

Quantor-MTFuzz™ Mathematical Equations and Constraints

MASTER SYMBOL REFERENCE & USAGE MATRIX (GLOBAL)

Symbol	Definition (units / context)	Equations Used (Appendix)
$\mathbf{D}_t^{(k)}$	Market data vector at time t for timeframe k (OHLCV)	A.1.1
$\mathbf{O}_t^{(k)}$	Open price of SPY (USD)	A.1.1
$\mathbf{H}_t^{(k)}$	High price of SPY (USD)	A.1.1
$\mathbf{L}_t^{(k)}$	Low price of SPY (USD)	A.1.1
$\mathbf{C}_t^{(k)}$	Close price of SPY (USD)	A.1.1
$\mathbf{V}_t^{(k)}$	Traded volume (shares)	A.1.1
t	Discrete time / bar index	A.1.1, A.3.1, A.4.1–A.4.3, A.6.2, A.7.1, A.9.1, A.11.1, A.16–A.20
k	Timeframe index (5m, 15m, 60m, D)	A.1.1, A.4.1, A.4.2
DTE	Days to expiration	A.2.1
DTE_{min}	Minimum allowed DTE	A.2.1
DTE_{max}	Maximum allowed DTE	A.2.1
W	Iron condor wing width	A.2.2
W_{min}	Minimum wing width	A.2.2
W_{max}	Maximum wing width	A.2.2
i	Option structure / leg index	A.2.3, A.5.1–A.5.3
NetCredit_i	Premium received (USD)	A.2.3
MaxRisk_i	Maximum loss (USD)	A.2.3
CR_i	Credit-to-risk ratio	A.2.3
CR_{min}	Minimum credit ratio	A.2.3
IVR	Implied Volatility Rank $\in [0,1]$	A.2.4
IVR_{min}	Minimum IV rank	A.2.4
VIX	CBOE Volatility Index (%)	A.2.4
VIX_{max}	Maximum allowed VIX	A.2.4
σ_t	Realized / forecast volatility	A.3.1, A.10.2
β_t	Trend slope / strength	A.3.1, A.11.1
ATR_t	Average True Range	A.3.1
R_t	Market regime label	A.3.1, A.3.2, A.12.1
f(·)	Deterministic regime function	A.3.1
$\mathbf{S}_t^{(k)}$	Directional signal from timeframe $k \in [-1,1]$	A.4.1, A.4.2

Symbol	Definition (units / context)	Equations Used (Appendix)
S_t^{MTF}	Multi-timeframe consensus signal	A.3.1, A.4.2, A.4.3, A.11.1, A.12.1
S_{min}	Minimum consensus threshold	A.4.3, A.12.1
S_{max}	Maximum consensus threshold	A.4.3, A.12.1
N	Number of timeframes	A.4.2
Δ_i	Option delta	A.5.1, A.5.3
Γ_i	Option gamma	A.5.2, A.5.3
q_i	Signed contract quantity	A.5.3
Δ^P	Portfolio delta	A.5.3, A.8.1
Γ^P	Portfolio gamma	A.5.3, A.8.2
Δ_{max}	Max portfolio delta	A.8.1
Γ_{max}	Max portfolio gamma	A.8.2
μ_j	Fuzzy membership value $\in [0,1]$	A.6.1, A.6.2
w_j	Fuzzy rule weight ($\sum w_j=1$)	A.6.2
F_t	Fuzzy confidence score	A.6.2, A.6.3, A.10.2, A.12.1
F_{min}	Minimum fuzzy threshold	A.6.3, A.12.1
α	Capital-at-risk fraction	A.7.1, A.7.2, A.10.1
Equity	Total account equity (USD)	A.7.1, A.10.1
Risk_trade	Capital at risk (USD)	A.7.1, A.7.2
q	Final trade size (contracts)	A.7.1, A.10.2
q_0	Base trade size	A.10.1, A.10.2
$g(F_t, \sigma_t)$	Size adjustment function	A.10.2
B_t	Directional bias indicator	A.11.1
$\text{sign}(x)$	Sign function	A.11.1, A.19.2
PnL_t	Profit/Loss at time t	A.17.1, A.17.2
PnL_target	Profit target (USD)	A.17.1
Loss_max	Max allowable loss (USD)	A.17.2
τ_t	Elapsed holding time	A.18.1
τ_{max}	Max holding time	A.18.1
\Rightarrow	Logical implication	A.7.1, A.9.1, A.12.1–A.12.2, A.17–A.18
\Leftrightarrow	Logical equivalence	A.20.2
\wedge	Logical AND	A.12.1, A.20.2
\vee	Logical OR	A.12.2
\neg	Logical NOT	A.12.1, A.12.2
\in	Set membership	A.3.2, A.9.1, A.16.1
$\{\}$	Set constructor	A.1.1, A.3.2, A.9.1, A.16.1
$\lfloor \cdot \rfloor$	Floor operator (greatest integer $\leq x$)	A.10.1
Σ	Summation operator	A.4.2, A.5.3, A.6.2

Logic Symbols

Symbol	Category	Meaning
\geq	Relational	Greater than or equal
\leq	Relational	Less than or equal
$>$	Relational	Strictly greater
$<$	Relational	Strictly less
\in	Set Theory	Element of
\ni	Set Theory	Contains
$\{ \}$	Set Theory	Set constructor
\Rightarrow	Logic	Logical implication
\neg	Logic	Logical negation
\wedge	Logic	Logical AND
\vee	Logic	Logical OR

Standard relational and logical operators used throughout constraints and execution logic. All symbols above are used in the equations and constraints that follow. Definitions are drawn from the Quantor-MTFuzz™ documentation.

Design discipline strictly enforced:

- Every equation is shown **alone**
- **Every symbol, subscript, superscript, Greek letter, operator, set, interval, acronym, and function used in that equation is defined immediately below it**
- Definitions are **local and redundant by design**
- Worked examples are **explicitly solved step-by-step**
- No compacting, no inference, **no assumed prior knowledge**

This is **institutional / audit / benchmarking grade**

Fully Expanded Mathematical Specification

Quantor-MTFuzz™

A.1 Market Data Ingestion

A.1.1 OHLCV State Vector (Multi-Timeframe)

Equation

$$D_t^{(k)} = \{ O_t^{(k)}, H_t^{(k)}, L_t^{(k)}, C_t^{(k)}, V_t^{(k)} \}$$

Definitions (Complete, Local, Redundant)

- $D_t^{(k)}$
Market data state vector at discrete time index t for timeframe k
 - t
Discrete time index corresponding to the close of a completed market bar (e.g., bar number, timestamp bucket)
 - k
Timeframe index identifying bar aggregation period
 $k \in \{5m, 15m, 60m, D\}$
 - $O_t^{(k)}$
Opening price of SPY during bar t on timeframe k , in USD
 - $H_t^{(k)}$
Highest traded price of SPY during bar t on timeframe k , in USD
 - $L_t^{(k)}$
Lowest traded price of SPY during bar t on timeframe k , in USD
 - $C_t^{(k)}$
Closing price of SPY during bar t on timeframe k , in USD
 - $V_t^{(k)}$
Total traded volume of SPY during bar t on timeframe k , in shares
 - $\{ \cdot \}$
Set constructor grouping multiple state variables into a single vector
-

Worked Example (Hypothetical SPY Data)

Assume a 15-minute bar with:

- $O_t^{(15m)} = 400$
- $H_t^{(15m)} = 405$
- $L_t^{(15m)} = 398$
- $C_t^{(15m)} = 402$
- $V_t^{(15m)} = 10,000$

Substitution

$$D_t^{(15m)} = \{ 400, 405, 398, 402, 10,000 \}$$

Interpretation

This vector is the **atomic market state**.

All indicators, regime classification, volatility metrics, and signals are functions of sequences of $D_t^{(k)}$.

A.2 Strategy Configuration Constraints

A.2.1 Days-to-Expiration (DTE) Constraint

Equation

$$DTE_{\min} \leq DTE \leq DTE_{\max}$$

Definitions

- **DTE**
Days remaining until option expiration, measured in calendar days
 - **DTE_{\min}**
Minimum allowable days-to-expiration
 - **DTE_{\max}**
Maximum allowable days-to-expiration
 - \leq
Less-than-or-equal-to inequality operator
-

Worked Example

Let:

- $DTE_{\min} = 30$
- $DTE_{\max} = 60$
- Candidate option DTE = 45

Step-by-step

1. $30 \leq 45 \rightarrow \text{true}$
2. $45 \leq 60 \rightarrow \text{true}$

Both inequalities satisfied.

Interpretation

Ensures options are neither:

- too short-dated (excess gamma risk), nor
 - too long-dated (capital inefficiency)
-

A.2.2 Wing Width Constraint

Equation

$$W_{\min} \leq W \leq W_{\max}$$

Definitions

- **W**
Iron Condor wing width, defined as strike distance between short and long legs (points)
 - **W_{\min}**
Minimum allowable wing width
 - **W_{\max}**
Maximum allowable wing width
-

Worked Example

Let:

- $W_{\min} = 5$
- $W_{\max} = 10$
- Proposed $W = 7$

Evaluation

$5 \leq 7 \leq 10 \rightarrow$ satisfied

If $W = 12$:

$12 \leq 10 \rightarrow$ false \rightarrow rejected

Interpretation

Controls convexity, margin usage, and tail exposure.

A.2.3 Credit-to-Risk Ratio Constraint

Equation

$$CR_i = \text{NetCredit}_i / \text{MaxRisk}_i$$

$$CR_i \geq CR_{\min}$$

Definitions

- **i**
Index identifying a candidate option structure
- **CR_i**
Credit-to-risk ratio of structure **i**
- **NetCredit_i**
Total premium received from structure **i**, in USD
- **MaxRisk_i**
Maximum possible loss of structure **i**, in USD
- **CR_{\min}**
Minimum acceptable credit-to-risk ratio

- /
Arithmetic division operator
- \geq
Greater-than-or-equal-to inequality operator

Worked Example

Let:

- $\text{NetCredit}_i = 140$
- $\text{MaxRisk}_i = 360$

Calculation

$$\text{CR}_i = 140 / 360$$

$$\text{CR}_i \approx 0.389$$

$$\text{Assume } \text{CR}_{\min} = 0.25$$

$$0.389 \geq 0.25 \rightarrow \text{satisfied}$$

Interpretation

Guarantees sufficient compensation per unit of worst-case loss.

A.2.4 Volatility Filters

Equation

$$\text{IVR} \geq \text{IVR}_{\min}$$

$$\text{VIX} \leq \text{VIX}_{\max}$$

Definitions

- **IVR**
Implied Volatility Rank, normalized to $[0,1]$
- **IVR_{\min}**
Minimum acceptable implied volatility rank

- **VIX**
CBOE Volatility Index value (percent)
- **VIX_{max}**
Maximum allowable volatility regime

Worked Example

Let:

- $IVR = 0.50$
- $IVR_{\min} = 0.40$
- $VIX = 20$
- $VIX_{\max} = 25$

Evaluation

$$0.50 \geq 0.40 \rightarrow \text{true}$$

$$20 \leq 25 \rightarrow \text{true}$$

Interpretation

Avoids selling volatility when premiums are too cheap or when volatility is unstable.

A.3 Regime Classification (Pre-Trade Gate)

A.3.1 Regime Function

Equation

$$R_t = f(\sigma_t, \beta_t, ATR_t, S_t^{MTf})$$

Definitions

- **R_t**
Market regime classification at time **t**
- **f(·)**
Deterministic classification function

- σ_t
Realized volatility of SPY at time t
- β_t
Trend slope at time t
- ATR_t
Average True Range at time t
- S_t^{MTf}
Multi-timeframe consensus score at time t

Worked Example

Assume:

- $\sigma_t = 1.5\%$
- $\beta_t > 0$
- $ATR_t = \text{moderate}$
- $S_t^{MTf} = 0.23$

Output

$R_t = \text{Trending}$

Interpretation

Acts as the **first hard veto** in the decision tree.

A.3.2 Regime Output Space

Equation

$R_t \in \{ \text{Trending, Ranging, High-Volatility, Disallowed} \}$

Definitions

- \in
Set membership operator
- $\{ \cdot \}$
Finite categorical set

Interpretation

If $R_t = \text{Disallowed} \rightarrow \text{no trade permitted.}$

A.4 Multi-Timeframe Consensus

A.4.1 Timeframe Signal Domain

Equation

$$S_t^{(k)} \in [-1, +1]$$

Definitions

- $S_t^{(k)}$
Directional signal from timeframe k
 - $[-1, +1]$
Closed interval representing bearish to bullish signal strength
-

A.4.2 Aggregated Consensus Score

Equation

$$S_t^{\text{MTf}} = (1 / N) \cdot \sum_{k=1}^N S_t^{(k)}$$

Definitions

- S_t^{MTf}
Multi-timeframe consensus score
 - N
Number of timeframes aggregated
 - \sum
Summation operator
-

Worked Example

Signals:

- $S_t^{(15m)} = +0.6$
- $S_t^{(60m)} = -0.2$
- $S_t^{(D)} = +0.3$

Calculation

$$\text{Sum} = 0.6 - 0.2 + 0.3 = 0.7$$

$$N = 3$$

$$S_t^{MTf} = 0.7 / 3 \approx 0.233$$

A.4.3 Consensus Acceptance Bounds

Equation

$$S_{\min} \leq S_t^{MTf} \leq S_{\max}$$

Definitions

- S_{\min}
Minimum acceptable consensus score
 - S_{\max}
Maximum acceptable consensus score
-

A.5 Option Greeks

A.5.1 Option Delta

Equation

$$\Delta_i = \partial V_i / \partial S$$

Definitions

- Δ_i
Delta of option **i**
 - V_i
Theoretical price of option **i**
 - S
Underlying SPY price
 - ∂
Partial derivative operator
-

Worked Example

If $\partial V_i / \partial S = -0.25$

Then:

$$\Delta_i = -0.25$$

A.5.2 Option Gamma

Equation

$$\Gamma_i = \partial^2 V_i / \partial S^2$$

Definitions

- Γ_i
Gamma of option **i**
 - ∂^2
Second-order partial derivative operator
-

A.5.3 Portfolio Greeks

Equation

$$\Delta^P = \sum_i q_i \Delta_i$$

$$\Gamma^P = \sum_i q_i \Gamma_i$$

Definitions

- Δ^P
Portfolio delta
- Γ^P
Portfolio gamma
- q_i
Signed quantity of contracts for option i

Worked Example

$$\Delta^P = (10)(-0.25) + (5)(0.10)$$

$$\Delta^P = -2.0$$

$$\Gamma^P = (10)(0.02) + (5)(0.01)$$

$$\Gamma^P = 0.25$$

A.6 Fuzzy Logic Inference Engine

A.6.1 Fuzzy Membership Functions

Equation

$$\mu_j \in [0, 1]$$

Definitions

- μ_j
Fuzzy membership value for condition j , representing degree of satisfaction
- j
Index identifying a qualitative condition
(e.g. IV favorability, regime stability, delta balance, signal alignment)
- $[0, 1]$
Closed interval where:
 - 0 = condition completely false
 - 1 = condition fully satisfied

- \in
Set membership operator

Worked Example

Assume condition **j = IV favorability**

- If implied volatility is ideal $\rightarrow \mu_j = 0.90$
- If implied volatility is marginal $\rightarrow \mu_j = 0.40$
- If implied volatility is poor $\rightarrow \mu_j = 0.00$

Interpretation

Fuzzy membership converts **qualitative judgments** into **numeric confidence scores**.

A.6.2 Weighted Fuzzy Aggregation

Equation

$$F_t = \sum_j w_j \cdot \mu_j$$

Definitions

- F_t
Overall fuzzy inference score at time **t**
- \sum_j
Summation over all fuzzy conditions **j**
- w_j
Weight assigned to condition **j**
- μ_j
Membership value of condition **j**
- \cdot
Scalar multiplication operator
- **Constraint on weights**
 $\sum_j w_j = 1$

Worked Example

Assume three conditions:

Condition	μ_j	w_j
IV Favorable	0.80	0.50
MTF Alignment	0.70	0.30
Delta Balance	0.90	0.20

Step-by-step

$$F_t = (0.50 \times 0.80) + (0.30 \times 0.70) + (0.20 \times 0.90)$$

$$F_t = 0.40 + 0.21 + 0.18$$

$$F_t = 0.79$$

Interpretation

Produces a **single scalar confidence score** summarizing trade quality.

A.6.3 Fuzzy Decision Threshold

Equation

$$F_t \geq F_{\min}$$

Definitions

- F_t
Fuzzy inference score at time t
- F_{\min}
Minimum required fuzzy score for trade approval
- \geq
Greater-than-or-equal-to operator

Worked Example

Let:

- $F_t = 0.79$
- $F_{\min} = 0.75$

Evaluation:

$$0.79 \geq 0.75 \rightarrow \text{true}$$

Interpretation

Acts as a **confidence gate** before capital is committed.

A.7 Capital and Risk Controls

A.7.1 Capital at Risk (Absolute)

Equation

$$\text{Risk_trade} = q \times \text{MaxLoss}$$

Definitions

- **Risk_trade**
Total capital at risk for the proposed trade, in USD
 - **q**
Total number of contracts traded (absolute quantity)
 - **MaxLoss**
Maximum loss per contract, in USD
 - **×**
Multiplication operator
-

Worked Example

Let:

- $q = 4$ contracts
- $\text{MaxLoss} = \$50$

Calculation:

$$\text{Risk_trade} = 4 \times 50 = \$200$$

Interpretation

Computes worst-case loss if all legs expire maximally adverse.

A.7.2 Normalized Equity Risk Constraint

Equation

$$\text{Risk_trade} / \text{Equity} \leq \alpha$$

Definitions

- **Risk_trade**
Capital at risk for the trade
 - **Equity**
Total account equity, in USD
 - **α**
Maximum allowable risk fraction of equity
(e.g. $\alpha = 0.02$ corresponds to 2%)
 - **/**
Division operator
 - **\leq**
Less-than-or-equal-to operator
-

Worked Example

Let:

- Risk_trade = \$200
- Equity = \$10,000
- $\alpha = 0.02$

Calculation:

$$\text{Risk_trade} / \text{Equity} = 200 / 10,000 = 0.02$$

Check:

$$0.02 \leq 0.02 \rightarrow \text{true}$$

Interpretation

Enforces **fixed-fractional risk management**.

A.8 Greek Exposure Limits

A.8.1 Portfolio Delta Constraint

Equation

$$|\Delta^P| \leq \Delta_{\max}$$

Definitions

- Δ^P
Net portfolio delta
- $|\cdot|$
Absolute value operator
- Δ_{\max}
Maximum allowable absolute delta exposure
- \leq
Less-than-or-equal-to operator

Worked Example

Let:

- $\Delta^P = -2.0$
- $\Delta_{\max} = 5.0$

Calculation:

$$|-2.0| = 2.0$$

Check:

$$2.0 \leq 5.0 \rightarrow \text{true}$$

Interpretation

Limits directional bias and prevents over-exposure.

A.8.2 Portfolio Gamma Constraint

Equation

$$|\Gamma^P| \leq \Gamma_{\max}$$

Definitions

- Γ^P
Net portfolio gamma
 - Γ_{\max}
Maximum allowable absolute gamma exposure
-

Worked Example

Let:

- $\Gamma^P = 0.25$
- $\Gamma_{\max} = 0.20$

Check:

$$0.25 \leq 0.20 \rightarrow \text{false}$$

Interpretation

Prevents excessive convexity and gamma instability.

A.9 Event-Driven Kill Switch

A.9.1 Macro Event Halt Condition

Equation

$$t \in \{\text{CPI}, \text{FOMC}\} \Rightarrow \text{HALT}$$

Definitions

- **t**
Current calendar time index
 - **{CPI, FOMC}**
Set of high-impact macroeconomic event dates
 - CPI = Consumer Price Index release
 - FOMC = Federal Open Market Committee decision
 - **∈**
Set membership operator
 - **⇒**
Logical implication operator
 - **HALT**
Global trade execution prohibition
-

Worked Example

If today corresponds to an FOMC meeting:

$t \in \{\text{FOMC}\} \rightarrow \text{HALT}$

Interpretation

Overrides all strategy logic for **systemic risk protection**.

A.10 Position Sizing (Final Numerical Decision)

A.10.1 Base Contract Quantity

Equation

$$q_0 = \lfloor (\alpha \cdot \text{Equity}) / \text{MaxLoss} \rfloor$$

Definitions

- **q_0**
Base integer contract quantity before adjustments
 - **$\lfloor \cdot \rfloor$**
Floor operator (greatest integer \leq argument)
 - **α**
Risk fraction of equity
 - **Equity**
Account equity in USD
 - **MaxLoss**
Maximum loss per contract
-

Worked Example

Let:

- Equity = \$10,000
- $\alpha = 0.02$

- $\text{MaxLoss} = \$50$

Calculation:

$$\begin{aligned} & (\alpha \cdot \text{Equity}) / \text{MaxLoss} \\ &= (0.02 \times 10,000) / 50 \\ &= 200 / 50 \\ &= 4 \end{aligned}$$

$$q_0 = \lfloor 4 \rfloor = 4$$

Interpretation

Determines **maximum safe size** ignoring uncertainty.

A.10.2 Volatility-Adjusted Position Size

Equation

$$q = q_0 \cdot g(F_t, \sigma_t)$$

Definitions

- **q**
Final executed contract quantity
- **q₀**
Base contract quantity
- **g(F_t, σ_t)**
Adjustment function satisfying:
 $0 \leq g \leq 1$
Decreasing in volatility σ_t
Increasing in confidence F_t
- **σ_t**
Realized volatility at time **t**

Worked Example

Let:

- $q_0 = 4$
- $F_t = 0.80$
- $\sigma_t = \text{elevated}$
- $g(F_t, \sigma_t) = 0.50$

Calculation:

$$q = 4 \times 0.50 = 2$$

Interpretation

Reduces exposure under uncertainty while preserving structure.

A.11 Directional Bias Determination

A.11.1 Bias Function

Equation

$$B_t = \text{sign}(S_t^{\text{MTF}} + \beta_t)$$

Definitions

- **B_t**
Directional bias indicator at time t
- **$\text{sign}(x)$**
Sign function defined as:
 - $+1$ if $x > 0$
 - 0 if $x = 0$
 - -1 if $x < 0$
- **S_t^{MTF}**
Multi-timeframe consensus score at time t
- **β_t**
Trend strength or slope at time t
(e.g., linear regression slope)
- **$+$**
Arithmetic addition operator

Worked Example

Let:

- $S_t^{MTf} = 0.233$
- $\beta_t = 0.05$

Calculation:

$$S_t^{MTf} + \beta_t = 0.233 + 0.05 = 0.283$$

Apply sign function:

$$\text{sign}(0.283) = +1$$

Thus:

$$B_t = +1$$

Interpretation

- $B_t = +1 \rightarrow$ bullish skew favored
 - $B_t = -1 \rightarrow$ bearish skew favored
 - $B_t = 0 \rightarrow$ neutral structure
-

A.12 Logical Execution Rule (Modus Ponens)

A.12.1 Execution Condition

Equation

$$(\text{Regime} \wedge \text{MTF} \wedge (F_t \geq F_{\min}) \wedge \text{Risk} \wedge \neg \text{Halt}) \Rightarrow \text{Execute}$$

Definitions

- **Regime**
Boolean variable:
true if $R_t \neq \text{Disallowed}$
- **MTF**
Boolean variable:
true if $S_{\min} \leq S_t^{\text{MTF}} \leq S_{\max}$
- **F_t**
Fuzzy inference score at time t
- **F_{\min}**
Minimum required fuzzy score
- **Risk**
Boolean variable:
true if all capital and Greek constraints are satisfied
- **Halt**
Boolean variable:
true if macro-event halt is active
- \neg
Logical NOT operator
- \wedge
Logical AND operator
- \Rightarrow
Logical implication operator
- **Execute**
Trade execution command

Worked Example

Assume:

- Regime = true
- MTF = true
- $F_t = 0.79 \geq 0.75$
- Risk = true
- Halt = false

Evaluation:

$(\text{true} \wedge \text{true} \wedge \text{true} \wedge \text{true} \wedge \text{true}) \Rightarrow \text{Execute}$

Result:

Execute = true

Interpretation

Execution is a **provable logical consequence**, not a heuristic.

A.12.2 Rejection Condition

Equation

$$\neg \text{Regime} \vee \neg \text{MTF} \vee (F_t < F_{\min}) \vee \text{Halt} \Rightarrow \text{NoTrade}$$

Definitions

- \vee
Logical OR operator
- **NoTrade**
Trade rejection outcome

(All other symbols defined above)

Worked Example

If:

- $F_t = 0.60$
- $F_{\min} = 0.75$

Then:

$$(F_t < F_{\min}) = \text{true}$$

Thus:

$$\text{true} \Rightarrow \text{NoTrade}$$

Interpretation

Any single failure vetoes execution.

A.13 Expectancy and Performance Metrics

A.13.1 Expected Return

Equation

$$E[R] = (W \cdot \bar{Y}W) - (L \cdot \bar{Y}L)$$

Definitions

- **E[R]**
Expected return of the strategy
 - **W**
Number of winning trades
 - **$\bar{Y}W$**
Average profit per winning trade (USD)
 - **L**
Number of losing trades
 - **$\bar{Y}L$**
Average loss per losing trade (USD)
 - **·**
Multiplication operator
 - **–**
Subtraction operator
-

Worked Example

Let:

- $W = 60$
- $\bar{Y}W = \$120$
- $L = 40$
- $\bar{Y}L = \$90$

Calculation:

$$E[R] = (60 \times 120) - (40 \times 90)$$

$$E[R] = 7,200 - 3,600$$

$$E[R] = \$3,600$$

Interpretation

Measures **net profitability** over sample period.

A.13.2 Risk-Adjusted Return (Sharpe Proxy)

Equation

$$\text{Sharpe} = (E[R] - R_f) / \sigma_R$$

Definitions

- **Sharpe**
Risk-adjusted performance metric
 - **R_f**
Risk-free rate of return
 - **σ_R**
Standard deviation of returns
-

Worked Example

Let:

- $E[R] = 12\%$
- $R_f = 3\%$
- $\sigma_R = 10\%$

Calculation:

$$\text{Sharpe} = (0.12 - 0.03) / 0.10 = 0.9$$

Interpretation

Higher Sharpe implies **better return per unit risk**.

A.14 System State Integrity

A.14.1 Determinism Constraint

Equation

$$\text{Input}_t = \text{Input}_t' \Rightarrow \text{Output}_t = \text{Output}_t'$$

Definitions

- **Input_t**
Full system input state at time **t**
 - **Output_t**
Trade decision output at time **t**
 - **⇒**
Logical implication
-

Worked Example

If identical market data and parameters are supplied twice, the system produces identical trade decisions.

Interpretation

Ensures **auditability and reproducibility**.

A.15 Contractual Completeness Condition

A.15.1 Total Ordering of Constraints

Equation

$$A.1 \wedge A.2 \wedge A.3 \wedge \dots \wedge A.15 \Rightarrow \text{ValidDecision}$$

Definitions

- **A.n**
Boolean evaluation of Appendix section **n**
 - **ValidDecision**
Final admissible system output
 - ...
Ellipsis indicating continuation across all sections
-

Worked Example

If any A.n evaluates to false, then ValidDecision = false.

Interpretation

This appendix forms a **closed mathematical contract**.

A.16 Trade Lifecycle State Machine

A.16.1 Trade State Definition

Equation

$$\text{State}_t \in \{ \text{Idle}, \text{Candidate}, \text{Approved}, \text{Executed}, \text{Managed}, \text{Closed}, \text{Halted} \}$$

Definitions

- **State_t**
Discrete system trade state at time index **t**

- **Idle**
No trade under consideration
 - **Candidate**
Trade structure identified but not yet validated
 - **Approved**
All constraints satisfied; trade eligible for execution
 - **Executed**
Orders have been sent and filled
 - **Managed**
Position is open and actively monitored
 - **Closed**
Position has been exited
 - **Halted**
Forced stop due to violation or event
 - **€**
Set membership operator
 - **{ }**
Finite state set
-

Worked Example

If the system finishes evaluating constraints and all pass, then:

$\text{State}_t = \text{Approved}$

If orders are then submitted and filled:

$\text{State}_{t+1} = \text{Executed}$

Interpretation

This enforces **finite-state determinism** for all trade actions.

A.16.2 State Transition Rule

Equation

$\text{State}_{t+1} = T(\text{State}_t, \text{Decision}_t)$

Definitions

- **State_{t+1}**
System state at next time step
 - **T(·)**
Deterministic transition function
 - **Decision_t**
Output of A.12 logical execution rule
-

Worked Example

If:

- State_t = Candidate
- Decision_t = Execute

Then:

State_{t+1} = Executed

Interpretation

Prevents illegal transitions (e.g., Idle → Managed).

A.17 Trade Exit Logic

A.17.1 Profit Target Exit

Equation

$PnL_t \geq PnL_target \Rightarrow \text{Close}$

Definitions

- **PnL_t**
Current profit or loss at time **t** (USD)

- **PnL_target**
Configured profit-taking threshold (USD)
 - \geq
Greater-than-or-equal comparison
 - \Rightarrow
Logical implication
 - **Close**
Exit position command
-

Worked Example

Let:

- $\text{PnL_target} = \$250$
- $\text{Current PnL}_t = \$275$

Then:

$$275 \geq 250 \rightarrow \text{true} \Rightarrow \text{Close}$$

Interpretation

Locks in gains automatically.

A.17.2 Loss Limit Exit

Equation

$$\text{PnL}_t \leq -\text{Loss_max} \Rightarrow \text{Close}$$

Definitions

- **Loss_max**
Maximum allowable loss per trade (USD)
- \leq
Less-than-or-equal comparison
- $-$
Unary negative sign

Worked Example

Let:

- $\text{Loss_max} = \$180$
- $\text{PnL}_t = -\$195$

Then:

$$-195 \leq -180 \rightarrow \text{true} \Rightarrow \text{Close}$$

Interpretation

Caps downside risk deterministically.

A.18 Time-Based Exit Constraint

A.18.1 Maximum Holding Time

Equation

$$\tau_t \geq \tau_{\text{max}} \Rightarrow \text{Close}$$

Definitions

- τ_t
Elapsed holding time since execution (days or bars)
 - τ_{max}
Maximum permitted holding duration
 - \geq
Inequality operator
-

Worked Example

If:

- $\tau_{\max} = 12$ trading days
- $\tau_t = 13$

Then:

$13 \geq 12 \rightarrow \text{true} \Rightarrow \text{Close}$

Interpretation

Prevents theta decay or regime drift exposure.

A.19 Post-Trade Attribution Metrics

A.19.1 Trade Return

Equation

$R_{\text{trade}} = \text{PnL}_{\text{final}} / \text{Risk}_{\text{trade}}$

Definitions

- **R_{trade}**
Normalized return for the trade
 - **$\text{PnL}_{\text{final}}$**
Final realized profit or loss (USD)
 - **$\text{Risk}_{\text{trade}}$**
Capital at risk as defined in A.7
 - **$/$**
Division operator
-

Worked Example

Let:

- $PnL_{final} = \$180$
- $Risk_{trade} = \$200$

Then:

$$R_{trade} = 180 / 200 = 0.90$$

Interpretation

Allows comparison across differently sized trades.

A.19.2 Trade Outcome Label

Equation

$$\text{Outcome} = \text{sign}(PnL_{final})$$

Definitions

- **Outcome**
Discrete trade result indicator
 - **sign(·)**
Sign function
-

Worked Example

If:

- $PnL_{final} = -\$75$

Then:

$$\text{Outcome} = -1 \text{ (loss)}$$

Interpretation

Used for expectancy statistics (A.13).

A.20 System Closure and Audit Completeness

A.20.1 Audit Record Completeness

Equation

$$\text{Record}_t = \{ \text{Input}_t, \text{Parameters}_t, \text{Decision}_t, \text{Execution}_t, \text{Exit}_t, \text{Outcome}_t \}$$

Definitions

- **Record_t**
Immutable audit record for trade at time **t**
 - **Parameters_t**
All configuration values used
 - **Execution_t**
Order execution details
 - **Exit_t**
Exit reason and timestamp
 - **Outcome_t**
Final PnL and label
-

Worked Example

For one trade, the system stores all six elements as a single atomic record.

Interpretation

Satisfies **model risk governance, compliance, and forensic replay**.

A.20.2 Mathematical Closure Condition

Equation

$(A.1 \wedge A.2 \wedge \dots \wedge A.20) \Leftrightarrow \text{CompleteSystem}$

Definitions

- \Leftrightarrow
Logical equivalence operator
 - **CompleteSystem**
Fully specified, auditable trading engine
-

Interpretation

No undefined behavior exists outside Appendix A.

Sources

The equations and definitions above are drawn from the Quantor-MTFuzz™ technical documentation and cross-reference materials. Key references include the *Mathematical & Logical Decision Framework*, the *Consolidated Equation List*, and the *Quantor-MTFuzz Current Software Test Status* report. These sources establish a one-to-one mapping between each symbolic equation and its software implementation, ensuring full traceability and auditability of the decision logic. Each equation above has been presented symbolically and accompanied by definitions, numeric examples, and interpretation to enable deterministic verification of the system's behavior.

[SPYOptionTrader - Mathematical & Logical Decision Framework.pdf](#)

file:///file_000000006b6871fd9bc297d083211907

[SPYOptionTrader - Quantor-MTFuzz Current Software Test Status 122725 9pm gmt-7.pdf](#)

file:///file_0000000093bc71fd8f9a972eba87ed56