1 1 0 1 0 0 0 0

1. 3322 1100$_{(4)}$ +

12303 2122$_{(4)}$

223132 22$_{(4)}$

1

it 1: $0_{(4)} + 0_{(4)} + 2_{(4)} = 0+0+2 = 2$

$2/4 = 0$ $\quad 2 \cdot 4 = 2$

it 2: $0_{(4)} + 0_{(4)} + 2_{(4)} = 0+0+2 = 2$

$2/4 = 0$ $\quad 2 \cdot 4 = 2$

it 3: $0_{(4)} + 1_{(4)} + 1_{(4)} = 0+1+1 = 2$

$2/4 = 0$ $\quad 2 \cdot 4 = 2$

it 4: $0_{(4)} + 1_{(4)} + 2_{(4)} = 0+1+2 = 3$

$3/4 = 0$ $\quad 3 \cdot 4 = 3$

it 5: $0_{(4)} + 2_{(4)} + 3_{(4)} = 0+2+3 = 5$

$5/4 = 1$ $\quad 5 \cdot 4 = 1$

2.

$$\begin{array}{r} {\scriptstyle -1\ -1\ -1\ -1\ -1\ 0\ 0} \\ 1\,2\,3\,0\,0\,5\,6_{(7)} \\ -\quad 4\,4\,5\,5\,6\,6_{(7)} \\ \hline 4\,5\,1\,1\,6\,0 \end{array}$$

i_1: $0_{(7)} + 6_{(7)} - 6_{(7)} = 0_{(7)} \geq 0$

i_2: $0_{(7)} + 5_{(7)} - 6_{(7)} = -1_{(7)} < 0$ $\Rightarrow b = -1$

$-1_{(7)} + 7_{(7)} = 6_{(7)}$

i_3: $-1_{(7)} + 0_{(7)} - 5_{(7)} = -6_{(7)} < 0$ $\Rightarrow b = -1$

$-6_{(7)} + 7_{(7)} = 1_{(7)}$

i_4: $-1_{(7)} + 0_{(7)} - 5_{(7)} = -6_{(7)} < 0$ $\Rightarrow b = -1$

$-6_{(7)} + 7_{(7)} = 1_{(7)}$

i_5: $-1_{(7)} + 3_{(7)} - 4_{(7)} = -2_{(7)} < 0$ $\Rightarrow b = -1$

$-2_{(7)} + 7_{(7)} = 5_{(7)}$

i_6: $-1_{(7)} + 2_{(7)} - 4_{(7)} = -3_{(7)} < 0$ $\Rightarrow b = -1$

$-3_{(7)} + 7_{(7)} = 4_{(7)}$

4 5 5 6 6 0 0

3. ABCDE1$_{(16)}$ *

7$_{(16)}$

4B2A187

i+1: $0_{(16)} + 1_{(16)} \cdot 7_{(16)} = 0 + 1 \cdot 7 = 7$

7 / 16 = 0 7 % 16 = 7

i+2: $0_{(16)} + E_{(16)} \cdot 7_{(16)} = 0 + 14 \cdot 7 = 105$

104 / 16 = 6 104 % 16 = 8

i+3: $6_{(16)} + D_{(16)} \cdot 7_{(16)} = 6 + 13 \cdot 7 = 97$

97 / 16 = 6 97 % 16 = 1

i+4: $6_{(16)} + C_{(16)} \cdot 7_{(16)} = 6 + 12 \cdot 7 = 90$

90 / 16 = 5 90 % 16 = 10 = $A_{(16)}$

i+5: $5_{(16)} + B_{(16)} \cdot 7_{(16)} = 5 + 11 \cdot 7 = 82$

82 / 16 = 5 82 % 16 = 2

i+6: $5_{(16)} + A_{(16)} \cdot 7_{(16)} = 5 + 10 \cdot 7 = 75$

75 / 16 = 4 75 % 16 = 11 = $B_{(16)}$

4.

$$
\begin{array}{r|l}
05321_{(6)} & 3_{(6)} \\
\hline
1 & \\
\hline
24 & 1530_4 \\
1 & \\
\hline
13 & \\
1 & \\
\hline
02 & \\
1 & \\
\hline
21 & \\
1 & \\
\hline
0 &
\end{array}
$$

$i=1$: $05_{(6)} = 0 \cdot 6 + 5 = 5$ $\quad 5/3 = 1$ $\quad 5 \% 3 = 2$

$i=2$: $24_{(6)} = 2 \cdot 6 + 4 = 16$ $\quad 16/3 = 5$ $\quad 16 \% 3 = 1$

$i=3$: $13_{(6)} = 1 \cdot 6 + 3 = 9$ $\quad 9/3 = 3$ $\quad 9 \% 3 = 0$

$i=4$: $02_{(6)} = 0 \cdot 6 + 2 = 2$ $\quad 2/3 = 0$ $\quad 2 \% 3 = 2$

$i=5$: $21_{(6)} = 2 \cdot 6 + 1 = 13$ $\quad 13/2 = 4$ $\quad 13 \% 2 = 1$

5.1 $12345,04_{(6)} = ?_{(8)}$ with 2 digits at the frac. part.

$b < h \Rightarrow$ we use the substitution method because we only need to do mult. and div by one digit.

$$12345,04_{(6)} = 1_{(6)}\cdot 6^4 + 2_{(6)}\cdot 6^3 + 3_{(6)}\cdot 6^2 + 4_{(6)}\cdot 6^1 + 5_{(6)}\cdot 6^0 + 0_{(6)}\cdot 6^{-1} + 4_{(6)}\cdot 6^{-2} = 1_{(8)}\cdot 6^4 + 2_{(8)}\cdot 6^3 + 3_{(8)}\cdot 6^2 + 4_{(8)}\cdot 6 + 5_{(8)}\cdot 6^0 + \underbrace{0_{(8)}\cdot 6^{-1}}_{=0} + 4_{(8)}\cdot 6^{-2} = 2420_{(8)} + 660_{(8)} + 154_{(8)} + 30_{(8)} + 5_{(8)} + 0,07_{(8)}$$

Calculations:

$4 \cdot 6_{(8)} = 44_{(8)}$

$3 \cdot 6_{(8)} = 330_{(8)}$

$220 \cdot 6_{(8)} = 2420_{(8)}$

$0 \cdot 6_{(8)} = 0$

$11 \cdot 6_{(8)} = 660_{(8)}$

$330_{(8)} : 2_{(10)} = 154_{(8)}$

$3 \cdot 6_{(8)} = 30_{(8)}$

$0+6\cdot 6 = 36$

$36/8 = 4 \quad 36 \% 8 = 4$

$0+4\cdot 6 = 24$

$24/8 = 3 \quad 24 \% 8 = 0$

$3+4\cdot 6 = 27$

$27/3 = 3 \quad 27 \% 3 = 3$

$4\cdot 6^{-2} = (4/6)/6$

4,00	6
0	0,52
40	
4	
20	
2	
4	

$0,52 = 52/100$

$52 : 6 = 0,07$

52	6
1	0,07
52	

$52 = 5\cdot 8 + 2 = 42$

$42/6 = 7 \quad 42 \% 6 = 0$

$40 = 4\cdot 8 + 0 = 32$

$32/6 = 5 \quad 32 \% 6 = 2$

$20 = 2\cdot 8 + 0 = 16$

$16/6 = 2 \quad 16 \% 6 = 4$

$3 \cdot 4_{(8)} = 30_{(8)}$

1 2 1

2420,00(8) +

660,00(8)

174,00(8)

30,00(8)

5,00(8)

0,07(8)

3511,07(8)

9.2. $ABCDE,1234_{(16)} = ?_{(4)}$

ABCDE	4	4	4	4	4	4	4	4	4
2B	2AF37	ABCD	2AF3	ABC	2AF	AB	2A	A	2
3C	2F	2B	2F	2B	2F	2B	2	2	
D	33	3C	33	3C	3	3			
1B	31	D	3	0					
2	3	2		1					

$A_{(16)} = 10$

$10/4 = 2 \quad 10 \cdot 4 = 2$

$2B_{(16)} = 2 \cdot 16 + 11 = 43$

$43/4 = 10 \quad 43 \cdot 4 = 3$

$3C_{(16)} = 3 \cdot 16 + 12 = 60$

$60/4 = 15 \quad 60 \cdot 4 = 0$

$D = 13$

$13/4 = 3 \quad 13 \cdot 4 = 1$

$1B = 1 \cdot 16 + 14 = 30$

$30/4 = 7 \quad 30 \cdot 4 = 2$

$2A = 2 \cdot 16 + 10 = 42$

$42/4 = 10 \quad 42 \cdot 4 = 2$

$2F_{(16)} = 2 \cdot 16 + 15 = 47$

$47/4 = 11 \quad 47 \cdot 4 = 3$

$33_{(16)} = 3 \cdot 16 + 3 = 51$

$51/4 = 12 \quad 51 \cdot 4 = 3$

$37_{(16)} = 3 \cdot 16 + 7 = 55$

$55/4 = 13 \quad 55 \cdot 4 = 3$

$A_{(16)} = 10 \quad 10/4 = 2 \quad 10 \cdot 4 = 2$

$$0,1234_{(16)} = ?_{(4)} = 0,0133_{(4)}$$

$$0,1234_{(16)} \cdot 4_{(16)} = 0,4936_{(4)}$$

$$0,4936_{(16)} \cdot 4_{(16)} = 1,9744_{(4)}$$

$$0,9744_{(16)} \cdot 4_{(16)} = 3,8976_{(4)}$$

$$0,8976_{(16)} \cdot 4_{(16)} = 3,5904_{(4)}$$

$$ABCDE,1234_{(16)} = 22233031 32,0133$$

$$11223300_{(4)} = ?_{(8)}$$

$$11223300_{(4)} = \overset{1}{0}\overset{1}{1}\overset{2}{0}\overset{2}{1}\overset{3}{1}\overset{3}{0}\overset{0}{0}\overset{0}{0}_{(2)} =$$

$$= 55360_{(8)}$$

5.3. $45078,63_{(9)} = ?_{(7)}$ with 2 digits at the f. part.

$45078_{(9)}$	$5_{(9)}$	$5_{(9)}$	$5_{(9)}$	$5_{(9)}$	$5_{(10)}$
1	8187	1571	285	57	1
10	1	1	1	1	
1	31	47	15	2	
47	1	1	1		
1	38	31	4		
38	1	1			
0	7	3			
	2				

$45 = 5 \cdot 9 + 5 = 41$

$41 / 5 = 8 \quad 41 \% 5 = 1$

$10 = 1 \cdot 9 + 0 = 9$

$9 / 5 = 1 \quad 9 \% 5 = 4$

$47 = 4 \cdot 9 + 7 = 43$

$43 / 5 = 8 \quad 43 \% 5 = 3$

$38 = 3 \cdot 9 + 8 = 35$

$35 / 5 = 7 \quad 35 \% 5 = 0$

$31 = 3 \cdot 9 + 1 = 28$

$28 / 5 = 5 \quad 28 \% 5 = 3$

$15 = 1 \cdot 9 + 5 = 14$

$14 / 5 = 2 \quad 14 \% 5 = 4$

$28 = 2 \cdot 9 + 8 = 2$

$26 / 5 = 5 \quad 26 \% 5 = 1.$

$15 = 1 \cdot 9 + 5 = 14$

$14 / 5 = 2 \quad 14 \% 5 = 4$

$0,63_{(9)} = ?_{(7)} = 0,303$

$0,63 \cdot 5 = 3,15$

$0,15 \cdot 5 = 0,75$

$0,75 \cdot 5 = 3,75$

$$C = 127 + 11 = 138 = 128 + 8 + 2 = 2^7 + 2^3 + 2^1 =$$
$$= 10001010_{(2)}$$

$$-3956,78 = -110110\,000\,000\,000\,,110\,001\,111\,010\,111\,000\,000_{(2)} \cdot 10^{11}$$
$$= -1,10111\,000\,000\,000\,110\,001\,111\,010\,111\,000\,000_{(2)} \cdot 10^{m}$$

S	c	m
1	10001010	10110000000110001111010100

$$0,24 \cdot 8 = 1,92$$
$$0,92 \cdot 8 = 7,36$$
$$0,36 \cdot 8 = 2,88$$
$$0,88 \cdot 8 = 7,04$$
$$0,04 \cdot 8 = 0,32$$

$$0,78 \cdot 8 = 6,24$$

$$3956 = 660\,0_{(8)} = 110\,110\,000\,000\,000$$

$$0,78 = 0,617270_{(8)} = 0,110\,001\,111\,010\,100\ldots$$

6.1. $-3956,78$, $sr)$, $m > 1$

3956	8	8	8	8
32	495	74	48	6
9	0		6	

6.2. $-0,241 = -0,173310_{(8)} = -0,0011110110011001_{(2)}$

$0,241 \cdot 8 = 1,928$

$0,928 \cdot 8 = 7,424$

$0,424 \cdot 8 = 3,392$

$0,392 \cdot 8 = 3,136$

$0,136 \cdot 8 = 1,088$

$0,088 \cdot 8 = 0,704$

$[-0,241]_{dir}$

S_1															
1	0	0	1	1	1	1	0	1	1	0	1	1	0	0	1

$[-0,241]_{inv}$

S_1															
1	1	1	0	0	0	0	1	0	0	1	0	0	1	1	0

$[-0,241]_{compl}$

S_1															
1	1	1	0	0	0	0	1	0	0	1	0	0	1	1	1

6.3. $55 = 32 + 16 + 4 + 2 + 1 = 2^5 + 2^4 + 2^2 + 2^1 + 2^0 =$

$= 110111_{(2)}$

6	4	3	2	1	0

$91 = 64 + 16 + 8 + 2 + 1 = 2 + 2 + 2 + 2 + 2 + 2 =$

$= 1011011_{(2)}$

$[55]_{dir} = [55]_{inv} = [55]_{compl}$

S
0 0 1 1 0 1 1 1

$[-55]_{compl}$

S
1 1 0 0 1 0 0 1

$[91]_{dir} = [91]_{inv} = [91]_{compl}$

S
0 1 0 1 1 0 1 1

$[-91]_{compl}$

S
1 0 1 0 0 1 0 1

$[55+91]_{compl} = [55]_{compl} \oplus [91]_{compl}$

S
0 0 1 1 0 1 1 1 ⊕

S
0 1 0 1 1 0 1 1

S
1 0 0 1 0 0 1 0 ⇒ Overflow

$[55-91]_{compl} = [55]_{compl} \oplus [-91]_{compl}$

S
0 0 1 1 0 1 1 1 ⊕

S
1 0 1 0 0 1 0 1

$[-35]_{compl}$ S 1 1 0 1 1 1 0 0 correct result

complement

S
0 0 1 0 0 0 1 1 $= [35]_{compl}$

$2^0 + 2 + 2^5 = 35$

$[-91-55]_{compl} = [-91]_{compl} \oplus [-55]_{compl}$

S							1
1	0	1	0	0	1	0	1

$\oplus$

S							
1	1	0	0	1	0	0	1

$[110]_{compl}$

S							
0	1	1	0	1	1	1	0

6.4. $-4501,33$, Sp.

$4501 = 725_{(8)} = 111010101_{(2)}$

4501	8	8
40	562	7
50	1	
48	2	
21		
5		

$0,33 = 0,250754_{(8)} = 0,010101000111101$

$0,33 \cdot 8 = 2,64$

$0,64 \cdot 8 = 5,12$

$0,12 \cdot 8 = 0,96$

$0,96 \cdot 8 = 7,68$

$0,68 \cdot 8 = 5,44$

$-4501,33 = -(111010101,010101000111101)_{(2)}$

$= -0,111010101010101000111101 \cdot 10^9$

S	c	p	m
1	10001000	11101010101010101000	11110

$c = 127 + 9 = 136 = 128 + 8 = 2^7 + 2^3 = 10001000$

6.5. 0,023 , sp

$0,023 \cdot 8 = 0,184$

$0,184 \cdot 8 = 1,472$

$0,472 \cdot 8 = 3,776$

$0,776 \cdot 8 = 6,208$

$0,208 \cdot 8 = 1,664$

$0,664 \cdot 8 = 5,312$

$0,312 \cdot 8 = 2,496$

$0,496 \cdot 8 = 3,968$

$0,968 \cdot 8 = 7,744$

$0,744 \cdot 8 = 5,952$

$0,023 = 0,0136152375_{(8)}$

$= 0,000001011110001101010\ 011111101$

$0,023 = 0,000001011110001101010\ 011111101_{(2)}$

$= 0,1011110001101010\ 011111101_{(2)} \cdot 10^{-5}$

$C = e + q = -5 + 127 = 122 = 64 + 32 + 16 + 8 + 2 =$

$= 2^6 + 2^5 + 2^4 + 2^3 + 2^1 = 1111010$

S	e								m
0	0	1	1	1	1	0	1	0	10111100011010100111111

6.8 BOCDA000 , i=18 bits F=13 bits

S	i	F
1	011000001100110110	0000000000000

$X = -1100000110011011 =$

$= -(2^{16}+2^{15}+2^{10}+2^{9}+2^{6}+2^{5}+2^{3}+2^{2}+2^{0}) =$

$= -(65536 + 32768 + 1024 + 512 + 64 + 32 + 8 + 4 + 1) =$

$= -99949$

6.9. BOCDA000 , SP , m>1

S	C	f	m
1	01100001	10011011010	0000000000000

C = 127 + l

$C = 1100001 = 2^{6}+2^{5}+2^{0} = 64+32+1 = 97$

$l = 97 - 127 = -30$

$X = -1,1001101101 \cdot 10^{-30} =$

$= -1,\underbrace{00\ldots0}_{30}1001101101 = -(1+2^{-31}+2^{-34}+2^{-35}+2^{-37}+2^{-38}+2^{-40}) =$

$=$

6.7. $7/16 = 0,43 = 0,334121_{(8)} =$

$0,43 \cdot 8 = 3,44$ $= 0,011011100001010001_{(2)}$

$0,44 \cdot 8 = 3,52$

$0,52 \cdot 8 = 4,16$

$0,16 \cdot 8 = 1,28$

$0,28 \cdot 8 = 2,24$

$0,24 \cdot 8 = 1,92$

$[7/16]_{dir} = [7/16]_{inv} = [7/16]_{compl} =$

$=$ $\overset{S}{\boxed{0}}\boxed{0110111000010100}$

$[-7/16]_{dir} =$ $\overset{S}{\boxed{1}}\boxed{0110111000010100}$

$[-7/16]_{inv} =$ $\overset{S}{\boxed{1}}\boxed{1001000111101011}$

$[-7/16]_{compl} =$ $\overset{S}{\boxed{1}}\boxed{1001000111101100}$

$0,71 = 0,533412_{(8)} = 0,101101011100001010_{(2)}$

$0,71 \cdot 8 = 5,68$ $0,16 \cdot 8 = 1,28$

$0,68 \cdot 8 = 5,44$ $0,28 \cdot 8 = 2,24$

$0,44 \cdot 8 = 3,52$

$0,52 \cdot 8 = 4,16$