Design of a Reactive Agent for the Pickup and Delivery Problem : Implementation of a Markov Decision Process

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1 Problem definition

1.1 Markov Decision Process

In a MDP current state is know with certainty, but the reward of transition is not. A MDP is defined by :

$$\textit{A reward function:} \quad \overbrace{R(s,a) \rightarrow \mathbb{R}}^{\textit{Where } s \textit{ denotes a state and } a \textit{ an action}}$$

$$\textit{A probabilistic state transition table:} \quad \overbrace{T(s,a,s') = p(s'|s,a)}^{\textit{Where } s' \textit{ denotes the state the action leads to}}$$

The goal of the process is to find a policy $\pi : S \rightarrow A$ (a mapping from state to action) such that *the average reward is maximized.*

1.2 The Pickup and Delivery Problem

Agents exist in a static environment (a model of Switzerland's road network) described by a graph. Nodes of the graph are called *cities* and it's (weighted) edges are called *roads*.

The pickup and delivery problem is described by a series of tasks spread over the topology, the transportation tasks are described by:

1. Pickup city (and it's position)
2. Delivery city (and it's position)
3. Reward in CHF

1.3 Definitions

1.3.1 Existing tables

The dataset usable for learning is described by two probability tables :

1. $P_{table}(i, j)$: the probability of a task for city j to be present in city i
2. $R_{table}(i, j)$: the average reward given when a task is transported from city i to city j

1.3.2 State

The state an agent is in can be described by :

1. the city it is in
2. the task there is in the city (or the absence thereof)

1.3.3 Action

An action consists in the agent either:

1. going to another city with no task
2. taking a task in the city it's in (which amounts to moving to another city)

And always result in the agent being in a city (different or not from it's starting point, the agent can loop between two city if it maximizes reward) without a task, in another words in a (new) state.

The set A containing all actions is the set of all pairs of states.

1.3.4 Reward

Given a state s the agent is in and for and action a, the reward function for a single action is defined as follows :

$$R_{action}(s, a) = \begin{cases} 0 & \textit{if the agent moves to a city without taking a contract} \\ R_{table}(i(a), j(a)) & \textit{if the agent moves to a city after taking a contract} \end{cases}$$

1.3.5 Reward

Where $i(a)$ is the starting city of the action a and $j(a)$ it's ending city, note that $i(a) = s$.

1.3.6 Discount factor

We have, in order to ensure the convergence of $V(s)$ that :

$$\gamma \in [0, 1[$$

We want our system to optimize for time, not for number of actions take. Because an action may take more or less time. So let's define our $\gamma(a)$ not as a constant as a function of the action taken by the agent :

$$\gamma(a) = \frac{\min\limits_{\alpha \in \text{ actions}} (t(\alpha))}{t(a)}$$

1.3.7 Probability of transition $p(s'|s,a)$

For a given action a taken in the state s, the probability of ending up in the state s' is given by :

$$p(s'|s,a) = \begin{cases} P_{table}(i(s'), j(s')) & \text{if } i(s') = s' \\ 0 & \text{else} \end{cases}$$

Where the state s' is described by :

1. A city denoted $i(s')$
2. A contract to another city described by the city it goes to denoted $j(s')$

2 Solving the MDP

We denote *the value of a state* s as $V(s)$. This value represents "*the potential rewards from this state onwards*". In order to ensure $V(s_i) < \infty \; \forall i$ (and make the problem solvable) we introduce a *discount factor* $\gamma \in [0...1[$.

$$V(s_i) = R(s_i) + \gamma \cdot V(T(s_i), a(s_i))$$