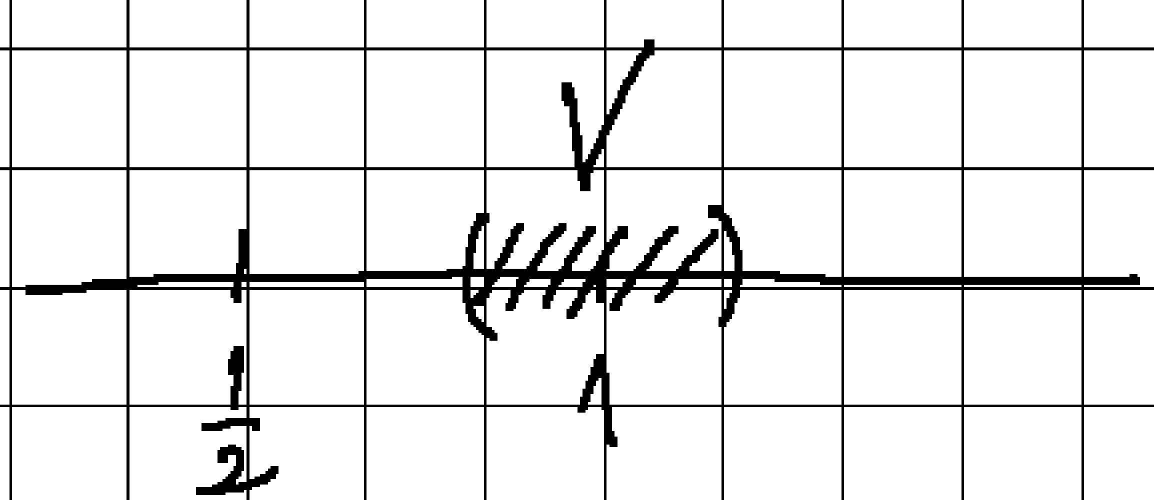
Accumulation points:

Ex: $A = \{1, \frac{1}{2}, \frac{1}{3}, \dots\}$

$\mathrm{cl}\, A = A \cup \{0\}$

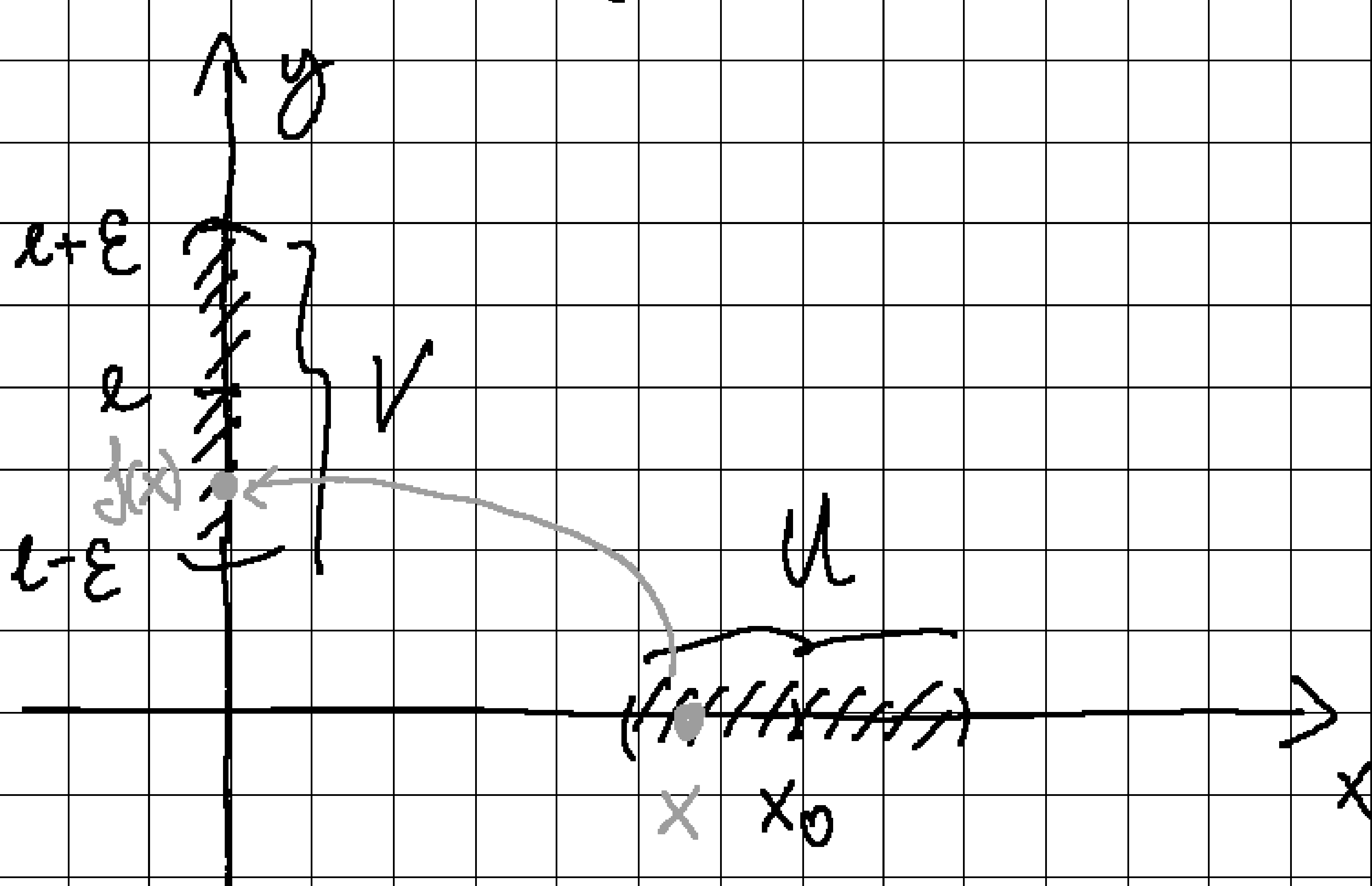
isolated points $= A$

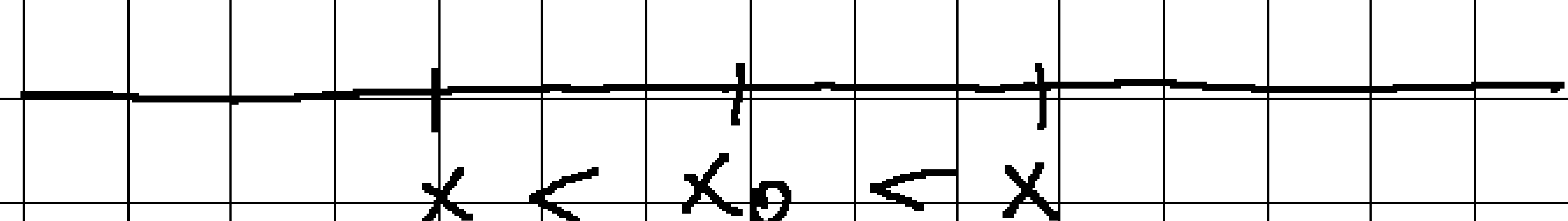
$A' = \{0\}$



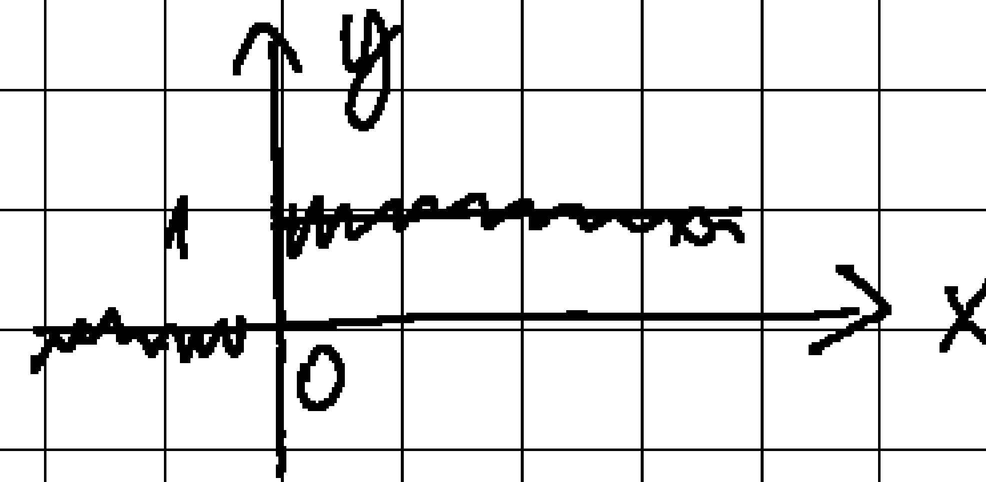
$V \cap \{A \setminus \{1\}\} \neq \emptyset$

Limit of a function:





Ex: $f(x) = \begin{cases} 0, & \text{if } x < 0 \\ 1, & \text{if } x \geq 1 \end{cases}$



$$\lim_{\substack{x \to 0 \\ x < 0}} f(x) = 0 \neq \lim_{\substack{x \to 0 \\ x > 0}} f(x) = 1$$

$$\Rightarrow \nexists \lim_{x \to 0} f(x)$$

$$\frac{\pi}{2} + 2n\pi$$

Ex: $\sin\left(\frac{1}{x}\right)$, $x_n = \frac{1}{2n\pi + \frac{\pi}{2}}$, $\sin x_n = 1$

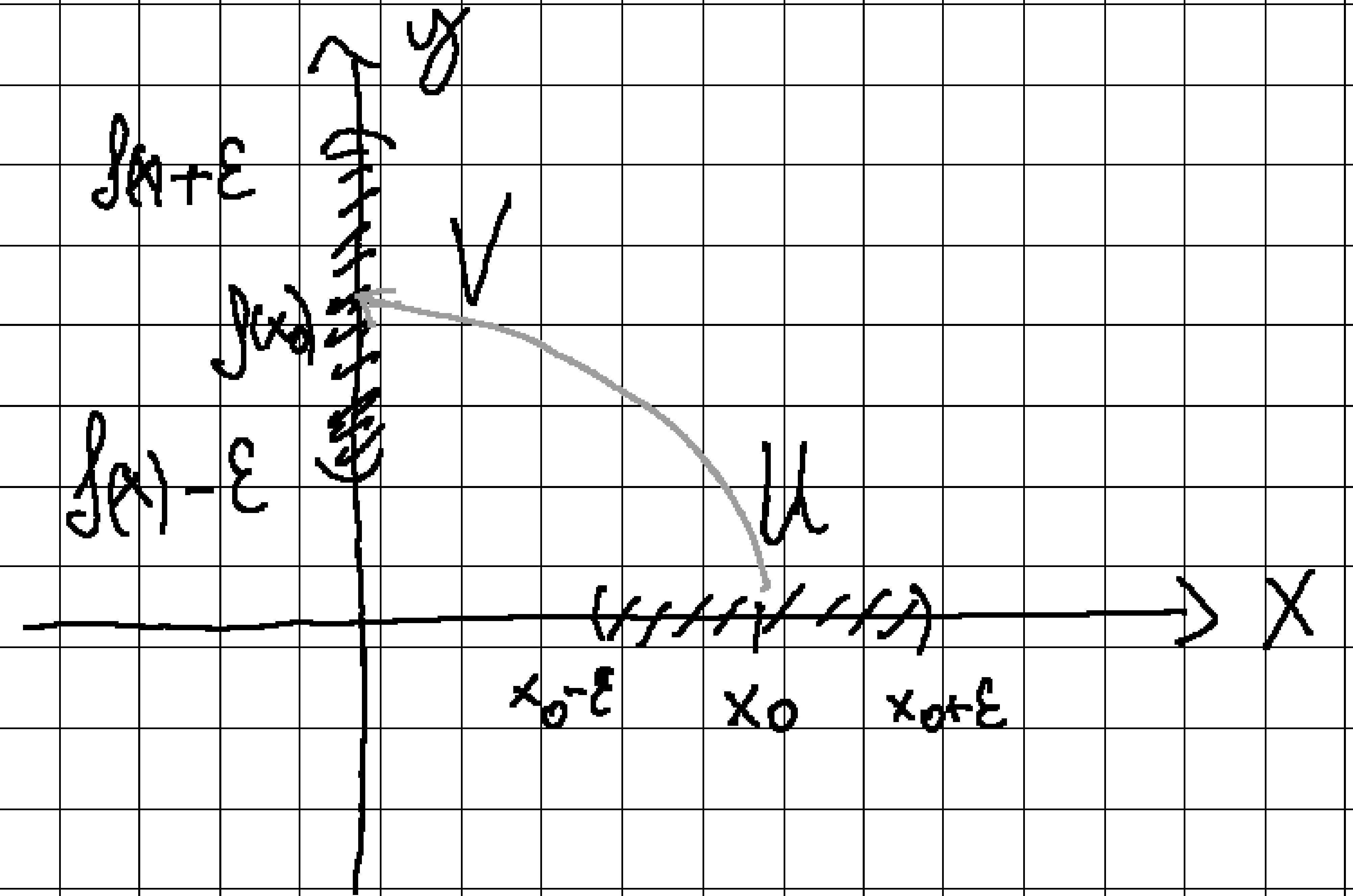
$y_n = \frac{1}{2n\pi - \frac{\pi}{2}}$, $\sin \frac{1}{y_n} = -1$

$$-\frac{\pi}{2} + 2n\pi$$

$x_n \to 0$, $y_n \to 0$, $\sin\left(\frac{1}{x_n}\right) \to 1$, $\sin\left(\frac{1}{y_n}\right) \to 1$

$$\Rightarrow \nexists \lim_{x \to 0} \sin \frac{1}{x}$$

Continuity



Ex: $f: \mathbb{R} \to \mathbb{R}$, $f(x) = \begin{cases} 1, & \text{if } x \in \mathbb{Q} \\ 0, & \text{if } x \notin \mathbb{Q} \end{cases}$

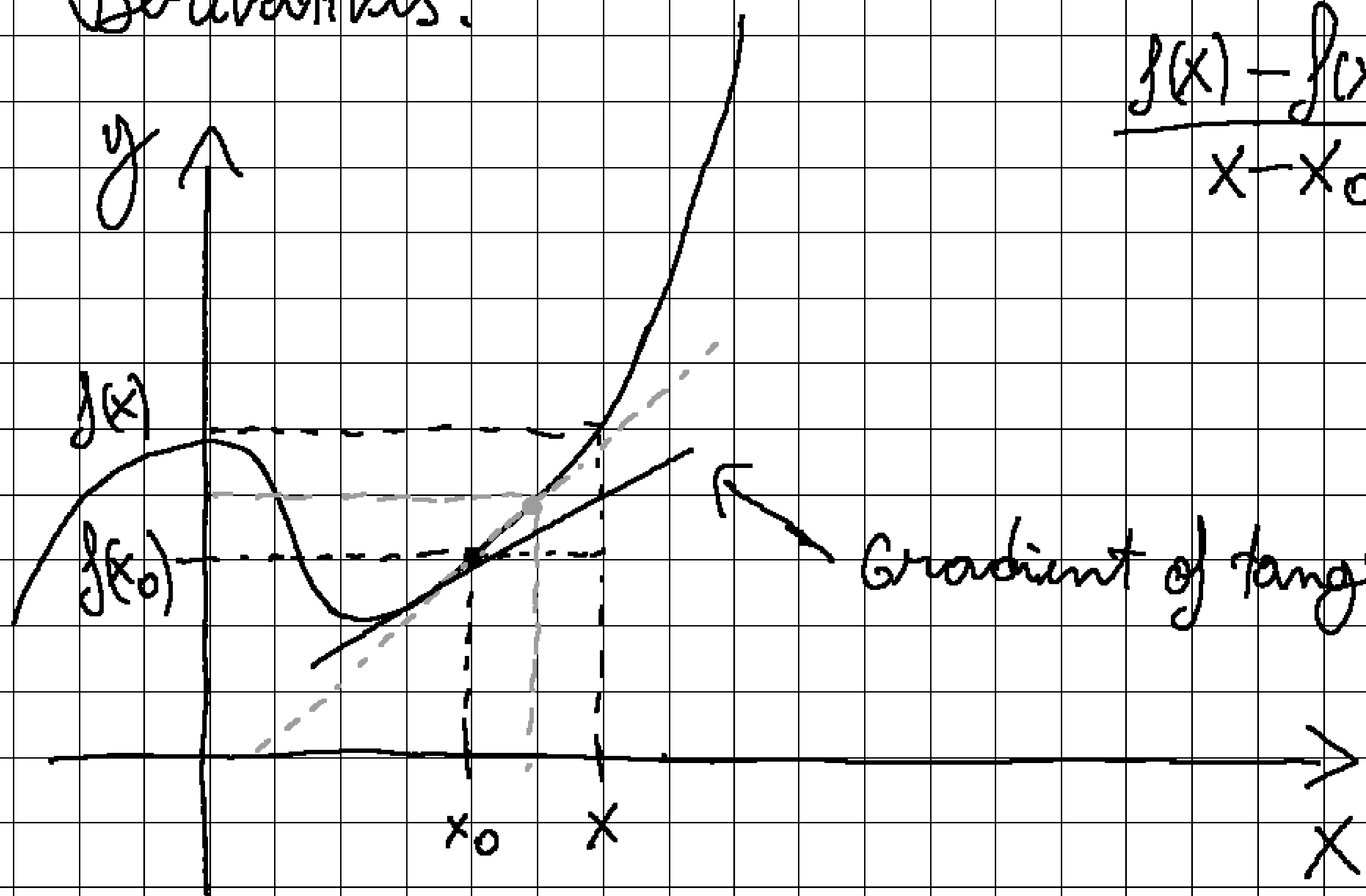
Let $x_0 \in \mathbb{R}$. Let $x_n \to x_0$, $x_n \in \mathbb{Q}$

$y_n \to x_0$, $y_n \notin \mathbb{Q}$

$$\left.\begin{aligned} x_n \to x_0, &\quad f(x_n) = 1 \\ y_n \to x_0, &\quad f(y_n) = 0 \end{aligned}\right\} \Rightarrow \nexists \lim_{x \to x_0} f(x)$$

$\Rightarrow f$ is <u>NOT</u> cont at x_0

Derivatives:



$\frac{f(x)-f(x_0)}{x-x_0}$ = gradient of a line

$= \frac{\text{Rise}}{\text{Run}} = \frac{\Delta y}{\Delta x}$

Gradient of tangent = $f'(x_0)$

$$\lim_{x\to x_0}\frac{f(x)-f(x_0)}{x-x_0} = f'(x_0) = \lim_{x\to x_0}\frac{\Delta y}{\Delta x} = \frac{dy}{dx}$$

Ex: $f(x) = \begin{cases} 0, & x<0 \\ 1, & x\geq 0 \end{cases}$

Discont. at 0 NOT Diff at 0

cont on $\mathbb{R}\setminus\{0\}$ Diff on $\mathbb{R}\setminus\{0\}$

Ex: $f(x) = \begin{cases} 0, & x<0 \\ x, & x\geq 0 \end{cases}$

$\lim_{x\to 0} f(x) = 0 = f(0) \Rightarrow$ CONT at 0

CONT on $\mathbb{R}$

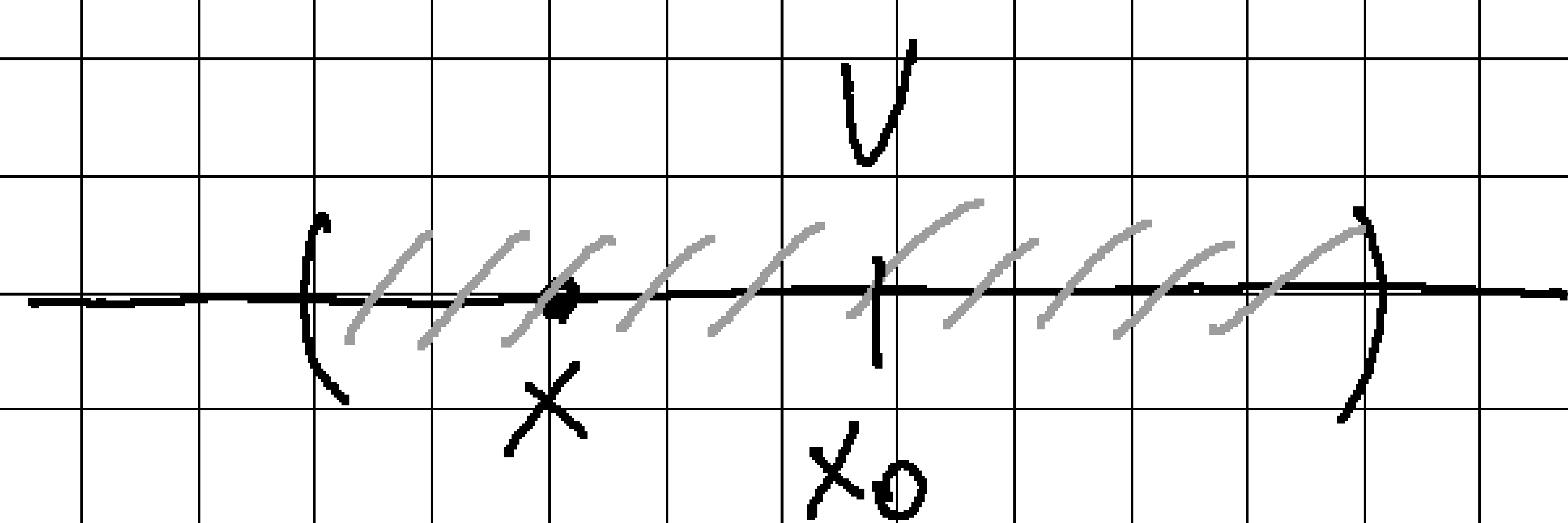
$$\left.\lim_{\substack{x\to 0\\ x>0}}\frac{f(x)-f(0)}{x-0} = \lim_{x\searrow 0}\frac{x}{x} = 1\right\} \Rightarrow$$

$$\lim_{x \nearrow 0} \frac{f(x)-f(0)}{x-0} = \lim_{x \nearrow 0} 0 = 0 \;/\; \Rightarrow \nexists\, f'(0)$$

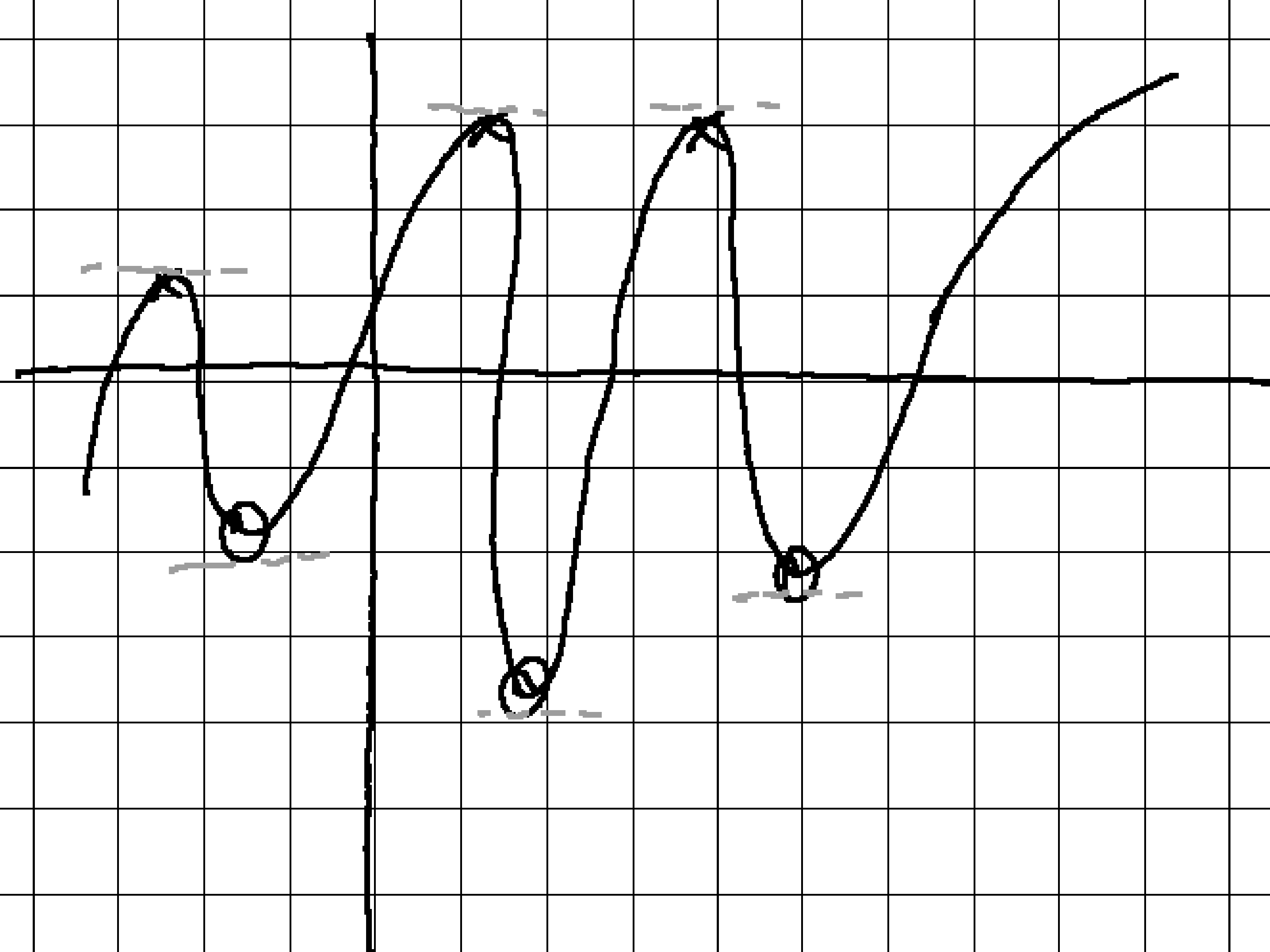
$$\frac{df}{dx} = \frac{df}{dg} \cdot \frac{dg}{dx}$$

CHAIN RULE

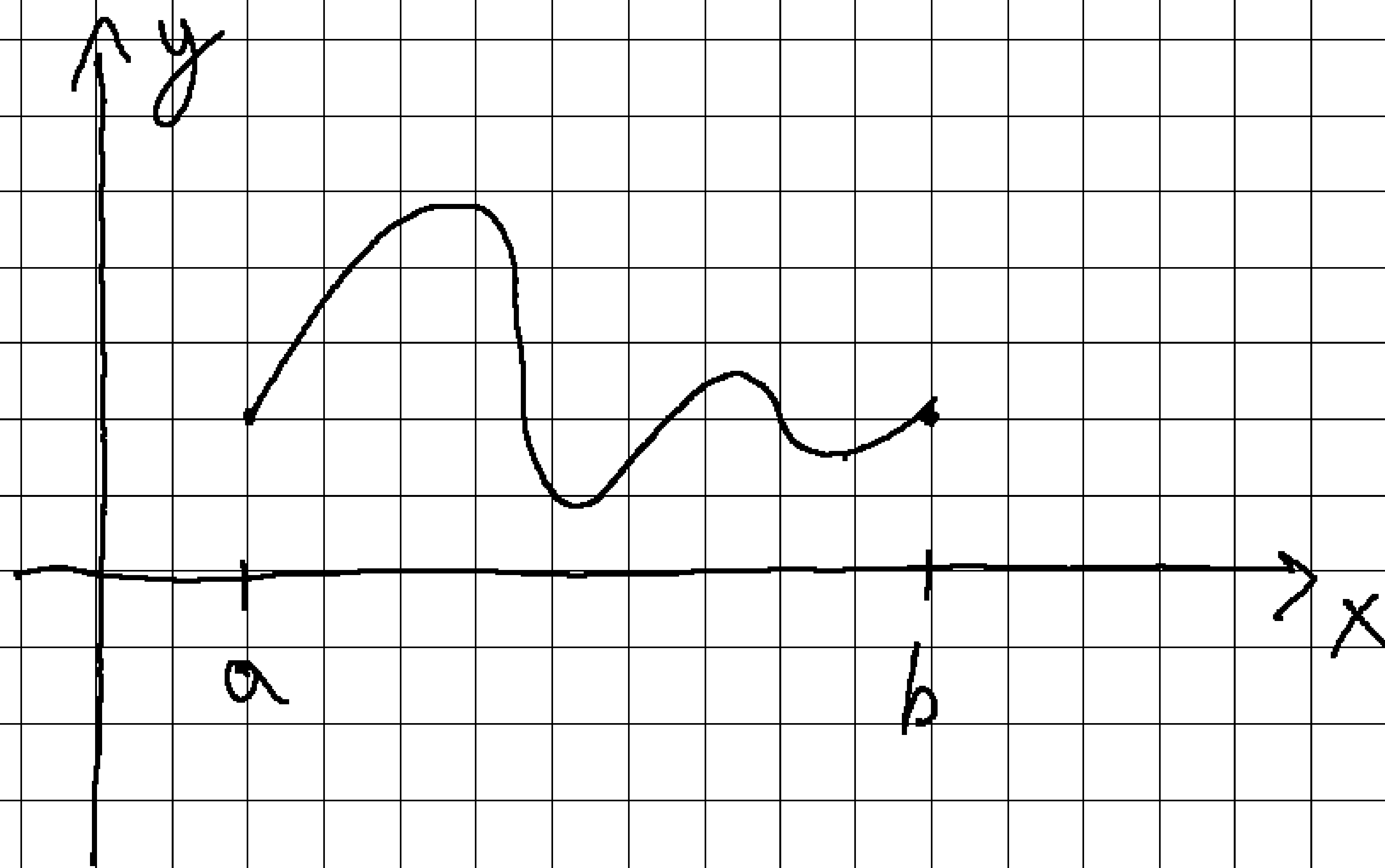
Def 20:



$f(x_0) \le f(x)$, $\forall x \in V$: x_0 is a local MIN



Theorem 22 (Rolle):



Theorem 23 (Mean value theorem)