1. Let $A \in \mathbb{R}^{m \times m}$, $b \in \mathbb{R}^m$

$g : \mathbb{R}^m \to \mathbb{R}$, $g(v) = \frac{1}{2} v^T A \cdot v + b^T v \to \min$

A - symmetric, positive definite.

Prove that the minimum of g is given by

$v = -A^{-1} \cdot b$

$$\left.\begin{array}{l} \nabla g = \left[\frac{\partial g}{\partial v_1}, \frac{\partial g}{\partial v_2}, \ldots, \frac{\partial g}{\partial v_m} \right] = Av + b \\ \\ \nabla g = 0 \end{array}\right\} \Rightarrow$$

$$\Rightarrow \quad Av + b = 0 \quad \Leftrightarrow$$

$$\Leftrightarrow A^{-1} \cdot / \quad Av = -b \quad \Leftrightarrow$$

$$\Leftrightarrow \quad v = -A^{-1} \cdot b$$

2. $f : \mathbb{R}^n \to \mathbb{R}$, $x \in \mathbb{R}^n$, $H(x)$ - positive definite.

$f(x+h) \simeq f(x) + \nabla f(x)^T \cdot h + \frac{1}{2} \cdot h^T H(x) \cdot h \to \min$

Prove that the direction h that minimizes $f(x+h)$ is given by $h = -H^{-1}(x) \nabla f(x)$

$$\left.\begin{array}{l} \nabla f(x+h) = \nabla f(x) + H(x) \cdot h \\ \nabla f(x+h) = 0 \end{array}\right\} \Rightarrow \nabla f(x) + H(x) \cdot h = 0 \Leftrightarrow$$

$$\Leftrightarrow H(x)^{-1} \cdot / \quad H(x) h = -\nabla f(x) \quad \Leftrightarrow \quad h = -H(x)^{-1} \nabla f(x)$$