SAARLAND UNIVERSITY ARTIFICIAL INTELLIGENCE

Prof. Jana Koehler, Dr. Álvaro Torralba

Exercise Sheet 10

Solution

Exercise 36: Bayesian inference. (3 Points)

Consider the Bayesian network BN from Figure 1.

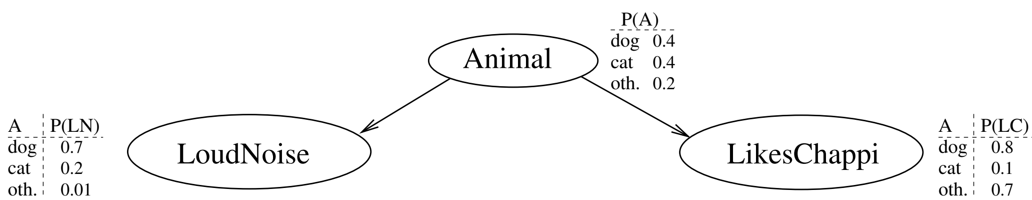


Figure 1: Bayesian network for Exercise 36.

Use inference by enumeration to compute the following probabilities:

(a) $P(dog \mid loudNoise, likesChappi)$.

(b) $P(loudNoise \mid \neg likesChappi)$.

(c) $P(likesChappi)$.

Include intermediate steps at a level of granularity as in the examples on the lecture slides. In particular, for each part, state what the query variable, evidence, and hidden variables are; and write down *which* probabilities provided in BN can be combined, and *how* to obtain the demanded probability P.

(Solution)

As the variable ordering consistent with BN, we choose $X_1 = Animal$, $X_2 = LoudNoise$, $X_3 = LikesChappi$.

(a) The query variable X here is $Animal$. The evidence $\mathbf{e}$ is $loudnoise, likeschappi$. There are no hidden variables, $\mathbf{Y} = \emptyset$. Using Normalization+Marginalization $\mathbf{P}(X \mid \mathbf{e}) = \alpha \sum_{\mathbf{y} \in \mathbf{Y}} \mathbf{P}(X, \mathbf{e}, \mathbf{y})$ we get $\mathbf{P}(Animal \mid loudnoise, likeschappi) = \alpha \mathbf{P}(Animal, loudnoise, likeschappi)$. By the Chain rule and exploiting conditional independence, we get $\alpha \mathbf{P}(Animal, loudnoise, likeschappi) = \alpha \mathbf{P}(likeschappi \mid Animal) * \mathbf{P}(loudnoise \mid Animal) * \mathbf{P}(Animal) = \alpha \langle 0.8 * 0.4 * 0.7, 0.1 * 0.2 * 0.4, 0.7 * 0.01 * 0.2 \rangle \approx \langle 0.96, 0.03, 0.01 \rangle$. Thus $P(dog \mid loudnoise, likeschappi) \approx 0.96$. ("If your animal likes Chappi and makes loud noise, chances are good it's a dog.")

(b) The query variable X here is $LoudNoise$. The evidence $\mathbf{e}$ is $\neg likeschappi$. The hidden variables $\mathbf{Y}$ are $\{Animal\}$. Using Normalization+Marginalization $\mathbf{P}(X \mid \mathbf{e}) = \alpha \sum_{\mathbf{y} \in \mathbf{Y}} \mathbf{P}(X, \mathbf{e}, \mathbf{y})$ we get $\mathbf{P}(LoudNoise \mid \neg likeschappi) = \alpha \sum_{\mathbf{y} \in \{Animal\}} \mathbf{P}(\mathbf{y}, LoudNoise, \neg likeschappi)$. By the Chain rule and exploiting conditional independence, we get $\alpha \sum_{\mathbf{y} \in \{Animal\}} \mathbf{P}(\mathbf{y}, LoudNoise, \neg likeschappi) = \alpha \sum_{\mathbf{y} \in \{Animal\}} P(\neg likeschappi \mid \mathbf{y}) * \mathbf{P}(LoudNoise \mid \mathbf{y}) * P(\mathbf{y})$. That is, for each truth value v of $LoudNoise$, we need to sum over each possible animal $\mathbf{y}$ the product of the likelihood of $\mathbf{y}$ not liking Chappi multiplied with the likelihood of $\mathbf{y}$ having value v for $LoudNoise$ multiplied with the likelihood of $\mathbf{y}$. This gives us $\mathbf{P}(LoudNoise \mid \neg likeschappi) = \alpha \langle 0.2 * 0.7 * 0.4 + 0.9 * 0.2 * 0.4 + 0.3 * 0.01 * 0.2, 0.2 * 0.3 * 0.4 + 0.9 * 0.8 * 0.4 + 0.3 * 0.99 * 0.2 \rangle = \alpha \langle 0.1286, 0.3714 \rangle = \langle 0.2572, 0.7428 \rangle$. Thus $P(loudnoise \mid \neg likeschappi) = 0.2572$. ("If your animal does not like Chappi, chances are good it is quiet.")

(c) The query variable X here is $LikesChappi$. There is no evidence this time, and the hidden variables are $\mathbf{Y} = \{Animal, LoudNoise\}$. Using Normalization+Marginalization $\mathbf{P}(X) = \alpha \sum_{a \in v_{Animal}} \sum_{b \in v_{LoudNoise}} \mathbf{P}(X, a, b)$. By the Chain rule and avoiding irrelevant computation as shown on the slides, $\mathbf{P}(X) = \alpha \sum_{a \in v_{Animal}} \mathbf{P}(a) \mathbf{P}(X \mid a) \sum_{b \in v_{LoudNoise}} \mathbf{P}(b \mid a) = \alpha \sum_{a \in v_{Animal}} \mathbf{P}(a) \mathbf{P}(X \mid a) = \alpha \langle 0.4 * 0.8 + 0.4 * 0.1 + 0.2 * 0.7, 0.4 * 0.2 + 0.4 * 0.9 + 0.2 * 0.3 \rangle = \langle 0.5, 0.5 \rangle$. Thus $\mathbf{P}(likesChappi) = 0.5$.

(/Solution)

Exercise 37: Construction of Bayesian networks. (2 Bonus Points)

Assume it is your responsibility to monitor volcanic eruptions. You receive data from two different stations (seismometers), S_1 and S_2. Each S_i is modeled as a Boolean variable where "true" stands for "I detected an eruption" and "false" stands for "I did not detect an eruption". The seismometers are not fully reliable, however; they may not detect an eruption even though there was one, and they may mistake an earthquake for an eruption of a volcano. We model this situation with two additional Boolean variables: V for volcanic eruption, and E for Earthquake.

Use the algorithm from the lecture to construct a Bayesian network for these 4 variables. Do so for the following two variable orders:

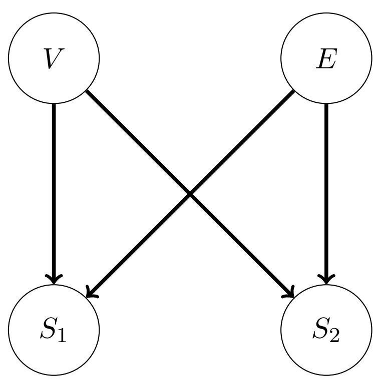
(a) $X_1 = V$, $X_2 = E$, $X_3 = S_1$, $X_4 = S_2$.

(b) $X_1 = S_1$, $X_2 = S_2$, $X_3 = E$, $X_4 = V$.

For each of these orders, draw the resulting Bayesian network. Justify your design, i.e., for each variable X_i added to the network explain why the set of parents you give X_i are needed, and why they are sufficient.

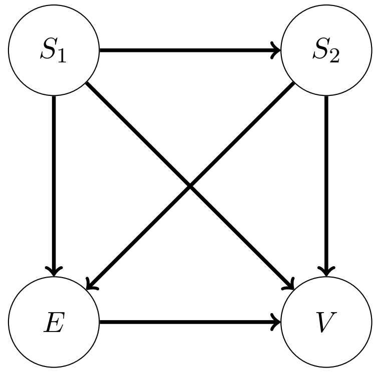
(Solution)

(a) With this variable order, we get the following network:



$X_2 = E$ does not need $X_1 = V$ as a parent because Earthquakes are independent from volcano tests. $x_3 = S_1$ needs both $X_1 = V$ and $X_2 = E$ as parents because each of these may influence the measurement; same for $X_4 = S_2$, i.e., here we also need the parents $X_1 = V$ and $X_2 = E$. However, given the values of V and E, the measurements of $X_3 = S_1$ and $x_4 = S_2$ are independent. So $X_4 = S_2$ does not require the parent $X_3 = S_1$.

(b) With this variable order, we get the following network:



$X_2 = S_2$ needs $X_1 = S_1$ as a parent because, if S_1 detects a seismic phenomenon, then chances are higher S_2 will detect one as well. $X_3 = E$ needs each of $X_1 = S_1$ and $X_2 = S_2$ as parents because, if a station detects a seismic phenomenon, then chances are higher there was an earthquake; same for $X_4 = V$, i.e., here we also need the parents $X_1 = S_1$ and $X_2 = S_2$ because measurements indicate volcano tests as well. Finally, say we already know that S_1 and S_2 are true; then the value of E still has an influence on the value of V: If there was an earthquake, then there is a chance that the seismic measurements were caused by the earthquake rather than a volcano test. Thus V is *not* conditionally independent of E given S_1 and S_2, and we need $X_3 = E$ as a parent of $X_4 = V$ as well.

(/Solution)

Exercise 38: $\mathcal{ALC}$. (7 Points)

1. Formulate the following $\mathcal{ALC}$ concept descriptions in natural language:

   (a) $\text{Person} \sqcap \exists\text{gives}.(\text{Talk} \sqcap \forall\text{topic}.\text{DL})$

   (b) $\text{Person} \sqcap \forall\text{gives}.(\text{Talk} \sqcap \exists\text{topic}.\text{DL})$

2. Using only concept names

   Vehicle, Boat, Bicycle, Car, Device, Wheel, Engine, Axle, Rotation, Water, Human, Driver, Adult, Child

   and the role names

   hasPart, poweredBy, capableOf, travelsOn, controls

   build an $\mathcal{ALC}$ - TBox $\mathcal{T}$ that captures each of the following statements in a suitable concept definition.

   (a) Cars are exactly those vehicles that have wheels and are powered by an engine.

   (b) Bicycles are exactly those vehicles that have wheels and are powered by a human.

   (c) Boats are exactly those vehicles that travel on water.

   (d) Boats have no wheels.

   (e) Cars and bicycles do not travel on water.

   (f) Wheels are exactly those devices that have an axle and are capable of rotation.

   (g) Drivers are exactly those humans who control a vehicle.

   (h) Drivers of cars are adults.

   (i) Humans are not vehicles.

   (j) Wheels and engines are not humans.

   (k) Humans are either adults or children.

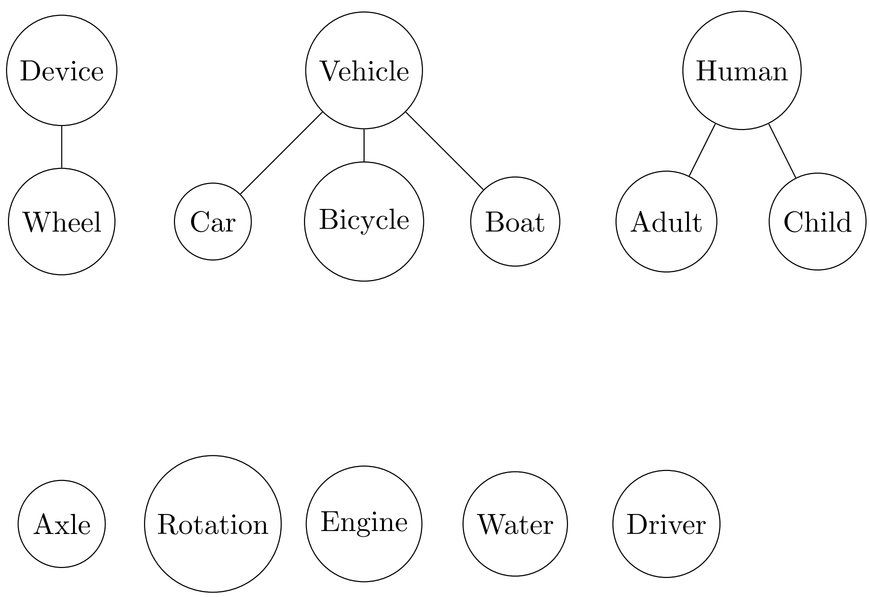
   (l) Adults are not children.

3. Draw the subsumption hierarchy for the concept definitions created in part 2.

(Solution)

1. (a) A Person that gives talks only about DL.
   (b) A Person that exclusively gives talks, some about DL.

2. (a) Car $\equiv$ Vehicle $\sqcap$ $\exists$hasPart.Wheel $\sqcap$ $\exists$poweredBy.Engine
   (b) Bicycle $\equiv$ Vehicle $\sqcap$ $\exists$hasPart.Wheel $\sqcap$ $\exists$poweredBy.Human
   (c) Boat $\equiv$ Vehicle $\sqcap$ $\exists$travelsOn.Water
   (d) Boat $\sqsubseteq$ $\forall$hasPart.$\neg$Wheel
   (e) Car $\sqcup$ Bicycle $\sqsubseteq$ $\forall$travelsOn.$\neg$Water
   (f) Wheel $\equiv$ Device $\sqcap$ $\exists$hasPart.Axle $\sqcap$ $\exists$capableOf.Rotation
   (g) Driver $\equiv$ Human $\sqcap$ $\exists$controls.Vehicle
   (h) Driver $\sqcap$ $\exists$controls.Car $\sqsubseteq$ Adult (alternative: Person $\sqcap$ $\exists$controls.Car $\sqsubseteq$ Adult)
   (i) Human $\sqsubseteq$ $\neg$Vehicle
   (j) Wheel $\sqcup$ Engine $\sqsubseteq$ $\neg$Human
   (k) Human $\sqsubseteq$ Adult $\sqcup$ Child
   (l) Adult $\sqsubseteq$ $\neg$ Child



3.

(/Solution)