Example 1: of reasoning modeling (check lecture 4)

$$U, U \to V \vdash_{mp} V$$

$$H_5, H_3 \vdash_{mp} J \quad (J6)$$

$$J6, H_2 \vdash_{mp} L \quad (J7)$$

$$J7, H_3 \vdash_{conj} L \wedge 7G \quad (J8$$

$$J8, H_1 \vdash_{mp} M \quad (J9)$$

$(H_1, \ldots, H_5, J6, \ldots, J9)$ the deduction of M from the hypotheses

Example 3: (duck lecture)

$R, Kl, K_S, C_S, Jl, C_{sick}$

H_1: $R \Rightarrow Kl \vee K_S$

H_2: $K_S \Rightarrow C_S$

H_3: Jl

H_4: $R \wedge C_{sick} \wedge \neg C_S$

C: $Kl \wedge Jl$

$H_4 \vdash_{\text{simplification}} R \; (f_5)$

$H_4 \vdash_{\text{simplification}} \neg C_S \; (f_6)$

$f_5, H_1 \vdash_{mp} Kl \vee K_S \; (f_7)$

$f_2, f_6 \vdash_{mt} \neg K_S \; (f_8)$

$f_8, f_7 \vdash_{\text{resolution}} Kl \; (f_9)$

$f_9, H_3 \vdash_{\text{conjunction}} Kl \wedge Jl : C$

(duck proportional inference rules from lecture)

Example 4: (Theorem of deduction) (check lecture)

$$\vdash (p \to r) \to ((p \wedge r \to q) \to (p \to q))$$

reverse theorem of deduction:

Step 1:

if $\vdash (p \to r) \to ((p \wedge r \to q) \to (p \to q))$

then $p \to r \vdash (p \wedge r \to q) \to (p \to q)$

then $p \to r, p \wedge r \to q \vdash p \to q$

then $p \to r, p \wedge r \to q, p \vdash q$ (*)

Step 2: Prove (*)

$f_1 = p \to r$

$f_2 = p \wedge r \to q$

$f_3 = p$

$f_1, f_3 \vdash_{mp} r$ (f_4)

$f_3, f_4 \vdash_{conj} p \wedge r$ (f_5)

$f_5, f_2 \vdash_{mp} q$

$$\frac{}{u, u \to v \vdash v}$$

Step 3 ⑧ we apply theorem of deduction

if $\rho \Rightarrow \kappa$, $\rho \wedge \kappa \Rightarrow \varrho$, $\rho \vdash \varrho$

then $\rho \Rightarrow \kappa$, $\rho \wedge \kappa \Rightarrow \varrho \vdash \rho \Rightarrow \varrho$

then $\rho \Rightarrow \kappa \vdash (\rho \wedge \kappa \Rightarrow \varrho) \Rightarrow (\rho \Rightarrow \varrho)$

then $\vdash (\rho \Rightarrow \kappa) \Rightarrow ((\rho \wedge \kappa \Rightarrow \varrho) \Rightarrow (\rho \Rightarrow \varrho))$

$\rho \Rightarrow \kappa$, $\rho \wedge \kappa \Rightarrow \varrho$, $\rho \vdash \varrho$

$\vdash T_1: (\rho \Rightarrow \kappa) \Rightarrow ((\rho \wedge \kappa \Rightarrow \varrho) \Rightarrow (\rho \Rightarrow \varrho))$

$\vdash T_2: (\rho \Rightarrow \kappa) \Rightarrow (\rho \Rightarrow (\rho \wedge \kappa \Rightarrow \varrho) \Rightarrow \varrho)$

$\vdash T_3: \rho \Rightarrow ((\rho \Rightarrow \kappa) \Rightarrow ((\rho \wedge \kappa \Rightarrow \varrho) \Rightarrow \varrho))$

$\vdash T_4: \rho \Rightarrow ((\rho \wedge \kappa \Rightarrow \varrho) \Rightarrow ((\rho \Rightarrow \kappa) \Rightarrow \varrho))$

$\vdash T_5: \rho \wedge \kappa \Rightarrow \varrho \Rightarrow \rho \Rightarrow ((\rho \Rightarrow \kappa) \Rightarrow \varrho)$

$\vdash T_6: (\rho \wedge \kappa \Rightarrow \varrho) \Rightarrow ((\rho \Rightarrow \kappa) \Rightarrow (\rho \Rightarrow \varrho))$