1.

c) $\displaystyle\int_0^{\frac{\pi}{2}}\int_x^{\frac{\pi}{2}} \frac{\sin y}{y}\,dy\,dx = J$

$0 \le x \le \frac{\pi}{2}$

$x \le y \le \frac{\pi}{2}$

$\Rightarrow \displaystyle\int_0^{\frac{\pi}{2}}\int_0^{y} \frac{\sin y}{y}\,dx\,dy = J$

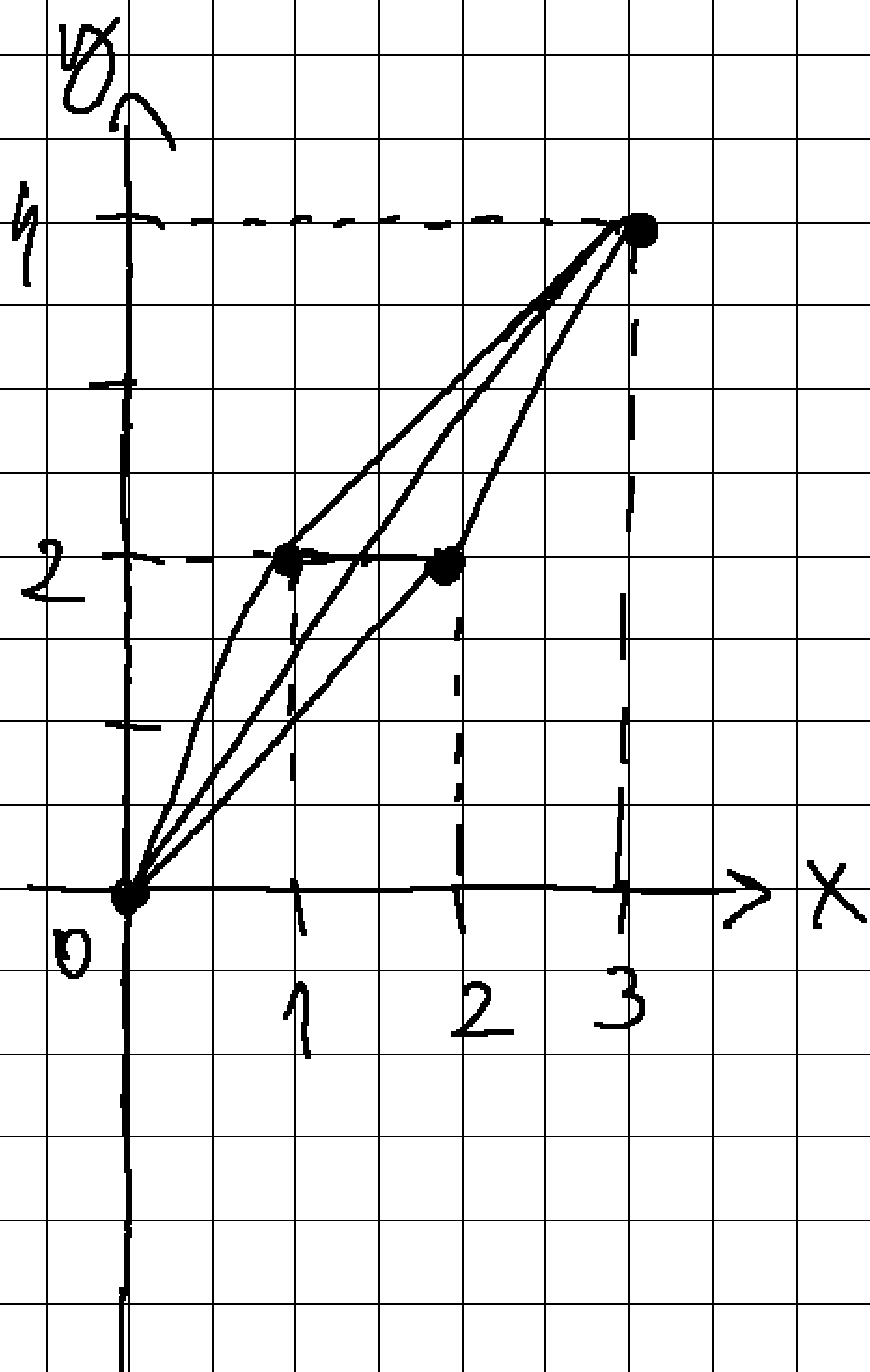
$\displaystyle\int_0^{y} \frac{\sin y}{y}\,dx = x \cdot \frac{\sin y}{y}\Bigg|_{x=0}^{x=y} = y \cdot \frac{\sin y}{y} - 0 =$

$= \sin y$

$J = \displaystyle\int_0^{\frac{\pi}{2}} \sin y\,dy = -\cos y\Bigg|_{y=0}^{y=\frac{\pi}{2}} = -\cos\frac{\pi}{2} + \cos 0 =$

$= 1$

2. $\iint\limits_D xy\,dx\,dy$, D is the parallelogram with verticies $(0,0),(2,2),(1,2),(3,4)$



$u = x+y$

$v = x$

$$J = \begin{vmatrix} \frac{\partial u}{\partial x} & \frac{\partial u}{\partial y} \\ \frac{\partial v}{\partial x} & \frac{\partial v}{\partial y} \end{vmatrix} = \begin{vmatrix} 1 & 1 \\ 1 & 0 \end{vmatrix} = -1$$

$$\iint\limits_D xy\,dx\,dy = \iint\limits_{D'} (v(u-v))\,dv\,du =$$

$0 \le u \le 6$

$0 \le v \le u$

$$= \int_0^6 \int_0^u (v(u-v))\,dv\,du = I$$

$$\int_0^u v(u-v)\,dv = \frac{1}{2}uv^2 - \frac{1}{3}v^3 \Big|_0^u = \frac{1}{2}u^3 - \frac{1}{3}u^3 = \frac{1}{6}u^3$$

$$I = \int_0^6 \frac{1}{6}u^3\,du = \frac{1}{24}u^4 \Big|_0^6 = 27$$

3. $\iint_D \ln(x^2+y^2)\,dx\,dy$, D is the region in the first quadrant between the circles $x^2+y^2=a^2$ and $x^2+y^2=b^2$, with $0<a<b$

$x = r\cos\theta$ $\qquad J = r \qquad a \le r \le b$

$y = r\sin\theta \qquad\qquad 0 \le \theta \le \frac{\pi}{2}$

$d_A = dx\,dy \Rightarrow d_{A_{polar}} = r\,dr\,d\theta$

$$\iint_D \ln(x^2+y^2)\,dx\,dy = \int_0^{\frac{\pi}{2}}\int_a^b (\ln r^2)\cdot r\,dr\,d\theta$$

$$\int_a^b r\ln r^2\,dr = \frac{1}{2}\int_{a^2}^{b^2} \ln u\,du = \frac{1}{2}(u\cdot\ln u - u)\Big|_{a^2}^{b^2} =$$

$u = r^2 \Rightarrow du = 2r\,dr$

$$= \frac{1}{2}\left(b^2\ln b^2 - a^2\ln a^2 - b^2 + a^2\right)$$

$$\int_0^{\frac{\pi}{2}} 1\,d\theta = \frac{\pi}{2}$$

$$\Rightarrow \int_0^{\frac{\pi}{2}}\int_a^b (\ln r^2)\,r\,dr\,d\theta = \frac{\pi}{4}\left(b^2\ln b^2 - a^2\ln a^2 - b^2 + a^2\right)$$