

A function that is continuous everywhere but differentiable nowhere is the Weierstrass function:

$$f(x) = \sum_{n=0}^{\infty} a^n \cos(b^n \pi x)$$

where $0 < a < 1$ and $b > 0$ s.t. $a \cdot b > 1 + \frac{3\pi}{2}$
 $b \cdot 2 = 1$

this condition ensures that the function is differentiable nowhere

Each term in the Weierstrass series is a continuous function \Rightarrow is continuous everywhere
here is the graph: