Basic properties

(i) $\underbrace{k\cdot 0 + k\cdot v}_{} = k\cdot(0+v) = \underbrace{k\cdot v}_{} \quad \Big| -(k\cdot v)$

$k\cdot 0 = 0$

(iii) $\underbrace{k\cdot(v-v') + k\cdot v'}_{} = k\cdot(v-\cancel{v'}+\cancel{v'}) = \underbrace{k\cdot v}_{} \Big| -k\cdot v'$

$k\cdot(v-v') = k\cdot v - k\cdot v'$

$\boxed{\Rightarrow}$ $k\cdot v = 0$

I $k=0$ ✓

II $k\neq 0 \Rightarrow \exists k^{-1} \Rightarrow k^{-1}\cdot(k\cdot v) = k^{-1}\cdot 0 \Rightarrow$

$\Rightarrow (k^{-1}\cdot k)\cdot v = 0 \Rightarrow 1\cdot v = 0 \Rightarrow v = 0$

$\boxed{\Leftarrow}$ $k=0$ or $v=0 \Rightarrow k\cdot v = 0$ by the previous property

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S subgroup of $(V,+) \Leftrightarrow \begin{cases} S \neq \emptyset \\ \forall v_1, v_2 \in S,\ v_1 - v_2 \in S \end{cases}$

$S = \{(x,y,z) \in \mathbb{R}^3 \mid x+y+z = 0\}$

We show that $S \leq_{\mathbb{R}} \mathbb{R}^3$

- $(0,0,0) \in S \neq \emptyset$
- let $k_1, k_2 \in \mathbb{R}$ and $v_1 = (x_1, y_1, z_1), v_2 = (x_2, y_2, z_2) \in S$

$\Rightarrow k_1 \cdot v_1 + k_2 \cdot v_2 = k_1(x_1, y_1, z_1) + k_2(x_2, y_2, z_2) =$

$= (k_1 \cdot x_1, k_1 \cdot y_1, k_1 \cdot z_1) + (k_2 x_2, k_2 y_2, k_2 z_2) =$

$= (k_1 x_1 + k_2 x_2, k_1 y_1 + k_2 y_2, k_1 z_1 + k_2 z_2) \in \mathbb{R}^3$

and $(k_1 x_1 + k_2 x_2) + (k_1 y_1 + k_2 y_2) + (k_1 z_1 + k_2 z_2) =$

$= k_1 \underbrace{(x_1 + y_1 + z_1)}_{=0} + k_2(x_2 + y_2 + z_2) = 0 \quad \Rightarrow k_1 v_1 + k_2 v_2 \in S$