Example Solutions for Classroom Assignment 11 (C11)

Problem 1: (Otsu's Threshold Selection Method)

By using the identities $\mu_0(T) = \frac{\mu(T)}{\omega(T)}$ and $\mu_1(T) = \frac{\mu_{\text{tot}} - \mu(T)}{1-\omega(T)}$, it suffices to take the definition of $\sigma_B^2(T)$ and to plug these values in.

$$
\begin{aligned}
\sigma_B^2(T) &= \omega(T)(\mu_0(T) - \mu_{\text{tot}})^2 + (1-\omega(T))(\mu_1(T) - \mu_{\text{tot}})^2 \\
&= \omega(T)\left(\frac{\mu(T)}{\omega(T)} - \mu_{\text{tot}}\right)^2 + (1-\omega(T))\left(\frac{\mu_{\text{tot}} - \mu(T)}{1-\omega(T)} - \mu_{\text{tot}}\right)^2 \\
&= \omega(T)\left(\frac{\mu(T) - \mu_{\text{tot}}\omega(T)}{\omega(T)}\right)^2 + (1-\omega(T))\left(\frac{\mu_{\text{tot}}\omega(T) - \mu(T)}{1-\omega(T)}\right)^2 \\
&= \frac{(\mu(T) - \mu_{\text{tot}}\omega(T))^2}{\omega(T)} + \frac{(\mu_{\text{tot}}\omega(T) - \mu(T))^2}{1-\omega(T)} \\
&= (\mu_{\text{tot}}\omega(T) - \mu(T))^2\left(\frac{1}{\omega(T)} + \frac{1}{1-\omega(T)}\right) \\
&= (\mu_{\text{tot}}\omega(T) - \mu(T))^2 \frac{1-\omega(T)+\omega(T)}{\omega(T)(1-\omega(T))} \\
&= \frac{(\mu_{\text{tot}}\omega\,(T) - \mu\,(T))^2}{\omega\,(T)\,(1-\omega\,(T))}
\end{aligned}
$$

Problem 2: (Toboggan Watershed Algorithm)

First of all we compute the magnitude of the first derivative of the signal $\boldsymbol{f}$ by using forward differences:

$$\begin{aligned} \boldsymbol{f} &= (2,7,1,7,7,4,3,0,4,6,1,2,0,5,2,3)^\top \\ \boldsymbol{g} := |f_{i+1} - f_i|_{i=1\ldots8} &= (5,6,6,0,3,1,3,4,2,5,1,2,5,3,1,0)^\top \end{aligned}$$

In each pixel of $\boldsymbol{g}$ we follow the direction of the steepest descend until a local minimum (in bold font) is reached and keep track of all pixels on the way to this minimum. The tracked pixels are set to the value f_i of the original signal that corresponds the local minimum:

$$\begin{aligned} \boldsymbol{f} &= (2,7,1,7,7,4,3,0,4,6,1,2,0,5,2,3)^\top \\ \boldsymbol{g} &= (\underline{5,6},\underline{6,0,3},\underline{1,3},\underline{4,2},\underline{5,1,2,5},\underline{3,1,0})^\top \\ \boldsymbol{f}_{\text{filtered}} &= (2,2,7,7,7,4,4,4,4,1,1,1,1,3,3,3)^\top \end{aligned}$$
