University of California, Los Angeles
Department of Statistics

Statistics C173/C273 **Instructor: Nicolas Christou**

Homework 3

Exercise 1
Answer the following questions:

a. Consider ordinary lognormal kriging. Find the unbiased predictor of $Z(s_0)$. Note: We predict $Y(s_0)$, where $Y(s_0) = ln(Z(s_0))$.

b. Consider simple lognormal kriging. We discussed in class the unbiased predictor of $Z(s_0)$. Note: We predict $Y(s_0)$, where $Y(s_0) = ln(Z(s_0))$. Find $E[Z(s_0) - \hat{Z}^*(s_0)]^2$, where $\hat{Z}^*(s_0)$ is the unbiased predictor of $Z(s_0)$.

Exercise 2
Consider universal kriging. In matrix/vector form universal kriging minimizes

$$C(0) - 2\mathbf{c}'\mathbf{w} + \mathbf{w}'\boldsymbol{\Sigma}\mathbf{w},$$

subject to the set of constraints $\mathbf{X}'\mathbf{w} = \mathbf{x}$. Find explicit solutions for $\mathbf{w}$ and $\boldsymbol{\lambda}$, where $\mathbf{w} = (w_1, w_2, \ldots, w_n)'$ and $\boldsymbol{\lambda} = (\lambda_0, \lambda_1, \ldots, \lambda_k)'$ is the vector of the Lagrange multipliers.

Exercise 3
Show that using the simple kriging weights plus the generalized least squares estimate of $\boldsymbol{\beta}$ we obtain the universal kriging weights. The generalized least squares estimate of $\boldsymbol{\beta}$ is given by $\hat{\boldsymbol{\beta}}_{gls} = (\mathbf{X}'\boldsymbol{\Sigma}^{-1}\mathbf{X})^{-1}\mathbf{X}'\boldsymbol{\Sigma}^{-1}\mathbf{Z}$.