2.5 $C_1 = P(x) \vee Q(x)$

$C_2 = \neg P(a)$

$Res_\theta(C_1, C_2) = Q(a)$

$\theta = [x \leftarrow a] =$

$= mgu(P(x), P(a))$

$\downarrow$

most general unifiers

Substitutions

$\theta = [x \leftarrow a,\ y \leftarrow z,\ t \leftarrow f(b)]$ $x, y, z, t \in Var$

$a, b \in Const$

$x \leftarrow a$ ✓ $x \leftarrow f(a)$ ✓

$x \leftarrow y$ ✓ $x \leftarrow f(y)$ ✓

$\cancel{a \leftarrow x}$ $\cancel{x \leftarrow f(x)}$

$\cancel{a \leftarrow b}$

$\cancel{f(x) \leftarrow a}$

most general unifiers

Find mgu of l_1 and l_2

a) $l_1 = P(a, x, g(f(y)))$ $l_2 = P(x, y, g(f(b)))$

$\theta = \varepsilon$

$\theta(l_1) = P(a, x, g(f(y)))$

$\theta(l_2) = P(x, y, g(f(b)))$

it 1: $\lambda = [x \leftarrow a]$

$\theta = \theta\lambda = [x \leftarrow a]$

$\theta(l_1) = P(a, \underline{a}, g(f(y)))$
$\theta(l_2) = P(a, \underline{y}, g(f(b)))$

it 2: $\lambda = [y \leftarrow a]$

$\theta = \theta\lambda = [x \leftarrow a][y \leftarrow a] =$
$= [x \leftarrow a, y \leftarrow a]$

$\theta(l_1) = P(a, a, g(f(a)))$
$\theta(l_2) = P(a, a, g(f(b)))$

The constants a, b are not unifiable so the literals l_1, l_2 are not unifi

$l_1 = P(a, h(x,u), g(f(z)))$
$l_2 = P(y, h(u, f(z)), g(x))$

$\theta = \varepsilon$

$\theta(l_1) = P(\underline{a}, h(x,u), g(f(z)))$
$\theta(l_2) = P(\underline{y}, h(u, f(z)), g(x))$

it1: $\lambda = [y \leftarrow a]$

$\theta = \theta\lambda = [y \leftarrow a]$

$\theta(l_1) = P(a, h(x, u), g(f(z)))$

$\theta(l_2) = P(a, h(u, f(z)), g(x))$

it2: $\lambda = [x \leftarrow u]$

$\theta = \theta\lambda = [y \leftarrow a][x \leftarrow u] = [y \leftarrow a, x \leftarrow u]$

$\theta(l_1) = P(a, h(u, u), g(f(z)))$

$\theta(l_2) = P(a, h(u, f(z)), g(u))$

it3: $\lambda = [u \leftarrow f(z)]$

$\theta = \theta\lambda = [y \leftarrow a, x \leftarrow u][u \leftarrow f(z)] =$

$= [y \leftarrow a, x \leftarrow f(z), u \leftarrow f(z)]$

$\theta(l_1) = P(a, h(f(z), f(z)), g(f(z)))$

$\theta(l_2) = P(a, h(f(z), f(z)), g(f(z)))$

$\theta(l_1) = \theta(l_2) =$ the common instance of l_1 and l_2

3. Prove the inconsistency of the following set of clauses using lock resolution.

$$S_5 = \{ P(x) \vee Q(x),\ \neg P(a) \vee W(x),\ \neg Q(y) \vee R(y),\ \neg R(x) \vee W(x),\ \neg W(a) \}$$

$C_1 = {}_{(1)}P(x) \vee {}_{(2)}Q(x)$

$C_3 = {}_{(5)}\neg Q(y) \vee {}_{(6)}R(y)$

$C_2 = {}_{(3)}\neg P(a) \vee {}_{(4)}W(x)$

$C_4 = {}_{(7)}\neg R(x) \vee {}_{(8)}W(x)$

$C_5 = {}_{(9)}\neg W(a)$

$$C_6 = Res^{lock}_{\theta_1 = [x \leftarrow a]}(C_1, C_2) = {}_{(2)}Q(a) \vee {}_{(4)}W(a)$$

$$C_7 = Res^{lock}_{\theta_2 = [y \leftarrow a]}(C_3, C_6) = {}_{(4)}W(a) \vee {}_{(6)}R(a)$$

$$C_8 = Res^{lock}(C_7, C_5) = {}_{(6)}R(a)$$

$$C_9 = Res^{lock}_{\theta_1}(C_8, C_4) = {}_{(8)}W(a)$$

$$C_{10} = Res^{lock}(C_9, C_5) = \square$$

$S_5 \vdash_{Res}^{lock} \square$ so S_5 is inconsistent

5.5 Check whether the following formulas are theorems or not using predicate resolution

$$U = (\exists y)(\exists x)P(x,y) \leftrightarrow (\forall x)(\exists y)P(x,y)$$

$$U \equiv \overbrace{\left((\exists y)(\exists x)P(x,y) \to (\forall x)(\exists y)P(x,y)\right)}^{U_1} \wedge \underbrace{\left((\forall x)(\exists y)P(x,y) \to (\exists y)(\exists z)P(x,y)\right)}_{U_2}$$

$$\neg U_1 = \neg\left((\exists y)(\exists x)P(x,y) \to (\forall x)(\exists y)P(x,y)\right) \equiv$$

$$\underset{\text{Replace } \neg(\to)}{\equiv} (\exists y)(\exists x)P(x,y) \wedge (\exists \overset{u}{x})(\forall \overset{t}{y})\neg P(\overset{u}{x},\overset{t}{y}) \equiv$$

$$\underset{\substack{\text{rename} \\ u \leftarrow x,\, t \leftarrow y}}{\equiv} (\exists y)(\exists x)P(x,y) \wedge (\exists u)(\forall t)\neg P(u,t)$$

$$(\neg U_1)^P = (\exists y)(\exists x)(\exists u)(\forall t)\left(P(x,y) \wedge \neg P(u,t)\right)$$

Skolem form

$[y \leftarrow a,\ x \leftarrow b,\ u \leftarrow c]$

$$(\neg U_1)^S = (\forall t)[P(b,a) \wedge \neg P(c,t)]$$

$$(\neg U_1)^C = \underbrace{P(b,a)}_{C_1} \wedge \underbrace{\neg P(c,t)}_{C_2}$$

$$A \Leftrightarrow B \equiv (A \Rightarrow B) \wedge (B \Rightarrow A)$$

$$A \Rightarrow B \equiv \neg A \vee B$$

$$\neg(A \Rightarrow B) \equiv A \wedge \neg B$$

$$T \quad \vdash U \quad \text{iff} \quad (\neg U)^C \vdash_{\text{res}} \square$$

literals $P(b,a)$ and $P(c,t)$ are not unifiable because b, c dif. constants so C_1, C_2 cannot resolve and the empty clause couldn't be derived

$$(\neg U_1)^C \not\vdash_{\text{res}} \square \quad \text{so} \quad \not\vdash U_2$$

$$\neg U_2 = \neg(\forall x)(\exists y)P(x,y) \rightarrow (\exists y)(\exists x)P(x,y) \equiv$$

$$\equiv (\forall x)(\exists y)P(x,y) \wedge (\forall y)(\forall x)\neg P(x,y) \equiv$$

replace $\neg(\rightarrow)$

$t \quad u \quad u \quad t$

$$\equiv (\forall x)(\exists y)P(x,y) \wedge (\forall t)(\forall u)\neg P(u,t)$$

$(\neg U_2)^P = (\forall x)(\exists y)(\forall t)(\forall u)(P(x,y) \wedge \neg P(u,t))$

$[y \leftarrow f(x)]$

$(\neg U_2)^S = (\forall x)(\forall t)(\forall u)(P(x,f(x)) \wedge \neg P(u,t))$

$$(\neg U_2)^C = \underbrace{P(x,f(x))}_{C_1} \wedge \underbrace{\neg P(u,t)}_{C_2}$$

$$mgu(P(x,f(x)), P(u,t)) = [u \leftarrow t, t \leftarrow f(x)] = \theta$$

$$C_3 = Res_\theta(C_1, C_2) = \square \Rightarrow (\neg U_2)^C \vdash \square \Rightarrow$$

$\Rightarrow U_2$ is a theorem

4. Using a refinement of predicate resolution prove

5. the distributivity of '$\forall$' over '$\wedge$'

$$\vdash (\forall x)P(x) \wedge (\forall x)Q(x) \leftrightarrow (\forall x)(P(x) \wedge Q(x))$$

$$\nvdash ((\exists x)P(x) \rightarrow (\exists x)Q(x)) \rightarrow (\forall x)(P(x) \rightarrow Q(x))$$

$$U_1 = (\forall x)(P(x) \overset{1}{\rightarrow} Q(x)) \overset{2}{\rightarrow} ((\exists x)P(x) \overset{3}{\rightarrow} (\exists x)Q(x))$$

$$U_2 = ((\exists x)P(x) \rightarrow (\exists x)Q(x)) \rightarrow (\forall x)(P(x) \rightarrow Q(x))$$

(T) $\vdash \mathcal{U}_1$ iff $(\neg \mathcal{U}_1)^c \vdash^{\text{Prud}}_{\text{Res}} \square$

$\nvdash \mathcal{U}_2$ iff $(\neg \mathcal{U}_2)^c \vdash^{\text{Prud}}_{\text{Res}} \square$

replace $\neg(\to)2$

$\neg \mathcal{U}_1 \equiv (\forall x)(P(x) \overset{1}{\to} Q(x)) \wedge \neg((\exists x) P(x) \overset{3}{\to} (\exists x) Q(x))$

replace $(\to)1$

$\equiv (\forall x)(\neg P(x) \vee Q(x)) \wedge (\exists x) P(x) \wedge \neg (\exists x) Q(x)$

replace $\neg(\to)3$
DeMorgans Law

$\equiv (\forall x)(\neg P(x) \vee Q(x)) \wedge (\exists x) P(x) \wedge (\forall x) \neg Q(x)$

y y z z

rename

$\equiv (\forall x)(\neg P(x) \vee Q(x)) \wedge (\exists y) P(y) \wedge (\forall z) \neg Q(z)$

$(\neg \mathcal{U}_1)^P = (\exists y)(\forall x)(\forall z)(\neg P(x) \vee Q(x)) \wedge P(y) \wedge \neg Q(z)$

$(\neg \mathcal{U}_1)^S = (\forall x)(\forall z)((\neg P(x) \vee Q(x)) \wedge P(a) \wedge \neg Q(z))$
$[y \leftarrow a]$

$(\neg \mathcal{U}_1)^C = \underbrace{(\neg P(x) \vee Q(x))}_{C_1} \wedge \underbrace{P(a)}_{C_2} \wedge \underbrace{\neg Q(z)}_{C_3}$

$C_1 = \neg P(x) \vee Q(x)$

$C_2 = P(a)$

$C_3 = Q(z)$

$$C_4 = \mathrm{Res}_{\theta_1 = [x \leftarrow a]}(C_1, C_2) = Q(a)$$

$$C_5 = \mathrm{Res}_{\theta_2 = [z \leftarrow a]}(C_3, C_4) = \square$$

$$\theta_1 = mgu(P(x), P(a))$$

$$\theta_2 = mgu(Q(z), Q(a))$$

$(\neg U_1)^C \vdash_{\mathrm{Res}} \square$ so $\vdash U_1$