Example Solutions for Classroom Assignment 10 (C10)

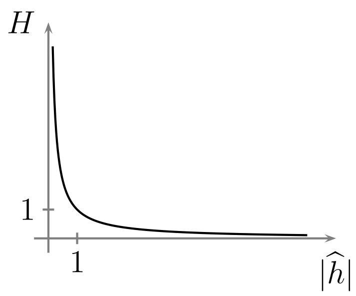
Problem 1 (Inverse Filtering)

(a) For each deconvolution method we are interested in the function H that is multiplied with $\hat{f}$ in dependence of $\hat{h}$. Note that $\hat{h}$ is a complex number and thus the domain of H is in $\mathbb{C}$ as well. By considering H for each case, we see that the codomain of H will also be in $\mathbb{C}$. However, for the analysis of lowpass and highpass effects we always considered the Fourier spectra in the lecture. Therefore we sketch here H in dependence of $|\hat{h}|$.

- *Inverse filtering* is defined as

$$\widehat{u} = \underbrace{\frac{1}{\widehat{h}}}_{=:H(\widehat{h})} \widehat{f}.$$

The following figure depicts the function $H(|\widehat{h}|) = \frac{1}{|\widehat{h}|}$:



- If h is a lowpass filter $|\widehat{h}|$ decreases for high frequencies. For *inverse filtering* $H = \frac{1}{|\widehat{h}|}$ increases for small values of $|\widehat{h}|$. Thus, H amplifies high frequencies. In contrast, low frequencies are not much attenuated by the lowpass filter h. Consequently, the values for $|\widehat{h}|$ remain relatively large and that is why H becomes small.

Therefore, inverse filtering acts like a highpass filter if h is a lowpass filter, even though it is theoretically the optimal solution if there is no noise. In natural images there is usually always a bit noise, such that this filter is often of no use.

Problem 1 (Cooccurrence Matrices)

The given image is

0	1	1	0
0	2	3	2
1	3	1	1
2	3	1	2

Using the vector $\boldsymbol{d} = (-1,1)^\top$ (i.e. the lower left neighbour) yields the following pairs

$$(1,0),(2,1),(3,2),(1,2),(3,3),(1,3),(0,3),(2,1),(1,1)$$

The entries are entered in row-column ordering, e.g the coordinate pair (i,j) is placed in row i and column j. Note that this differs from the indexing scheme usually used for images.

$$\frac{1}{9}\begin{pmatrix} 0 & 0 & 0 & 1 \\ 1 & 1 & 1 & 1 \\ 0 & 2 & 0 & 0 \\ 0 & 0 & 1 & 1 \end{pmatrix}$$

With the definition from the lecture, the cooccurence matrix contains the relative frequency of the pixel pairs. Note that other definitions exist. Some authors use the absolute frequency and some others again use the probability of the considered pixel pair. All these definitions only differ at most by a scaling factor and yield the same results when applied in a consistent way. Clearly the highest probability is given by $\frac{2}{9}$. To compute the contrast, let us first determine the various distances required for this. They are given by

$$((i-j)^2)_{ij} = \begin{pmatrix} 0 & 1 & 4 & 9 \\ 1 & 0 & 1 & 4 \\ 4 & 1 & 0 & 1 \\ 9 & 4 & 1 & 0 \end{pmatrix}$$

Since this distance matrix only depends on the indices, not on the actual grey values in the image, it is the same for all images of equal size. Note that this is a so-called *Toeplitz* Matrix. These matrices have special properties and are well studied in mathematics. A pointwise multiplication with the cooccurence matrix leads us to

$$\frac{1}{9}\begin{pmatrix} 0 & 0 & 0 & 9 \\ 1 & 0 & 1 & 4 \\ 0 & 2 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{pmatrix}$$

In order to compute the contrast we just need to sum up all the coefficients. This gives us $\sum p_{ij}(i-j)^2 = \frac{18}{9} = 2$.

If we had used the vector $\boldsymbol{d} = (1, -1)^\top$ instead, then all the pairs would appear in inverted order, which would yield the transpose of our cooccurence matrix. The highest relative frequency is of course not affected by this change and remains the same. As for the contrast, we note that the distance matrix is symmetric. Thus the pointwise multiplication yields again the transpose of our results from above, such that summing all the entries up would yield the same value for the contrast.
