Lecture 26:
3-D Reconstruction II:
Stereo

Contents

Introduction

- So far, we have only investigated the projective geometry in the *monocular* case. It uses a *single* pinhole camera.
- Today we consider the *binocular* case with *two* cameras. This allows us to reconstruct the depth of a scene from the displacements between the two stereo images.
- This problem resembles the correspondence problem from optic flow estimation.
- However, we will see that the stereo geometry (also called *epipolar geometry*) creates an additional constraint.

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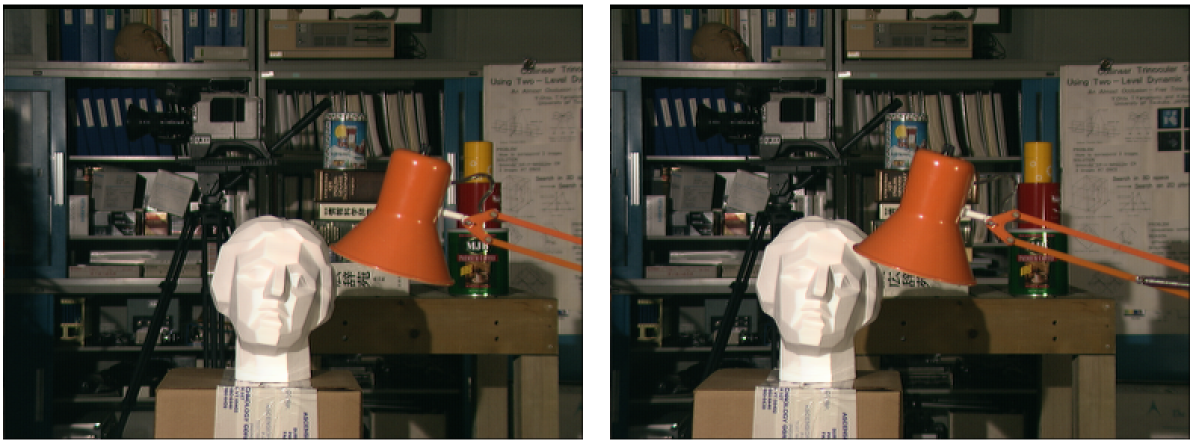
Introduction (2)



A stereo image pair from the Middlebury web page. The goal is to reconstruct the 3-D scene. Source: `http://cat.middlebury.edu/stereo/data.html`.

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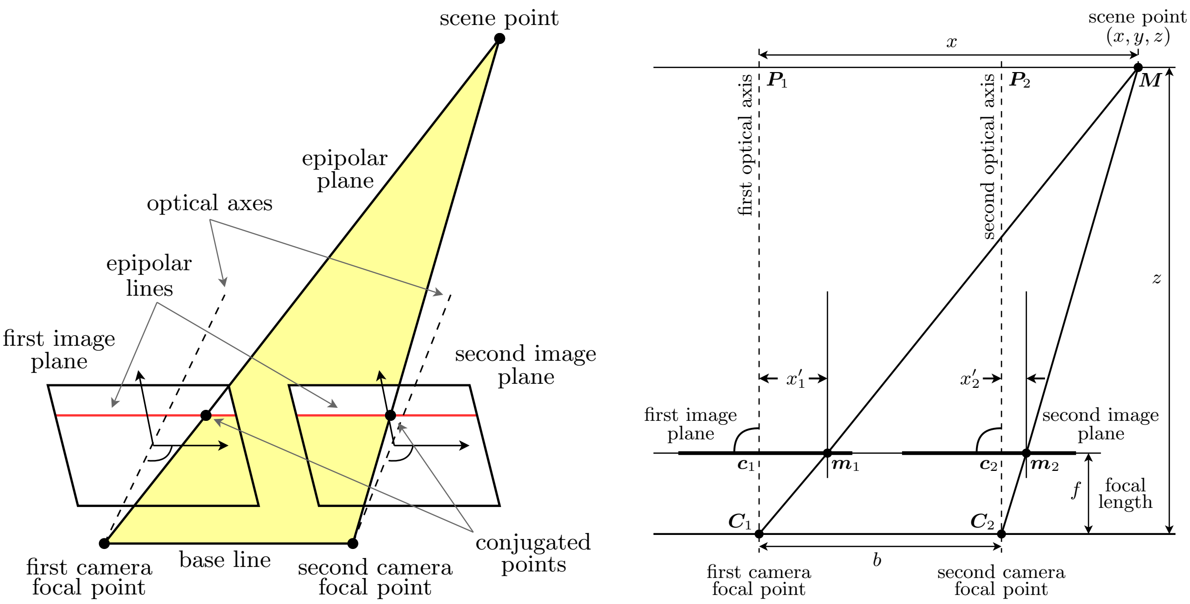
Stereo Geometry for Orthoparallel Cameras (1)

Stereo Geometry for Orthoparallel Cameras



Stereo geometry for two identical pinhole cameras with parallel optical axes (orthoparallel cameras). Author: M. Mainberger.

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Stereo Geometry for Orthoparallel Cameras (2)

A Simplified Model: Orthoparallel Cameras

- *orthoparallel cameras:*
  two identical cameras with parallel optical axes
- *base line (Basislinie):*
  connecting line between both optical centres (focal points)
- *base line distance (Basisliniendistanz) b:*
  distance between both optical centres
- *conjugated points (konjugierte Punkte):*
  two points in different images that depict the same 3-D scene point
- *epipolar plane (Epipolarebene):*
  plane through the scene point and both optical centres
- *epipolar lines (Epipolarlinien):*
  intersecting lines of the epipolar plane with both image planes;
  contain conjugated points
- *disparity (Disparität):*
  distance between two conjugated points, if both images are superposed

Stereo Geometry for Orthoparallel Cameras (3)

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Depth Computation

- Consider the figure on Page 4, right:
  Place the origin of the coordinate system in the left camera lens centre $\boldsymbol{C}_1$.
- From the similarity of the triangles $\boldsymbol{C}_1\,\boldsymbol{P}_1\,\mathbf{M}$ and $\boldsymbol{C}_1\,\boldsymbol{c}_1\,\boldsymbol{m}_1$ it follows that

$$\frac{x}{z} = \frac{x_1'}{f}.$$

- From the similarity of the triangles $\boldsymbol{C}_2\,\boldsymbol{P}_2\,\boldsymbol{M}$ and $\boldsymbol{C}_2\,\boldsymbol{c}_2\,\boldsymbol{m}_2$ one obtains

$$\frac{x-b}{z} = \frac{x_2'}{f}.$$

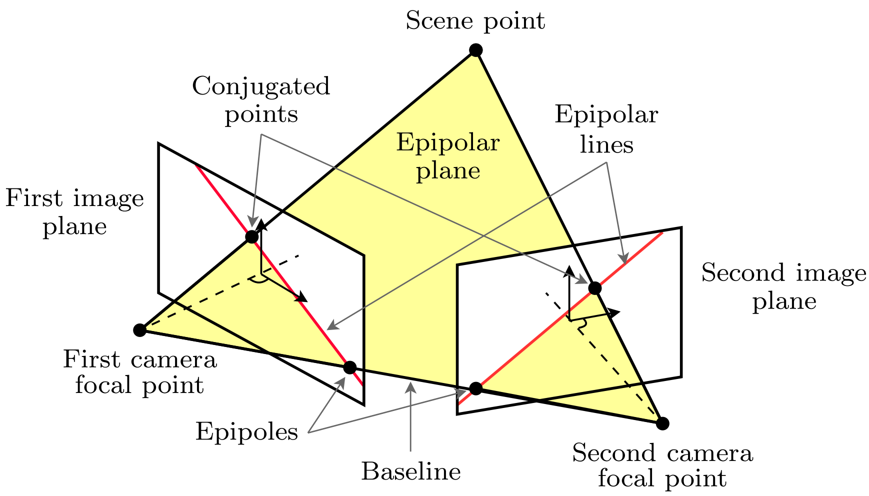
- Eliminating x in both equations and using $x_1' > x_2'$ gives

$$z = \frac{bf}{x_1' - x_2'}.$$

- This shows:
  If the baseline distance b and the focal length f are known in the orthoparallel case, the disparity $|x_1' - x_2'|$ allows to compute the depth z.

- The main problem is the reliable estimation of the disparity:
  - Often disparities can only be measured with pixel precision.
    Keeping the disparity error low suggests to choose a large baseline distance.
  - On the other hand, this may lead to more occlusions.
    Moreover, it becomes more difficult to find correspondences.

Stereo Geometry for Converging Cameras



Two converging cameras in arbitrary position and orientation. Author: M. Mainberger.

- Instead of orthoparallel cameras, one can also consider *converging cameras.*

- Conjugated points can still be found along the epipolar lines.
  In general, however, the two epipolar lines are no longer parallel.

Epipolar Constraint and Fundamental Matrix

- The conjugated points $\boldsymbol{m}_1$ and $\boldsymbol{m}_2$ cannot lie everywhere:
  Both of them are located on the epipolar plane.

- One can show that $\boldsymbol{m}_1$ and $\boldsymbol{m}_2$ must satisfy the *epipolar constraint*

$$\tilde{\boldsymbol{m}}_2^\top \boldsymbol{F} \, \tilde{\boldsymbol{m}}_1 = 0$$

  with a suitable 3×3 matrix $\boldsymbol{F}$ *(fundamental matrix).*
  The tilde denotes homogeneous coordinates with $\tilde{\boldsymbol{m}}_i := (\boldsymbol{m}_i, 1)^\top$.

- One can also show that $\boldsymbol{F}$ has rank 2 (one eigenvalue 0). Thus, it is not invertible.

- Obviously, $\boldsymbol{F}$ is only determined up to a scaling factor:
  If $\boldsymbol{F}$ satisfies epipolar constraint, then also $\alpha \boldsymbol{F}$ does.

- Thus, $\boldsymbol{F}$ has 7 degrees of freedom: 9 minus 2 for rank and scale.

- Hence, it is not possible to extract the 11 intrinsic and extrinsic parameters of each of the two cameras from $\boldsymbol{F}$.

- A system where only the fundamental matrix is known is called *weakly calibrated.*

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Epipolar Lines for a Known Fundamental Matrix

- Let us denote an epipolar line $ax + by + c = 0$ by a vector $\boldsymbol{\ell} = (a, b, c)^\top$.
  Thus, any point $\boldsymbol{x} = (x, y)^\top$ on the line $\boldsymbol{\ell}$ satisfies $\boldsymbol{\ell}^\top \tilde{\boldsymbol{x}} = 0$.

- Then for a known fundamental matrix $\boldsymbol{F}$, the following holds:
  - For each point $\boldsymbol{m}_1$ in the first frame, the corresponding epipolar line $\boldsymbol{\ell}_2$ in the second frame is given by

$$\boldsymbol{\ell}_2 = \boldsymbol{F} \, \tilde{\boldsymbol{m}}_1 \, .$$

    Note that the epipolar constraint $\tilde{\boldsymbol{m}}_2^\top (\boldsymbol{F} \, \tilde{\boldsymbol{m}}_1) = 0$ implies that $\tilde{\boldsymbol{m}}_2^\top \boldsymbol{\ell}_2 = 0$.
    Thus, $\boldsymbol{m}_2$ lies on the epipolar line $\boldsymbol{\ell}_2$.
  - For each point $\boldsymbol{m}_2$ in the second frame, the corresponding epipolar line $\boldsymbol{\ell}_1$ in the first frame is given by

$$\boldsymbol{\ell}_1 = \boldsymbol{F}^\top \tilde{\boldsymbol{m}}_2 \, .$$

    Here the epipolar constraint $(\tilde{\boldsymbol{m}}_2^\top \boldsymbol{F}) \, \tilde{\boldsymbol{m}}_1 = 0$ implies that $\boldsymbol{\ell}_1^\top \tilde{\boldsymbol{m}}_1 = 0$.
    Thus, $\boldsymbol{m}_1$ lies on the epipolar line $\boldsymbol{\ell}_1$.

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- This creates a *reduced search space (1-D)* for a stereo matching algorithm: One only has to search along epipolar lines.
- Orthoparallel cameras even yield horizontal epipolar lines (search in x-direction).
- If the fundamental matrix is not known (*uncalibrated system*), one can estimate it from point correspondences. Let us study now how this can be done.

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Estimation of the Fundamental Matrix

- Let us consider the epipolar constraint given by the equation

$$\begin{aligned} 0 \;&=\; \tilde{\boldsymbol{m}}_2^\top \, \boldsymbol{F} \, \tilde{\boldsymbol{m}}_1 \;=\; \begin{pmatrix} x_2 \\ y_2 \\ 1 \end{pmatrix}^\top \begin{pmatrix} f_{1,1} & f_{1,2} & f_{1,3} \\ f_{2,1} & f_{2,2} & f_{2,3} \\ f_{3,1} & f_{3,2} & f_{3,3} \end{pmatrix} \begin{pmatrix} x_1 \\ y_1 \\ 1 \end{pmatrix} \\ &=\; \quad x_1 x_2 \, f_{1,1} + y_1 x_2 \, f_{1,2} + x_2 \, f_{1,3} \\ &\quad\; + x_1 y_2 \, f_{2,1} + y_1 y_2 \, f_{2,2} + y_2 \, f_{2,3} \\ &\quad\; + x_1 \, f_{3,1} + y_1 \, f_{3,2} + f_{3,3}. \end{aligned}$$

- Defining the following two vectors for correspondences and matrix entries,

$$\begin{aligned} \boldsymbol{s} &:= (x_1 x_2,\, y_1 x_2,\, x_2,\, x_1 y_2,\, y_1 y_2,\, y_2,\, x_1,\, y_1,\, 1)^\top, \\ \boldsymbol{f} &:= (f_{1,1},\, f_{1,2},\, f_{1,3},\, f_{2,1},\, f_{2,2},\, f_{2,3},\, f_{3,1},\, f_{3,2},\, f_{3,3})^\top, \end{aligned}$$

  we can write the epipolar constraint as an inner product:

$$0 \;=\; \tilde{\boldsymbol{m}}_2^\top \, \boldsymbol{F} \, \tilde{\boldsymbol{m}}_1 \;=\; \boldsymbol{s}^\top \boldsymbol{f}.$$

Total Least Squares Fit (Longuet-Higgins 1981)

- For each known correspondence $(x_1^i, y_1^i) \leftrightarrow (x_2^i, y_2^i)$ we obtain one linear constraint

$$\boldsymbol{s}_i^\top \boldsymbol{f} = 0$$

with $\boldsymbol{s}_i \;=\; (x_1^i x_2^i,\; y_1^i x_2^i,\; x_2^i,\; x_1^i y_2^i,\; y_1^i y_2^i,\; y_2^i,\; x_1^i,\; y_1^i,\; 1)^\top$.

- Sum up the squared deviations from $N \geq 8$ correspondence constraints, and minimise the resulting quadratic form

$$E(\boldsymbol{f}) \;=\; \sum_{i=1}^{N} (\boldsymbol{s}_i^\top \boldsymbol{f})^2 \;=\; \boldsymbol{f}^\top \Big(\sum_{i=1}^{N} \boldsymbol{s}_i \boldsymbol{s}_i^\top \Big) \boldsymbol{f} \qquad (1)$$

with explicit constraint $|\boldsymbol{f}| = 1$ to avoid the trivial solution $\boldsymbol{f} = \boldsymbol{0}$.

- The solution to this problem is given by the normalised eigenvector to the smallest eigenvalue of the symmetric 9×9 matrix

$$\sum_{i=1}^{N} \boldsymbol{s}_i \boldsymbol{s}_i^\top .$$

Correlation-Based Methods

Basic Problem

- estimation of the disparity is a correspondence problem:
  find conjugated points

- similar to the optic flow problem

- main difference:
  search space is smaller, since conjugated points are located on the epipolar line

Correlation-Based Methods (2)

Correlation Method

- searches for corresponding local structures in both images
- The search can be restricted along epipolar lines.
  Often one also limits the largest displacement.
- The so-called correlation coefficient may serve as a quality measure of the match.
  It compares the neighbourhoods around two points (x_0, y_0) and $(x_0+u,\, y_0+v)$.
- Let f_1, f_2 denote the two images and $B_\rho(x_0, y_0)$ a disk-shaped neighbourhood around (x_0, y_0) with radius ρ.
  Then the *correlation coefficient (Korrelationskoeffizient)* is given by

$$\frac{\int_{B_\rho(x_0,y_0)} \left(f_1(x,y) - \bar{f}_1\right)\, \left(f_2(x+u,\, y+v) - \bar{f}_2\right)\, dx\, dy}{\sqrt{\int_{B_\rho(x_0,y_0)} \left(f_1(x,y) - \bar{f}_1\right)^2\, dx\, dy}\; \sqrt{\int_{B_\rho(x_0,y_0)} \left(f_2(x+u,\, y+v) - \bar{f}_2\right)^2\, dx\, dy}}$$

  where $\bar{f}_1$ is the mean value of f_1 in $B_\rho(x_0, y_0)$,
  and $\bar{f}_2$ is the mean of f_2 in $B_\rho(x_0+u,\, y_0+v)$.
- Similar to a normalised inner product $\frac{\boldsymbol{a}^\top \boldsymbol{b}}{|\boldsymbol{a}|\,|\boldsymbol{b}|}$ with values in $[-1, 1]$, the correlation coefficient attains values in $[-1, 1]$. Higher values indicate a better match.

Correlation-Based Methods (3)

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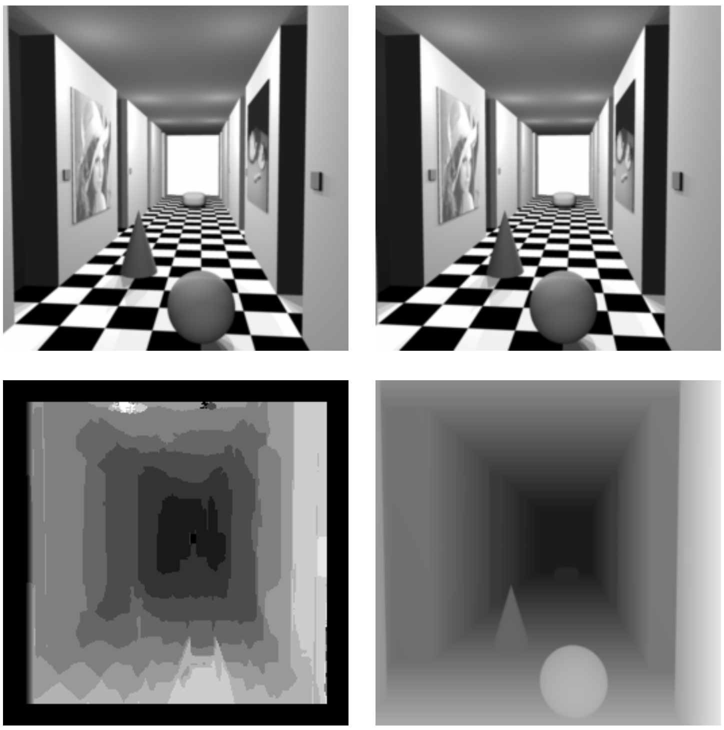


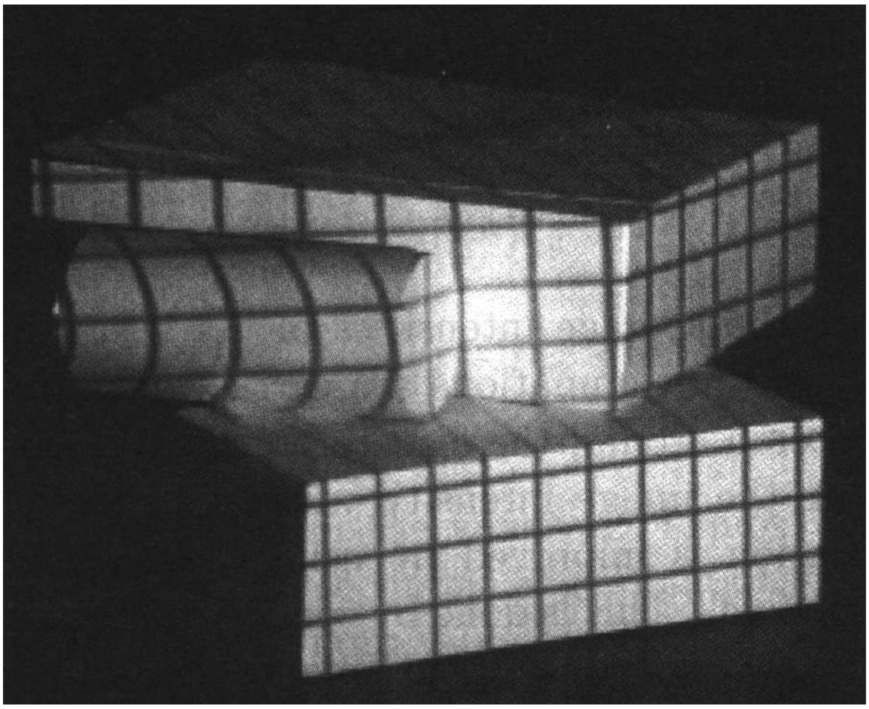
Top left: Left image (from `http://www-dbv.cs.uni-bonn.de/stereo_data/`). **Top right:** Right image. **Bottom left:** Disparity computed with a correlation window of size 17×17. **Bottom right:** Ground truth. Authors: A. Bruhn, J. Weickert.

Correlation-Based Methods (4)

Advantages and Shortcomings of Correlation-Based Methods

- simple and fast
- local method:
  gives unreliable results in flat regions
- possibility of improving the quality:
  structured illumination

Correlation-Based Methods (5)



Structured illumination: Light patterns are projected onto the object in order to create additional features for matching both images. Authors: R. Jain, R. Kasturi, B. G. Schunck.

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Variational Methods

Assumptions

- For simplicity of notation, consider the orthoparallel case:
  The epipolar lines are parallel to the x-axis, i.e. no disparities occur in y-direction.
- Grey value constancy assumption:
  Conjugated points have the same grey values.
- Smoothness assumption:
  The disparity u varies smoothly in space.

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Energy Functional

$$E(u) \;=\; \int_{\Omega} \left(\big(f_2(x+u,\, y) - f_1(x,y)\big)^2 + \alpha\, |\boldsymbol{\nabla} u|^2 \right) dx\, dy\,.$$

Problem

- The data term $(f_2(x+u,\, y) - f_1(x,y))^2$ cannot guarantee convexity.
  Since the disparities can be large, a Taylor linearisation is inappropriate.
- Thus, the energy functional may have many local minima.
- No global convergence of numerical methods:
  Different initialisations may lead to different local minima.

Variational Methods (3)

Basic Idea Behind a Possible Numerical Continuation Strategy

- Smooth the data in order to reduce the number of local minima in the energy: Replace the images f_1, f_2 in the Euler–Lagrange equation

$$\Big(f_2(x+u,y) - f_1(x,y)\Big)\,\partial_x f_2(x+u,y) - \alpha\,\Delta u \;=\; 0$$

  by Gaussian-smoothed versions $K_{\sigma_0} * f_1$ and $K_{\sigma_0} * f_2$.

- Solve this problem with an iterative numerical method (next page). If σ_0 is sufficiently large, the convergence is nonlocal. A good value for σ_0 is the expected order of magnitude for the disparity.

- Reduce the width of the Gaussian (e.g. $\sigma_1 := 0.95\,\sigma_0$) and solve the problem with the solution for σ_0 as initialisation.

- Continue this strategy until σ_k reaches the pixel scale.

Variational Methods (4)

Numerical Solution of the Euler–Lagrange Equation

- Discretise the derivatives with simple finite differences (cf. Lecture 12).

- Solve the resulting system of equations with a suitable iterative method.

Advantages and Shortcomings of Variational Methods

- filling-in effect creates dense disparity maps of reasonable quality

- requires relatively high computational effort

Possible Model Refinements

- nonquadratic smoothness terms that reduce smoothing across discontinuities of the image or its disparities.

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Variational Methods (5)

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A stereo image pair from INRIA Sophia-Antipolis. The epipolar lines are not horizontal in this case.

Variational Methods (6)

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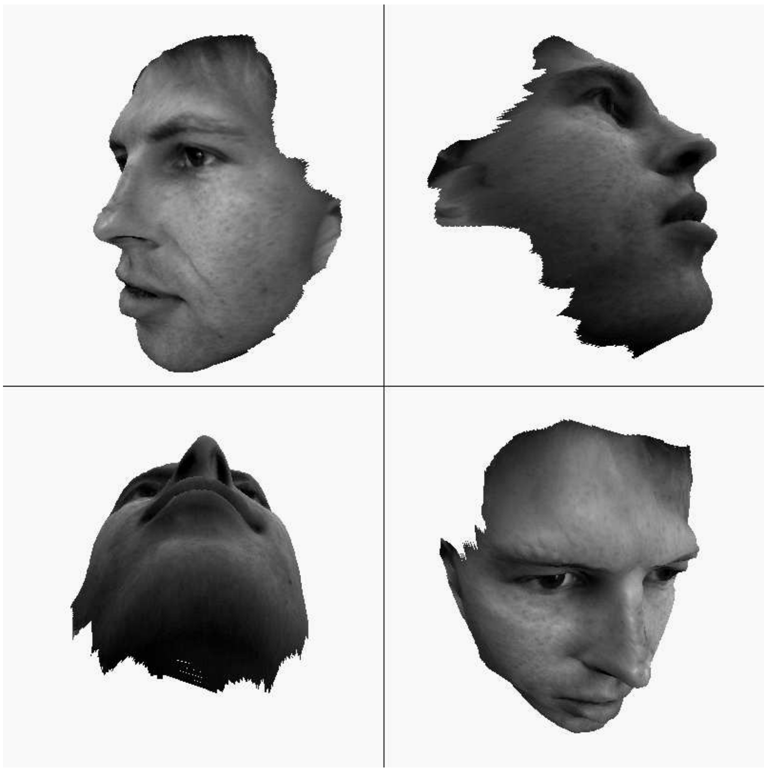

Four views of a stereo reconstruction with a variational method that reduces smoothing across image edges. Authors: L. Alvarez, R. Deriche, J. Sánchez, J. Weickert.

Extensions

- One can also consider stereo sequences.
- Sophisticated variational models allow the simultaneous estimation of
  - the fundamental matrix
  - the 3-D surface
  - its 3-D motion over time *(scene flow)*

left, time t | right, time t

left, time $t+1$ | right, time $t+1$

Four images of a stereo image sequence (340×480 pixels). Authors: L. Valgaerts et al.

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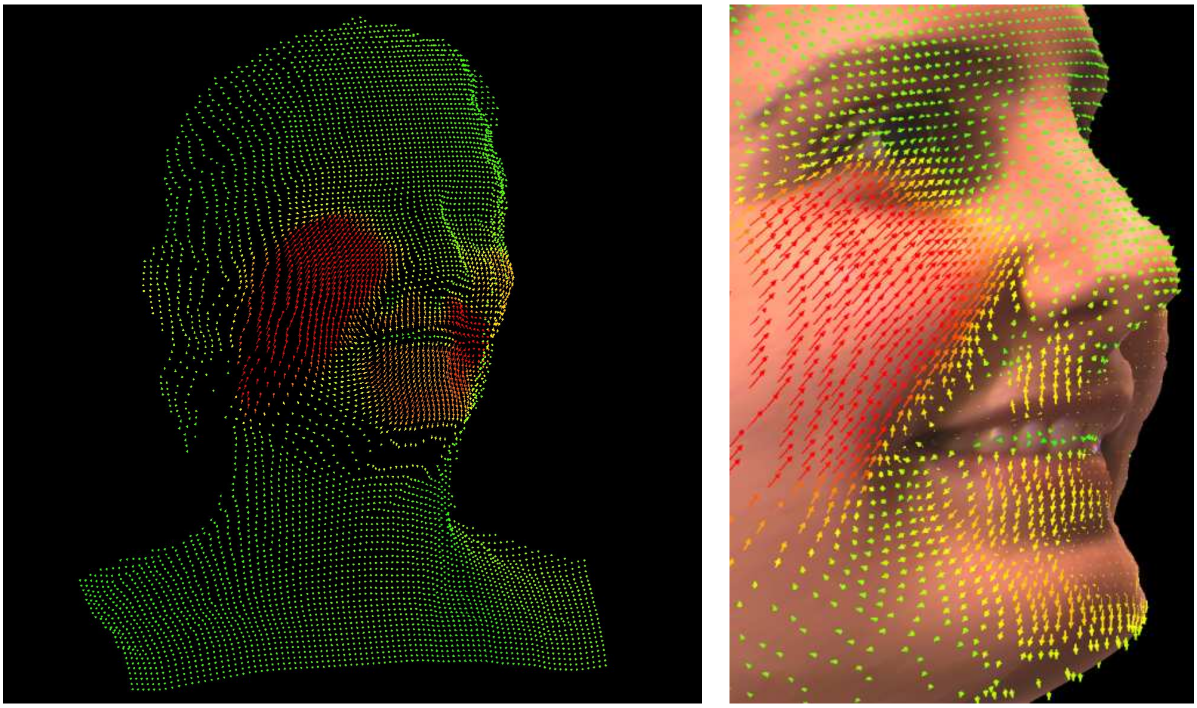
Extensions (3)



Variational 3D reconstruction und scene flow estimation. **Left:** Computed scene flow. It magnitude is colour-coded and increases from green to red. **Right:** Zoom into the 3-D surface reconstruction with texture mapping and superposed scene flow. Authors: L. Valgaerts et al.

Summary

Summary

- Conjugated points in two stereo images are located on the epipolar lines (epipolar constraint).
- The fundamental matrix allows to specify the epipolar line to a given point.
- It is a 3×3 matrix of rank 2 that is determined up to a scale factor.
  It can be estimated from ≥ 8 point correspondences via a total least squares approach.
- In the orthoparallel case, the disparity allows to compute the depth information if the focal length and the baseline distance is known.
- Estimating the disparity is a correspondence problem similar to optic flow estimation with large displacements.
  However, the search space is restricted due to the epipolar constraint.
- Correlation methods are simple and fast, but may give poor results in flat regions.
  Structured illuminatation can increase the quality.
- Global methods based on variational approaches create dense disparity maps, but are computationally more demanding.

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References

- R. Jain, R. Kasturi, B. G. Schunck: *Machine Vision*. McGraw-Hill, New York, 1995.
  (see Sections 11.1 and 11.2)
- L. Alvarez, R. Deriche, J. Sánchez, J. Weickert: Dense disparity map estimation respecing image discontinuities: a PDE and scale-space based approach. *Journal of Visual Communication and Image Representation*, Vol. 13, No. 1/2, pp. 3–21, 2002.
  (`http://www-sop.inria.fr/rapports/sophia/RR-3874`)
  (example of a variational approach)
- G. Xu, Z. Zhang: *Epipolar Geometry in Stereo, Motion and Object Recognition: A Unified Approach*. Kluwer, Dordrecht, 1996.
  (excellent book on stereo)
- O. Schreer: *Stereoanalyse und Bildsynthese*. Springer, Berlin, 2008.
  (German book on stereo geometry and nonvariational stereo reconstruction methods)
- R. Klette, K. Schlüns, A. Koschan: *Computer Vision: Three-Dimensional Data from Images*. Springer, Singapore, 1998.
  (Chapter 4 describes geometrical foundations and stereo reconstruction methods.)
- O. Faugeras, Q.-T. Luong, T. Papadopoulo: *The Geometry of Multiple Images*. MIT Press, Cambridge, MA, 2001.
  (Chapters 5-7 deal with stereo geometry)

- R. Hartley, A. Zisserman: *Multiple View Geometry in Computer Vision*. Cambridge University Press, Cambridge, UK, 2000.
  (Part II on stereo geometry)
- Y. Ma, S. Soatto, J. Kosecká, S. S. Sastry: *An Invitation to 3-D Vision*. Springer, New York, 2002.
  (Chapters 4–6 on stereo geometry)
- Middlebury Stereo Vision Page: `http://vision.middlebury.edu/stereo/`.
  (compares many algorithms)
- L. Valgaerts, A. Bruhn, H. Zimmer, J. Weickert, C. Stoll, C. Theobalt: Joint estimation of motion, structure and geometry from stereo sequences. In K. Daniilidis, P. Maragos, N. Paragios (Eds.): *Computer Vision – ECCV 2010*. Lecture Notes in Computer Science, Vol. 6314, 568–581, Springer, Berlin, 2010.
  (3-D reconstruction and scene flow from stereo sequences)

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1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25 26 27 28 29 30 31 32 33 34 35

Assignment C13 – Classroom Work

Problem 1 (Transformation Matrices)

Compute a transformation matrix in homogeneous coordinates which describes a rotation around the x-axis through an angle of 45 degrees followed by a translation with a vector $(2, 4, -1)^\top$ and a rotation around the y-axis through an angle of -60 degrees.

Problem 2 (3-D Rotation and Euler Angles)

A sequence of three rotations can be used to give a 3-D-object a specific orientation. To this end we first rotate α degrees around the z-axis. This is followed by a rotation around the new x-axis by an angle of β. Finally we rotate γ degrees around the z-axis that was created by the previous rotations.

State the final rotation matix that depends only on α, β and γ.

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Assignment H13 – Homework

Problem 1 (Homogeneous Coordinates) (1+2+2 points)

Let $\boldsymbol{m}_1 = (x_1, y_1)^\top$ and $\boldsymbol{m}_2 = (x_2, y_2)^\top$ be two different points (i.e. $\boldsymbol{m}_1 \neq \boldsymbol{m}_2$) in Euclidean coordinates. Moreover, let $\boldsymbol{\ell}_1 = (a_1, b_1, c_1)^\top$ and $\boldsymbol{\ell}_2 = (a_2, b_2, c_2)^\top$ be nonparallel lines, where the notation $\boldsymbol{\ell}_1$ means that $a_1 x + b_1 y + c_1 = 0$.

(a) Show that $\boldsymbol{m}_1$ lies on $\boldsymbol{\ell}_1$ if and only if $\tilde{\boldsymbol{m}}_1^\top \boldsymbol{\ell}_1 = 0$.

(b) Let $\boldsymbol{m}_1$ be the intersection of $\boldsymbol{\ell}_1$ and $\boldsymbol{\ell}_2$. Show that $\boldsymbol{\ell}_1 \times \boldsymbol{\ell}_2 = \tilde{\boldsymbol{m}}_1$ holds.

(c) Let $\boldsymbol{\ell}_1$ be the line that connects $\boldsymbol{m}_1$ and $\boldsymbol{m}_2$. Show that $\tilde{\boldsymbol{m}}_1 \times \tilde{\boldsymbol{m}}_2 = \boldsymbol{\ell}_1$.

Remarks: For a point $\boldsymbol{x} = (x_1, x_2)^\top$ in Euclidean coordinates, $\tilde{\boldsymbol{x}} = (x_1, x_2, 1)^\top$ is its counterpart in homogeneous coordinates. The operator $\times$ denotes the cross product

$$\boldsymbol{x} \times \boldsymbol{y} \;:=\; (x_2 y_3 - x_3 y_2,\; x_3 y_1 - y_1 x_3,\; x_1 y_2 - x_2 y_1)^\top \qquad (\boldsymbol{x}, \boldsymbol{y} \in \mathbb{R}^3).$$

Problem 2 (Fundamental Matrix for the Orthoparallel Camera Setup) (4 points)

For the orthoparallel case, the fundamental matrix $\boldsymbol{F}$ is uniquely defined up to a scaling factor. Determine the fundamental matrix for the orthoparallel case.

Hint: Exploit the formula $\boldsymbol{\ell}_2 = \boldsymbol{F}\tilde{\boldsymbol{m}}_1$, using the homogeneous points $(0, 0, 1)^\top$, $(1, 0, 1)^\top$, and $(0, 1, 1)^\top$, and their corresponding lines. Recall that the vector $\boldsymbol{\ell}_2 = (a, b, c)^\top$ describes the epipolar line $ax + by + c = 0$ in the second frame which corresponds a given point $\boldsymbol{m}_1$ in the first frame.

Assignment H13 (2)

Problem 3 (Stereo Reconstruction) (2+2+2 points)

Let us assume that we have two cameras in $\boldsymbol{C}_1$ and $\boldsymbol{C}_2$. The extrinsic and intrinsic parameters of both cameras are given in the form of the matrices

$$\boldsymbol{A}_1^{\text{int}} = \boldsymbol{A}_2^{\text{int}} = \begin{pmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \quad \boldsymbol{A}_1^{\text{ext}} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}, \quad \boldsymbol{A}_2^{\text{ext}} = \begin{pmatrix} \frac{1}{\sqrt{2}} & 0 & -\frac{1}{\sqrt{2}} & -\sqrt{2} \\ 0 & 1 & 0 & 2 \\ \frac{1}{\sqrt{2}} & 0 & \frac{1}{\sqrt{2}} & -\sqrt{2} \\ 0 & 0 & 0 & 1 \end{pmatrix}.$$

Moreover, the focal length of both cameras is given by $f_1 = f_2 = 1$. Let us assume that we have found the following correspondence:

$$\boldsymbol{x}_1 = \left(2,\ \frac{6}{5}\right)^\top, \quad \boldsymbol{x}_2 = \left(\frac{3}{4},\ \sqrt{2}\right)^\top.$$

Here, $\boldsymbol{x}_1$ and $\boldsymbol{x}_2$ denote a point in pixel locations in the first and in the second frame, respectively. In order to restore the depth of the original scene point, perform the following steps:

(a) Compute the corresponding optical rays in the 3-D coordinate system of the cameras.
(b) Compute the corresponding optical rays in the 3-D Euclidean world coordinate system.
(c) Intersect these rays to recover the 3-D world coordinates of the 3-D point that was projected on both image planes.

Assignment H13 (3)

Problem 4 (Correlation-Based Stereo Method) (4+3+2 points)

Please download the required files from the web page

`http://www.mia.uni-saarland.de/Teaching/ipcv19.shtml`

into your own directory. You can unpack them with the command `tar xvzf Ex13.tar.gz`.

The program `stereocorr.c` should be extended to the stereo correlation method from the lecture (orthoparallel case). As correlation window B_ρ it uses a square of size $(2n+1) \times (2n+1)$ pixels. Moreover, it limits the search space by defining a maximum disparity `max_disp`.

(a) Supplement the subroutines `mean_window`, `sum_window`, and `correlation` accordingly. The routine `sum_window` plays the main role, since it computes expressions of type

$$\int_{B_\rho(x_0,y_0)} \Big(f_1(x,y) - \bar{f}_1(x,y)\Big)\Big(f_2(x-s,y) - \bar{f}_2(x-s,y)\Big)\, dx\, dy.$$

The code is compiled via `gcc -O2 -o stereocorr stereocorr.c -lm`.

(b) Use the *Tsukuba* stereo pair `tsu_l.pgm` and `tsu_r.pgm` from the Middlebury College as input data and compare your results to the ground truth `tsu_t.pgm`. Use a maximal disparity of 6 pixels. What happens if you increase the size of the correlation window? Try to find a suitable value for n.
(c) Replace the right image `tsu_r.pgm` by its modified variant `tsu_r_mod.pgm`. In this variant all grey values have been shifted by 50. What can you observe? What is the explanation for this behaviour?

Assignment H13 (4)

Submission

The theoretical Problems 1, 2, and 3 should be submitted in handwritten form before the lecture. For the practical Problem 4, please submit the files as follows: Rename the main directory `Ex13` to Ex13_<your_name> and use the command

```
tar czvf Ex13_<your_name>.tar.gz Ex13_<your_name>
```

to pack the data. The directory that you pack and submit should contain the following files:

- the source code of Problem 4(a)
- two images created in Problem 4(b) for different sizes of the correlation window.
- one image created in Problem 4(c) for the modified variant using one of the window sizes chosen in Problem 4(b).
- a text file README that
  - states the used parameters
  - contains a brief discussion about the pros and cons of different window sizes n
  - answers the questions of Problem 4(c)
  - contains information on all people working together for this assignment.

Please make sure that only your final version of the programs and images are included.
Submit the file via e-mail to your tutor via the address:

```
ipcv-xx@mia.uni-saarland.de
```

where xx is either `t1`, `t2`, `t3`, `t4`, `t5`, `w1`, `w2`, `w3` or `w4` depending on your tutorial group.

Deadline for submission: Friday, July 12, 10 am (before the lecture)