Computational logic :

I Numeration systems

II Logic

III Logic circuits

Positional systems :

- binary $10100_{(2)}$, digits : 0, 1
- octal $765_{(8)}$, digits : 0, 1 ... 7
- decimal $1390_{(10)}$, digits : 0, 1 9
- hexadecimal $A096_{(16)}$ digits : 0, ... 10, A^{10}, B^{11}, C^{12}, D^{13}, E^{14}, F^{15}

Conversions:

1) substitution method : $N_{(b)} = ?_{(10)}$

Example: $232_{(10)} = 2 \cdot 10^0 + 3 \cdot 10^1 + 2 \cdot 10^2 = 232$ (210)

$276_{(8)} = ?_{(10)}$

$= 6 \cdot 8^0 + 7 \cdot 8^1 + 2 \cdot 8^2 = 6 + 56 + 128 = 190$

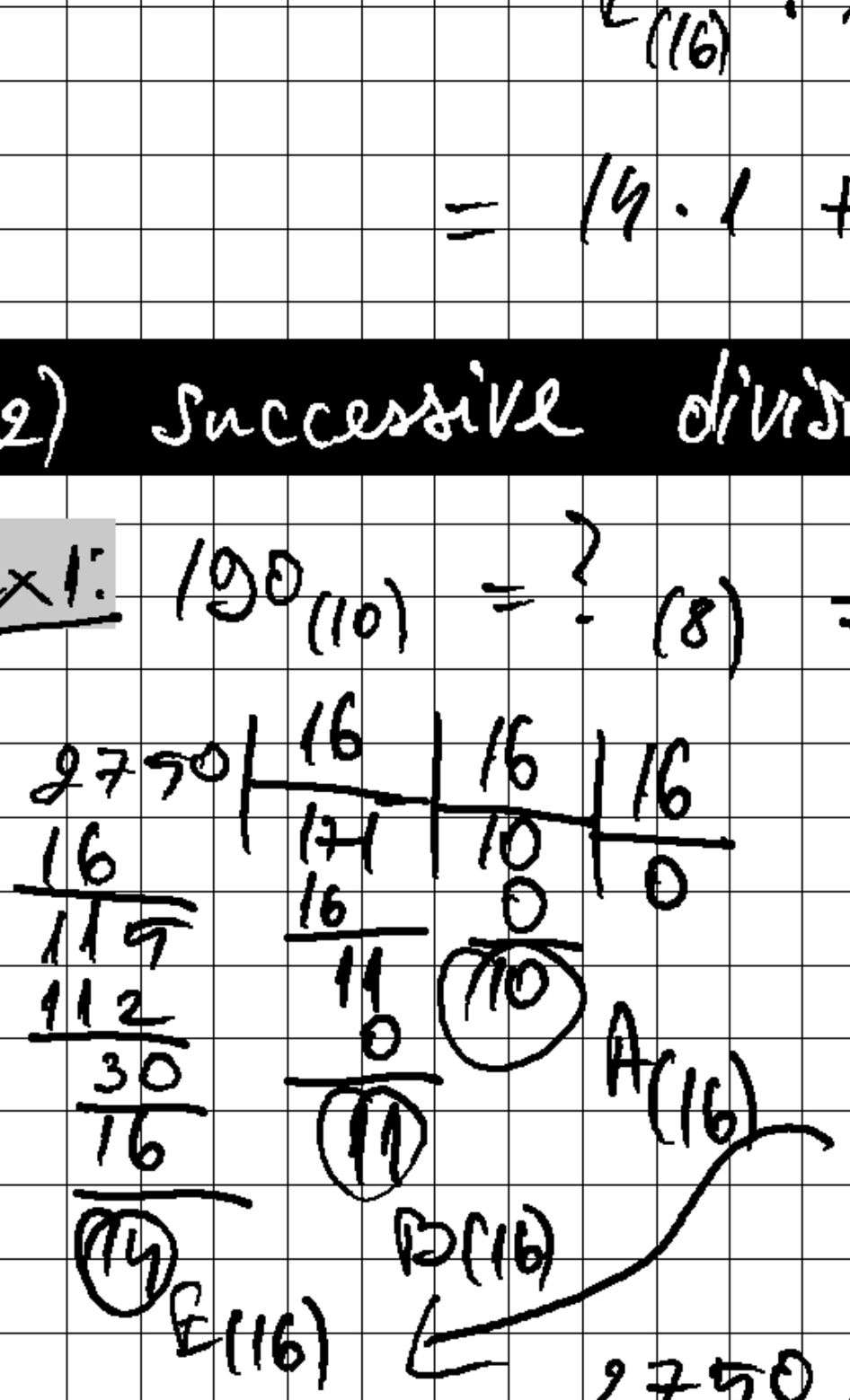
$ABE_{(16)} = ?_{(10}$

$= E_{(16)} \cdot 16^0 + B_{(16)} \cdot 16^1 + A_{(16)} \cdot 16^2$

$= 14 \cdot 1 + 11 \cdot 16 + 10 \cdot 16^2 = 2750$

2) Successive divisions : $N_{(10)} = N'_{(b)}$

Ex1: $190_{(10)} = ?_{(8)} = 276_{(8)}$



$2750_{(10)} = ?_{(16)} = ABE_{(16)}$

Aoldition:

Ex1:

Coonies 1 1 1 0
3 2 1 +
8 9 9
---
1 2 2 0

it:

0+1+9 = 10

1=10/10

0=10÷10

Ex2:

1 1 1 1 1 0 0
1 0 1 1 1 0 (2) +
0 1 0 1 1 1 (2)
---
1 0 0 0 1 0 1

it:

0(2) + 1(2) + 1(2) =

= 0+1+1=2

2/2=1

2÷2=0

1(2) + 1(2) = 10(2)

7(8) + 1(8) = 10(8)

9(10) + 1(10) = 10(10)

F(16) + 1(16) = 10(16)

Ex3:

$$\begin{array}{r} {\scriptstyle 1\,1\,0\,1\,0} \\ 7426_{(8)} + \\ 3513_{(8)} \\ \hline 13141_{(8)} \end{array}$$

it1: $6_{(8)} + 6_{(8)} + 3_{(8)} =$

$= 0 + 6 + 3 = 9_{(10)}$

$1 = 9/8 = 1$

$9\%.8 = 1$

it2: $1_{(8)} + 2_{(8)} + 1_{(8)} = 1 + 2 + 1 = 4_{(10)}$

$4/8 = 0$ $\quad 4\%.8 = 4$

it3: $3_{(8)} + 4_{(8)} + 5_{(8)} = 0 + 4 + 5$

$= 9_{(10)}$

$9/8 = 1$, $9\%.8 = 1$

it4: $1_{(8)} + 7_{(8)} + 3_{(8)} =$

$= 1 + 7 + 3 = 11_{(10)}$

$11/8 = 1$; $11\%.8 = 3$

Ex4:

$$\begin{array}{r} {\scriptstyle 0\,1\,1\,1\,0} \\ A35C_{(16)} + \\ 0E7_{(16)} \\ \hline B143_{(16)} \end{array}$$

it1: $C_{(16)} + 7_{(16)} + D_{(16)} =$

$= 12 + 7 + 13 = 32$

$32/16 = 2$ $\quad 32\%.16 = 0$

it2: $5_{(16)} + E_{(16)} + 4_{(16)} =$

$= 5 + 14 + 4 = 23$

$23/16 = 1$ $\quad 23\%.16 = 7$

it3: $3_{(16)} + A_{(16)} + 0_{(16)} =$

$= 3 + 10 + 0 = 13$

$13/16 = 0$ $\quad 13\%.16 = 13$

it4: $1_{(16)} + A_{(16)} + 0_{(16)}$

$1 + 10 + 0 = 11$

$11/16 = 0$ $\quad 11\%.16 = 11$

Multiplication by one digit:

Ex 1:

carries 1 3 3 0

$$\begin{array}{r} 376 \cdot \\ 5 \\ \hline 1880 \end{array}$$

it1: $0+6\cdot 5=30$ $30/10=3$

$30\%10=0$

Ex 2:

9 7 B 2 0

$$\begin{array}{r} A7C3_{(16)} \\ F_{(16)} \\ \hline 9D46D_{(16)} \end{array}$$

it1: $0_{(16)}+3_{(16)}\cdot F_{(16)}=$

$=0+3\cdot 15=45$

$45/16=2$

$45\%16=13=D_{(16)}$

it2: $2_{(16)}+F_{(16)}\cdot C_{(16)}=2+15\cdot 12=182$

$182/16=11$ $182\%16=6$

$=B_{(16)}$

it3: $B_{(16)}+7_{(16)}\cdot F_{(16)}=11+7\cdot 15=116$

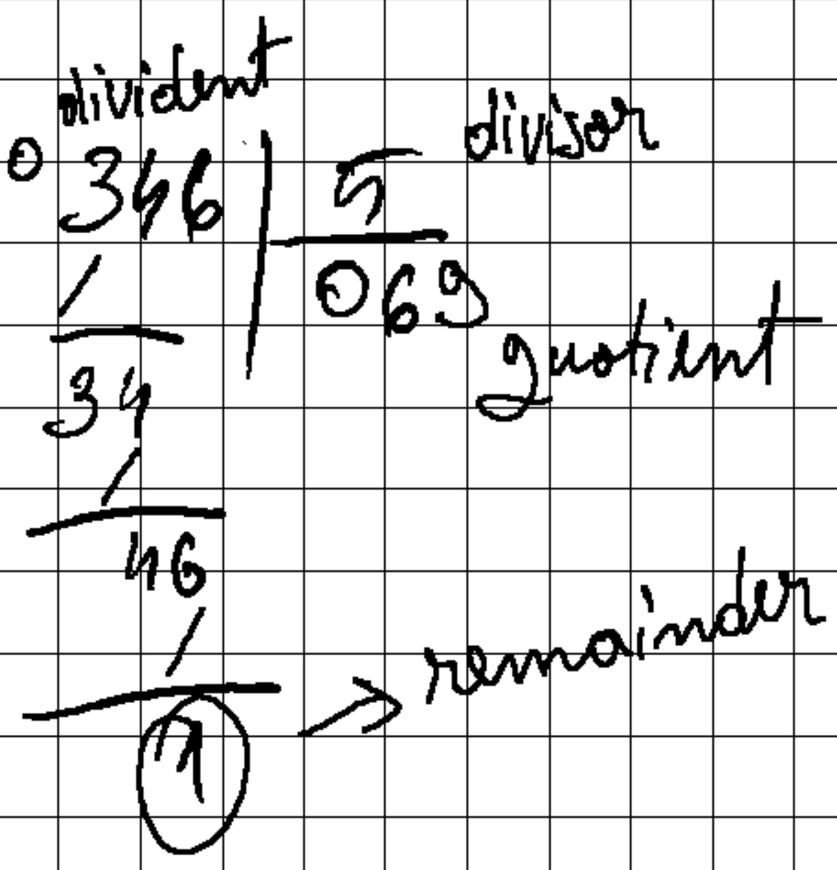
$116/16=7$ $116\%16=4$

it4: $7_{(16)}+A_{(16)}\cdot F_{(16)}=7+10\cdot 15=157$

$157/16=9$ $157\%16=13=D_{(16)}$

Division by one digit

Ex 1:



it1: $03 \rightarrow 0 \cdot 10 + 3 = 3$, $3 / 5 = 0$

$3 \% 5 = 3$

it2: $34 = 3 \cdot 10 + 4$, $34 / 5 = 6$

$34 \% 5 = 4$

Ex 2:

$$
\begin{array}{l|l}
0 4571_{(8)} & 6_{(8)} \\
\underline{\ \ /\ \ } & \overline{0624} \rightarrow \text{quotient} \\
45 & \\
\underline{\ \ /\ \ } & \\
17 & \\
\underline{\ \ /\ \ } & \\
31 & \\
\underline{\ \ /\ \ } & \\
1 \rightarrow \text{remainder} &
\end{array}
$$

it1: $04_{(8)} = 0 \cdot 8 + 4 = 4$, $4 / 6 = 0$
$4 \% 6 = 4$

it2: $45_{(8)} = 4 \cdot 8 + 5 = 37$, $37 / 6 = 6$
$37 \% 6 = 1$

it2: $17_{(8)} = 1 \cdot 8 + 7 = 15$, $15 / 6 = 2$
$15 \% 6 = 3$

it4: $31_{(8)} = 3 \cdot 8 + 1 = 25$, $25 / 6 = 4$
$25 \% 6 = 1$

Ex 3:

^{0}BEOF$_{(16)}$ | A$_{(16)}$

1301 → quotient

1E

1

00

1

07

1

(7) → remainder

i1: OB$_{(16)}$ = 0·16 + 11 = 11 , 11 / 10 = 1

i2: 1E$_{(16)}$ = 1·16 + 14 = 30 , 30 / 10 = 3

30 :. 10 = 0

i3: 00$_{(16)}$ = 0·16 + 0 = 0 , 0 / 10 = 0

0 :. 10 = 0

i4: 0F$_{(16)}$ = 0·16 + 15 = 15 , 15 / 10 = 1

15 :. 10 = 5

Ex4:

$$ {}^{0}B8AC_{(16)} \;\Big|\; 8_{(16)} $$

Quotient: 1715 → quotient

Long division steps:

B8AC
/
38
/
0A
/
2C
/
4 → remainder

it1: $0B_{(16)} = 0 \cdot 16 + 11 = 11$, $11 / 8 = 1$
$11 \% 8 = 3$

it2: $38_{(16)} = 3 \cdot 16 + 8 = 56$, $56 / 8 = 7$
$56 \% 8 = 0$

it3: $0A_{(16)} = 0 \cdot 16 + 10 = 10$, $10 / 8 = 1$
$10 \% 8 = 2$

it4: $2C = 2 \cdot 16 + 12 = 44$, $44 / 8 = 5$
$44 \% 8 = 4$