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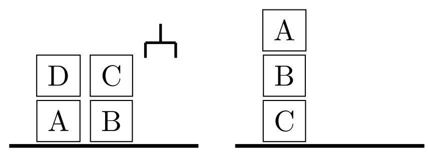
Exercise Sheet 8.

Solutions due Tuesday, **June 28**, 16:00 – 16:15, in the lecture hall.[1]

Exercise 29. (3 Points)



(a) Initial state (b) Goal

Figure 1: Blocksworld Example of Exercise 29

Consider the Blocksworld problem with initial state and goal shown in Figure 1. There are overall four different actions to move blocks around: a block can be unstacked from another block, it can be picked up from the table, it can be stacked on top of another block, and it can be put onto the table. The problem is formalized as the following STRIPS planning problem $\Pi = (P, A, I, G)$:

$$
\begin{aligned}
P = &\{stacked(x, y) |\ x, y \in \{A, B, C, D\}\} \\
&\cup \{ontable(x) |\ x \in \{A, B, C, D\}\} \\
&\cup \{clear(x) |\ x \in \{A, B, C, D\}\} \\
&\cup \{holding(x) |\ x \in \{A, B, C, D\}\} \\
&\cup \{empty\}
\end{aligned}
$$

$$
\begin{aligned}
A = &\{stack(x, y), unstack(x, y) \mid x, y \in \{A, B, C, D\}, x \neq y\} \\
&\cup \{putdown(x), pickup(x) \mid x \in \{A, B, C, D\}\}
\end{aligned}
$$

[1]Solutions in paper form only, and solution submission only at the stated time at the stated place. At most 3 authors per solution. All authors must be in the same tutorial group. All sheets of your solution must be stapled together. At the top of the first sheet, you must write the names of the authors and the name of your tutor. Your solution must be placed into the correct box for your tutorial group. If you don't comply with these rules, 3 points will be subtracted from your score for this sheet.

where

- $stack(x, y)$ for $x, y \in \{A, B, C, D\}$, $x \neq y$
  - $pre : \{holding(x), clear(y)\}$
  - $add : \{stacked(x, y), clear(x), empty\}$
  - $del : \{holding(x), clear(y)\}$
- $unstack(x, y)$ for $x, y \in \{A, B, C, D\}$, $x \neq y$
  - $pre : \{stacked(x, y), clear(x), empty\}$
  - $add : \{holding(x), clear(y)\}$
  - $del : \{stacked(x, y), clear(x), empty\}$
- $putdown(x)$ for $x \in \{A, B, C, D\}$
  - $pre : \{holding(x)\}$
  - $add : \{ontable(x), clear(x), empty\}$
  - $del : \{holding(x)\}$
- $pickup(x)$ for $x \in \{A, B, C, D\}$
  - $pre : \{ontable(x), clear(x), empty\}$
  - $add : \{holding(x)\}$
  - $del : \{ontable(x), clear(x), empty\}$

All actions have uniform cost of 1.
$I = \{stacked(D, A), stacked(C, B), ontable(A), ontable(B), clear(C), clear(D), empty\}$
$G = \{stacked(A, B), stacked(B, C), ontable(C)\}$

a) Compute the h^{FF} value for the initial state. Write down the action and fact sets for each iteration of the RPG algorithm. Then, iteratively compute the h^{FF} value. For each step t, write down which actions are selected and the resulting G_{t-1} you obtain. What is the final $h^{\mathrm{FF}}(I)$ value?

b) Run A* search on the problem and use h^+ as heuristic. If several nodes have the same $g + h$ value, expand the node with largest g value. In each search node, mention the literals that are true (for example by drawing the states as in Figure 1), the g and h values as well as the expansion time. Note each applicable action and mention the action name.

(Solution)

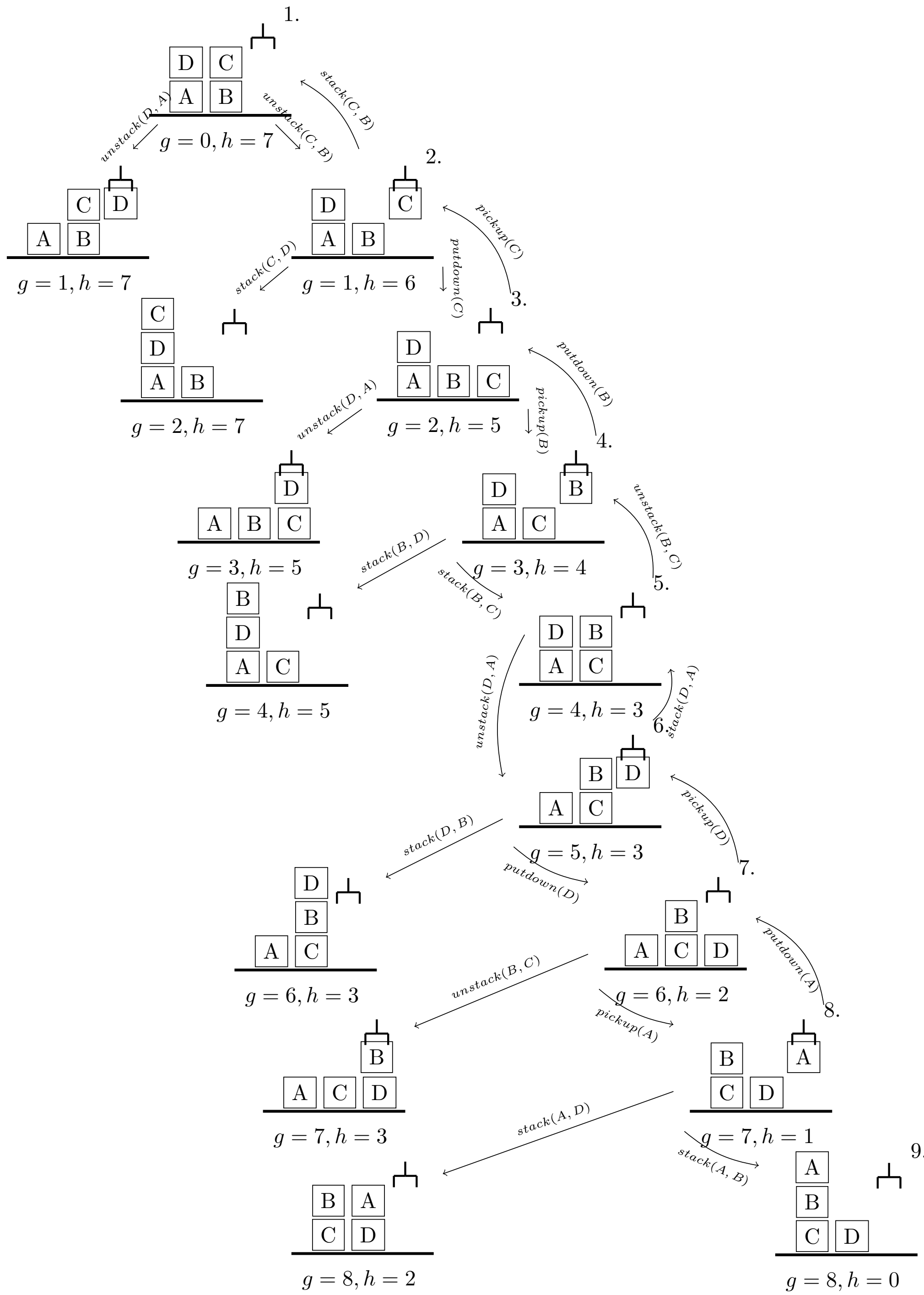
a) RPG:

- $F_0 = \{stacked(D, A), stacked(C, B), ontable(A), ontable(B), clear(C), clear(D), empty\}$
- $A_0 = \{unstack(D, A), unstack(C, B)\}$
  $F_1 = F_0 \cup \{clear(A), clear(B), holding(C), holding(D)\}$
- $A_1 = A_0 \cup \{stack(C, A), stack(C, B), stack(C, D), putdown(C), stack(D, A), stack(D, B), stack(D, C), putdown(D), pickup(A), pickup(B)\}$
  $F_2 = F_1 \cup \{stacked(C, A), stacked(C, D), ontable(C), stacked(D, B), stacked(D, C), ontable(D), holding(A), holding(B)\}$
- $A_2 = A_1 \cup \{stack(A, B), stack(A, C), stack(A, D), putdown(A), stack(B, A), stack(B, C), stack(B, D), putdown(B), unstack(C, A), unstack(C, D), pickup(C), unstack(D, B), unstack(D, C), pickup(D)\}$
  $F_3 = F_2 \cup \{stacked(A, B), stacked(A, C), stacked(A, D), stacked(B, A), stacked(B, C), stacked(B, D)\}$
- $G \subseteq F_3 \rightarrow$ stop

Plan extraction:

- $G_3 = \{stacked(A, B), stacked(B, C)\}$
  $stacked(A, B)$: $stack(A, B)$
  $stacked(B, C)$: $stack(B, C)$
- $G_2 = \{ontable(C), holding(A), holding(B)\}$
  $ontable(C)$: $putdown(C)$
  $holding(A)$: $pickup(A)$
  $holding(B)$: $pickup(B)$
- $G_1 = \{clear(A), clear(B), holding(C)\}$
  $clear(A)$: $unstack(D, A)$
  $clear(B)$: $unstack(C, B)$
  $holding(C)$: $unstack(C, B)$
- $G_0 = I$

Total number of selected (different) actions is 7 and thus $h^{\mathrm{FF}}(I) = 7$.



1.
D C
A B
$g = 0, h = 7$
$unstack(D, A)$
$unstack(C, B)$
$stack(C, B)$
C D
A B
$g = 1, h = 7$
2.
D C
A B
$g = 1, h = 6$
$stack(C, D)$
$putdown(C)$
$pickup(C)$
C
D
A B
$g = 2, h = 7$
3.
D
A B C
$g = 2, h = 5$
$unstack(D, A)$
$pickup(B)$
$putdown(B)$
D
A B C
$g = 3, h = 5$
4.
D B
A C
$g = 3, h = 4$
$stack(B, D)$
$stack(B, C)$
$unstack(B, C)$
B
D
A C
$g = 4, h = 5$
5.
D B
A C
$g = 4, h = 3$
$unstack(D, A)$
$stack(D, A)$
6.
B D
A C
$g = 5, h = 3$
$stack(D, B)$
$putdown(D)$
$pickup(D)$
D
B
A C
$g = 6, h = 3$
7.
B
A C D
$g = 6, h = 2$
$unstack(B, C)$
$pickup(A)$
$putdown(A)$
B
A C D
$g = 7, h = 3$
8.
B A
C D
$g = 7, h = 1$
$stack(A, D)$
$stack(A, B)$
B A
C D
$g = 8, h = 2$
9.
A
B
C D
$g = 8, h = 0$

b)

(/Solution)

Exercise 30. (2 Points)

a) Consider again the Blocksworld problem from last exercise, but with different initial state and goal, as shown in Figure 2. The problem is formalized as the following STRIPS planning problem $\Pi = (P, A, I, G)$:

(a) Initial state (b) Goal

Figure 2: Blocksworld Example of Exercise 30

- $P = \{stacked(x, y), ontable(x), clear(x), holding(x), empty \mid x, y \in \{A, B, C\}\}$
- $A = \{stack(x, y), unstack(x, y) \mid x, y \in \{A, B, C\}, x \neq y\} \cup \{putdown(x), pickup(x) \mid x \in \{A, B, C\}\}$ as defined as in the previous exercise.
- $I = \{stacked(C, A), ontable(A), ontable(B), clear(B), clear(C), empty\}$
- $G = \{stacked(A, B), stacked(B, C)\}$

Compare $h^*(I)$ with $h^{\max}(I, G)$ and $h^{\text{add}}(I, G)$. For this, give the value of $h^*(I)$, fill the table below, and give the values of $h^{\max}(I, G)$ and $h^{\text{add}}(I, G)$.

g	$h^{\max}(I, g)$	$h^{\text{add}}(I, g)$
$empty$	0	0
$clear(A)$		
$clear(B)$	0	0
$clear(C)$	0	0
$holding(A)$		
$holding(B)$		
$holding(C)$		
$ontable(A)$	0	0
$ontable(B)$	0	0
$ontable(C)$		
$stacked(A, B)$		
$stacked(B, A)$		
$stacked(A, C)$		
$stacked(C, A)$	0	0
$stacked(B, C)$		
$stacked(C, B)$		

b) Provide a STRIPS planning task $\Pi = (P, A, I, G)$ so that h^{add} is not an admissible estimation, i.e. $h^{\text{add}}(I, G) > h^*(I)$. Justify your answer (e.g. by computing and comparing $h^{\text{add}}(I, G)$ and $h^*(I)$ in your planning task).

(Solution)

a) $h^*(I) = 6$, $h^{\max}(I, G) = 3$, $h^{\text{add}}(I, G) = 5$

g	$h^{\max}(I, g)$	$h^{\text{add}}(I, g)$
empty	0	0
$clear(A)$	1	1
$clear(B)$	0	0
$clear(C)$	0	0
$holding(A)$	2	2
$holding(B)$	1	1
$holding(C)$	1	1
$ontable(A)$	0	0
$ontable(B)$	0	0
$ontable(C)$	2	2
$stacked(A, B)$	3	3
$stacked(B, A)$	2	3
$stacked(A, C)$	3	3
$stacked(C, A)$	0	0
$stacked(B, C)$	2	2
$stacked(C, B)$	2	2

b) Consider the planning task $\Pi = (P, A, I, G)$ with facts $P = \{g_1, g_2\}$, initial state $I = \emptyset$, goal $G = \{g_1, g_2\}$, and actions $A = \{a\}$ where $pre(a) = \emptyset$, $add(a) = \{g_1, g_2\}$, $del(a) = \emptyset$, and a has a cost of 1. Then, $h^{\text{add}}(I, \{g_1\}) = 1$ and $h^{\text{add}}(I, \{g_2\}) = 1$, and therefore $h^{\text{add}}(I, \{g_1, g_2\}) = 1 + 1 = 2 > h^*(I) = 1$.

(/Solution)

Exercise 31. (2.5 Points)

Prove that, given any planning task $\Pi = (P, A, I, G)$ and any state s, if $\langle a_1, \dots, a_n \rangle$ is a plan for (P, A, s, G), then $\langle a_1^+, \dots, a_n^+ \rangle$ is a plan for $(P, A, s, G)^+$.
Tip: Denote the state-action sequence traversed by $\langle a_1, \dots, a_n \rangle$ as $seq = s_0, a_1, s_1, \dots, a_n, s_n$. Denote the state-action sequence traversed by $\langle a_1^+, \dots a_n^+ \rangle$ as $seq^+ = s_0^+, a_1^+, s_1^+, \dots, a_n^+, s_n^+$. Then show that *(*) for* $1 \leq i \leq n$*,* a_i^+ *is applicable to* s_{i-1}^+*; and for* $0 \leq i \leq n$

we have $s_i \subseteq s_i^+$. Once (*) is proved, use it to prove that $\langle a_1^+, \dots, a_n^+ \rangle$ is a plan for $(P, A, s, G)^+$.

Exercise 32. (2.5 Points)

As specified in the lecture, PlanLen^+ is the problem of deciding, given a STRIPS planning task Π and an integer B, whether or not there exists a delete-relaxed plan for Π of length at most B. Prove that PlanLen^+ is a member of **NP**.
Tip: There is a simple upper bound on the length of a relaxed plan, if one exists.