Image Processing and Computer Vision (IPCV)

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Example Solutions for Classroom Assignment 3 (C3)

Problem 1: Intepretation of Fourier spectrum

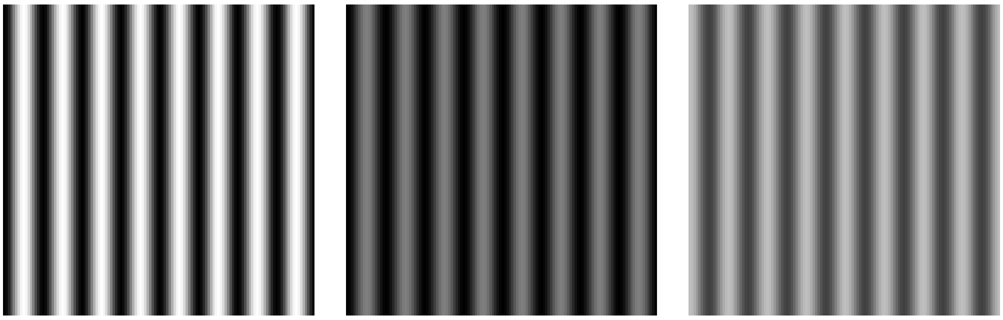
(a) Our image contains a texture consisting of parallel sine waves propagating in horizontal direction.

Taking a look at the grey value representation of the logarithmically rescaled Fourier spectrum, one observes a three point spectrum. One of the three points is located in the centre of the spectral image. Since the low frequencies have been shifted to the middle of the image, this point corresponds to frequency zero, i.e. the rescaled average grey value. The other two points, located at positions symmetric to the centre, correspond to one single frequency namely that of the "wave" that fills the image.

A frequency here is a vector, having a direction and a norm. The direction corresponds to the "front propagation" direction of the wave (and is therefore perpendicular to the direction of the stripes) while the norm (distance to the centre) expresses the density of the periodic stripes or the wavelenght of the sine.

There are two explanations why the points come in pairs. First, one verifies using the formula of the DFT that a real-valued image must have a point-symmetric Fourier spectrum (Fourier coefficients at symmetric positions are complex conjugates of each other). Second, it is impossible to distinguish frequencies which are equal up to the sign.

(b) If the frequencies obtained by the DFT are all halved one obtains the same sine wave structures as in the original image, but with all values rescaled by half. The reason for this is that the lowest frequency corresponds to the average grey value and influences therefore the whole image if altered. If the lowest frequency is unchanged, and we only halve all the other frequency coefficients, we obtain an image with the same texture and average grey value. What changed is the fact that the range of the grey values shrinks by half. This concludes that the coefficients of the frequency describe the amplitude of the frequency.



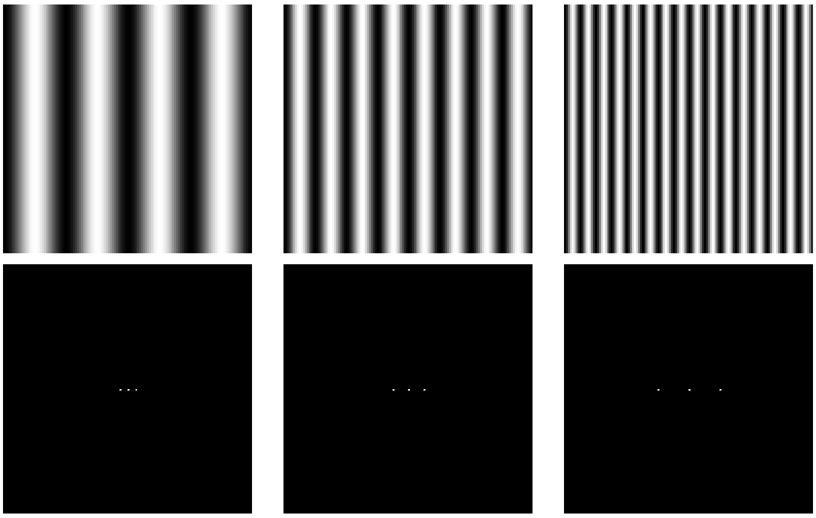
b) Intensity values of image. *Left:* original image with values in $[0, 255]$. *Middle:* image after all frequencies were halved with values in $[0, 127]$. *Right:* image after all but the lowest frequency were halved with values in $[64, 191]$.

(c) The distance between the point at the centre and the other points describes the frequency of the sine wave or the period of the wave respectively. If we move the points further away, we move to a higher frequency such that the sine pattern repeats more often. On the other hand, we can decrease the frequency of the sine pattern if we move the points more towards the centre.

(d) A translation of the sine wave doesn't influence the Fourier spectrum, as only the magnitude of the Fourier coefficients is presented in there.

(e) If we rotate the image we change the the propagation direction of the sine wave. The ordering of the three-point spectrum behaves accordingly. Nonetheless, boundary artifacts can arise if we rotate the images. The reason is the implicitly assumed periodicity in both space and frequency domain. By rotating the image, a phase jump can occur at the boundaries which leads to artifacts.

(f) Like we showed in homework 2 for the continuous Fourier transfrom, the discrete Fourier transfrom of a Gaussian gives us a Gaussian-like function with reciprocal variance.



c) Spacing in Fourier spectrum. *Top Row:* different spacing of single frequency in Fourier spectrum. *Bottom Row:* respective sine pattern. Shorter distances between points corresponds to lower frequencies.

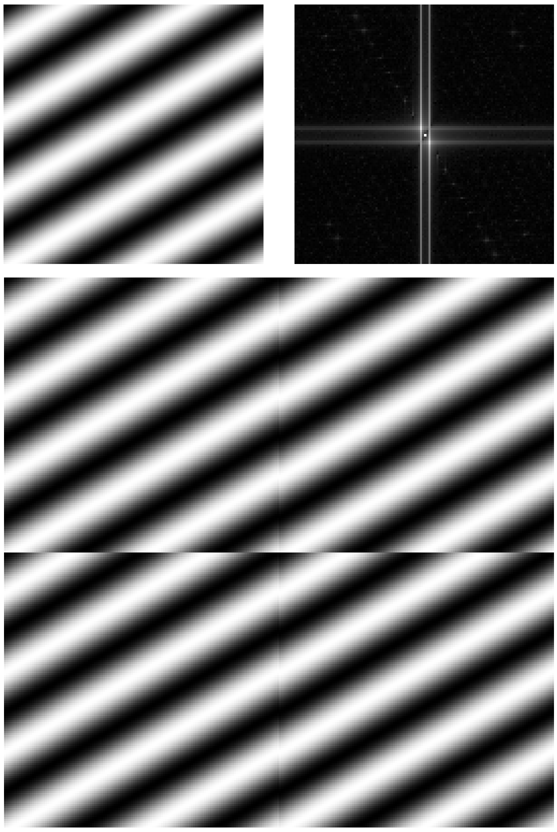
(g) The lines in `tile.pgm` induce in the Fourier spectrum visible beams starting off in the centre and directed perpendicular to the corresponding lines.

The reason is that a single edge in the spatial domain is represented by a superposition of wave-like patterns of different frequencies but equal direction. The phases and amplitudes of these waves are adjusted such that their slopes add up to give the edge at the specified location but cancel elsewhere.

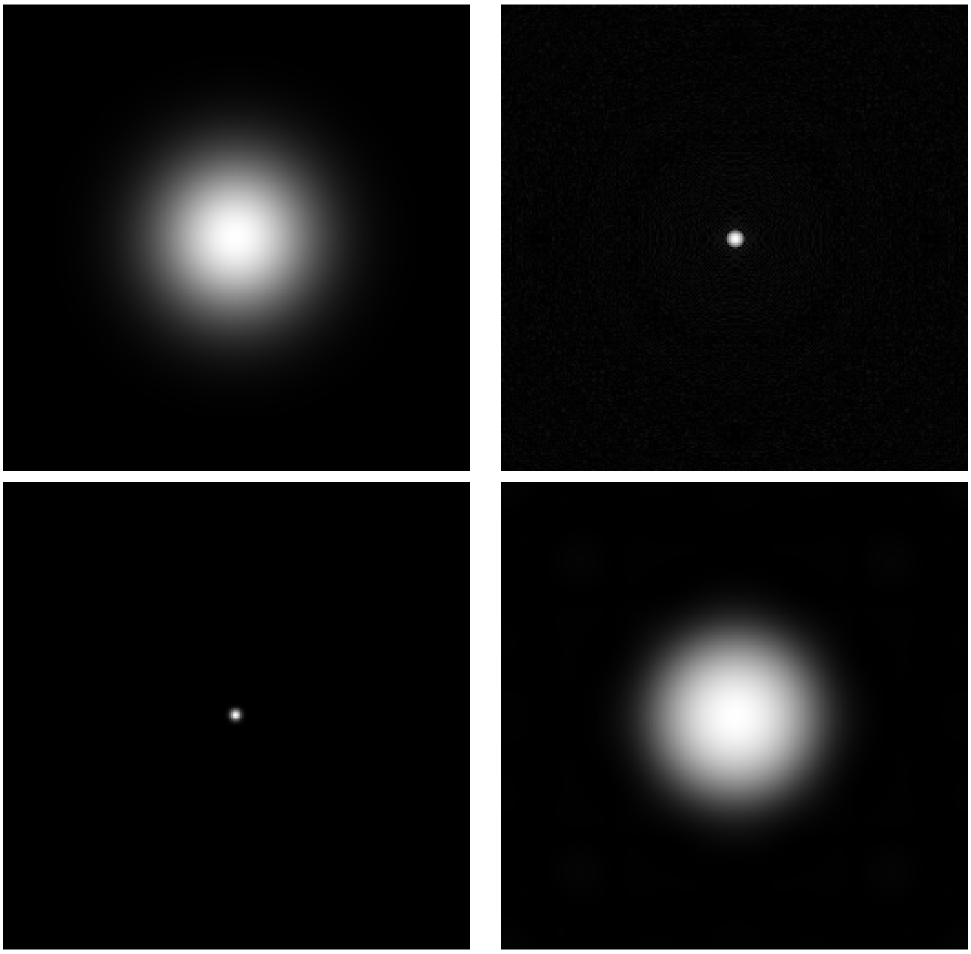
A close look at the spectrum of `tile.pgm` reveals that there are beams that do not go out radially from the centre but from the image boundary. These are traces of aliasing effects.

Remembering that images and also Fourier spectra are treated periodically by the DFT we see that some of the radial lines extending from the centre do not end at the image boundaries but are prolongated beyond that boundary, wrapping around to the opposite image boundaries. Translated into frequencies: These lines depict high frequencies which don't fit in our Fourier spectrum but are represented in it by lower frequencies. This is aliasing.

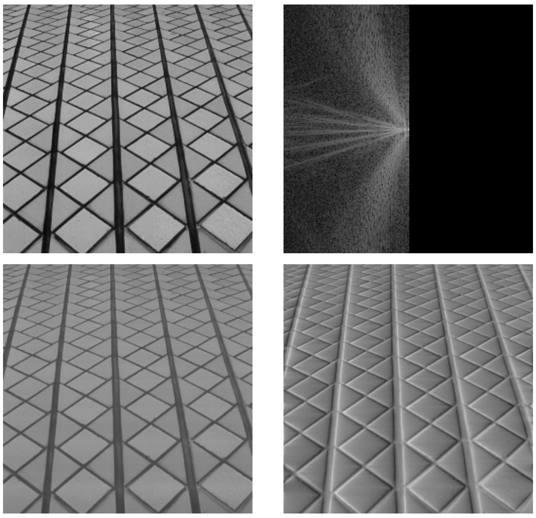
(h) If we break the symmetry of the Fourier spectrum we will get a non-zero imaginary part in the spatial domain.



e). *Left:* rotated image. *Right:* Fourier spectrum with boundary artifacts. *Bottom: period extended images with discontinuity at boundary*



f). *Top left:* original image. *Top right:* original Fourier spectrum. *Bottom left:* Gaussian with smaller variance. *Bottom right:* Fourier spectrum with larger variance



h). **Top Left:** Downsampled Image of tiles. **Top Right:** The corresponding Fourier spectrum without symmetry. **Bottom Left:** Real part of the back transformed image. **Bottom Right:** Imaginary part of the back transformed image (shifted to have 0 at grey value 127).