Lecture 28:
Object Recognition I:
Hough Transform and Invariants

Contents

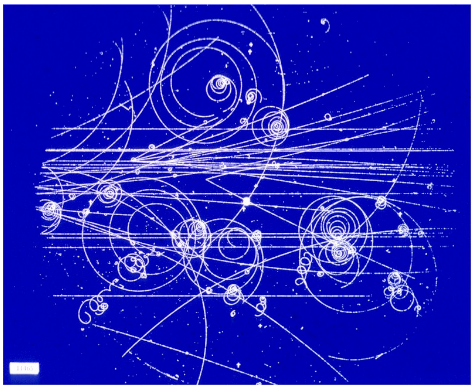
Introduction

- Object recognition is crucial for image understanding.
- Often one knows the object classes that can be expected, e.g.
  - simple geometric primitives such as lines, circles, ellipses
  - translated and scaled versions of certain basic patterns
    (example: optical character recognition (OCR))
- Object recognition should also be robust under varying illumination conditions.
- Which tools can be used in order to address these problems?

The Hough Transform

Historical Origin

- method for line detection, proposed by Hough in a 1962 patent
- used for analysing bubble chamber images from high energy physics



Particle tracks in a hydrogen bubble chamber. A pion particle has interacted with a proton that created a number of new particles. Their motion in the magnetic field depends on their energy and their charge. Source: `http://www.particlephysics.ac.uk`

M I A 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25 26 27 28 29 30 31 32 33

Basic Idea

- **Goal:**
  - detect simple geometric objects that can be represented by a few parameters (e.g. lines, circles)
- **Assumptions:**
  - Object boundaries can be detected by large gradient magnitudes: $|\boldsymbol{\nabla} f| \geq T$.
  - The object boundary satisfies $g(x, y, p_1, ..., p_m) = 0$.
    Here p_1,...,p_m are the parameters we want to determine.
- **Examples:**
  - A line is represented by a normal vector $\boldsymbol{n} = (\cos\phi, \sin\phi)^\top$ and a number d:
    $x\cos\phi + y\sin\phi - d = 0$.
    Two parameters: $\phi \in (-\frac{\pi}{2}, \frac{\pi}{2}]$, $d \in \mathbb{R}$.
    (In contrast to the formula $y = mx + b$, this also allows vertical lines.)
  - Circle with centre (a, b) and radius r: $(x-a)^2 + (y-b)^2 - r^2 = 0$.
    Three parameters: $a, b \in \mathbb{R}$, $r \in (0, \infty)$.

M I A 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25 26 27 28 29 30 31 32 33

Hough Transform (3)

- **Voting Strategy:**
  - Every significant contour point (where $|\nabla f| \geq T$) votes for all parameters p_1,...,p_m that allow it to satisfy the equation $g(x, y, p_1, ..., p_m) = 0$.
  - The majority rules:
    Assume that a specific parameter setting $(p_1, ..., p_m)$ gets the most votes.
    Then it describes the most likely contour.

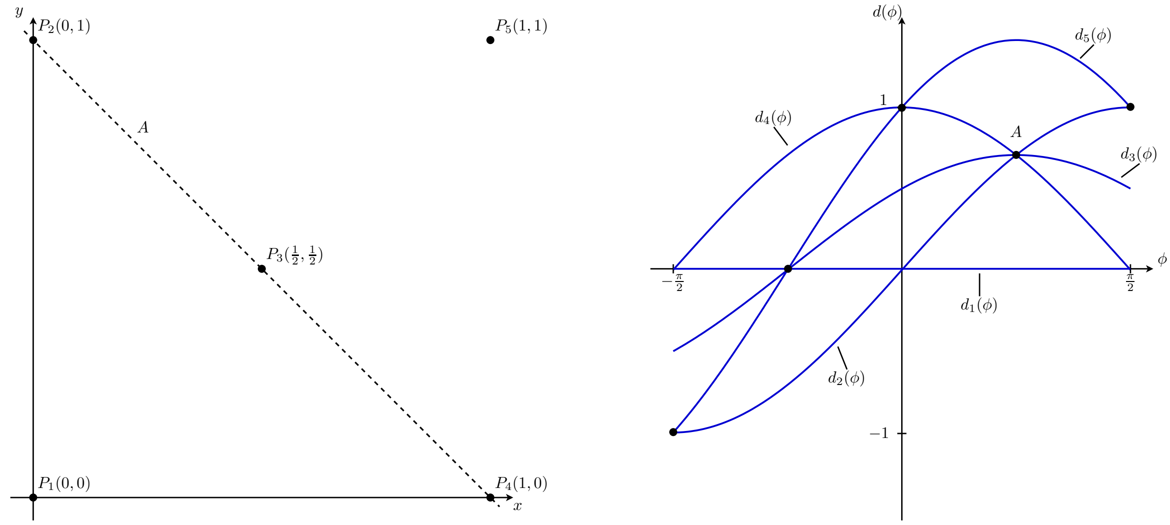
Hough Transform (4)

MIA 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25 26 27 28 29 30 31 32 33

The Hough Transform for Lines

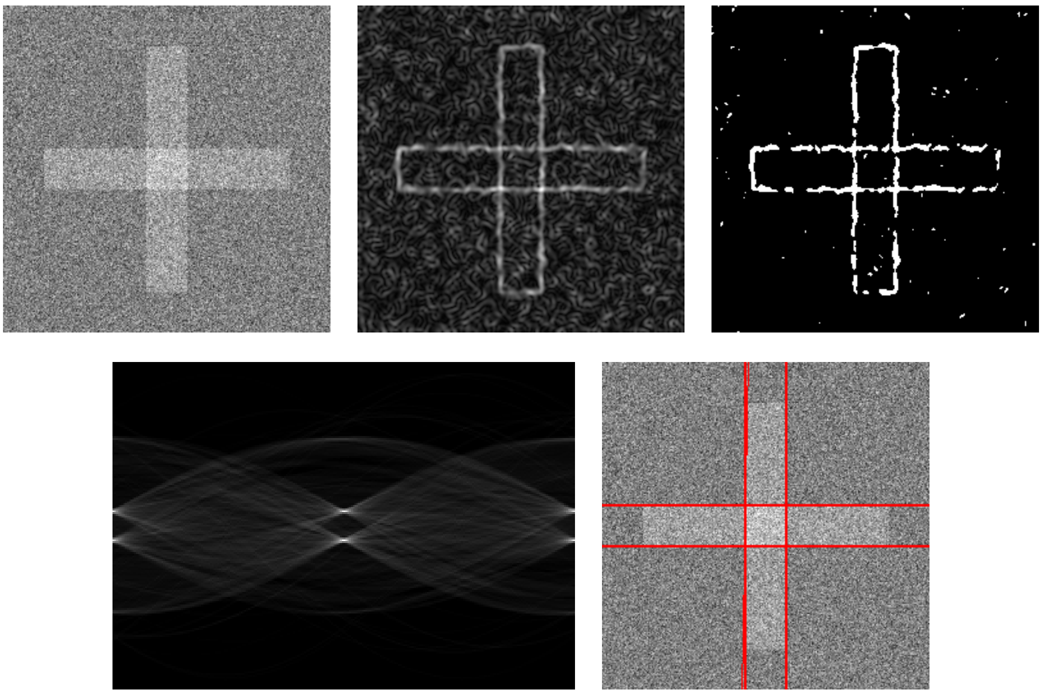
- based on the equation $x \cos\phi + y \sin\phi - d = 0$ with $\phi \in (-\frac{\pi}{2}, \frac{\pi}{2}]$ and $d \in \mathbb{R}$
- Discretise the two-dimensional parameter space *(Hough space)* (ϕ, d).
- Initialise each cell with 0 votes.
- For every pixel (x_i, y_i) with $|\nabla f(x_i, y_i)| \geq T$:
  Increment the counter of all compatible parameter cells (ϕ, d) by 1.
- What does this mean in Hough space?
  - A point (x_i, y_i) lies on a line with parameters (ϕ, d), if it satisfies
    $$x_i \cos\phi \; + \; y_i \sin\phi - d \; = \; 0\,.$$
  - In the Hough space (ϕ, d), this can be seen as a trigonometric curve
    $$d_i(\phi) \; = \; x_i \cos\phi \; + \; y_i \sin\phi\,.$$
  - Thus, the Hough transform for lines maps points to trigonometric curves.
- What happens if several points belong to the same line?
  Then their trigonometric curves intersect at the parameters (ϕ, d) of this line.

Hough Transform (5)



Left: Five points $P_i(x_i, y_i)$ with $i = 1,\ldots,5$. **Right:** Their curves $d_i(\phi) = x_i \cos\phi + y_i \sin\phi$ in the Hough space (ϕ, d). Note that the curves to the points P_2, P_3 and P_4 intersect in one point A in (ϕ, d)-space. This shows that they belong to the same line. Since A has the coordinates $\phi = \frac{\pi}{4}$ and $d = d_2(\frac{\pi}{4}) = \sin(\frac{\pi}{4}) = \frac{1}{2}\sqrt{2}$, this line is given by $x \cos\frac{\pi}{4} + y \sin\frac{\pi}{4} - \frac{1}{2}\sqrt{2} = 0$. This can be simplified to $y = 1 - x$. Author: T. Schneevoigt.

Hough Transform (6)



Top left: Original image, 256×256 pixels. **Top middle:** Gradient magnitude of its Gaussian-smoothed version ($\sigma = 2$). **Top right:** Thresholded gradient magnitude ($T = 5$). **Bottom left:** Representation in (ϕ, d)-space with resolution 256×181 pixels. **Bottom right:** The most significant lines (with at least 280 votes). Author: J. Weickert.

Hough Transform (7)

The Hough Transform for Circles

- based on the equation $(x-a)^2+(y-b)^2-r^2=0$
- Hough space given by the three-dimensional parameter space (a, b, r)
- Discretise the parameter space for a, b and r.
- For every pixel (x_i, y_i) with $|\nabla f(x_i, y_i)| \geq T$:
  - Increment the counter of all compatible cells (a, b, r) by 1.
- The cells with the most votes give the desired parameters for the circles.

Hough Transform (8)

Advantages of the Hough Transform

- no need for a fully connected contour:
  Object contours may even be interrupted or partially occluded.
- extremely robust:
  A simple majority of votes is sufficient, and everything is fully automised.

Main Disadvantage of the Hough Transform

- Memory requirements and computational effort increase rapidly with the number of parameters.
- Coarse-to-fine pyramid-like approaches can help to address this problem.

Important Application

- lane detection in driver assistant systems uses Hough transform for lines

M I A
1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25 26 27 28 29 30 31 32 33

M I A
1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25 26 27 28 29 30 31 32 33

Moment Invariants (Momenteninvarianten)

Goal

- We want to check if a (binary or greyscale) 2-D image object is identical to a 2-D object in another image.
- The second object may have undergone several transformations such as translations, scalings and rotations.
- Potential applications include
  - optical character recognition (OCR)
  - recognition of workpieces (Werkzeugteile)
- Are there certain expressions that are invariant under such transformations?
- Such invariances would be very useful for object recognition.

M I A
1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25 26 27 28 29 30 31 32 33

Moments

- For a continuous image $f : \mathbb{R}^2 \to \mathbb{R}$ one can define the *moment of order $p+q$* by

$$m_{p,q} := \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} x^p \, y^q \, f(x,y) \, dx \, dy \qquad (p,q = 0,1,2,...).$$

- *Uniqueness Theorem for Moments (Papoulis 1991):*
  - If $f(x,y)$ is piecewise continuous and is vanishing outside a bounded subdomain of $\mathbb{R}^2$, then all moments do exist.
  - The sequence of moments $(m_{p,q})$ is not only uniquely determined by $f(x,y)$, but even the reverse holds: If the moments $(m_{p,q})$ satisfy a certain growth condition, then they uniquely characterise the image $f(x,y)$.

Moment Invariants (3)

- For a discrete image $(f_{i,j})$ with $i \in \{1, ..., N\}$ and $j \in \{1, ..., M\}$ one defines the moments in a similar way:

$$m_{p,q} \ := \sum_{i=1}^{N} \sum_{j=1}^{M} i^p \, j^q \, f_{i,j}$$

with $p = 0,...,N-1$ and $q = 0,...,M-1$.

- These NM moments $(m_{p,q})$ are sufficient to uniquely determine the discrete image $(f_{i,j})$ with NM pixels.

Moment Invariants (4)

M I A
1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25 26 27 28 29 30 31 32 33

Incorporating Translation Invariance

- Goal: Modify the moments such that they are invariant under translations.
- The *central moments* of a continuous image $f(x, y)$ are defined as

$$\mu_{p,q} \ := \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} (x - \bar{x})^p \, (y - \bar{y})^q \, f(x, y) \, dx \, dy$$

where $\bar{x} := \frac{m_{1,0}}{m_{0,0}}$ and $\bar{y} := \frac{m_{0,1}}{m_{0,0}}$ denote the x and y coordinates of the *centre of gravity* (also called *centroid*).

- By construction these expressions are shift invariant.
- For a discrete image $(f_{i,j})$ one defines the central moments as

$$\mu_{p,q} \ := \sum_{i=1}^{N} \sum_{j=1}^{M} (i - \bar{i})^p \, (j - \bar{j})^q \, f_{i,j}$$

where $\bar{i} := \frac{m_{1,0}}{m_{0,0}}$ and $\bar{j} := \frac{m_{0,1}}{m_{0,0}}$ with discrete moments $m_{0,0}$, $m_{1,0}$, and $m_{0,1}$.

Remark

- Note the difference between the central moments in this lecture and in Lecture 20.
- Here we regard an image $(f_{i,j})$ as a 2D histogram:
  A grey value $f_{i,j}$ is interpreted as the probability that a photon hits the sensor in pixel cell (i,j).

M I A
1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25 26 27 28 29 30 31 32 33

From Moments to Central Moments

- It is not difficult to show that the central moments can be computed directly from the moments:

$$
\begin{aligned}
\mu_{0,0} &= m_{0,0}, \\
\mu_{1,0} &= 0, \\
\mu_{0,1} &= 0, \\
\mu_{2,0} &= m_{2,0} - \bar{i} m_{1,0}, \\
\mu_{1,1} &= m_{1,1} - \bar{j} m_{1,0}, \\
\mu_{0,2} &= m_{0,2} - \bar{j} m_{0,1}, \\
\mu_{3,0} &= m_{3,0} - 3\bar{i} m_{2,0} + 2\bar{j}^2 m_{1,0}, \\
\mu_{1,2} &= m_{1,2} - 2\bar{j} m_{1,1} - \bar{i} m_{0,2} + 2\bar{j}^2 m_{1,0}, \\
\mu_{2,1} &= m_{2,1} - 2\bar{i} m_{1,1} - \bar{j} m_{2,0} + 2\bar{i}^2 m_{0,1}, \\
\mu_{0,3} &= m_{0,3} - 3\bar{j} m_{0,2} + 2\bar{j}^2 m_{0,1}.
\end{aligned}
$$

M I A
1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25 26 27 28 29 30 31 32 33

Incorporating Scale Invariance

- Goal: Modify the central moments such that they are not only invariant under translations, but also under (spatial) rescalings.

- The *normalised central moments* are defined as

$$\eta_{p,q} := \frac{\mu_{p,q}}{\mu_{0,0}^{\gamma}}$$

  with $\gamma := \frac{p+q}{2} + 1$ and $p + q = 2, 3, \ldots$

- Obviously they are invariant under translations.
  One can show that they are also invariant under scalings.

What about rotations?

M I A
1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25 26 27 28 29 30 31 32 33

Incorporating Rotation Invariance

- The *seven moment invariants of Hu* (1962) have been derived as combinations of normalised central moments such that they are also invariant under rotations:

$$\begin{aligned}
\phi_1 &= \eta_{2,0} + \eta_{0,2}, \\
\phi_2 &= (\eta_{2,0} - \eta_{0,2})^2 + 4\eta_{1,1}^2, \\
\phi_3 &= (\eta_{3,0} - 3\eta_{1,2})^2 + (3\eta_{2,1} - \eta_{0,3})^2, \\
\phi_4 &= (\eta_{3,0} + \eta_{1,2})^2 + (\eta_{2,1} + \eta_{0,3})^2, \\
\phi_5 &= (\eta_{3,0} - 3\eta_{1,2})(\eta_{3,0} + \eta_{1,2})\left((\eta_{3,0} + \eta_{1,2})^2 - 3(\eta_{2,1} + \eta_{0,3})^2\right) + \\
&\quad (3\eta_{2,1} - \eta_{0,3})(\eta_{2,1} + \eta_{0,3})\left(3(\eta_{3,0} + \eta_{1,2})^2 - (\eta_{2,1} + \eta_{0,3})^2\right), \\
\phi_6 &= (\eta_{2,0} - \eta_{0,2})\left((\eta_{3,0} + \eta_{1,2})^2 - (\eta_{2,1} + \eta_{0,3})^2\right) + \\
&\quad 4\eta_{1,1}(\eta_{3,0} + \eta_{1,2})(\eta_{2,1} + \eta_{0,3}), \\
\phi_7 &= (3\eta_{2,1} - \eta_{0,3})(\eta_{3,0} + \eta_{1,2})\left((\eta_{3,0} + \eta_{1,2})^2 - 3(\eta_{2,1} + \eta_{0,3})^2\right) + \\
&\quad (3\eta_{1,2} - \eta_{3,0})(\eta_{2,1} + \eta_{0,3})\left(3(\eta_{3,0} + \eta_{1,2})^2 - (\eta_{2,1} + \eta_{0,3})^2\right).
\end{aligned}$$

- Moreover, ϕ_1,...,ϕ_6 remain also invariant under mirroring, and ϕ_7 changes only sign.

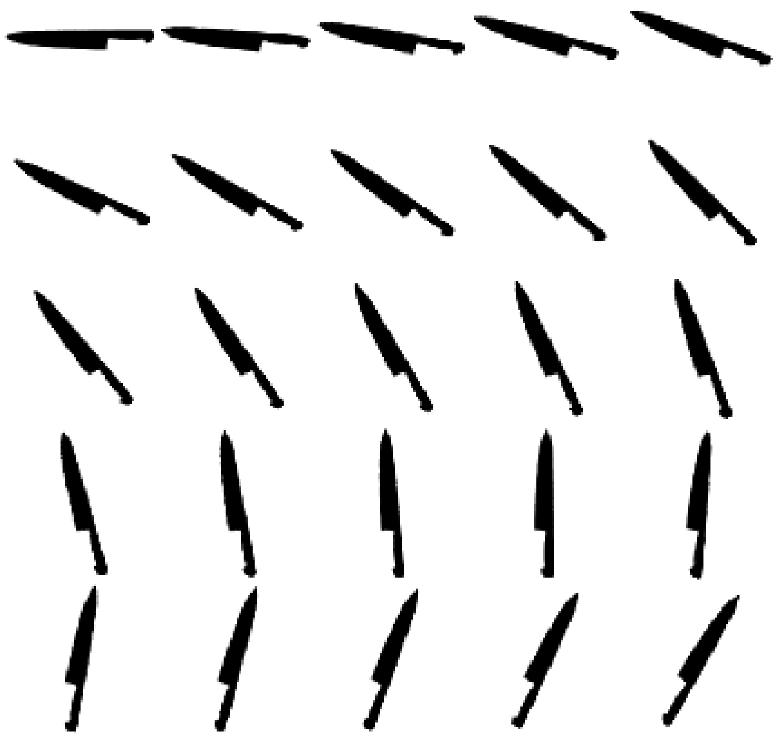
Moment Invariants (9)

Application to Object Recognition

Low resolution set representing 3 objects: scissors, cleaver, and knife. Author: M. J. Andrews.

MIA 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25 26 27 28 29 30 31 32 33

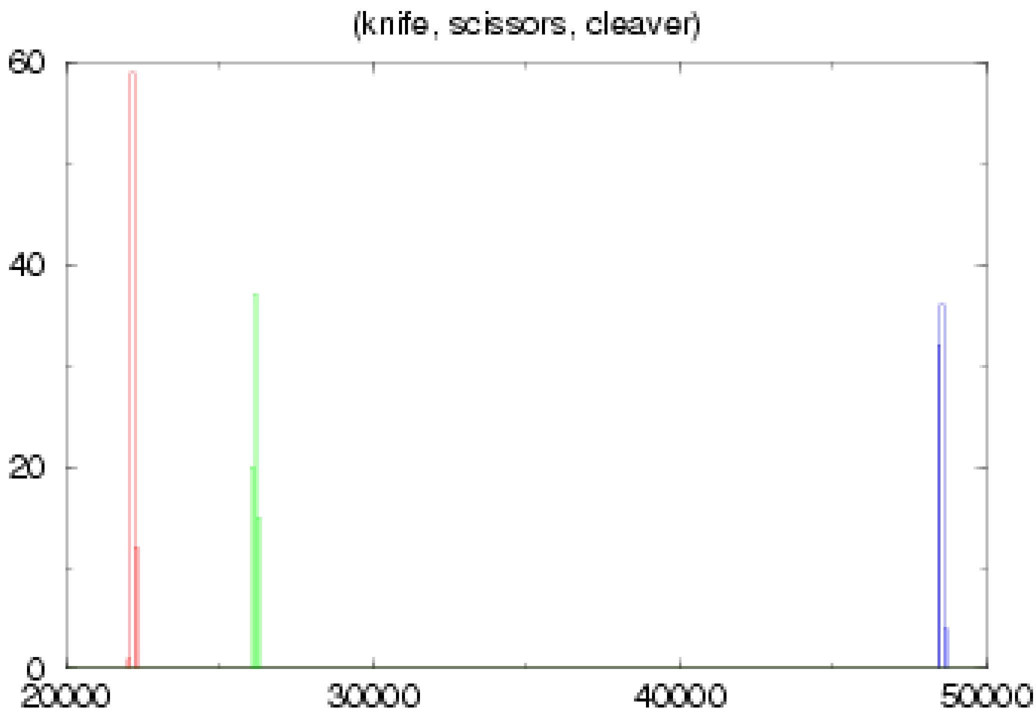
Moment Invariants (10)



Rotation of the knife by some angles between 0 and 120 degrees. Author: M. J. Andrews.

MIA 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25 26 27 28 29 30 31 32 33

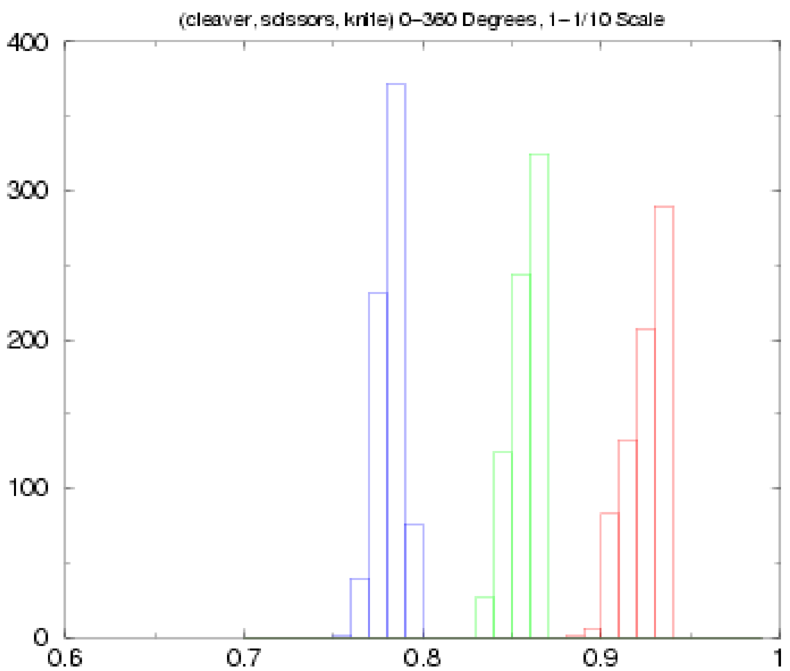
Moment Invariants (11)



Histogram of the first moment invariant ϕ_1 of the rotated three objects. Already the first moment invariant is sufficient for discriminating all three object classes. One observes that the rotation does hardly lead to a broadening of the clusters due to discretisation artifacts. Author: M. J. Andrews.

M I A

Moment Invariants (12)



If one does not only rotate the objects but also rescales them by factors between 0.1 and 1, then the moment invariant ϕ_2 does still allow to distinguish all classes. The broadening of the clusters results from discretisation effects that become particularly dangerous if the object is not very large compared to the pixel size. Author: M. J. Andrews.

1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25 26 27 28 29 30 31 32 33

Incorporating Affine Invariance

- In 1993, Flusser and Suk derived a set of four moment invariants that are even invariant under affine transformations (of type $\boldsymbol{u} = \boldsymbol{A}\boldsymbol{f} + \boldsymbol{b}$ with $\boldsymbol{u}$, $\boldsymbol{f}$, $\boldsymbol{b} \in \mathbb{R}^2$ and a nonsingular matrix $\boldsymbol{A} \in \mathbb{R}^{2\times 2}$; includes translations, rotations, scaling).
- In terms of central moments, they are given by

$$\begin{aligned}
I_1 &= \frac{\mu_{2,0}\mu_{0,2} - \mu_{1,1}^2}{\mu_{0,0}^4}, \\
I_2 &= \frac{\mu_{3,0}^2\mu_{0,3}^2 - 6\mu_{3,0}\mu_{2,1}\mu_{1,2}\mu_{0,3} + 4\mu_{3,0}\mu_{1,2}^3 + 4\mu_{2,1}^3\mu_{0,3} - 3\mu_{2,1}^2\mu_{1,2}^2}{\mu_{0,0}^{10}}, \\
I_3 &= \frac{\mu_{2,0}(\mu_{2,1}\mu_{0,3} - \mu_{1,2}^2) - \mu_{1,1}(\mu_{3,0}\mu_{0,3} - \mu_{2,1}\mu_{1,2}) + \mu_{0,2}(\mu_{3,0}\mu_{1,2} - \mu_{2,1}^2)}{\mu_{0,0}^7}, \\
I_4 &= \big(\mu_{2,0}^3\mu_{0,3}^2 - 6\mu_{2,0}^2\mu_{1,1}\mu_{1,2}\mu_{0,3} - 6\mu_{2,0}^2\mu_{0,2}\mu_{2,1}\mu_{0,3} + 9\mu_{2,0}^2\mu_{0,2}\mu_{1,2}^2 \\
&\quad + 12\mu_{2,0}\mu_{1,1}^2\mu_{2,1}\mu_{0,3} + 6\mu_{2,0}\mu_{1,1}\mu_{0,2}\mu_{3,0}\mu_{0,3} - 18\mu_{2,0}\mu_{1,1}\mu_{0,2}\mu_{2,1}\mu_{1,2} \\
&\quad - 8\mu_{1,1}^3\mu_{3,0}\mu_{0,3} - 6\mu_{2,0}\mu_{0,2}^2\mu_{3,0}\mu_{1,2} + 9\mu_{2,0}\mu_{0,2}^2\mu_{2,1}^2 \\
&\quad + 12\mu_{1,1}^2\mu_{0,2}\mu_{3,0}\mu_{1,2} - 6\mu_{1,1}\mu_{0,2}^2\mu_{3,0}\mu_{2,1} + \mu_{0,2}^3\mu_{3,0}^2\big) \;/\; \mu_{0,0}^{11}.
\end{aligned}$$

Incorporating Invariance under Linear Motion Blur

- In 1996, Flusser, Suk and Saic have investigated how one can construct expressions that are invariant under translation and under linear motion blur.
- They have derived an explicit formula how one can compute invariants of arbitrary high order (and computed the first 30 invariants).
- Experiments show that these invariants can also be well approximated in a discrete setting and that they are also fairly robust under noise.
- In their original formulation, these moments are not invariant under rotations and scalings.
- However, there are possibilities how to incorporate rotation and scale invariance.
- In this way Flusser et al. have shown that the translation, rotation and scale invariant Hu moments ϕ_3, ϕ_4, ϕ_5 and ϕ_7 are even invariant under linear motion blur.

Moment Invariants (15)

Test images for evaluating invariance under linear motion blur. **(a) Top left:** Lena. **(b) Top middle:** Horizontal motion blur, 30 pixels. **(c) Top right:** Additional Gaussian noise with $\sigma = 10$. **(d) Bottom left:** Image (b) with Gaussian noise with $\sigma = 30$. **(e) Bottom middle:** Horizontal motion blur, 80 pixels. **(f) Bottom right:** Lisa. Authors: Flusser et al.

Moment Invariants (16)

M I A
1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25 26 27 28 29 30 31 32 33

	Lena orig.	Motion 30 pix.	Motion 30 STD = 10	Motion 30 STD = 30	Motion 80 pix.	Lisa
$M_0[10^6]$	6.4586	6.4589	6.4607	6.4674	6.4590	5.0860
$M_1[10^{10}]$	0.2560	0.2559	0.2553	0.2612	0.2560	-0.6036
$M_2[10^{10}]$	3.4740	3.4742	3.4750	3.4816	3.4743	2.3215
$M_3[10^{11}]$	-1.1022	-1.1024	-1.0968	-1.0616	-1.1017	1.4777
$M_4[10^{11}]$	1.0456	1.0462	1.0570	1.0639	1.0450	-2.9602
$M_5[10^{11}]$	2.2436	2.2434	2.2313	2.2541	2.2444	0.4081
$M_6[10^{11}]$	-5.3930	-5.3925	-5.3901	-5.4370	-5.3934	6.8344
$M_7[10^{14}]$	3.3918	3.3920	3.3918	3.4093	3.3921	2.0830
$M_8[10^{14}]$	0.1776	0.1775	0.1772	0.1811	0.1776	-0.5208
$M_9[10^{14}]$	-0.0513	-0.0513	-0.0507	-0.0544	-0.0513	0.0426
$M_{10}[10^{14}]$	-0.2293	-0.2292	-0.2293	-0.2277	-0.2292	0.5672
$M_{11}[10^{15}]$	-1.5087	-1.5088	-1.4996	-1.4809	-1.5081	2.7966
$M_{12}[10^{15}]$	4.7607	4.7606	4.7382	4.7451	4.7622	0.7743
$M_{13}[10^{15}]$	-3.1958	-3.1954	-3.1843	-3.2365	-3.1957	0.6806
$M_{14}[10^{15}]$	0.0734	0.0739	0.0985	0.0330	0.0732	-2.8659
$M_{15}[10^{16}]$	-0.2281	-0.2283	-0.2280	-0.2314	-0.2282	0.9275
$M_{16}[10^{16}]$	1.7525	1.7524	1.7540	1.7645	1.7528	-3.5346
$M_{17}[10^{18}]$	3.9858	3.9860	3.9817	4.0315	3.9863	2.3215
$M_{18}[10^{18}]$	0.1484	0.1484	0.1483	0.1505	0.1485	-0.5753
$M_{19}[10^{18}]$	-0.0520	-0.0520	-0.0503	-0.0552	-0.0519	0.0603
$M_{20}[10^{18}]$	-0.2045	-0.2044	-0.2058	-0.2041	-0.2044	0.5220
$M_{21}[10^{18}]$	0.1186	0.1186	0.1191	0.1229	0.1185	-0.1292
$M_{22}[10^{18}]$	0.7187	0.7186	0.7195	0.7164	0.7187	-2.3919

The values of the first 22 motion invariants computed for the images from the previous slide. We observe that all five Lena images yield almost identical numbers, wheras the Lisa image differs quite strongly from these numbers. Authors: Flusser et al.

Photometric Invariants

Problem

- Many computer vision methods are not robust under realistic illumination changes such as globally varying illumination, shadow/shading, highlights.
- This can create severe problems in real world applications.

Classification of Illumination Changes

- global multiplicative changes
- local multiplicative changes: shadow, shading
- local additive changes: highlights due to specular reflections

One can construct so-called *photometric invariants* that are not influenced by these changes. In particular colour images offer a number of possibilities.

M I A
1 2
3 4
5 6
7 8
9 10
11 12
13 14
15 16
17 18
19 20
21 22
23 24
25 26
27 28
29 30
31 32
33

Examples of Photometric Invariants

- **Log-Derivative Transform:**

$$f \;\mapsto\; \big((\ln f)_x,\; (\ln f)_y\big)^\top$$

  Invariant under global multiplicative illumination changes.

- **Chromaticity Space:**

$$(R,G,B)^\top \;\mapsto\; \left(\frac{R}{N}, \frac{G}{N}, \frac{B}{N}\right)^\top$$

  with normalisation $N := \frac{1}{3}(R+G+B)$.
  Invariant under global and local multiplicative illumination changes.

- **HSV Colour Space:**

  The hue component H (cf. Lecture 3) is invariant under global and local multiplicative illumination changes as well as under local additive changes.

Summary

- The Hough transform is useful for detecting simple geometric objects such as lines or circles:
  - Every significant point votes for all compatible parameter sets.
  - The parameter sets with the most votes specify the desired objects.
- Moment invariants are important for recognising objects that have undergone transformations such as translations, rotations and scalings:
  - The moment invariants of Hu are a set of seven expressions.
    They are invariant under translations, scalings and rotations.
  - It is possible to derive similar expressions if other invariances are needed.
    This includes invariance under affine transformations or under linear motion blur.
  - Due to discretisation effects, invariances can be slightly violated.
- Photometric invariants can be used to make computer vision systems invariant under illumination changes.

References

- R. C. Gonzalez, R. E. Woods: *Digital Image Processing*. Addison-Wesley, Reading, Fourth Global Edition, 2017.
  (covers the Hough transform and moment invariants)
- E. R. Davies: *Machine Vision*. Academic Press, San Diego, 1997.
  (contains an extended chapter on the Hough transform; other chapters are less recommendable)
- P. V. C. Hough: *Methods and means for recognizing complex pattens.* U.S. Patent 3069654, Dec. 1962.
  (original patent that gave rise to the Hough transform)
- M. Sonka, V. Hlaváč, R. Boyle: *Image Processing, Analysis, and Machine Vision*. Thomson Learning, London, 1999.
  (Section 6.3.2 includes affine invariant moments.)
- M. J. Andrews: *Moment representation of blobs in 2-D intensity images.*
  (`http://www.gweep.net/~rocko/Moment/`)
  (describes experiments)
- J. Shutler: *Statistical moments – An introduction.*
  (`homepages.inf.ed.ac.uk/rbf/CVonline/LOCAL_COPIES/SHUTLER3/`)
  (good overview)
- M. K. Hu: Visual pattern recognition by moment invariants. *IRE Transactions on Information Theory*, Vol. 8, pp. 179–187, Feb. 1962.
  (original paper on the seven Hu invariants)

References (2)

- J. Flusser, T. Suk: Pattern recognition by affine moment invariants. *PatternRecognition*, Vol. 26, pp. 167–174, 1993.
(https://www.utia.cas.cz/biblio?author=cav_un_auth*2A0101087)
(introduced affine moment invariants)
- J. Flusser, T. Suk, S. Saic: Recognition of images degraded by linear motion blur without restoration. *Computing*, Supplement 11, pp. 37–51, 1996.
(https://www.utia.cas.cz/biblio?author=cav_un_auth*2A0101087)
(moment invariants for linear motion blur)
- A. G. Mamistvalov: n-dimensional moment invariants and conceptual mathematical theory of recognition of n-dimensional data sets. *IEEE Transactions on Pattern Analysis and Machine Intelligence*, Vol. 20, No. 8, pp. 819–831, 1998.
(generalises the Hu theory to arbitrary dimensions)
- K. Voss, H. Süße: *Adaptive Modelle und Invarianten für zweidimensionale Bilder*. Shaker, Aachen, 1995.
(recommendable book on moment invariants)
- J. Flusser, T. Suk, B. Zitová: *Moments and Moment Invariants in Pattern Recognition*. Wiley, 2009.
(comprehensive book on moment invariants)
- A. Papoulis, S. U. Pillai: *Probability, Random Variables and Stochastic Processes*. McGraw-Hill, New York, 2002.
(uniqueness theorem for moments)

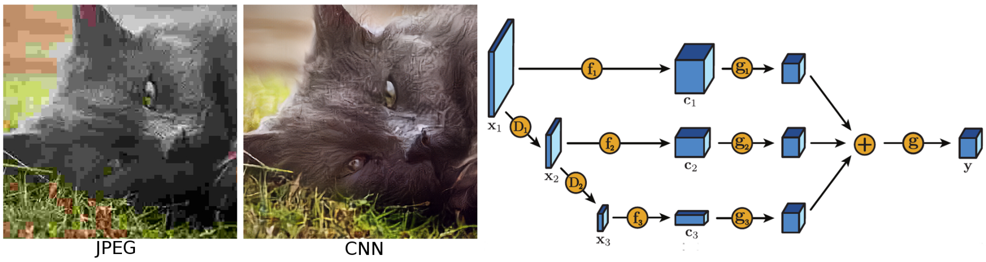
Self Test Problems

Self Test Problems ("Probeklausur")

- You can download an IPCV self test problem sheet from our webpage. It
  - contains 6 problems covering different topics
  - is similar in style and difficulty to a 180-minute written exam
  - is neither submitted to nor corrected by the tutors
- Use these problems to test yourself. Opportunity to ask questions related to the problems of this test is given in the tutorials next week.

Seminar Announcement
Deep Learning: From Mathematical Foundations to Image Compression

Pascal Peter and Joachim Weickert



O. Rippel and L. Bourdev: Real-time adaptive image compression.

- more information and registration:
  `https://www.mia.uni-saarland.de/Teaching/dl19.shtml`
- introductory meeting: E1.7, Room 4.10 on Friday, July 19, 2019, 4:15 pm.
- meetings during the semester: TBA
- registration from Fri, July 12, 2019, 6:00 pm to Wed, July 17, 2019, 6:00 pm.