1. $X\begin{pmatrix} 0 & 1 & 2 \\ 0.7 & 0.2 & 0.1 \end{pmatrix}$ $\quad X^2\begin{pmatrix} 0^2 & 1^2 & 2^2 \\ 0.7 & 0.2 & 0.1 \end{pmatrix}$

\$500

a) $E(500 \cdot X) = 500 \cdot E(X) = 500 \cdot \sum_{i=0}^{2} x_i p_i =$

$= 500 \cdot (0 \cdot 0.7 + 1 \cdot 0.2 + 2 \cdot 0.1) = 500 \cdot 0.4 = 200$

b) $\sigma(X) = Std(X) = \sqrt{V(X)}$

$V(X) = E(X^2) - (E(X))^2 = 0{,}6 - 0{,}16 = 0{,}44$

$E(X^2) = 0 \cdot 0.7 \cdot 1 \cdot 0.2 \cdot 4 \cdot 0.1 = 0{,}6$

$V(500X) = 500^2 \cdot V(X) = 250000 \cdot 0{,}44 = 110.000$

$\sigma(500X) = \sqrt{V(500X)} = 331{,}66$

3. $f(x) = \begin{cases} \dfrac{3}{x^4} & \text{for } x \geq 1 \\ 0 & \text{for } x < 1 \end{cases}$

f – continuous $\Rightarrow E(X) = \int_{\mathbb{R}} x\, f(x)\, dx$

$$E(X) = \int_1^\infty x \cdot \frac{3}{x^4}\, dx = 3 \int_1^\infty \frac{1}{x^3}\, dx = 3 \cdot \left(-\frac{1}{2x^2}\right)\Big|_1^\infty =$$

$= 3 \cdot \left(0 + \frac{1}{2}\right) = \frac{3}{2}$ years

2. $X \sim \text{geometric}\left(\frac{1}{10}\right)$

$$E(X) = \frac{1-p}{p} = \frac{1-\frac{1}{10}}{\frac{1}{10}} = \frac{9}{10} \cdot 10 = 9$$

4. Risk = amount of variability from the expected return = variance

Optimal = high expected return and low variance

$A \begin{pmatrix} -1 & 2 & 0 \\ 0,2 & 0,6 & 0,2 \end{pmatrix}$ $A^2 \begin{pmatrix} 1 & 4 & 0 \\ 0,2 & 0,6 & 0,2 \end{pmatrix}$

$B \begin{pmatrix} -1 & 3 & 0 \\ 0,3 & 0,6 & 1 \end{pmatrix}$ $B^2 \begin{pmatrix} 1 & 9 & 0 \\ 0,3 & 0,6 & 1 \end{pmatrix}$

$E(A) = 1$ $\qquad E(A^2) = 2{,}6$

$E(B) = 1{,}5$ $\qquad E(B^2) = 5{,}7$

$V(A) = 1{,}6$

$V(B) = 3{,}45$

a) 30 shares in A

$E(30A) = 30 \cdot E(A) = 30$

$V(30A) = 900 \cdot V(A) = 1440$

b) 20 shares in B

$E(20B) = 20 \cdot E(B) = 20$

$V(20B) = 400 \cdot V(B) = 1380$

c) 15 shares in A & 10 shares in B

$E(15A + 10B) = E(15A) + E(10B) = 15 \cdot E(A) + 10 \cdot E(B) = 30$

$V(15A + 10B) = V(15A) + V(10B) = 15 \cdot V(A) + 10 \cdot V(B) =$

$= 705$

5. $y = \dfrac{X - E(X)}{\sigma(X)}$

$$E(y) = E\left(\frac{X-E(X)}{\sigma(X)}\right) = \frac{1}{\sigma(X)}\left(E(X) - E(X)\right) = 0$$

$$V(y) = E(y^2) - \left(E(y)\right)^2 = E(y^2) = E\left(\left(\frac{X-E(X)}{\sigma(X)}\right)^2\right) =$$

$$= \frac{1}{(\sigma(X))^2} E\left(\left(X - E(X)\right)^2\right) = \frac{V(X)}{V(X)} = 1$$

6. Obs: $E(h(X,Y)) = \int_{\mathbb{R}}\int_{\mathbb{R}} h(x,y)\cdot f(x,y)\,dx\,dy$

$f(x,y) = x+y \quad (x,y) \in [0,1]\times[0,1]$

$$E(X) = \int_{\mathbb{R}}\int_{\mathbb{R}} x f(x,y)\,dx\,dy = \int_0^1\int_0^1 x(x+y)\,dx\,dy =$$

$$= \int_0^1\int_0^1 x^2 + xy\,dx\,dy = \int_0^1 \left(\frac{x^3}{3} + \frac{x^2}{2}y\right)\Big|_0^1 dy =$$

$$= \int_0^1 \frac{1}{3} + \frac{1}{2}y\,dy = \frac{1}{3} + \frac{1}{4}y^2\Big|_0^1 = \frac{7}{12} = E(Y)$$

$$E(X^2) = \int_{\mathbb{R}}\int_{\mathbb{R}} x^2 f(x,y)\,dx\,dy = \int_0^1\int_0^1 x^3 + x^2 y\,dx\,dy =$$

$$= \int_0^1 \left(\frac{x^4}{4} + \frac{x^3}{3}y\right)\Big|_0^1 dy = \frac{1}{4} + \frac{1}{3}\int_0^1 y\,dy = \frac{1}{4} + \frac{1}{6} = \frac{5}{12} = E(Y^2)$$

$$V(X) = \frac{5}{12} - \left(\frac{7}{12}\right)^2 = \frac{11}{144} = V(Y)$$

$$E(X \cdot Y) = \iint_{\mathbb{R}\mathbb{R}} xy\, f(x,y)\, dx\, dy = \int_0^1 \int_0^1 x^2 y + y^2 x \, dx\, dy =$$

$$= \int_0^1 \left(\frac{x^3}{3} y + \frac{x^2}{2} y^2 \right) \Big|_0^1 dy = \int_0^1 \frac{y}{3} + \frac{y^2}{2} dy = \frac{5}{6} \cdot \frac{y^3}{3} \Big|_0^1 =$$

$$= \frac{1}{3}$$

$$Cov(X,Y) = E(X \cdot Y) - E(X) \cdot E(Y) = \frac{1}{3} - \frac{49}{144} = -\frac{1}{144}$$

$$\rho(X,Y) = \frac{Cov(X,Y)}{\sqrt{V(X)} \cdot \sqrt{V(Y)}} = \frac{-\frac{1}{144}}{\frac{11}{144}} = -\frac{1}{11}$$

Since $\rho(X,Y) = -\frac{1}{11}$ there is a very weak linear relationship between X and Y