University of California, Los Angeles
Department of Statistics

Statistics C173/C273 **Instructor: Nicolas Christou**

Exam 2
28 February 2014

Name: ______________________________

Problem 1 (25 points)
Consider the coal-ash data:

```
a <- read.table("http://www.stat.ucla.edu/~nchristo/statistics_c173_c273/coal_ash.txt", header=TRUE)
```

There are 208 measurements on coal-ash. The first six rows of the data set and the spatial locations of the data points are given below:

```
> head(a)
  x  y coalash
1 1 14   10.21
2 1 15    9.92
3 1 16   11.17
4 2  8   10.01
5 2 10   11.15
6 2 11   11.31
```

Things to do using only `gstat`:

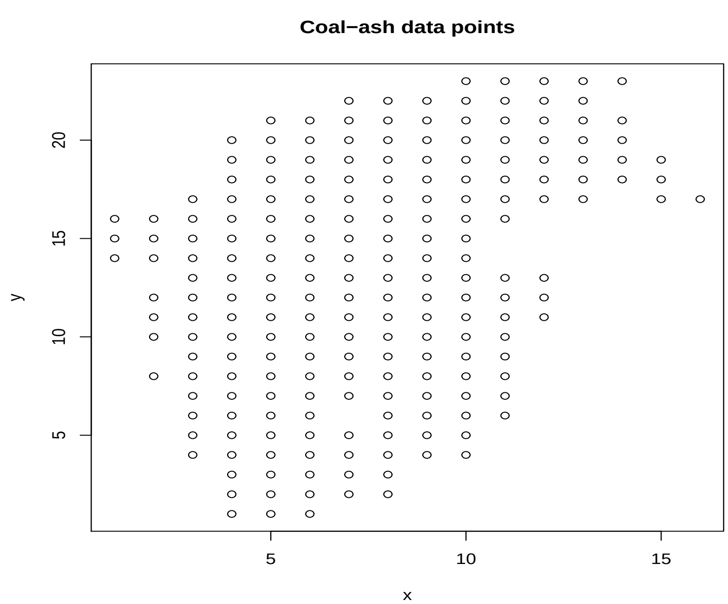
a. Comput the sample variogram.

b. Fit a model to the sample variogram.

c. Create a grid (use `by=0.1`).

d. Perform ordinary kriging predictions.

e. Construct the raster map using the predicted values from (d).



Problem 2 (25 points) - Due on Monday, 03 March

Consider the 4-point layout given on the graph below. The $z(s_1), z(s_2), z(s_3), z(s_4)$ are the observed values of the Z process which is described by the exponential semivariogram $\gamma(h) = c_0 + c_1(1 - e^{-\frac{h}{\alpha}})$, with $c_0 = 0, c_1 = 3.5, \alpha = 4.5$. Our goal is to predict the value $z(s_0)$ at location s_0. The coordinates of these 5 points are:

s_i	x_i	y_i	$z(s_i)$
s_0	1	2	???
s_1	1	1	513
s_2	1	3	531
s_3	2	1	516
s_4	2	3	537

Answer the following questions:

a. Compute the distance matrix of these 5 points.

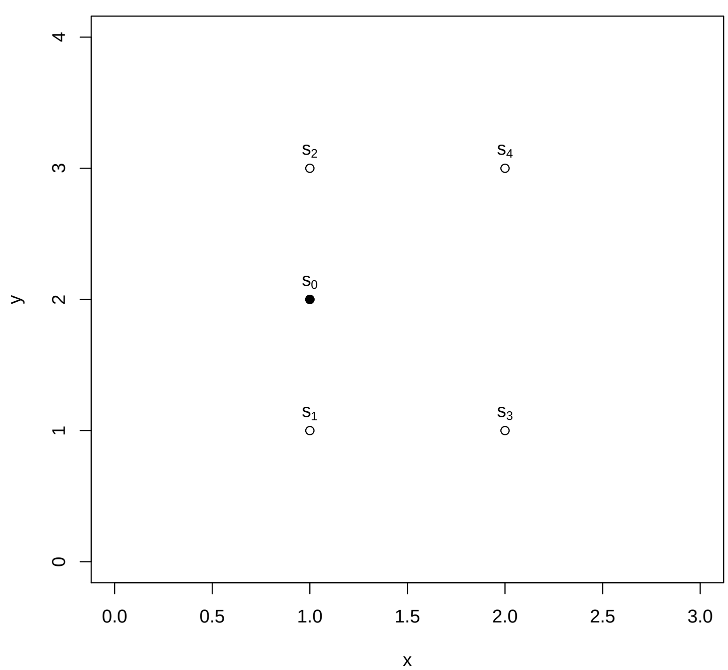
b. Ordinary kriging: Using the variogram, compute the matrix $\mathbf{\Gamma}$ and the vector $\boldsymbol{\gamma}$ needed for the calculation of the ordinary kriging weights. You can also use the covariance function and solve it using matrix $\boldsymbol{C}$ and vector $\boldsymbol{c}$.

c. Simple kriging: Using the covariance, compute the matrix $\boldsymbol{C}$ and the vector $\boldsymbol{c}$ needed for the calculation of the simple kriging weights.

d. Universal kriging: Assume a linear trend as a function of the coordinates x an y. Using the variogram, compute the matrix $\mathbf{\Gamma_1}$ and the vector $\boldsymbol{\gamma_1}$ needed for the calculation of the universal kriging weights.

e. Use `geoR` and the function `ksline` to estimate the value of Z at s_0 and its variance with ordinary kriging.

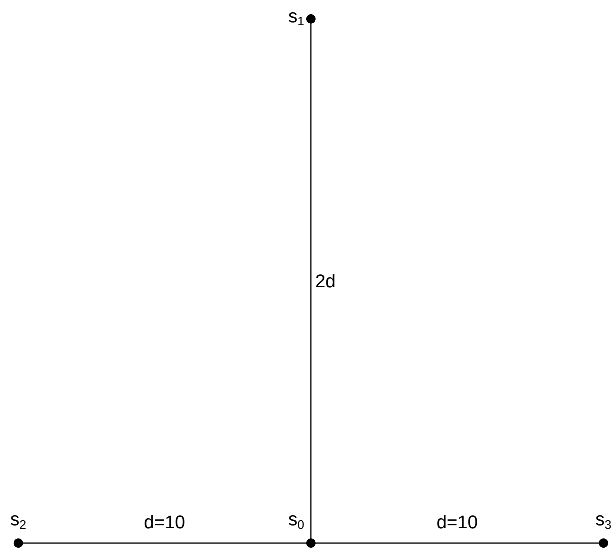
f. Use `geoR` and the function `krige.conv` to estimate the value of Z at s_0 and its variance with universal kriging.



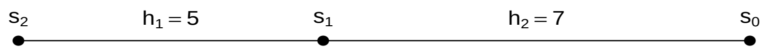
Problem 3 (25 points)
Consider the points in the figure below. Locations s_1, s_2, s_3 are the data points and location s_0 is the location of the point to be predicted. Assume the spherical variogram with $c_0 = 0, c_1 = 1, \alpha = 50$. If $d = 10$ set up the ordinary kriging system and compute:

a. The weights w_1, w_2, w_3.

b. The ordinary kriging variance.



Problem 4 (25 points)
In general, kriging assigns larger weights to the closer points than data farther away from the point to be predicted. Let's examine here what happens if the point to be predicted and two data points are in line as shown in the figure below.



This is called the "screening" effect of kriging. It means that the weight of a data point which is screened by another data point is reduced dramatically when the two points are in line with the point to be predicted. Use simple kriging and assume exponential covariance function with $c_0 = 0, c_1 = 10$, and $\alpha = 40$ to find the weights w_1 and w_2.

For home - due on Monday, 03 March
Do this problem again as follows: Assume that the distance from s_1 to s_2 is h_1 and the the distance from s_0 to s_1 is h_2. So the distance from s_0 to s_2 is $h_1 + h_2$. Use the exponential covariance function with sill c_1 and range α. You will need to find the inverse of a 2×2 matrix in order to compute the weights. What do you find?

For home - due on Monday, 03 March
Consider ordinary lognormal kriging (we work with $Y(s_0) = log(Z(s_0))$. Find the variance of the prediction error in terms of the original scale of the data, i.e. find $E(Z(s_0) - \hat{Z}(s_0))^2$.