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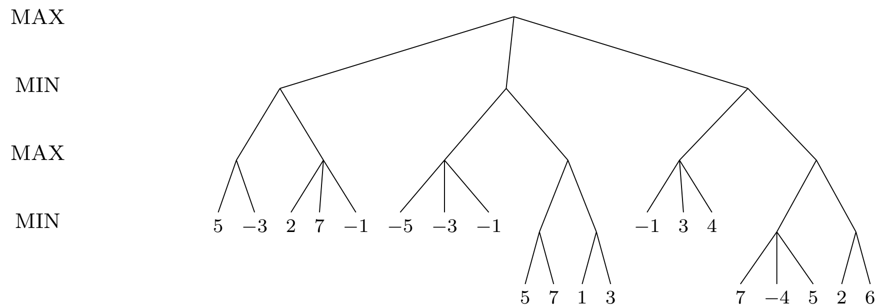
Exercise Sheet 7.

Solution

Exercise 26: Alpha-Beta pruning. (3 Points)

Consider the following game tree corresponding to a two-player zero-sum game as specified in the lecture. As usual, **Max is to start in the initial state (i.e., the root of the tree)**. For the following algorithms, the expansion order is **from left to right, i.e., in each node the left-most branch is expanded first.**



1. Perform Minimax search, i.e., annotate all internal nodes with the correct Minimax value. Which move does Max choose?

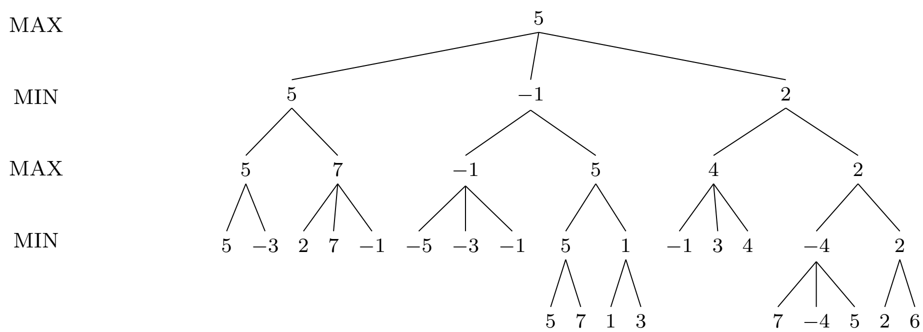
2. Perform Alpha-Beta search. Annotate all internal nodes (that are not pruned) with the value that will be propagated to the parent node as well as the final $[\alpha, \beta]$ window before propagating the value to the parent (similar to slides 37+38 in Chapter 11).

   Mark which edges will be pruned. How many leaf nodes are pruned?

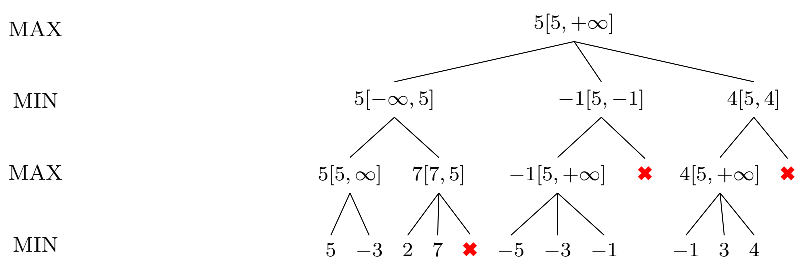
3. There are move orderings that yield more effective pruning (compare, e.g., slides 33+41 in Chapter 11). Reorder the nodes of the tree such that **at least 12 leaf nodes are pruned**. A node counts as pruned in this context if it is not expanded or evaluated. Perform Alpha-Beta search on this tree as in the previous exercise.

(Solution)

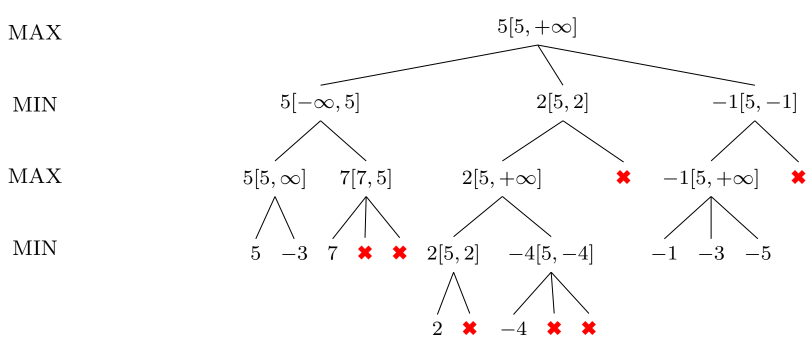
1. The Minimax tree:



2. The Alpha-Beta tree (10 leaf nodes are pruned):



3. The Alpha-Beta tree with 12 pruned leaf nodes:



The basic idea of this tree is as follows:

- Place the optimal goal value on the leftmost branch.
- Place the children of Min-nodes in order of increasing Minimax value.
- Place the children of Max-nodes in order of decreasing Minimax value.

(/Solution)

Exercise 27: STRIPS. (2.5 Points)

Consider the following problem: XinYue wants to bake moon cakes for the moon festival. She has three different types of stamps: Luck, Wealth, and Health. She bakes 3 different types of cakes: veggie, chicken, rose. In order to stamp on a cake she has to be focusing it. Initially, XinYue does not focus anything and she can only ever focus a single cake at a time. She can hold a stamp in either hand: left, right. She wants to stamp Health on the veggie cake, Wealth on the chicken cake, and Luck and Health on the rose cake.

(a) Give a STRIPS formalization of the initial state and the goal.

(b) Give a STRIPS formalization of the actions: LookAt, PickUp, Drop, Stamp. Please use "object variables", i.e., write the actions up in a parameterized way and indicate, for each parameter, by which objects it can be instantiated (e.g. $move(x, y)$ for all $x, y \in \{loc1, loc2, loc3\}$).

Notes:

- The actions should be written in a parameterized way. However, you can also specify special cases separately.
- In both (a) and (b), you should use the following predicates, but you are free to add new predicates if necessary:
  - $Focusing(c)$: To indicate that XinYue is focusing cake $c \in \{veggie, chicken, rose\}$.
  - $Available(s)$: To indicate that stamp $s \in \{Luck, Wealth, Health\}$ is available, so can be picked up and is not currently held in a hand.
  - $Empty(h)$: To indicate that hand $h \in \{left, right\}$ is empty.
  - $Holding(h, s)$: To indicate that XinYue is holding stamp $s \in \{Luck, Wealth, Health\}$ in hand $h \in \{left, right\}$.
  - $Stamped(c, s)$: To indicate that stamp $s \in \{Luck, Wealth, Health\}$ has been stamped on cake $c \in \{veggie, chicken, rose\}$.

(Solution)

(a) Initial state description:

$$I = \{Empty(left), Empty(right), Available(Luck), Available(Wealth), Available(Health)\}$$
$$\cup \{NotStamped(c, s) \mid c \in \{veggie, chicken, rose\},\ \ s \in \{Luck, Health, Wealth\}\}$$

Goal description:

$$G = \{Stamped(veggie, Health), Stamped(chicken, Wealth), Stamped(rose, Luck), Stamped(rose, Health)\}$$
$$\cup \{NotStamped(veggie, s) \mid s \in \{Wealth, Luck\}\}$$
$$\cup \{NotStamped(chicken, s) \mid s \in \{Health, Luck\}\}$$
$$\cup \{NotStamped(rose, Wealth)\}$$

(b) Action descriptions:

- $LookAt(c)$:

$$pre : \{\}$$
$$add : \{Focusing(c1)\}$$
$$del : \{Focusing(c2), Focusing(c3)\}$$

  for all $c1, c2, c3 \in \{veggie, chicken, rose\}$, $c1 \neq c2 \neq c3$.

- $PickUp(h, s)$:

$$pre : \{Available(s), Empty(h)\}$$
$$add : \{Holding(h, s)\}$$
$$del : \{Available(s), Empty(h)\}$$

  for all $s \in \{Luck, Wealth, Health\}$, $h \in \{left, right\}$.

- $Drop(s, h)$:

$$pre : \{Holding(h, s)\}$$
$$add : \{Available(s), Empty(h)\}$$
$$del : \{Holding(h, s)\}$$

  for all $s \in \{Luck, Wealth, Health\}$, $h \in \{left, right\}$.

- $Stamp(c, s, h)$:

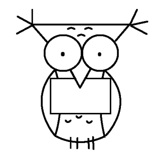
$$pre : \{NotStamped(c, s), Focusing(c), Holding(h, s)\}$$
$$add : \{Stamped(c, s)\}$$
$$del : \{NotStamped(c, s)\}$$

  for all $c \in \{veggie, chicken, rose\}$, $s \in \{Luck, Wealth, Health\}$, $h \in \{left, right\}$.

(/Solution)

Exercise 28: Planning. (2.5 Points)

The university mascot Steffi wants to deliver a letter containing the result of the student council elections from the CS student council to the AStA building. After delivering the results, she wants to go back to her nest in the student council. Steffi is currently located at the Mensa building. She can fly between any two locations.



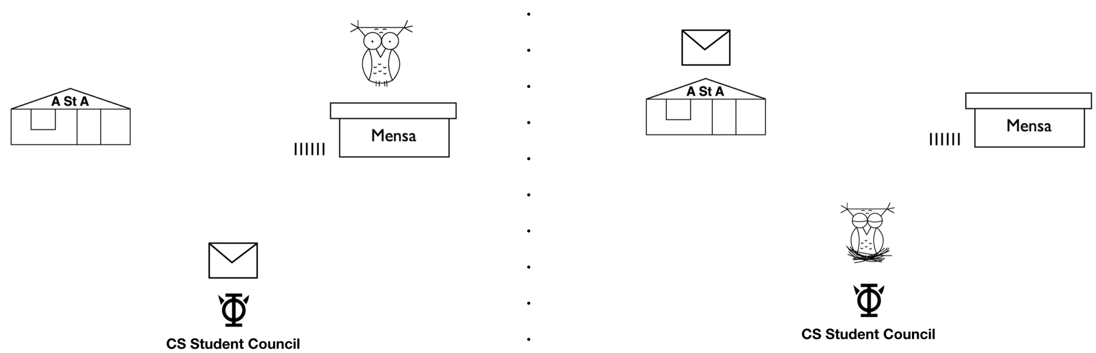


Figure 1: Initial state (left) and goal of the planning task

(a) (i) Complete the formalization of the planning task. Fill in all gaps indicated by a question mark (?). You may introduce new predicates if necessary.

$$
\begin{aligned}
P =& \{at(Steffi, x), at(Letter, x), carrying(Letter), empty\} \\
& \text{for } x \in \{Mensa, AStA, Council\} \\
I =& \{at(Steffi, Mensa), at(Letter, Council), empty\} \\
G =& \{?\} \\
A =& \{fly(x, y) | (x, y) \in \{?\}\} \\
& \cup \{pickup(i, x) \mid x \in \{?\}, i \in \{?\}\} \\
& \cup \{drop(i, x) \mid x \in \{?\}, i \in \{?\}\}
\end{aligned}
$$

- $fly(x, y)$: pre: $\{?\}$, add: $\{?\}$, del: $\{?\}$
- $pickup(i, x)$: pre: $\{?\}$, add: $\{?\}$, del: $\{?\}$
- $drop(i, x)$: pre: $\{?\}$, add: $\{?\}$, del: $\{?\}$

(ii) Give an optimal plan for the task. What is the value of $h^*(I)$?

(b) This year the AStA decided to no longer believe the student councils and wants Steffi to bring all voting slips to the AStA building. In total, there are 20 voting slips. Unfortunately Steffi can only carry up to 2 letters at a time.

   (i) Formalize the new initial and goal state and the new actions. You can add more parameters to the existing predicates/actions or add new predicates (but no new actions) if needed.

   (ii) Consider the modification where there are only 3 voting slips. Give an optimal plan for the task. What is the value of $h^*(I)$?

(c) Consider the following planning task, in which we have only one letter, Steffi is already holding the letter and Steffi can only move between the AStA and the Student Council:

$P = \{atS(C), atS(A), atL(C), atL(A), atL(S)\}$
$I = \{atS(C), atL(S)\}$
$G = \{atS(C), atL(A)\}$
$A = \{fly(x,y), pickup(x), drop(x)\}$, where $x, y \in \{A, C\}$; and $x \neq y$ for $fly(x,y)$

$$\begin{array}{lcll} fly(x,y) & : & pre: & \{atS(x)\} \\ & & add: & \{atS(y)\} \\ & & del: & \{atS(x)\} \\ \\ pickup(x) & : & pre: & \{atL(x), atS(x)\} \\ & & add: & \{atL(S)\} \\ & & del: & \{atL(x)\} \\ \\ drop(x) & : & pre: & \{atL(S), atS(x)\} \\ & & add: & \{atL(x)\} \\ & & del: & \{atL(S)\} \end{array}$$

Draw the reachable part of the state space. Denote every state by the facts that are currently true, i.e., the current positions of Steffi and the Letter (for example, the initial state can be denoted by CS).

(Solution)

(a) (i) $I = \{at(Steffi, Mensa), at(Letter, Council), empty\}$

$G = \{at(Letter, AStA), at(Steffi, Council)\}$

$$\begin{array}{rcl} A & = & \{fly(x,y) \mid x, y \in \{Mensa, AStA, Council\}, x \neq y\} \\ & \cup & \{pickup(i,x) \mid x, y \in \{Mensa, AStA, Council\}, i \in \{Letter\}\} \\ & \cup & \{drop(i,x) \mid x, y \in \{Mensa, AStA, Council\}, i \in \{Letter\}\} \end{array}$$

- $fly(x,y)$:

$$pre: \{at(Steffi, x)\}$$
$$add: \{at(Steffi, y)\}$$
$$del: \{at(Steffi, x)\}$$

for all $x, y \in \{Mensa, AStA, Council\}$, $x \neq y$

- $pickup(i, x)$:

$$pre: \{at(i, x), at(Steffi, x), empty\}$$
$$add: \{carrying(i)\}$$
$$del: \{at(i, x), empty\}$$

for all $x \in \{Mensa, AStA, Council\}$, $i \in \{Letter\}$

- $drop(i, x)$:

$$pre: \{carrying(i), at(Steffi, x)\}$$
$$add: \{at(i, x), empty\}$$
$$del: \{carrying(i)\}$$

for all $x \in \{Mensa, AStA, Council\}$, $i \in \{Letter\}$

(ii) $\pi = \langle fly(Mensa, Council), pickup(Letter, Council), fly(Council, AStA),$
$drop(Letter, AStA), fly(AStA, Council)\rangle$
$h^*(I) = 5$

(b) (i) $I = \{at(Steffi, Mensa), capacity(2)\} \cup \{at(i, Council) \mid i \in \{1, ..., 20\}\}$
$G = \{at(Steffi, Council)\} \cup \{at(i, AStA) \mid i \in \{1, ..., 20\}\}$

- $pickup(i, x, c)$:

$$pre: \{at(i, x), at(Steffi, x), capacity(c)\}$$
$$add: \{carrying(i), capacity(c - 1)\}$$
$$del: \{at(i, x), capacity(c)\}$$

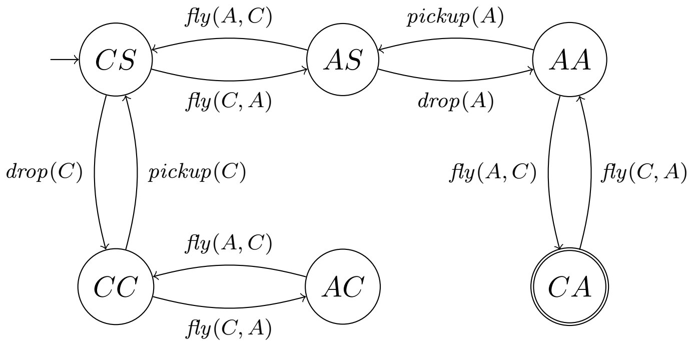
for all $x \in \{Mensa, AStA, Council\}$, $i \in \{1, ...20\}$, $c \in \{1, 2\}$

- $drop(i, x, c)$:

$$pre: \{carrying(i), at(Steffi, x), capacity(c)\}$$
$$add: \{at(i, x), capacity(c + 1)\}$$
$$del: \{carrying(i), capacity(c)\}$$

for all $x \in \{Mensa, AStA, Council\}$, $i \in \{1, ...20\}$, $c \in \{0, 1\}$

(ii) $\pi = \langle fly(Mensa, Council), pickup(1, Council, 2), pickup(2, Council, 1),$
$fly(Council, AStA), drop(2, AStA, 0), drop(1, AStA, 1),$
$fly(AStA, Council), pickup(3, Council, 2),$
$fly(Council, AStA), drop(3, AStA, 1), fly(AStA, Council)\rangle$
$h^* = 11$

(c) 

(/Solution)

Exercise 29: Complexity of Planning. (2 Points)

As specified in the lecture, PolyPlanLen is the problem of deciding, given a STRIPS planning task Π and an integer B bounded by a polynomial in the size of Π, whether or not there exists a plan for Π of length at most B.

Prove that PolyPlanLen is **NP**-hard. Tip: Use a polynomial reduction from SAT.