10.5 $\vdash (p \to (q \to r)) \to (q \to (p \to r))$

Reverse theorem deduction

If $U_1, U_{n-1} \vdash U_n \to U$ then $U_1 \dots U_{n-1}, U_n \vdash U$

I Deduction

If $U_1, \dots, U_{n-1}, U_n \vdash U$ then $U_1, \dots U_{n-1} \vdash U_n \to U$

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Check homework

1.2. In a plane there are lines parallel to a constant line $d$ and there are lines perpendicular to $d$.

$\mathcal{D}$ - set of all the lines in a plane $P$,

$d \in \mathcal{D}$, $d$ - constant

Binary predicat symbols

$par : \mathcal{D} \times \mathcal{D} \to \{T, F\}$, $par(x, y) = T$ if $x \parallel y$

$per : \mathcal{D} \times \mathcal{D} \to \{T, F\}$, $per(x, y) = T$ if $x \perp y$

$(\exists x)_{x \in \mathcal{D}} (par(x, d)) \wedge (\exists y)_{y \in \mathcal{D}} (per(y, d))$

Symmetry: $(\forall x)(\forall y)(par(x, y) \Rightarrow par(y, x))$

$(\forall x)(\forall y)(per(x, y) \to per(y, x))$

Transitivity: $(\forall x)(\forall y)(\forall z)(par(x, y) \wedge par(y, z) \to par(x, z))$

$(\forall x)(\forall y)(\forall z)(per(x, y) \wedge per(y, z) \to par(x, z))$

1.5. if x and y are positive prime numbers, $x \neq y$, $x \neq 2$ and $y \neq 2$, their sum and difference are even numbers and their product is an odd number

$D$ = the set of all $\mathbb{N}$, $2 \in D$

Unary predicate symbols

prime : $D \to \{T, F\}$, prime(x) = T if x is prime

even : $D \to \{T, F\}$, even(x) = T if x is even

Binary function symbols

sum : $D \times D \to D$, sum(x, y) = x + y

diff : $D \times D \to D$, diff(x, y) = x − y

prod : $D \times D \to D$, prod(x, y) = x * y

eq : $D \times D \to \{T, F\}$, eq(x, y) = T if x = y

$(\forall x)(\forall y)$ (prime(x) $\wedge$ prime(y) $\wedge \neg$ eq(x, y) $\wedge \neg$ eq(x, z) $\wedge$
$\wedge$ eq(y, z) $\to$ even(sum(x, y)) $\wedge$ even(diff(x, y)) $\wedge$
$\wedge \neg$ even(prod(x, y)))

commutativity : $(\forall x)(\forall y)$ eq(sum(x, y), sum(y, x))
$(\forall x)(\forall y$ eq(prod(x, y), prod(y, x))

associativity : $(\forall x)(\forall y)(\forall z)$ eq(sum(x, sum(y, z)),
, sum(sum(x, y), z))

neutral element: $(\forall x)\ eq(sum(x,0),x)$

distributivity: $(\forall x)(\forall y)(\forall z)\ eq(prod(x, sum(y,z)),$
$, sum(prod(x,y), prod(x,z)))$

2.3. CS students like either algebra or logic, all of them like java but only Bill likes history

D1 - set of all students

D2 - set of all disciplines.

algebra (A), logic (L), history (H) $\in D_2$

$CS: D_1 \to \{T,F\} \mid CS(x) = T$ if $x$ is a CS student

$LK: D_1 \times D_2 \to \{T,F\} \mid LK(x,y) = T$ if $x$ likes $y$

$$(\forall x)_{x \in D_1} \left[ CS(x) \to LK(x,A) \oplus LK(x,L)\right) \wedge$$
$$\wedge\ LK(x,J) \wedge (\neg eq(x, Bill) \to \neg LK(x,H))\Big] \wedge$$
$$\wedge\ Likes(Bill,H)$$

3. $H_1$: if x is the King and y is his oldest son, then y can become King

$H_2$: if x is the King and y defeats x, then y will become the King

$H_3$: Richard III is the King

$H_4$: Henry VII defeated Richard III

$H_5$: Henry VIII is Henry's VII's oldest son

C: Can Henry VIII become king?

$$H_1, H_2, H_3, H_4, H_5 \vdash C$$

$H_1$: $(\forall x)(\forall y)(\text{King}(x) \wedge \text{oldestson}(y, x) \rightarrow \text{King}(y))$

$H_2$: $(\forall x)(\forall y)(\text{King}(x) \wedge \text{Defeats}(y, x) \rightarrow \text{King}(y))$

$H_3$: $\text{King}(R_3)$

$H_4$: $\text{Defeats}(H_7, R_3)$

$H_5$: $\text{Oldestson}(H_8, H_7)$

C: $\text{King}(H_8)$ ?

$$(\forall x)\, U(x) \xrightarrow[\substack{\text{univ.}\\ \text{inst.}}\ ;\ [x \leftarrow t]]{} U(t)$$

$$H_2 \xrightarrow[\substack{\text{univ.}\\ \text{inst.}}\ [X \leftarrow R_3]]{} (\forall y)\,(King(R_3) \wedge Defeats(y, R_3) \rightarrow$$

$$\rightarrow King(y)) \ : \ f_6$$

$$H_1 \xrightarrow[\substack{\text{univ.}\\ \text{inst.}}\ [y \leftarrow H_7]]{} King(R_3) \wedge Defeats(H_7, R_3) \rightarrow King(H_7) : f_7$$

$$f_8 = H_3 \wedge H_4 = King(R_3) \wedge Defeats(H_7, R_3)$$

$$f_8, f_7 \xrightarrow[mp]{} King(H_7) \ : \ f_9$$

$$H_1 \xrightarrow[\substack{\text{univ.}\\ \text{inst.}}\ [X \leftarrow H_7]]{} (\forall y)\,(King(H_7) \wedge oldestson(y, H_7) \rightarrow$$

$$\rightarrow King(y)) \ : \ f_{10}$$

$$f_{10} \xrightarrow[\substack{\text{univ.}\\ \text{inst.}}\ [y \leftarrow H_8]]{} King(H_7) \wedge oldestson(H_8, H_7) \rightarrow$$

$$\rightarrow King(H_8) \ : \ f_{11}$$

$$f_{12} : f_9 \wedge H_5 = King(H_7) \wedge oldestson(H_8, H_7) \ : \ f_{13}$$

$$f_{13}, f_{11} \xrightarrow[mp]{} King(H_8) \ : \ C$$