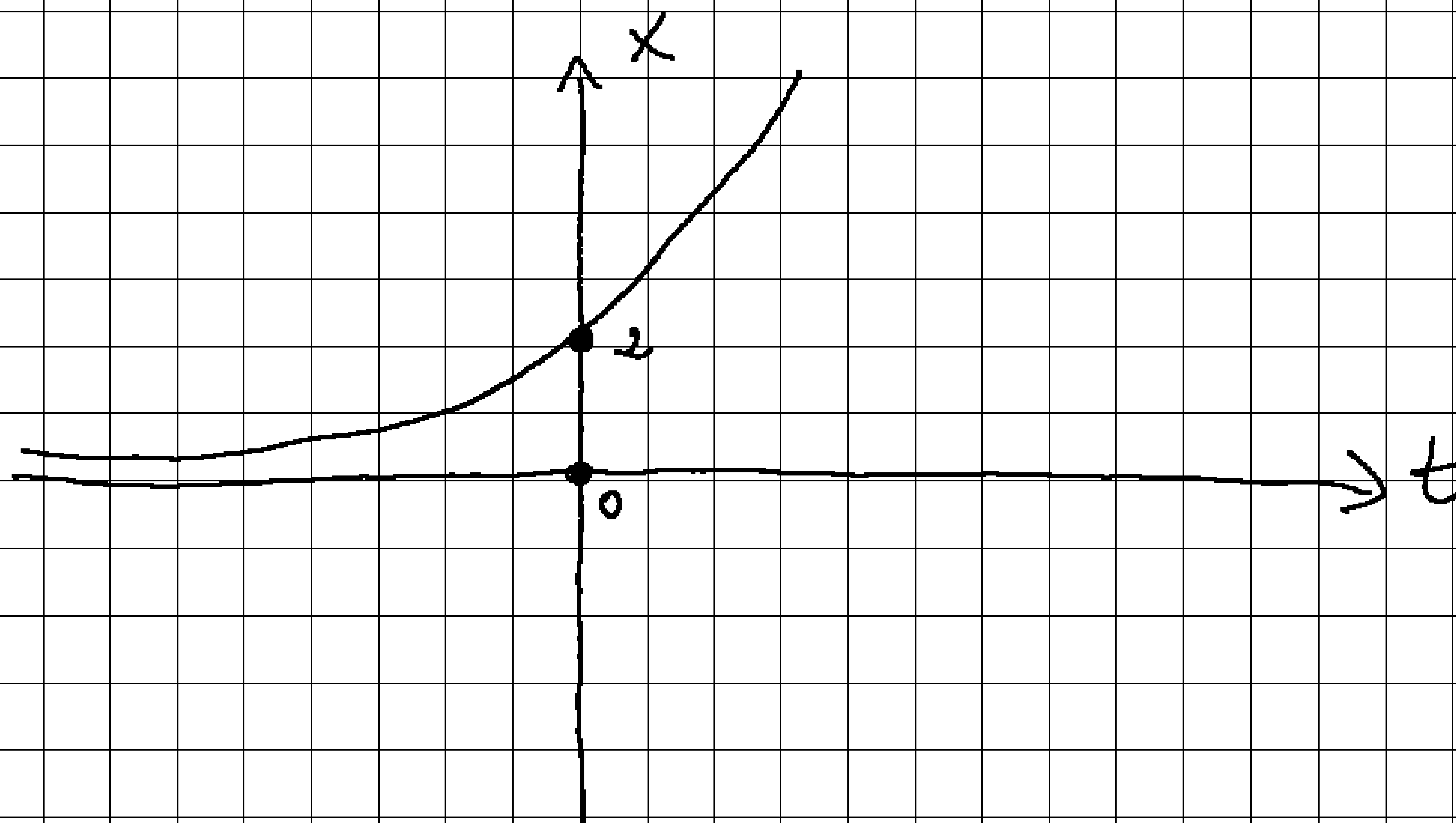
1.1.1. $\varphi : \mathbb{R} \to \mathbb{R}$, $\varphi(t) = 2e^{3t}$ - sol. of IVP

$$\begin{cases} x' = 3x \iff x'(t) = 3x(t) \\ x(0) = 2 \end{cases}$$

$x = x(t)$ - unknown

$$\left.\begin{aligned} \varphi'(t) &= 6e^{3t} \\ 3\varphi(t) &= 3 \cdot 2e^{3t} = 6e^{3t} \end{aligned}\right\} \Rightarrow \varphi'(t) = 3\varphi(t)$$



φ is not periodic because

$\varphi(t) \neq \varphi(t+\underbrace{T}_{\text{period}}), \forall t \in \mathbb{R}$ $\quad \left(e^{3t} \neq e^{3(t+T)} \right)$

φ is increasing

$\lim_{t \to -\infty} \varphi(t) = 0$

$\lim_{t \to \infty} \varphi(t) = +\infty$

1.1.2. $\eta \in \mathbb{R}^*$

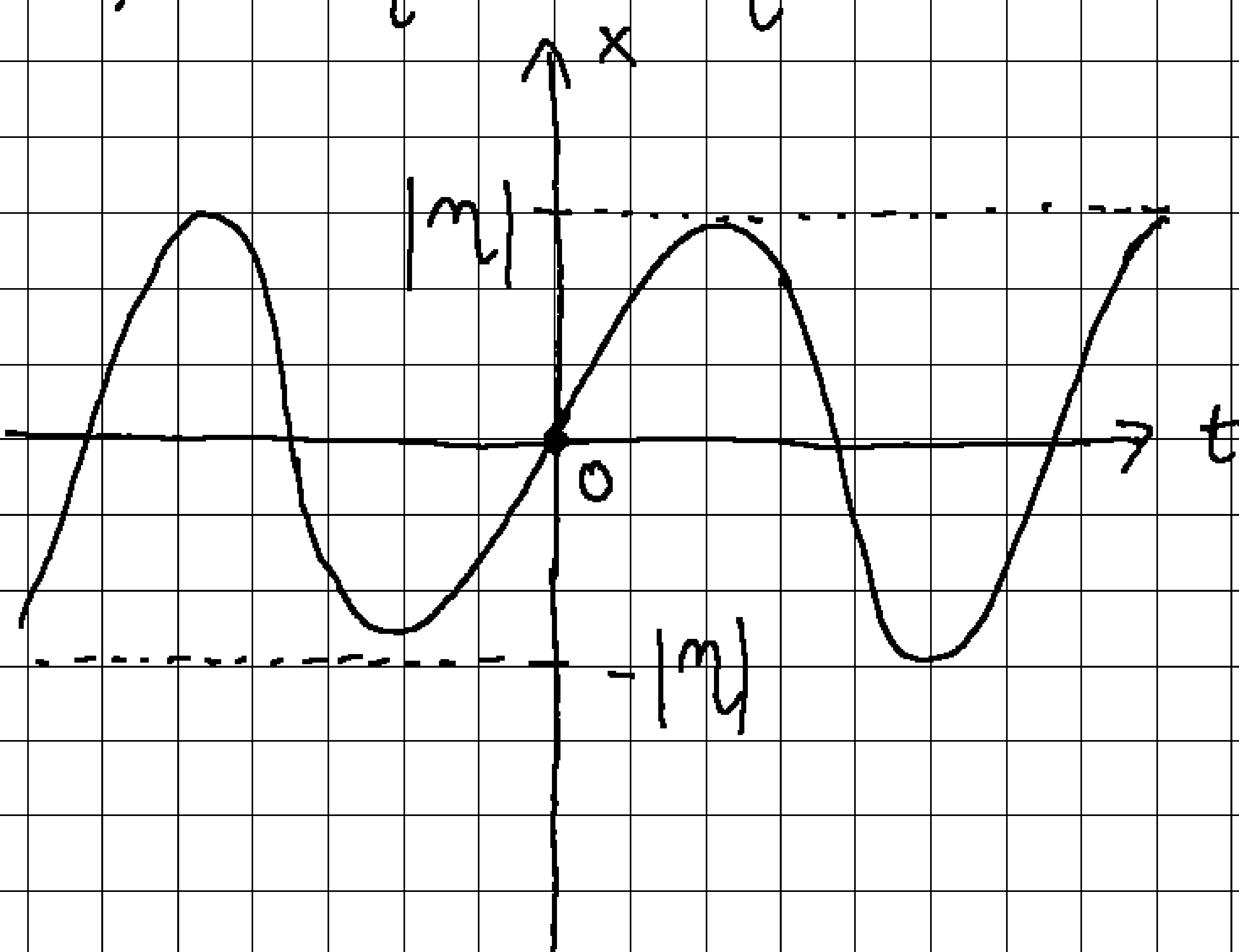
$\varphi : \mathbb{R} \to \mathbb{R}$, $\varphi(t) = \eta \sin t$

sol of IVP: $\begin{cases} x'' + x = 0 \\ x(0) = 0 \\ x'(0) = \eta \end{cases}$

$$\left.\begin{array}{l} \varphi''(t) = -\eta \sin t \\ \varphi(t) = \eta \sin t \end{array}\right\} \Rightarrow \varphi''(t) + \varphi(t) = 0$$

$\varphi(0) = \eta \sin 0 = 0$

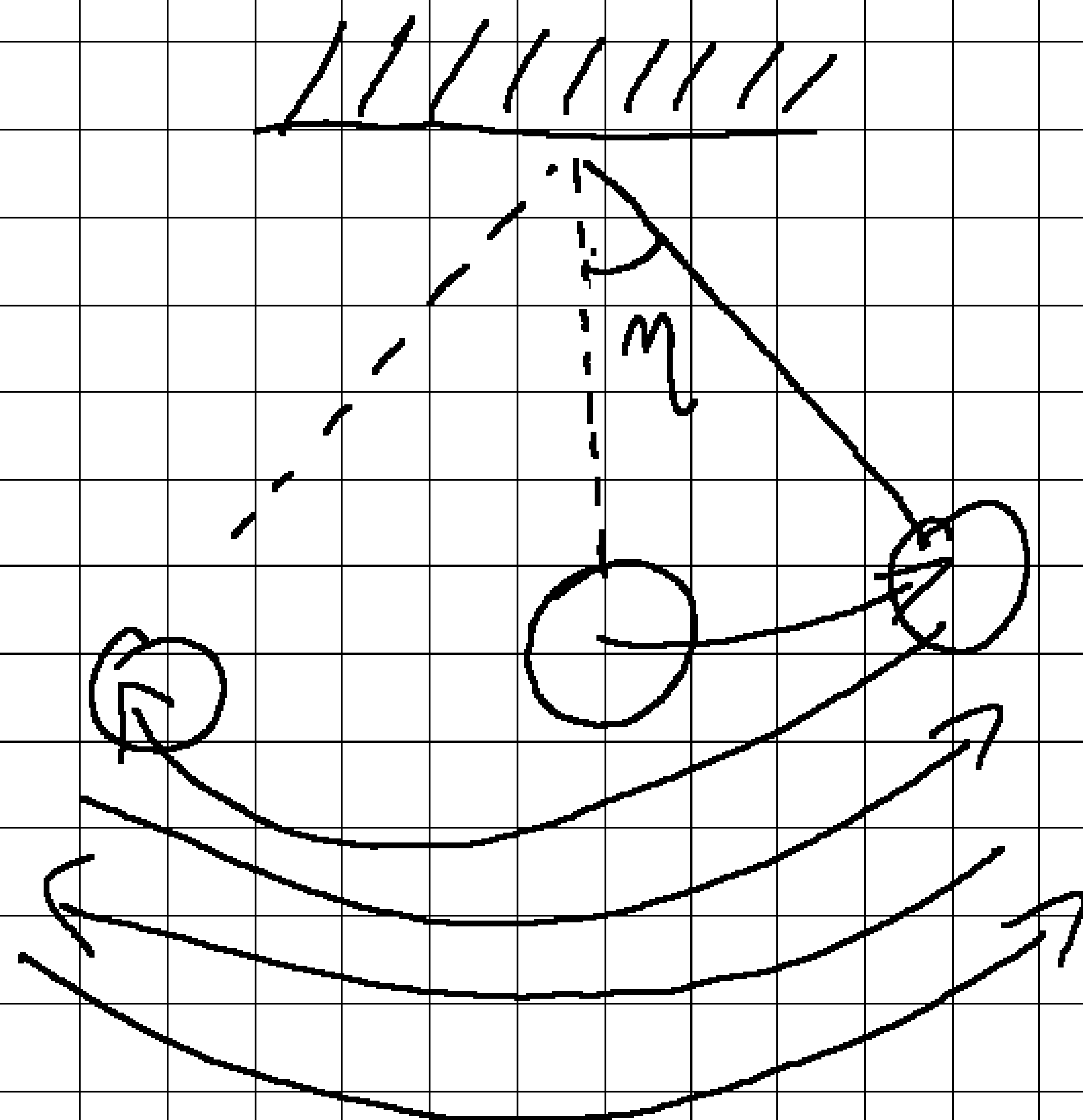
$\varphi'(0) = \eta \cos 0 = \eta$



φ is periodic

φ is bounded, $\varphi(t) \in [-|\eta|, |\eta|]$

oscill + const + amplitude $\Leftrightarrow$ periodic



1.1.3. $\varphi : \mathbb{R} \to \mathbb{R}$, $\varphi(t) = e^{-2t}\cos t$

sol of IVP $\begin{cases} x'' + 4x' + 5x = 0 \\ x(0) = 1 \\ x'(0) = -2 \end{cases}$

$$\varphi'(t) = -2e^{-2t}\cos t - e^{-2t}\sin t = -e^{-2t}(2\cos t + \sin t)$$

$$\varphi''(t) = 2e^{-2t}(2\cos t + \sin t) - e^{-2t}(-2\sin t + \cos t) =$$

$$= e^{-2t}(3\cos t + 4\sin t)$$

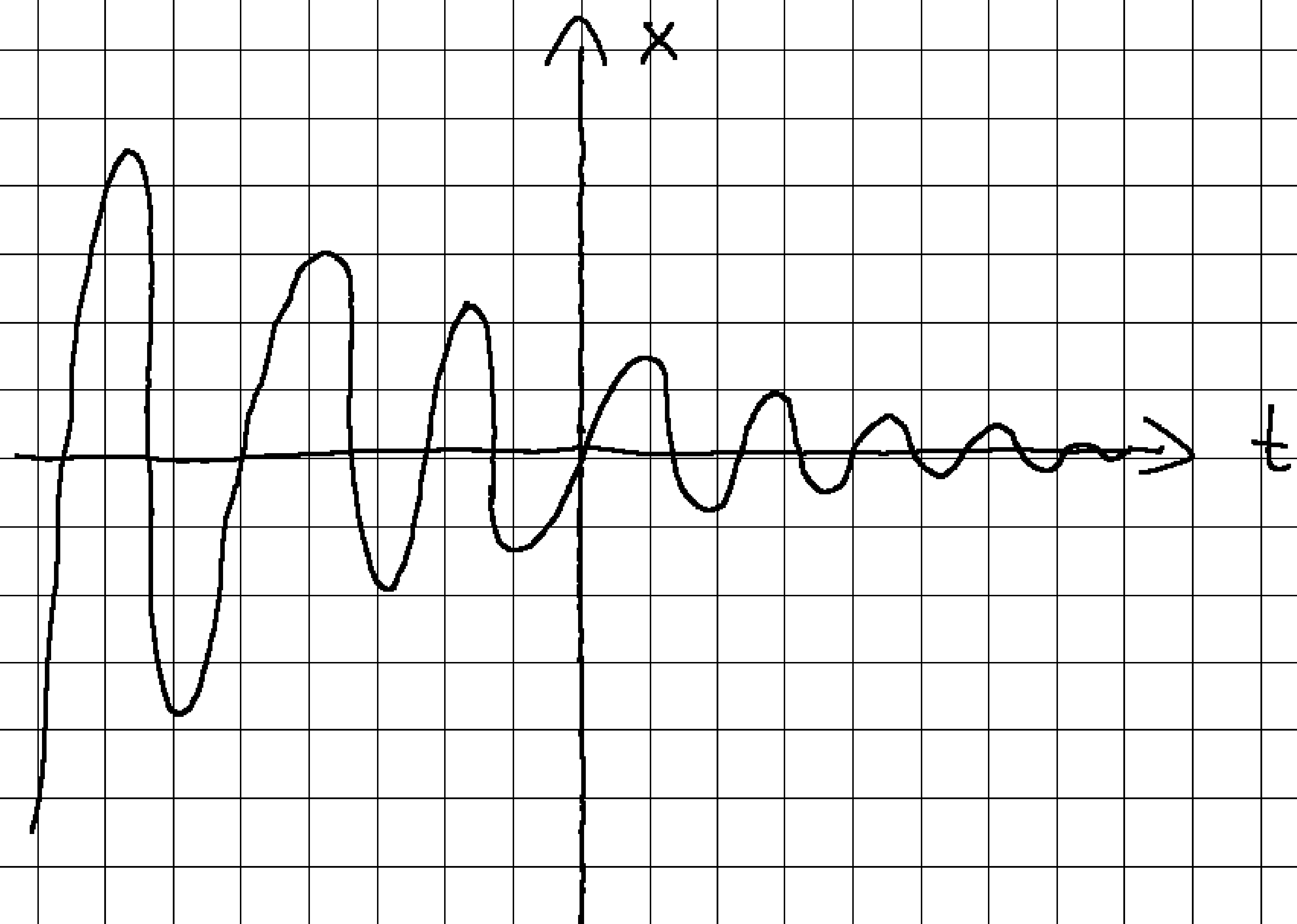
$$\varphi''(t) + 4\varphi'(t) + 5\varphi(t) =$$

$$= e^{-2t}(3\cos t + 4\sin t) - 4e^{-2t}(2\cos t + \sin t) + 5e^{-2t}\cos t =$$

$$= 0$$

$\varphi(0) = e^{-0} \cdot \cos 0 = 1$

$\varphi'(0) = -e^{0}(2-0) = -2$



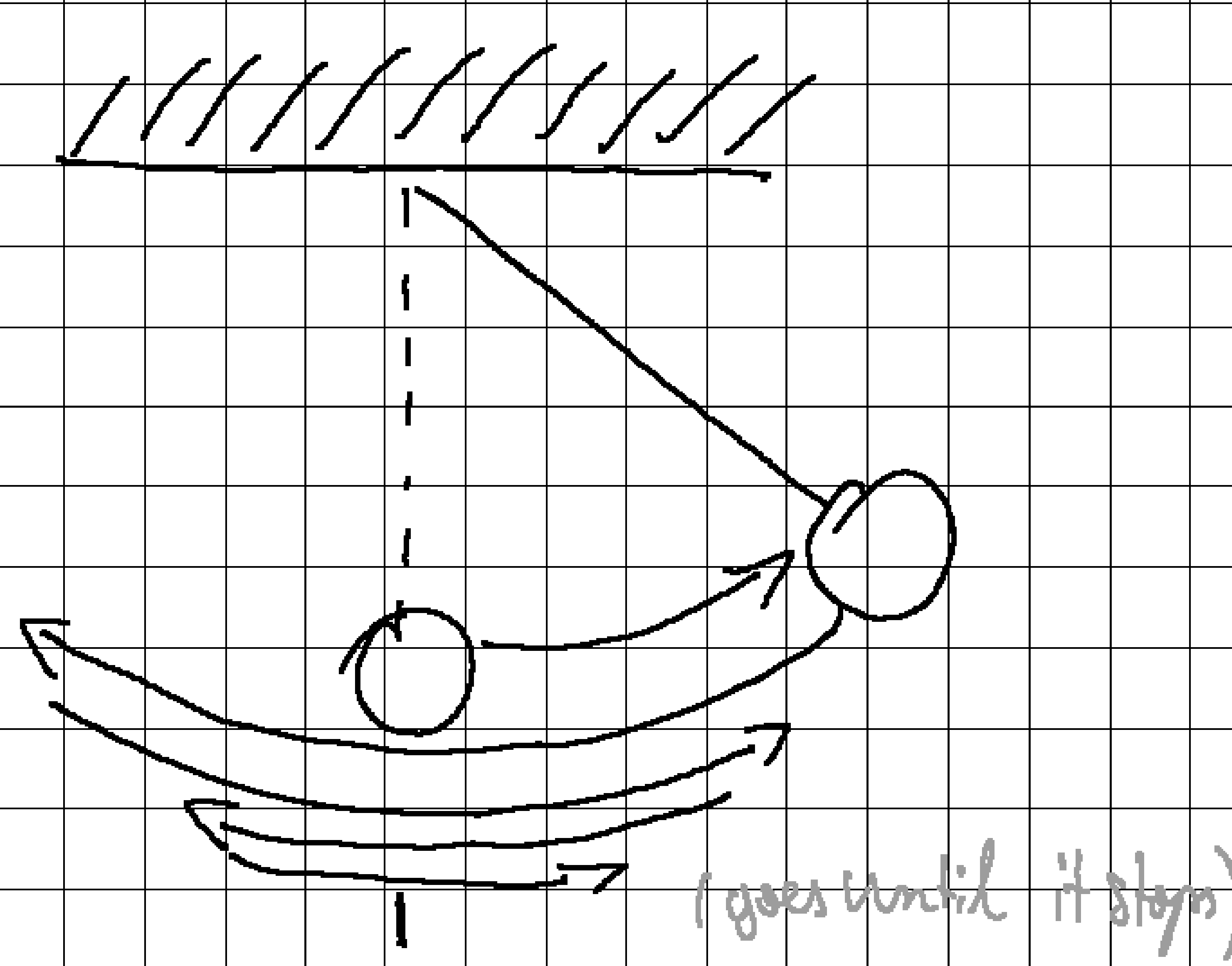
$G_\varphi \cap O_t$ equals an infinite number of sol $\Rightarrow$

$\Rightarrow \varphi$ is oscillatory

1.1.4, 5, 6 homework
extra

$\lim_{t \to -\infty} \varphi(t) = \pm \infty$ (divergent)

$\lim_{t \to \infty} \varphi(t) = 0$



1.1.7. $\varphi: \mathbb{R} \to \mathbb{R}$, $\varphi(t) = \cos t$

$x'' - x = 0$

$\varphi'(t) = -\sin t$

$\varphi''(t) = -\cos t$

$\varphi''(t) - \varphi(t) = -\cos t - \cos t = -2\cos t \Rightarrow$

$\Rightarrow \varphi(t)$ is not a solution

$x''' + x' = 0$

$\varphi'''(t) = \sin t$

$\varphi'''(t) + \varphi'(t) = \sin t - \sin t = 0 \Rightarrow$

$\Rightarrow \varphi(t)$ is a solution

1.1.8. a) $x' = x - x^3 \Leftrightarrow 0 = c - c^3$

$\varphi(t) = c \qquad \Leftrightarrow c(1-c^2) = 0$

$\varphi'(t) = 0$

$c_1 = 0$

$c_2 = 1$

$c_3 = -1$

b) $x' = \sin x \qquad \sin c = 0$

$\varphi(t) = c \qquad c = \{k\pi \mid k \in \mathbb{Z}\}$

$\varphi'(t) = 0$

1.1.11. $r = ?$, $r \in \mathbb{R}$ s.t. $x(t) = e^{rt}$ sol of

$x'' - 5x' + 6x = 0$

$\varphi'(t) = r e^{rt}$

$\varphi''(t) = r^2 e^{rt}$

$x''(t) - 5x'(t) + 6x(t) = 0 \Leftrightarrow$

$r^2 e^{rt} - 5r e^{rt} + 6 e^{rt} = 0$

$$\left.\begin{array}{l} e^{rt}(r^2 - 5r + 6) = 0 \\ e^{rt} > 0 \end{array}\right\} \Rightarrow r^2 - 5r + 6 = 0$$

$\Rightarrow (r-2)(r-3) = 0$

$r \in \{2, 3\}$

$x_1(t) = e^{2t}$ and $x_2(t) = e^{3t}$ $\Rightarrow$

$\Rightarrow$ $x(t) = C_1 x_1 + C_2 x_2 = C_1 e^{2t} + C_2 e^{3t}$ = general solution

1.1.12. $r \in \mathbb{R}$ $\quad x(t) = t^r$ sol on $(0, \infty)$

for $t^2 x'' - 4t x' + 6x = 0$

$$x' = r t^{r-1}$$

$$x'' = r(r-1) t^{r-2}$$

$$t^2 r(r-1) t^{r-2} - 4t r t^{r-1} + 6t^r = 0$$

$$t^r (r^2 - r - 4r + 6) = 0$$

$$t^r \neq 0 \Rightarrow r^2 - 5r + 6 = 0$$

$$\Delta = 1 \Rightarrow \begin{matrix} r_1 = 3 \\ r_2 = 2 \end{matrix} \Rightarrow \left.\begin{matrix} x_1 = t^3 \\ x_2 = t^2 \end{matrix}\right\} \Rightarrow x = c_1 t^3 + c_2 t^2$$

$c_1, c_2 \in \mathbb{R}$