5.8. Using Quine's method, simplify the following boolean functions given by their values 0.

$$f_8(0,0,1) = f_8(1,0,1) = f_8(1,1,0) = 0$$

$$DCF\big(f_8(x_1,x_2,x_3)\big) = \underbrace{\overline{x_1}\,\overline{x_2}\,\overline{x_3}}_{m_0} \vee \underbrace{\overline{x_1}\,x_2\,\overline{x_3}}_{m_2} \vee \underbrace{\overline{x_1}\,x_2\,x_3}_{m_4} \vee$$

$$\vee \underbrace{x_1\,\overline{x_2}\,\overline{x_3}}_{m_3} \vee \underbrace{x_1\,x_2\,x_3}_{m_7}$$

$$S(f_8) = \{\underbrace{(0,0,0)}_{m_0}, \underbrace{(0,1,0)}_{m_2}, \underbrace{(1,0,0)}_{m_4}, \underbrace{(0,1,1)}_{m_3}, \underbrace{(1,1,1)}_{m_7}\} -$$

- ordered ascending by num. of '1'

Simple factorization ↓

	x_1	x_2	x_3	
I	0	0	0	m_0
II	0	1	0	m_2
	1	0	0	m_4
III	0	1	1	m_3
IV	1	1	1	m_7
V = I + II	0	–	0	$m_0 \vee m_2 = \overline{x_1}\,\overline{x_2} = max_1$
	–	0	0	$m_0 \vee m_4 = \overline{x_2}\,\overline{x_3} = max_2$
VI = II + III	0	1	–	$m_2 \vee m_3 = \overline{x_1}\,x_2 = max_3$
VII = III + IV	–	1	1	$m_3 \vee m_7 = x_2 x_3 = max_4$

there are no groups with at most one X in common for V, VI, VII

↑ We can't continue to apply double factorization

$M(f_8) = \{ max_1, max_2, max_3, max_4 \}$

g covers all of this (we also need to cover m_2)

max monoms \ minterm	max_1	max_2	max_3	max_4
$\cancel{m_0}$	*	*		
m_2	*		*	
$\cancel{m_4}$		*		
$\cancel{m_5}$			*	*
$\cancel{m_7}$				*

that's why we have 2 simplified forms, we can chose either max_1 or max_3 to cover m_2

we circle it if we find a unique star on a line

$C(f_8) = \{ max_2, max_4 \}$

$M(f_8) \neq C(f_8) \neq \emptyset$, second simplification code

$g = max_2 \vee max_4$

2 simplified forms

$f_8^{S_1} = g \vee max_1 = \bar{x}_2\bar{x}_3 \vee x_2 x_3 \vee \bar{x}_1\bar{x}_3$

$f_8^{S_2} = g \vee max_3 = \bar{x}_2\bar{x}_3 \vee x_2 x_3 \vee \bar{x}_1 x_2$

6.5 Using Quine's method, simplify the following Boolean functions given in DCF (disjunction of minterms)

$$f_5(X_1,X_2,X_3) = m_0 \vee m_1 \vee m_2 \vee m_4 \vee m_6 \vee m_7 =$$

$$= \bar X_1 \bar X_2 \bar X_3 \vee \bar X_1 \bar X_2 X_3 \vee \bar X_1 X_2 \bar X_3 \vee X_1 \bar X_2 \bar X_3 \vee X_1 X_2 \bar X_3 \vee X_1 X_2 X_3$$

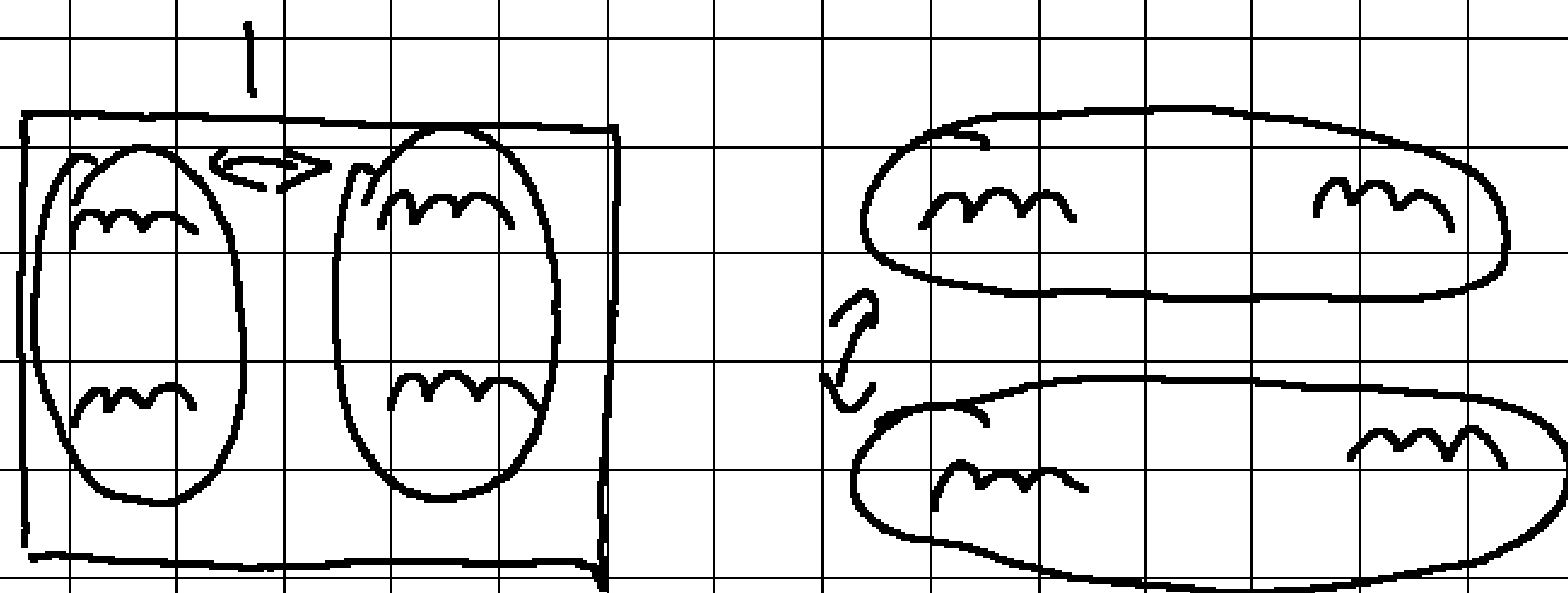
$$S_{f_5} = \{ \underbrace{(0,0,0)}_{m_0}, \underbrace{(0,0,1)}_{m_1}, \underbrace{(0,1,0)}_{m_2}, \underbrace{(1,0,0)}_{m_4}, \underbrace{(1,1,0)}_{m_6}, \underbrace{(1,1,1)}_{m_7} \}$$

		X_1	X_2	X_3	
representation	I v	0	0	0	
	v	0	0	1	
	II v	0	1	0	
	v	1	0	0	
	III v	1	1	0	
	IV v	1	1	1	
simple factorization		0	0	—	$m_0 \vee m_1 = \bar X_1 \bar X_2 = max_1$
	I+II=V	0	—	0	$m_0 \vee m_2 = \bar X_1 \bar X_3$
		—	0	0	$m_0 \vee m_4 = \bar X_2 \bar X_3$
		—	1	0	$m_2 \vee m_6 = X_2 \bar X_3$
	II+III=VI	1	—	0	$m_4 \vee m_6 = X_1 \bar X_3$
	III+IV=VII	1	1	—	$m_6 \vee m_7 = X_1 X_2 = max_2$

double factorization

$\overline{V} + \overline{VI} = \overline{VIII}$ $\boxed{- \quad - \quad 0}$ $m_0 \vee m_2 \vee m_4 \vee m_6 = \overline{x_3} = max_3$

$M(f_5) = \{max_1, max_2, max_3\}$



two ways of obtaining double factorization but we only write it once

max monoms / minterms	max_1	max_2	max_3
m_0	*		*
m_1	*		
m_2			*
m_4			*
m_6		*	*
m_7		*	

$\Rightarrow$ 1 simplified form

$f_5 = max_1 \vee max_2 \vee max_3 = \overline{x_1}\,\overline{x_2} \vee x_1 x_2 \vee \overline{x_3}$

7.5. Simplify the following Boolean functions of 4 variables given by their values 1, using Quine's method

$$f_5(1,1,1,1) = f_5(0,1,0,1) = f_5(0,1,1,1) = f_5(1,1,1,0) =$$
$$= f_5(1,1,0,0) = f_5(1,0,0,0) = f_5(1,0,0,1) = f_5(0,0,0,1) = 1$$

$$S f_5 = \{ \underset{m_{15}}{(1,1,1,1)}, \underset{m_5}{(0,1,0,1)}, \underset{m_7}{(0,1,1,1)}, \underset{m_{14}}{(1,1,1,0)}, \underset{m_{12}}{(1,1,0,0)},$$
$$\underset{m_8}{(1,0,0,0)}, \underset{m_9}{(1,0,0,1)}, \underset{m_1}{(0,0,0,1)} \}$$

Group		x_1	x_2	x_3	x_4	
representation	I	0	0	0	1	m_1
		1	0	0	0	m_8
	II	0	1	0	1	m_5
		1	0	0	1	m_9
		1	1	0	0	m_{12}
	III	1	1	1	0	m_{14}
		0	1	1	1	m_7
	IV	1	1	1	1	m_{15}
$V = I + II$		0	–	0	1	$m_1 \vee m_5 = \bar{x}_1 \bar{x}_3 x_4 = max_1$
		–	0	0	1	$m_1 \vee m_9 = \bar{x}_2 \bar{x}_3 x_4 = max_2$
		1	0	0	–	$m_8 \vee m_9 = x_1 \bar{x}_2 \bar{x}_3 = max_3$
		1	–	0	0	$m_8 \vee m_{12} = x_1 \bar{x}_3 \bar{x}_4 = max_4$

VI = II + III	0	1	–	1	$m_3 \vee m_7 = \bar{x}_1 x_2 x_4 = max_5$
	1	1	–	0	$m_{12} \vee m_{14} = x_1 x_2 \bar{x}_4 = max_6$
VII = III + IV	1	1	1	–	$m_{14} \vee m_{15} = x_1 x_2 x_3 = max_7$
	–	1	1	1	$m_7 \vee m_{15} = x_2 x_3 x_4 = max_8$

$M(f_5) = \{ max_1, max_2, max_3, max_4, max_5, max_6, max_7, max_8 \}$

max monoms / minterms	max_1	max_2	max_3	max_4	max_5	max_6	max_7	max_8
~~m_{15}~~							X	X
~~m_5~~	X				X			
~~m_7~~					X			X
~~m_{14}~~						X	X	
~~m_{12}~~				X		X		
~~m_8~~			X	X				
~~m_9~~		X	X					
~~m_1~~	X	X						

○ or □ ⇒ 2 ways

There is no (*) which is unique on a certain row each minterm is covered by 2 max monoms

$C(f_5) = \emptyset$ third case of simplification $\Rightarrow$

$\Rightarrow$ 2 simplified forms of f_5

$$f_5^{S_1}(x_1, x_2, x_3, x_4) = max_1 \vee max_3 \vee max_6 \vee max_8 =$$

$$= \overline{x_1} x_3 x_4 \vee x_1 \overline{x_2}\,\overline{x_3} \vee x_1 x_2 \overline{x_4} \vee x_2 x_3 x_4$$

$$f_5^{S_2}(x_1, x_2, x_3, x_4) = max_2 \vee max_4 \vee max_5 \vee max_6 =$$

$$= \overline{x_2}\,\overline{x_3} x_4 \vee x_1 \overline{x_3}\,\overline{x_4} \vee \overline{x_1} x_2 x_4 \vee x_1 x_2 x_3$$

8.5. Using Moisil's method simplify the following Boolean functions of 3 variables.

$f_5(x_1,x_2,x_3) = m_1 \vee m_2 \vee m_5 \vee m_6 \vee m_7 =$

$= \bar{x}_1\bar{x}_2x_3 \vee \bar{x}_1x_2\bar{x}_3 \vee x_1\bar{x}_2x_3 \vee x_1x_2\bar{x}_3 \vee x_1x_2x_3$

$S_f = \{(0,0,1), (0,1,0), (1,0,1), (1,1,0), (1,1,1)\} \Rightarrow$ ascending order

Quine's method

	x_1	x_2	x_3	
I	0	0	1	m_1
	0	1	0	m_2
II	1	0	1	m_5
	1	1	0	m_6
III	1	1	1	m_7
IV = I + II	–	0	1	$m_1 \vee m_5 = \bar{x}_2x_3 = max_1$
	–	1	0	$m_2 \vee m_6 = x_2\bar{x}_3 = max_2$
V = II + III	1	–	1	$m_5 \vee m_7 = x_1x_3 = max_3$
	1	1	–	$m_6 \vee m_7 = x_1x_2 = max_4$

$M(f) = \{max_1, max_2, max_3, max_4\}$

P_i "max_1 belongs to a simplified form of f",
$i = \overline{1,4}$

m_1 is covered by max_1 $\Rightarrow P_1 \equiv T$
m_2 is covered by max_2 $\Rightarrow P_2 \equiv T$
m_5 is covered by max_1 or max_3 $\Rightarrow P_1 \vee P_3 \equiv T$
m_6 is covered by max_2 or max_4 $\Rightarrow P_2 \vee P_4 \equiv T$
m_7 is covered by max_3 or max_4 $\Rightarrow P_3 \vee P_4 \equiv T$ $\Rightarrow$

CNF
$\Rightarrow$ $P_1 \wedge P_2 \wedge (P_1 \vee P_3) \wedge (P_2 \vee P_4) \wedge (P_3 \vee P_4) \equiv T$ | apply absorbtion law

$a \wedge (a \vee b) \equiv a$

CNF
$P_1 \wedge P_2 \wedge (P_3 \vee P_4) \equiv T$ distribution

$(P_1 \wedge P_2 \wedge P_3) \vee (P_1 \wedge P_2 \wedge P_4) \equiv T$
DNF

$P_1 \wedge P_2 \wedge P_3 \equiv T$ provides the simplified form

$f^{S_1} = max_1 \vee max_2 \vee max_3 = \overline{x_2} x_3 \vee x_2 \overline{x_3} \vee x_1 x_3$

$P_1 \wedge P_2 \wedge P_4 \equiv T$ provides the simplified form

$f^{S_2} = max_1 \vee max_2 \vee max_4 = \overline{x_2} x_3 \vee x_2 \overline{x_3} \vee x_1 x_2$