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ARTIFICIAL INTELLIGENCE

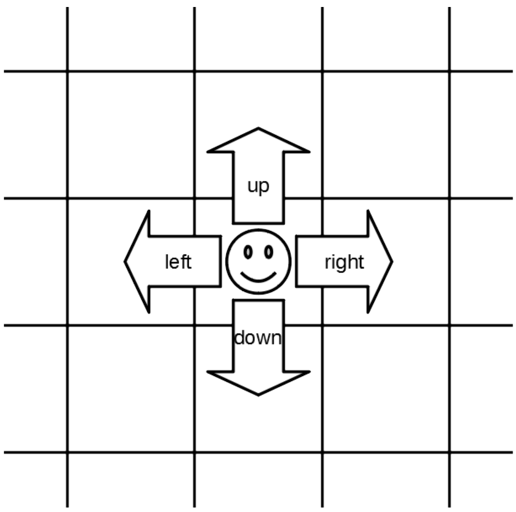
Theoretical Exercise Sheet 2.

Solutions due Tuesday, May 19, 23:59.

Total points of the sheet: 22

Exercise 1: Heuristic Function — 5 Points

Consider an unbounded regular 2D grid. The start state is at the origin, $(0, 0)$, and the goal state is at (x, y). You can move left, right, up and down. For example, if you are at the start and move to the right, you get to position $(1, 0)$. Moving down gets you to position $(0, -1)$. Each move has unit cost.



1. What is the branching factor b for each state in this state space?
2. Is $h((u, v)) = |u - x| + |v - y|$ an admissible heuristic for a state at (u, v)? Justify your answer.
3. At most how many nodes are expanded by A* graph search using the Manhattan distance h_{Man} as your heuristic function? *You may assume that the Manhattan distance is admissible for this state space.*
4. Does h_{Man} remain admissible if some links are removed?

5. Does h_{Man} remain admissible if some links are added between nonadjacent states?

Solution:

1. *The branching factor (number of neighbors of each location) is equal to 4 for every state.*

2. *Yes, this is an admissible heuristic; it is the Manhattan distance metric.*

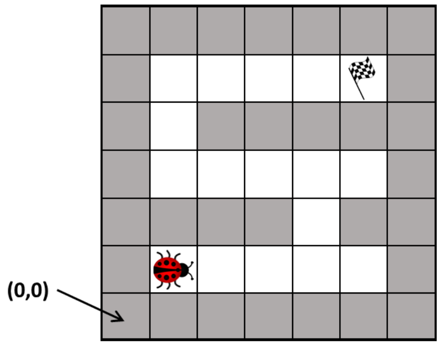
3. *All nodes in the rectangle spanned by the corner nodes* $(0, 0)$ *and* (x, y) *are candidates for the optimal path and could be explored by A* graph search. Three are exactly* $(|x| + 1) \cdot (|y| + 1)$ *states in this rectangle and in the worst case, A* may explore every single one of them.*

4. *Yes; removing links may induce detours, which require more steps. So the heuristic h is an underestimate and hence still admissible.*

5. *No; nonlocal links can reduce the actual path length below the Manhattan distance. For example, if* $(1, 1)$ *was the goal state and an extra link between* $(0, 0)$ *and* $(1, 1)$ *has been added: then* $h((0, 0)) = 2$*, but the actual shortest path has length 1.*

Total points: 5

Exercise 2: Ladybird in a maze 6 Points



We are considering the state space shown above. You control the single ladybird shown in the maze, which must reach a designated target location marked by the finish flag. The ladybird starts at position $(1,1)$ and aims to reach its target in position $(5,5)$. The ladybird can move into any of its four adjacent cells (left, right, up or down). It can not move out of the state space shown above (so will always be in the rectangle between $(0,0)$ and $(6,6)$). The ladybird can either walk along the maze (white square), fly up a wall to a neighbouring grey cell, walk along the wall (grey cell) if it is already on a wall or jump back down from the wall into the maze (in each case only to a neighbouring square). The action cost of moving from a white cell (in the maze) to a grey cell (on the wall) is 2, all other action costs are equal to 1.

1. Give a minimal state representation for the above search problem.

2. Give the size of the state space for this search problem.

3. Is the following heuristic admissible?

$$h_{\text{Man}} = \text{the Manhattan distance from the ladybird to the goal}$$

   If it is admissible, explain why it is. If it is not admissible, give an example of why not and provide a nontrivial admissible heuristic in the space below.

4. Is the h_{Man} heuristic consistent? Either explain why it is or give an example of why it is not.

Solution:

1. *The state is the location of the ladybird as an (x, y) coordinate. The map is known, including walls and the goal, and the actions of the ladybird depend only on its location.*

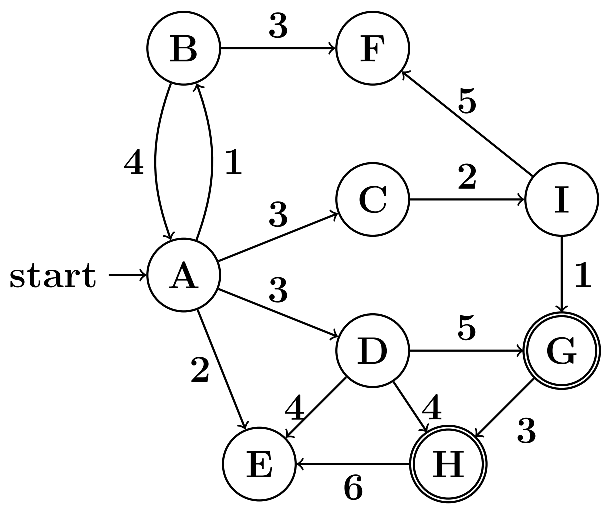
2. *The size of the state space is $7 \cdot 7 = 49$. The ladybird can occupy any free location in a given maze, and any square might be free or a wall in a maze, so any of the 49 locations are possible.*

3. *It is yielded by the relaxed problem where the ladybird passes through walls. It never overestimates because 1. a wall can never decrease the length of a path to the goal and 2. the cost of the ladybird jumping up a wall (2) is higher than the cost of moving.*

4. *To show consistency we must show that for all transitions (s, a, s') the following inequality holds:*
$$h(s) - h(s') \leqslant c(s, a)$$
To do so, we first note that for any successor state s' of s there are two possible cases: either s' is closer in Manhattan distance to the goal or it is further in Manhattan distance from the goal. In the first case then $h(s') = h(s) - 1$ and in the second case $h(s') = h(s) + 1$ and either way $h(s) - h(s') \leqslant 1$. Note further that the cost of moving from s to s' for the ladybird is either 2 (if s is in the maze and s' is on the wall) or 1 (otherwise). In either case $h(s) - h(s') \leqslant 1 \leqslant 1 \leqslant c(s, a)$ as required.

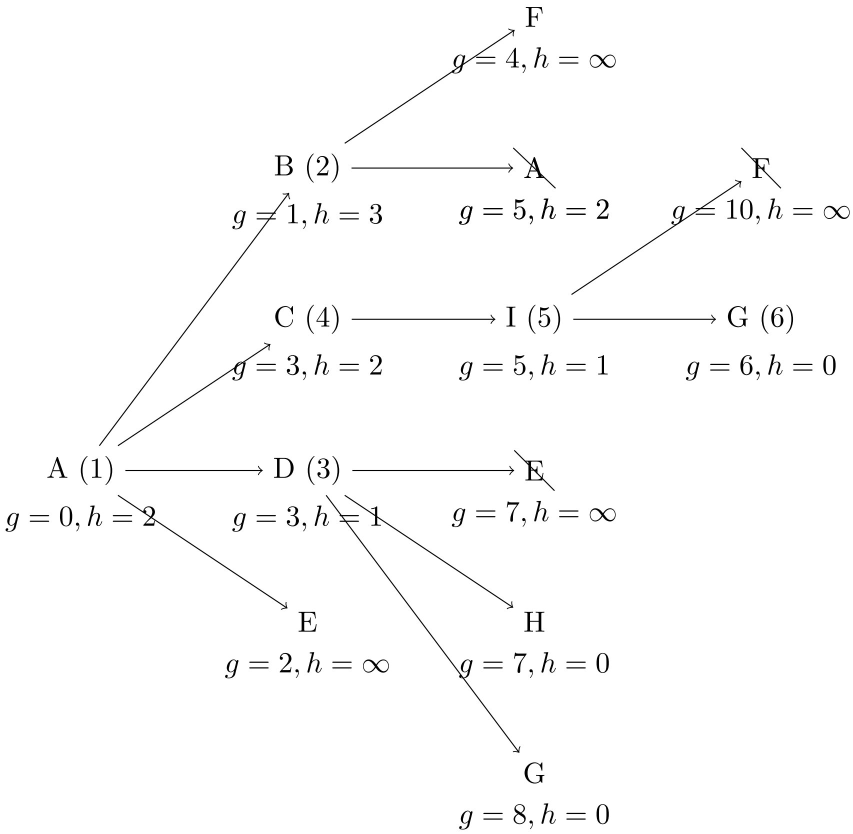
Total points: 6

Exercise 3: A* 7 Points



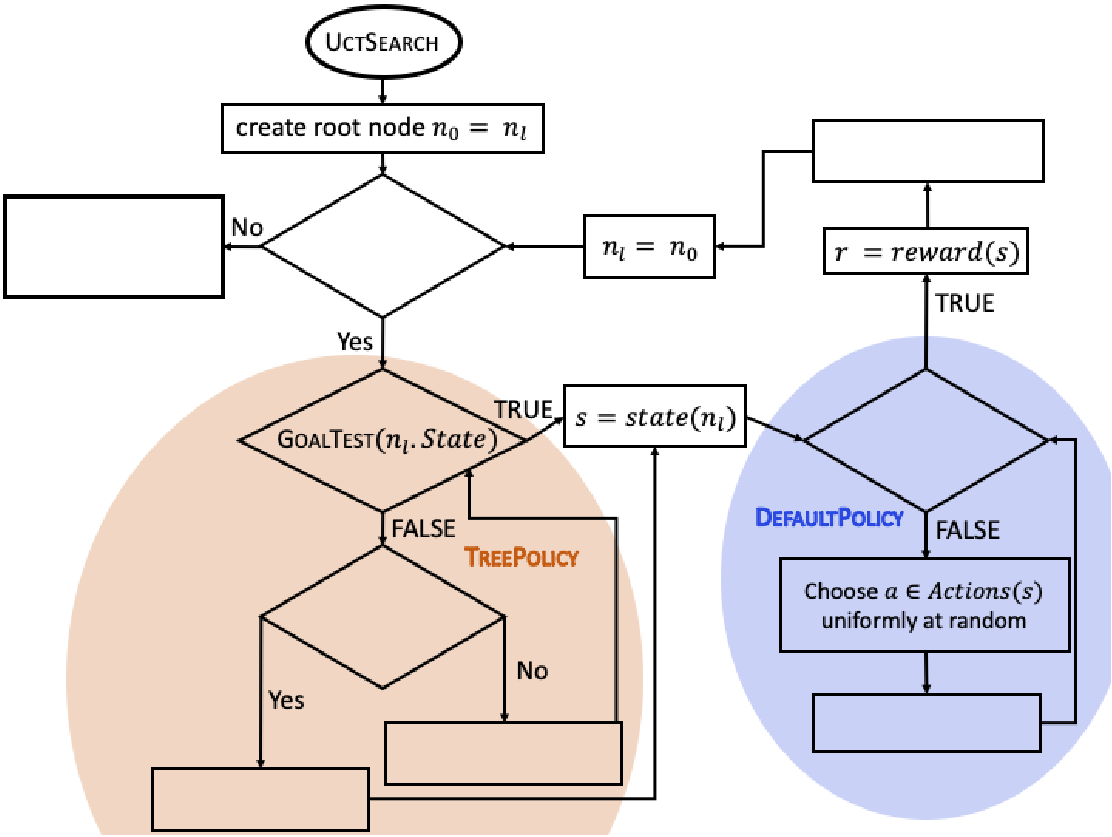
Given is the state space shown above, where A is the initial state and G and H are the two goal states. Run A* Search on this state space. As your heuristic function for a state s, use the minimal number of edges that are needed to reach the goal, or ∞ if s is unsolvable (so for example h(C) = 2). You may assume that this heuristic is both admissible and consistent. Draw the search tree and annotate each node with the g and h value, and the order in which they get expanded. Also draw duplicate nodes and mark them as such by crossing them out (you are still required to note down the values of g and h for these nodes). If the choice of the next node to expand is not unique, expand the lexicographically first node. Give the solution found by A*. Is it optimal? Justify your answer.

Solution:

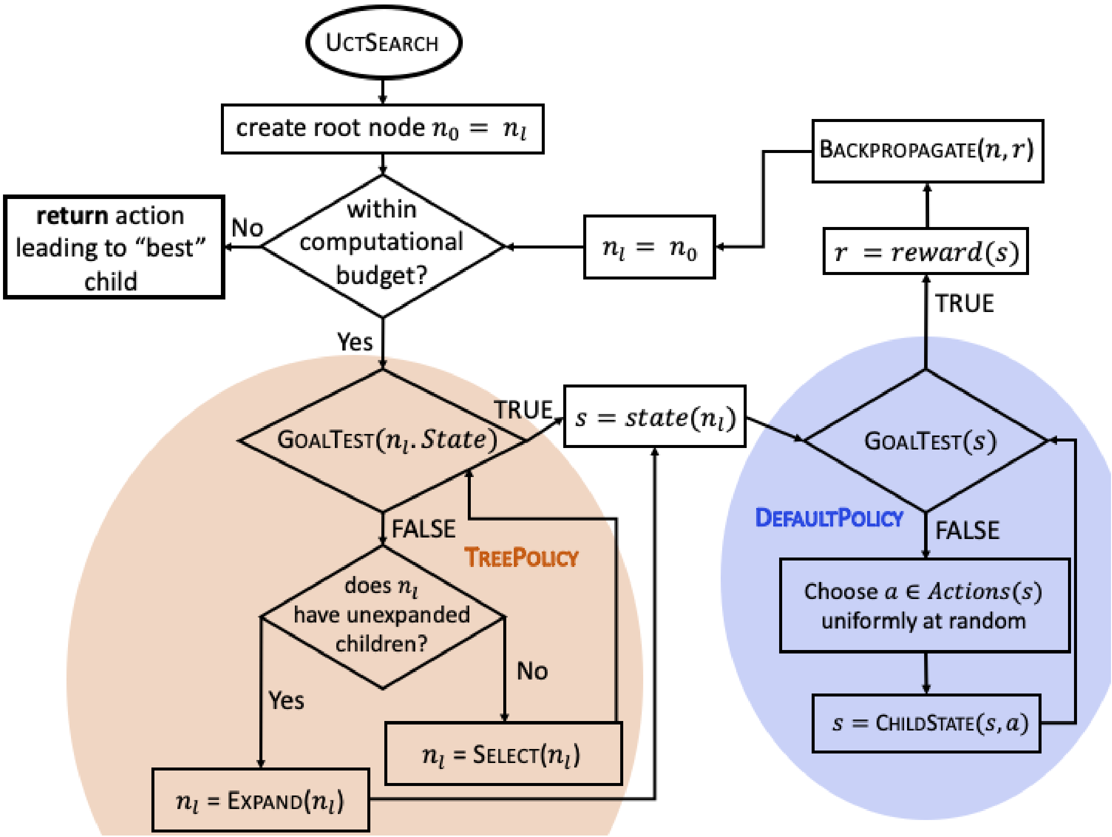

F
$g = 4, h = \infty$
B (2)
$g = 1, h = 3$
A
$g = 5, h = 2$
F
$g = 10, h = \infty$
C (4)
$g = 3, h = 2$
I (5)
$g = 5, h = 1$
G (6)
$g = 6, h = 0$
A (1)
$g = 0, h = 2$
D (3)
$g = 3, h = 1$
E
$g = 7, h = \infty$
E
$g = 2, h = \infty$
H
$g = 7, h = 0$
G
$g = 8, h = 0$

Exercise 4: UCT Flowchart 4 Points

Fill out the empty boxes in the following Flowchart of the UCT Algorithm. Note that you can find the pseudocode of the UCT Algorithm on the Materials page of CMS, which should prove useful in answering this question.



Solution:



UctSearch
create root node $n_0 = n_l$
within computational budget?
No
return action leading to "best" child
Yes
$n_l = n_0$
Backpropagate(n, r)
$r = reward(s)$
TRUE
GoalTest$(n_l.State)$
TRUE
$s = state(n_l)$
FALSE
TreePolicy
does n_l have unexpanded children?
Yes
No
n_l = Select(n_l)
n_l = Expand(n_l)
GoalTest(s)
DefaultPolicy
FALSE
Choose $a \in Actions(s)$ uniformly at random
s = ChildState(s, a)

Submission Instructions

Solutions need to be packaged into a `.zip` file and uploaded in the AI CMS. The `.zip` file has to contain a single folder with name: `AI2020_TE2_mat1_mat2_mat3` where `mat1, mat2, mat3` are the matriculation numbers of the students who submit together. This folder must contain the following files:

- `authors.txt` listing the names and matriculation numbers of all students who submit together. Use one line per student and no spaces: Name;Matriculation number.
- The `.pdf` file containing your solutions.

Do not add any other folder or sub folder, this means place all files directly into `AI2020_TE2_mat1_mat2_mat3`. Do not place any file outside of `AI2020_TE2_mat1_mat2_mat3`.

Only one student of each group needs to do the submission! Remember that this sheet can be submitted in groups of up to three members (all members of the group must however be assigned to the same tutorial). If you are still looking for submission partners, it is recommended to ask in the forum of the CMS in the category "Student Room".