University of California, Los Angeles
Department of Statistics

Statistics C173/C273 **Instructor: Nicolas Christou**

Kriging revisited

We observe $\mathbf{Z} = (Z(s_1), Z(s_2), \ldots Z(s_n))'$ and we want to predict $Z(s_0)$.

Theorem 1
As discussed, kriging minimizes the mean square prediction error, $MSE(\hat{Z}(s_0)) = E(Z(s_0) - \hat{Z}(s_0))^2$. An important result is the following: $MSE(\hat{Z}(s_0))$ takes its minimum value when $\hat{Z}(s_0) = E(Z(s_0)|\mathbf{Z})$.

Theorem 2
Suppose that $\mathbf{Y}$, $\boldsymbol{\mu}$, and $\boldsymbol{\Sigma}$ are partitioned as follows $\mathbf{Y} = \left(\begin{array}{c} \mathbf{Y_1} \\ \mathbf{Y_2} \end{array}\right), \boldsymbol{\mu} = \left(\begin{array}{c} \boldsymbol{\mu}_1 \\ \boldsymbol{\mu}_2 \end{array}\right), \boldsymbol{\Sigma} = \left(\begin{array}{cc} \boldsymbol{\Sigma}_{11} & \boldsymbol{\Sigma}_{12} \\ \boldsymbol{\Sigma}_{21} & \boldsymbol{\Sigma}_{22} \end{array}\right)$, and $\mathbf{Y} \sim MVN(\boldsymbol{\mu}, \boldsymbol{\Sigma})$. It can be shown that the conditional distribution of $\mathbf{Y_1}$ given $\mathbf{Y_2}$ is also multivariate normal, $\mathbf{Y_1}|\mathbf{Y_2} \sim MVN(\boldsymbol{\mu}_{1|2}, \boldsymbol{\Sigma}_{1|2})$, where
$\boldsymbol{\mu}_{1|2} = \boldsymbol{\mu}_1 + \boldsymbol{\Sigma}_{12}\boldsymbol{\Sigma}_{22}^{-1}(\mathbf{Y_2} - \boldsymbol{\mu}_2)$, and $\boldsymbol{\Sigma}_{1|2} = \boldsymbol{\Sigma}_{11} - \boldsymbol{\Sigma}_{12}\boldsymbol{\Sigma}_2 2^{-1}\boldsymbol{\Sigma}_{21}$.

To apply this theorem in the spatial prediction problem assume the distribution of $\left(\begin{array}{c} Z_(s_0) \\ \mathbf{Z} \end{array}\right)$ is multivariate normal with mean vector $\mu\mathbf{1}$ and variance covariance matrix $\left(\begin{array}{cc} \sigma^2 & \mathbf{c}' \\ \mathbf{c} & \mathbf{C} \end{array}\right)$.

Result
Using the previous theorems, the predictor that minimizes the mean square prediction error (see Theorem 1) will be (see Theorem 2) $\hat{Z}(s_0) = \mu + \mathbf{c}'\mathbf{C}^{-1}(\mathbf{Z} - \mu\mathbf{1})$, which is the simple kriging predictor. The prediction variance (also see Theorem 2) will be $\sigma^2 - \mathbf{c}'\boldsymbol{\Sigma}^{-1}\mathbf{c} = C(0) - \mathbf{c}'\boldsymbol{\Sigma}^{-1}\mathbf{c}$, which is the simple kriging variance.