9.5. Using the definition of deduction, prove the following deductions

$U \to V \equiv \neg U \vee V$

Step 2)

$\neg p \vee \neg q \vee r, q, p \vdash r$

$f_1: \neg p \vee \neg q \vee r \equiv \neg p \vee (\neg q \vee r) \equiv p \to (\neg q \vee r) \equiv$

$f_2: q$ $\qquad \equiv p \to (q \to r)$

$f_3: p$

$f_3, f_1 \vdash_{mp} q \to r \quad (f_4)$

$f_4, f_2 \vdash_{mp} r \quad (f_5)$

Step 3: $(f_1, f_2, f_3, f_4, f_5)$ is the deduction of r from the hypotheses

12. H_1: it is not sunny this afternoon and it is colder than yesterday

H_2: We will go swimming only if it is sunny

H_3: if we do not go swimming, then we will take a canoe trip.

H_4: if we take a canoe trip, then we will be home by sunset

C: We will be home by sunset

is C deducible from the set of hypotheses $\{H_1, H_2, H_3, H_4\}$?

if yes build it's deduction.

$U, U \to V \vdash_{mp} V$

$U \land V \vdash_{Simplif} U$

$\neg V, U \to V \vdash_{mt} \neg U$

Notations:

S = Sunny   Co - Colder

SW = Swimming   HBS - home by sunset.

CT = canoe trip

H_1: $\neg S \land Co$ C: HBS

H_2: $SW \to S$

H_3: $\neg SW \to CT$

H_4: $CT \to HBS$

$H_1 \vdash_{\text{simplif.}} \neg S : H_5$

$H_5, H_2 \vdash_{mt} \neg SW : H_6$

$H_6, H_3 \vdash_{mp} CT : H_7$

$H_7, H_4 \vdash_{mp} HBS : C$

The sequence $(H_1, H_2, H_3, H_4, H_5, H_6, H_7, C)$ is the deduction of C from the hypothesis $\{H_1, H_2, H_3, H_4\}$