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Lecture 18: Global Filters II: Continuous Variational Methods

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Motivation

- In the last lecture we discussed discrete variational models of type

$$E_{\boldsymbol{f}}(\boldsymbol{u}) := \frac{1}{2} \sum_{k=1}^{N} \left((u_k - f_k)^2 + \frac{\alpha}{|\mathcal{N}(k)|} \sum_{\ell \in \mathcal{N}(k)} (u_\ell - u_k)^2 \right).$$

- Today we consider their continuous counterparts.
- The continuous formulation can offer some advantages:
  - Some continuous models are more transparent and elegant.
  - One continuous model may give rise to a number of discrete models (depending on the specific discretisation).
  - Invariance under rotations is easily satisfied in the continuous setting.
- However, this requires to minimise a continuous energy.
  Therefore, we must learn some mathematics from the calculus of variations.

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A Continuous Variational Method

- Consider the 2-D setting with a continuous image $f : \Omega \to \mathbb{R}$ with $\Omega \subset \mathbb{R}^2$.
- We want to obtain a smoothed image $u : \Omega \to \mathbb{R}$ as minimiser of

$$E_f(u) \;:=\; \frac{1}{2} \int\limits_{\Omega} \left((u-f)^2 + \alpha\, |\boldsymbol{\nabla} u|^2 \right) dx\, dy$$

  with $\boldsymbol{\nabla} u := (u_x, u_y)^\top$.
- As in the discrete case, this energy involves a data and a smoothness term.
- The minimising image u is no longer a vector like in the discrete case:
  It is a function.

How can we find this minimising function?
To this end, we make an excursion to the calculus of variations.

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Calculus of Variations

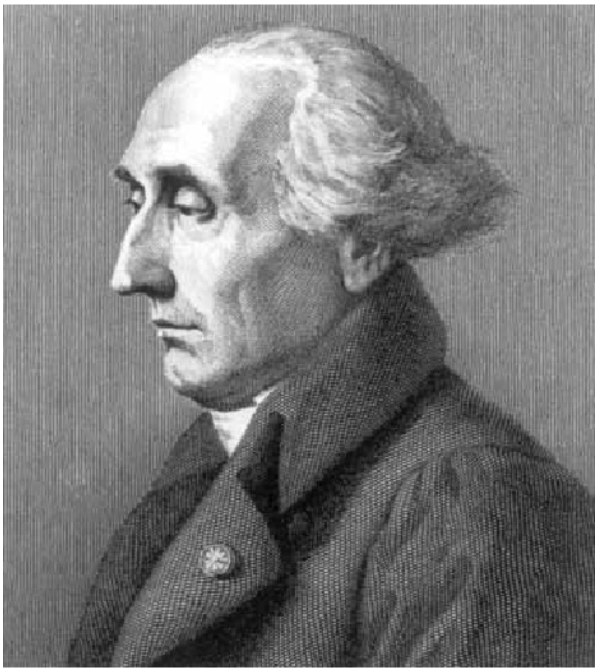
Calculus of Real Numbers (reellwertige Analysis)

- considers real-valued *functions* $f(x)$:
  mappings from a *real number* x to a real number
- If f has a minimum in ξ, then ξ necessarily satisfies $f'(\xi) = 0$.
- If f is strictly convex, then ξ is the unique minimum.

Calculus of Variations (Variationsrechnung)

- considers real-valued *functionals* $E(u)$:
  mappings from a *function* u to a real number
- We will learn the following result:
  If E is minimised by a function v, then v necessarily satisfies the corresponding *Euler–Lagrange equation,* a differential equation in v.
- If E is strictly convex, then v is the unique minimiser.

Calculus of Variations (2)

The mathematicians Euler and Lagrange belong to the founders of the calculus of variations. **Left:** Leonhard Euler (1707–1783). **Right:** Joseph–Louis Lagrange (1736–1813). Source: `http://www-gap.dcs.st-and.ac.uk/~history/PictDisplay/`.

Calculus of Variations (3)

Theorem (Euler–Lagrange Equation in 1-D):

A smooth function $u : [a, b] \to \mathbb{R}$ that minimises the functional

$$E(u) \;=\; \int_a^b F(x, u, u')\, dx$$

necessarily satisfies the *Euler–Lagrange equation*

$$F_u - \frac{d}{dx} F_{u'} \;=\; 0$$

with so-called *natural boundary conditions*

$$F_{u'} = 0$$

in $x = a$ and $x = b$. (Subscripts denote partial derivatives.)

Calculus of Variations (4)

Proof

- Idea: Reduce the problem to a minimisation problem of a scalar-valued function.
- Let us assume that $v(x)$ is a sufficiently often differentiable minimiser of E. We embed $v(x)$ into the family

$$u(x, \varepsilon) \;:=\; v(x) + \varepsilon\, r(x)$$

with some smooth perturbation function $r(x)$.

- Since $v(x)$ minimises $E(u)$, we know that the scalar-valued function

$$g(\varepsilon) \;:=\; E(u(x, \varepsilon)) \;=\; E(v + \varepsilon r)$$

has a minimum in $\varepsilon = 0$. Therefore, we have

$$0 \;=\; g'(0) \;=\; \frac{d}{d\varepsilon}\, E(v + \varepsilon r)\Big|_{\varepsilon=0} \;=\; \frac{d}{d\varepsilon} \int_a^b F(x, \underbrace{v + \varepsilon r}_{u(.,\varepsilon)}, \underbrace{v' + \varepsilon r'}_{u'(.,\varepsilon)})\, dx\, \Big|_{\varepsilon=0}$$

where u' denotes the derivative $\partial_x u$.

Calculus of Variations (5)

- With the chain rule it follows that

$$\begin{aligned} 0 \;&=\; \int_a^b \Big(F_u(x, u, u')\, \partial_\varepsilon u \;+\; F_{u'}(x, u, u')\, \partial_\varepsilon u' \Big)\, dx\, \Big|_{\varepsilon=0} \\ &=\; \int_a^b \Big(F_u(x, v, v')\, r(x) \;+\; F_{u'}(x, v, v')\, r'(x) \Big)\, dx. \end{aligned}$$

- Partial integration of the second term yields

$$0 \;=\; \int_a^b \left(F_u(x, v, v')\, r(x) - \frac{d}{dx} F_{u'}(x, v, v')\, r(x) \right) dx + \left[F_{u'}(x, v, v')\, r(x) \right]_{x=a}^{x=b} \quad (1)$$

- v is a minimiser within our family of competing functions $u(\varepsilon, x) = v(x) + \varepsilon r(x)$. Therefore, v is also a minimiser within the smaller class of functions where the perturbation $r(x)$ satisfies $r(a) = 0 = r(b)$. Thus,

$$0 \;=\; \int_a^b \left(F_u(x, v, v') - \frac{d}{dx} F_{u'}(x, v, v') \right) r(x)\, dx.$$

Calculus of Variations (6)

- This gives the Euler–Lagrange equation

$$F_u - \frac{d}{dx} F_{u'} \;=\; 0.$$

Note that this equation does not depend on the perturbation r.

- Plugging it into Equation (1) gives

$$\big[F_{u'}(x, v, v')\, r(x)\big]_{x=a}^{x=b} \;=\; 0.$$

which holds for arbitrary perturbations r (also with $r(a) \neq 0$ and $r(b) \neq 0$).

- Thus, one obtains the natural boundary conditions

$$F_{u'} \;=\; 0$$

for $x = a$ und $x = b$. This concludes the proof.

Calculus of Variations (7)

Extensions to the 2-D Case

$$E(u) \;=\; \int_\Omega F(x, y, u, u_x, u_y)\, dx\, dy$$

yields the Euler–Lagrange equation

$$F_u - \partial_x F_{u_x} - \partial_y F_{u_y} \;=\; 0$$

with the natural boundary condition

$$\boldsymbol{n}^\top \begin{pmatrix} F_{u_x} \\ F_{u_y} \end{pmatrix} \;=\; 0$$

on the image boundary $\partial\Omega$ with normal vector $\boldsymbol{n}$.

Extensions to higher dimensions are analog.

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Extensions to Vector-Valued Functions

$$E(u,v) \;=\; \int_a^b F(x,u,v,u',v')\,dx$$

creates a set of Euler–Lagrange equations:

$$\begin{aligned} F_u - \frac{d}{dx}F_{u'} &= 0, \\ F_v - \frac{d}{dx}F_{v'} &= 0 \end{aligned}$$

with natural boundary conditions for u and v.

This will be needed in a later lecture on optic flow computation.
Extensions to vector-valued functions with more components are straightforward.

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Application to the Continuous Model

- For the integrand

  $$F(x,y,u,u_x,u_y) \;:=\; \frac{1}{2}(u-f)^2 \;+\; \frac{\alpha}{2}\left(u_x^2+u_y^2\right)$$

  we obtain the partial derivatives

  $$\begin{aligned} F_u &= u-f, \\ F_{u_x} &= \alpha\,u_x, \\ F_{u_y} &= \alpha\,u_y \end{aligned}$$

- This leads to the Euler–Lagrange equation

  $$\begin{aligned} 0 &= F_u - \partial_x F_{u_x} - \partial_y F_{u_y} \\ &= u - f - \partial_x(\alpha u_x) - \partial_y(\alpha u_y) \\ &= u - f - \alpha\,\Delta u \end{aligned}$$

- The natural boundary condition

$$0 \;=\; \boldsymbol{n}^\top \begin{pmatrix} F_{u_x} \\ F_{u_y} \end{pmatrix}$$

on the image boundary $\partial\Omega$ gives

$$0 \;=\; \boldsymbol{n}^\top \boldsymbol{\nabla} u \;=\; \partial_{\boldsymbol{n}} u$$

where we have divided by α.

- Such a vanishing normal derivative at the boundaries of the image domain comes down to reflecting boundary conditions (or an extension by mirroring).

- The Euler–Langrange equation contains partial derivatives of the unknown function $u(x,y)$. Therefore, it is a partial differential equation.
Usually such equations have to be solved numerically.

- The discrete model from the previous lecture can be regarded as such a numerical approximation. Thus, we only need solvers for linear systems of equations.

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Relations to Discrete Variational Methods

- For simplicity, consider the 1-D discrete case from the previous lecture:

$$E(\boldsymbol{u}) \;:=\; \frac{1}{2}\sum_{k=1}^{N}(u_k - f_k)^2 \;+\; \frac{\alpha}{2}\sum_{k=1}^{N-1}(u_{k+1}-u_k)^2 \,.$$

Minimisation yielded

$$\begin{aligned} 0 \;&=\; u_1 - f_1 - \alpha\,(u_2 - u_1), \\ 0 \;&=\; u_i - f_i - \alpha\,(u_{i+1} - 2u_i + u_{i-1}) \qquad (i = 2, ..., N-1), \\ 0 \;&=\; u_N - f_N - \alpha\,(-u_N + u_{N-1}). \end{aligned}$$

- This can be seen as a discretisation of the 1-D Euler–Lagrange equation

$$0 \;=\; u - f - \alpha\, u''$$

that arises from the energy functional

$$E(u) \;=\; \frac{1}{2}\int_a^b \left((u-f)^2 + \alpha (u')^2\right)\, dx \,.$$

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Relations to Discrete Variational Methods (2)

- In this case, u'' was approximated in pixel i in the following way:

$$u''|_i \;\approx\; \frac{u_{i+1} - 2u_i + u_{i-1}}{h^2}$$

  with spatial grid size (pixel distance) $h = 1$.

- The natural boundary conditions $u'(a) = 0$ and $u'(b) = 0$ were discretised by mirrored dummy boundary pixels $u_0 := u_1$ and $u_{N+1} := u_N$:

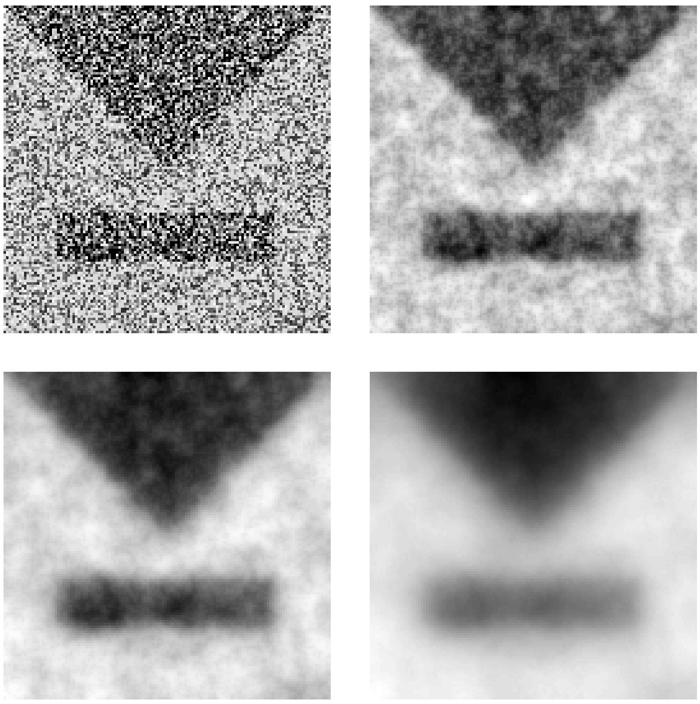
$$0 \;=\; u'_{1/2} \;\;\approx\;\; \frac{u_1 - u_0}{h}$$

$$0 \;=\; u'_{N+1/2} \;\;\approx\;\; \frac{u_{N+1} - u_N}{h}$$

  Note that we had a pixel centred grid where u_i approximates $u\left(a + (i - \frac{1}{2})h\right)$. Thus, $u'_{1/2}$ approximates $u'(a)$, and $u'_{N+1/2}$ approximates $u'(b)$.

Relations to Discrete Variational Methods (3)

original, 128×128 $\qquad$ $\alpha = 5$



$\alpha = 20$ $\qquad$ $\alpha = 100$

Results with the quadratic regularisation method. For improving visibility, an affine greyscale transformation to $[0, 255]$ has been performed. Author: J. Weickert.

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Nonquadratic Variational Methods

Problem

- The methods discussed so far blur edges.

Remedy

- Modify the smoothness term $|\nabla u|^2$ such that large gradients are less severely penalised than before.
- Replace $|\nabla u|^2$ by a function $\Psi(|\nabla u|^2)$ that is still convex in $|\nabla u|$, but for large gradients it increases slower than $|\nabla u|^2$, e.g. the *Charbonnier penaliser*

$$\Psi(|\nabla u|^2) \;:=\; 2\lambda^2 \sqrt{1 + |\nabla u|^2/\lambda^2}\,.$$

Euler-Lagrange Equation

- becomes a *nonlinear* PDE (homework assignment)
- discretisation yields a nonlinear system of equations (more cumbersome to solve)

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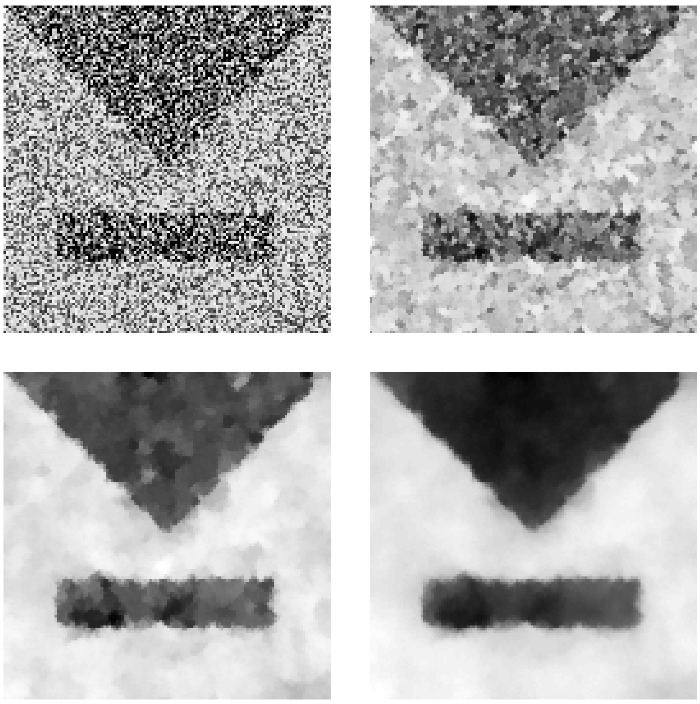
original, 128×128 | $\alpha = 20$



$\alpha = 50$ | $\alpha = 100$

Results with a nonquadratic regularisation method with Charbonnier penaliser ($\lambda = 1$). An affine greyscale transformation to $[0, 255]$ has been performed. Author: J. Weickert.

From Variational Denoising to Image Compression

Variational Denoising Revisited

- Instead of minimising

$$E_f(u) := \frac{1}{2} \int_\Omega \left(1 \cdot (u-f)^2 + \alpha\, |\boldsymbol{\nabla} u|^2\right) dx\, dy$$

we can just as well minimise the rescaled functional

$$E_f(u) := \frac{1}{2} \int_\Omega \left(c \cdot (u-f)^2 + (1-c) \cdot |\boldsymbol{\nabla} u|^2\right) dx\, dy$$

with $\frac{\alpha}{1} = \frac{1-c}{c}$ (which leads to $c := \frac{1}{1+\alpha}$).

- This yields the Euler-Langrange equation

$$c \cdot (u-f) - (1-c) \cdot \Delta u \;=\; 0.$$

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Application to Inpainting and Compression

- Making c space-variant,

$$c(\boldsymbol{x}) := \begin{cases} 1 & \text{if } \boldsymbol{x} \in K, \\ 0 & \text{else,} \end{cases}$$

in the Euler-Lagrange equation gives

$$c(\boldsymbol{x}) \cdot (u-f) - (1-c(\boldsymbol{x})) \cdot \Delta u \;=\; 0.$$

- This can be used for PDE-based interpolation:
  The interpolated image u is identical to f in K (where the data are known), and it solves the Laplace equation $\Delta u = 0$ in $\Omega \setminus K$ (where the data are unknown).

- This fills in missing information in $\Omega \setminus K$ *(inpainting)*.

- One can use this for image compression by specifying data only at some points, e.g. next to edge contours.

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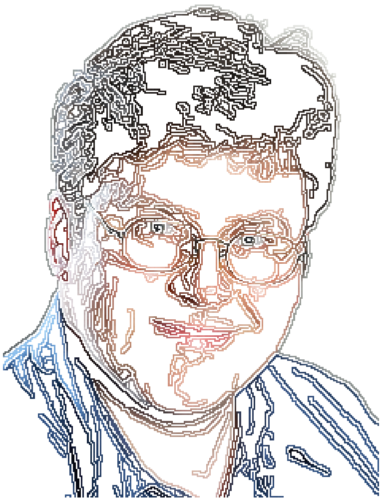
From Variational Denoising to Image Compression (3)

Example: Inpainting with Sparse Data


original


data near edges kept


inpainted

Image reconstruction from edges. **(a) Left:** Original image, 237×316 pixels. **(b) Middle:** Edge set. Only the left and right neighbours of each Canny edge are stored. **(c) Right:** Interpolation of (b) by solving the Laplace equation $\Delta u = 0$ in each RGB channel. Author: J. Weickert.

From Variational Denoising to Image Compression (4)

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Inpainting-based Codecs

original

data kept

reconstruction

Left: Original image. **Middle:** Stored data with colour value optimisation. The data are subsampled along the contour, requantised and entropy coded. **Right:** Reconstruction by solving the Laplace equation in each RGB channel. Compression rate: 200:1. Author: Mainberger.

Comparison to JPEG and JPEG 2000 at Compression Rate 200 : 1

original image

JPEG, MSE=441.20

JPEG 2000, MSE=164.53

inpainting, MSE=25.97

Overview of Denoising Methods

- We have discussed a large number of denoising methods so far:
  - convolution filters such as box filters or Gaussian convolution
  - median filters
  - wavelet shrinkage
  - bilateral filtering
  - NL means
  - nonlinear diffusion filtering
  - quadratic and nonquadratic regularisation
- None of them is always best.
- Each of them has its pros and cons.
- Knowing them allows to choose the best one in a specific situation.

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Overview of Denoising Methods

Comparison of filters for structure-preserving denoising. Wavelet = wavelet shrinkage, NL = nonlocal means, NDiff = nonlinear diffusion filtering, NRegul = nonquadratic regularisation. (no) means "no in original model, but can be cured with reasonable efforts".

	Box	Gauss	Median	Wavelet	Bilateral	NL	NDiff	NRegul
Quality	–	– to o	o to +	o to +	+	++	+	+
Linearity	yes	yes	no	no	no	no	no	no
Separability	yes	yes	no	yes	no	no	(no)	no
Shift Invar.	yes	yes	yes	(no)	yes	yes	yes	yes
Rotation Inv.	no	yes	(no)	no	yes	(no)	yes	yes
Pres. of Mean	yes	yes	no	yes	no	no	yes	yes
Max.-Min. Pr.	yes	yes	yes	no	yes	yes	yes	yes
Parameters	1	1	1	1	2	3	2	2
Flexibility	–	–	o	o	+	+	++	++
Speed	++	++	o to +	++	o to +	–	o to +	o to +

Summary

- Like their discrete counterparts, continuous variational methods minimise an energy with data and smoothness term.
  However, sums are replaced by integrals, and differences to neighbours by derivatives.

- The minimising function satisfies the Euler-Lagrange equation, a partial differential equation, supplemented with natural boundary conditions.

- Discretising the Euler–Lagrange equation leads to a
  - linear system of equations for a quadratic functional
  - nonlinear system for a nonquadratic functional.

  Such systems also arise directly from discrete variational methods.

- Nonquadratic variational methods allow edge-preserving denoising.

- Variational methods can also be used for image inpainting and compression.

References

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- M. Bertero, T. A. Poggio, V. Torre: Ill-posed problems in early vision. *Proceedings of the IEEE*, Vol. 76, No. 8, 869–889, August 1988.
  (survey on quadratic regularisation in image processing and computer vision)
- M. Mainberger, A. Bruhn, J. Weickert, S. Forchhammer: Edge-based compression of cartoon-like images with homogeneous diffusion. *Pattern Recognition*, Vol. 44, No. 9, 1859-1873, Sept. 2011.
  (`http://www.mia.uni-saarland.de/Publications/mainberger-pp269.pdf`)
  (application to image compression)
- I. M. Gelfand, S. V. Fomin: *Calculus of Variations*. Dover, New York, 2000.
  (well-readable and inexpensive book on the calculus of variations)
- R. Courant, D. Hilbert: *Methods of Mathematical Physics*. Vol. 1, Interscience, New York, 1953.
  (excellent classical book with a large chapter on variational concepts; also in German)

Assignment C9 – Classroom Work

Problem 1 (Convexity of a Discrete Energy Function)

Consider the following discrete energy function:

$$E(\boldsymbol{u}) := \frac{1}{2}\sum_{k=1}^{N}(u_k - f_k)^2 + \frac{\alpha}{2}\sum_{k=1}^{N-1}(u_{k+1} - u_k)^2$$

with $\boldsymbol{u}, \boldsymbol{f} \in \mathbb{R}^N$ and $\alpha > 0$. Verify that this energy is strictly convex. What can you say about possible minimisers of this energy?

Problem 2 (Euler-Lagrange Equations)

Let $\Omega \subset \mathbb{R}^3$ and consider the continuous functional for a 3-D image $u : \Omega \to \mathbb{R}$:

$$E(u) = \int_{\Omega} \left(\frac{1}{2}(u-f)^2 - \alpha\lambda^2 \exp\left(\frac{-|\boldsymbol{\nabla} u|^2}{2\lambda^2}\right)\right) dx\, dy\, dz.$$

with parameter $\alpha > 0$.
Derive the Euler-Lagrange equation associated with this functional.

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Assignment H9 – Homework

Problem 1 (Continuous Nonquadratic Variational Methods) (2+1+2 points)

Let $f(x)$ be a noisy 1-D signal on the interval $[a, b]$. For discontinuity-preserving denoising, we seek a function $u(x)$ that minimises the energy functional

$$E(u) \;:=\; \int_a^b \left(\frac{1}{2}(u-f)^2 + \alpha\lambda^2\sqrt{1+u_x^2/\lambda^2} \right) \, dx \, .$$

(a) Write down the Euler-Lagrange equation of this energy functional and simplify it as much as possible.

(b) Specify the boundary conditions.

(c) Show that the Euler-Lagrange equation has a unique solution, and that this solution is indeed a minimiser (and not a maximiser) of the energy functional.

Problem 2 (Discrete Variational Methods) (2+2 points)

(a) Write down a discrete analogue to the energy functional from Problem 1. Use forward differences to approximate derivatives. Do **not** introduce artificial boundary values u_0 and u_{N+1}.

(b) Which nonlinear system of equations has to be satisfied necessarily in its minimum?

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Problem 3 (Fourier Analysis of Linear Filters) (5+5 points)

(a) Consider the following filters and perform a continous Fourier analysis. Rewrite the results in the form $\hat{u} = g \cdot \hat{f}$, where $\hat{u}$ and $\hat{f}$ are the Fourier transform of u and f, and g is a function that differs from filter to filter.

(i) 1-D discrete regularisation with grid size h:

$$-\frac{\alpha}{h^2}u(x-h) + \left(1 + 2\frac{\alpha}{h^2}\right) u(x) - \frac{\alpha}{h^2}u(x+h) = f(x).$$

(ii) 1-D continous regularisation:

$$0 = u(x) - f(x) - \alpha u''(x).$$

(iii) Convolution with a 1-D Gaussian kernel:

$$u = K_\sigma * f \qquad \text{with} \quad K_\sigma(x) := \frac{1}{\sqrt{2\pi}\sigma} \exp\left(-\frac{x^2}{2\sigma^2}\right).$$

(iv) Result after one iteration step using the 1-D explicit scheme for linear diffusion (i.e. nonlinear diffusion with diffusivity $g(x) = 1$):

$$u(x) = \frac{\tau}{h^2}f(x-h) + \left(1 - 2\frac{\tau}{h^2}\right) f(x) + \frac{\tau}{h^2}f(x+h).$$

(b) Show that the corresponding functions for g in (i), (ii), (iii), and (iv) have the same quadratic Taylor polynomial in 0, if $\alpha = \tau = \frac{1}{2}\sigma^2$ and $h \to 0$.

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Assignment H9 (3)

Please download the required files from the webpage

```
http://www.mia.uni-saarland.de/Teaching/ipcv19.shtml
```

into your own directory. You can unpack them with the command tar xvzf Ex09.tar.gz.

Problem 4 (Whittaker-Tikhonov Regularisation and Unsharp Masking) (2+2+1 points)

(a) The method regularise in the file wtr-um.c is supposed to apply a Whittaker-Tikhonov regularisation to an image $\boldsymbol{f}$ as presented in the lecture. Supplement the missing code.

(b) So called *unsharp masking* can be used to increase the apparent sharpness of a blurred image $\boldsymbol{f}$: One blurs the given image $\boldsymbol{f}$ and gets an image $\boldsymbol{u}$. Then one computes $\boldsymbol{f} - (\boldsymbol{u} - \boldsymbol{f})$. Supplement the code in the routine unsharp_masking. For that purpose use the function regularise (see (a)) to blur $\boldsymbol{f}$.

(c) Compile the program wtr-um using the command

```
gcc -O2 -o wtr-um wtr-um.c -lm
```

Use the program wtr-um to deblur the image girl-blurred.pgm. There are two parameters to choose:

- The regularisation parameter α for the Whittaker-Tikhonov regularisation.
- The unsharp masking factor that says how often unsharp masking is repetitively applied.

Assignment H9 (4)

Submission

Please submit the theoretical Problems 1, 2, and 3 in handwritten form before the lecture. For the practical Problem 4, please submit the files as follows: Rename the main directory Ex09 to Ex09_<your_name> and use the command

```
tar czvf Ex09_<your_name>.tar.gz Ex09_<your_name>
```

to pack the data. The directory that you pack and submit should contain the following files:

- the file wtr-um.c which contains the supplemented code,
- one reasonable deblurred version of girl-blurred.pgm,
- a text file README that contains the parameters which are used for the creation of the submitted image as well as information on all people working together for this assignment.

Please make sure that only your final version of the programs and images are included.
Submit the file via e-mail to your tutor via the address:

```
ipcv-xx@mia.uni-saarland.de
```

where xx is either t1, t2, t3, t4, t5, w1, w2, w3 or w4 depending on your tutorial group.

Deadline for submission: Friday, June 14, 10 am (before the lecture)

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