Image Processing and Computer Vision (IPCV)

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Example Solutions for Classroom Assignment 5 (C5)

Problem 1 (Point Transformations)

(a) In order to perform an affine grey scale transformation to the interval $[0, 7]$, we have to find an affine function $f(x) = ax + b$ which maps the largest grey value in our image (in our case 6) to the new maximum grey value (in our case 7) and the smallest value (1) to the new minimum grey value (0). This means that we have to solve the following linear system.

$$
\begin{aligned}
7 &= a \cdot 6 + b \\
0 &= a \cdot 1 + b
\end{aligned}
$$

The solution of this system can easily be found and it is given by $a = \frac{7}{5}$ and $b = -\frac{7}{5}$. Therefore, we obtain the following mapping

original	1	2	3	4	5	6
mapped	0	1.4	2.8	4.2	5.6	7
quantised	0	1	3	4	6	7

which results in the image

1	0	1	4	4
0	1	6	1	1
0	3	7	1	0
1	1	0	1	4
4	3	1	0	1
3	1	0	0	1

(b) First, we calculate the histogram of the image, i.e. we count the occurrence frequency of all grey values from the 3bit range $(0, \dots, 7)$.

Grey value index i	1	2	3	4	5	6	7	8
Grey value v_i	0	1	2	3	4	5	6	7
Frequency p_i	0	8	13	3	4	1	1	0

The image consists of $N = 30$ pixels and we have $2^3 = 8$ different grey values. Thus the desired number of pixels for each value is $q_i = \frac{30}{8} = 3.75$.

To apply the histogram equalisation we first compute the accumulated histogram:

Grey value index i	1	2	3	4	5	6	7	8
$s_i := \sum_{j=1}^{i} p_j$	0	8	21	24	28	29	30	30

For $r = 1, \dots, 8$, we are looking for the largest index k_r such that $s_{k_r} \leq r \cdot 3.75$:

Index r	1	2	3	4	5	6	7	8
k_r	1	1	2	2	2	3	4	8
s_{k_r}	0	8	21	24	28	29	30	30
$r \cdot 3.75$	3.75	7.5	11.25	15	18.75	22.5	26.25	30

Let $k_0 := 0$. We map the grey values $v_{k_{r-1}+1}, \dots, v_{k_r}$ to new grey value

w_r for $r = 1, \ldots, 8$:

$$
\begin{array}{lll}
r = 1: & \displaystyle\sum_{i=1}^{k_1} p_i \leqslant 1 \cdot 3.75 \Leftrightarrow k_1 = 1 \quad \Rightarrow & v_1, \ldots, v_1 \text{ are mapped to } w_1 = 0 \\
r = 2: & \displaystyle\sum_{i=1}^{k_2} p_i \leqslant 2 \cdot 3.75 \Leftrightarrow k_2 = 1 \quad \Rightarrow & v_2, \ldots, v_1 : \lightning \text{ no mapping to } w_2 = 1 \\
r = 3: & \displaystyle\sum_{i=1}^{k_3} p_i \leqslant 3 \cdot 3.75 \Leftrightarrow k_3 = 2 \quad \Rightarrow & v_2, \ldots, v_2 \text{ are mapped to } w_3 = 2 \\
r = 4: & \displaystyle\sum_{i=1}^{k_4} p_i \leqslant 4 \cdot 3.75 \Leftrightarrow k_4 = 2 \quad \Rightarrow & v_3, \ldots, v_2 : \lightning \text{ no mapping to } w_4 = 3 \\
r = 5: & \displaystyle\sum_{i=1}^{k_5} p_i \leqslant 5 \cdot 3.75 \Leftrightarrow k_5 = 2 \quad \Rightarrow & v_3, \ldots, v_2 : \lightning \text{ no mapping to } w_5 = 4 \\
r = 6: & \displaystyle\sum_{i=1}^{k_6} p_i \leqslant 6 \cdot 3.75 \Leftrightarrow k_6 = 3 \quad \Rightarrow & v_3, \ldots, v_3 \text{ are mapped to } w_6 = 5 \\
r = 7: & \displaystyle\sum_{i=1}^{k_7} p_i \leqslant 7 \cdot 3.75 \Leftrightarrow k_7 = 4 \quad \Rightarrow & v_4, \ldots, v_4 : \text{are mapped to } w_7 = 6 \\
r = 8: & \displaystyle\sum_{i=1}^{k_8} p_i \leqslant 8 \cdot 3.75 \Leftrightarrow k_8 = 8 \quad \Rightarrow & v_5, \ldots, v_8 \text{ are mapped to } w_8 = 7
\end{array}
$$

The equalised image is given by

5	2	5	7	7
2	5	7	5	5
2	6	7	5	2
5	5	2	5	7
7	6	5	2	5
6	5	2	2	5

and its histogram is

Grey value index i	1	2	3	4	5	6	7	8
Grey value w_i	0	1	2	3	4	5	6	7
Frequency	0	0	8	0	0	13	3	6

(c) Looking at the histograms before and after the equalisation, we notice that initially most of the grey values were to be found on the lower end of the interval $[0, 7]$ whereas after the equalisation they are more "spread" over the whole domain. The affine rescaling did not achieve this effect (due to the fact that it is a bijection, it cannot perform one-to-many or many-to-one mappings, which would be required to obtain such spreadings of the pixel values). In this sense, the histogram equalisation is superior to the affine rescaling. However, the affine rescaling contains much more different pixel values (histogram equalisation has reduced the number of colours actually appearing in the image to 4) and in that context, the rescaling performs far better because it preserves a lot more information from the initial image.