1. Sol:

$$X\begin{pmatrix} 0 & 1 & 2 \\ 0{,}42 & 0{,}46 & 0{,}12 \end{pmatrix}$$

$P(X=0) = P(1\ \text{no}\ \&\ 2\ \text{no}) =$

$= P(1\ \text{no}) \cdot P(2\ \text{no}) = 0{,}42$

$P(X=2) = P(1\ \text{yes}\ \&\ 2\ \text{yes}) = P(1\ \text{yes}) \cdot P(2\ \text{yes}) = 0{,}12$

$P(X=1) = 1 - P(X=0) - P(X=2) = 0{,}46$

2.

$X \sim Bino(3, 0.5)$

$$X\begin{pmatrix} 0 & 1 & 2 & 3 \\ \frac{1}{8} & \frac{3}{8} & \frac{3}{8} & \frac{1}{8} \end{pmatrix}$$

$$X\begin{pmatrix} K \\ C_3^K \cdot (0{,}5)^K \cdot (0{,}5)^{3-K} \end{pmatrix} \quad K \in 0,1,2,3$$

$P(X \leq 2) = 1 - P(X=3) = \frac{7}{8}$

$= \text{Binocdf}(2, 3, 0.5)$

$P(X<2) = P(X=0) + P(X=1) = \frac{1}{2}$

$= P(X \leq 1) = \text{Binocdf}(1, 3, 0.5)$

3. Sol:

New account init = rare events = discrit events observed over a period of time $\Rightarrow$

$\Rightarrow$ Poisson Distribution $\lambda = 10$

$$X \begin{pmatrix} k \\ \frac{10^k}{k!} e^{-10} \end{pmatrix} \quad k \in 0, 1 \dots$$

a) $P(X>8) = 1 - P(X \leq 8) = 1 - \sum_{k=0}^{8} \frac{10^k}{k!} e^{-10} =$

$= 1 - \text{poisscdf}(8, 10) = 0.6672$

b) Y = "no. of acc. established in two days"

$\Rightarrow Y \sim \text{Poisson}(20)$

$$P(Y \le 16) = \sum_{k=0}^{16} \frac{20^k}{k!} \cdot e^{-20} = \text{poisscdf}(16, 20) = 0.2211$$

4. X = "no. of attempts made in order to login"

X can take the values: $1, 2 \ldots$

$P(X=1) = 0{,}7$

$P(X=2) = 0{,}3 \cdot 0{,}7$

$P(X=3) = 0{,}3^2 \cdot 0{,}7$

$P(X=k) = 0{,}3^{k-1} \cdot 0{,}7$

a)

$$X \begin{pmatrix} k \\ 0{,}3^{k-1} \cdot 0{,}7 \end{pmatrix} \quad k = 1, 2 \ldots$$

$$X-1 \begin{pmatrix} k-1 \\ 0{,}3^{k-1} \cdot 0{,}7 \end{pmatrix} \quad k = 1, 2 \ldots$$

$$X-1 \begin{pmatrix} l \\ 0{,}3^{l} \cdot 0{,}7 \end{pmatrix} \quad l = 0, 1 \ldots$$

$P(X \le 4) = P(X-1 \le 3) = \text{geocdf}(3, 0.7) \simeq 0.9919$

$P(X \ge 3) = P(X-1 \ge 2) = 1 - P(X-1 \le 1) = 1 - \text{geocdf}(1, 0.7) \simeq$

$\simeq 0.09$

5. $X \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ \frac{1}{5} & \frac{1}{5} & \frac{1}{5} & \frac{1}{5} & \frac{1}{5} \end{pmatrix}$

$Y \begin{pmatrix} 1 & 2 & 3 \\ \frac{1}{5} & \frac{3}{5} & \frac{1}{5} \end{pmatrix}$

$X+Y \begin{pmatrix} 2 & 3 & 4 & 5 & 6 & 7 & 8 \\ \frac{1}{5} & 0 & \ldots & & & & \end{pmatrix}$

$$P(X+Y=2) = P(X=1, Y=1)$$
$$= P(X=1) = \frac{1}{5}$$

$$P(X+Y=3) = P((X=1, Y=2) \cup (X=2, Y=1))$$
$$= P(X=1, Y=2) + P(X=2, Y=1) = 0+0=0$$

$$X \cdot Y = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 8 & 9 & 10 & 15 \\ \ldots & & & & & & & & & \end{pmatrix}$$

$$P(X \cdot Y = 1) = \ldots\ldots$$

6. Homework

7.

X \ Y	1	2	3
1	0	0.10	0.40
2	0.06	0.10	0.10
3	0.06	0.04	0
4	0.10	0.04	0

a) $X\begin{pmatrix} 1 & 2 & 3 & 4 \\ 0.5 & 0.26 & 0.1 & 0.14 \end{pmatrix}$

$Y\begin{pmatrix} 1 & 2 & 3 \\ 0.22 & 0.28 & 0.5 \end{pmatrix}$

b) $P((X,Y)=(2,1)) = 0.06$

c) $P(X \geq 3, Y \leq 2) = P((X,Y)=(3,1)) + \underset{(3,1)}{0.04} + \underset{(4,1)}{0.10} +$

$+ \underset{(4,2)}{0.04} = 0.24$

d) $Z = X \cdot Y \begin{pmatrix} 1 & 2 & 3 & 4 & 6 & 8 & 9 & 12 \\ 0 & \ldots & & & & & & \end{pmatrix}$

$P(Z=1) = P(X \cdot Y = 1) = P((X,Y) = (1,1)) = 0$