Forward difference:

$$f'(x) = \frac{f(x+h) - f(x)}{h} + O(h)$$

$$f(x+h) = f(x) + h f'(x) + \frac{h^2}{2} f''(x) + \frac{h^3}{6} f'''(x) + \dots \quad \Big| -f(x)$$

$$\Leftrightarrow f(x+h) - f(x) = h f'(x) + \frac{h^2}{2} f''(x) + \frac{h^3}{6} f'''(x) + \dots \quad \Big| : h$$

$$\Leftrightarrow \frac{f(x+h) - f(x)}{h} = f'(x) + \frac{h}{2} f''(x) + \frac{h^2}{6} f'''(x) + \dots$$

$\Rightarrow$ The error is $O(h)$, because all of the other terms involve higher powers of h

Centered difference:

$$f'(x) = \frac{f(x+h) - f(x-h)}{2 \cdot h} + O(h^2)$$

$$\left.\begin{aligned} f(x+h) &= \cancel{f(x)} + h f'(x) + \cancel{\frac{h^2}{2} f''(x)} + \frac{h^3}{6} f'''(x) + \dots \\ f(x-h) &= \cancel{f(x)} - h f'(x) + \cancel{\frac{h^2}{2} f''(x)} - \frac{h^3}{6} f'''(x) + \dots \end{aligned}\right\} \ominus \Rightarrow$$

$$\Rightarrow f(x+h) - f(x-h) = 2 h f'(x) + \frac{2 \cdot h^3}{6} f'''(x) + \dots \quad \Big| : 2h$$

$$\Leftrightarrow \frac{f(x+h) - f(x-h)}{2h} = f'(x) + \frac{h^2}{6} f'''(x) + \dots$$

$\Rightarrow$ The error is $O(h^2)$, because all of the other terms involve higher powers of h