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Theoretical Exercise Sheet 10.

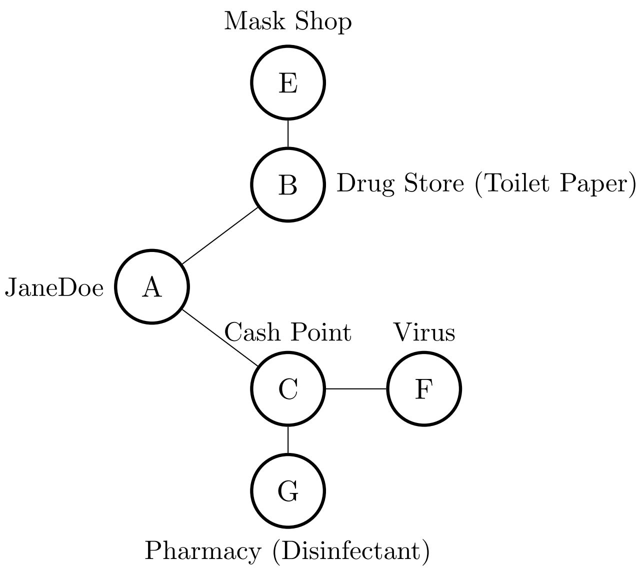
***Disclaimer**: This exercise sheet is voluntary and will not be graded. However, the topics covered in this exercise sheet **are still relevant for the exam**, and we want to give you the opportunity to test your understanding of the topics.*

Exercise 1: h^{FF} and h^+

Recall the following scenario from theoretical sheet 9:

Jane Doe's country was hit by a virus. She wants to fight the virus. In order to do so, she either needs disinfectant, or a mask and toilet paper. Currently, Jane Doe is at home at location A, ToiletPaper can be bought at the Drug Store at location B, a mask can be bought at location E, and disinfectant can be bought at the pharmacy at location G. Note that she first needs to withdraw money at the cashpoint at location C.

This time, we want to study the h^{FF} and h^+ heuristics using this scenario. The problem was formalized as (P not given explicitly):



$$
\begin{aligned}
I &= \{at(A)\} \\
G &= \{\mathit{VirusDead}\} \\
A &= \{\mathit{buyDisinfectant}, \mathit{buyMask}, \mathit{buyToiletPaper}, \mathit{withdrawMoney}, \\
&\quad \mathit{useToiletPaper}, \mathit{useDisinfectant}\} \\
&\quad \cup \{\mathit{walk}(x,y) \mid x, y \in \{A, \ldots, G\} \text{ are connected on the map}\}
\end{aligned}
$$

$\mathit{walk}(x,y)$:
- $\mathit{pre} : \{at(x)\}$
- $\mathit{add} : \{at(y)\}$
- $\mathit{del} : \{at(x)\}$

$\mathit{buyDisinfectant}$:
- $\mathit{pre} : \{at(G), \mathit{has2Money}\}$
- $\mathit{add} : \{\mathit{hasDisinfectant}\}$
- $\mathit{del} : \{\mathit{has2Money}\}$

$\mathit{buyMask2}$:
- $\mathit{pre} : \{at(E), \mathit{has2Money}\}$
- $\mathit{add} : \{\mathit{hasMask}, \mathit{has1Money}\}$
- $\mathit{del} : \{\mathit{has2Money}\}$

$\mathit{buyMask1}$:
- $\mathit{pre} : \{at(E), \mathit{has1Money}\}$
- $\mathit{add} : \{\mathit{hasMask}\}$
- $\mathit{del} : \{\mathit{has1Money}\}$

$\mathit{buyToiletPaper2}$:
- $\mathit{pre} : \{at(B), \mathit{has2Money}\}$
- $\mathit{add} : \{\mathit{hasToiletPaper}, \mathit{has1Money}\}$
- $\mathit{del} : \{\mathit{has2Money}\}$

$\mathit{buyToiletPaper1}$:
- $\mathit{pre} : \{at(B), \mathit{has1Money}\}$
- $\mathit{add} : \{\mathit{hasToiletPaper}\}$
- $\mathit{del} : \{\mathit{has1Money}\}$

$\mathit{withdrawMoney}$:
- $\mathit{pre} : \{at(C)\}$
- $\mathit{add} : \{\mathit{has2Money}\}$
- $\mathit{del} : \{\mathit{has1Money}\}$

$\mathit{useDisinfectant}$:
- $\mathit{pre} : \{at(F), \mathit{hasDisinfectant}\}$
- $\mathit{add} : \{\mathit{VirusDead}\}$
- $\mathit{del} : \{\mathit{hasDisinfectant}\}$

$\mathit{useToiletPaper}$:
- $\mathit{pre} : \{at(F), \mathit{hasToiletPaper}, \mathit{hasMask}\}$
- $\mathit{add} : \{\mathit{VirusDead}\}$
- $\mathit{del} : \{\mathit{hasToiletPaper}\}$

a) Compute h^{FF} for the initial state. Write down the action and fact sets (A_i and F_i) for each iteration of the RPG algorithm. In the plan extraction, for each step t, write down the fact set G_t, which actions are selected for each fact in G_t, and the resulting fact set updates for each selected action. The order in which you select the actions that generate the facts is not relevant. **Try to avoid using the disinfectant during plan extraction.** What is the value of $h^{FF}(I)$?

b) Run A* search on the problem using h^+ as the heuristic. If several nodes have the same $g + h$ value, rely on the heuristic and **choose the one with the smaller h value**. If there are several nodes with same g and same h value, break ties by using the alphabetical order on location names. In each search node, mention the literals that are true, the g and h values as well as the expansion order. In the search graph, mark duplicate nodes (e.g. by crossing them out).

Note: Use the following abbreviations.
h2M := has2Money, hD := hasDisinfectant, VD := VirusDead

Solution:

a) Forward search to compute the RPG:

- $F_0 = I = \{at(A)\}$
- $A_0 = \{walk(A,B), walk(A,C)\}$
  $F_1 = F_0 \cup \{at(B), at(C)\}$
- $A_1 = A_0 \cup \{walk(B,E), walk(B,A), walk(C,A), walk(C,F), walk(C,G), withdrawMoney\}$
  $F_2 = F_1 \cup \{at(E), at(F), at(G), has2Money\}$
- $A_2 = A_1 \cup \{walk(E,B), walk(F,C), walk(G,C), buyToiletPaper2, buyMask2, buyDisinfectant\}$
  $F_3 = F_2 \cup \{hasToiletPaper, hasMask, hasDisinfectant, has1Money\}$
- $A_3 = A_2 \cup \{useToiletPaper, useDisinfectant, buyToiletPaper1, buyMask1\}$
  $F_4 = F_3 \cup \{VirusDead\} \subseteq G \rightarrow stop$

Backward search to extract a plan:

Initially: $G_0, G_1, G_2, G_3 = \emptyset,\ G_4 = \{VirusDead\}$

Layer	Selected fact	Inserted Action	Marked facts
4	$VirusDead$	$useToiletPaper$	$G_4 = \{VirusDead\}$
3	$hasToiletPaper$	$buyToiletPaper2$	$G_3 = \{hasToiletPaper, hasMask\}$
	$hasMask$	$buyMask2$	
2	$has2Money$	$withdrawMoney$	$G_2 = \{has2Money, at(E), at(F)\}$
	$at(E)$	$walk(B,E)$	
	$at(F)$	$walk(C,F)$	
1	$at(B)$	$walk(A,B)$	$G_1 = \{at(B), at(C)\}$
	$at(C)$	$walk(A,C)$	

$h^{FF}(I) = 8$ since eight actions were selected during plan extraction.

b) The search graph is depicted in figure 1.

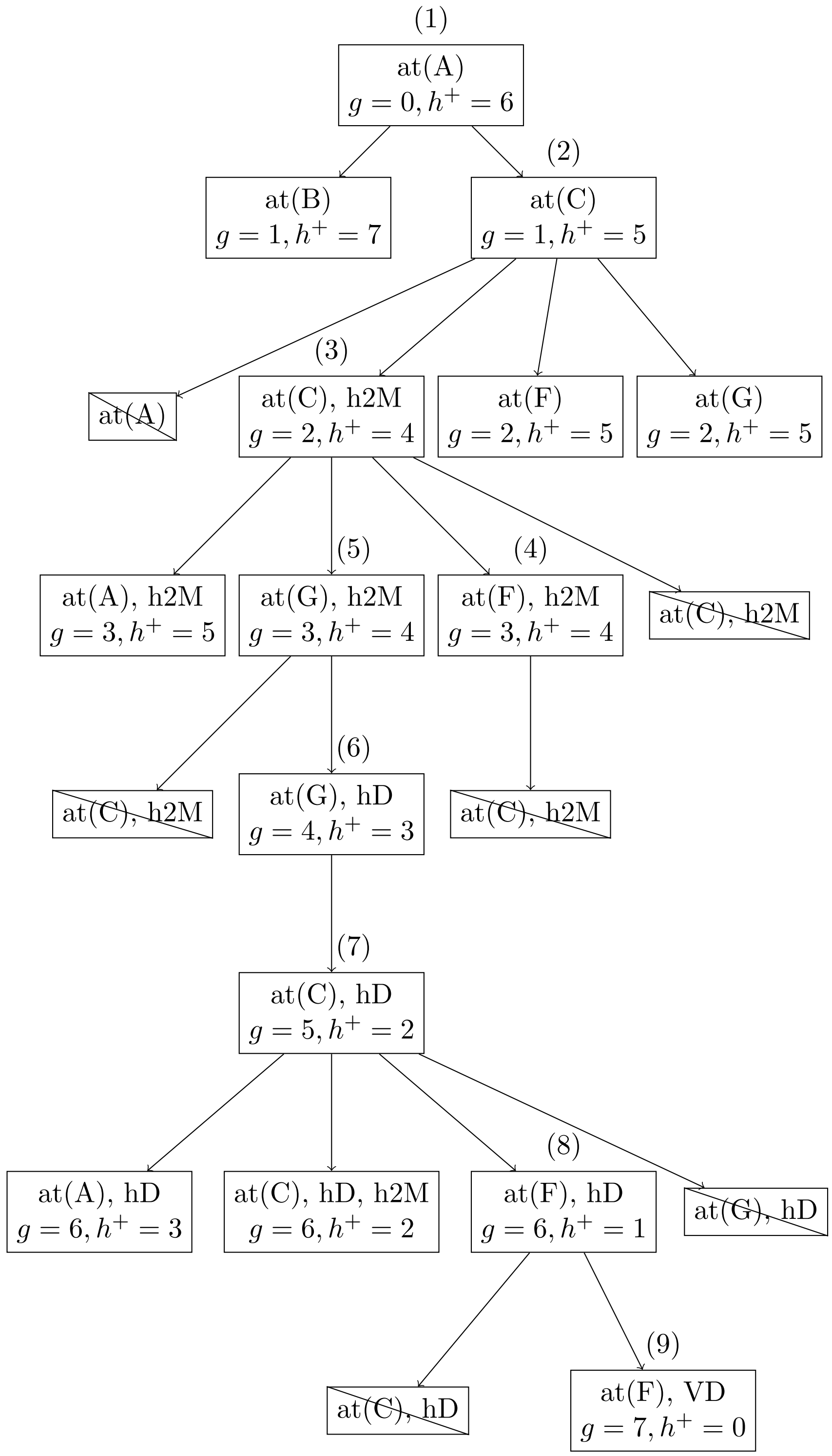


Figure 1: The search graph of Exercise 2 (c).

Exercise 2: True or False?

For all of the following statements, decide if they are correct. Briefly justify your answer (1–3 sentences).

1. In the delete relaxation, an optimal plan (if existent) has length at most $|P|$.
2. Our implementation of h^{FF} is an admissible heuristic.
3. Let $\mathcal{R}$ be a relaxation mapping that drops all preconditions in a given planning task. Then the heuristic defined as $h^{\mathcal{R}} := h^* \circ \mathcal{R}$ is an admissible heuristic.
4. For every state it holds that if it is unsolvable in the original problem, it is unsolvable in the delete relaxation.
5. For every state it holds that if it is solvable in the original problem, it is solvable in the delete relaxation.
6. Let $\Pi = (P, A, I, G)$ be a STRIPS planning task with empty delete lists, i.e. $\text{del}_a = \emptyset$ for all action $a \in A$. The task $\Pi' = (P, A, I, G \setminus I)$ in which all facts that are initially true have been removed from the set of goal facts is solvable if and only if Π is solvable.

Solution:

1. False. This was true for the only-adds relaxation, but only becuase we did not have any preconditions. Consider the following counterexample in which the optimal plan in the delete relaxation has length $2 > |G| = 1$.

$$\begin{aligned} P &= \{p, q\} \\ A &= \{a_1, a_2\} \\ I &= \{\} \\ G &= \{q\} \end{aligned}$$

$$\begin{aligned} \text{pre}_{a_1} &= \{\}, & \text{add}_{a_1} &= \{p\}, & \text{del}_{a_1} &= \{\} \\ \text{pre}_{a_2} &= \{p\}, & \text{add}_{a_2} &= \{q\}, & \text{del}_{a_2} &= \{\} \end{aligned}$$

2. False. Our implementation gives us **some** plan for the delete-relaxation, not the shortest. The claim would only hold if h^{FF} always computed the shortest plan in the delete-relaxation, since $h^+(s) \leq h^*(s)$.

3. True. Any plan in the original problem would still be a plan in this relaxation, as all action are applicable because of absent preconditions.

4. False. Consider this task in which our agent shall be at two locations (•) **at the same time**. He can move left or right if he stays inside the 3 squares.

Agent	•	•

Obviously this is impossible. However, in the delete relaxation the agent can move to the very right. After doing this, the agent is everywhere in the delete-relaxation, as our previous positions have never been erased. Therefore, $h^+(I) = 2$ for this example.

5. True. Any plan in the original problem is a plan in the delete relaxation.

6. True. Since we have no deletes, the goal facts that were initially true stay true forever. Therefore, any plan in this relaxation is also a plan in the original problem.