MFToE Vacuum Dynamics:
From Toy/EFT Integrator to DESI DR2 BAO Comparison

Documentation & Reproducibility Notes (2025)

Christof Krieg

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Abstract

We present a compact, reproducible pipeline to explore a single-field slice of the *Maat Field Theory of Everything* (MFToE) with a dynamic vacuum field χ, including optional renormalization-group (RG) running and Ornstein-Uhlenbeck (OU) noise. The background dynamics are integrated in $N = \ln a$ using a robust RK4 scheme. For observational comparison, we construct a flat ΛCDM baseline via `Astropy` and confront the model with baryon acoustic oscillation (BAO) measurements from **DESI DR2 (2025)**. We report sub-percent deviations in $H(z)$ and $d_L(z)$ relative to ΛCDM, and compute χ^2 against DESI DR2 compressed BAO summaries (with diagonal or synthetic correlated covariance). The codebase provides scripts for baseline runs, relaxion scans, and BAO checks, enabling full end-to-end reproducibility.

1 Overview

Goal. Provide a fast exploration tool for MFToE-inspired late-time dynamics with a dynamic vacuum sector χ that can (i) self-adjust a residual vacuum level and (ii) introduce mild, testable departures from ΛCDM, while remaining consistent with current BAO constraints (DESI DR2, 2025).

Key features.

- Single scalar ϕ with potential $V(\phi) = \frac{1}{2}m^2\phi^2 + \lambda\phi^4$ plus a vacuum controller χ.
- Modes: `off`, `sequester`, `targetH0`, `relaxion`.
- Optional RG running (m^2, λ as functions of H) and OU noise.
- Numerical integration in $N = \ln a$ from $z \approx 0$ to $z \approx 3$.
- Observables: $H(z)/H_0$, $d_L(z)$, $w_\phi(z)$, $w_{\rm tot}(z)$ and comparison to flat ΛCDM (either analytic or Astropy).
- BAO comparison: D_M/r_d, D_H/r_d, D_V/r_d with χ^2 evaluation vs DESI DR2 compressed tables.

2 Model and Dynamics

We consider a homogeneous scalar ϕ on an FLRW background, with canonical kinetic term and potential

$$V(\phi) = \frac{1}{2}m^2\phi^2 + \lambda\phi^4, \tag{1}$$

and a dynamic vacuum field χ entering the effective vacuum level as $V_{\rm vac}(\chi) = V_0^{\rm bare} + g\,\chi$. The total effective potential is

$$V_{\rm eff}(\phi, \chi) = V_{\rm vac}(\chi) + V(\phi) + \xi_{\rm noise}, \tag{2}$$

where ξ_{noise} can be realized by an OU process (optional).

We evolve in $N = \ln a$ with state vector $y = (\phi, \pi, \chi)$ and $\pi \equiv d\phi/dN$. The Hubble equation (in dimensionless units with $H_0 = 1$ internally) reads

$$H^2 = \frac{\rho_m a^{-3} + \rho_r a^{-4} + V_{\text{eff}}}{1 - \frac{1}{2}\pi^2}, \tag{3}$$

with $a = e^N$. The equation of state of the scalar is

$$w_\phi = \frac{K - V}{K + V}, \qquad K = \frac{1}{2} H^2 \pi^2, \; V \equiv V(\phi), \tag{4}$$

and the total w_{tot} includes the vacuum piece V_{vac}.

Modes for χ evolution.

- `off`: $d\chi/dN = 0$.
- `sequester/targetH0`: gradient descent to minimize $(V_{\text{eff}} - \rho_{\text{target}})^2/(2M_{\text{pen}}^4)$ with rate γ/H.
- `relaxion`: adds a coupling to the ϕ-sector gradient to *nudge* towards smaller $V(\phi)$ (toy behavior).

RG running (optional). A simple one-step flow with scale $\mu \sim H$:

$$m^2(\mu) \approx m^2 + a_\beta m^2 \ln \frac{\mu}{\mu_{\text{ref}}}, \qquad \lambda(\mu) \approx \lambda + b_\beta \lambda^2 \ln \frac{\mu}{\mu_{\text{ref}}}. \tag{5}$$

Noise (optional). OU process in N:

$$dX = -\frac{X}{\tau} dN + \sqrt{\frac{2\sigma^2}{\tau}}\, dW_N, \quad \xi_{\text{noise}} = X(N). \tag{6}$$

3 Numerics and Code

Integrator. Classic RK4 step in N with step size $h = (N_{\text{end}} - N_{\text{start}})/(N_{\text{steps}} - 1)$. We clip the kinetic term to avoid $1 - \pi^2/2 \to 0$.

Astropy baseline. For fair comparison we compute a flat ΛCDM reference using `Astropy`:

$$H_{\Lambda\text{CDM}}(z)/H_0 \text{ via } \texttt{FlatLambdaCDM}, \tag{7}$$

$$d_L(z) = (1+z)\, D_C(z), \quad D_C \text{ from Astropy}, \quad d_L^{\text{dimless}} = d_L(\text{Mpc}) \cdot H_0^{\text{phys}}/c. \tag{8}$$

Internally, the MFToE solver uses $H_0 = 1$; the Astropy baseline uses H_0^{phys} (e.g. $67.36\,\text{km}\,\text{s}^{-1}\,\text{Mpc}^{-1}$).

4 Data and Licensing (DESI DR2, 2025)

We use DESI Data Release 2 (DR2), October 2025, specifically the BAO compressed measurements (effective redshifts, D_M/r_d, D_H/r_d, D_V/r_d) and best-fit cosmology summaries (e.g. `iminuit/base/desi-bao-all/bestfit.minimum`).

5 Results

We summarize two representative runs (your logs reproduced):

5.1 Baseline: `targetH0`, no RG/noise, DESI bestfit

Astropy baseline set by DESI DR2 bestfit: $\Omega_m \simeq 0.297179$, $H_0^{\rm phys} \simeq 67.36\,\mathrm{km\,s^{-1}\,Mpc^{-1}}$, $r_d \simeq 150.754\,\mathrm{Mpc}$.

Relative deviations vs. ΛCDM (Astropy):

z	$\Delta H/H_{\Lambda\rm CDM}$	$\Delta d_L/d_{L,\Lambda\rm CDM}$
0.20	-0.116%	$+0.127\%$
0.50	-0.091%	$+0.114\%$
1.00	-0.067%	$+0.098\%$
2.00	-0.058%	$+0.084\%$
3.00	-0.066%	$+0.080\%$

EoS means: $\overline{w}_\phi \simeq -0.919$, $\overline{w}_{\rm tot} \simeq -0.282$.
BAO χ^2 (no covariance, 19 points): $\chi^2 = 16.122$ (reduced $\chi^2 = 0.849$).

Figures. The code produces `runs/mftoe_vacuum_astropy.png` and `runs/bao_check.png`. Include them here after you have them in Overleaf:

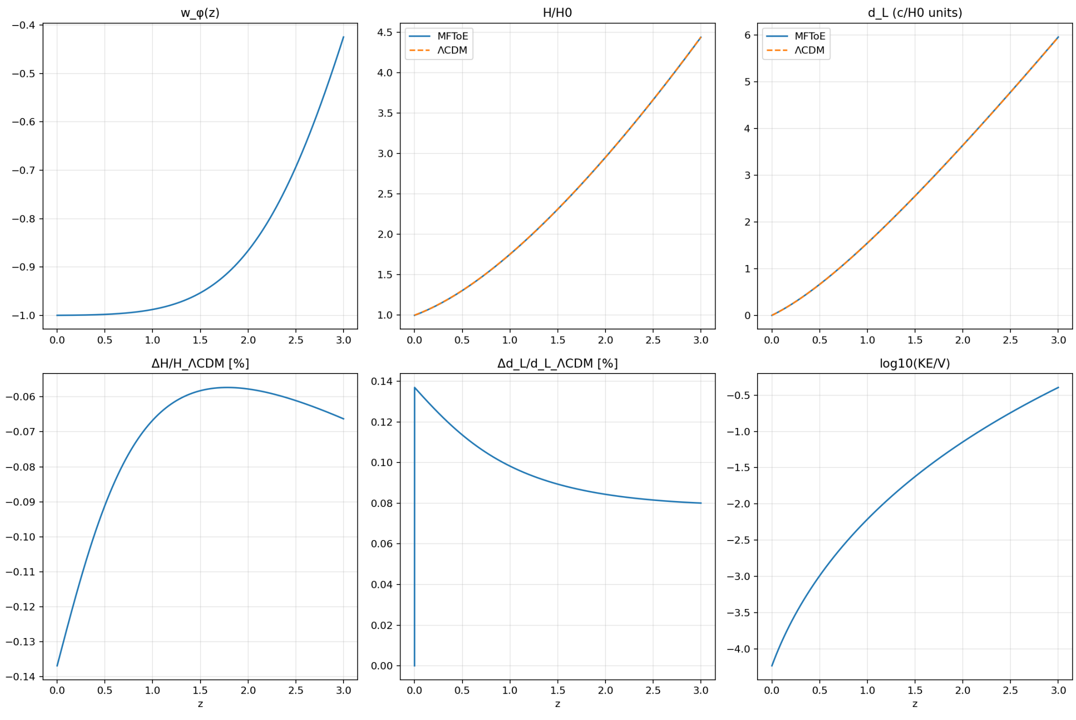

Figure 1: MFToE baseline vs. : $w_\phi(z)$, H/H_0, $d_L(z)$, and relative residuals.

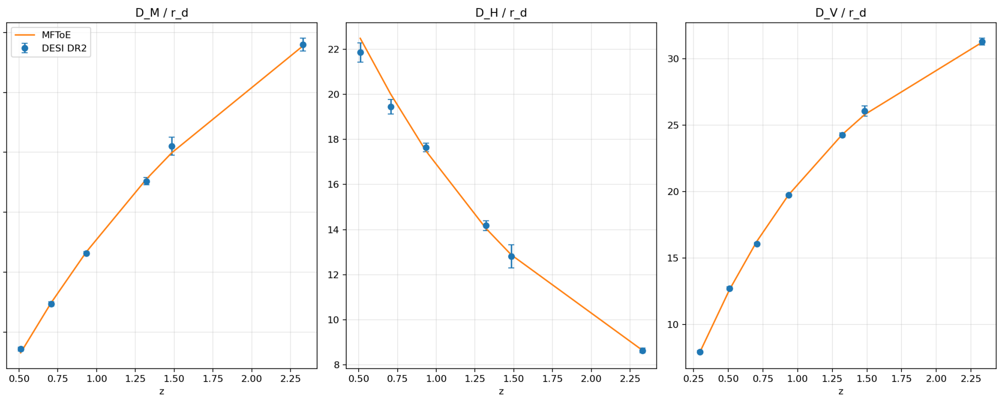

Figure 2: BAO comparison (DESI DR2 compressed) for D_M/r_d, D_H/r_d, D_V/r_d and residuals.

5.2 Relaxion mode: RG+Noise on

Same DESI baseline, but `-mode relaxion -rg on -noise on`.

Relative deviations:

z	$\Delta H/H_{\Lambda\text{CDM}}$	$\Delta d_L/d_{L,\Lambda\text{CDM}}$
0.20	+0.388%	−0.152%
0.50	+0.659%	−0.380%
1.00	+0.575%	−0.491%
2.00	+0.246%	−0.461%
3.00	+0.084%	−0.406%

EoS means: $\overline{w}_\phi \simeq -0.917$, $\overline{w}_{\text{tot}} \simeq -0.289$.
BAO χ^2 (no covariance, 19 points): $\chi^2 = 19.171$ (reduced $\chi^2 = 1.009$).

Figures. (add when available)

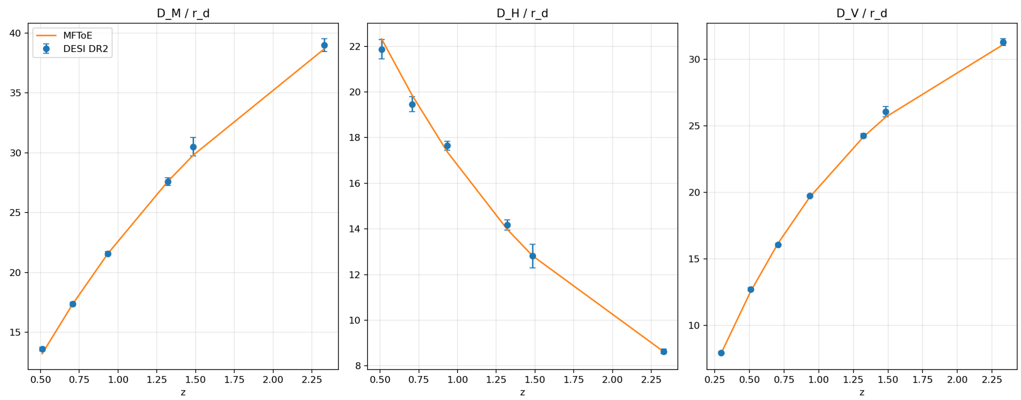

Figure 3: BAO comparison for relaxion run (DESI DR2 compressed).

5.3 Covariance sweep (synthetic)

In absence of an official published covariance file in the same layout as our CSV, we generated a synthetic covariance with uniform correlation coefficient ρ across the 19 compressed quantities. The sweep yields:

ρ	0.0	0.2	0.3	0.4	0.5
χ^2 (baseline)	16.12	16.66	17.79	19.60	22.42

All are consistent (reduced $\chi^2 \sim 0.85$–1.18).

Note on H_0 tension test. Running the baseline with $H_0^{\text{phys}} = 73\,\text{km}\,\text{s}^{-1}\,\text{Mpc}^{-1}$ (keeping DESI DR2 BAO r_d) yields a huge mismatch: $\chi^2 \approx 852.4$, as expected. MFToE can accommodate dynamics, but BAO alone strongly constrains the H_0–r_d combination.

6 Reproducibility & Project Layout

```
project/
      mftoe_vacuum_astropy.py      # main integrator + Astropy baseline
      analysis/
```

```
        bao_compare.py            # BAO comparison &
        make_cov_from_csv.py      # synthetic covariance builder
        compare_runs.py           # quick overlay plots
    data/
        desi_dr2/
          bao_summary.csv         # compressed DESI DR2 BAO (z, DM/DH/DV, errors)
          bao_cov.npy             # optional covariance (synthetic or real)
          iminuit/base/desi-bao-all/bestfit.minimum
    runs/                       # outputs (CSV/PNG)
    scripts/
      run_baselines.sh          # baseline run + BAO check
      cov_sweep.sh              # rho-sweep for synthetic covariance
      scan_relaxion.sh          # small grid over (gamma, sigma_noise)
```

Quick start (conda or venv recommended).

```
pip install -r requirements.txt
bash scripts/run_baselines.sh
# results in runs/: mftoe_vacuum_astropy.{csv,png}, bao_check.png
```

BAO comparison directly.

```
python analysis/bao_compare.py \
  --model-csv runs/mftoe_vacuum_astropy.csv \
  --bao-csv  data/desi_dr2/bao_summary.csv \
  --H0phys 67.36 --rd 150.754 \
  --out runs/bao_check
```

Relaxion scan.

```
bash scripts/scan_relaxion.sh
# outputs many runs/relaxion_g{...}_s{...}.{csv,png}
```

7 Discussion

The single-field MFToE slice with a dynamic vacuum controller χ can sit *comfortably* within DESI DR2 constraints: sub-percent shifts in H and d_L lead to reduced χ^2 within ~ 1. The `relaxion` mode plus RG and noise slightly increases χ^2 (still consistent), modeling a gently evolving $w_{\rm tot} \gtrsim -1$ compatible with hints for dynamical dark energy.

Interpretation. Within this toy/EFT approach, MFToE-inspired principles (e.g. balance/creativity encoded via χ and mild running) can describe late-time dynamics *without* running afoul of BAO data. A multi-field generalization and a joint $\{\mathrm{BAO}, \mathrm{SNIa}, \mathrm{CMB}\}$ fit would be the natural next step.

8 Future Work

- Multi-field extension ($\phi_i + \chi$ with $i = 1 \dots 5$), with a small field-space metric G_{ij}.
- Include curvature Ω_k and full covariance from DESI DR2 likelihoods; add SNIa (Pantheon+, Union3) and CMB priors.
- Early-Universe consistency: r_d from a Boltzmann code (CLASS/CAMB) instead of fixed input.
- Robust parameter scans and MCMC, e.g. using `emcee` or `cobaya`, exported to GetDist.

Acknowledgments & Licensing

We acknowledge the DESI Collaboration for making these data publicly available.

Software. We use `Astropy` [1, 2, 3] for cosmology baselines.

References

[1] Astropy Collaboration et al., *Astropy: A community Python package for astronomy*, A&A **558**, A33 (2013).

[2] Astropy Collaboration et al., *The Astropy Project: Building an Open-science Project and Status of the v2.0 Core Package*, AJ **156**, 123 (2018).

[3] Astropy Collaboration et al., *The Astropy Project: Sustaining and Growing a Community-oriented Open-source Project and the Latest Major Release (v5.0) of the Core Package*, ApJ **935**, 167 (2022).

Appendix A: CLI Cheat Sheet

```
# Baseline with DESI bestfit, Astropy on
python mftoe_vacuum_astropy.py \
  --astropy on \
  --desi-bestfit data/desi_dr2/iminuit/base/desi-bao-all/bestfit.minimum \
  --out runs/mftoe_vacuum_astropy

# Relaxion + RG + Noise
python mftoe_vacuum_astropy.py \
  --mode relaxion --rg on --noise on --astropy on \
  --desi-bestfit data/desi_dr2/iminuit/base/desi-bao-all/bestfit.minimum \
  --out runs/relaxion

# BAO comparison (no covariance)
python analysis/bao_compare.py \
  --model-csv runs/mftoe_vacuum_astropy.csv \
  --bao-csv  data/desi_dr2/bao_summary.csv \
  --H0phys 67.36 --rd 150.754 \
  --out runs/bao_check

# Synthetic covariance and BAO
python analysis/make_cov_from_csv.py data/desi_dr2/bao_summary.csv data/desi_dr2/
    bao_cov.npy 0.3
python analysis/bao_compare.py \
  --model-csv runs/mftoe_vacuum_astropy.csv \
  --bao-csv  data/desi_dr2/bao_summary.csv \
  --H0phys 67.36 --rd 150.754 \
  --cov data/desi_dr2/bao_cov.npy \
  --out runs/bao_check_cov
```