Substraction:

Ex1:

$$\begin{array}{r} \overset{2}{\cancel{3}}\overset{1}{0}\overset{3}{4}\overset{1}{5} - \\ 826 \\ \hline 2219 \end{array} \quad \begin{array}{l} \text{minuend} \\ \text{subtrahend} \\ \text{difference} \end{array}$$

borrows

$$\begin{array}{r} -1\,0\,-1\,0 \\ 3045 - \\ 0826 \\ \hline 2219 \end{array}$$

it1: $0+5-6 = -1 < 0$ ↗ $b=-1$ ↘ $-1+10=9$

it2: $-1+4-2 = 1 \geq 0 \Rightarrow b=0$

it3: $0+0-8 = -8 < 0 \Rightarrow b=-1$ ↘ $-8+10=2$

it4: $-1+3-0 = 2 \geq 0$

Ex2:

$$\begin{array}{r} -1\,-1\,-1\,0 \\ 5204_{(7)} - \\ 436 \\ \hline 4435 \end{array}$$

it1: $0_{(7)}+4_{(7)}-6_{(7)} = -2_{(7)} < 0$ ↗ $b=-1$ ↘ $-2+7=5_{(7)}$

it2: $-1_{(7)}+0_{(7)}-3_{(7)} = -4_{(7)} < 0 \Rightarrow b=-1$ ↘ $-4+7 = 3_{(7)}$

it3: $-1_{(7)}+2_{(7)}-4_{(7)} = -3_{(7)} < 0 \Rightarrow b=-1$ ↘ $-3+7 = 4_{(7)}$

it4: $-1_{(7)}+5_{(7)}-0_{(7)} = 4_{(7)} \geq 0$

Ex3:

$$\begin{array}{r} -1\ -1\ -1\ 0 \\ D\ 7\ 3\ A_{(16)} \\ 0\ B\ C\ F_{(16)} \\ \hline c\ C\ 6\ B \end{array}$$

i1: $0_{(16)} + A_{(16)} - F_{(16)} = 0 + 10 - 15 = -5 < 0$ → $b=-1$ → $-5+16=11 = B_{(16)}$

i2: $-1_{(16)} + 3_{(16)} - C_{(16)} = -1+3-12 = -10 < 0$ → $b=-1$ → $-10+16=6$

i3: $-1_{(16)} + 7_{(16)} - B_{(16)} = -1+7-11 = -4 < 0$ → $b=-1$ → $-4+16=12 = C_{(16)}$

i4: $-1_{(16)} + D_{(16)} - 0_{(16)} = -1+13-0 = 12 > 0 = C_{(16)}$

Rapid conversions:

$N_{(b)} = N'_{(h)}$ $\quad b = 2^k$ $\quad h = 2^l$

Ex1: $72430,65_{(8)} = ?_{(16)}$

Example

$6 = 4+2 = 2^2+2^1 = 110_{(2)}$

$D_{(16)} = 13 = 8+4+1 = 2^3+2^2+2^0 = 1101_{(2)}$

$72430,65_{(8)} = \underbrace{0111}_{7}\ \underbrace{010}_{2}\ \underbrace{100}_{4}\ \underbrace{011}_{3}\ \underbrace{000}_{0},\ \underbrace{110}_{6}\ \underbrace{10100}_{5}$

→ left to right ← right to left

$= 7518,D4_{(16)}$

$D_{(16)} = 13 = 3\cdot 4^1 + 1\cdot 4^0 = 31_{(4)}$

$8_{(16)} = 2\cdot 4^1 + 0\cdot 4^0 = 20_{(4)}$

$7_{(16)} = 1\cdot 4^1 + 3\cdot 4^0 = 13_{(4)}$

$7518,D4_{(16)} = \underbrace{13}\ \underbrace{11}\ \underbrace{01}\ \underbrace{20},\ \underbrace{31}\ \underbrace{10}_{(4)}$

Substitution method:

$N_{(b)} = N'_{(h)}$

- $b < h$, calculations in base h
- multiplication and divisions by one digit

Ex 1: $3421,31_{(5)} = ?_{(8)}$ with 2 digits of the fractional part in base h

$$3421,31_{(5)} = 3_{(5)} \cdot 5^3 + 4_{(5)} \cdot 5^2 + 2_{(5)} \cdot 5^1 + 1_{(5)} \cdot 5^0$$

$$+ 3_{(5)} \cdot 5^{-1} + 1_{(5)} \cdot 5^{-2} = \underbrace{3_{(8)} \cdot 5_{(8)}^3}_{567_{(8)}} + \underbrace{4_{(8)} \cdot 5_{(8)}^2}_{144_{(8)}} + \underbrace{2_{(8)} \cdot 5_{(8)}^1}_{12_{(8)}} +$$

$$+ \underbrace{1_{(8)} \cdot 5_{(8)}^0}_{1_{(8)}} + 3_{(8)} \cdot 5_{(8)}^{-1} + 1_{(8)} \cdot 5_{(8)}^{-2} =$$

$3_{(5)} = 3_{(8)}$

$4_{(5)} = 4_{(8)}$

$2_{(5)} = 2_{(8)}$

$1_{(5)} = 1_{(8)}$

$5_{(5)} = 5_{(8)}$

Calculations:

$2_{(8)} \cdot 5_{(8)}$ / $12_{(8)}$

$2_{(8)} \cdot 5_{(8)} = 2 \cdot 5 = 10$

$10 / 8 = 1$, $10 \% 8 = 2$

....

$3_{(8)} \cdot 5_{(8)}^{-1} = 3_{(8)} / 5_{(8)}$

$300_{(8)}$ | $5_{(8)}$

30 | $0,46$

40

2

$30_{(8)} = 3 \cdot 8 + 0 = 24$

$24 / 5 = 4$

$24 \% 5 = 4$

$40_{(8)} = 4 \cdot 8 = 32$

$32 / 5 = 6$

$32 \% 5 = 2$

$$1_{(8)} \cdot 5^{-2}_{(8)} = (1_{(8)} / 5_{(8)}) / 5_{(8)} = 0,14_{(8)} / 5_{(8)}$$

$1,00_{(8)} \mid 5_{(8)}$

$1 \quad\quad 0,14$

10

1

30

$10_{(8)} = 1 \cdot 8 + 0 = 8$

$8 / 5 = 1 \qquad 8 \cdot 5 = 3$

$30_{(8)} = 3 \cdot 8 + 0 = 24$

$24 < 5 = 4$

$24 \cdot 5 = 4$

$0,14_{(8)} \mid 5_{(8)}$

$1 \quad\quad 0,02$

1

1

14

1

$14_{(8)} = 1 \cdot 8 + 4 = 12$

$12 / 5 = 2 \qquad 12 \cdot 5 = 2$

$1\ 1\ 0\ 1$

$567,00_{(8)} +$

$144,00$

$12,00$

$1,00$

$0,46$

$0,02$

$746,50_{(8)}$

Ex 2:

$$1345,62_{(7)} = ?_{(16)}$$

with 2 digits of the fractional part in base 16

$$1345,62_{(7)} = 1_{(7)} \cdot 7^3 + 3_{(7)} \cdot 7^2 + 4_{(7)} \cdot 7^1 + 5_{(7)} \cdot 7^0 +$$

$$+ 6_{(7)} \cdot 7^{-1} + 2_{(7)} \cdot 7^{-2} = \underbrace{1_{(16)} \cdot 7^3}_{157_{(16)}} + \underbrace{3_{(16)} \cdot 7^2}_{93_{(16)}} + \underbrace{4_{(16)} \cdot 7^1}_{1C_{(16)}} +$$

$$+ \underbrace{5_{(16)} \cdot 7^0}_{5_{(16)}} + \underbrace{6_{(16)} \cdot 7^{-1}}_{0,DB_{(16)}} + \underbrace{2_{(16)} \cdot 7^{-2}}_{0,0A_{(16)}}$$

$1_{(7)} = 1_{(16)}$

$3_{(7)} = 3_{(16)}$

$4_{(7)} = 4_{(16)}$

$5_{(7)} = 5_{(16)}$

$6_{(7)} = 6_{(16)}$

$2_{(7)} = 2_{(16)}$

a) $\dfrac{4_{(16)} \cdot}{7_{(16)}}$

$\overline{1C_{(16)}}$

b) $\dfrac{7_{(16)}}{7_{(16)}}$

$\overline{31_{(16)}}$

$\dfrac{31_{(16)} \cdot}{3_{(16)}}$

$\overline{93_{(16)}}$

c) $\dfrac{31_{(16)} \cdot}{7_{(16)}}$

$\overline{157_{(16)}}$

d) $6_{(16)} \cdot 7_{(16)}^{-1} = 6_{(16)} / 7_{(16)}$

6,00	$7_{(16)}$
1	0,DB
60	
1	
50	

$60_{(16)} = 6 \cdot 16 + 0 = 96$

$96 / 7 = 13 = D_{(16)}$

$96 \times 7 = 5$

$50_{(16)} = 5 \cdot 16 + 0 = 80$

$80 / 7 = 11 = B_{(16)}$

$80 \times 7 = 3_{(16)}$

e) $2_{(16)} \cdot 7_{(16)}^{-2} = \left(2_{(16)} / 7_{(16)}\right) / 7_{(16)}$

$$\begin{array}{l|l} 2,00_{(16)} & 7_{(16)} \\ \hline 1 & 0,49 \\ 20 & \\ 1 & \\ 40 & \end{array}$$

$20 = 2 \cdot 16 + 0 = 32$

$32 / 7 = 4 \qquad 32 \% 7 = 4$

$40 = 4 \cdot 16 + 0 = 64$

$64 / 7 = 9$

$$\begin{array}{l|l} 0,49_{(16)} & 7_{(16)} \\ \hline 1 & 0,0A_{(16)} \\ 4 & \\ 1 & \\ 49 & \end{array}$$

$49_{(16)} = 4 \cdot 16 + 9 = 73$

$73 / 7 = 10 = A_{(16)}$

$$\begin{array}{r} 157,00_{(16)} \; + \\ 93,00_{(16)} \\ 1C,00_{(16)} \\ 5,00_{(16)} \\ 0,DB_{(16)} \\ 0,0A_{(16)} \\ \hline 20B,E5 \end{array}$$

Succesive div and multipl:

$N_{(b)} = N'_{(h)}$

- when $b > h$, calculations in b

Ex 1: $746{,}32_{(8)} = ?_{(5)}$ with 2 digits

$746_{(8)} \div 5_{(8)}$: $746 - 1 \to 24$, $24 - 1 \to 06$, $06 - 1 \to$ (1); quotient $141_{(8)}$

$141_{(8)} \div 5_{(8)}$: $14 - 1 \to 14$, $14 - 1 \to 21$, $21 - 1 \to$ (2); quotient 023

$023 \div 5_{(8)}$: $23 - 1 \to$ (4); quotient 3

$3 \div 5_{(8)}$: $3 - 1 \to$ (3); quotient 0

$24_{(8)} = 2 \cdot 8^1 + 4 = 20$

$20 / 5 = 4 \qquad 20 \% 5 = 0$

$14_{(8)} = 1 \cdot 8 + 4 = 12$

$12 / 5 = 2 \qquad 12 \% 5 = 2$

$21_{(8)} = 2 \cdot 8 + 1 = 17$

$17 / 5 = 3 \qquad 17 \% 5 = 2$

$23_{(8)} = 2 \cdot 8 + 3 = 19$

$19 / 5 = 3 \qquad 19 \% 5 = 4$

$746_{(8)} = 3421_{(5)}$

$0{,}32_{(8)} \cdot 5_{(8)} = 2{,}02$

$0{,}02_{(8)} \cdot 5_{(8)} = 0{,}12$

$\Rightarrow 0{,}32_{(8)} = 0{,}2_{(5)}$

$746{,}32_{(8)} \simeq 3421{,}20_{(5)}$