x	y	z	f_5	m_1	M
0	0	0	0	0	0
0	0	1	1	1	1
0	1	0	1	0	1
0	1	1	0	0	1
1	0	0	0	0	1
1	0	1	1	0	1
1	1	0	1	0	1
1	1	1	0	0	1

$DCF(f_5) = m_1 \vee m_2 \vee m_5 \vee m_6 =$

$= \overset{0}{\bar{x}}\overset{0}{\bar{y}}\overset{1}{z} \vee \overset{0}{x}\overset{1}{\bar{y}}\overset{0}{z} \vee \overset{1}{x}\overset{0}{\bar{y}}\overset{1}{z} \vee \overset{1}{x}\overset{1}{y}\overset{0}{z} =$

$= \bar{x}\bar{y}z \vee x\bar{y}\bar{z} \vee x\bar{y}z \vee xy\bar{z}$

$\overset{0}{\bar{x}}\overset{1}{y}\overset{0}{z}$ $\overset{0}{\bar{x}}\overset{1}{y}\overset{1}{z}$

$m_2 = x\bar{y}\bar{z}$, $m_3 = x\bar{y}z$

$m_2 \vee m_3 = \overset{0}{x}\overset{1}{\bar{y}}\overset{0}{\bar{z}} \vee \overset{0}{x}\overset{1}{\bar{y}}\overset{1}{z} = \overset{0}{x}\overset{1}{\bar{y}}(\overset{0}{\bar{z}} \vee \overset{1}{z}) = x\bar{y}(\bar{z} \vee z) = x\bar{y}$

Karnaugh diagram

xyz	00	01	11	10
0		m_1		m_2
1		m_5		m_6

Fact

$max_1 = m_1 \vee m_5 = \bar{y}z$

$max_2 = m_2 \vee m_6 = y\bar{z}$

$M(f_5) = \{max_1, max_2\}$ maximal monoms

$C(f_5)$ = the set of central monoms

$M(f_5) = C(f_5) \Rightarrow$ 1st simplified case

$f_5^{DS} = max_1 \vee max_2 = \bar{y}z \vee y\bar{z}$

$CCF(f_5) = M_0 \wedge M_3 \wedge M_4 \wedge M_7 =$

$= (x^{\bar{0}} \vee y^{\bar{0}} \vee z^{\bar{0}}) \wedge (x^{\bar{0}} \vee y^{\bar{1}} \vee z^{\bar{1}}) \wedge (x^{\bar{1}} \vee y^{\bar{0}} \vee z^{\bar{0}}) \wedge (x^{\bar{1}} \vee y^{\bar{1}} \vee z^{\bar{1}})$

$= (x \vee y \vee z) \wedge (x \vee \bar{y} \vee \bar{z}) \wedge (\bar{x} \vee y \vee z) \wedge (\bar{x} \vee \bar{y} \vee \bar{z})$

x \ yz	00	01	11	10
0	M_0		M_3	
1	M_4		M_7	

We apply a dual simplification alg.

Dual factorization

$maxd_1 = M_0 \wedge M_2 = y^{\bar{0}} \vee z^{\bar{0}} = y \vee z$

$maxd_2 = M_3 \wedge M_7 = y^{\bar{1}} \vee z^{\bar{1}} = \bar{y} \vee \bar{z}$

$Md(f_5) = \{ \max d_1, \max d_2 \}$

maximal disjunction

$Cd(f_5) = Md(f_5) = 1^{st}$ case

↳ central disjunctions

$f_5^{CS} = \max d_1 \wedge \max d_2 = (y \vee z) \wedge (\bar{y} \vee \bar{z})$

2. $f_5(x_1, x_2, x_3, x_4) = x_1x_2x_3x_4 \vee x_1x_2\bar{x}_3x_4 \vee x_1x_2x_3\bar{x}_4 \vee$
$\vee \bar{x}_1\bar{x}_2\bar{x}_3\bar{x}_4 \vee \bar{x}_1x_2\bar{x}_3\bar{x}_4 \vee \bar{x}_1x_2x_3\bar{x}_4 \vee x_1\bar{x}_2x_3\bar{x}_4 \vee x_1\bar{x}_2\bar{x}_3\bar{x}_4 \vee$
$\vee x_1x_2\bar{x}_3\bar{x}_4 \vee x_1\bar{x}_2\bar{x}_3x_4 =$

$= m_{15} \vee m_{13} \vee m_{14} \vee m_4 \vee m_6 \vee m_{10} \vee m_8 \vee m_{12} \vee m_9$

$x_1x_2 \backslash x_3x_4$	00	01	11	10
00				
01	m_4			m_6
11	m_{12}	m_{13}	m_{15}	m_{14}
10	m_8	m_9		m_{10}

$\max_1 = m_{12} \vee m_{13} \vee m_8 \vee m_9 = x_1\bar{x}_3$

$\max_2 = m_4 \vee m_6 \vee m_{12} \vee m_{14} = x_2\bar{x}_4$

$\max_3 = m_{12} \vee m_{14} \vee m_8 \vee m_{10} = x_1\bar{x}_4$

$\max_4 = m_{12} \vee m_{13} \vee m_{15} \vee m_{14} =$

$= x_1x_2\bar{x}_3\bar{x}_4 \vee x_1x_2\bar{x}_3x_4 \vee x_1x_2x_3x_4 \vee x_1x_2x_3\bar{x}_4 = x_1x_2$

$M(f) = \{ max_1, max_2, max_3, max_4 \}$

$C(f) = \{ max_1, max_2, max_3, max_4 \}$

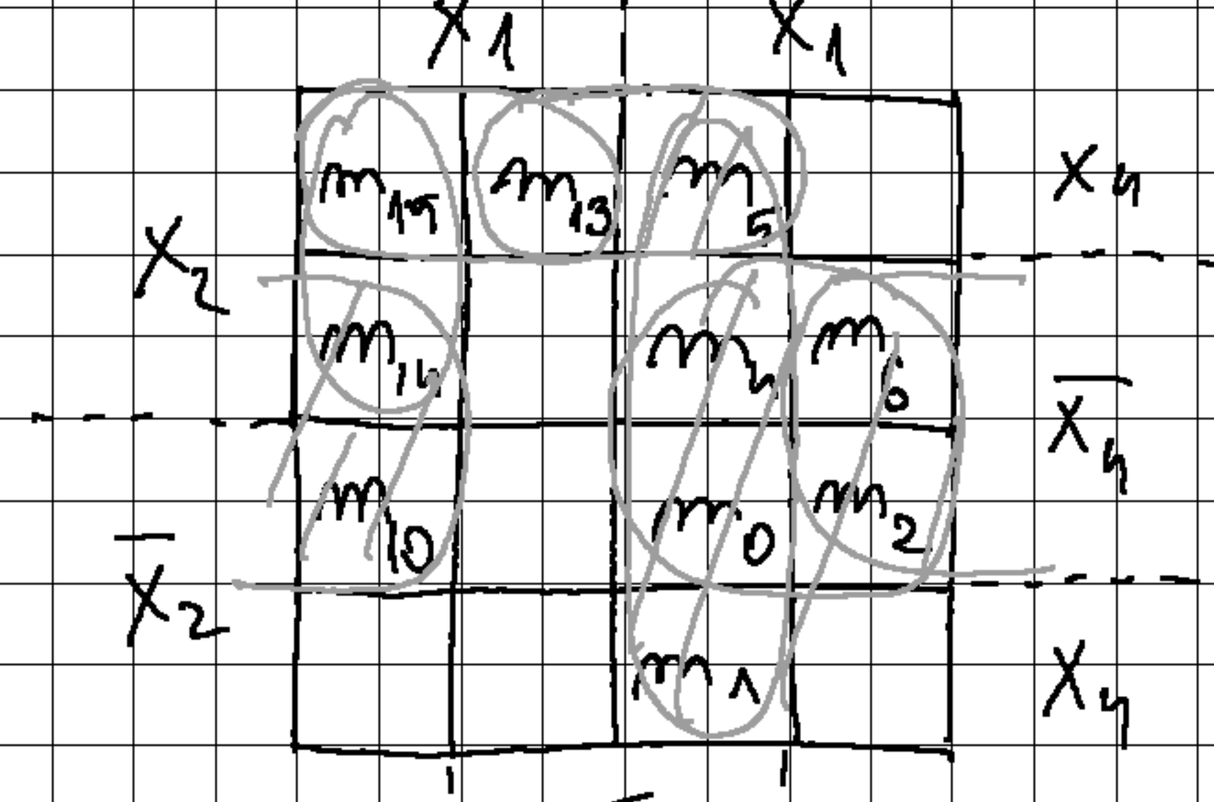
$M(f) = C(f) \Rightarrow$ 1st simplification case

$f_{\overline{S}} = max_1 \vee max_2 \vee max_3 \vee max_4 =$

$= x_1\overline{x_3} \vee x_2\overline{x_4} \vee x_1\overline{x_4} \vee x_1x_2$

4. $f_T(x_1, x_2, x_3, x_4) = x_3\overline{x_4} \vee \overbrace{x_1x_2x_3x_4}^{max_{15}} \vee \overline{x_3}x_2x_4 \vee$

$\vee \overline{x_1}\,\overline{x_3} \vee \overline{x_1}\,\overline{x_4}$



$= m_{13} \vee m_{15} \vee m_7 \vee m_{14} \vee$

$\vee m_4 \vee m_6 \vee m_{10} \vee m_0 \vee$

$\vee m_2 \vee m_1$

factorization:

$max_1 = m_9 \vee m_4 \vee m_0 \vee m_1 = \overline{x_1}\,\overline{x_3}$

$max_2 = m_4 \vee m_6 \vee m_0 \vee m_2 = \overline{x_1}\,\overline{x_4}$

$max_3 = m_2 \vee m_6 \vee m_{14} \vee m_{10} = x_3\overline{x_4}$

$max_4 = m_5 \vee m_{14} = x_1 x_2 x_3$

$max_5 = m_{15} \vee m_{13} = x_1 x_2 x_4$

$max\,6 = m_{13} \vee m_5 = x_2 \bar{x}_3 x_4$

$M(f_5) = \{ max_1, max_2 \dots max_6 \}$

$C(f_5) = \{ max_1, max_3 \}$ (must contain a cell that is circled once)

$M(f_5) \neq C(f_5) \neq 0$ this is a 2nd case - we take $g = max_1 \vee max_3$

$f_5^S = \emptyset \vee max_5 = \bar{x}_1 \bar{x}_3 \vee x_3 \bar{x}_4 \vee x_1 x_2 x_4$

3. $f_5(x_1, x_2, x_3) = x_1 (x_2 \downarrow x_3) \vee \bar{x}_1 \bar{x}_2 x_3 \vee (\overline{\bar{x}_1 \vee (\bar{x}_2 \uparrow x_3)}) \vee$

$\vee \bar{x}_1 x_2 \bar{x}_3 = x_1 (\overline{x_2 \vee x_3}) \vee \bar{x}_1 \bar{x}_2 x_3 \vee (\overline{\bar{x}_1 \vee (\bar{x}_2 \wedge x_3)}) \vee \bar{x}_1 x_2 \bar{x}_3 =$

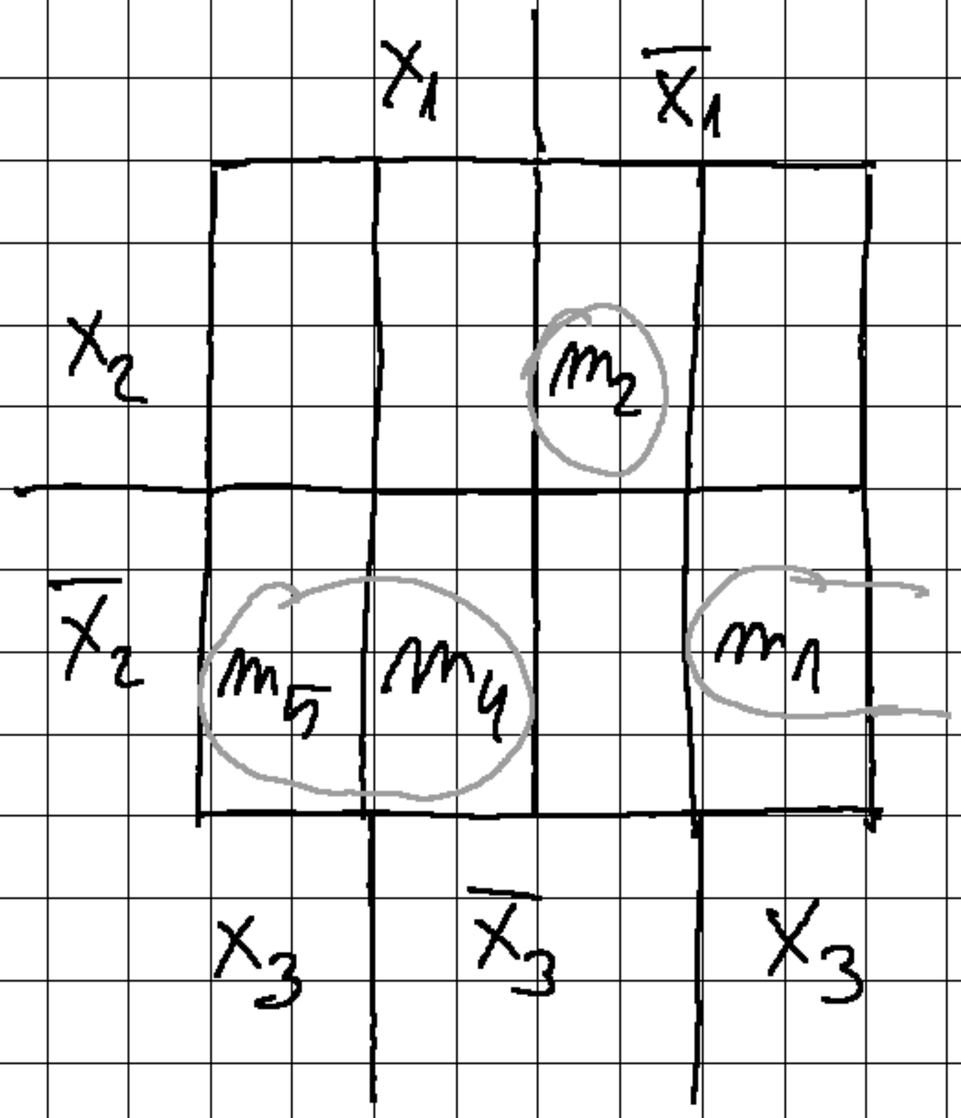
$a \downarrow b = (\overline{a \vee b})$

$a \uparrow b = (\overline{a \wedge b})$

$a \oplus b = \bar{a} b \vee a \bar{b}$

$= x_1 \bar{x}_2 \bar{x}_3 \vee \bar{x}_1 \bar{x}_2 x_3 \vee x_1 \bar{x}_2 x_3 \vee \bar{x}_1 x_2 \bar{x}_3 =$

$= m_4 \vee m_1 \vee m_5 \vee m_2$



$max_1 = m_4 \vee m_5 = x_1\bar{x}_2$

$max_2 = m_1 \vee m_5 = \bar{x}_2 x_3$

0-fact:

$max_3 = m_2 = x_1 x_2 \bar{x}_3$

$M(f_5) = \{max_1, max_2, max_3\}$

$C(f_5) = M(f_5) \Rightarrow 1^{st}$ case

DS

$f_5 = max_1 \vee max_2 \vee max_3 = x_1x_2 \vee x_2x_3 \vee x_1x_2x_3$