2. a) $\langle x, y \rangle = 0$

$\langle x, y \rangle = x_1 y_1 + x_2 y_2 + \dots + x_n y_n$

$x_1 y_1 + x_2 y_2 + \dots + x_n y_n = 0 \Rightarrow x, y$ are orthogonal vectors

$\Leftrightarrow \langle x, y \rangle = \|x\| \|y\| \cdot \underbrace{\cos(\theta)}_{=0}$

$\Rightarrow \langle x, y \rangle = \|x\| \cdot \|y\| \cdot 0 = 0$

b) $\cdot \|x+y\| = \|x-y\|$

x, y - orthogonal ?

We assume that $\|x+y\| = \|x-y\| \;/ ()^2 \Leftrightarrow$

$\Leftrightarrow (x+y) \cdot (x+y) = (x-y)(x-y) \Leftrightarrow$

$\Leftrightarrow \cancel{x^2} + 2xy + \cancel{y^2} = \cancel{x^2} - 2xy + \cancel{y^2} \Leftrightarrow 2xy = -2xy \Leftrightarrow$

$\Leftrightarrow xy = -xy \Rightarrow x \cdot y = 0 \Rightarrow x, y$ - orthogonal

Since $x \cdot y = 0 \Rightarrow \begin{cases} \|x+y\|^2 = x^2 + y^2 \\ \|x-y\|^2 = x^2 + y^2 \end{cases} \Rightarrow$

$\Rightarrow \|x+y\|^2 = \|x-y\|^2 \;/ \sqrt{\ } \Leftrightarrow \|x+y\| = \|x-y\|$

c) $\|x+y\|^2 = \|x\|^2 + \|y\|^2$

$(x+y)(x+y) = \|x\|^2 + 2xy + \|y\|^2$

$\Rightarrow \|x\|^2 + 2xy + \|y\|^2 = \|x\|^2 + \|y\|^2 \quad / -(\|x\|^2 + \|y\|^2)$

$2 \cdot x \cdot y = 0 \quad \Rightarrow \quad x \cdot y = 0 \quad \Rightarrow x, y$ - orthogonal

$\|x+y\|^2 = (x+y)(x+y) = x \cdot x + 2xy + y \cdot y^2 = \|x\|^2 + \|y\|^2$

$\Rightarrow \|x+y\|^2 = \|x\|^2 + \|y\|^2$