$$DNF(U) = (\neg p \wedge q \wedge r) \vee (\neg p \wedge q) \vee (q \wedge r)$$

$$\equiv (\neg p \wedge q) \vee (q \wedge r)$$

$(A \wedge B) \vee A \equiv A$ (absorption law)

Example 3:

$$DFF(U) = (\underbrace{\neg q \wedge q}_{F}) \vee (\underbrace{\neg q \wedge \neg p \wedge q}_{F}) \vee (\underbrace{\neg q \wedge \neg p \wedge p}_{F})$$

Example 2:

$H_1 \wedge H_2 \wedge H_3 \wedge H_4 \wedge \neg C$ (1) ✓

| α for (1)

H_1: $K \wedge S \to M$ (2) ✓

|

H_2: $KE \to K$ (3) ✓

|

H_3: $\neg KE$

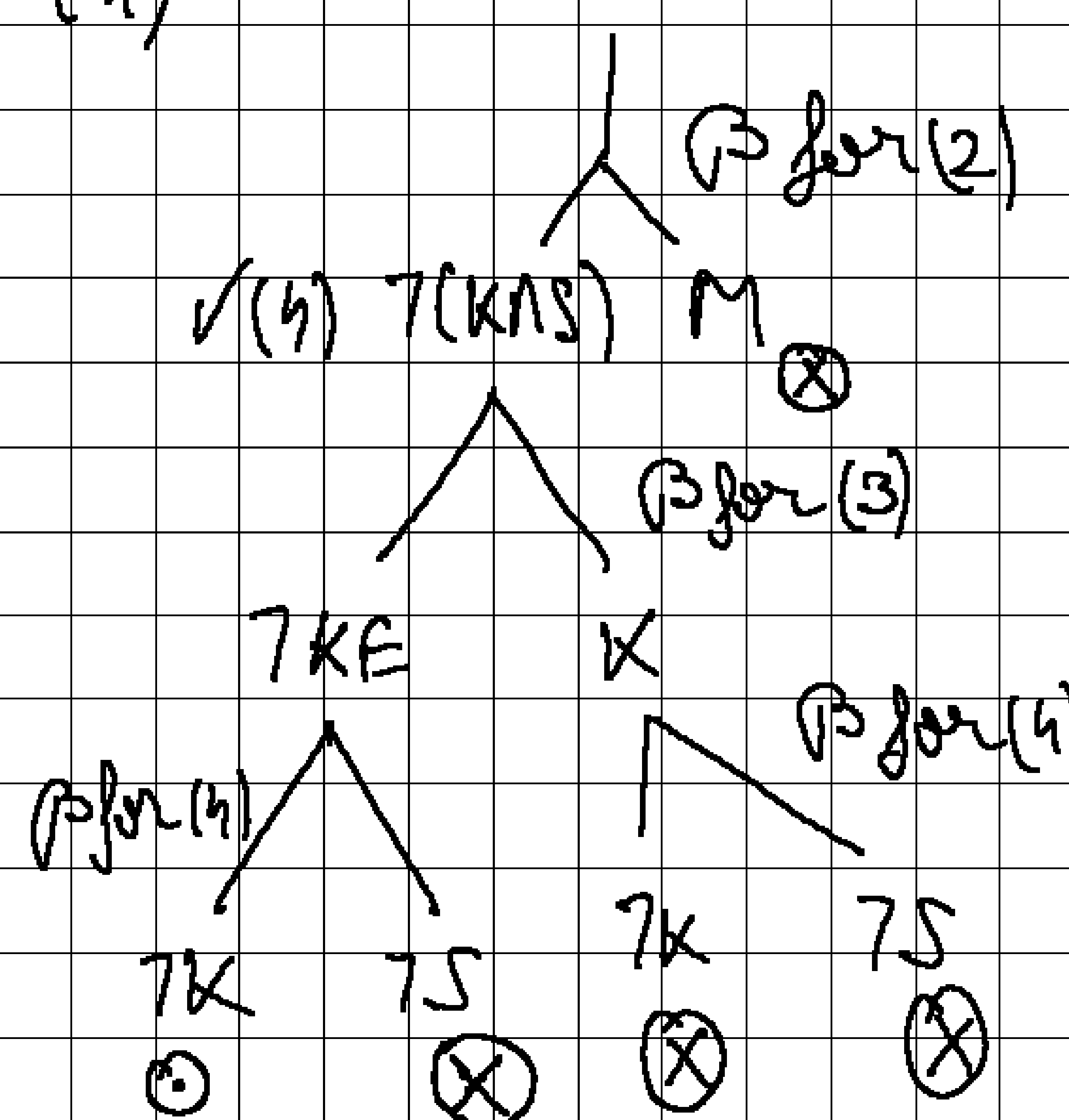
|

H_4: S

|

$\neg C$: $\neg M$

|



$(\forall x)\, A(x) \equiv A(a) \wedge A(b) \wedge A(c)$

$x \in D = \{a, b, c\}$

$U \equiv V$ iff

$U \models V$ and $V \models U$

$\neg(A \to B)$

$|$ α rule

A

$|$

$\neg B$

$\neg U_2 = \neg\big(((\forall x)P(x) \to (\forall x)Q(x)) \to (\forall x)(P(x) \to Q(x))\big)$ (1)

| α for (1)

$(\forall x)P(x) \to (\forall x)Q(x)$ (2) ✓

$(\exists x)\neg(P(x) \to Q(x)) \equiv \neg(\forall x)(P(x) \to Q(x))$ (3) ✓

| δ for (3), a-new const

$\neg(P(a) \to Q(a))$ 4 ✓

| α for (4) ✓

$P(a)$

$\neg Q(a)$

| β for (2)

$\neg(\forall x)P(x)$ (5) ✓ $\equiv$ $\equiv (\exists x)\neg P(x)$	$(\forall x)Q(x)$ (6)
| δ for (5) b-new const	| γ for (6) a-for inst.
$\neg P(b)$	$Q(a)$
$\odot$	(6) - copy of formula (6)
	$\otimes$

$\neg(A \to B)$
| α rule
A
$\neg B$

Models of $\neg U_2$ are provided by the open branch

$i = \langle D, m \rangle$

$D = \{a, b\}$

$m(P) : \{a, b\} \to \{T, F\}$

$m(Q) : \{a, b\} \to \{T, F\}$

$m(P)(a) = T \quad , \quad m(P)(b) = F$

$m(Q)(a) = F \quad , \quad m(Q)(b) = T$

$v^i(\neg U_2) = T \quad , \quad v^i(U_2) = F$

i - generic model that generates concrete models

$i_1 - \langle D_1, m_1 \rangle$

$D_1 = \{2, 3\}$

$m(P)(X)$: "X is even"

$m(Q)(X)$: "X is odd"