Lecture 10: Point Operations

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What are Point Operations?

Image processing operations transform an image into another one that is more useful for a human– or computer–based interpretation.

Point Operations

- simplest transformations for image enhancement: global greyscale modification
- do not take into account grey value configuration in the neighbourhood
- For a greyscale image $f(x, y)$, all point operations have the structure

$$\phi : \quad f(x,y) \quad \longmapsto \quad g(x,y) = \phi(f(x,y)) \, .$$

- Note that the location (x, y) does not matter for ϕ, only its grey value $f(x, y)$.
- Often ϕ is *nonlinear*, i.e. it violates the superposition principle

$$\phi\big(\alpha\, f_1(x,y) + \beta\, f_2(x,y)\big) \;=\; \alpha\, \phi(f_1(x,y)) \;+\; \beta\, \phi(f_2(x,y)) \qquad \forall\, \alpha, \beta \in \mathbb{R}.$$

- Point operations can be useful in many applications and should be tried first.

Affine Greyscale Transformations

- have a simple structure: $g(x,y) \;=\; a \cdot f(x,y) + b \quad (a, b \in \mathbb{R})$.
- For $b = 0$, they are also called *linear* greyscale transformations.
- allow numerous possibilities:

(1)	*identity:*	$a = 1,$	$b = 0$
(2)	*contrast enhancement:*	$a > 1$	
(3)	*contrast attenuation:*	$0 \le a < 1$	
(4)	*brightening:*	$a = 1,$	$b > 0$
(5)	*darkening:*	$a = 1,$	$b < 0$
(6)	*greyscale reversion:*	$a = -1,$	$b = g_{\max}$

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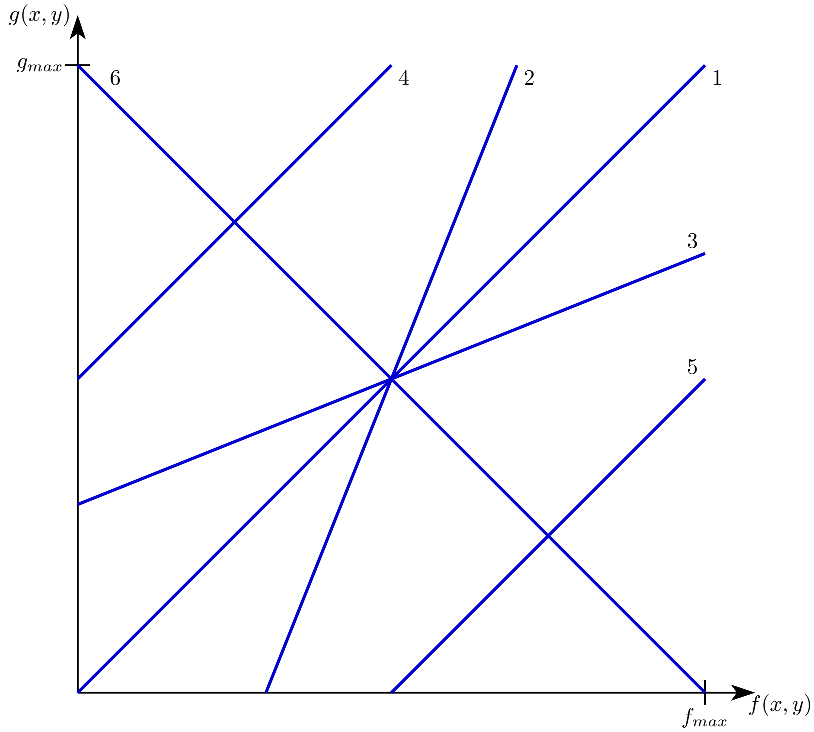

Affine greyscale transformations. **(1)** Identity. **(2)** Contrast enhancement. **(3)** Contrast attenuation. **(4)** Brightening. **(5)** Darkening. **(6)** Greyscale reversion. Author: T. Schneevoigt.

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Important Affine Transformation

- greyscale transformation to the interval $[0, 255]$ (which is easy to display and to store)
- If $f(x, y)$ has the range $[f_{\min}, f_{\max}]$, this transformation is defined as

$$g(x, y) \;=\; 255 \cdot \frac{f(x, y) - f_{\min}}{f_{\max} - f_{\min}} \,.$$

- disadvantage:
  - does not take into account how often a grey value is present
  - single outlier can spoil result
- remedy:
  - histogram equalisation (later)

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Left: Underexposed original image. **Right:** After affine greyscale mapping to the interval $[0, 255]$. Author: J. Weickert.

Thresholding (Binarisation, Schwellwertbildung)

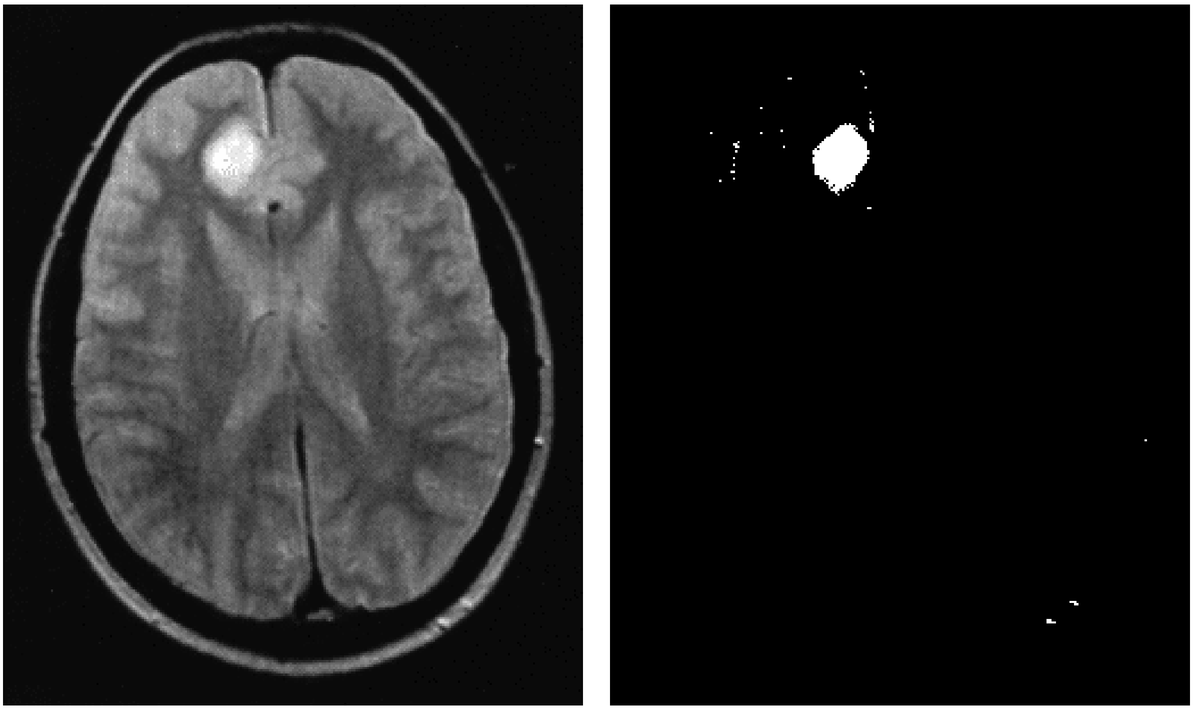
- can be described by the nonlinear transformation

$$\phi(f(x,y)) := \begin{cases} g_{\max} & \text{for } f(x,y) \geq T, \\ 0 & \text{else}, \end{cases}$$

  where $g_{\max}$ is usually 1 or 255

- simplest method for segmentation

- most difficult part: finding a good threshold parameter T
  (more details in Lecture 21)



Thresholding. **Left:** MR image of a human brain with a tumour. Greyscale range: $[0, 255]$. **Right:** Thresholding with $T = 180$ allows to segment the tumour. Author: J. Weickert.

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Logarithmic Dynamic Compression

- useful transformation if
  - the greyscale ranges over many orders of magnitude
  - the ratio between two grey values is more important than their difference
- example: visualisation of the Fourier spectrum (cf. Lecture 5)
- For some image $f(x,y)$ with range $[0, f_{\max}]$, one computes

$$\phi(f(x,y)) \;:=\; c \ln\left(1 + f(x,y)\right) \qquad (c > 0).$$

  Adding 1 ensures that $\phi(0) = 0$.

- Often c is chosen such that $\phi(f_{\max}) = 255$:

$$c \;:=\; \frac{255}{\ln\left(1 + f_{\max}\right)}\,.$$

- Thus, all transformed grey values are in $[0, 255]$.

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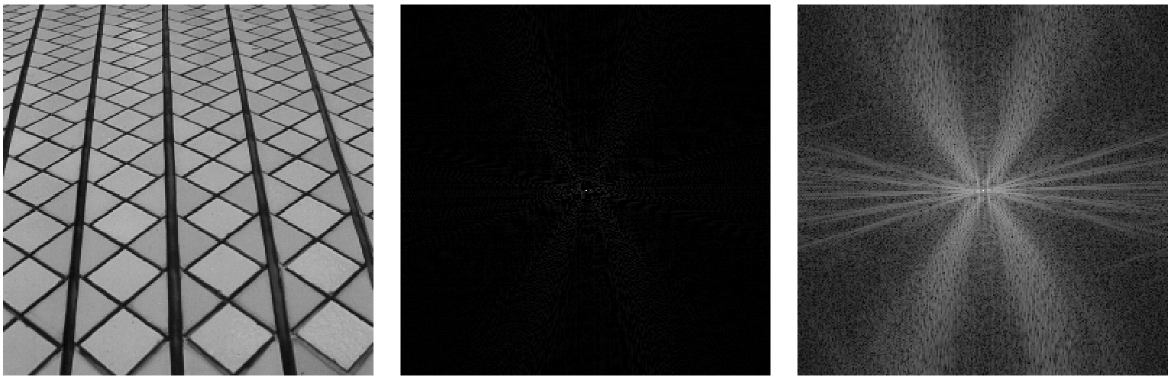

Logarithmic dynamic compression. **Left:** Original image, 256×256 pixels. **Middle:** Fourier spectrum without logarithmic dynamic compression. The white pixel in the centre corresponds to the sum of all grey values. It dominates over all other Fourier coefficients. **Right:** After logarithmic dynamic compression, the entire Fourier spectrum is well visible. The constant c is chosen such that the range of the transformed spectrum coincides with the interval [0,255]. Author: J. Weickert.

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Gamma Correction (Gammakorrektur)

- Many video cameras transform a light intensity I into a grey value f that is proportional to I^γ.
  Often $\gamma \approx 0.4$ (compresses dynamic range, similar to our visual system).

- Similar transformations are also used in computer monitors.
  Here the value for γ may vary from brand to brand.

- Sometimes the γ value is even changed by software.

- As a result, an image may look unpleasant on a specific monitor or printer.

- To compensate these effects, a so-called *gamma correction* can be used.
  For an image $f(x,y)$ with greyscale range $[0, f_{\max}]$ it is defined as

$$\phi(f(x,y)) \; := \; f_{\max} \cdot \left(\frac{f(x,y)}{f_{\max}} \right)^{1/\gamma} \qquad (\gamma > 0).$$

- Thus, the transformed image has the same range $[0, f_{\max}]$.

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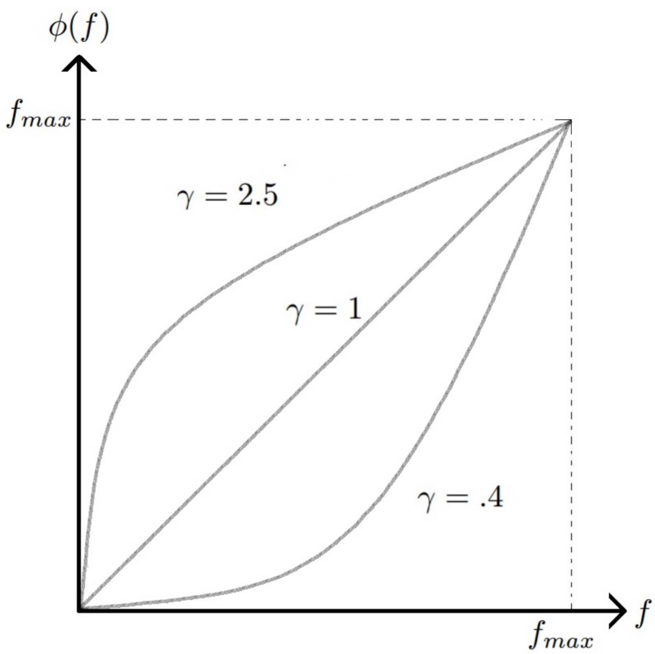

Visualisation of the gamma correction curve. Author: A. Goswami.

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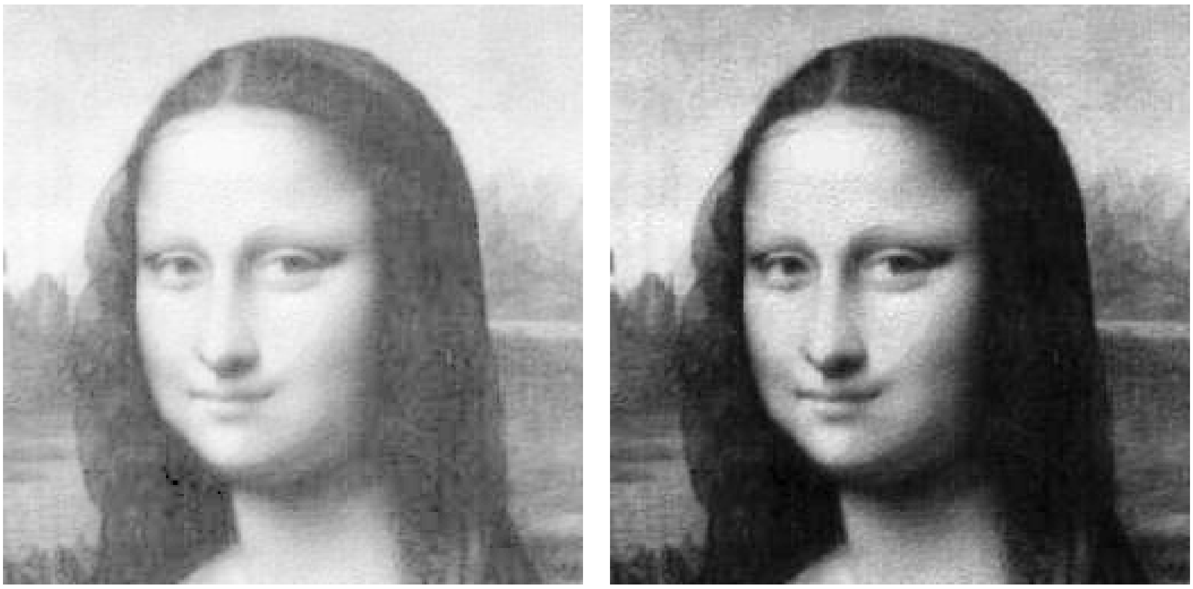

Gamma correction. **Left:** Although the entire greyscale range $[0, 255]$ is used, the Mona Lisa image appears pale and not very rich in contrast. **Right:** A gamma correction with $\gamma = 0.4$ is a remedy. Authors: L. da Vinci, J. Weickert.

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Histogram Equalisation (Histogrammegalisierung)

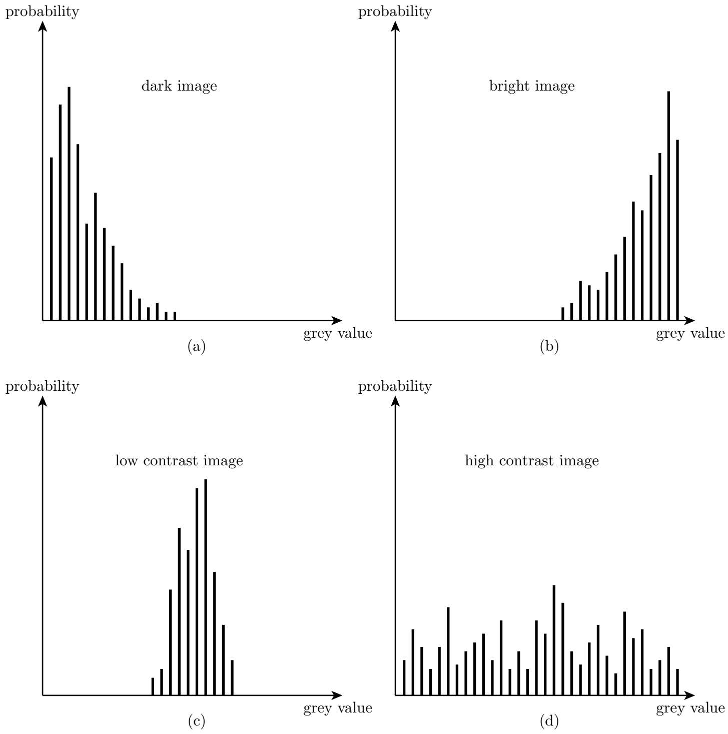
Basic Idea

- another important nonlinear point operation
- goal: transformation such that all grey values occur equally often
- may give dramatic improvements in the subjective image quality

Histogram

- specifies how often a certain grey value appears within an image
- spatial context does not matter: any pixel permutation gives same histogram

Histogram Equalisation (2)



Histograms of different types of images. **(a)** Dark image. **(b)** Bright image. **(c)** Low contrast image. **(d)** High contrast image. Author: T. Schneevoigt.

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Histogram Equalisation (3)

Algorithm for Direct Histogram Equalisation of a Discrete Image

- Basic idea: area-based mapping from a histogram (p_i) to a histogram (q_j)
- Given: p_i: number of pixels of image f having grey value v_i $(i = 1, ..., m)$
  q_j: desired number of pixels of g with grey value w_j $(j = 1, ..., n)$
  (for N pixels and 256 grey scales: $q_j := \frac{N}{256}$)
- Set $k_0 := 0$.
- For $r = 1, ..., n$:
  /* fill bin number r in the target histogram by comparing cumulative histograms */
  Find the largest index $k_r \le m$ with

$$\sum_{i=1}^{k_r} p_i \;\le\; \sum_{j=1}^{r} q_j$$

  and map the grey values $v_{k_{r-1}+1}, ..., v_{k_r}$ to w_r.

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Histogram Equalisation (4)

Remarks

- The algorithm does not only perform histogram equalisation:
  - It can also transform a histogram to any other histogram: *histogram specification*.
  - All one has to do is to use other values for $q_1,...,q_n$.
- For a general discrete image, histogram equalisation can only be approximated (see next page).

Histogram Equalisation (5)

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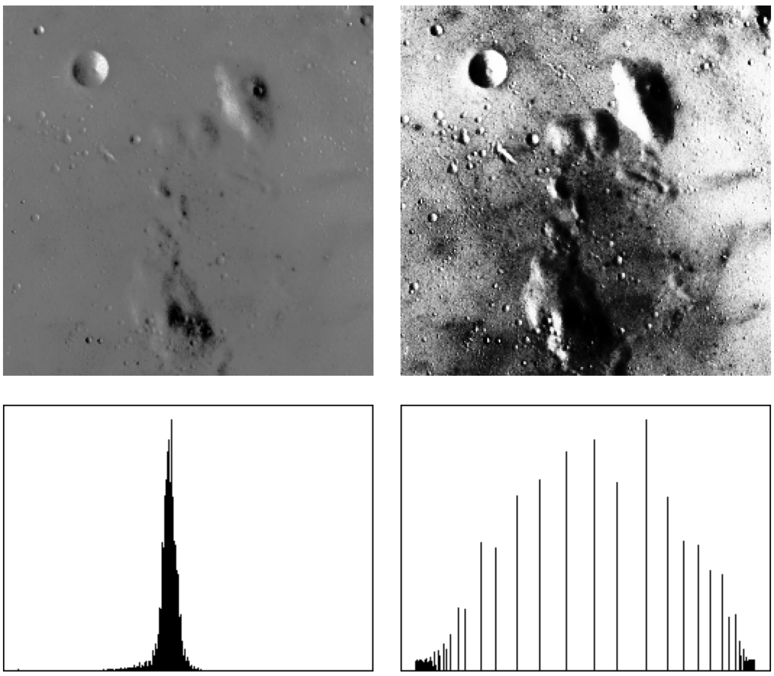


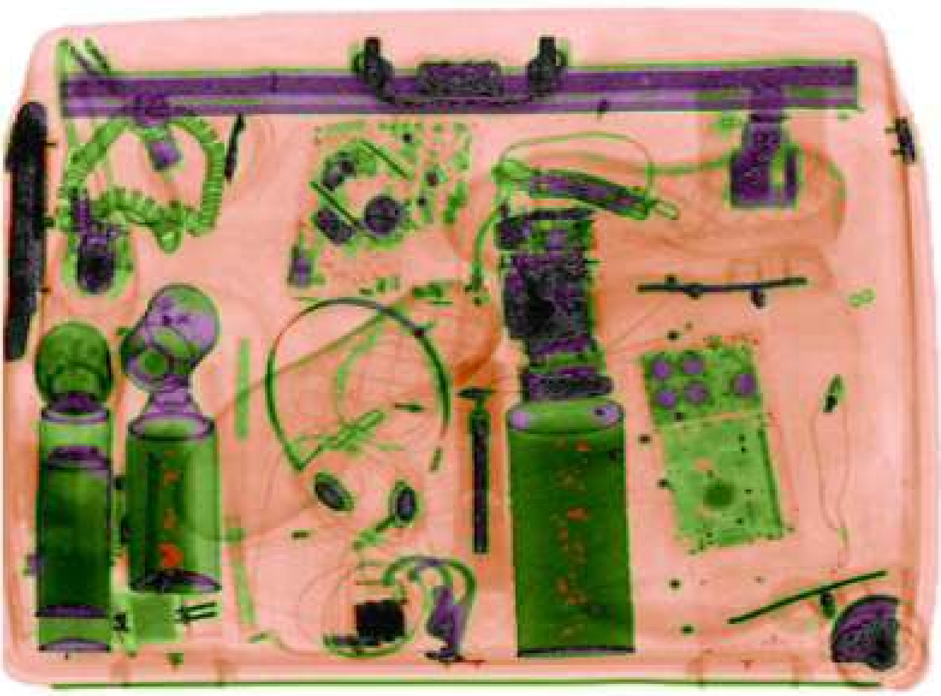
Histogram equalisation. **Top left:** Original image of the surface of the moon. **Top right:** After discrete histogram equalisation. **Bottom left:** Histogram of the original image. **Bottom right:** Histogram of the equalised image. Since the heights of the histogram bars cannot be decreased, the algorithm tries to equalise the histogram by spreading their distances. Author: J. Weickert.

Pseudocolour Representation of Greyscale Images

- Humans can distinguish only ca. 40 greyscales, but 2 million colours.
- Thus, colouring grey values allows better visual discrimination.
- There are numerous possibilities to design mappings of type

$$f(x,y) \quad \longmapsto \quad \begin{pmatrix} \phi_r(f(x,y)) \\ \phi_g(f(x,y)) \\ \phi_b(f(x,y)) \end{pmatrix}.$$

- used e.g. in X-ray scanners at airports and for thermographic images



Colouring X-ray images at airport security checks allows a human observer to distinguish objects in a better way. Source: `http://static.howstuffworks.com/gif/airport-security-xray2.jpg`.

Pseudocolour Representation of Greyscale Images (3)

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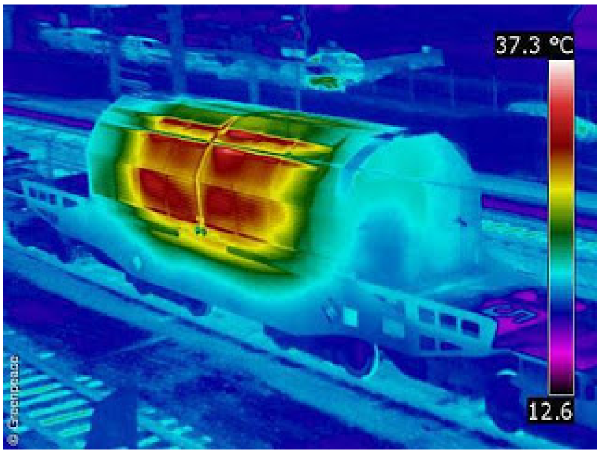

Thermography allows to measure the temperature of objects by their infrared radiation. Typically one depicts low temperatures in blue and high temperatures in red. **Left:** Pseudocolour representation of the transport of a nuclear waste container on a train. Source: Greenpeace. **Right:** The pseudocolour representation reveals that the woman freezes at her hands and her nose. Source: R. Reischuk.

Pseudocolour Representation of Greyscale Images (4)

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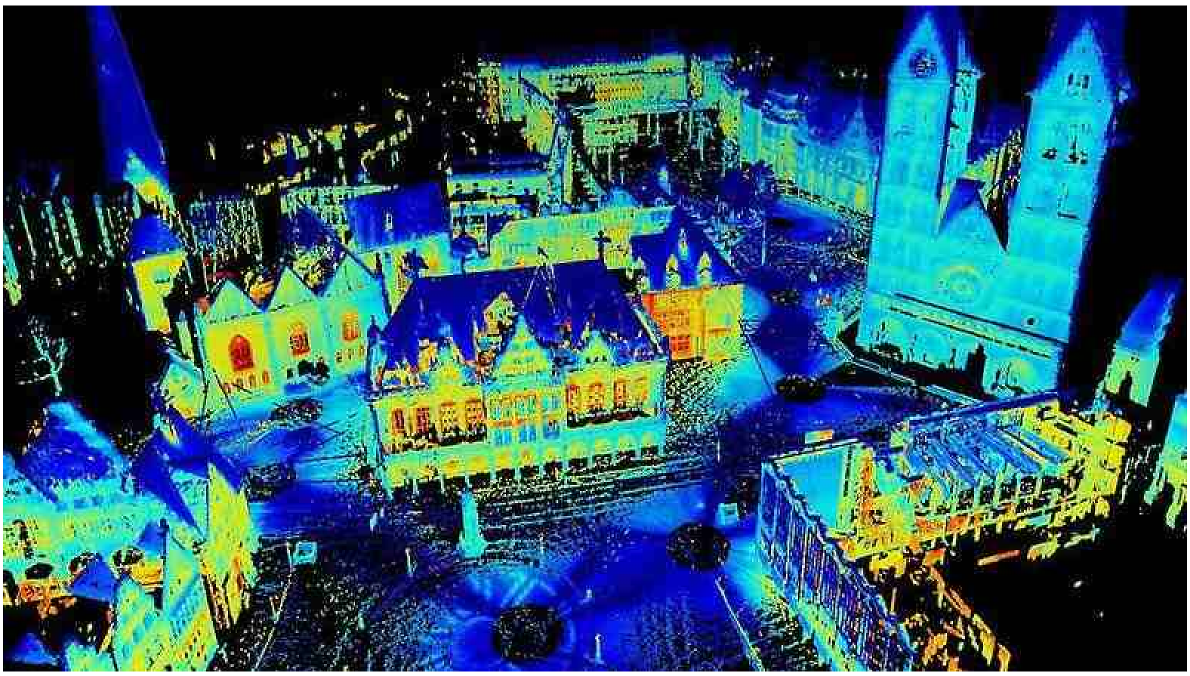
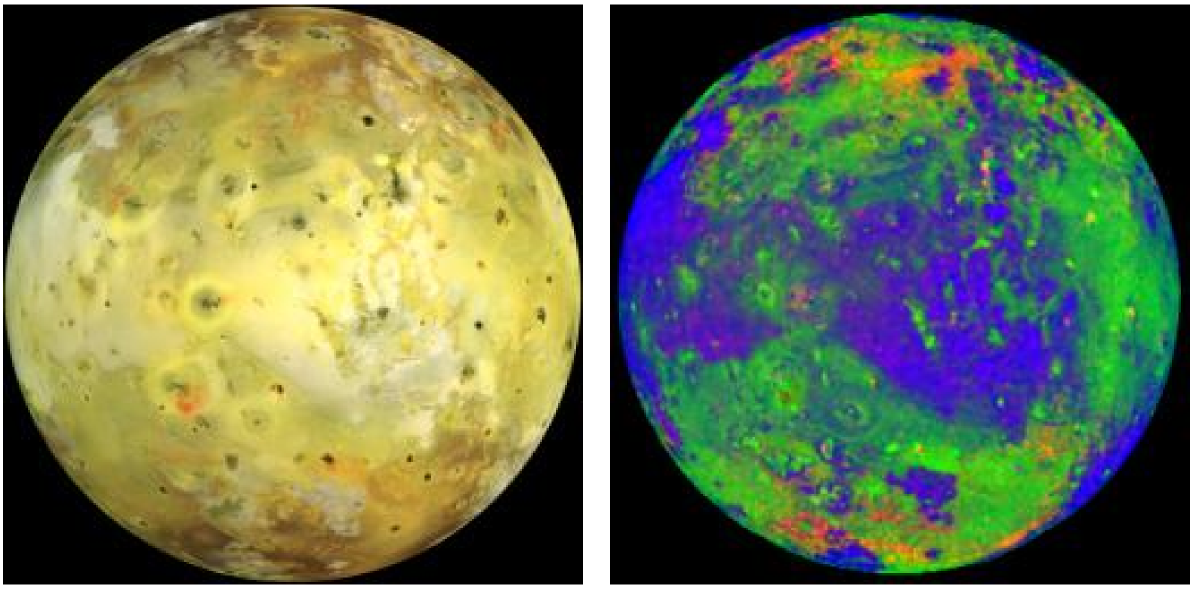


A pseudocolour representation of a thermographic measurement of the market square of Bremen. One can see that the facades of many historical buildings and in particular their windows lose too much heat. Source: A. Nüchter.

False Colour Representation of Vectorial Images

- transforms a multichannel image to a colour image
- often used in remote sensing and astronomical imaging:
  frequencies outside the visible spectrum are mapped to colours



Left: True colour representation of Jupiter's moon Io. **Right:** False colour representation that combines information from two visible and two infrared frequency bands. The depicted colours red, green and blue show the fraction between two of the four channels. They allow a better interpretation of the surface structure: Red depicts hot vulcanoes, green presumably characterises regions with much sulphur, and blue indicates frozen sulphur dioxide. Source: NASA, `http://www.jpl.nasa.gov/galileo/images/io/iocolor.html`.

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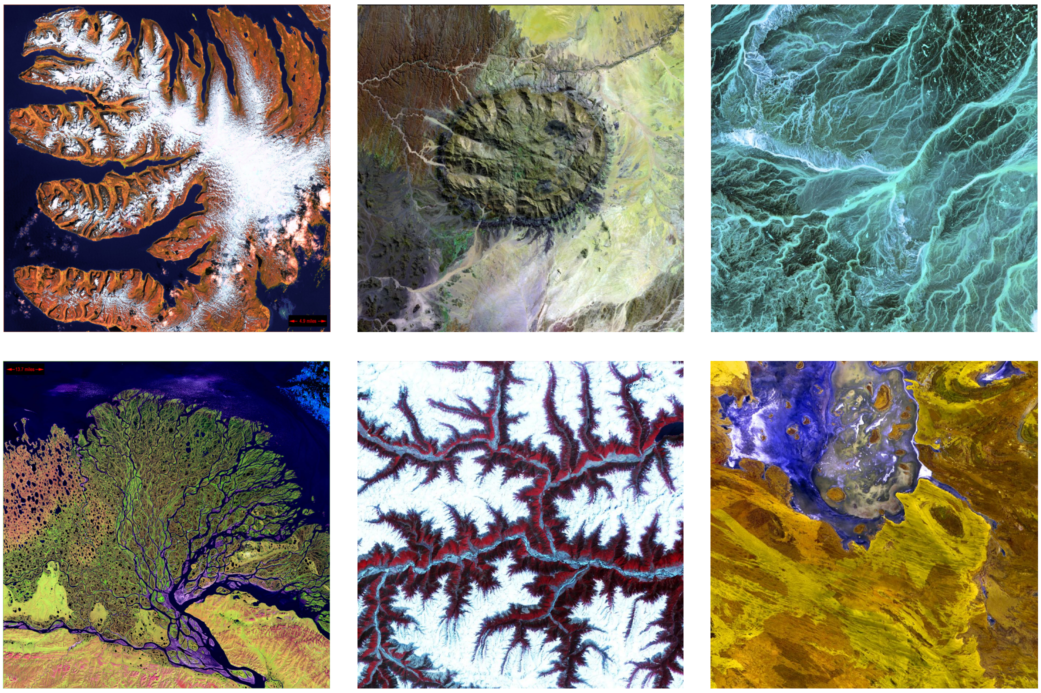

Six remote sensing images in false colour representation, such that they look like art. **Top left:** West Fjords, Iceland. **Top middle:** Brandberg Massif, Namibia. **Top right:** Jordan desert. **Bottom left:** Lena delta, Russia. **Bottom middle:** Himalaya region. **Bottom right:** Lake Disappointment, Australia. Source: NASA, `http://earthasart.gsfc.nasa.gov/`.

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Adding Images

Problem

- Some imaging methods (e.g. electron microscopy) create very noisy images:

$$\underbrace{f(x,y)}_{\text{noisy}} = \underbrace{v(x,y)}_{\text{no noise}} + \underbrace{n(x,y)}_{\text{noise}}$$

Solution

- If (!) images of the object can be taken multiple times under the same conditions, one can average these images to reduce noise.
- Let us assume that the noise has mean 0 and is uncorrelated to the image. Then averaging M images $f_i : \Omega \to \mathbb{R}$ with noise variance

$$\sigma^2 = \frac{1}{|\Omega|} \int_\Omega |n(x,y)|^2 \, dx \, dy$$

  creates a reduced noise variance $\bar{\sigma}^2 = \sigma^2/M$.
- Thus, if one wants to reduce the standard deviation $\bar{\sigma}$ of the averaged image to $1/10$ of its original value σ, one needs $10^2 = 100$ images !

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original, 323×279 pixels

Gaussian noise, $\sigma^2 = 52.29$

averaged 4 times

averaged 16 times

averaged 64 times

averaged 256 times

Denoising by averaging. Author: J. Weickert.

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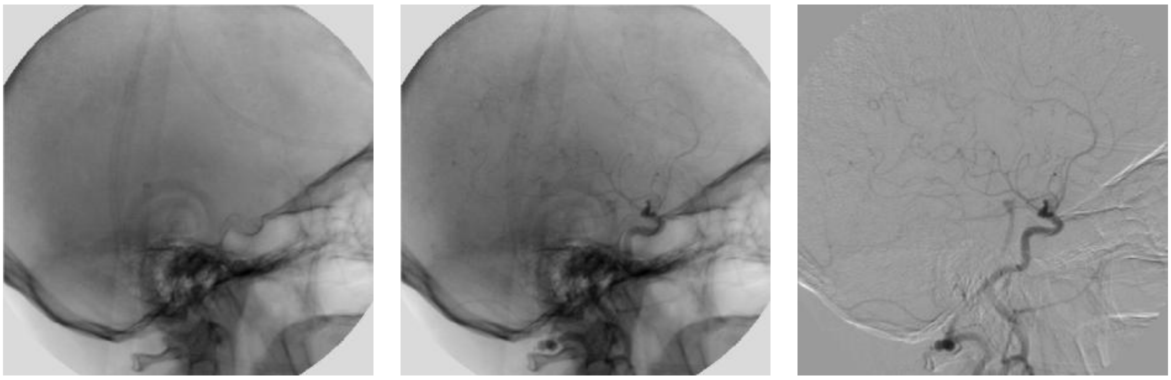
Subtracting Images

Example: Digital Subtraction Angiography (DSA)

- medical imaging method for visualising the blood flow through the vessels
- X ray images are taken before and after giving a flourescent contrast agent
- difference image makes active vessels visible

Subtracting Images (2)



Digital subtraction angiography. **Left:** Initial image (so-called mask). **Middle:** After giving a contrast agent. **Right:** The difference image removes the background and visualises vessel structures with blood flow. Source: `http://www.isi.uu.nl/Research/Gallery/DSA/`

Summary

Summary

- A point operation performs a global transformation of the greyscales.
- typical application:
  new representation of the grey values with better visibility for humans
- The most important point transforms include:
  – affine rescaling
  – thresholding
  – logarithmic dynamics compression
  – gamma correction
  – histogram equalisation
- Pseudo- and false colour representations further improve the visible information content for humans.
- Pixelwise averaging of images reduces noise.
- Subtraction of images allows background elimination.

References

- R. C. Gonzalez, R. E. Woods: *Digital Image Processing*. Pearson, Upper Saddle River, Global Edition, 2017.
  (Sections 3.2 to 3.3 describe point transformations.)
- R. Jain, R. Kasturi, B. G. Schunck: *Machine Vision*. McGraw-Hill, New York, 1995.
  (see in particular Chapter 4)
- Web page on histogram equalisation:
  `http://fourier.eng.hmc.edu/e161/lectures/contrast_transform/node2.html`
  (provides more details using cumulative histograms)

Assignment C5 – Classroom Work

Problem 1 (Point Transformations)

Consider the 8×5 image below. Its grey values are quantised to 3 bits per pixel, i.e. they have the range $[0, 7]$. However, most pixel values lie in the range $[1, 5]$. In the following we will improve the grey scale representation of this image.

6	5	5	1	2	2	2	3
4	4	4	4	5	3	2	3
4	4	4	3	6	3	3	3
4	4	3	3	5	2	2	2
1	1	5	5	1	2	2	2

(a) Apply the discrete histogram equalisation.

(b) Sketch the histograms for the original image as well as for the transformation.

Assignment H5 – Homework

Problem 1 (Keys Interpolation) (5 points)

Consider the 1-D Keys synthesis function (with $a = -\frac{1}{2}$):

$$\varphi_{\text{int}}(x) := \begin{cases} \frac{3}{2}|x|^3 - \frac{5}{2}x^2 + 1, & \text{if } |x| < 1, \\ -\frac{1}{2}|x|^3 + \frac{5}{2}x^2 - 4|x| + 2, & \text{if } 1 \leqslant |x| < 2, \\ 0, & \text{else.} \end{cases}$$

Show that the Keys synthesis function satisfies the "Partition of Unity"-property, i.e.

$$\sum_{k \in \mathbb{Z}} \varphi_{\text{int}}(x-k) = 1 \qquad \text{for all } x \in \mathbb{R}\,.$$

Hint: Make a case distinction between $x \in \mathbb{Z}$ and $x \notin \mathbb{Z}$.

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Problem 2 (Quadratic B-Spline Interpolation) (2+2+2+1 points)

Consider 1-D interpolation data $f_1 = 30,\ f_2 = 25,\ f_3 = 120$ given at the points $x_1 = 1$, $x_2 = 2,\ x_3 = 3$. The goal of this assignment is to perform interpolation with *quadratic B-splines*. Quadratic B-splines are splines of degree 2 that are based on the synthesis function

$$\beta_2(x) := \begin{cases} \frac{3}{4} - x^2 & \text{for } |x| < \frac{1}{2}, \\ \frac{1}{2}(\frac{3}{2} - |x|)^2 & \text{for } \frac{1}{2} \leq |x| < \frac{3}{2} \\ 0 & \text{else.} \end{cases}$$

(a) Set up the linear system that has to be solved in order to determine the interpolation coefficients c_1, c_2, and c_3. Verify that the system is solved by $(c_1, c_2, c_3)^\top = (40, 0, 160)^\top$.

(b) Give the analytic expression of the interpolant defined by quadratic B-spline interpolation with the coefficients from (a). Use it to determine the values at the locations 0, $\frac{3}{2}$, 2, and 4.

(c) The above data points were obtained by sampling the function $f(x) = 50x^2 - 155x + 135$. Evaluate the function at the locations 0, $\frac{3}{2}$, 2, and 4, and compare the results to the interpolated values from (b). Explain your findings.

(d) Quadratic B-Splines are unpopular in practice. Explain why this is the case.

Assignment H5 (3)

Please download the required files from the webpage

`http://www.mia.uni-saarland.de/Teaching/ipcv19.shtml`

into your own directory. You can unpack them with the command `tar xvzf Ex05.tar.gz`.

Problem 3 (Affine Rescaling) (2+1 points)

The program `pointtrans.c` contains the subroutine `rescale`. It is supposed to perform an affine greyscale transformation such that the rescaled image has the greyscale range $[a, b]$.
As for all point transformations, this can be realised by specifying the entries of an integer 1-D mapping array `g[i]` that assigns to each possible input grey value `i` $\in [0, 255]$ its new output grey value `g[i]`.

(a) Supplement the missing code and compile your program with

`gcc -O2 -o pointtrans pointtrans.c -lm`.

Note that the image `u[i][j]` is defined in the index range `i=1,...,nx` and `j=1,...,ny`.

(b) Test the routine with the image `machine.pgm`. In order to determine if the rescaling works correctly use at least one setting with $a \neq 0$.

Assignment H5 (4)

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Problem 4 (Gamma Correction) (1+2 points)

Gamma correction is an important nonlinear point operation. For an image with greyscale range $[0, 255]$, it has the structure

$$\phi(f(x,y)) := 255 \left(\frac{f(x,y)}{255} \right)^{1/\gamma} .$$

(a) Implement the method in the subroutine `gamma_correct` of program `pointtrans.c`.

(b) Validate it with the image `asbest.pgm`. What values for γ give reasonable results ?

Problem 5 (Histogram Equalisation) (5+1 points)

The third point transformation that shall be addressed in our assignments is the equalisation of an image histogram.

(a) Complete the subroutine `hist_equal` in the program `pointtrans.c` such that it performs this task in accordance with the algorithm presented in the lecture.

(b) Test your implementation with the image `office.pgm`.

Assignment H5 (5)

Submission

Please submit the theoretical Problems 1 and 2 in handwritten form before the lecture. For the practical Problems 3, 4, and 5 please submit the files as follows: Rename the main directory Ex05 to Ex05_<your_name> and use the command

```
tar czvf Ex05_<your_name>.tar.gz Ex05_<your_name>
```

to pack the data. The directory that you pack and submit should contain the following files:

- the source code for pointtrans.c with the subroutines for the Problems 3–5
- the corresponding test images with applied point operations
- a text file README that contains the answer to the question in Problem 4 as well as information on all people working together for this assignment

Please make sure that only your final version of the programs and images are included.
Submit the file via e-mail to your tutor via the address:

ipcv-xx@mia.uni-saarland.de

where xx is either t1, t2, t3, t4, t5, w1, w2, w3 or w4

Deadline for submission: Friday, May 17, 10 am (before the lecture)