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Exercise Sheet 1.
Solutions due Tuesday, **May 10**, 16:00 – 16:15, in the lecture hall.[1]

Exercise 1. (5 Points)

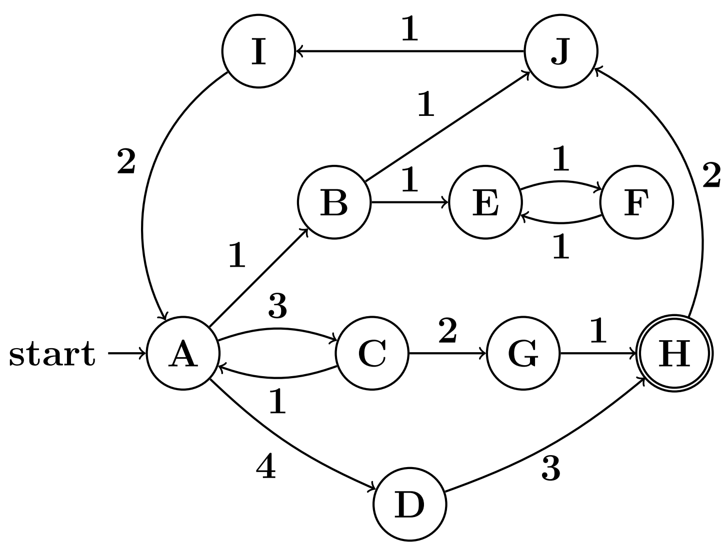


Figure 1: Road map of Exercise 1.

Consider the road map depicted in Figure 1. Assume that you start at city A and you want to find a route to city H. The labels on the edges represent the cost of using the corresponding road. Note that the edges are directed.

[1]Solutions in paper form only, and solution submission only at the stated time at the stated place. At most 3 authors per solution. All authors must be in the same tutorial group. All sheets of your solution must be stapled together. At the top of the first sheet, you must write the names of the authors and the name of your tutor. Your solution must be placed into the correct box for your tutorial group. If you don't comply with these rules, 3 points will be subtracted from your score for this sheet.

(a) Run uniform-cost search on this problem. Draw the search graph and annotate each node with its g value and the order of expansion. Draw duplicate nodes as well, and mark them accordingly. If the choice of the next state to be expanded is not unique, expand the state with smallest lexicographical ordering. Give the solution found by uniform-cost search. Is this solution optimal? Justify your answer.

(b) Run A^* search on this problem. As a heuristic estimate for a state s, use the minimal number of edges that are needed to reach H from s (or ∞ if there is no path from s to H). Again, draw the search graph and annotate each node with the g and h value as well as the order of expansion. Draw duplicate nodes as well, and mark them accordingly. If the choice of the next state to be expanded is not unique, expand the state with smallest lexicographical ordering. Give the solution found by A^* search. Is this solution optimal? Justify your answer.

(c) 1) Run the hill climbing algorithm, as stated in the lecture, on this problem. Use the heuristic function from part (b). For each state provide all applicable actions and the states reachable using these actions. Annotate states with their heuristic value. Specify which node is expanded in each iteration of the algorithm. Give the solution found by hill climbing. Is this solution optimal? Justify your answer.

2) Now design a new heuristic $h : \{A, B, \ldots, J\} \to \mathbb{N}_0^+ \cup \{\infty\}$ for the path finding problem such that hill climbing reaches a local minimum. Run the hill climbing algorithm using your new heuristic.

(d) Run iterative-deepening depth-first-search until it finds a solution. For each depth depict the corresponding search tree. Annotate each state with the order of expansion. If the choice of the next state to be expanded is not unique, expand the state with smallest lexicographical ordering. Give the solution found by iterative-deepening search. Is this solution optimal? Justify your answer.

(Solution)

(a) Solution A, C, G, H. It is optimal because uniform cost search is guaranteed to find the optimal solution.

(b) Solution A, C, G, H. A^* is guaranteed to return an optimal solution if the heuristic that is being used is consistent. Thus, it remains to show that the described heuristic h is consistent, i.e., that for every transition $s \xrightarrow{a} s'$, it holds that $h(s) - h(s') \leq c(a) \Leftrightarrow h(s) \leq h(s') + c(a)$. Since $h(s)$ gives the minimal number of edges of any path from s to H, and there is an edge from s to s', it must hold $h(s) \leq h(s') + 1$. Since $c(a) \geq 1$ for every action a in this example, it follows that $h(s) \leq h(s') + 1 \leq h(s') + c(a)$. Hence, h is consistent.

(c) 1) Solution found A, D, H. It is not optimal, cf. previous exercises.

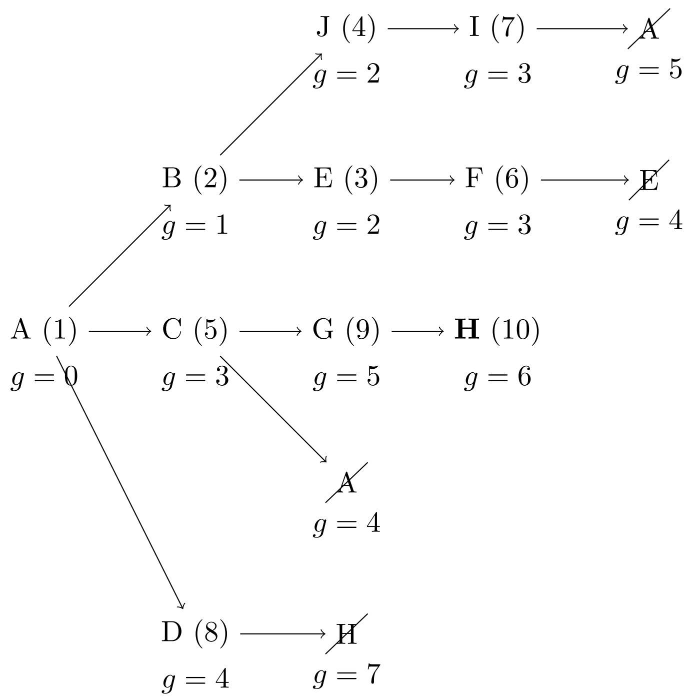


Figure 2: Solution to Exercise 1a)

- State A
  $A \to B$: $h(B) = 5$
  $A \to C$: $h(C) = 2$
  $A \to D$: $h(D) = 1 \Leftarrow$ expand
- State D
  $D \to H$: $h(H) = 0 \Leftarrow$ expand
- State H
  $H \to J$: $h(J) = 4 \Leftarrow$
  $h(J) > h(H) \Rightarrow$ return the path to H (which is A, D, H)

2) Consider the following (admissible) heuristic function:

	A	B	C	D	E	F	G	H	I	J
h	6	2	3	3	1	0	0	0	0	9

Hill climbing results in the following expansion order:

- State A
  $A \to B$: $h(B) = 2 \Leftarrow$ expand

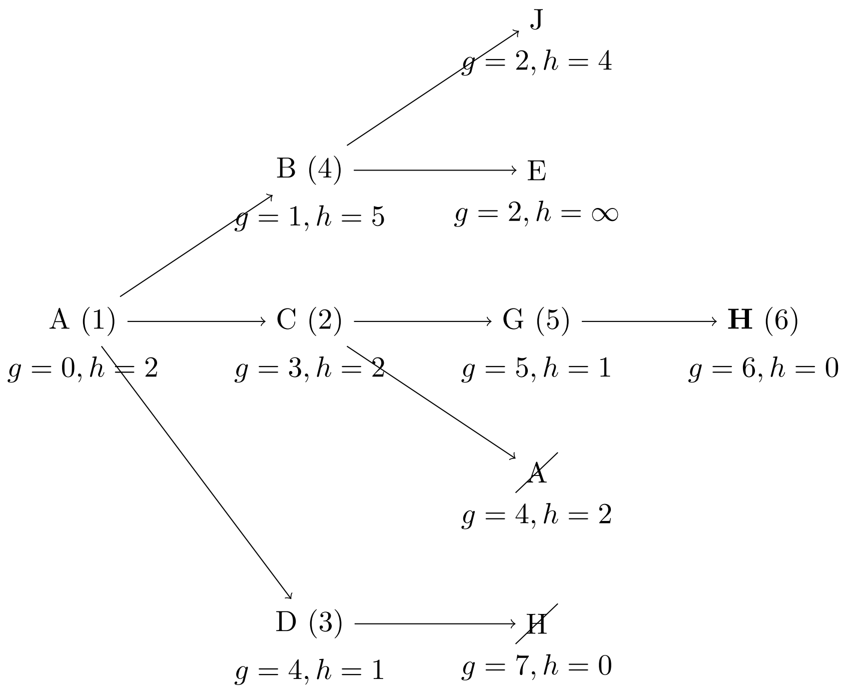


Figure 3: Solution to Exercise 1b)

  $A \to C$: $h(C) = 3$
  $A \to D$: $h(D) = 3$

- State B
  $B \to E$: $h(E) = 1 \Leftarrow$ expand
  $B \to J$: $h(J) = 9$
- State E
  $E \to F$: $h(F) = 0 \Leftarrow$ expand
- State F
  $F \to E$: $h(E) = 1 \Rightarrow F$ is a local minimum $\Rightarrow$ Stop

(d) Solution A, D, H. It is not optimal, see above.

(/Solution)

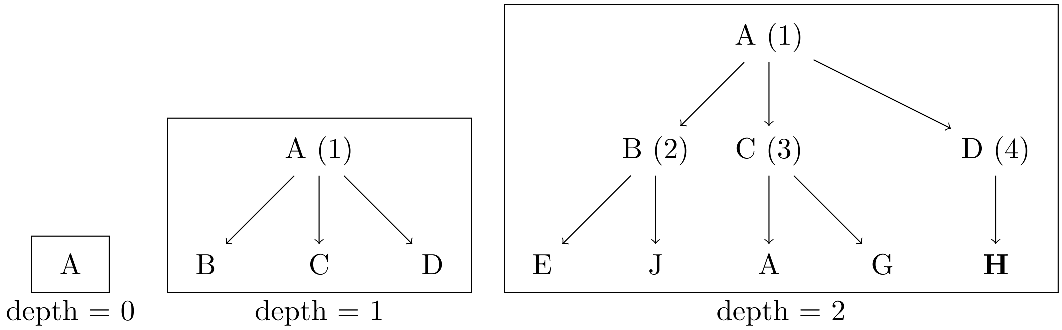


Figure 4: Solution to Exercise 1d)

Exercise 2. (2.5 Points)

Consider a variant of the Vacuum Cleaner problem from the lecture where a robot has to clean a 5×5 square room (see Figure 5). There are four possible actions: up, left, right, and down. There is no suck action since the robot automatically cleans any dirty spot it stands on. Hence, its task is moving around the room and visit all dirty spots. Throughout the exercise, we use Manhattan distance.

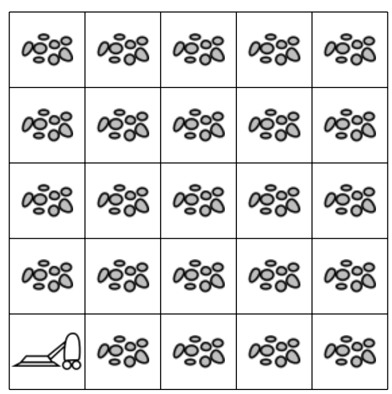


Figure 5: Illustration of the Vacuum Cleaner problem

1. Which of these heuristics are admissible? Why / Why not? Give clear proof arguments. (Formal proofs are not needed.)

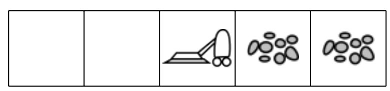
   Note: To prove a heuristic admissible, you need to show that it is admissible for an arbitrary input state of the illustrated 5×5 example. To show that a heuristic is not admissible, a counter example is sufficient.

- h_1 = Distance from the robot to the closest dirty spot.
- h_2 = Distance from the robot to the farthest dirty spot.
- h_3 = Sum of the distances from the robot to all the dirty spots.
- h_4 = Number of dirty spots.
- $h_5 = h_1 + h_2$.

2. Is any of the heuristics perfect? Justify your answer.

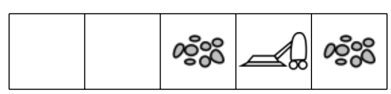
(Solution)

1.
   - h_1: Admissible because the robot needs to reach at least one dirty spot.
   - h_2: Admissible because the robot needs to reach all dirty spots, so it must at some point go to the one farthest away.
   - h_3: Not admissible. Consider the following counterexample, where $h_3 = 1 + 2 = 3 > 2 = h^*$ (assume that the rest of the board has already been cleaned):



   - h_4: Admissible. Across any one move action, h_4 can decrease by at most 1. Thus h_4 is consistent and, consequently admissible.
   - h_5: Not admissible. Consider again the counter example from h_3. It is $h_5 = h_1 + h_2 = 1 + 2 = 3$, but $h^* = 2$.

2. No. h_3 is not admissible so it cannot be the perfect heuristic. For the rest, consider the following counterexample where $h_1 = h_2 = 1$, $h_4 = 2$, and $h^* = 3$:



(/Solution)

Exercise 3. (2.5 Points)

Given a solvable problem with finite branching factor as input, which of the following statements are correct? Provide a proof for each of your claims.

(i) Depth-first search guarantees to find a solution.

(ii) Iterative-deepening search guarantees to find a solution.

Note: For both algorithms, consider the version specified on the lecture slides.