Lecture 19:
Global Filters III:
Fourier Methods and Deconvolution

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Motivation

- So far, we have mainly investigated filters for denoising tasks.
- However, there are also other perturbations in images, such as
  - periodic artifacts due to rasterisation
  - space-variant illumination
  - blur
- Removal of these artifacts requires specifically adapted filters.
- Let us study some of the most popular methods for these problems.
  We will see that the Fourier transform is highly useful again.

Removal of Periodic Perturbations

- Periodic perturbations appear e.g. as grid patterns in rastered newspaper images.
- Periodic artifacts are well-localised in the Fourier domain.
- Thus, we can remove them easily in the Fourier domain.

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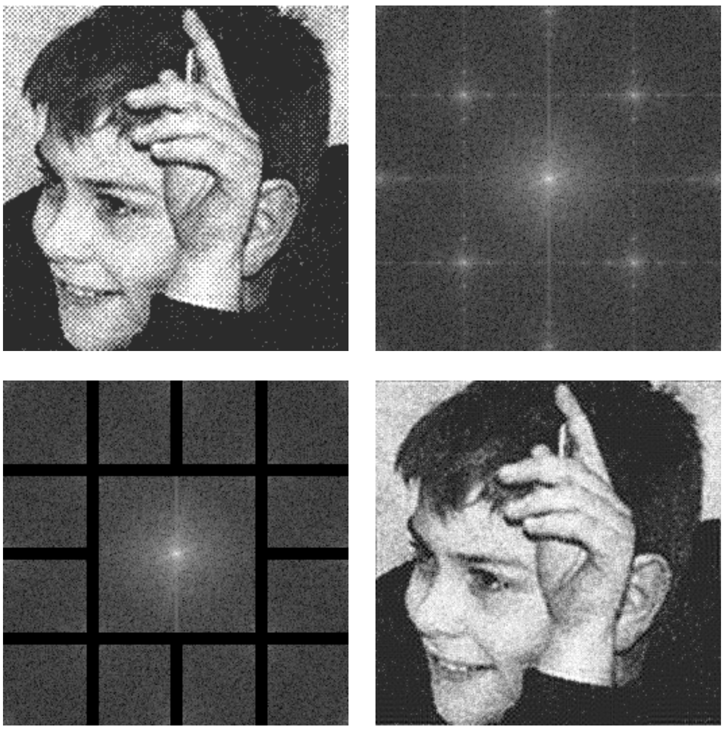


Top left: Newspaper image with rastered structures, 256×256 pixels. **Top right:** Fourier spectrum (logarithmically scaled). **Bottom left:** Removal of the rastered structures in the Fourier domain. **Bottom right:** Backtransformation of the filtered image. Author: J. Weickert (2002).

Homomorphic Filtering (Homomorphe Filterung)

Goal

- restore a photo that has been taken under space-variant illumination conditions

What Do the Grey Values Describe?

- A grey value $f(x, y)$ is the product of two factors:
  – the illumination intensity $I(x, y)$ of the light source,
  – the reflectance $R(x, y)$ of the illuminated object.

$$f(x, y) = I(x, y)\, R(x, y)\,.$$

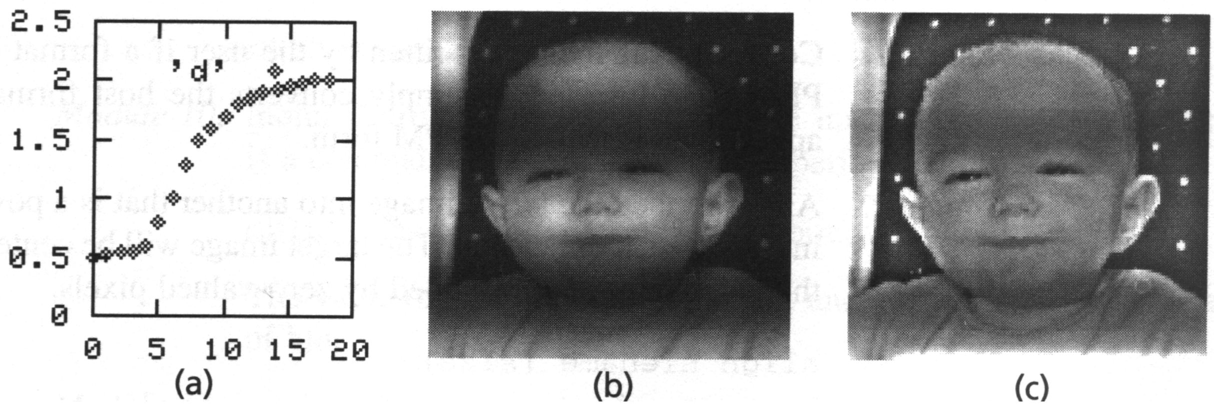
- Often the illumination intensity varies slowly and smoothly in space.
  Thus, it contributes a *multiplicative, low-frequent perturbation.*
  We want to remove this perturbation.
- The reflectance can change rapidly, e.g. at edges.
  Thus, it contains more high-frequent components.
  We want to keep (and perhaps even enhance) these frequencies.

Basic Idea Behind Homomorphic Filtering

- *Forward transformation:*
  - Taking the logarithm turns a multiplicative perturbation into an additive one:

$$\ln f(x, y) = \ln I(x, y) + \ln R(x, y).$$

  - Compute the Fourier transform of $\ln f(x, y)$.
    (The additive splitting allows to exploit the linearity of the Fourier transform.
    A multiplicative perturbation would give a convolution in the Fourier domain.)
- *High-pass filtering:*
  - Multiply in the Fourier domain with a function that
    – damps low frequencies (reduces illumination changes)
    – boosts high frequencies (enhances reflectance effects)
- *Backtransformation:*
  - Compute the inverse Fourier transform.
  - Compute the exponential function of the result.



Homomorphic filtering. **Left:** High pass filter that is used in the Fourier domain. **Middle:** Original image with space-variant illumination. **Right:** Reduction of illumination variations and enhancement of reflectance effects. Author: J. R. Parker (1997).

Inverse Filtering

Image Degradation Model (cf. Lecture 2):

- image degradation model with shift-invariant blur and additive noise:

$$f(x,y) \;=\; (h * u)(x,y) \;+\; n(x,y)$$

  - f: observed degraded image
  - u: (unknown) ideal image
  - h: (known) shift-invariant convolution kernel *(point spread function (PSF))*
  - n: noise (caused by the sensor, transmission, quantisation, ...)

- The convolution kernel h can model e.g.
  - a defocused optical system (cylinder-like shape)
  - motion blur (oriented box function)
  - atmospheric perturbations in telescope (almost Gaussian)

Inverse Filtering (2)

How Can u be Restored ?

- If (!) noise is negligible, one obtains in the Fourier domain:

$$\hat{f} = \hat{h} \cdot \hat{u}.$$

- If (!) $\hat{h}$ does not vanish, u can be reconstructed as follows:
  Compute

$$\hat{u} \; = \; \frac{\hat{f}}{\hat{h}}$$

  and apply the inverse Fourier transform.

- This process is called *inverse filtering*.

- It acts like a highpass filter, if h is a lowpass filter.

Inverse Filtering (3)

Problem

- Usually h is a smoothing kernel (lowpass filter).

- Thus, for high frequencies, $\hat{h}$ is close to 0.

- In this case inverse filtering massivly amplifies even tiny high-frequent noise (e.g. quantisation noise).

- Thus, without additional stabilisation, inverse filtering is often of no use.

Simplest Remedy

- remove all dangerous frequencies, where $|\hat{h}|$ is too small:

$$\hat{u} \; = \; \begin{cases} \frac{\hat{f}}{\hat{h}} & \text{if } |\hat{h}| > \varepsilon, \\ 0 & \text{else.} \end{cases}$$

- This stabilised inverse filtering is called *pseudoinverse filtering.*

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Left: Original image of size 256×256 pixels. **Middle:** Perturbed by simulated horizontal motion blur with a length of 31 pixels. **Right:** Deblurring with pseudoinverse filtering. Source: `http://www.owlnet.rice.edu/∼elec431/projects95/lords/wolf.html`.

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Wiener Filtering

- Pseudoinverse filtering stabilises inverse filtering with a discontinuous cut-off.
- Wiener filtering aims at a smoother stabilisation of the inverse filtering $\hat{u} = \frac{\hat{f}}{\hat{h}}$.
- It approximates $\hat{u}$ by

$$\hat{u} \approx \underbrace{\frac{|\hat{h}|^2}{|\hat{h}|^2 + K}}_{\text{stabilisation}} \frac{\hat{f}}{\hat{h}}$$

  with a positive parameter K.
- cannot diverge for $\hat{h} \to 0$:

$$\frac{|\hat{h}|^2}{|\hat{h}|^2 + K} \frac{1}{\hat{h}} \hat{f} \to 0 \quad \text{for} \quad \hat{h} \to 0.$$

- excellent linear method for deconvolution problems with additive Gaussian noise

Wiener Filtering (2)

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Influence of the Parameter K

- $K \to 0$ gives simple inverse filtering.
- Larger values for K create a stronger damping effect for small $|\hat{h}|$.
  This is particularly desirable if f suffers from massive high-frequent noise.
- Thus, K should increase with the estimated noise variance σ^2,
  e.g. by choosing
  $$K := 2\sigma^2.$$

Wiener Filtering (3)

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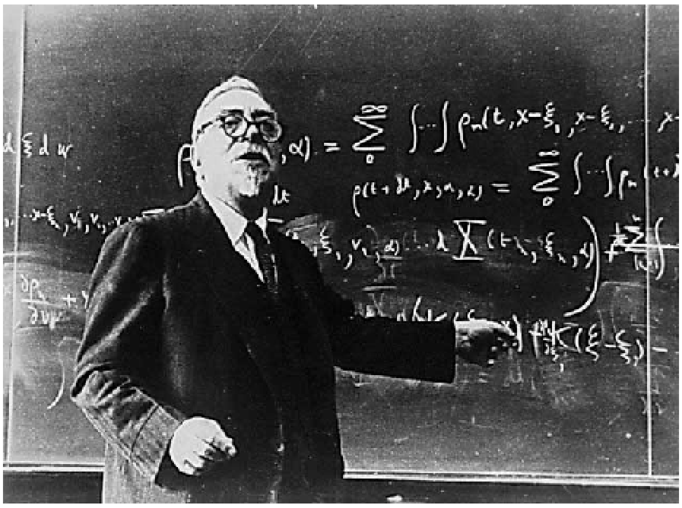

The American mathematician Norbert Wiener (1894–1964) studied problems of control theory and communication. He is regarded as the founder of cybernetics and a co-founder of information theory (with Claude Shannon). Source: `http://perso.wanadoo.fr/metasystems/Cybernetics.html`.

> *"He drove 150 miles to a math conference at Yale University.*
> *After the conference he forgot that he came by car and returned home by bus.*
> *The next morning, he went to his garage to get his car, found it was missing,*
> *and complained to the police that while he was away, someone stole his car."*

Wiener Filtering (4)

Top left: Original image (taken from the U2 album *Joshua Tree*). **Top right:** With simulated motion blur and some Gaussian noise. **Bottom left:** Deblurring with Wiener filtering. Source: http://www.owlnet.rice.edu/∼elec431/projects95/lords/wiener.html.

Variational Deconvolution Methods (1)

Variational Deconvolution Methods

Basic Model

- variational image restoration studied so far (Lecture 18):

$$E(u) \;:=\; \frac{1}{2}\int_{\Omega} \left((u-f)^2 + \alpha\,|\boldsymbol{\nabla} u|^2\right)\,dx\,dy\,.$$

- modification by introducing a symmetric shift-invariant blur kernel h:

$$E(u) \;:=\; \frac{1}{2}\int_{\Omega} \left((h*u-f)^2 + \alpha\,|\boldsymbol{\nabla} u|^2\right)\,dx\,dy\,.$$

- resulting Euler–Lagrange equation can be shown to be

$$h*(h*u-f) - \alpha\,\Delta u \;=\; 0\,.$$

Variational Deconvolution Methods (2)

Numerical Aspects

- A discrete model like in Lecture 17 creates a linear system of equations. However, due to the convolution it is no longer sparse in most cases. Iterative methods (based on matrix–vector multiplications) become expensive.
- more efficient alternative for such shift-invariant linear problems with convolution: Fourier transform
- Mirror the image f in x and y direction and extend it periodically.
- The Fourier transform of the Euler–Lagrange equation

$$h * (h * u - f) - \alpha\,(\partial_{xx}u + \partial_{yy}u) \;=\; 0$$

gives immediately

$$\hat{h}\left(\hat{h}\hat{u} - \hat{f}\right) \;-\; \alpha\Big(-\,4\pi^2\,\xi_1^2\,\hat{u} - 4\pi^2\,\xi_2^2\,\hat{u}\Big) \;=\; 0\,,$$

where ξ_1 and ξ_2 denote the frequencies in x and y direction.

Variational Deconvolution Methods (3)

- Solving for $\hat{u}$ yields

$$\hat{u} \;=\; \frac{\hat{h}\hat{f}}{\hat{h}^2 + 4\pi^2\alpha(\xi_1^2 + \xi_2^2)}\,.$$

- Transforming $\hat{u}$ back to the space domain gives u.

Modifications and Extensions

- If discontinuity-preserving deconvolution is required, one can use a nonquadratic regulariser $\Psi(|\nabla u|^2)$ (cf. Lecture 18).
- gives a nonlinear Euler-Lagrange equation
- Discretisation leads to a nonlinear system of equations.
- In this nonlinear case, no efficient alternatives in the Fourier domain exist.
- The method can also be modified to treat nonsymmetric kernels and colour images.

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From left to right: Left: Original colour image, 240×320 pixels. **Middle left:** Blurred with a nonsymmetric kernel. **Middle right:** Blur kernel, 61×52 pixels. **Right:** Result of a variational deconvolution approach with nonquadratic penalisation. Authors: M. Welk, D. Theis, T. Brox, J. Weickert.

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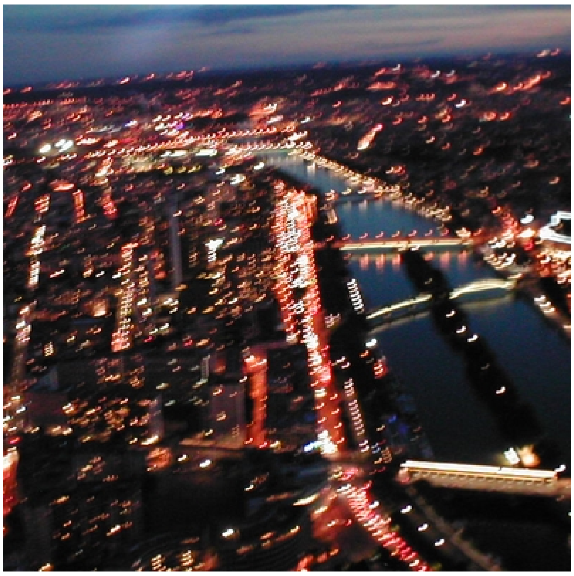
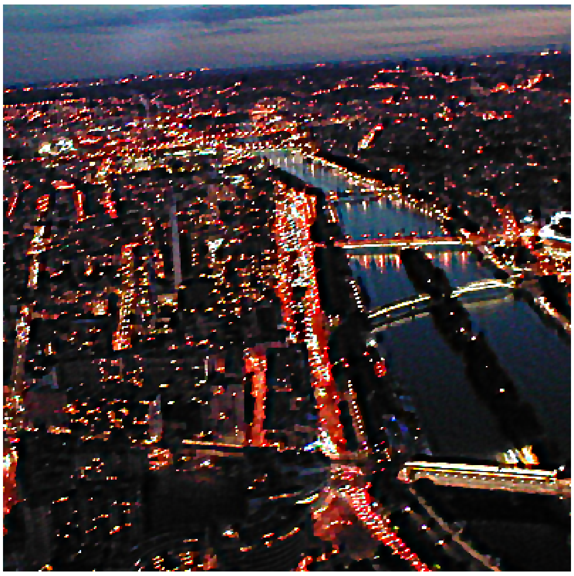

Left: Real-world image with motion blur, 400×400 pixels. The motion kernel can be estimated from the blurred point light sources. **Right:** After applying a variational deconvolution method with nonquadratic data and smoothness term. Author: M. Welk.

Blind Deconvolution

Problem

- Often also the blurring kernel is unknown.
- Is it possible to estimate
  - a shift-invariant convolution kernel h
  - and the unknown image u

  at the same time ?
- This task is called *blind deconvolution*.

Blind Deconvolution (2)

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Variational Blind Deconvolution

- One minimises an energy with regularisers for both u and h:

$$E(u,h) \;:=\; \frac{1}{2}\int_{\Omega} \left((h*u-f)^2+\alpha\,|\boldsymbol{\nabla} u|^2+\beta\,|\boldsymbol{\nabla} h|^2\right)\,dx\,dy$$

- creates two coupled Euler–Lagrange equations (cf. Lecture 18):

$$\begin{aligned} 0 &= h*(h*u-f)-\alpha\,\Delta u, \\ 0 &= u*(h*u-f)-\beta\,\Delta h. \end{aligned}$$

- Its discretisations can be solved in an iterative and alternating manner.
- Also here one can use nonquadratic regularisers.

Blind Deconvolution (3)

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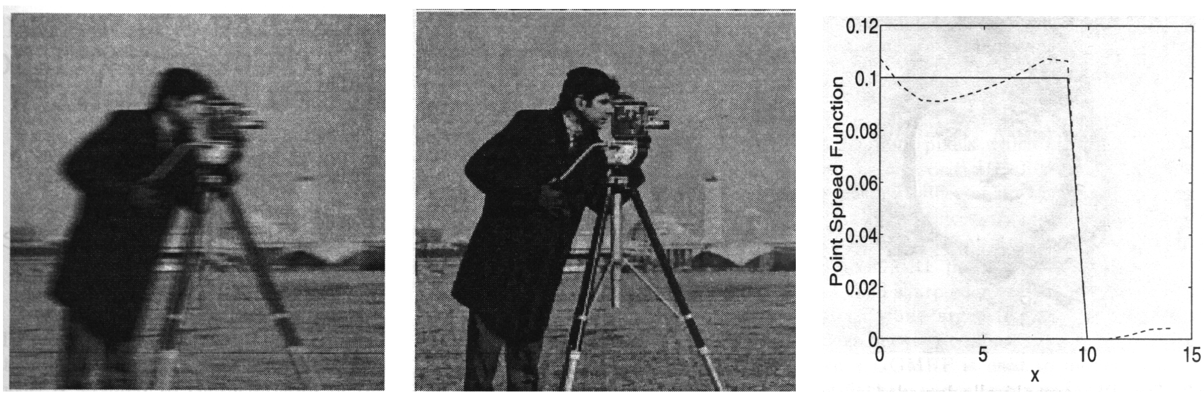

Left: Original image with noise and motion blur. **Middle:** Blind deconvolution with nonquadratic regularisers. **Right:** Estimated kernel (dotted line) compared with the correct kernel. Authors: Y.-L. You, M. Kaveh.

Summary

- Fourier methods allow to remove periodic perturbations.
- Homomorphic filtering reduces illumination inhomogeneities.
  It uses a logarithmic transform with subsequent highpass filtering.
- For a known shift-invariant blurring kernel, deconvolution can be performed in the Fourier domain.
- A naive inverse filtering is usually unstable.
  It is better to use pseudoinverse filtering.
- Wiener filtering belongs to the best linear deconvolution methods.
  It can also cope with a moderate amount of additive Gaussian noise.
- Alternatively one can use variational methods.
  For quadratic penalisers, efficient solvers in the Fourier domain exist.
  For nonquadratic penalisers, one must solve nonlinear systems of equations.
- Blind deconvolution simultaneously estimates also the unknown convolution kernel.

References

- F. M. Wahl: *Digitale Bildsignalverarbeitung.* Springer, Berlin, 1984.
  (Homomorphic filtering is treated in Section 3.3.3.
  Chapter 4 gives a good description of image deconvolution in the Fourier domain.)
- J. M. Parker: *Algorithms for Image Processing and Computer Vision*. Wiley, New York, 1997.
  (usually not a recommendable book, but the image restoration part is okay)
- K. R. Castleman: *Digital Image Processing.* Prentice Hall, Upper Saddle River, 1996.
  (for deconvolution in the Fourier domain)
- M. Petrou, P. Bosdogianni: Image Processing: The Fundamentals. Wiley, Chichester, 1999.
  (Chapter 6 gives many mathematical details about deconvolution in the Fourier domain.)
- M. Welk, D. Theis, T. Brox, J. Weickert: PDE-based deconvolution with forward-backward diffusivities and diffusion tensors. In R. Kimmel, N. Sochen, J. Weickert (Eds.): *Scale-Space and PDE Methods in Computer Vision*. Lecture Notes in Computer Science, Vol. 3459, Springer, Berlin, pp. 585–597, 2005.
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  (variational deconvolution)
- Y.-L. You, M. Kaveh: Anisotropic blind image restoration. *Proc. IEEE International Conference on Image Processing* (ICIP–96, Lausanne, Sept. 16–19, 1996), Vol. 2, pp. 461–464, 1996.
  (variational blind deconvolution)