Ex: Write the equivalent DNF of the formula and find it's models

order for solving $\xrightarrow{\quad}$
$\neg, \wedge, \vee, \rightarrow, \leftrightarrow$

$$X = (p \wedge q \overset{1}{\rightarrow} r) \overset{2}{\rightarrow} (p \overset{3}{\rightarrow} r) \wedge q \equiv$$

replace 1,3

$$\equiv (\neg(p \wedge q) \vee r) \overset{2}{\rightarrow} (\neg p \vee r) \wedge q \equiv$$

replace 2

$$\equiv \neg(\neg(p \wedge q) \vee r) \vee ((\neg p \vee r) \wedge q) \equiv$$

Apply De Morgan laws

$$\equiv ((p \wedge q) \wedge \neg r) \vee ((\neg p \vee r) \wedge q) \equiv$$

$A \rightarrow B \equiv \neg A \vee B$

distributive laws

$$\equiv ((p \wedge q) \wedge \neg r) \vee ((\neg p \vee q) \vee (r \wedge q))$$

DNF with 3 cubes

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1) Cube $p \wedge q \wedge \neg r \equiv T$ provides 1 model

$i_1 : \{p, q, r\} \rightarrow \{T, F\}$

$i_1(p) = T$, $i_1(q) = T$, $i_1(r) = F$,

$i_1(p \wedge q \wedge \neg r) = T$, $i_1(X) = T$

2) Cube $\neg p \wedge q \equiv T$ provides 2 models

$i_2, i_3 : \{p, q, r\} \rightarrow \{T, F\}$

$i_2, i_3(p) = F$, $i_2, i_3(q) = T$

$i_2(r) = T$, $i_3(r) = F$

$i_2, i_3(\neg p \wedge q) = T$

$i_2, i_3(X) = T$

3) Cube $r \wedge q \equiv T$ provides 2 models

$i_4, i_5 : \{p, q, r\} \to \{T, F\}$

$i_4(p) = T$

$i_5(p) = F$, $i_4, i_5(q) = T$, $i_4, i_5(r) = T$

$i_2 = i_5$

X has 4 models : i_1, i_2, i_3, i_4

4 anti-models

<u>Ex2:</u> DNF: $p \vee (\neg q \wedge r) \vee (\neg p \wedge \neg r) \equiv$ distributive laws

$\equiv \overbrace{(p \vee \neg q \vee \neg p)}^{\equiv T} \wedge (p \vee \neg q \vee \neg r) \wedge (\underbrace{p \vee r \vee \neg p}_{\equiv T}) \wedge$

$\wedge (\underbrace{p \vee r \vee \neg r}_{\equiv T}) \equiv$

$\equiv p \vee \neg q \vee \neg r \quad \to$ CNF

clause $p \vee \neg q \vee \neg r \equiv F$

provides 1 anti-model

$i : \{p, q, r\} \to \{T, F\}$

$i(p) = F$, $i(q) = T$, $i(r) = T$