$H = n\mathbb{Z} \overset{?}{\subseteq} (\mathbb{Z},+)$

- $n\mathbb{Z} = \{ n\cdot k \mid k \in \mathbb{Z} \} \neq \emptyset \quad (0 = n\cdot 0 \in n\cdot\mathbb{Z})$
- Let $x, y \in n\mathbb{Z} \Rightarrow x = n\cdot k \;, y = n\cdot l$

for some $k, l \in \mathbb{Z}$

$\Rightarrow \; x - y = n(k-l) \in n\mathbb{Z}$

$\Rightarrow$ The set $n\mathbb{Z}$ is a subgroup of $(\mathbb{Z},+)$

$n\mathbb{Z}$ is a subring of $(\mathbb{Z},+)$

- $n\mathbb{Z} \neq \emptyset \quad (0 \in n\mathbb{Z})$
- $\forall\, x, y \in n\mathbb{Z} \;, x - y \in n\mathbb{Z}$
- $\forall\, x, y \in n\mathbb{Z} \;, x = n\cdot k \;, y = n\cdot l \quad (k, l \in \mathbb{Z}) \Rightarrow$

$\Rightarrow \; x\cdot y = n\cdot(n\cdot k\cdot l) \in n\mathbb{Z}$

Let $f: \mathbb{C}^* \to \mathbb{R}^*$, $f(z) = |z|$. Then f is a group homomorphism between $(\mathbb{C}^*, \cdot)$ and $(\mathbb{R}^*, \cdot)$. But $f: \mathbb{C} \to \mathbb{R}$, $f(z) = |z|$ is not a group homomorphism between $(\mathbb{C}, +)$ and $(\mathbb{R}, +)$

$$f(z_1 \cdot z_2) = f(z_1) \cdot f(z_2), \ \forall z_1, z_2 \in \mathbb{C}^*$$

$$|z_1 \cdot z_2| = |z_1| \cdot |z_2|$$

$$|z_1 + z_2| \neq |z_1| + |z_2|$$

$(2\mathbb{Z}, +, \cdot)$ ring without indentity