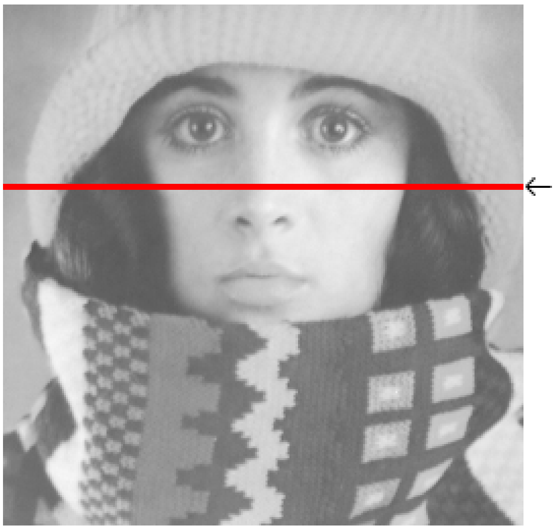
Lecture 12:
Linear Filters II:
Derivative Filters

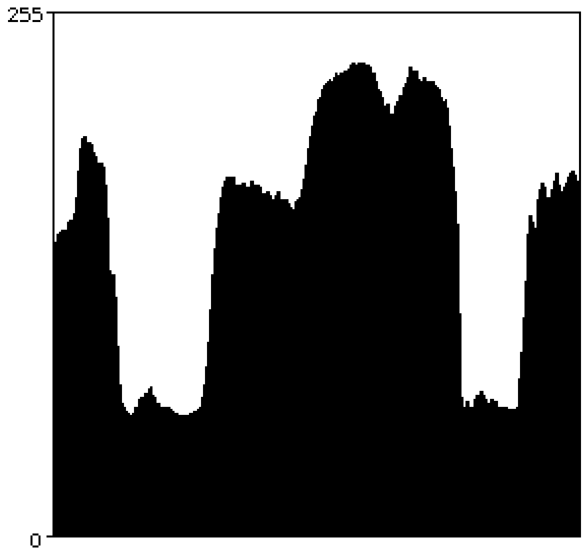
Contents

Introduction

Why Do We Need Derivative Filters ?

- Derivative filters are very important examples of linear shift invariant filters.
- Derivatives tell us something about local grey value changes in images.
- They can be used for detecting semantically important image features such as edges and corners (Lecture 13).
- To compute derivatives in images, we need some mathematical insights:
  - What are the dangers when we want to compute derivatives?
  - What are useful derivative expressions for a continuous image $f(x, y)$?
  - How can we discretise these ideas to make them applicable to discrete images?





Left: Image of size 256×256, from which a 1-D signal along a horizontal scanline has been extracted. **Right:** Intensity profile along this scanline. The largest intensity jumps mark the boundaries of the hair region. Author: T. Schneevoigt.

Why is it Dangerous to Compute Derivatives?

Explanation in the Spatial Domain

- Small, but high-frequent fluctuations in the original signal can create very large perturbations in its derivatives.

- Example: The high-frequent 1D perturbation

$$f(x) \;=\; \varepsilon \,\sin\left(\frac{x}{\varepsilon^2}\right)$$

becomes arbitrarily small in magnitude for $\varepsilon \to 0$. However, its derivative

$$f'(x) \;=\; \frac{1}{\varepsilon}\,\cos\left(\frac{x}{\varepsilon^2}\right)$$

exceeds all bounds !!!

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Explanation in the Fourier Domain

- Lecture 4:
  Derivatives in the spatial domain become multiplications with the frequency in the Fourier domain:

$$\mathcal{F}\left[\frac{\partial^{n+m} f}{\partial x^n\,\partial y^m}\right](u,v) \;=\; (i2\pi u)^n\,(i2\pi v)^m\,\mathcal{F}[f](u,v)\,.$$

- Thus, high-frequent perturbations (e.g. noise) are massively amplified!

Remedy

- Perform lowpass filtering before computing derivatives.
- This damps the dangerous high frequent components.
- frequently used: Gaussian convolution (Lecture 11)

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Useful Concepts from Calculus in 2D

Partial Derivatives

- Consider a sufficiently smooth function of several variables.
  Then we can compute its *partial derivatives* w.r.t. each of these variables.
  To this end, one regards it as a function of a single variable.
  The other variables are treated like constants.
- Example:

$$\begin{aligned}
f(x,y) &= \sin(xy^2) + x^3, \\
\frac{\partial f}{\partial x}(x,y) &= y^2\cos(xy^2) + 3x^2, \\
\frac{\partial f}{\partial y}(x,y) &= 2xy\cos(xy^2)\,.
\end{aligned}$$

Useful Concepts from Calculus in 2D (2)

- Equivalent notations:
$$\frac{\partial f}{\partial x} \;=\; \partial_x f \;=\; f_x\,.$$

- Higher order partial derivatives can be computed consecutively:
$$\frac{\partial^2 f}{\partial x\, \partial y} \;:=\; \frac{\partial}{\partial x}\left(\frac{\partial f}{\partial y}\right).$$

- Under suitable smoothness assumptions (which we always assume to hold) one may exchange the order of partial differentiation:
$$f_{xy} = f_{yx}\,.$$

Useful Concepts from Calculus in 2D (3)

Nabla Operator

- The column vector of the partial derivatives is called *nabla operator* or *gradient.* In 2D:
$$\boldsymbol{\nabla} \;:=\; \begin{pmatrix} \partial_x \\ \partial_y \end{pmatrix}.$$

- Often it is possible to work with $\boldsymbol{\nabla}$ as if it were an ordinary vector. For instance, for a scalar-valued function $f(x,y)$, one gets
$$\boldsymbol{\nabla} f \;=\; \begin{pmatrix} \partial_x f \\ \partial_y f \end{pmatrix}.$$

- $\boldsymbol{\nabla} f$ points in the direction of the steepest ascend of f.

- $|\boldsymbol{\nabla} f| = \sqrt{(\partial_x f)^2 + (\partial_y f)^2}$ is invariant under rotations.

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Divergence and Laplacian

- The inner product of the nabla operator and a vector-valued function $\boldsymbol{j}(x,y) = (j_1(x,y),\ j_2(x,y))^\top$ is called the *divergence (Divergenz)* of $\boldsymbol{j}$:

$$\operatorname{div} \boldsymbol{j} \ := \ \boldsymbol{\nabla}^\top \boldsymbol{j} \ = \ (\partial_x, \partial_y) \begin{pmatrix} j_1 \\ j_2 \end{pmatrix} \ = \ \partial_x j_1 + \partial_y j_2 \,.$$

We will need this in Lecture 16 (Nonlinear Diffusion Filtering).

- The inner product of the divergence and the gradient is called *Laplacian (Laplace-Operator):*

$$\Delta f \ := \ \operatorname{div}(\boldsymbol{\nabla} f) \ = \ (\partial_x, \partial_y) \begin{pmatrix} \partial_x f \\ \partial_y f \end{pmatrix} \ = \ \partial_{xx} f + \partial_{yy} f \,.$$

It is invariant under rotations of f.

- It is straightforward to generalise all previous operators to higher dimensions.

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Taylor Expansion

- **One-Dimensional Taylor Expansion**

A function $f : \mathbb{R} \to \mathbb{R}$ that is $n{+}1$ times continuously differentiable with bounded derivatives can be represented in $x{+}h$ by its Taylor expansion around x:

$$f(x+h) \ = \ \sum_{k=0}^{n} \frac{h^k}{k!} f^{(k)}(x) \ + \ \mathcal{O}(h^{n+1}).$$

- **Two-Dimensional Taylor Expansion**

A function $f : \mathbb{R}^2 \to \mathbb{R}$ that is $n{+}1$ times continuously differentiable with bounded derivatives can be represented in $\boldsymbol{x} + \boldsymbol{h}$ by

$$f(\boldsymbol{x}+\boldsymbol{h}) \ = \ \sum_{k=0}^{n} \frac{1}{k!} \langle \boldsymbol{h}, \boldsymbol{\nabla} \rangle^k f(\boldsymbol{x}) \ + \ \mathcal{O}(|\boldsymbol{h}|^{n+1})$$

where we have e.g.

$$\begin{aligned} \langle \boldsymbol{h}, \boldsymbol{\nabla} \rangle^2 \ &= \ (h_1\, \partial_{x_1} + h_2\, \partial_{x_2})^2 \\ &= \ h_1^2\, \partial_{x_1 x_1} \ + \ 2\, h_1 h_2\, \partial_{x_1 x_2} \ + \ h_2^2\, \partial_{x_2 x_2} \,. \end{aligned}$$

Numerical Approximation of Derivatives

Finite difference approximations of derivatives are obtained by a Taylor expansion with subsequent comparison of the coefficients.

Example

Approximate the second derivative f_i'' in pixel i with a stencil that takes into account the three pixels $i-1$, i, and $i+1$. The grid size is h. Compute the stencil weights.

Taylor expansion around pixel i:

$$\begin{aligned} f_{i-1} &= f_i - hf_i' + \tfrac{h^2}{2}f_i'' - \tfrac{h^3}{6}f_i''' + \tfrac{h^4}{24}f_i'''' - \tfrac{h^5}{120}f_i''''' + \mathcal{O}(h^6), \\ f_i &= f_i, \\ f_{i+1} &= f_i + hf_i' + \tfrac{h^2}{2}f_i'' + \tfrac{h^3}{6}f_i''' + \tfrac{h^4}{24}f_i'''' + \tfrac{h^5}{120}f_i''''' + \mathcal{O}(h^6). \end{aligned}$$

(We always assume that all required derivatives exist and are bounded.)

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Comparison of the coefficients in

$$\begin{aligned} 0\cdot f_i + 0\cdot f_i' + 1\cdot f_i'' &\overset{!}{=} \alpha_{-1} f_{i-1} + \alpha_0 f_i + \alpha_1 f_{i+1} \\ &= (\alpha_{-1}+\alpha_0+\alpha_1)\cdot f_i \\ &+ h\,(-\alpha_{-1}+\alpha_1)\cdot f_i' \\ &+ \tfrac{h^2}{2}(\alpha_{-1}+\alpha_1)\cdot f_i'' + \mathcal{O}(h^3) \end{aligned}$$

leads to a linear system in the unknown weights α_{-1}, α_0, and α_1:

$$\begin{pmatrix} 1 & 1 & 1 \\ -1 & 0 & 1 \\ 1 & 0 & 1 \end{pmatrix} \begin{pmatrix} \alpha_{-1} \\ \alpha_0 \\ \alpha_1 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ \frac{2}{h^2} \end{pmatrix}.$$

Its solution is given by

$$\alpha_{-1} = \frac{1}{h^2}, \qquad \alpha_0 = -\frac{2}{h^2}, \qquad \alpha_1 = \frac{1}{h^2}.$$

Let us now study the error of our approximation

$$f_i'' \approx \frac{1}{h^2} f_{i-1} - \frac{2}{h^2} f_i + \frac{1}{h^2} f_{i+1} .$$

Replacing f_{i-1} and f_{i+1} by their Taylor expansions gives

$$\frac{1}{h^2} f_{i-1} - \frac{2}{h^2} f_i + \frac{1}{h^2} f_{i+1} = f_i'' + \underbrace{\frac{h^2}{12} f_i'''' + \mathcal{O}(h^4)}_{\text{error}} .$$

Since f_i'''' is bounded, the leading error term is quadratic in the grid size h.
Thus, we call it an approximation with *consistency order (Konsistenzordnung)* 2.

Remarks

- Higher consistency orders give better accuracies.
- Discretisations must have at least the consistency order 1.
  This ensures that for $h \to 0$ the desired expression is indeed approximated.
- Otherwise, the method is called *inconsistent.*

Inconsistent discretisations are inacceptable!

Derivative Filters in 1D

The Most Important Approximations

first derivative:

$$f_i' = \frac{f_{i+1} - f_i}{h} + \mathcal{O}(h) \qquad \text{forward difference}$$

$$f_i' = \frac{f_i - f_{i-1}}{h} + \mathcal{O}(h) \qquad \text{backward difference}$$

$$f_i' = \frac{f_{i+1} - f_{i-1}}{2h} + \mathcal{O}(h^2) \qquad \text{central difference}$$

second derivative:

$$f_i'' = \frac{f_{i+1} - 2f_i + f_{i-1}}{h^2} + \mathcal{O}(h^2) \qquad \text{central difference}$$

Usually central differences have a higher order of consistency:
The symmetry causes cancellation effects of some Taylor coefficients.

Derivative Filters in 1D (2)

Filter Analysis in the Frequency Domain

- The continuous Fourier transform from Lecture 4 is also useful for analysing the frequency behaviour of linear shift invariant filters.
- Assume we want to analyse the frequency behaviour of the central difference approximation of f':

$$g(x) \;=\; \frac{1}{2h}\Big(f(x{+}h) - f(x{-}h)\Big)$$

  where h denotes the grid size. To this end, we express $\hat{g}$ in terms of $\hat{f}$.

- Using the linearity of the Fourier transform and the shift theorem we obtain

$$\begin{aligned}\hat{g}(u) \;&=\; \frac{1}{2h}\Big(\mathcal{F}[f(x{+}h)](u) \;-\; \mathcal{F}[f(x{-}h)](u)\Big)\\ &=\; \frac{1}{2h}\Big(e^{i2\pi hu} - e^{-i2\pi hu}\Big)\,\hat{f}(u).\end{aligned}$$

Derivative Filters in 1D (3)

- With $e^{i2\pi hu} - e^{-i2\pi hu} = 2i\,\sin(2\pi hu)$ this gives the frequency behaviour:

$$\hat{g}(u) \;=\; \frac{i}{h}\,\sin(2\pi hu)\,\hat{f}(u).$$

- With the Taylor expansion $\sin(2\pi hu) \;\approx\; 2\pi hu + \mathcal{O}(h^3u^3)$ we obtain

$$\hat{g}(u) \;=\; i2\pi u\,\hat{f}(u) \;+\; \mathcal{O}(h^2u^3).$$

- Remembering that $\mathcal{F}[f'] = i2\pi u\,\hat{f}(u)$ shows that

$$\hat{g}(u) \;=\; \mathcal{F}[f'](u) \;+\; \mathcal{O}(h^2u^3).$$

  Thus, the filter approximates the derivative f'.

- For a fixed frequency u, the approximation order is $\mathcal{O}(h^2)$.
  For a fixed grid size h, the approximation order is $\mathcal{O}(u^3)$.
- Thus, in contrast to a filter analysis in the spatial domain, the Fourier analysis gives also frequency-dependent results on the approximation quality.

Derivative Filters in 1D (4)

Improving the Order of Consistency with Larger Stencils

- By extending the stencil size, we can increase the order of consistency. However, this also increases the computational effort.
- Example: Central difference approximations of f' with stencil sizes 3, 5, and 7:

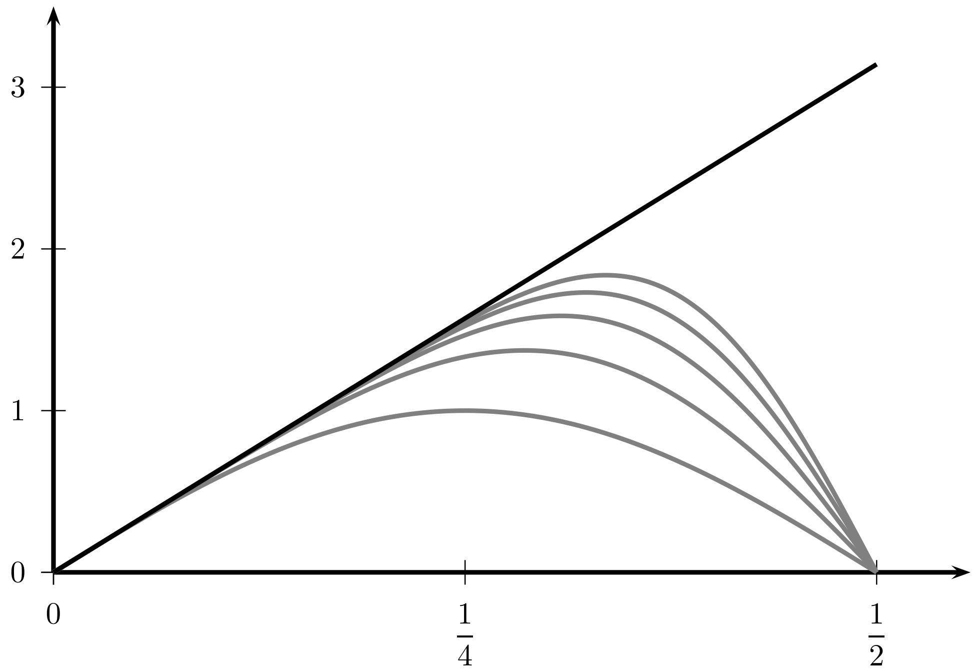
$$f'_i = \frac{f_{i+1} - f_{i-1}}{2h} + \mathcal{O}(h^2),$$

$$f'_i = \frac{-f_{i+2} + 8f_{i+1} - 8f_{i-1} + f_{i-2}}{12h} + \mathcal{O}(h^4),$$

$$f'_i = \frac{f_{i+3} - 9f_{i+2} + 45f_{i+1} - 45f_{i-1} + 9f_{i-2} - f_{i-3}}{60h} + \mathcal{O}(h^6).$$

- The approximation quality can be visualised in the Fourier domain:
  - The ideal derivative operator gives $\hat{g}(u) = 2\pi i u \hat{f}(u) =: w(u)\hat{f}(u)$. Thus, the Fourier spectrum is amplified by $|w(u)| = |2\pi i u| = 2\pi|u|$.
  - We have seen that the $\mathcal{O}(h^2)$ approximation amplifies the Fourier spectrum by $|w(u)| = \frac{1}{h}|\sin(2\pi h u)|$.

Derivative Filters in 1D (5)



Amplification function $|w(u)|$ of the ideal derivative operator (black) and some of its central difference approximations (grey) for $h = 1$. The numerical approximations show a lowpass filter effect, i.e. there is some smoothing along the direction of the derivative. Increasing the mask size from 3 to 5, 7, 9, and 11 pixels one gets closer to the ideal derivative operator. Author: M. Mainberger.

Improving the Order of Consistency without Larger Stencils

- We just saw that stencils with low consistency order create additional smoothing.
- By explicitly modelling this smoothing, we can increase the consistency order.
- **Example:** We can regard the central difference $\frac{1}{2h}(f_{i+1} - f_{i-1})$
  - either as an $\mathcal{O}(h^2)$ approximation of the derivative f_i'
  - or as an $\mathcal{O}(h^4)$ approximation of the smoothed derivative $\frac{1}{6}(f_{i+1}' + 4f_i' + f_{i-1}')$. This can be verified by a Taylor expansion.
- The latter interpretation requires to solve a linear system of equations of type

$$\tfrac{1}{6}\,(f_{i+1}' + 4f_i' + f_{i-1}') \;=\; \tfrac{1}{2h}\,(f_{i+1} - f_{i-1})$$

  in order to obtain fourth order approximations of all f_i' .
- Such approximations – which do not give explicit access to f_i' – are called *implicit*.
- Thus, solving linear systems can be an alternative to using larger stencils. Note the similarities to classical versus generalised interpolation (Lecture 9).

Derivative Filters in 2D

First-Order Derivatives

- In principle, the 1D masks can also be used in 2D.
- For the first order derivatives, the following stencils have consistency order 2:

$$\partial_x \;\approx\; \frac{1}{2h}\,\boxed{\begin{array}{c|c|c} -1 & 0 & 1 \end{array}}\,, \qquad \partial_y \;\approx\; \frac{1}{2h}\,\boxed{\begin{array}{c} 1 \\ \hline 0 \\ \hline -1 \end{array}}$$

  (where the x axis goes from left to right, and the y axis from bottom to top)
- Problem:
  - The masks smooth in the direction of the derivative, but not orthogonal to it.
  - This suggests to introduce some smoothing perpendicular to the derivative direction, if one is interested in better isotropy.

Derivative Filters in 2D (2)

- The *Sobel operators* create such a perpendicular smoothing by convolving with the binomial kernel $(\frac{1}{4}, \frac{1}{2}, \frac{1}{4})$:

$$\partial_x \approx \frac{1}{4}\begin{bmatrix}1\\2\\1\end{bmatrix} * \frac{1}{2h}\begin{bmatrix}-1 & 0 & 1\end{bmatrix} = \frac{1}{8h}\begin{bmatrix}-1 & 0 & 1\\-2 & 0 & 2\\-1 & 0 & 1\end{bmatrix}$$

$$\partial_y \approx \frac{1}{4}\begin{bmatrix}1 & 2 & 1\end{bmatrix} * \frac{1}{2h}\begin{bmatrix}1\\0\\-1\end{bmatrix} = \frac{1}{8h}\begin{bmatrix}1 & 2 & 1\\0 & 0 & 0\\-1 & -2 & -1\end{bmatrix}$$

- One can show that this does not deteriorate the consistency order (still 2).
- However, Sobel operators approximate the rotation invariance of $|\boldsymbol{\nabla} f|$ better.

Derivative Filters in 2D (3)

Second Order Derivatives

- Standard approximation of the Laplacian $\Delta f = \partial_{xx} f + \partial_{yy} f$:

$$\Delta f_{i,j} = \frac{1}{h^2}\begin{bmatrix}0 & 1 & 0\\1 & -4 & 1\\0 & 1 & 0\end{bmatrix} f_{i,j}$$
$$- \frac{1}{12} h^2 (\partial_{xxxx} f_{i,j} + \partial_{yyyy} f_{i,j}) + O(h^4).$$

- Problem: Although the Laplacian is rotationally invariant, the derivative expression $\partial_{xxxx} f_{i,j} + \partial_{yyyy} f_{i,j}$ in its leading error term is not.

- Better results are obtained with

$$\Delta f_{i,j} = \frac{1}{6h^2} \begin{array}{|c|c|c|} \hline 1 & 4 & 1 \\ \hline 4 & -20 & 4 \\ \hline 1 & 4 & 1 \\ \hline \end{array} \; f_{i,j}$$

$$- \frac{1}{12} h^2 (\partial_{xxxx} f_{i,j} + 2\,\partial_{xxyy} f_{i,j} + \partial_{yyyy} f_{i,j}) \; + \; O(h^4)$$

where the leading error term involves a rotationally invariant derivative expression, namely $\Delta(\Delta f)_{i,j}$.

Summary

- Image derivatives are useful for detecting features such as edges.
- Differentiation is dangerous. It can be stabilised with a lowpass filter.
- The weights of the discrete derivative approximations can be computed via a Taylor expansion with subsequent comparision of coefficients.
- The order of consistency can be increased by larger stencils or implicit approximations.
- The continuous Fourier transform allows to analyse the frequency-dependent approximation quality.
- 2D derivative operators should have good rotation invariance.
  Example: Sobel operators.

References

- M. Wolff, P. Hauck und W. Küchlin: *Mathematik für Informatik und Bioinformatik*. Springer, Berlin, 2004.
  (German textbook covering also calculus in 2D)
- E. Kreyszig: *Advanced Engineering Mathematics*. Wiley, Chichester, 2010.
  (English textbook covering also calculus in 2D)
- H. R. Schwarz, N. Köckler: *Numerische Mathematik*. Achte Auflage, Teubner, Stuttgart, 2011.
  English Edition:
  H. R. Schwarz, J. Waldvogel: *Numerical Analysis: A Comprehensive Introduction*. Wiley, 1989.
  (recommendable numerical analysis textbook dealing also with finite difference approximations)
- A. Belyaev: On implicit image derivatives and their applications. In J. Hoey, S. McKenna, and E. Trucco (Eds.): *Proc. 2011 British Machine Vision Conference* (BMVC 2011, Aug. 29 – Sept. 2, 2011, Dundee, UK), pp. 72.1–72.12, 2011.
  `(http://www.bmva.org/bmvc/2011/proceedings/paper72/).`
  (one of the few papers on implicit derivative approximations in image processing)

Announcement

- The IPCV lecture next Tuesday will take place in E1.3, Lecture Hall 1.

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Assignment C6 – Classroom Work

Problem 1 (Linear Filters)

(a) Write down the stencil of a two-dimensional separable binomial filter that is based on the one-dimensional binomial mask $\frac{1}{4}(1, 2, 1)$.

(b) Construct a corresponding highpass filter.

Problem 2 (1-D Derivative Filters)

Consider a 1-D discrete signal $\boldsymbol{f} = (f_i)$ that is sampled from a sufficiently smooth function $f(x)$ on a uniform grid with step size h.

(a) How many equations do you need to uniquely determine the coefficients for the finite difference approximation of $f'(x)$ in pixel i using the values f_{i-1}, f_i, f_{i+1} and f_{i+2}? What happens when you use more/less equations? Derive the linear system of equations that has to be solved to find the finite difference approximation.

(b) The solution of the linear system from (a) yields the approximation

$$(f')_i \approx \frac{-f_{i+2} + 6f_{i+1} - 3f_i - 2f_{i-1}}{6h}.$$

Derive the order of consistency.

(c) Is there, in general, a lower bound of the order p of consistency, if a derivative of order d is approximated with n points? Use your findings to check if your result from (b) is plausible.

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Assignment H6 – Homework

Problem 1 (Fourier Analysis of Derivative Filters) (2+3+1 points)

Consider a function $f(x)$ and the approximation $g(x)$ of its first order derivative with grid size h:

$$f'(x) \approx g(x) = \frac{-f(x+2h) + 8f(x+h) - 8f(x-h) + f(x-2h)}{12h}.$$

(a) Compute the Fourier transform of $g(x)$ and bring it to the form $\mathcal{F}[g](u) = w(u)\,\mathcal{F}[f](u)$.

(b) Compute $\mathcal{F}[g](u) - \mathcal{F}[f'](u)$ to determine the order of consistency.
Hint: Use a Taylor expansion around 0 of the trigonometic terms you obtain in your computation.

(c) What can you learn from the expansion of $w(u)$ computed in (b) about the relation between frequency and approximation error?

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Assignment H6 (2)

Problem 2 (2-D Derivative Filters) (5+5+2 points)

Consider the following stencil for the approximation of the Laplacian:

$$\Delta u_{i,j} \approx (1-\alpha)\,\frac{1}{h^2}\begin{array}{|c|c|c|}\hline 0 & 1 & 0 \\ \hline 1 & -4 & 1 \\ \hline 0 & 1 & 0 \\ \hline \end{array}\; u_{i,j} + \alpha\,\frac{1}{(\sqrt{2}h)^2}\begin{array}{|c|c|c|}\hline 1 & 0 & 1 \\ \hline 0 & -4 & 0 \\ \hline 1 & 0 & 1 \\ \hline \end{array}\; u_{i,j}, \qquad \alpha \in [0,1]$$

(a) Show that the stencil is a linear combination of two separate derivative approximations for the (rotationally invariant) Laplacian $\Delta u = \partial_{xx}u + \partial_{yy}u$.
(Hint: Use a 2D Taylor expansion around pixel (i,j).)

(b) Use a 2D Taylor expansion around pixel (i,j) to determine the leading error term as well as the approximation order depending on the value α.

(c) Show that there exists a value of α for which the leading error term is given by

$$\tfrac{h^2}{12}\,\Delta(\Delta u)_{i,j} + \mathcal{O}(h^4).$$

Why can one expect to obtain better results for this choice of α than for the standard approximation given by $\alpha = 0$?

Assignment H6 (3)

Problem 3 (Linear Filters) (2+4 points)

Please download the required files from the webpage

`http://www.mia.uni-saarland.de/Teaching/ipcv19.shtml`

into your own directory. You can unpack them with the command `tar xvzf Ex06.tar.gz`.
The file `linear_filters.c` implements low-, high- and bandpass filters.

(a) Supplement the missing code in the routines `highpass` and `bandpass` which are supposed to compute filters of respective type by using Gaussian lowpass filters as basic building blocks.
Compile the program using the command
`gcc -O2 -o linear_filters linear_filters.c -lm` .

(b) Use the program to perform the following tasks. Use each filter exactly once.

- Remove noise from `leopard.pgm`.
- Remove background structures from `angiogram.pgm`.
- Isolate the dark gaps between the tiles in `tile.pgm`.

Assignment H6 (4)

Submission

Please submit the theoretical Problems 1 and 2 in handwritten form before the lecture. For the practical Problem 3 submit the files as follows: Rename the main directory Ex06 to Ex06_<your_name> and use the command

```
tar czvf Ex06_<your_name>.tar.gz Ex06_<your_name>
```

to pack the data. The directory that you pack and submit should contain the following files:

- the source code for the file linear_filters.c
- one image for each task from (b).
- a text file README that contains the parameters you have chosen in (b) as well as a short explanation of your filter choice for each task.

Please make sure that only your final version of the programs and images are included.
Submit the file via e-mail to your tutor via the address:

```
ipcv-xx@mia.uni-saarland.de
```

where xx is either t1, t2, t3, t4, t5, w1, w2, w3 or w4

Deadline for submission: Friday, May 24, 10 am.