its pollution by 30 units at a cost of $20 × 30 = $600. After buying 40 permits, B has 80 permits and does not need to reduce pollution. Therefore, the total cost of pollution reduction is $1,100.

If the permits could not be traded, then A would have to reduce its pollution by 30 units at a cost of $20 × 30 = $600, B would have to reduce its pollution by 40 units at a cost of $25 × 40 = $1,000, and C would have to reduce its pollution by 10 units at a cost of $10 × 10 = $100. The total cost of pollution reduction would be $1,700, $600 higher than in the case in which the permits could be traded.

Problem 3 (4 Points)

The following table shows the marginal costs for each of four firms (A, B, C, and D) to eliminate units of pollution from their production processes. For example, for Firm A to eliminate one unit of pollution, it would cost \$54, and for Firm A to eliminate a second unit of pollution it would cost an additional \$67.

Firm	A	B	C	D
1st unit	54	57	54	62
2nd unit	67	68	66	73
3rd unit	82	86	82	91
4th unit	107	108	107	111

1. If the government charges a pollution tax of \$69 per unit, how many units of pollution would the firms eliminate altogether? (2 Points)

   `Firm A would eliminate 2 units. Firm B 2 units. Firm C 2 units. Firm D 1 unit. Altogether, 7 units.`

2. If the government wants to reduce pollution from 16 units to 6 units, what levels of pollution tax would achieve that goal? (2 Points)

   `Tax` $\in (82, 86)$ `(i.e., any tax that is` > 82 `and` < 86 `would accomplish this goal)`

Problem 4 (6 Points)

Four roommates are planning to spend the weekend in their dorm room watching old movies, and they are debating how many to watch. If it costs \$8 to rent a movie. Here is their willingness to pay for each film:

	Judd	Joel	Gus	Tim
First film	\$7	\$5	\$3	\$2
Second film	6	4	2	1
Third film	5	3	1	0
Fourth film	4	2	0	0
Fifth film	3	1	0	0

1. What is the total surplus if they rent 1 movie? What is the total surplus if they rent 5 movies? (4 Points)

   `1 movie:` $7+5+3+2-8=9$
   `5 movie:` $7+5+3+2+6+4+2+1+5+3+1+4+2+3+1-8\times 5=9$

2. How many movies should the roommates rent to maximize total surplus? (2 Points)
   3

3. If they choose the optimal number from 2. and then split the cost of renting the movies equally, how much surplus does each person obtain from watching the movies? (2 Points)

   `Judd:` $7+6+5-2\times 3=12$
   `Joel:` $5+4+3-2\times 3=6$
   `Gus:` $3+2+1-2\times 3=0$
   `Tim:` $2+1-2\times 3=-3$

4. Is there any way to split the cost to ensure that everyone benefits? (2 Points)

   `Split according to each person's willingness to pay`[2].

[2] `However, in reality, it is hard to know each person's WTP, as people may not have the incentive to reveal their true WTP.`

Problem 5 (22 Points)

The Laffer curve, named after Economist Arthur Laffer, is a representation of the theoretical relationship between rates of taxation and the resulting levels of government revenue. In this exercise, we derive the Laffer curve for a hypothetical labor market. Suppose the labor market is described by the following supply and demand equations:

$$\begin{aligned} \text{Supply: } Q_S &= 2W \\ \text{Demand: } Q_D &= 100 - 8W \end{aligned}$$

, where W denotes hourly wage, Q_S is the quantity of labor supplied (in hours), and Q_D is the quantity of labor demanded (in hours).

1. What are the equilibrium wage and hours of employment in this market? (2 Points)

   $W = 10, Q = 20$

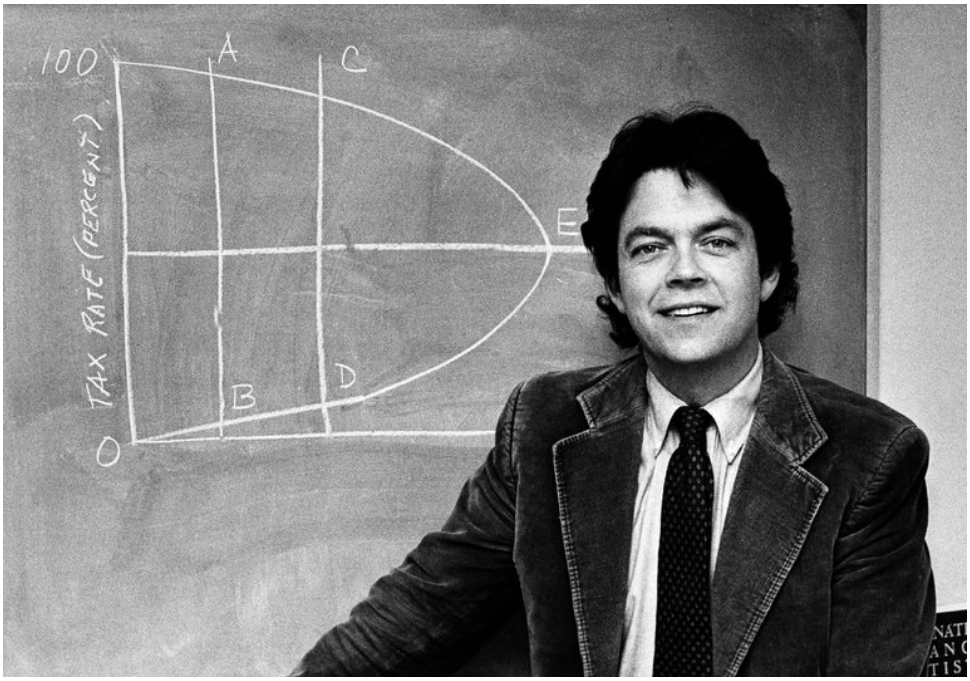
2. Now suppose we impose an ad-valorem wage tax $\tau \in (0, 1)$ on the workers. Let W^b denote before-tax wage and let W^f denote after-tax wage[3]. Solve for equilibrium W^b and W^f as a function of τ. (2 Points)

$$Q_S = 2W^f \tag{1}$$
$$Q_D = 100 - 8W^b \tag{2}$$
$$W^f = W^b(1 - \tau) \tag{3}$$

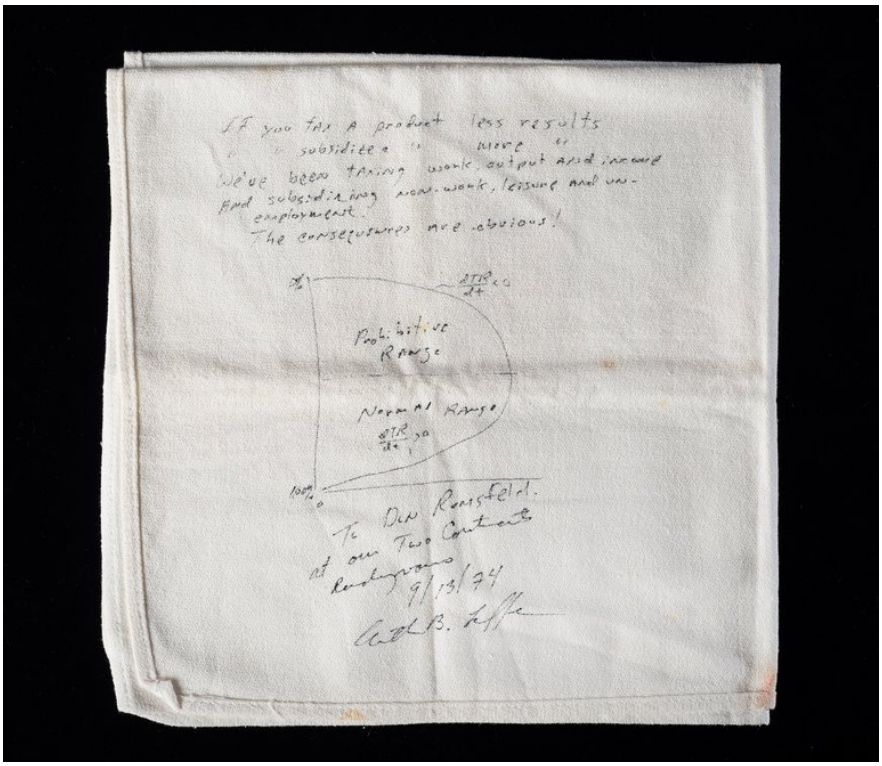
   $\Rightarrow$

$$W^b = \frac{50}{5 - \tau}$$
$$W^f = \frac{(1 - \tau)\,50}{5 - \tau}$$

[3]For example, suppose $\tau = 0.1$ (a 10% tax rate), then $W^f = (1 - \tau)\,W^b = 0.9W^b$.

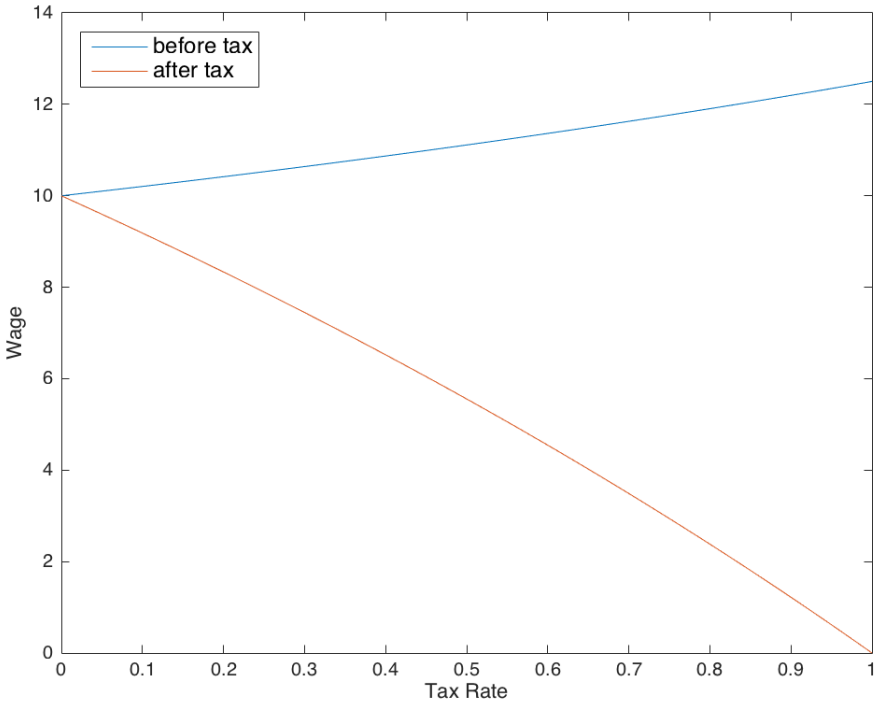


Arthur Laffer in 1981



The Laffer Curve napkin is on display at the National Museum of American History . For the story behind it, read this article.

3. In the same graph, plot the relationship between τ and W^b, and the relationship between τ and W^f[4]. (2 Points)



4. Solve for tax revenue as a function of τ. (2 Points)

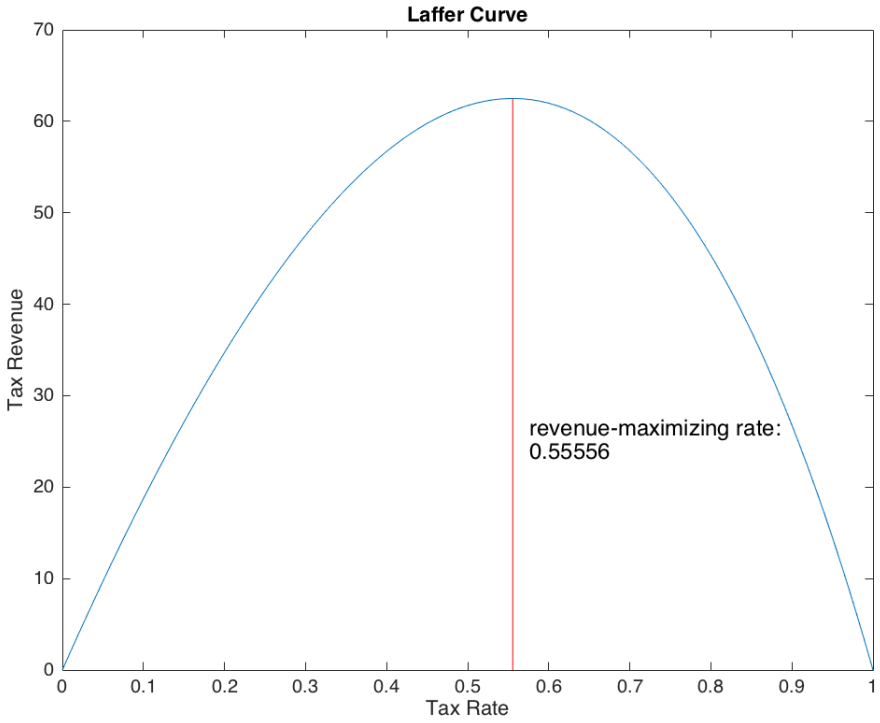
$$Q = \frac{(1-\tau)\,100}{5-\tau}$$

$$TR = \tau W^b Q = \frac{\tau\,(1-\tau)\,5000}{(5-\tau)^2}$$

, where TR denotes tax revenue.

[4]For this question and question 5 and 8, plot τ on the horizontal axis.

5. Plot the relationship between the tax rate τ and tax revenue – This is the Laffer curve. (2 Points)



6. Let τ^* denote the tax rate at which tax revenue is maximized. Calculate τ^*. (2 Points)

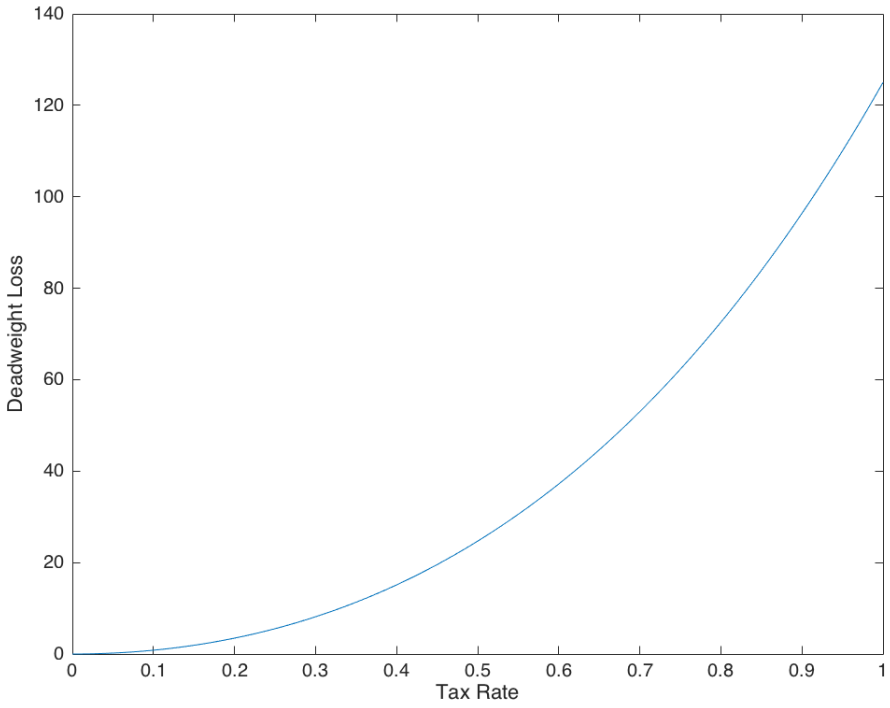
$$\left.\frac{dTR}{d\tau}\right|_{\tau=\tau^*} = 0 \Rightarrow \tau^* = \frac{5}{9}$$

7. Solve for deadweight loss as a function of τ. (2 Points)

$$\begin{aligned} DWL &= \frac{1}{2}\left(W^b - W^f\right)(20 - Q) \\ &= \frac{2000\tau^2}{(5-\tau)^2} \end{aligned}$$

, where 20 is the equilibrium quantity of labor supply before the wage tax is imposed.

8. Plot the relationship between τ and deadweight loss. (2 Points)



The Laffer curve shows that at high tax rates ($\tau > \tau^*$), cutting tax can lead to higher tax revenue. Some people, such as Laffer himself, have therefore advocated cutting U.S. income taxes for many years, believing that U.S. income taxes have always been too high and that cutting income taxes can lead to more, not less, government revenue. This is sometimes called the Laffer Hypothesis. Most economists, however, disagree[5].

In this exercise, let us look at what happened to U.S. government revenue after two of the largest tax cuts in recent U.S. history: (a) The Economic Recovery Tax Act of 1981 (ERTA), a.k.a. the 1981 Reagan tax cut, which, among other things, reduced top marginal income tax rate from 70% to 50%; and (b) The Economic Growth and Tax Relief Reconciliation Act of 2001 (EGTRRA), a.k.a. the 2001 Bush tax cut, which, among other things, reduced top marginal rate from 39.6% to 35%[6].

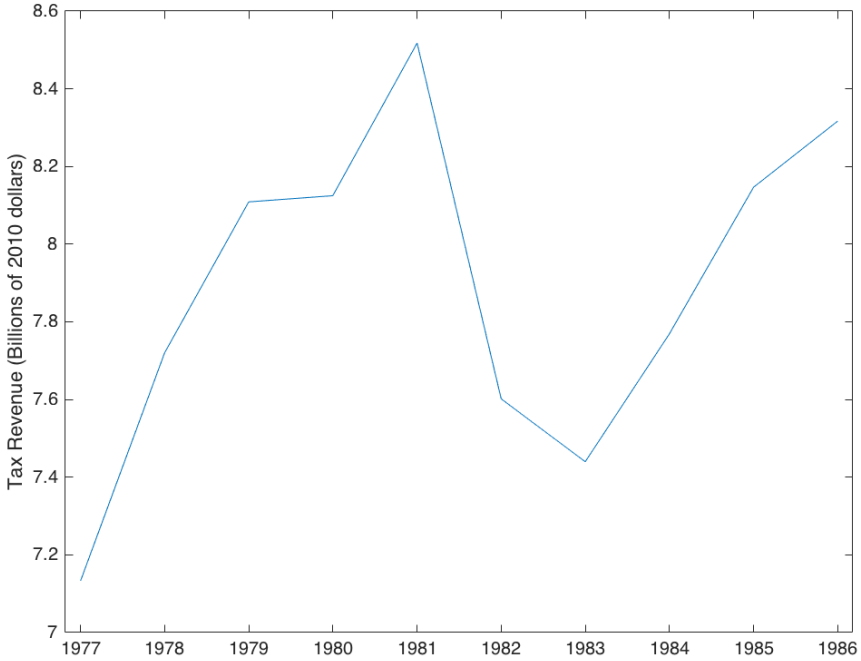
[5]See responses to Question B in the linked article. David Autor, for example, responds: "Not aware of any evidence in recent history where tax cuts actually raise revenue."

[6]Reagan himself believed in the Laffer Hypothesis. Here is what he said before signing the ERTA:

> "...our kind of tax cut will so stimulate the economy that we will actually increase government revenues..." July 7, 1981 speech.

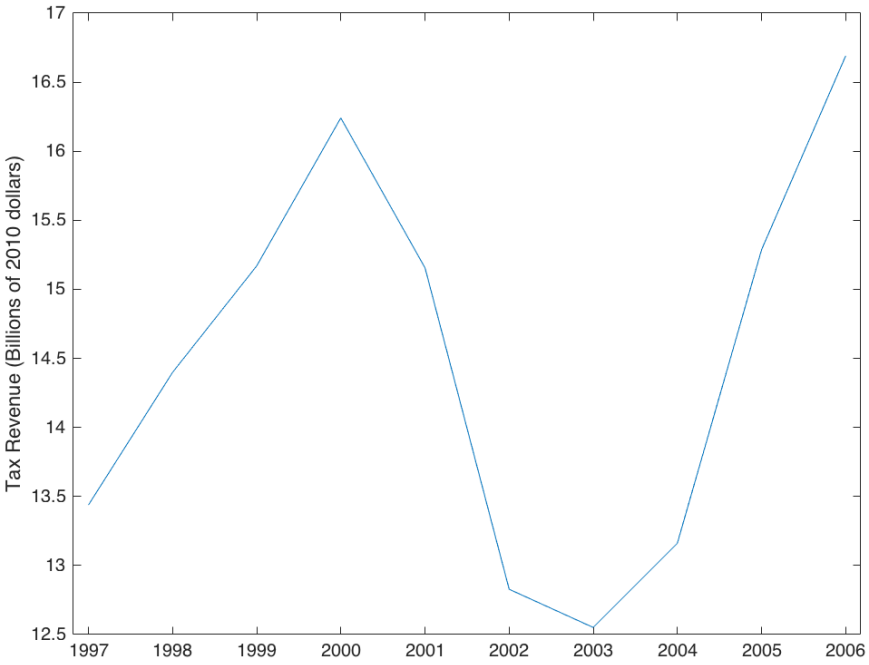
The FRED database at the Federal Reserve Bank of St. Louis contains data on federal government tax receipts. To look at the impact of the 1981 Reagan tax cut, we look at government tax receipts from 1977 to 1986. To look at the impact of the 2001 Bush tax cut, we look at government tax receipts from 1997 to 2006. To adjustment for inflation, divide tax receipts by the GDP Implicit Price Deflator[7]. We will call tax receipts that are not adjusted for inflation "*nominal* tax receipts," and those that have been adjusted for inflation "*real* tax receipts."

9. Plot *real* U.S. government tax receipts from 1977 to 1986. What does the data suggest about the effect of the 1981 Reagan tax cut on government revenue? (2 Points)



[7]We will talk about inflation and how to adjust for it later in this course. For now, you can just assume that by dividing tax revenue by the GDP deflator, we are able to "get rid of" inflation, which allows us to better compare tax revenues in different time periods.

10. Plot *real* U.S. government tax receipts from 1997 to 2006. What does the data suggest about the effect of the 2001 Bush tax cut on government revenue? (2 Points)



11. Do the experiences of these two major tax cuts validate the Laffer Hypothesis[8]? (2 Points)

No.

[8]Our analysis here is of course not rigorous – many things other than tax cuts happened during those years. A careful analysis needs to parcel out the effects of various causes. For more rigorous analysis of the revenue impact of major tax cuts in U.S. history, see the literature summarized here.