Image Processing and Computer Vision (IPCV)

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Example Solutions for Classroom Assignment 2 (C2)

Problem 1 (Colour Spaces)

We should first mention that

1. the C_g channel measures deviation from Z in green–magenta direction.
2. the C_r channel measures deviation from Z in red–cyan direction.

As known from the lecture, the RGB representation of a grey value $c \in [0,1]$ is given by $R = G = B = c$, as the grey values have to lie on the diagonal of the RGB-Cube. If you plug these values in our transformation formula, we obtain the ZCgCr representation $(c, 1/2, 1/2)$. This makes sense as the weights for the luma channel sum up to 1, and the chroma channels measure the deviation in either direction towards the respective colour components. If we have a grey value, there is no deviation and we obtain the value $1/2$, which we needed to shift the values into the interval $[0,1]$.

We are looking for the coefficients that satisfy the following equations.

$$\begin{array}{ccccccccc} Z & = & 0 & + & rR & + & gG & + & bB \\ C_g & = & \frac{1}{2} & + & g_r R & + & g_g G & + & g_b B \\ C_r & = & \frac{1}{2} & + & r_r R & + & r_g G & + & r_b B \end{array}$$

We construct the formula for transformation in the same way as for YCbCr in the lecture. For the luma component, Z, an explicit formula in terms of R, G and B is already given:

$$Z = \frac{1}{3}R + \frac{1}{3}G + \frac{1}{3}B$$

Let us first consider chroma channel C_g. Since we want to move in a linear fashion from green to magenta in an interval from $[0,1]$ (deviation is shifted by $1/2$), for $(R,G,B) = (0,1,0)$ (green) we have to fulfil

$$\frac{1}{2} + g_r \cdot 0 + g_g \cdot 1 + g_b \cdot 0 = 1$$

and for $(R,G,B) = (1,0,1)$ (magenta) we want to have

$$\frac{1}{2} + g_r \cdot 1 + g_g \cdot 0 + g_b \cdot 1 = 0.$$

From the first equation we can uniquely define $g_g = \frac{1}{2}$. For the remaining unknowns we seemingly have only one equation, but we also have to incorporate the ratio between R and B in the formula for the luma channel. The ratio of the weights for the both channels should be the same, i.e.

$$\frac{g_r}{g_b} = \frac{r}{b} = \frac{\frac{1}{3}}{\frac{1}{3}}$$

Solving that, we get $g_r = g_b$ and finally $g_r = -\frac{1}{4}$ and $g_b = -\frac{1}{4}$.

Treating the red–cyan direction in the same way yields the equations

$$\begin{aligned}
\frac{1}{2} + r_r \cdot 1 + r_g \cdot 0 + r_b \cdot 0 = 1 \\
\frac{1}{2} + r_r \cdot 0 + r_g \cdot 1 + r_b \cdot 1 = 0
\end{aligned}$$

and we again get $r_r = \frac{1}{2}$, $r_g = r_b = -\frac{1}{4}$.

The transformation formula in matrix-vector notation is given by

$$\begin{pmatrix} Z \\ C_g \\ C_r \end{pmatrix} = \begin{pmatrix} 0 \\ \frac{1}{2} \\ \frac{1}{2} \end{pmatrix} + \begin{pmatrix} \frac{1}{3} & \frac{1}{3} & \frac{1}{3} \\ -\frac{1}{4} & \frac{1}{2} & -\frac{1}{4} \\ \frac{1}{2} & -\frac{1}{4} & -\frac{1}{4} \end{pmatrix} \begin{pmatrix} R \\ G \\ B \end{pmatrix}$$

(compare Slide 26 of Lecture 3). Note that we can also identify magenta with 1 in the C_g channel and green with 0 which might be a more intuitive representation of the direction green–magenta, but this notation is consistent with the definition from the Lecture.

We can also compute the transformation formula for other YCbCr-like spaces using the general formulas

$$\begin{aligned}
Z &= rR + gG + (1 - g - r)B \\
C_g &= \frac{1}{2} + \frac{1}{2}\frac{G - Z}{1 - g} \\
C_r &= \frac{1}{2} + \frac{1}{2}\frac{R - Z}{1 - r}
\end{aligned}$$

We fix the weight for two of the RGB-channels (in this example G and R) and determine by this the weight of the third channel. The deviation of a color channel in respect to the luma channel (e.g. $G - Z$) is normalised to the interval $[-1/2, 1/2]$ (e.g. by multiplication with $\frac{1}{2(1-g)}$) and shifted to the interval $[0, 1]$. The colour space in this exercise results from the choice $g = \frac{1}{3}$ and $r = \frac{1}{3}$.

Remark: In the literature, one usually distinguishes a colour space with continuous chroma channels (YPgPr) and its discrete counter part (YCgCr). For simplicity reasons, we don't make this notational distinction here.

Problem 2 (Continuous Fourier Transform of a Hat Function)

The function f from this exercise corresponds to a convolution $f(x) = g(x) * g(x) * g(x)$ where g is the box function from H1, Problem 1:

$$g(x) := \begin{cases} \frac{1}{2} & -1 \leq x \leq 1 \\ 0 & \text{else} \end{cases}$$

From the lecture we know what form the Fourier transform of a box function h with height A on the interval $[-R, R]$ is:

$$\mathcal{F}[h](u) = \frac{A}{\pi u} \sin(2\pi u R)$$

In the case of our box function g we have $A = \frac{1}{2}$ and $R = 1$, thus yielding

$$\mathcal{F}[g](u) = \frac{1}{2\pi u} \sin(2\pi u) = \operatorname{sinc}(2\pi u)$$

Finally, we apply the convolution theorem and get:

$$\mathcal{F}[f](u) = \mathcal{F}[g * g * g](u) = (\mathcal{F}[g](u))^3 = \operatorname{sinc}^3(2\pi u)$$
