University of California, Los Angeles
Department of Statistics

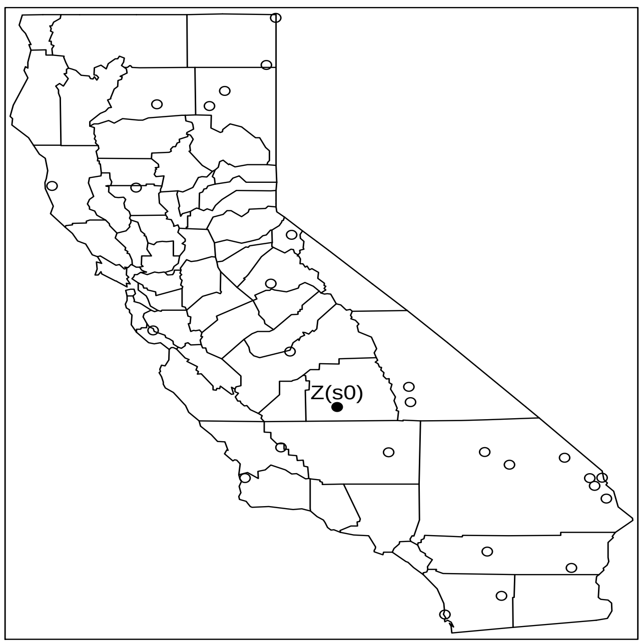
Statistics C173/C273 **Instructor: Nicolas Christou**

Exam 2
20 February 2015

Name: ______________________________

Problem 1 (25 points)
The following data represent 27 spatial locations in California.

```
a <- read.table("http://www.stat.ucla.edu/~nchristo/statistics_c173_c273/ca_exam2_w15.txt",
header=TRUE)
```



Answer the following questions:

a. Use the spectral decomposition method to generate spatial data on the 27 locations. Assume the spherical covariance function with parameters $c_0 = 0, c_1 = 1000, \alpha = 6$.

b. Use ordinary kriging to predict the value of $Z(s_0)$ (as shown on the map) and its variance. The coordinates of this location are (-119, 36). You must use the R code. Do not use `geoR` or `gstat`.

c. Refer to part (a). Compute the experimental variogram and fit the spherical model to it by estimating the parameters using one of the methods discussed in class..

d. Fit by eye the spherical variogram with parameters the same as the ones used to generate the data in part (a).

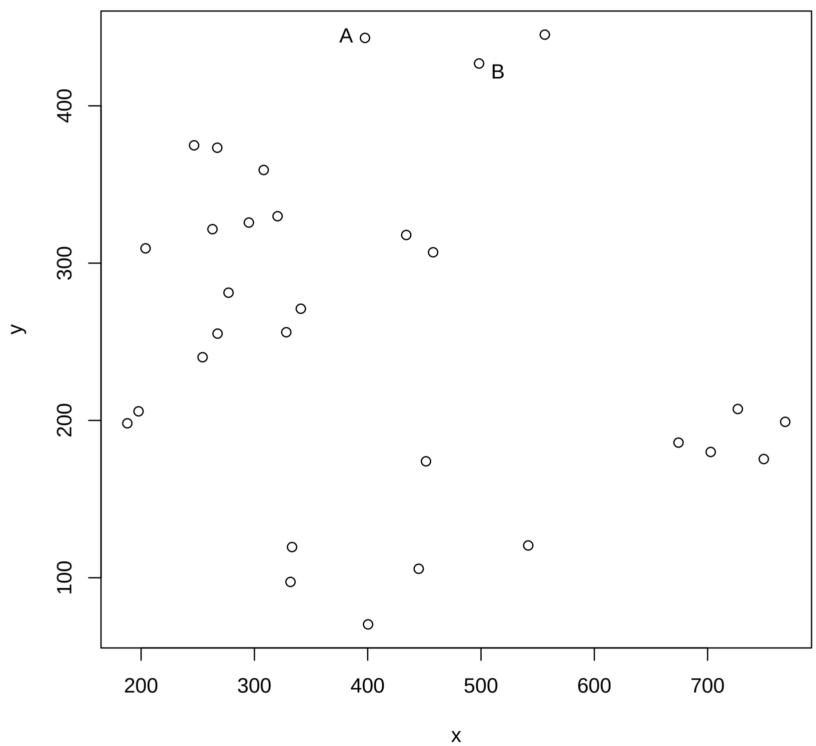
e. Create a dense grid for predictions: Use `by=0.1`. Use `gstat` to make predictions at the grid locations. Construct a raster map using the predicted values and add contours to it.

Problem 2 (25 points)

You have computed several semivariograms and you discovered that the process shows geometric anisotropy. The largest range (major axis of the ellipse) is positioned at $\theta = 20°$ from the `y` axis. The length of this range is 400 meters while the range of the minor axis of the ellipse is equal to 200 meters. The sill is $c_1 = 1$. Answer the following questions:

a. Describe the steps needed to obtain an isotropic process. No calculations are needed but you need to describe clearly the steps.

b. Suppose the data points are as shown in the figure below. Find the distance between points `A`(397.6, 443.2) and `B`(498.3, 426.9) of the transformed data.



c. Consider the gaussian semivariogram model

$$\gamma(h) = c_1(1 - e^{-\frac{h^2}{\alpha^2}}).$$

What is the value of $\gamma(20)$ of the isotropic model?

Problem 3 (25 points)
Suppose that Z is a second-order stationary process with $E[Z(s)] = 0$ and with covariance function:

$$c(h) = \begin{cases} 8 - \sqrt{h}, & 0 < h \leq 64 \\ 0, & h > 64 \\ 10, & h = 0 \end{cases}$$

a. What is the sill of Z?

b. What is the nugget effect of Z?

c. Draw this covariance function (approximately). Make sure you place some important numbers on the graph.

d. Write the semivariogram function $\gamma(h)$ that corresponds to the covariance function above.

e. Consider three spatial locations s_1, s_2, s_3 with distances $d_{12} = 2, d_{13} = 3, d_{23} = 4$. Compute

$$E\left[Z(s_1) - Z(s_3)\right]\left[Z(s_2) - Z(s_3)\right].$$

Hint: The expectation above can be written as $\frac{1}{2}\left[E(Z_1 - Z_3)^2 + E(Z_2 - Z_3)^2 - E(Z_1 - Z_2)^2\right]$.

Problem 4 (25 points)

Consider the 4-point layout given on the graph on the previous page. The $z(s_1), z(s_2), z(s_3), z(s_4)$ are the observed values of the Z process which is described by the exponential semivariogram $\gamma(h) = c_0 + c_1(1 - e^{-\frac{h}{\alpha}})$, with $c_0 = 0, c_1 = 3.5, \alpha = 4.5$. Our goal is to predict the value $Z(s_0)$ at location s_0. The coordinates of these 5 points are:

s_i	x_i	y_i	$z(s_i)$
s_0	1	2	???
s_1	1	1	513
s_2	1	3	531
s_3	2	1	516
s_4	2	3	537

a. Compute the distance matrix of these 5 points.

b. Ordinary kriging: Using the variogram, compute the matrix $\mathbf{\Gamma}$ and the vector $\boldsymbol{\gamma}$ needed for the calculation of the ordinary kriging weights.

c. Simple kriging: Using the covariance, compute the matrix $\boldsymbol{C}$ and the vector $\boldsymbol{c}$ needed for the calculation of the simple kriging weights.

d. Use the function `ksline` of `geoR` to predict the value of $Z(s_0)$ at s_0 and its variance with ordinary kriging.