SAARLAND UNIVERSITY ARTIFICIAL INTELLIGENCE
Prof. Jörg Hoffmann and Prof. Wolfgang Wahlster
Dr. Álvaro Torralba, Daniel Gnad, Marcel Steinmetz
Christian Bohnenberger, Cosmina Croitoru, Akram Elkorashy, Sophian Guidara,
Daniel Heller, Björn Mathis, Lukas Schaal, Julia Wichlacz

Exercise Sheet 4.

Solutions due Tuesday, **May 31**, 16:00 – 16:15, in the lecture hall.[1]

Exercise 13. (2 Points)

Transform the following formulas to CNF. To do so, follow the steps from the lecture (Chapter 9 slide 21) and give the intermediate results (you may skip those steps where there is nothing to do for the formula at hand).

(a) $\neg((Q \vee R) \rightarrow (\neg U \leftrightarrow P))$

(b) $((P \rightarrow Q) \wedge (\neg Q \vee R)) \rightarrow (R \wedge P)$

(Solution)

(a) 1. Eliminate $\leftrightarrow$: $\neg((Q \vee R) \rightarrow ((\neg U \rightarrow P) \wedge (P \rightarrow \neg U)))$
2. Eliminate $\rightarrow$: $\neg(\neg(Q \vee R) \vee ((U \vee P) \wedge (\neg U \vee \neg P)))$
3. Move $\neg$ inwards: $(Q \vee R) \wedge ((\neg U \wedge \neg P) \vee (U \wedge P))$
4. Distribute $\vee$ over $\wedge$: $(Q \vee R) \wedge (\neg U \vee P) \wedge (U \vee \neg P)$

CNF: $(Q \vee R) \wedge (\neg U \vee P) \wedge (U \vee \neg P)$

(b) 1. Eliminate $\leftrightarrow$: nothing to do
2. Eliminate $\rightarrow$: $\neg((\neg P \vee Q) \wedge (\neg Q \vee R)) \vee (R \wedge P)$
3. Move $\neg$ inwards: $(P \wedge \neg Q) \vee (Q \wedge \neg R) \vee (R \wedge P)$
4. Distribute $\vee$ over $\wedge$: $(P \vee Q \vee R) \wedge (P \vee Q) \wedge (P \vee \neg R) \wedge (P \vee \neg Q \vee \neg R)$

CNF: $(P \vee Q \vee R) \wedge (P \vee Q) \wedge (P \vee \neg R) \wedge (P \vee \neg Q \vee \neg R)$

(/Solution)

[1] Solutions in paper form only, and solution submission only at the stated time at the stated place. At most 3 authors per solution. All authors must be in the same tutorial group. All sheets of your solution must be stapled together. At the top of the first sheet, you must write the names of the authors and the name of your tutor. Your solution must be placed into the correct box for your tutorial group. If you don't comply with these rules, 3 points will be subtracted from your score for this sheet.

Exercise 14. (2 Points)

For the two CNF formulas below, use resolution to prove that the formulas are unsatisfiable. To do so, first give a set of clauses Δ that is equivalent to the formula and second, use resolution to prove that it is unsatisfiable. Write the resolution process in the form of a tree for easier readability.

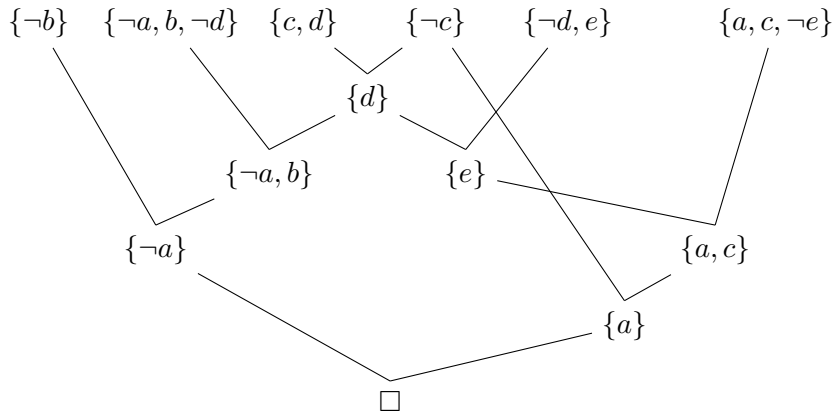
(a) $(\neg a \vee b \vee \neg d) \wedge (\neg a \vee \neg c) \wedge (c \vee d) \wedge (\neg d \vee e) \wedge \neg c \wedge (a \vee c \vee \neg e) \wedge \neg b$

(b) $(\neg b \vee d) \wedge (a \vee c \vee \neg d) \wedge (b \vee c) \wedge (\neg a \vee e) \wedge (\neg c \vee e) \wedge (\neg d \vee \neg e) \wedge (b \vee \neg e)$

(Solution)

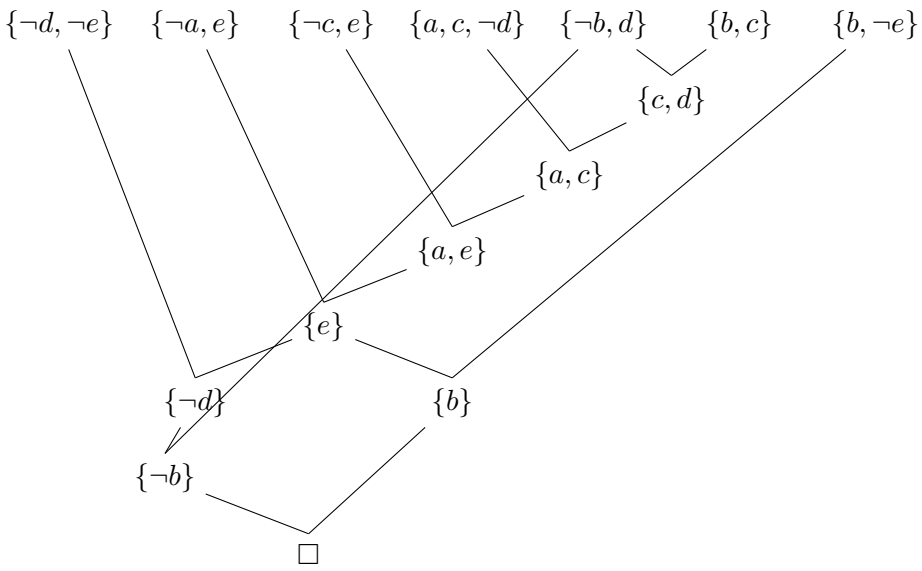
(a) $\Delta = \big\{\{\neg a, b, \neg d\}, \{\neg a, \neg c\}, \{c, d\}, \{\neg d, e\}, \{\neg c\}, \{a, c, \neg e\}, \{\neg b\}\big\}$

Resolution tree:



(b) $\Delta = \big\{\{\neg b, d\}, \{a, c, \neg d\}, \{b, c\}, \{\neg a, e\}, \{\neg c, e\}, \{\neg d, \neg e\}, \{b, \neg e\}\big\}$

Resolution tree:


$\{\neg d, \neg e\}$
$\{\neg a, e\}$
$\{\neg c, e\}$
$\{a, c, \neg d\}$
$\{\neg b, d\}$
$\{b, c\}$
$\{b, \neg e\}$
$\{c, d\}$
$\{a, c\}$
$\{a, e\}$
$\{e\}$
$\{\neg d\}$
$\{b\}$
$\{\neg b\}$
$\square$

(/Solution)

Exercise 15. (3 Points)

For each of the following formulas use the DPLL procedure to determine whether ϕ is satisfiable or unsatisfiable. Give a complete trace of the algorithm, showing the simplified formula for each recursive call of the DPLL function. Assume that DPLL selects variables in alphabetical order (i.e., A, B, C, D, E), and that the splitting rule first attempts the value False (F) and then the value True (T).

(a) $\phi_1 = (A \vee B) \wedge (C \vee D) \wedge (C \vee \neg D) \wedge (\neg B \vee \neg C) \wedge (\neg A \vee E) \wedge (\neg D \vee \neg E)$

(b) $\phi_2 = (D \vee \neg C) \wedge (\neg A \vee \neg B \vee \neg E) \wedge (C \vee D) \wedge (\neg B \vee E \vee \neg D) \wedge (\neg A \vee C) \wedge (B \vee \neg D) \wedge (A \vee \neg E) \wedge (B \vee C)$

(Solution)
a) DPLL trace:

$$\{\{A, B\}, \{C, D\}, \{C, \neg D\}, \{\neg B, \neg C\}, \{\neg A, E\}, \{\neg D, \neg E\}\}$$

1. Splitting rule:

1a. $A \mapsto F$

$\{\{B\}, \{C, D\}, \{C, \neg D\}, \{\neg B, \neg C\}, \{\neg D, \neg E\}\}$

2a. UP rule: $B \mapsto T$

$\{\{C, D\}, \{C, \neg D\}, \{\neg C\}, \{\neg D, \neg E\}\}$

3a. UP rule: $C \mapsto F$

$\{\{D\}, \{\neg D\}, \{\neg D, \neg E\}\}$

4a. UP rule: $D \mapsto F$

$\{\square\}$

1b. $A \mapsto T$

$\{\{C, D\}, \{C, \neg D\}, \{\neg B, \neg C\}, \{E\}, \{\neg D, \neg E\}\}$

2b. UP rule: $E \mapsto T$

$\{\{C, D\}, \{C, \neg D\}, \{\neg B, \neg C\}, \{\neg D\}\}$

3b. UP rule: $D \mapsto F$

$\{\{C\}, \{\neg B, \neg C\}\}$

4b. UP rule: $C \mapsto T$

$\{\{\neg B\}\}$

5b. UP rule: $B \mapsto F$

$\{\}$

Satisfying assignment: $A, \neg B, C, \neg D, E$

b) DPLL trace:

$\{\{D, \neg C\}, \{\neg A, \neg B, \neg E\}, \{C, D\}, \{\neg B, E, \neg D\}, \{\neg A, C\}, \{B, \neg D\}, \{A, \neg E\}, \{B, C\}\}$

1. Splitting rule:

1a. $A \mapsto F$

$\{\{D, \neg C\}, \{C, D\}, \{\neg B, E, \neg D\}, \{B, \neg D\}, \{\neg E\}, \{B, C\}\}$

2a. UP rule: $E \mapsto F$

$\{\{D, \neg C\}, \{C, D\}, \{\neg B, \neg D\}, \{B, \neg D\}, \{B, C\}\}$

3aa. $B \mapsto F$

$\{\{D, \neg C\}, \{C, D\}, \{\neg D\}, \{C\}\}$

3ab. $B \mapsto T$

$\{\{D, \neg C\}, \{C, D\}, \{\neg D\}\}$

4aa. UP rule: $C \mapsto T$

$\{\{D\}, \{\neg D\}\}$

4ab. UP rule: $D \mapsto F$

$\{\{\neg C\}, \{C\}\}$

5aa. UP rule: $D \mapsto F$

$\{\square\}$

5ab. UP rule: $C \mapsto F$

$\{\square\}$

1. Splitting rule:

1b. $A \mapsto T$

$\{\{D, \neg C\}, \{\neg B, \neg E\}, \{C, D\}, \{\neg B, E, \neg D\}, \{C\}, \{B, \neg D\}, \{B, C\}\}$

2b. UP rule: $C \mapsto T$

$\{\{D\}, \{\neg B, \neg E\}, \{\neg B, E, \neg D\}, \{B, \neg D\}\}$

3b. UP rule: $D \mapsto T$

$\{\{\neg B, \neg E\}, \{\neg B, E\}, \{B\}\}$

4b. UP rule: $B \mapsto T$

$\{\{\neg E\}, \{E\}\}$

5b. UP rule: $E \mapsto F$

$\{\square\}$

There is no satisfying assignment.

(/Solution)

Exercise 16. (3 Points)

Prove the contraposition theorem: $\mathbf{KB} \cup \{\varphi\} \models \neg\psi$ iff $\mathbf{KB} \cup \{\psi\} \models \neg\varphi$.