

1	2
3	4
5	6
7	8
9	10
11	12
13	14
15	16
17	18
19	20
21	22
23	24
25	26
27	28
29	30
31	32
33	34
35	36
37	38
39	40
41	42
43	44

# Image Processing and Computer Vision

Joachim Weickert

Faculty of Mathematics and Computer Science  
Saarland University, Saarbrücken, Germany

weickert@mia.uni-saarland.de  
www.mia.uni-saarland.de/weickert

Summer Term 2019

[www.mia.uni-saarland.de/Teaching/ipcv19.shtml](http://www.mia.uni-saarland.de/Teaching/ipcv19.shtml)

© 2000–2019 Joachim Weickert

## Organisational Issues (1)

1	2
3	4
5	6
7	8
9	10
11	12
13	14
15	16
17	18
19	20
21	22
23	24
25	26
27	28
29	30
31	32
33	34
35	36
37	38
39	40
41	42
43	44

## Organisational Issues

*Welcome to this class!*

*Please do not hesitate to pose questions (in English or German).*

### Why is this Class Important?

- ◆ broad introduction to mathematically well-founded areas of image processing and computer vision
- ◆ important in numerous applications, e.g. medical imaging, computer-aided quality control, driver assistance systems, robotics, bioinformatics, computer graphics, multimedia, and artificial intelligence
- ◆ nice application area for almost all branches of mathematics
- ◆ qualifies for starting a bachelor thesis in our group;  
for a master thesis, one also has to attend  
*Differential Equations in Image Processing and Computer Vision*

## Organisational Issues (2)



### How Can I Make Use of These Classes?

- ◆ 4 hours classroom lectures, 2 hours tutorials (9 ECTS points)
- ◆ can be used in numerous bachelor or master programmes, e.g.:

Study Programme	Usage
Visual Computing	core area visual computing / subarea image analysis
Computer Science	core class (Stammvorlesung) or mathematics class if minor is mathematics
Mathematics	applied mathematics core class or computer science class if minor is computer science
Mathematics and Computer Science	core class in mathematics or core class in computer science
Embedded Systems	master core class
Media Informatics	master core class
Bioinformatics	master core class
Physics	non-physical free elective master class

1	2
3	4
5	6
7	8
9	10
11	12
13	14
15	16
17	18
19	20
21	22
23	24
25	26
27	28
29	30
31	32
33	34
35	36
37	38
39	40
41	42
43	44

## Organisational Issues (3)



### Which Prerequisites are Required?

- ◆ **undergraduate mathematics (e.g. Mathematik für Informatiker I–III)**  
examples of suitable textbooks:
  - E. Kreyszig: *Advanced Engineering Mathematics*. Wiley, Chichester, 2010.
  - M. Wolff, P. Hauck und W. Küchlin: *Mathematik für Informatik und Bioinformatik*. Springer, Berlin, 2004.
- ◆ **elementary C knowledge (for the programming assignments)**  
can be learned e.g. in
  - R. Kirsch, U. Schmitt: *Programmieren in C – Eine mathematikorientierte Einführung*. Springer, Berlin, 2013.
  - *C Programming*  
[http://en.wikibooks.org/wiki/C\\_Programming](http://en.wikibooks.org/wiki/C_Programming)
- ◆ **reasonable working knowledge of English**  
(solutions of assignments can be in German for four tutorial groups)

1	2
3	4
5	6
7	8
9	10
11	12
13	14
15	16
17	18
19	20
21	22
23	24
25	26
27	28
29	30
31	32
33	34
35	36
37	38
39	40
41	42
43	44

## Organisational Issues (4)



### Tutorials

- ◆ combined theoretical and programming assignments
- ◆ coordinated by Jón Arnar Tómasson  
(E1.7, Room 4.16, [tomasson@mia.uni-saarland.de](mailto:tomasson@mia.uni-saarland.de))
- ◆ six groups:
  - Group T1: Tue, 12–14, E1.3, Seminar Room 015 (in English)
  - Group T2: Tue, 14–16, E1.3, Seminar Room 015 (in German)
  - Group T3: Tue, 16–18, E1.3, Seminar Room 015 (in English only)
  - Group W1: Wed, 12–14, E2.5, Seminar Room 1 (U37) (in English only)
  - Group W2: Wed, 14–16, E2.5, Seminar Room 1 (U37) (in English)
  - Group W3: Wed, 16–18, E2.5, Seminar Room 1 (U37) (in English)
- ◆ Optional Guided Programming: Tue, 18–20, E1.3, CIP-Room 104
- ◆ start: next week (with classroom assignments)

1	2
3	4
5	6
7	8
9	10
11	12
13	14
15	16
17	18
19	20
21	22
23	24
25	26
27	28
29	30
31	32
33	34
35	36
37	38
39	40
41	42
43	44

## Organisational Issues (5)



### Registration and Lecture Notes

- ◆ Please register for tutorials between Tue, April 9, 14:00 and Fri, April 12, 13:00:  
[www.mia.uni-saarland.de/Teaching/ipcv19.shtml](http://www.mia.uni-saarland.de/Teaching/ipcv19.shtml)
- ◆ You can also download freshly prepared lecture notes from there.  
They are password-protected and available 30 minutes before each lecture.  
To browse similar slides earlier, just go to last year's IPCV website.
- ◆ You must also register in the HISPOS-LSF system (apart from Erasmus students).

1	2
3	4
5	6
7	8
9	10
11	12
13	14
15	16
17	18
19	20
21	22
23	24
25	26
27	28
29	30
31	32
33	34
35	36
37	38
39	40
41	42
43	44

## Organisational Issues (6)



### The Rules

- ◆ There will be weekly homework assignments (theory and programming) as well as classroom assignments.
- ◆ For the homework assignments you can obtain up to 24 points per week, depending on the correctness of your homework solutions.
- ◆ Active participation in the classroom assignments gives you 12 more points per week, regardless of the correctness of your classroom solutions.
- ◆ To qualify for the exams you need two third of all possible points.  
For 13 weeks, this comes down to  $13 \cdot \frac{2}{3} \cdot (24 + 12) = 312$  points.
- ◆ These points are your entry ticket for the exams, but do not influence your grade. If you have qualified for last year's IPCV exams, you must requalify.
- ◆ Working in groups of up to 3 people is fine, if all are in the same tutorial group.
- ◆ Each group submits the theoretical assignments in handwritten form, and the programming assignments via e-mail. One submission per group.
- ◆ Sample solutions for the homework and classroom assignments can be downloaded afterwards.

1	2
3	4
5	6
7	8
9	10
11	12
13	14
15	16
17	18
19	20
21	22
23	24
25	26
27	28
29	30
31	32
33	34
35	36
37	38
39	40
41	42
43	44


## Organisational Issues (7)



### Exams

- ◆ Self-test problems (Probeklausur) with sample solutions will be available.
- ◆ **First exam: Thu, July 25, 2019 from 14:00 to 17:00.**  
**Second exam: Wed, October 9, 2019 from 14:00 to 17:00.**
- ◆ Two exams count as two attempts.  
If you participate in both, the better grade counts.
- ◆ Both exams will be **closed book exams**.  
You are allowed and obliged to bring three things to your desk only:
  - your student ID card (Studierendenausweis)
  - a ball-pen that has no function other than writing
  - one double-sided, hand-written A4 cheat sheet  
(will be collected after the exam and returned at the inspection meeting)
- ◆ All questions will be available both in English and in German.  
Your solutions can be in English or German, too.


1	2
3	4
5	6
7	8
9	10
11	12
13	14
15	16
17	18
19	20
21	22
23	24
25	26
27	28
29	30
31	32
33	34
35	36
37	38
39	40
41	42
43	44

M	I
	A
1	2
3	4
5	6
7	8
9	10
11	12
13	14
15	16
17	18
19	20
21	22
23	24
25	26
27	28
29	30
31	32
33	34
35	36
37	38
39	40
41	42
43	44

## Questions and Problems

- ◆ Please feel free to ask me during the lectures, in the break, or in the office hour (Tue, 14:15–15:15, E1.7, Room 4.23).
- ◆ Also your tutor will be happy to help and offers an office hour. The times will be given at the IPCV web page.
- ◆ Jón Arnar Tómasson's office hour is Wed, 14:30–15:30 (E1.7, Room 4.16).

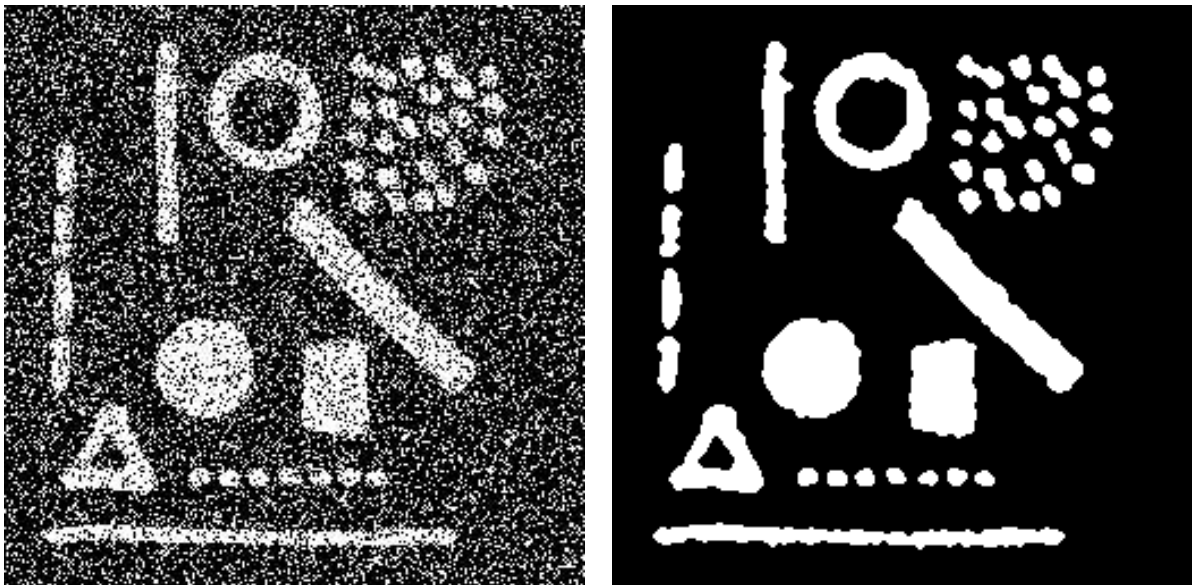
# Important Areas within Visual Computing (1)

M	I
	A
1	2
3	4
5	6
7	8
9	10
11	12
13	14
15	16
17	18
19	20
21	22
23	24
25	26
27	28
29	30
31	32
33	34
35	36
37	38
39	40
41	42
43	44

## Important Areas within Visual Computing

- ◆ **Image Processing (Bildverarbeitung):**  
transforms a digital image to another digital image that allows a better interpretation by humans or computers  
*Examples:* denoising, deblurring
- ◆ **Computer Vision, Image Understanding (Rechnersehen, Bildverstehen):**  
extraction of information about the 3-D world from 2-D images  
*Examples:* stereographic reconstruction, object recognition
- ◆ **Pattern Recognition (Mustererkennung):**  
labelling of image structures to certain classes  
*Examples:* character recognition, cell classification

Important Areas within Visual Computing (2)



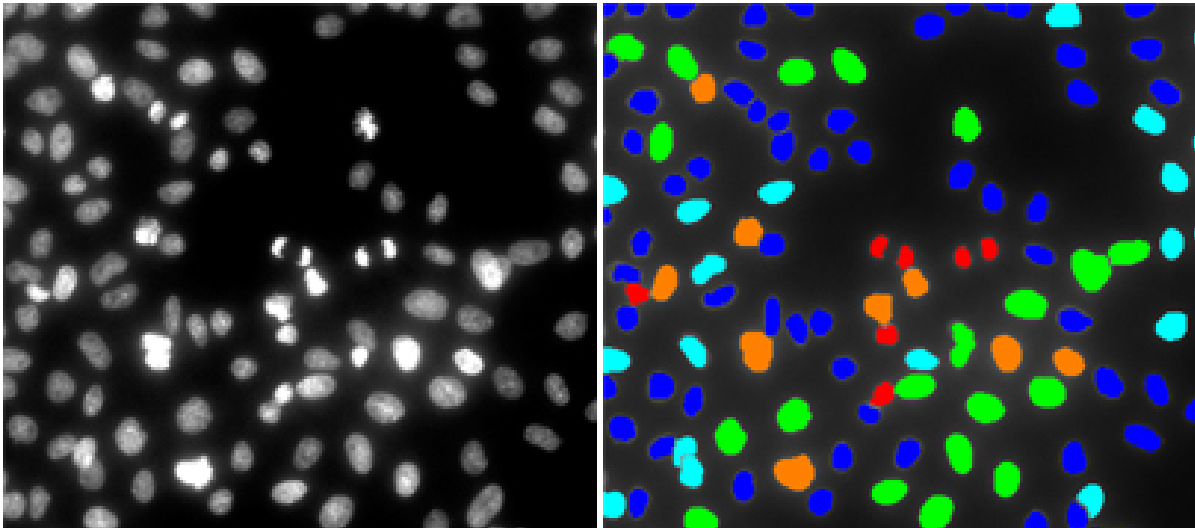
Example of an image processing application. **Left:** Noisy original image. **Right:** Filtered. Author: J. Weickert.

Important Areas within Visual Computing (3)



Example of a computer vision application. **Left:** Stereo image pair. **Right:** 3-D reconstruction. Authors: L. Alvarez, R. Deriche, J. Sánchez, J. Weickert.

## Important Areas within Visual Computing (4)



Example of a pattern recognition application. **Left:** Cells. **Right:** Classification into different cell cycle phases. Source: MetaMorph Analysis, <http://www.moleculardevices.com/>.

## Important Areas within Visual Computing (5)

### ◆ Computer Graphics (Computergrafik):

synthesis of a digital image that is supposed to look like an image of a 3-D scene

*Example:* computer games

### ◆ Geometric Modelling (Geometrische Modellierung):

mathematical representation of curves and surfaces for synthesising 2-D and 3-D objects on a computer

*Example:* CAD-based design of a car coachwork

### ◆ Scientific Visualisation (Wissenschaftliche Visualisierung):

useful visual representation of data from real experiments or computer simulations

*Example:* flow visualisation

M	I
	A
1	2
3	4
5	6
7	8
9	10
11	12
13	14
15	16
17	18
19	20
21	22
23	24
25	26
27	28
29	30
31	32
33	34
35	36
37	38
39	40
41	42
43	44


M	I
	A
1	2
3	4
5	6
7	8
9	10
11	12
13	14
15	16
17	18
19	20
21	22
23	24
25	26
27	28
29	30
31	32
33	34
35	36
37	38
39	40
41	42
43	44



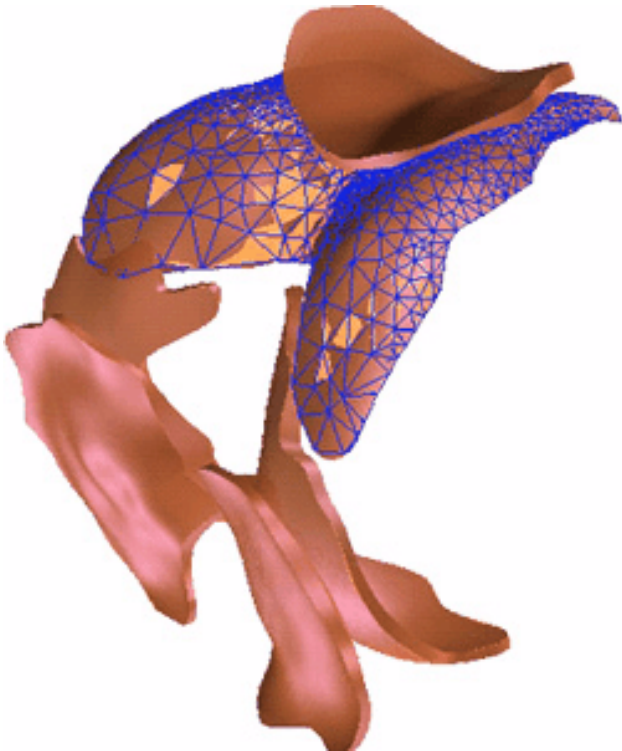
Important Areas within Visual Computing (6)




Example of a computer graphics application: Realistically looking simulation of a forest using ray tracing.  
Author: P. Slusallek (2006).

M	I
	A
1	2
3	4
5	6
7	8
9	10
11	12
13	14
15	16
17	18
19	20
21	22
23	24
25	26
27	28
29	30
31	32
33	34
35	36
37	38
39	40
41	42
43	44

Important Areas within Visual Computing (7)

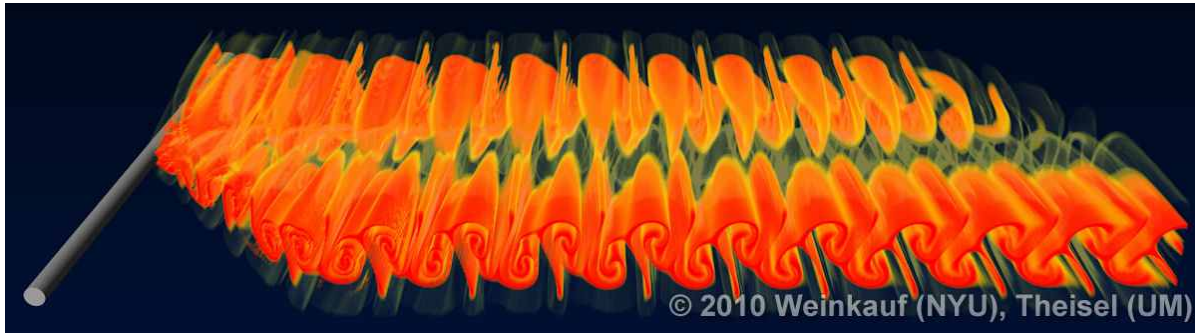


Example of a geometric modelling application: A surface model of a tissue in a human knee using a triangulation. Source: [http://www.scorec.rpi.edu/research\\_biomechanical.html](http://www.scorec.rpi.edu/research_biomechanical.html).


M	I
	A
1	2
3	4
5	6
7	8
9	10
11	12
13	14
15	16
17	18
19	20
21	22
23	24
25	26
27	28
29	30
31	32
33	34
35	36
37	38
39	40
41	42
43	44



## Important Areas within Visual Computing (8)



Example of a scientific visualisation application: Three-dimensional flow visualisation. Authors: T. Weinkauff, H. Theisel.

M	I
	A
1	2
3	4
5	6
7	8
9	10
11	12
13	14
15	16
17	18
19	20
21	22
23	24
25	26
27	28
29	30
31	32
33	34
35	36
37	38
39	40
41	42
43	44

## Important Areas within Visual Computing (9)

### ◆ Image Acquisition, Imaging (Bildgebende Verfahren):


deals with all aspects of image creation

*Example:* tomographic reconstruction

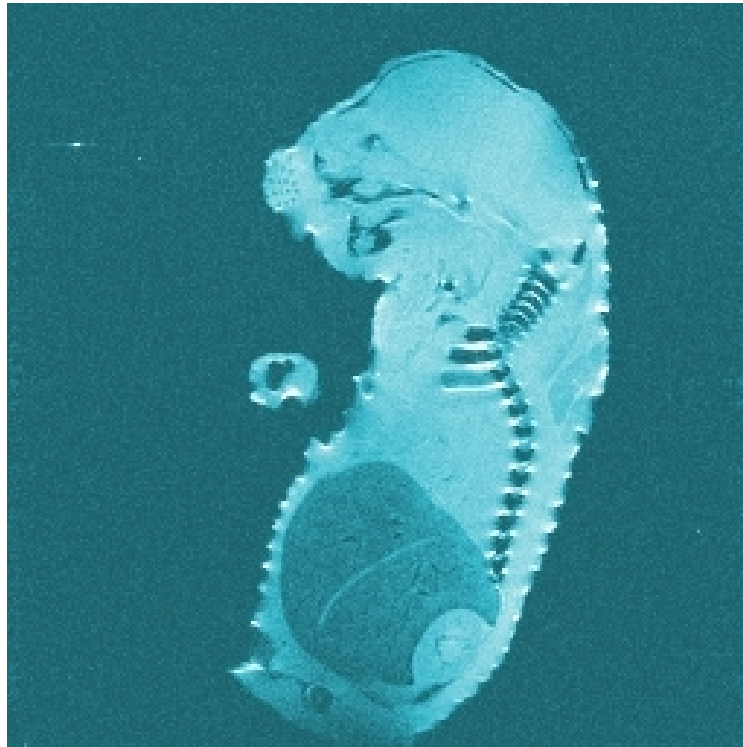
### ◆ Computational Photography:

combines image capture, processing and manipulation

*Example:* fusing differently exposed images to a single, tone mapped image

M	I
	A
1	2
3	4
5	6
7	8
9	10
11	12
13	14
15	16
17	18
19	20
21	22
23	24
25	26
27	28
29	30
31	32
33	34
35	36
37	38
39	40
41	42
43	44

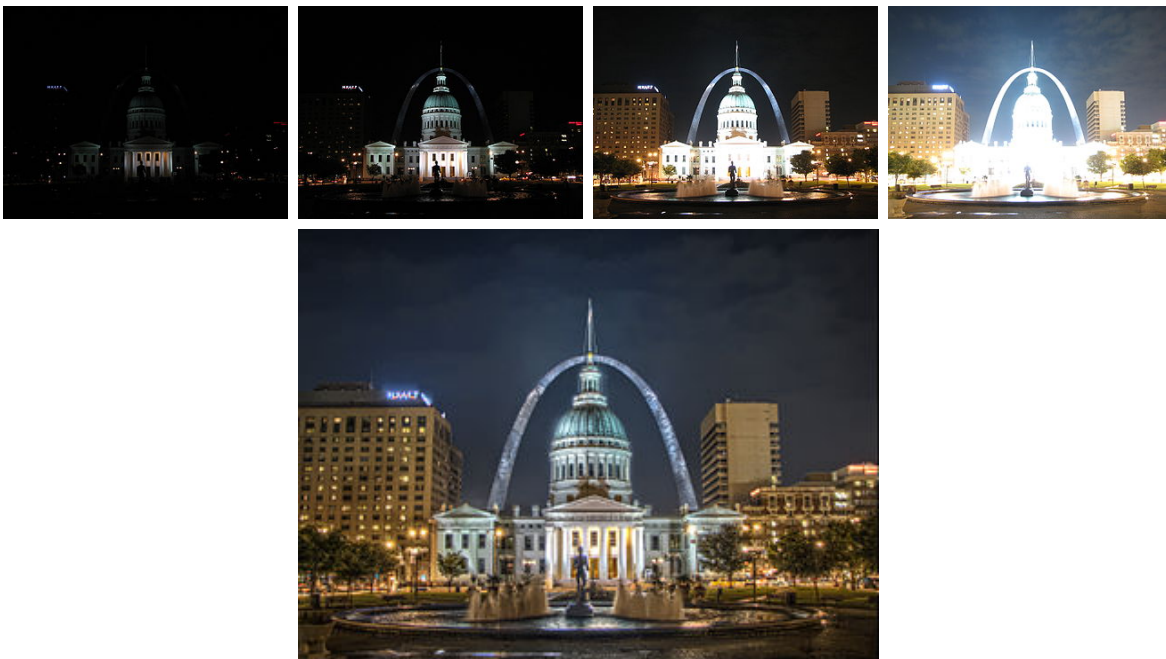
## Important Areas within Visual Computing (10)



Example of an imaging application: Magnetic resonance image of a mouse embryo. Author: F. Volke.

1	2
3	4
5	6
7	8
9	10
11	12
13	14
15	16
17	18
19	20
21	22
23	24
25	26
27	28
29	30
31	32
33	34
35	36
37	38
39	40
41	42
43	44

## Important Areas within Visual Computing (11)



Example of a computational photography application: **Top row:** Four differently exposed images. **Bottom row:** Fusion to a single image with local tone mapping. Source: [en.wikipedia.org/wiki/High\\_dynamic\\_range\\_imaging](https://en.wikipedia.org/wiki/High_dynamic_range_imaging).

1	2
3	4
5	6
7	8
9	10
11	12
13	14
15	16
17	18
19	20
21	22
23	24
25	26
27	28
29	30
31	32
33	34
35	36
37	38
39	40
41	42
43	44

## Collective Terms

### ◆ Image Analysis (Bildanalyse):

includes image processing, computer vision, pattern recognition

### ◆ Image Synthesis (Bildsynthese):

includes computer graphics, geometric modelling, scientific visualisation

### ◆ Visual Computing, Image Sciences (Bildwissenschaften):

comprises image acquisition and the analysis and synthesis of digital images

We focus on image analysis, in particular on image processing and computer vision.

There is a growing confluence between the individual areas:

- ◆ Human motion capture and embedding into virtual scenes combines image analysis and image synthesis.
- ◆ Computational photography combines image acquisition, image analysis, and image synthesis.

1	2
3	4
5	6
7	8
9	10
11	12
13	14
15	16
17	18
19	20
21	22
23	24
25	26
27	28
29	30
31	32
33	34
35	36
37	38
39	40
41	42
43	44

# Contents (1)

## Contents




Foundations and Transformations


Image Processing


Computer Vision and Image Understanding


The IPCV class resembles a triptychon, consisting of three parts of roughly equal size. Image source: <https://commons.wikimedia.org/wiki/File:Welterbe-Wein-Triptychon.jpg>.

1	2
3	4
5	6
7	8
9	10
11	12
13	14
15	16
17	18
19	20
21	22
23	24
25	26
27	28
29	30
31	32
33	34
35	36
37	38
39	40
41	42
43	44

Contents (2)		M	I
			A
<b>Part I: Foundations and Transformations</b>		1	2
		3	4
		5	6
<b>1. Foundations</b>		7	8
1.1 Definitions, Image Types, Discretisation		9	10
1.2 Degradations in Digital Images		11	12
1.3 Colour Perception and Colour Spaces		13	14
		15	16
<b>2. Image Transformations</b>		17	18
2.1 Continuous Fourier Transform		19	20
2.2 Discrete Fourier Transform		21	22
2.3 Image Pyramids		23	24
2.4 Wavelet Transform		25	26
		27	28
<b>3. Image Compression</b>		29	30
		31	32
<b>4. Image Interpolation</b>		33	34
		35	36
		37	38
		39	40
		41	42
		43	44


Contents (3)		M	I
			A
<b>Part II: Image Processing</b>		1	2
		3	4
		5	6
<b>5. Point Operations</b>		7	8
		9	10
<b>6. Linear Filters</b>		11	12
6.1 System Theory		13	14
6.2 Edge Detection		15	16
6.3 Corner Detection		17	18
		19	20
<b>7. Nonlinear Filters</b>		21	22
7.1 Morphology and Median Filters		23	24
7.2 Wavelet Shrinkage		25	26
7.3 Bilateral Filters		27	28
7.4 NL Means		29	30
7.5 Nonlinear Diffusion Filtering		31	32
		33	34
<b>8. Global Filters</b>		35	36
8.1 Discrete Variational Methods		37	38
8.2 Continuous Variational Methods		39	40
8.3 Deconvolution Methods		41	42
		43	44
<b>9. Texture Analysis</b>			

Contents (4)		M	I
			A
<b>Part III: Computer Vision and Image Understanding</b>		1	2
		3	4
10. Segmentation		5	6
10.1 Classical Methods		7	8
10.2 Optimisation Methods		9	10
		11	12
11. Image Sequence Analysis		13	14
11.1 Local Methods		15	16
11.2 Variational Methods		17	18
		19	20
12. 3-D Reconstruction		21	22
12.1 Camera Geometry		23	24
12.2 Stereo		25	26
12.3 Shape-from-Shading		27	28
		29	30
13. Object Recognition		31	32
13.1 Hough Transform		33	34
13.1 Moment Invariants		35	36
13.2 Eigenspace Methods		37	38
		39	40
		41	42
		43	44

References (1)		M	I
			A
<b>References</b>		1	2
		3	4
		5	6
We do <b>not</b> rely on a single textbook:		7	8
Our didactic concept differs from all of them.		9	10
However, here are some of the more useful books:		11	12
		13	14
<b>Textbooks on Image Processing</b>		15	16
		17	18
◆ J. Bigun: <i>Vision with Direction</i> . Springer, Berlin, 2010.		19	20
<i>One of most systematic books on image processing, with a clear focus on matrix representations.</i>		21	22
		23	24
		25	26
◆ R. C. Gonzalez, R. E. Woods: <i>Digital Image Processing</i> . Global Edition, Pearson Prentice Hall, Upper Saddle River, 2017.		27	28
<i>A classical book on image processing. Comprehensive and fairly well readable.</i>		29	30
		31	32
		33	34
◆ K. D. Tönnies: <i>Grundlagen der Bildverarbeitung</i> . Pearson Studium, München, 2005.		35	36
<i>One of the better German books on image processing. Contains no computer vision.</i>		37	38
		39	40
		41	42
		43	44



## References (2)


M	I
	A
1	2
3	4
5	6
7	8
9	10
11	12
13	14
15	16
17	18
19	20
21	22
23	24
25	26
27	28
29	30
31	32
33	34
35	36
37	38
39	40
41	42
43	44

### Textbooks on Computer Vision

- ◆ R. Klette: *Concise Computer Vision*. Springer, London, 2014.  
*Covers also some more recent methods, but not everything.*
- ◆ R. Szeliski: *Computer Vision: Algorithms and Applications*. Springer, New York, 2010.  
[http://szeliski.org/Book/drafts/SzeliskiBook\\_20100903\\_draft.pdf](http://szeliski.org/Book/drafts/SzeliskiBook_20100903_draft.pdf)  
*Up to date, covers many things, but not very detailed.*
- ◆ E. Trucco, A. Verri: *Introductory Techniques for 3-D Computer Vision*. Prentice-Hall, Upper Saddle River, 1998.  
*Good selection of important aspects, but not fully up to date anymore.*

These and other textbooks can be found in the course reading (Semesterapparat) of our Computer Science and Mathematics Library (Building E2.3).

## References (3)

M	I
	A
1	2
3	4
5	6
7	8
9	10
11	12
13	14
15	16
17	18
19	20
21	22
23	24
25	26
27	28
29	30
31	32
33	34
35	36
37	38
39	40
41	42
43	44

### Useful Internet Resources

- ◆ **CV Online**  
<http://homepages.inf.ed.ac.uk/rbf/CVonline/>  
Online compendium on numerous image processing and computer vision topics.  
Often very useful.
- ◆ **Annotated Computer Vision Bibliography**  
<http://www.visionbib.com/bibliography/contents.html>  
helpful when searching for specific references for a certain topic
- ◆ **Google Scholar**  
<http://scholar.google.com>  
provides links to highly cited articles of all scientific areas

## Lecture 1:

# Foundations I: Image Types and Discretisation

### Contents

1. Sampling and Quantisation
2. Types of Images

© 2000–2019 Joachim Weickert

1	2
3	4
5	6
7	8
9	10
11	12
13	14
15	16
17	18
19	20
21	22
23	24
25	26
27	28
29	30
31	32
33	34
35	36
37	38
39	40
41	42
43	44

## Sampling and Quantisation (1)

## Sampling and Quantisation

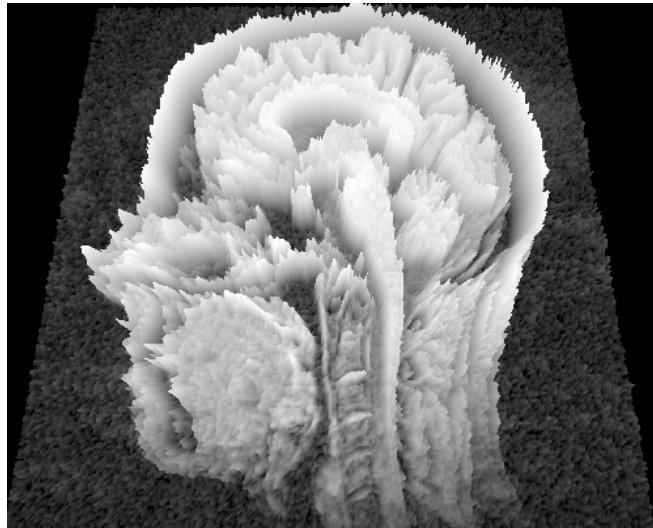
### Continuous Greyscale Image (Kontinuierliches Grauwertbild)

- ◆ mapping  $f$  from a rectangular domain (Definitions-bereich)  $\Omega = (0, a_1) \times (0, a_2)$  to a co-domain (Wertebereich)  $\mathbb{R}$ :

$$f : \mathbb{R}^2 \supset \Omega \rightarrow \mathbb{R}$$

- ◆ domain  $\Omega$  is called *image domain* or *image plane*
- ◆ co-domain specifies *grey value*
- ◆ Usually low grey values are dark and high grey values bright.

1	2
3	4
5	6
7	8
9	10
11	12
13	14
15	16
17	18
19	20
21	22
23	24
25	26
27	28
29	30
31	32
33	34
35	36
37	38
39	40
41	42
43	44



**Left:** Magnetic resonance (MR) image of a human head. **Right:** Representation as a function  $f(x, y)$  over a rectangular image domain  $\Omega$ . Authors: J. Weickert, C. Schnörr.

1	2
3	4
5	6
7	8
9	10
11	12
13	14
15	16
17	18
19	20
21	22
23	24
25	26
27	28
29	30
31	32
33	34
35	36
37	38
39	40
41	42
43	44

### Sampling (Abtastung)

- ◆ discretisation of the *domain*  $\Omega$
- ◆ Image data are only given on a rectangular point grid within the image domain  $\Omega$ .
- ◆ creates a digital image

$$\{f_{i,j} \mid i = 1, \dots, N; j = 1, \dots, M\}$$

- ◆ The grid point / cell  $(i, j)$  is called *pixel* (picture element).
- ◆ 2-D images often have equal pixel distances in both directions.
- ◆ Image processing people often normalise these grid sizes to 1.  
This can be dangerous and is **not** recommended!
- ◆ If the sampling is too coarse, the image quality degrades severely.

1	2
3	4
5	6
7	8
9	10
11	12
13	14
15	16
17	18
19	20
21	22
23	24
25	26
27	28
29	30
31	32
33	34
35	36
37	38
39	40
41	42
43	44

## Sampling and Quantisation (4)

256 × 256 pixels



128 × 128 pixels



64 × 64 pixels



32 × 32 pixels

Digital test image with different sampling rates. Author: J. Weickert.

M	I
1	2
3	4
5	6
7	8
9	10
11	12
13	14
15	16
17	18
19	20
21	22
23	24
25	26
27	28
29	30
31	32
33	34
35	36
37	38
39	40
41	42
43	44

## Sampling and Quantisation (5)

### Quantisation (Quantisierung)

- ◆ discretisation of the *co-domain*
- ◆ saves disk space
- ◆ If a grey value is encoded with a single byte, the discrete co-domain is given by  $\{0, 1, \dots, 255\}$ .
- ◆ Binary images have the co-domain  $\{0, 1\}$ .
- ◆ Humans can distinguish only about 40 greyscales.  
They are even very good in recognising the content of binary images.

M	I
1	2
3	4
5	6
7	8
9	10
11	12
13	14
15	16
17	18
19	20
21	22
23	24
25	26
27	28
29	30
31	32
33	34
35	36
37	38
39	40
41	42
43	44

256 greyscales



32 greyscales



8 greyscales



2 greyscales

Digital test image (256 × 256 pixels) with different quantisation rates. Author: J. Weickert.

1	2
3	4
5	6
7	8
9	10
11	12
13	14
15	16
17	18
19	20
21	22
23	24
25	26
27	28
29	30
31	32
33	34
35	36
37	38
39	40
41	42
43	44

## Types of Images (1)

### Types of Images

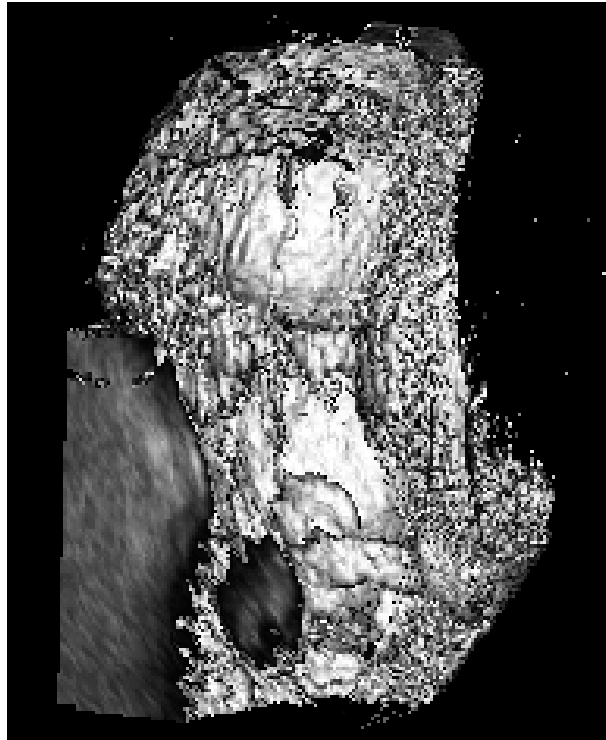
#### *m*-Dimensional Images

- ◆ have their domain in  $\mathbb{R}^m$
- ◆  $m = 1$ : signals
- ◆  $m = 2$ : (two-dimensional) images
- ◆  $m = 3$ : three-dimensional images
  - important in medical imaging, e.g. computerised tomography (CT, Computertomographie), magnetic resonance imaging (MRI, Kernspintomographie).
  - Image points/cells in 3-D are called *voxels* (volume elements).
  - Voxel dimensions usually differ in different directions!

1	2
3	4
5	6
7	8
9	10
11	12
13	14
15	16
17	18
19	20
21	22
23	24
25	26
27	28
29	30
31	32
33	34
35	36
37	38
39	40
41	42
43	44



## Types of Images (2)



Rendering of a 3-D ultrasound image of a human fetus in its 10th week. Authors: J. Weickert, K. Zuiderveld, B.M. ter Haar Romeny, W. Niessen.

M	I
	A
1	2
3	4
5	6
7	8
9	10
11	12
13	14
15	16
17	18
19	20
21	22
23	24
25	26
27	28
29	30
31	32
33	34
35	36
37	38
39	40
41	42
43	44

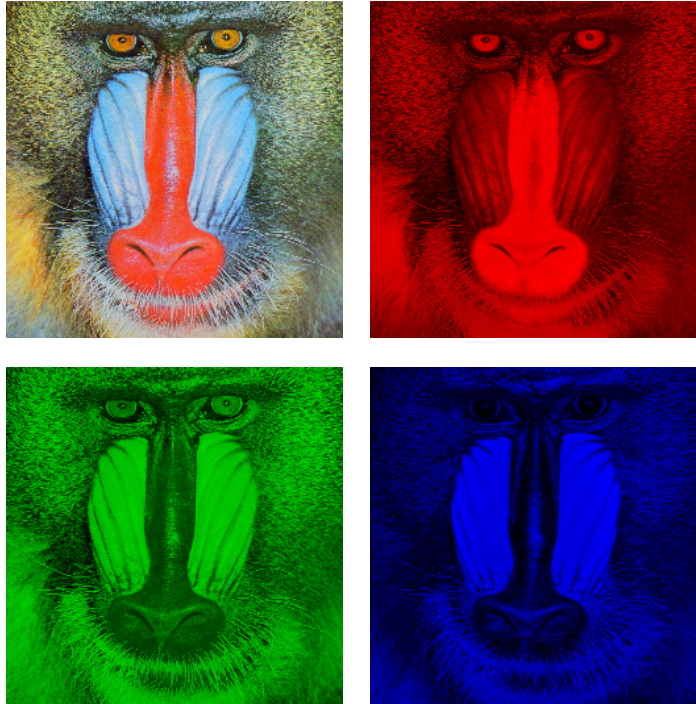
## Types of Images (3)

### Vector-Valued Images

- ◆ have their co-domain in  $\mathbb{R}^n$ , containing  $n$  channels
- ◆ Example 1: Colour Images
  - three channels: R (red), G (green), B (blue)
  - Humans can distinguish two million colours!
- ◆ Example 2: Satellite Images
  - Different channels represent different frequency bands.
  - Multispectral images use only a few channels.  
Hyperspectral images involve hundreds or thousands of channels.

M	I
	A
1	2
3	4
5	6
7	8
9	10
11	12
13	14
15	16
17	18
19	20
21	22
23	24
25	26
27	28
29	30
31	32
33	34
35	36
37	38
39	40
41	42
43	44

## Types of Images (4)



Colour image as example for a vector-valued image. **Top left:** Original image. **Top right:** Red channel. **Bottom left:** Green channel. **Bottom right:** Blue channel. Author: J. Weickert.

M	I
1	2
3	4
5	6
7	8
9	10
11	12
13	14
15	16
17	18
19	20
21	22
23	24
25	26
27	28
29	30
31	32
33	34
35	36
37	38
39	40
41	42
43	44

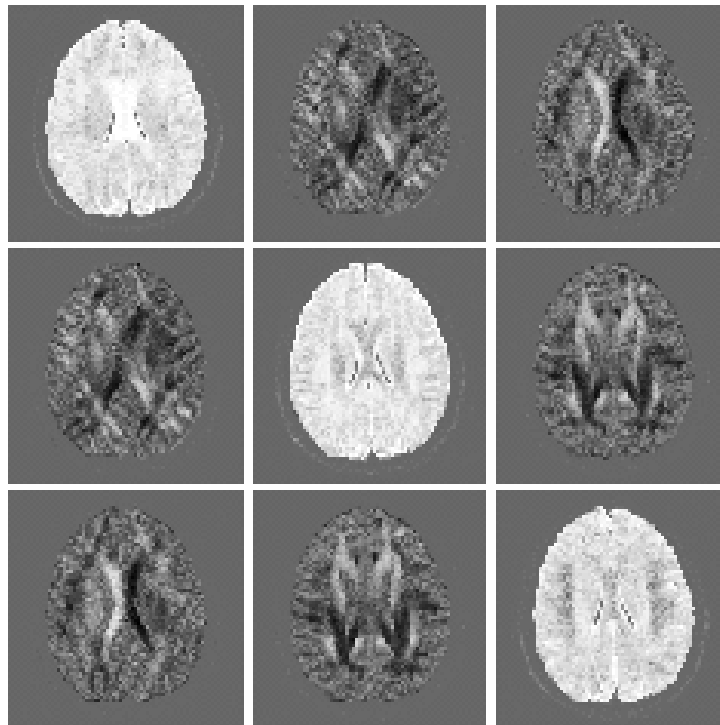
## Types of Images (5)

### Matrix-Valued Images

- ◆ have their co-domain in  $\mathbb{R}^{n \times n}$
- ◆ Example: Diffusion Tensor Magnetic Resonance Imaging (DT-MRI):  
measures in each voxel of a 3-D image domain a symmetric positive definite  $3 \times 3$  matrix
- ◆ may create additional constraints, e.g.:  
A reasonable filter should not destroy relevant properties such as positive definiteness of the matrix field.

M	I
1	2
3	4
5	6
7	8
9	10
11	12
13	14
15	16
17	18
19	20
21	22
23	24
25	26
27	28
29	30
31	32
33	34
35	36
37	38
39	40
41	42
43	44

## Types of Images (6)



Images of the nine coefficients of a DT-MRI data set of a human brain. Since the diffusion matrix is a symmetric  $3 \times 3$  matrix, only 6 out of 9 images differ. Authors: D. Weinstein, G. Kindlmann, E. Lundberg.

M	I
A	
1	2
3	4
5	6
7	8
9	10
11	12
13	14
15	16
17	18
19	20
21	22
23	24
25	26
27	28
29	30
31	32
33	34
35	36
37	38
39	40
41	42
43	44

## Types of Images (7)

### Image Sequences (Bildfolgen)

- ◆ One can consider image sequences for any of the above mentioned types of images.
- ◆ This increases the dimensionality of the domain from  $m$  to  $m + 1$ .
- ◆ Example: 3-D echo cardiography (Echokardiographie) creates a sequence of 3-D scalar-valued images (can be regarded as 4-D image)

### Relevant Images in This Class

- ◆ We mainly focus on 2-D scalar-valued (i.e. greyscale) images and their image sequences.
- ◆ This keeps the description as simple as possible.
- ◆ Many of our methods can be generalised to other types of images. Often this is not very challenging.

M	I
A	
1	2
3	4
5	6
7	8
9	10
11	12
13	14
15	16
17	18
19	20
21	22
23	24
25	26
27	28
29	30
31	32
33	34
35	36
37	38
39	40
41	42
43	44

## Types of Images (8)



**Form left to right and from top to bottom:** Four subsequent frames of an image sequence (the famous Hamburg taxi scene,  $256 \times 190$  pixels). Can you recognise what has moved in which direction?

Source: [http://i21www.ira.uka.de/image\\_sequences/](http://i21www.ira.uka.de/image_sequences/)

## Summary

### Summary

- ◆ Digital images are discretised in two ways:  
in the domain (sampling) and the co-domain (quantisation).
- ◆ generalisation of the domain:  
 $m$ -dimensional images, image sequences
- ◆ generalisation of the co-domain:  
vector-valued images, matrix-valued images

M	I
1	2
3	4
5	6
7	8
9	10
11	12
13	14
15	16
17	18
19	20
21	22
23	24
25	26
27	28
29	30
31	32
33	34
35	36
37	38
39	40
41	42
43	44

M	I
1	2
3	4
5	6
7	8
9	10
11	12
13	14
15	16
17	18
19	20
21	22
23	24
25	26
27	28
29	30
31	32
33	34
35	36
37	38
39	40
41	42
43	44