

2. Find the general solution of the following linear planar system using the reduction method

$$x' = 2x - 5y \Rightarrow 5y = 2x - x' \Rightarrow y = \frac{2x - x'}{5}$$

$$y' = x - 2y$$

$$\begin{aligned} x'' &= 2x' - 5y' = 2x' - 5(x - 2y) = 2x' - 5x + 10y = \\ &= 2x' - 5x + 10 \cdot \frac{2x - x'}{5} = 2x' - 5x + 4x - 2x' = -x \end{aligned}$$

$$\Rightarrow x'' - x = 0$$

$$\Rightarrow \lambda^2 - 1 = 0$$

$$\Rightarrow \lambda = \pm i$$

General solution: $x(t) = C_1 \cos t + C_2 \sin t$

$$x'(t) = -C_1 \sin t + C_2 \cos t$$

$$\Rightarrow y(t) = \frac{2x(t) - x'(t)}{5} = \frac{2(C_1 \cos t + C_2 \sin t) - (-C_1 \sin t + C_2 \cos t)}{5}$$

$$= \frac{2C_1 \cos t + 2C_2 \sin t + C_1 \sin t - C_2 \cos t}{5} =$$

$$= \frac{(2C_1 - C_2) \cos t + (2C_2 + C_1) \sin t}{5}$$

, C_1, C_2 constants