Image Processing and Computer Vision (IPCV)

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Example Solutions for Classroom Assignment 11 (C11)

Problem 1: (Otsu's Threshold Selection Method)

By using the identities $\mu_0(T) = \frac{\mu(T)}{\omega(T)}$ and $\mu_1(T) = \frac{\mu_{\text{tot}} - \mu(T)}{1-\omega(T)}$, it suffices to take the definition of $\sigma_B^2(T)$ and to plug these values in.

$$\begin{aligned}
\sigma_B^2(T) &= \omega(T)(\mu_0(T) - \mu_{\text{tot}})^2 + (1-\omega(T))(\mu_1(T) - \mu_{\text{tot}})^2 \\
&= \omega(T)\left(\frac{\mu(T)}{\omega(T)} - \mu_{\text{tot}}\right)^2 + (1-\omega(T))\left(\frac{\mu_{\text{tot}} - \mu(T)}{1-\omega(T)} - \mu_{\text{tot}}\right)^2 \\
&= \omega(T)\left(\frac{\mu(T) - \mu_{\text{tot}}\omega(T)}{\omega(T)}\right)^2 + (1-\omega(T))\left(\frac{\mu_{\text{tot}}\omega(T) - \mu(T)}{1-\omega(T)}\right)^2 \\
&= \frac{(\mu(T) - \mu_{\text{tot}}\omega(T))^2}{\omega(T)} + \frac{(\mu_{\text{tot}}\omega(T) - \mu(T))^2}{1-\omega(T)} \\
&= (\mu_{\text{tot}}\omega(T) - \mu(T))^2\left(\frac{1}{\omega(T)} + \frac{1}{1-\omega(T)}\right) \\
&= (\mu_{\text{tot}}\omega(T) - \mu(T))^2 \frac{1 - \omega(T) + \omega(T)}{\omega(T)(1-\omega(T))} \\
&= \frac{(\mu_{\text{tot}}\omega\,(T) - \mu\,(T))^2}{\omega\,(T)\,(1-\omega\,(T))}
\end{aligned}$$

Problem 2: (Toboggan Watershed Algorithm)

First of all we compute the magnitude of the first derivative of the signal $\boldsymbol{f}$ by using backward differences:

$$\begin{aligned} \boldsymbol{f} &= (6,1,5,5,3,5,6,3,4,4,2,7,7,6,3,2)^\top \\ \boldsymbol{g} := |f_i - f_{i-1}|_{i=1\ldots16} &= (0,5,4,0,2,2,1,3,1,0,2,5,0,1,3,1)^\top \ . \end{aligned}$$

In each pixel of $\boldsymbol{g}$ we follow the direction of the steepest descend until a local minimum (in bold font) is reached and keep track of all pixels on the way to this minimum. The tracked pixels are set to the value f_i of the original signal that corresponds the local minimum:

$$\begin{aligned} \boldsymbol{f} &= (6,1,5,5,3,5,6,3,4,4,2,7,7,6,3,2)^\top \\ \boldsymbol{g} &= (\underline{\mathbf{0},5},\underline{4,\mathbf{0},2},\underline{2,\mathbf{1},3},\underline{1,\mathbf{0},2},\underline{5,\mathbf{0},1,3},\underline{\mathbf{1}\ })^\top \\ \boldsymbol{f}_{\text{filtered}} &= (6,6,5,5,5,6,6,6,4,4,4,4,7,7,7,7,2)^\top \ . \end{aligned}$$