Lecture 23: Image Sequence Analysis I: Local Methods

Contents

Introduction

Basic Problem

- given: image sequence $f(x, y, z)$,
  where (x, y) specifies the location and z denotes time
- wanted: displacement vector field of the image structures:
  optic flow (optischer Fluss) $\begin{pmatrix} u(x,y,z) \\ v(x,y,z) \end{pmatrix}$

Such *correspondence problems* are key problems in computer vision.

Similar Correspondence Problems

- computing the displacements *(disparities)* between the two images of a stereo pair
- matching *(registration)* of medical images that are obtained with different imaging methods, parameter settings, or at different times

Introduction (2)

What is Optic Flow Good for?

- recognition of moving pedestrians in driver assistant systems
- tracking of moving objects, e.g. human body motion
- estimation of egomotion in robotics
- flow measurements by means of particle image velocimetry (PIV)
- video processing, e.g. frame interpolation
- efficient video coding

Introduction (3)

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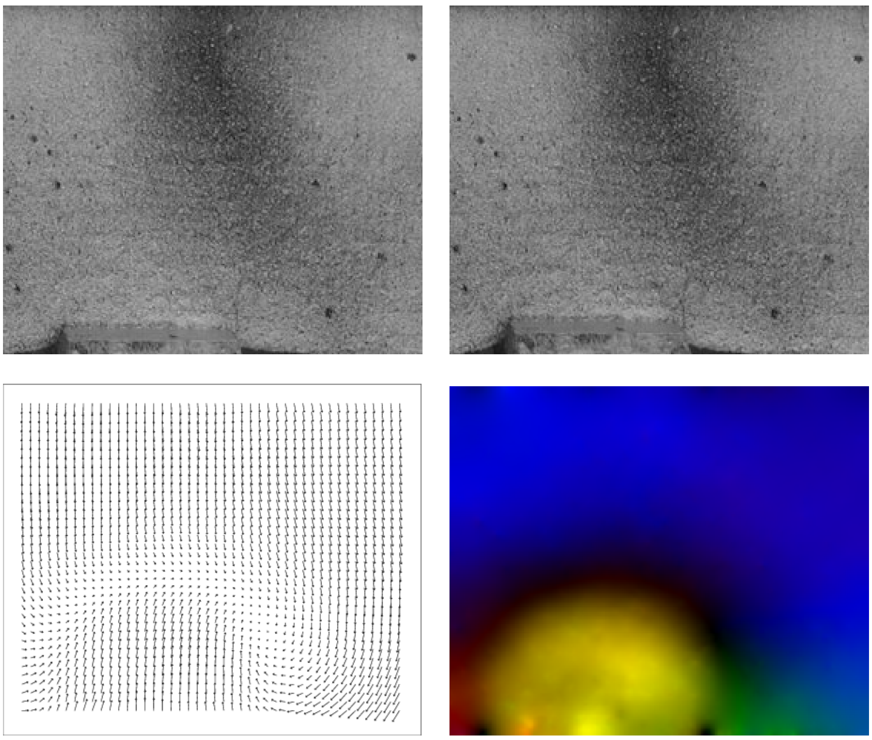

Deformation analysis of plastic foam using an optic flow method. **Top left:** Frame 1 of a deformation sequence. **Top right:** Frame 2. **Bottom left:** Vector plot of the displacement field. **Bottom right:** Colour-coded displacement field. Author: J. Weickert.

Introduction (4)

A Frequent Assumption: Grey Value Constancy

- Corresponding image structures should have the same grey value.
- Thus, the optic flow between subsequent frames z and $z+1$ satisfies

$$f(x+u,\, y+v,\, z+1) \;=\; f(x,y,z).$$

- Unfortunately the unknown flow field $(u,v)^\top$ is not directly accessible.

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Introduction (5)

Linearisation by Taylor Expansion

- Let us assume that (u,v) is small and f varies slowly and smoothly.
- Then a first order Taylor expansion around (x,y,z) gives a good approximation (cf. also Lecture 12):

$$\begin{aligned} 0 &= f(x+u,\, y+v,\, z+1) - f(x,y,z) \\ &\approx f_x(x,y,z)\, u \;+\; f_y(x,y,z)\, v \;+\; f_z(x,y,z) \end{aligned}$$

  where subscripts denote partial derivatives.
- This yields the *linearised optic flow constraint (OFC)*

$$f_x u + f_y v + f_z = 0$$

  where the unknown flow field $(u,v)^\top$ is directly accessible.

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Introduction (6)

How Realisitic are These Assumptions?

- The grey value constancy assumption is often surprisingly realistic:
  Many illumination changes happen very slowly, i.e. over many frames.
  More complicated models exist that take into account illumination changes.

- The linearisation assumption is violated more frequently:
  Conventional video cameras often suffer from temporal undersampling:
  They produce displacements over several pixels. Remedies:
  - use original OFC without linearisation (model becomes more difficult)
  - spatial downsampling (after lowpass filtering!)

- Another practical problem: *interlacing.*
  Some cameras record odd and even rows at different times.
  This creates artifacts for moving objects. Remedies:
  - consider only one of the two subimages
  - use cameras without interlacing *(progressive scan cameras)*

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Introduction (7)

Left: Original image taken with a progressive scan camera, 300×300 pixels. **Right:** When recording odd and even lines at different times, interlacing effects become visible for moving objects. Author: M. Ghodstinat.

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The Aperture Problem (Blendenproblem, Aperturproblem)

- The OFC $f_x u + f_y v + f_z = 0$ is one equation in two unknowns u, v.
  Thus, it cannot have a unique solution.

- To shed some more light on the nonuniqueness, we rewrite the OFC as

$$0 \;=\; f_x u + f_y v + f_z \;=\; (u,v)\,\boldsymbol{\nabla} f + f_z.$$

- This reformulation shows:
  - We can add arbitrary flow components orthogonal to $\boldsymbol{\nabla} f := (f_x, f_y)^\top$ without violating the OFC.
  - Thus, the OFC specifies only the flow component parallel to $\boldsymbol{\nabla} f$.

  This is called *aperture problem.*

- Additional assumptions are necessary to obtain a unique solution.
  Specifying different additional constraints leads to different methods.
  Before we investigate them, let us first analyse the flow component along $\boldsymbol{\nabla} f$.

The Normal Flow

- Let us express $(u,v)^\top$ in terms of the basis vectors $\boldsymbol{n} := \frac{\boldsymbol{\nabla} f}{|\boldsymbol{\nabla} f|}$ and $\boldsymbol{t} := \frac{\boldsymbol{\nabla}^\perp f}{|\boldsymbol{\nabla}^\perp f|}$
  where $\boldsymbol{\nabla}^\perp := (-\partial_y, \partial_x)^\top$.
  This gives the flow components normal and tangential to the edge of f:

$$\begin{pmatrix} u \\ v \end{pmatrix} \;=\; (u,v)\,\frac{\boldsymbol{\nabla} f}{|\boldsymbol{\nabla} f|}\,\frac{\boldsymbol{\nabla} f}{|\boldsymbol{\nabla} f|} + (u,v)\,\frac{\boldsymbol{\nabla}^\perp f}{|\boldsymbol{\nabla}^\perp f|}\,\frac{\boldsymbol{\nabla}^\perp f}{|\boldsymbol{\nabla}^\perp f|}$$
$$=: \begin{pmatrix} u_n \\ v_n \end{pmatrix} + \begin{pmatrix} u_t \\ v_t \end{pmatrix}.$$

- Since the OFC yields $(u,v)\,\boldsymbol{\nabla} f = -f_z$, the *normal flow* becomes

$$\begin{pmatrix} u_n \\ v_n \end{pmatrix} \;=\; -\frac{f_z}{|\boldsymbol{\nabla} f|}\cdot\frac{\boldsymbol{\nabla} f}{|\boldsymbol{\nabla} f|} \;=\; \frac{-1}{f_x^2+f_y^2}\begin{pmatrix} f_x f_z \\ f_y f_z \end{pmatrix}.$$

- The normal flow is the only flow component that can be computed from the OFC without additional constraints.

- Unfortunately, it gives poor results that are not very useful.

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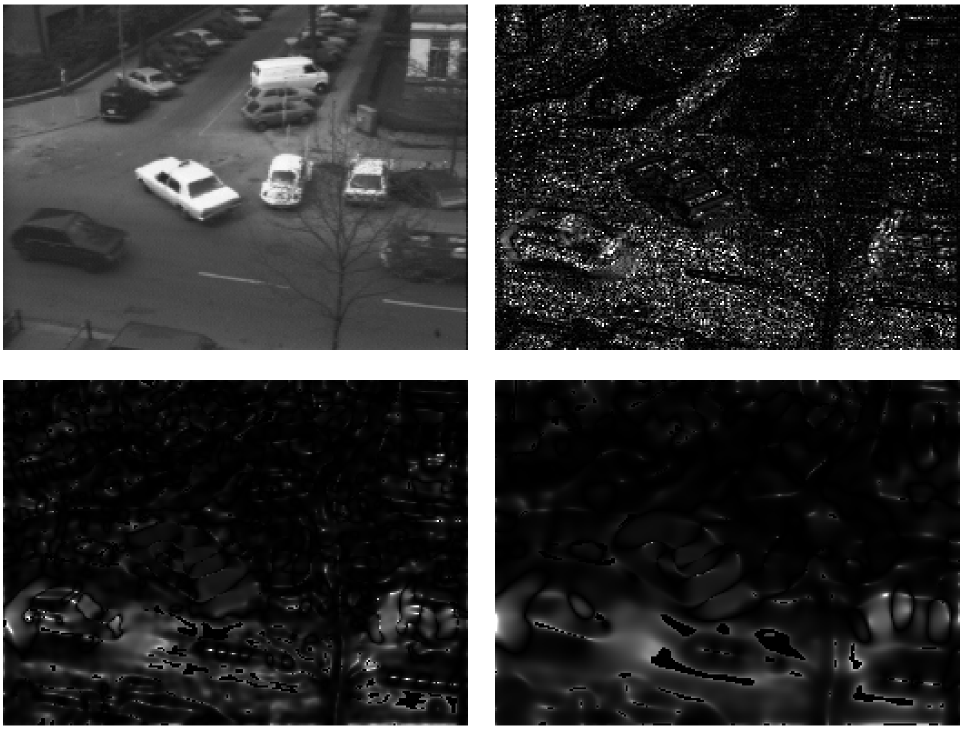

Top left: Image from the Hamburg taxi sequence: A taxi, a car, and a van are moving. **Top right:** Normal flow magnitude without presmoothing the derivatives of f. **Bottom left:** Presmoothing with a Gaussian with standard deviation $\sigma = 2$. **Bottom right:** $\sigma = 4$. Author: J. Weickert.

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The Spatial Approach of Lucas and Kanade (1981)

- Additional assumption for dealing with the aperture problem:
  The optic flow in (x_0, y_0) at time z_0 can be approximated by a *constant* vector (u, v) within some disk-shaped neighbourhood $B_\rho(x_0, y_0)$ of radius ρ.

- *least squares* model: flow in (x_0, y_0) minimises the *local* energy

$$E(u,v) = \frac{1}{2} \int\limits_{B_\rho(x_0,y_0)} (f_x u + f_y v + f_z)^2 \, dx \, dy.$$

- Computing the partial derivatives with respect to u and v gives

$$0 \stackrel{!}{=} \frac{\partial E}{\partial u} = \int\limits_{B_\rho} f_x (f_x u + f_y v + f_z) \, dx \, dy,$$

$$0 \stackrel{!}{=} \frac{\partial E}{\partial v} = \int\limits_{B_\rho} f_y (f_x u + f_y v + f_z) \, dx \, dy.$$

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The Spatial Approach of Lucas and Kanade (2)

- The unknowns u and v are constants that can be moved out of the integral. This yields the linear system

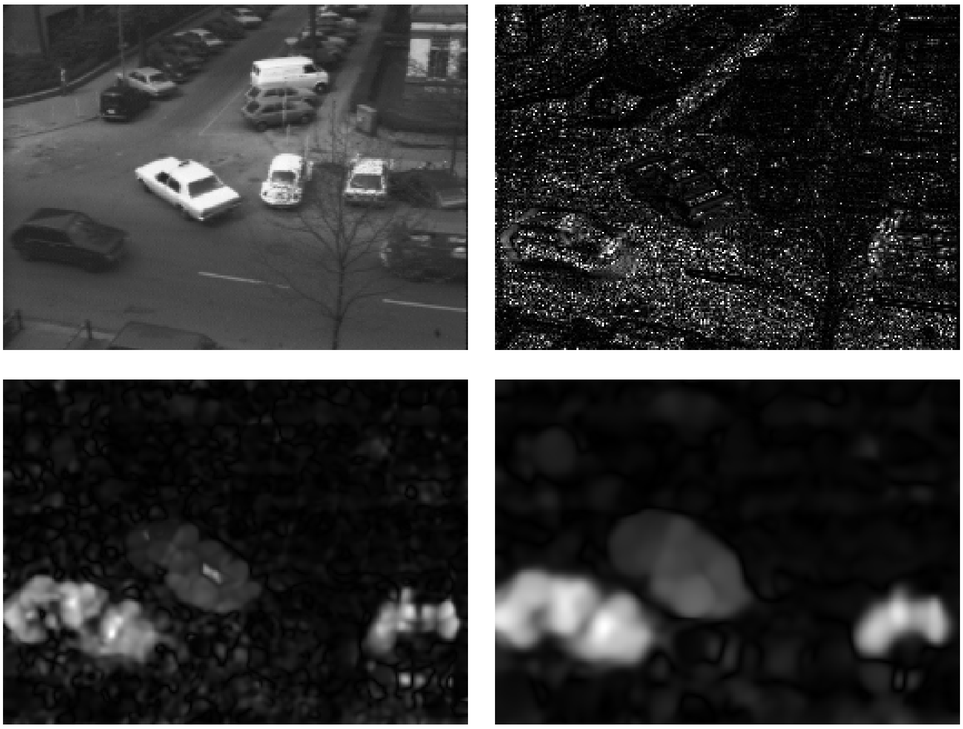
$$\begin{pmatrix} \int\limits_{B_\rho} f_x^2 \, dx \, dy & \int\limits_{B_\rho} f_x f_y \, dx \, dy \\ \int\limits_{B_\rho} f_x f_y \, dx \, dy & \int\limits_{B_\rho} f_y^2 \, dx \, dy \end{pmatrix} \begin{pmatrix} u \\ v \end{pmatrix} = \begin{pmatrix} -\int\limits_{B_\rho} f_x f_z \, dx \, dy \\ -\int\limits_{B_\rho} f_y f_z \, dx \, dy \end{pmatrix}.$$

- Often one replaces the pillbox filter with a “hard” window $B_\rho(x, y)$ by a “smooth” convolution with a Gaussian K_ρ:

$$\begin{pmatrix} K_\rho * (f_x^2) & K_\rho * (f_x f_y) \\ K_\rho * (f_x f_y) & K_\rho * (f_y^2) \end{pmatrix} \begin{pmatrix} u \\ v \end{pmatrix} = \begin{pmatrix} -K_\rho * (f_x f_z) \\ -K_\rho * (f_y f_z) \end{pmatrix}.$$

- Thus, the Lucas–Kanade method solves a 2×2 linear system of equations. The (spatial) structure tensor $\boldsymbol{J}_\rho$ serves as system matrix (cf. Lecture 13).

The Spatial Approach of Lucas and Kanade (3)



Top left: Image from the Hamburg taxi sequence. **Top right:** Normal flow magnitude. **Bottom left:** Optic flow magnitude using the Lucas-Kanade method with $\rho = 2$. **Bottom right:** Same with $\rho = 4$. Author: J. Weickert.

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The Spatial Approach of Lucas and Kanade (4)

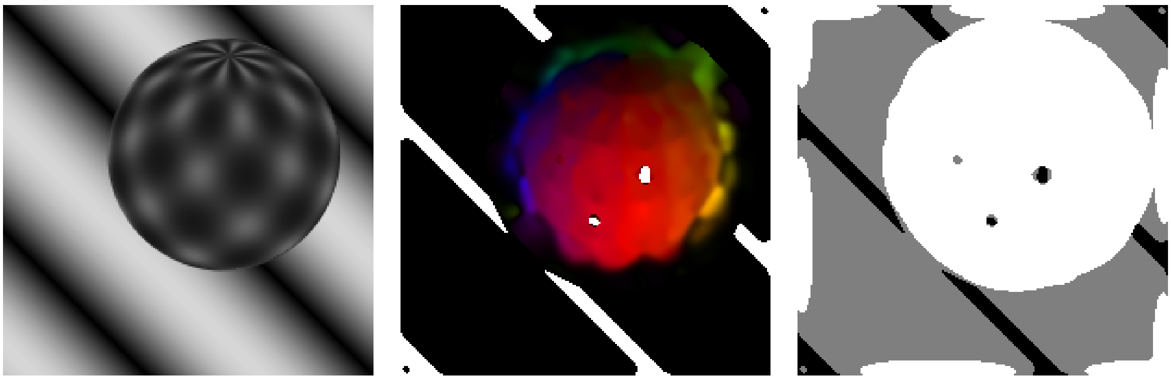
When Does the Linear System Have No Unique Solution ?

- $\mathrm{rank}(\boldsymbol{J}_\rho) = 0$ (two vanishing eigenvalues):
  Happens if the spatial gradient vanishes in the entire neighbourhood.
  Nothing can be said in this case.
  Simple criterion: $\mathrm{tr}\, \boldsymbol{J}_\rho \;=\; j_{1,1} + j_{2,2} \;\le\; \varepsilon.$
  (Recall that the trace is the sum of the eigenvalues.
  Moreover, $\boldsymbol{J}_\rho$ is positive semidefinite; cf. Lecture 13.)

- $\mathrm{rank}(\boldsymbol{J}_\rho) = 1$ (one vanishing eigenvalue):
  Happens if we have the same (nonvanishing) spatial gradient within the entire neighbourhood.
  Then both equations are linearly dependent (infinitely many solutions).
  Simple criterion: $\det \boldsymbol{J}_\rho \;=\; j_{1,1}\, j_{2,2} - j_{1,2}^2 \;\le\; \varepsilon$ (while $\mathrm{tr}\, \boldsymbol{J}_\rho > \varepsilon$).
  (Recall that the determinant is the product of the eigenvalues.)
  In this case the aperture problem persists. We can only compute the normal flow

$$\begin{pmatrix} u_n \\ v_n \end{pmatrix} = \frac{-1}{f_x^2 + f_y^2} \begin{pmatrix} f_x f_z \\ f_y f_z \end{pmatrix}.$$

The Spatial Approach of Lucas and Kanade (5)



Left: Image from a synthetic sequence: The sphere rotates in front of a static background. **Middle:** False colour representation of the optic flow using the Lucas–Kanade method. **Right:** Flow classification: black=no information (gradient too small, no flow given), grey=aperture problem (gradient too uniform, normal flow given), white=full flow (space-variant gradient). Author: J. Weickert.

Advantages

- simple and fast method
- requires only two frames (low memory requirements)
- good value for the money:
  results often superior to more complicated approaches

Disadvantages

- inaccurate at locations where the local constancy assumption is violated:
  flow discontinuities and non-translatory motion (e.g. rotation)
- local method that does not allow to compute the flow field at all locations

The Spatiotemporal Approach of Bigün et al. (1991)

- Optic flow is regarded as orientation in the space–time domain.
- It is obtained from an eigenvector problem of the structure tensor (cf. Lecture 13).
- We search for the spatiotemporal direction with the least grey value change.
  The search is done in a 3-D ball-shaped neighbourhood $B_\rho(x_0, y_0, z_0)$ of radius ρ.
- This direction is given by the unit vector $\boldsymbol{w} = (w_1, w_2, w_3)^\top$ that minimises

$$E(\boldsymbol{w}) \;=\; \int\limits_{B_\rho(x_0,y_0,z_0)} (f_x w_1 + f_y w_2 + f_z w_3)^2 \, dx \, dy \, dz.$$

- When renormalising the third component of the optimal $\boldsymbol{w}$ to 1, the first two components give the optic flow:

$$u = \frac{w_1}{w_3}, \qquad v = \frac{w_2}{w_3}.$$

The Spatiotemporal Approach of Bigün et al. (2)

- Note the differences between Lucas–Kanade and Bigün et al.:
  - Lucas–Kanade searches for a 2-D flow vector with a least squares approach.
  - Bigün et al. search for a 3-D vector, i.e. the time component is also optimised. This is a *total least squares* approach.
- Using the spatiotemporal gradient notation $\boldsymbol{\nabla}_3 f := (f_x, f_y, f_z)^\top$ one minimises

$$
\begin{aligned}
E(\boldsymbol{w}) &:= \int_{B_\rho} (\boldsymbol{w}^\top \boldsymbol{\nabla}_3 f)^2 \, dx\, dy\, dz \\
&= \int_{B_\rho} \boldsymbol{w}^\top \boldsymbol{\nabla}_3 f \, \boldsymbol{\nabla}_3 f^\top \boldsymbol{w} \, dx\, dy\, dz \\
&= \boldsymbol{w}^\top \Big(\int_{B_\rho} \boldsymbol{\nabla}_3 f \, \boldsymbol{\nabla}_3 f^\top \, dx\, dy\, dz \Big) \, \boldsymbol{w}
\end{aligned}
$$

under the constraint $|\boldsymbol{w}| = 1$.

The Spatiotemporal Approach of Bigün et al. (3)

- The desired vector $\boldsymbol{w}$ is the normalised eigenvector to the smallest eigenvalue of

$$
\int_{B_\rho} \boldsymbol{\nabla}_3 f \, \boldsymbol{\nabla}_3 f^\top \, dx\, dy\, dz.
$$

- Often the box filter is replaced by Gaussian convolution.
  This leads to a principal component analysis of the *spatiotemporal structure tensor*

$$
\begin{aligned}
\boldsymbol{J}_\rho &:= K_\rho * (\boldsymbol{\nabla}_3 f \, \boldsymbol{\nabla}_3 f^\top) \\
&= \begin{pmatrix} K_\rho * (f_x^2) & K_\rho * (f_x f_y) & K_\rho * (f_x f_z) \\ K_\rho * (f_x f_y) & K_\rho * (f_y^2) & K_\rho * (f_y f_z) \\ K_\rho * (f_x f_z) & K_\rho * (f_y f_z) & K_\rho * (f_z^2) \end{pmatrix}.
\end{aligned}
$$

The Spatiotemporal Approach of Bigün et al. (4)

Flow Classification with the Eigenvalues of the Structure Tensor

- Let $\mu_1 \geq \mu_2 \geq \mu_3 \geq 0$ be the eigenvalues of $\boldsymbol{J}_\rho$.
- $\operatorname{rank}(\boldsymbol{J}_\rho) = 0$ (three vanishing eigenvalues):
  If $\operatorname{tr} \boldsymbol{J}_\rho = j_{1,1} + j_{2,2} + j_{3,3} \leq \tau_1$, nothing can be said: The gradients are too small.
- $\operatorname{rank}(\boldsymbol{J}_\rho) = 3$ (no vanishing eigenvalues):
  If $\mu_3 \geq \tau_2$, then our assumption of a locally constant flow is violated.
  Either a flow discontinuity or noise dominates.
- $\operatorname{rank}(\boldsymbol{J}_\rho) = 1$ (two vanishing eigenvalues):
  If $\mu_2 \leq \tau_3$, we have two low-contrast eigendirections.
  No unique flow exists (aperture problem): One can only compute the normal flow

$$\begin{pmatrix} u_n \\ v_n \end{pmatrix} = \frac{-1}{f_x^2 + f_y^2} \begin{pmatrix} f_x f_z \\ f_y f_z \end{pmatrix}.$$

- $\operatorname{rank}(\boldsymbol{J}_\rho) = 2$ (one vanishing eigenvalue):
  The optic flow is determined by the eigenvector $\boldsymbol{w}$ to the smallest eigenvalue μ_3:
  Normalising its third component to 1, the first two components give u and v.

The Spatiotemporal Approach of Bigün et al. (5)

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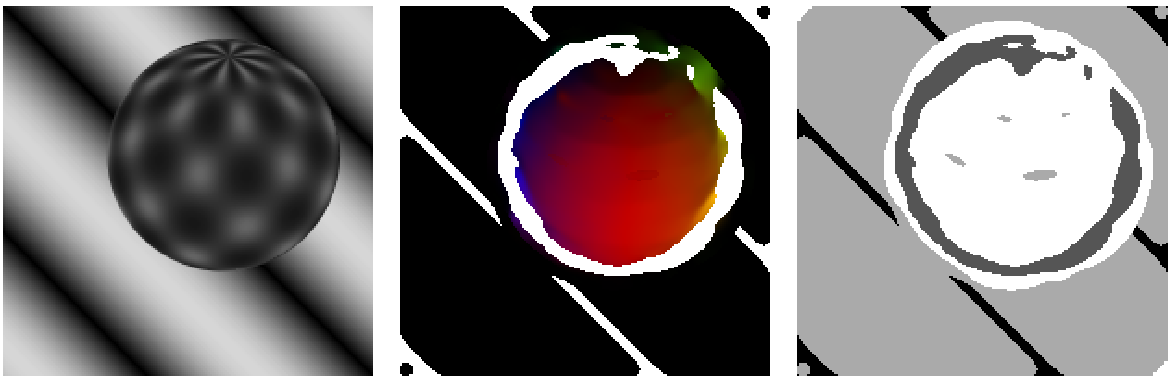

Left: Image from the sphere sequence. **Middle:** False colour representation of the optic flow using the method of Bigün et al. **Right:** Flow classification: black=no information (three small eigenvalues), dark grey=flow discontinuity or noise (three large eigenvalues), light grey=aperture problem (two small eigenvalues), white=full flow (one small eigenvalue). Author: J. Weickert.

The Spatiotemporal Approach of Bigün et al. (6)

Advantages

- fairly high robustness with respect to noise (due to 3-D integration)
- good results for translatory motion
- eigenvalues of the spatiotemporal structure tensors provide detailed information on the optic flow

Disadvantages

- more complicated than Lucas–Kanade:
  requires numerical algorithms for principal component analysis of a 3×3 matrix
  (suitable method: e.g. Jacobi transformations)
- problems at flow discontinuities and locations with non-translatory motion
  (such as divergent motion or rotations)
- local method that does not give full flow fields
- three threshold parameters

Summary

Summary

- Computing the optic flow is a key problem in computer vision.
- Assuming grey value constancy leads to the Optic Flow Constraint (OFC).
  It allows to compute the normal flow only (aperture problem).
  Computing the full flow requires additional assumptions.
- Lucas and Kanade assume a locally constant flow (in 2D).
  This requires to solve a linear system of equations.
  Its system mastrix is the spatial structure tensor.
- Bigün et al. estimate the flow as orientation in the spatiotemporal domain.
  It leads to a principal component analysis problem.
  The matrix is given by the spatiotemporal stucture tensor.

References

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