# Lecture 22:
# Segmentation II:
# Watersheds and Optimisation Methods

**Contents**



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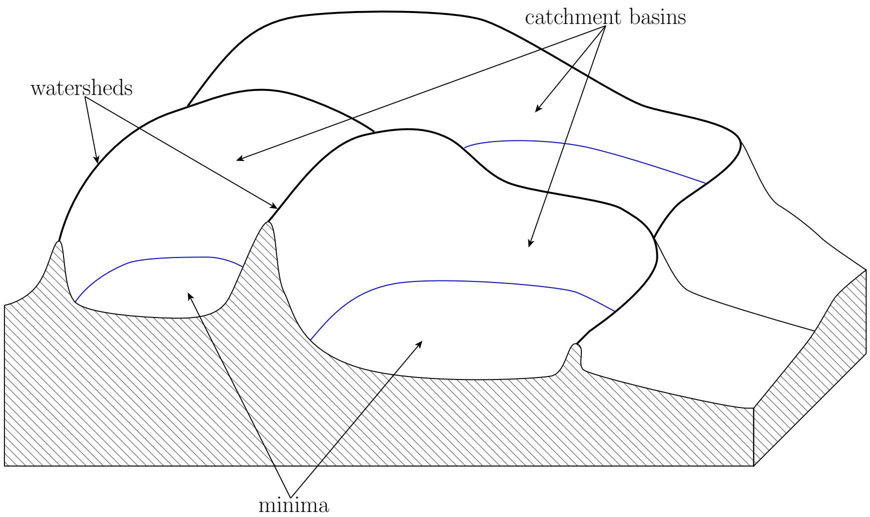
## The Watershed Transformation (Wasserscheidentransformation)




Topographic representation of watersheds, catchment basins, and minima. Each minimum creates a catchment basin. Watersheds are the boundaries between the influence zones of different catchment basins. Author: T. Schneevoigt.

## The Watershed Transformation (2)

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### Basic Idea

- morphological segmentation method, inspired by watersheds in nature
- extracts edges as watersheds in the gradient magnitude image $|\nabla f|$
- Consider $|\nabla f|$ in a topographic representation (landscape with hills and valleys).
- A water drop moves downwards to the valley.
  It chooses the direction of steepest descend.
- Regions where the rain flows to the same valley belong to the same basin (segment).
- boundaries between neighbouring basins: watersheds

## The Watershed Transformation (3)

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A toboggan is highly efficient for sliding downhill. This idea is also useful for image segmentation. Source: `http://www.madisonartshop.com/sledworks-mountain-boggan-4foot-toboggan.html`.

## The Watershed Transformation (4)

### A Toboggan Watershed Algorithm

- Proceed in a certain pixel ordering, e.g. from top left to bottom right.
- Track a toboggan on the image $g(x,y) = |\boldsymbol{\nabla} f(x,y)|$ on its way down to the valley:
  - Choose the most negative derivative of $g$ in the direction of the eight neighbours.
  - Be careful: If the distance to horizontal or vertical neighbours is $h$, it is $\sqrt{2}\,h$ for diagonal neighbours.
- Put all pixel coordinates along the tobbogan's path on a stack.
- Once the toboggan reaches a minimum of the *gradient* image $g = |\boldsymbol{\nabla} f|$ in $(p,q)$, assign the grey value $f(p,q)$ of the *original (!)* image $f$ to all pixels on the stack.
- Creates a piecewise constant image $f$:
  - Every region attains the grey value correponding to its extremum.
  - This image enhancement is also called *toboggan contrast enhancement.*
- Watersheds are extracted as discontinuities in the enhanced image.



## The Watershed Transformation (5)

### How about Efficiency?

- Points of the stack are marked and treated only once.
- If the toboggan reaches a pixel that has been treated already:
  - There is no need to follow its path downhill any further.
  - Assign the grey value of the treated pixel to the untreated pixels along the path.
- Thus, the algorithm is highly efficient: linear complexity in the pixel number.

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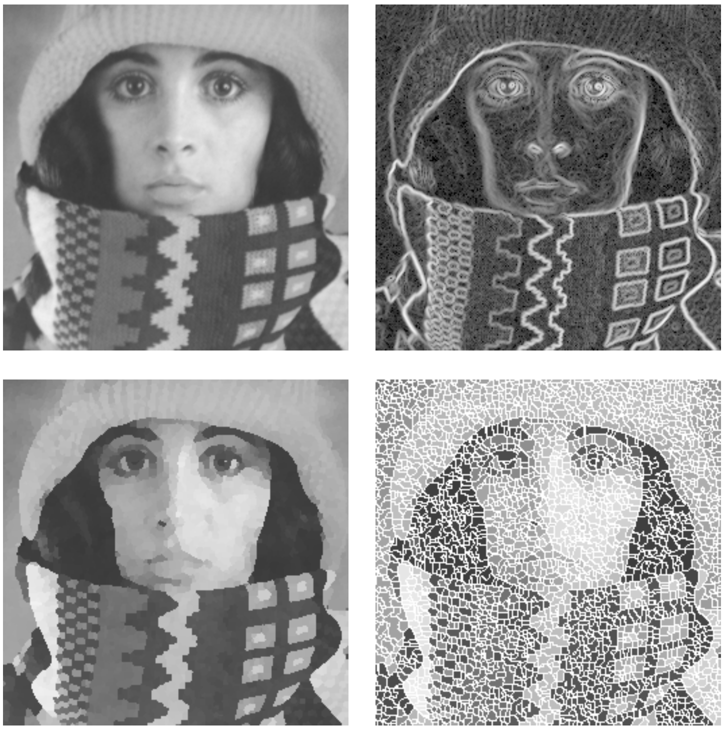
## The Watershed Transformation (6)



**Top left:** Original image. **Top right:** Gradient magnitude, with gamma correction ($\gamma = 3$) for better visibility. **Bottom left:** Toboggan contrast enhancement creates piecewise constant patches. **Bottom right:** Watersheds as boundaries between these patches. Author: J. Weickert.

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## The Watershed Transformation (7)

### Main Drawback of the Watershed Transformation

- Oversegmentation:
  - many small basins due to noise, image quantisation and irrelevant local minima
  - often neighbouring segments have fairly similar grey values

### Remedy

- preprocessing, e.g. by Gaussian convolution: less irrelevant segments
- postprocessing, e.g. by region merging: keeps only high contrast boundaries

However:

- Both steps introduce additional parameters.
- Often it is not sufficient to apply only one of these steps.
- Gaussian convolution may also delocalise structures.
  This can be addressed by downfocusing in scale-space (cf. Lecture 13).

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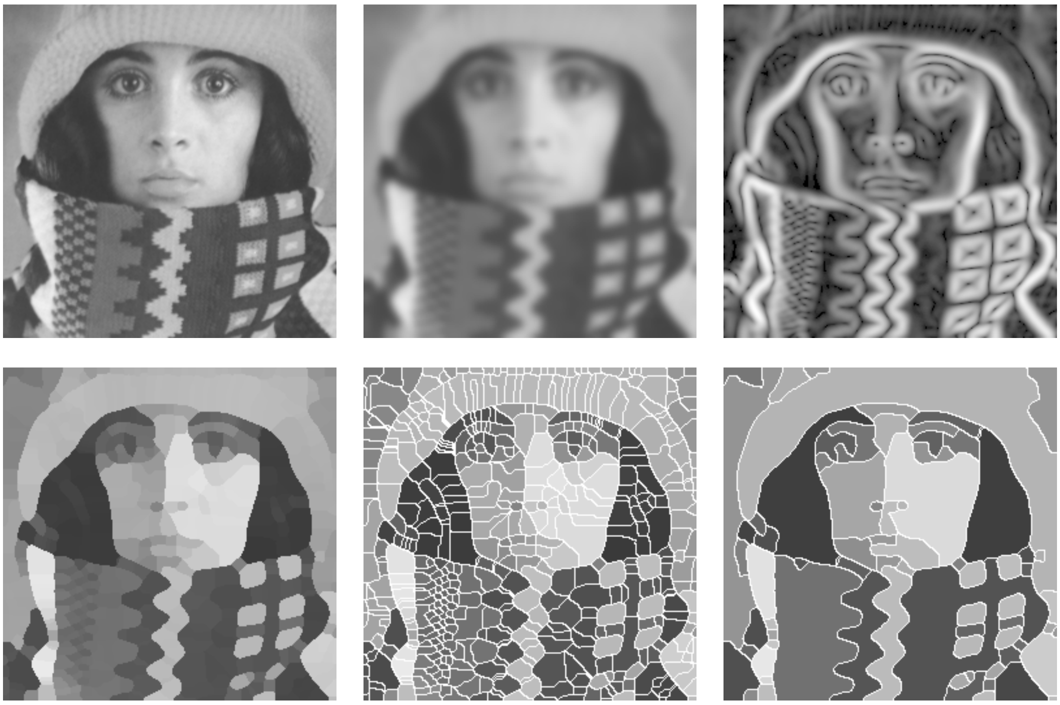
# The Watershed Transformation (8)

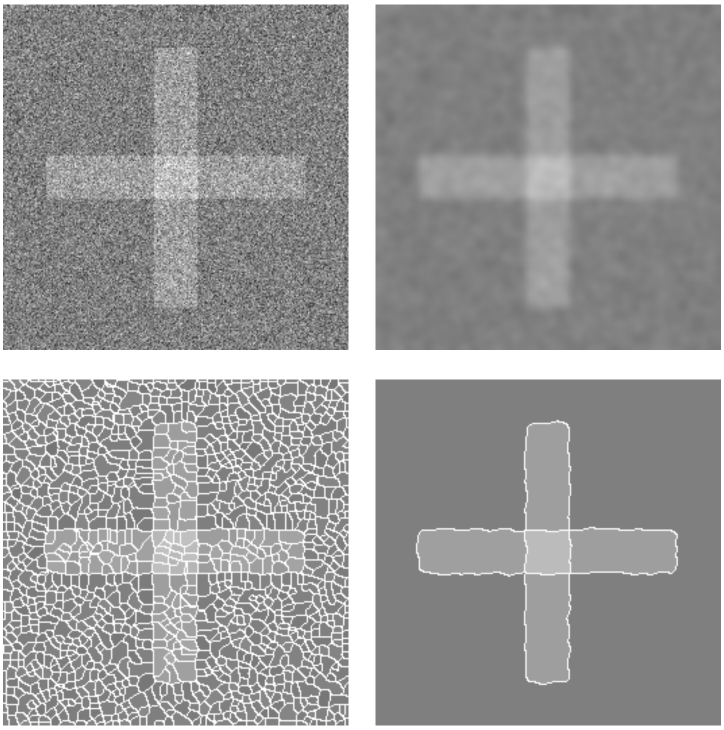




**(a) Top left:** Original image, $256 \times 256$ pixels. **(b) Top middle:** Preprocessing with Gaussian convolution ($\sigma = 3$). **(c) Top right:** Gradient magnitude of (b), with gamma correction ($\gamma = 3$). **(d) Bottom left:** Toboggan contrast enhancement. **(e) Bottom middle:** Watersheds of (d). **(f) Bottom right:** After region merging with threshold $\delta = 12$. Author: J. Weickert.

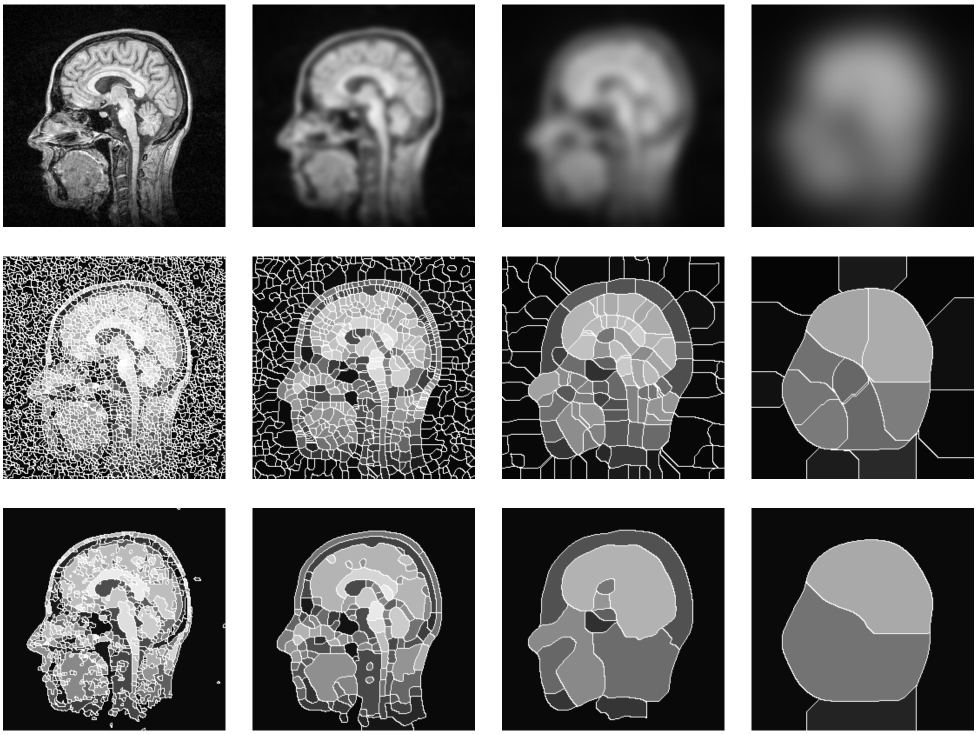
# The Watershed Transformation (9)





**(a) Top left:** Noisy test image, $256 \times 256$ pixels. **(b) Top right:** Preprocessing by Gaussian convolution with $\sigma = 3$. **(c) Bottom left:** Watershed segmentation of (b). **(d) Bottom right:** Postprocessing by region merging with threshold $\delta = 10$. Author: J. Weickert.

## The Watershed Transformation (10)



**Top row:** Preprocessing a $256 \times 256$ image using Gaussian convolution with $\sigma = 0$, $3$, $7$ and $19$. **Middle row:** Watershed segmentation. **Bottom row:** Postprocessing by region merging with thresholds $\delta = 12$, $12$, $25$ and $20$. Author: J. Weickert.



## The Mumford–Shah Cartoon Model (1)

# The Mumford–Shah Cartoon Model

### Disadvantages of Previous Segmentation Methods

- Methods with multiple parameters are burdensome (e.g. double thresholding).
- Methods with no or inappropriate parameters (e.g. contrast) may be too inflexible:
  - insufficient control over the size of segments (e.g. region merging)
  - no control over the number of segments (e.g. plain watersheds)
  - no control over length and smoothness of segment boundaries (e.g. watersheds)
- Avoidance of these effects requires pre-/postprocessing steps and new parameters (e.g. for watersheds).
- All our methods, apart from Otsu thresholding, lack an optimality interpretation. Hence, we have no tool for comparing the quality of different segmentations.

*Is there a conceptionally clean approach that avoids these shortcomings and requires only one single parameter (scale parameter)?*

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## Mumford-Shah Cartoon Model

- Let $f : \Omega \to \mathbb{R}$ be a continuous initial image.
- goal: cartoon-like segmentation $(u, K)$,
  where $u$ is constant in each segment,
  and $K$ represents the set of segment boundaries.
- variational approach: segmentation as minimiser of the energy functional

$$E(u,K) \;=\; \int_{\Omega \setminus K} (u-f)^2 \, dx \, dy \;+\; \alpha \, \ell(K),$$

  where $\ell(K)$ measures the length of the segment boundaries,
  and $\alpha > 0$ is a scale parameter.
- Term 1 controls the homogeneity within a segment.
  Term 2 controls the length of the boundaries.
  Increasing $\alpha$ creates larger segments, but with reduced homogeneity.
- sufficiently complicated to represent basic features of a good segmentation;
  sufficiently simple to allow a thorough mathematical analysis



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## Which Theoretical Properties can be Shown?

- **Preservation of Mean**

  Term 1 implies that within one segment, the constant $u$ is the mean of $f$.
  Thus, given $f$, the segmentation is uniquely determined by the boundary set $K$,
  and we may write $E(K)$ instead of $E(u, K)$.

- **Existence of a Global Minimiser, Regularity of the Boundaries**

  There exists (at least) one segmentation that minimises $E(K)$.
  The segment boundaries are either regular (at least $C^1$),
  or they are singular and of one of the following types:

  - Three edges meet at an angle of 120 degrees.
  - An edge meets the boundary of the image domain (at 90 degrees).

- **Many Local Minimisers**

  The energy landscape may have many local minimisers. There are two options:

  - extremely slow minimisation methods that guarantee to find a global minimum asymptotically *(e.g. simulated annealing).*
  - more efficient approximations that usually find reasonable local minima,
    but cannot guarantee to end up in a global minimum

## Restriction to 2-Normal Segmentations

We want to find good local minima of $E(K)$ in an efficient way.
To this end we consider so-called 2-normal segmentations.

### Definition (2-Normal Segmentation):

A segmentation $K$ is called *2-normal*, if every segmentation $K'$, that results from merging two neighbouring segments of $K$, has a larger energy:

$$E(K') \;>\; E(K).$$

Restricting to 2-normal segmentations allows to prove a number of useful properties.





## What are the Properties of 2-Normal Segmentations ?

### Limitation of the Number of Regions

$$n \;\le\; c_0(\Omega)\,\frac{|\Omega|\,(\sup f - \inf f)^4}{\alpha^2}$$

where $|\Omega|$ denotes the area of the image domain $\Omega$,
and $c_0$ is a known constant that depends only on the geometry of $\Omega$.

This shows:

- The larger the domain $\Omega$, the more segments can be expected.
- High image contrast increases the number of segments dramatically.
- Increasing $\alpha$ leads to less segments.

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- **Elimination of Small Regions**

  The area of each segment $\Omega_i$ is bounded from below by a positive constant:

$$|\Omega_i| \;\geq\; c_1(f, \alpha, \Omega).$$

- **Elimination of Thin Regions**

  The length $\ell(\partial\Omega_i)$ of a segment boundary $\partial\Omega_i$ cannot become arbitrary large compared to the square root of its area:

$$\ell(\partial\Omega_i) \;\leq\; c_2(f, \alpha, \Omega)\, |\Omega_i|^{1/2}.$$

  Since $|\Omega_i| \leq |\Omega|$, there exists an upper limit for the length of a segment boundary.

- **Smoothness of the Segment Boundaries**

  The segment boundaries are $C^1$ almost everywhere.



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## Algorithmic Realisation

### Basic Idea

- approximate global minimiser of cartoon model by local minimiser
- local minimiser results from a restriction to 2-normal segmentations
- create 2-normal segmentations by region merging
- enforce further region merging by successively increasing the scale parameter $\alpha$

## Useful Observation

- Merging two segments $\Omega_i$, $\Omega_j$ with means $u_i$, $u_j$ does not require to recompute the entire energy from scratch.
- You can show (Homework H11, Problem 2) that the new energy is given by

$$E(K \setminus \partial(\Omega_i, \Omega_j)) \;=\; E(K) \;+\; \frac{|\Omega_i| \cdot |\Omega_j|}{|\Omega_i| + |\Omega_j|}\,(u_i - u_j)^2 - \alpha\, \ell(\partial(\Omega_i, \Omega_j))$$

  where $\partial(\Omega_i, \Omega_j)$ is the joint border between $\Omega_i$ and $\Omega_j$.
- The blue term reduces the energy due to the smaller boundary length. The red term increases the energy due to higher fluctuations in $\Omega_i \cup \Omega_j$.
- Thus, merging $\Omega_i$ and $\Omega_j$ decreases the energy if

$$\alpha \;\geq\; \frac{|\Omega_i| \cdot |\Omega_j|}{|\Omega_i| + |\Omega_j|}\;\frac{(u_i - u_j)^2}{\ell(\partial(\Omega_i, \Omega_j))}\,.$$



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## Implementation as Region Merging Algorithm

1. Initialisation: Each pixel is one segment.
2. For each pair of neighbouring pixels:
   Compute the $\alpha$ value, for which a merging would decrease the energy.
3. Merge the pair with the smallest $\alpha$ value.
4. Compute the $\alpha$ values for merging the new segment with its neighbouring segments.
5. Repeat the Steps 3 and 4, until a desired number of regions or a specified value for $\alpha$ is reached.

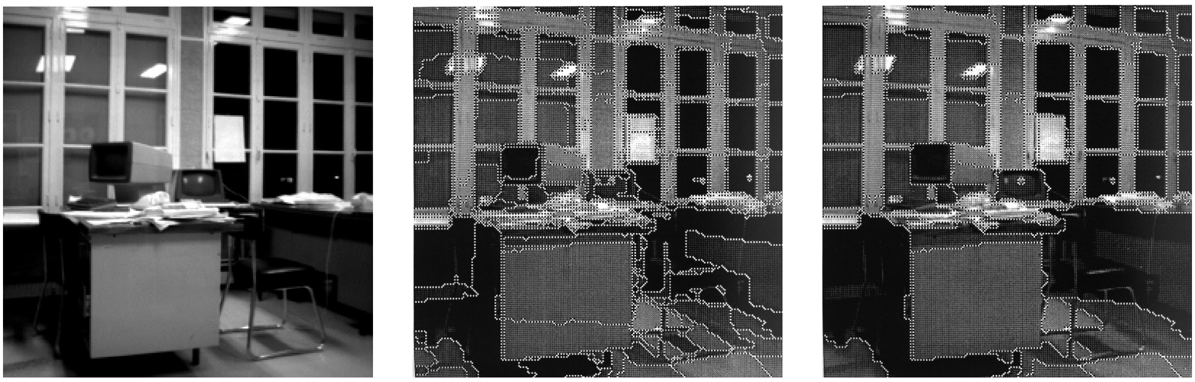
## Important Properties

- *Fixed Point Property:*
  For a piecewise constant image there exists some $\alpha_0$ such that it is a valid segmentation for all $\alpha < \alpha_0$.
- *Pyramid Property, Causality:*
  For $\alpha_2 > \alpha_1$, the $\alpha_2$-boundaries are a subset of the $\alpha_1$-boundaries.

## The Mumford–Shah Cartoon Model (10)



### Experiments



original image — segmentation, $\alpha = 1024$ — segmentation, $\alpha = 4096$

Influence of the parameter $\alpha$ on the segmentation result of the Mumford–Shah cartoon model. Authors: G. Koepfler, J.-M. Morel, S. Solimini.

## The Mumford–Shah Cartoon Model (11)

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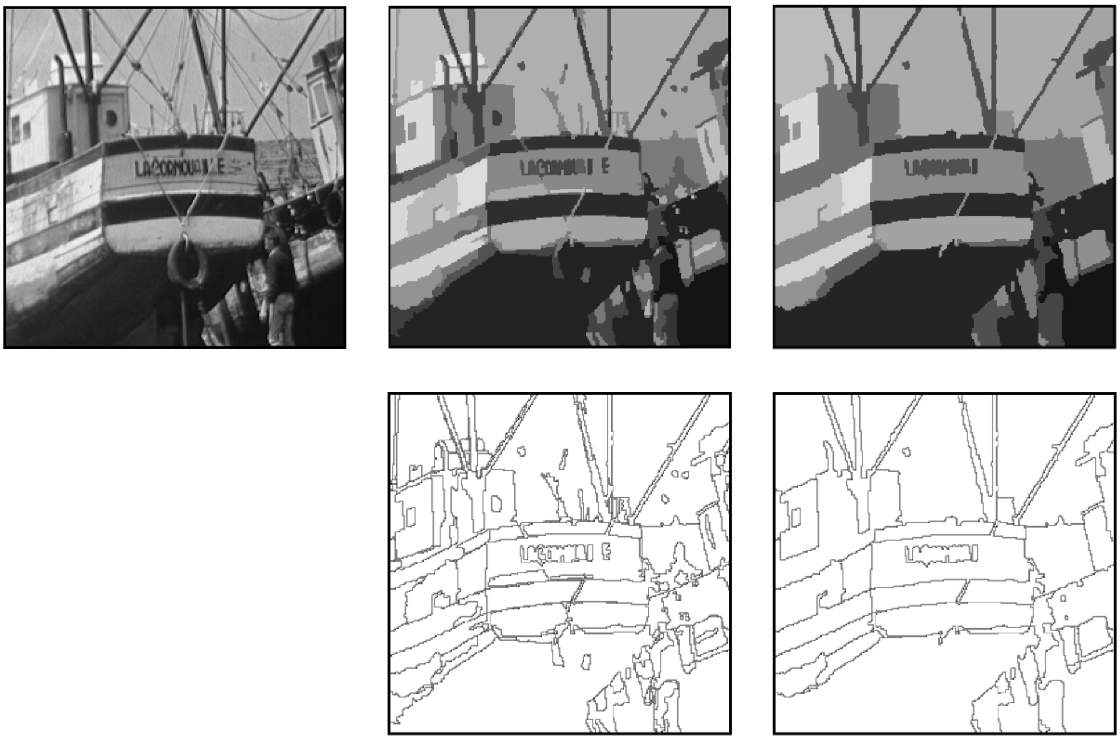
original image — segmentation, $\alpha = 2022$ — segmentation, $\alpha = 6173$



boundaries, $\alpha = 2022$ — boundaries, $\alpha = 6173$

Influence of the parameter $\alpha$. Author: G. Koepfler.

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# The Active Contour Model of Chan and Vese

- Chan and Vese considered the Mumford-Shah cartoon model under the restriction that the image is segmented into two phases only.

- The energy functional then becomes a function of a contour $C$:

$$E(C) \;=\; \int_{\text{inside } C} (f(x,y)-u_1)^2\,dx\,dy \;+\; \int_{\text{outside } C} (f(x,y)-u_2)^2\,dx\,dy \;+\; \alpha\,\ell(C)$$

  where $u_1$ and $u_2$ are the arithmetic means of $f(x,y)$ inside/outside $C$.

- One specifies some initial contour $C_0$ close to the segmentation one wants to find.

- Minimising $E(C)$ drives the curve towards the segment boundaries.
  (This requires e.g. so-called level set methods that even allow a contour to split.
  We omit the technical details here.)



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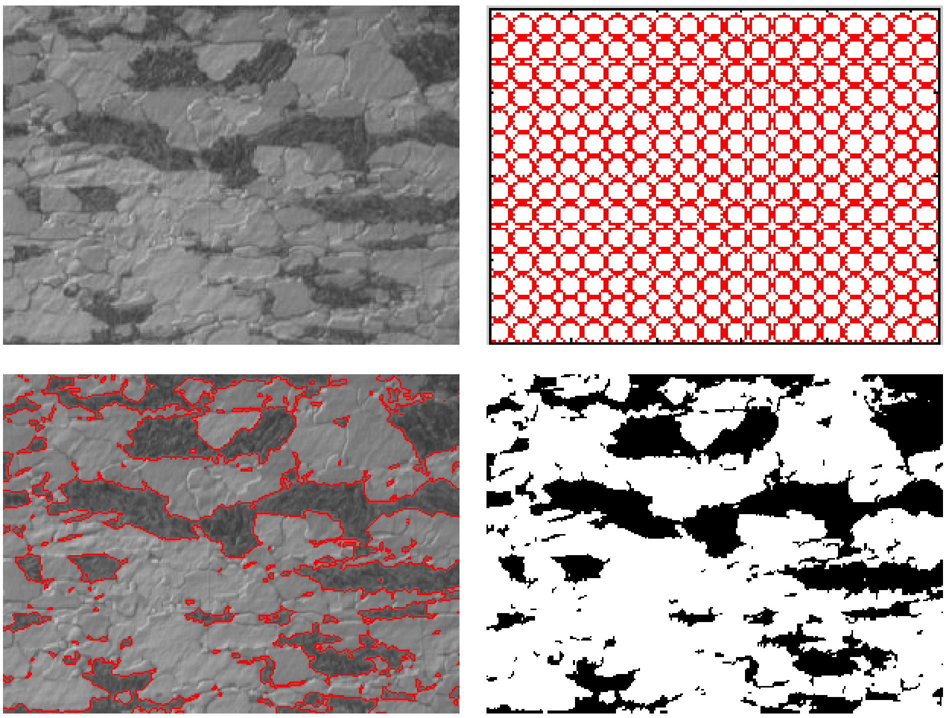
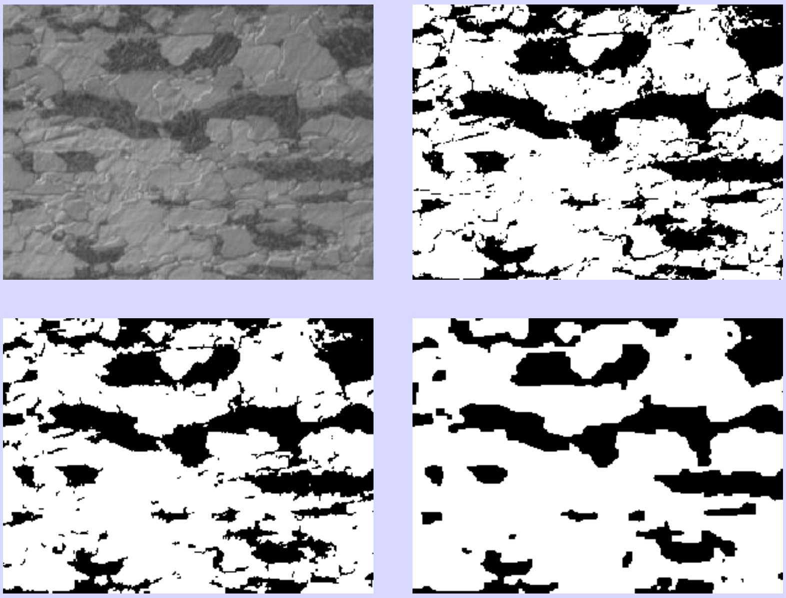


**Top left:** Metal surface ($300 \times 223$ pixel). **Top right:** Initialisation with many small curves. **Bottom left:** Chan-Vese segmentation with $\alpha = 0.1$. **Bottom right:** Corresponding two phases. The interior phase is depicted in black. Author: A. Chouikhi.

## The Active Contour Model of Chan and Vese (3)

metal surface, $300 \times 223$ Otsu thresholding, $T = 111$

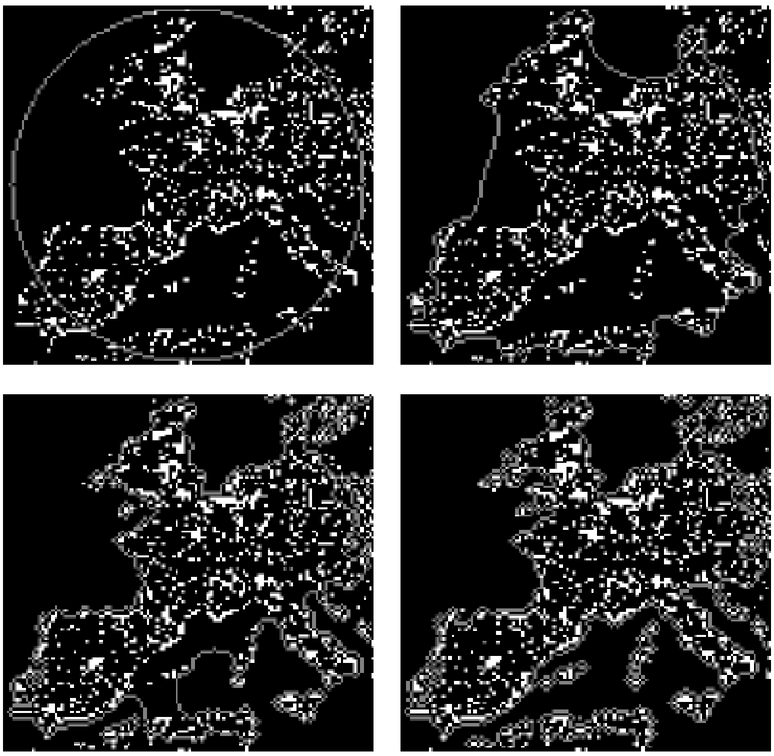


Chan–Vese, $\alpha = 0.1$ Chan–Vese, $\alpha = 0.6$

The Chan-Vese model allows a better control of the length of the segmentation boundaries than classical thresholding. This gives higher robustness under noise and small-scale perturbations. Authors: A. Chouikhi, J. Weickert.



## The Active Contour Model of Chan and Vese (4)



**Top left to bottom right:** Evolution of an active contour under the Chan-Vese model. The contour splits and creates a fairly good segmentation of Europe at night. Authors: T. Chan, L. Vese.

## Extensions of the Chan–Vese Model

- The Chan–Vese model has been modified in many ways:
  - more sophisticated features than grey values (colour channels, texture descriptors, motion information)
  - additional statistical characterisations of a region (not only mean, but also standard deviation, ...)
  - multiphase models that create more than two types of segments
  - sometimes even a-priori knowledge using a statistical characterisation of the shapes to be expected (Lecture 29)
- can yield excellent segmentation results

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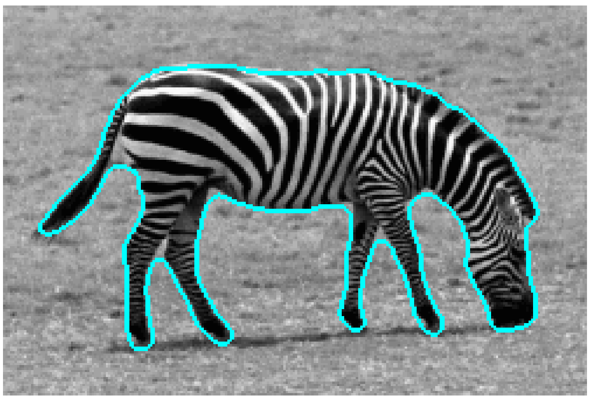

**Left:** Segmented image using grey values and texture descriptors as features. **Right:** Segmentation with colour and texture descriptors as feature channels. Author: T. Brox.

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# Summary

- The watershed transform is a morphological edge based segmentation.
  It suffers from oversegmentation.
  To get useful results, some pre- and postprocessing is recommendable.
- The cartoon model of Mumford and Shah is an energy based segmentation method.
- combines region-based and edge-based segmentation ideas:
  – rewards homogeneity within each region
  – encourages small length of segmentation boundaries
- requires only a single, intuitive parameter that is related to scale
- relatively simple and mathematically well understood
- restriction to 2-normal segmentations automatically generates many desirable features that have to be engineered in an ad hoc way for most other methods: The segments are neither too many, too small, too thin, nor do they have too irregular boundaries.
- algorithmic realisation via region merging
- Active contour variants exist that allow interactive segmentation of high quality.





# References


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  *(toboggan watershed algorithm)*
- L. Vincent, P. Soille: Watersheds in digital spaces: An efficient algorithm based on immersion simulations. *IEEE Transactions on Pattern Analysis and Machine Intelligence*, Vol. 13, pp. 583–598, 1991.
  *(popular alternative to the toboggan watershed algorithm)*
- O. F. Olsen: Multi-scale watershed transformation. In J. Sporring, M. Nielsen, L. Florack, P. Johansen (Eds.): *Gaussian Scale-Space Theory*. Kluwer, Dordrecht, 191–200, 1997.
  *(watershed segmentation in Gaussian scale-space)*
- J.-M. Morel, S. Solimini: Segmentation of images by variational methods: a constructive approach. *Revista Matematica de la Universidad Complutense de Madrid*, Vol. 1, No. 1–3, pp. 169–182, 1988.
  *(theoretical analysis of the cartoon model)*
- G. Koepfler, C. Lopez, J.-M. Morel: A multiscale algorithm for image segmentation by variational method. *SIAM Journal on Numerical Analysis*, Vol. 31, pp. 282–299, 1994.
  *(2-normal segmentation by region merging)*

## References (2)


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  *(an entire book on Mumford-Shah segmentation)*
- D. Mumford, J. Shah: Boundary detection by minimizing functionals, I. In *Proc. IEEE Computer Society Conference on Computer Vision and Pattern Recognition* (San Francisco, CA, June 1985), pp. 22–26, 1985.
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  *(introduced the Mumford-Shah functional)*
- D. Mumford, J. Shah: Optimal approximation by piecewise smooth functions and associated variational problems. *Communications in Pure and Applied Mathematics*, Vol. 42, pp. 577–685, 1989.
  (`http://www.math.neu.edu/~shah/publications.html`)
  *(very detailed mathematical analysis of the Mumford-Shah model)*
- T. F. Chan, L. A. Vese: Active contours without edges. *IEEE Transactions on Image Processing*, Vol. 10, No. 2, pp. 266–277, Febr. 2001.
  (`http://www.math.ucla.edu/~lvese/Publications.html`)
  *(Chan–Vese model)*
- T. Brox, M. Rousson, R. Deriche, J. Weickert: Unsupervised segmentation incorporating colour, texture, and motion. In N. Petkov, M. A. Westenberg: *Computer Analysis of Images and Patterns*. Springer LNCS Vol. 2756, pp. 353–360, Berlin, 2003.
  (`http://www.cs.berkeley.edu/~brox/publication_all.html`)
  *(extensions of the Chan–Vese model)*




## Assignment C11

# Assignment C11 – Classroom Work

**Problem 1 (Otsu's Threshold Selection Method)**

Using the definitions from Lecture 21, show that the between class variance

$$\sigma_B^2(T) \;:=\; \omega(T)\,(\mu_0(T)-\mu_{\text{tot}})^2 \;+\; (1-\omega(T))\,(\mu_1(T)-\mu_{\text{tot}})^2$$

can be computed by

$$\sigma_B^2(T) \;=\; \frac{(\mu_{\text{tot}}\,\omega(T)-\mu(T))^2}{\omega(T)\,(1-\omega(T))}\,.$$

**Problem 2 (Toboggan Watershed Algorithm)**

Use the Toboggan Watershed Algorithm to create a segmentation of the following discrete signal:

$$\boldsymbol{f} = (2,7,1,7,7,4,3,0,4,6,1,2,0,5,2,3)^\top\ .$$

Assume that the signal is mirrored at the boundaries and use forward differences with $h=1$ to approximate derivatives.


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# Assignment H11 – Homework

**Problem 1 (Segmentation Methods)** (2+2+2 points)

In this problem, segmentation is performed on the following discrete signal:

$$\boldsymbol{f} = (6, 4, 6, 2, 4, 5, 6, 1, 3, 3, \underline{5}, 5, 2, 4, 4, 7)^\top \; .$$

(a) Write down the stack of signals arising from region growing for segmentation of $\boldsymbol{f}$ and increase the threshold $T$ from its minimal value in steps of 1 to its maximal value. Use the underlined component of $\boldsymbol{f}$ as a seed. Mark regions by underlines.

(b) Perform the same task as in (a) with region merging instead of region growing.

(c) Use double thresholding to segment $\boldsymbol{f}$. Employ an upper threshold $T_2$ corresponding to the $75\%$ quantile, and a lower threshold $T_1$ resulting from the $50\%$ quantile.

**Problem 2 (Mumford–Shah Cartoon Model)** (8 points)

Let $\Omega_i, \Omega_j \subset \Omega$ denote two disjoint segments with mean $u_i$ resp. $u_j$. Furthermore, let $\partial(\Omega_i, \Omega_j)$ denote the common boundary between $\Omega_i$ and $\Omega_j$.
Show that merging $\Omega_i$ and $\Omega_j$ with the Mumford–Shah cartoon model changes the energy as follows:

$$E(K \setminus \partial(\Omega_i, \Omega_j)) - E(K) \;=\; \frac{|\Omega_i| \cdot |\Omega_j|}{|\Omega_i| + |\Omega_j|}\,(u_i - u_j)^2 - \alpha\, \ell(\partial(\Omega_i, \Omega_j)).$$

*Hint*: Determine the new value of the merged segment $\Omega_i \cup \Omega_j$.



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**Problem 3 (Region Merging, Toboggan Watershed Segmentation)** (4+4+2 points)

Please download the required files from the webpage

`http://www.mia.uni-saarland.de/Teaching/ipcv19.shtml`

into your own directory. You can unpack them with the command `tar xvzf Ex11.tar.gz`.

(a) The file `regionmerging.c` contains code which is supposed to perform region merging. Supplement the missing code and compile it with

`gcc -O2 -o regionmerging regionmerging.c -lm` ,

Apply the program to the images `house.pgm` and `trui.pgm` using different values for the threshold $T$. What are your findings?

(b) The file `watershed.c` contains an almost complete implementation of the Toboggan watershed segmentation. Supplement the missing code and compile it with

`gcc -O2 -o watershed watershed.c -lm` ,

Apply the program to the images `house.pgm` and `trui.pgm`. How do your results change with different values for $\sigma$? What happens for $\sigma = 0$?

(c) Use the program `regionmerging` from (a) to improve the results obtained in (b).

# Assignment H11 (3)

**Submission**

The theoretical Problems 1 and 2 should be submitted in handwritten form before the lecture. For the practical Problem 3, please submit the files as follows: Rename the main directory Ex11 to Ex11_<your_name> and use the command

```
tar czvf Ex11_<your_name>.tar.gz Ex11_<your_name>
```

to pack the data. The directory that you pack and submit should contain the following files:

- for 3(a): the supplemented source code and the filtered results for each of the given images using two different thresholds $T$;
- for 3(b): the supplemented source code and the segmented images with $\sigma = 0$ and one arbitrary value for $\sigma$;
- for 3(c): one improved segmentation per given image;
- a text file README that states the used parameters, that answers the question of problem 3 and that contains information on all people working together for this assignment.

Please make sure that only your final version of the programs and images are included.
Submit the file via e-mail to your tutor via the address:

```
ipcv-xx@mia.uni-saarland.de
```

where xx is either t1, t2, t3, t4, t5, w1, w2, w3 or w4 depending on your tutorial group.

**Deadline for submission:** Friday, June 28, 10 am (before the lecture)

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