10.5. Prove the following theorem using the theorem of deduction and it's reverse

$\vdash (p \to (q \to r)) \to (q \to (p \to r))$

Step 1: Apply the reverse of the theorem of deduction

if $\vdash (p \to (q \to r)) \to (q \to (p \to r))$ then

$p \to (q \to r) \vdash q \to (p \to r)$ then

$p \to (q \to r), q \vdash p \to r$ then

$p \to (q \to r), q, p \vdash r$

Step 2: Prove the deduction obtained at step 1

f_1: p - premise (hypothesis)

f_2: q - premise (hypothesis)

f_3: $p \to (q \to r)$ - premise

$f_1, f_3 \vdash_{mp} q \to r$

f_4: $q \to r$

$f_2, f_4 \vdash_{mp} r$

f_5: r

The sequence $(f_1, f_2, f_3, f_4, f_5)$ is the deduction of r from the premises $p \to (q \to r), q, p$

Step 3: We begin the deduction $p \to (q \to r), q, p \vdash r$ proved at Step 2.

if $p \to (q \to r), q, p \vdash r$ then

$p \to (q \to r), q \vdash p \to r$ then

$p \to (q \to r) \vdash q \to (p \to r)$ then

$\vdash (p \to (q \to r)) \to (q \to (p \to r))$