6. Prove inconsistency using lock resolution

$$S_5 = \{ \overbrace{r \vee q}^{C_1}, \overbrace{r \vee \neg q \vee \neg p}^{C_2}, \overbrace{\neg p \vee \neg r}^{C_3}, \overbrace{p}^{C_4} \}$$

First indexing:

a) $C_1 = {}_{(2)}r \vee {}_{(1)}q$, $C_2 = {}_{(4)}r \vee {}_{(3)}\neg q \vee {}_{(5)}\neg p$

$C_3 = {}_{(6)}\neg p \vee {}_{(7)}\neg r$, $C_4 = {}_{(8)}p$

Lock resolution:

T.s.c. S - inconsistent iff $S \vdash^{lock}_{Res} \square$

T_1 lock res + level saturation strategy

a) if $\square \in S^k$ then S - inconsistent

b) if $S^k = \emptyset$ then S - consistent.

$C_5 = Res_q^{lock}(C_1, C_2) = {}_{(2)}r \vee {}_{(5)}\neg p$

$C_6 = Res_r^{lock}(C_5, C_3) = {}_{(5)}\neg p$

$C_7 = Res_p^{lock}(C_6, C_4) = \square \Rightarrow S_5$ is inconsistent

Second indexing: + level saturation strategy

b) $C_1 = \underset{(2)}{r} \vee \underset{(1)}{q}$, $C_2 = \underset{(4)}{r} \vee \underset{(3)}{\neg q} \vee \underset{(5)}{\neg p}$

$C_3 = \underset{(6)}{\neg p} \vee \underset{(7)}{\neg r}$ $\qquad C_4 = \underset{(8)}{p}$

$$S^0 = S$$

$$S^1 = \{ Res^{lock}(C_i, C_j) \mid C_i \in S^0, C_j \in S^0 \} =$$

$$= \{ C_5^1, C_6^1 \}$$

$$C_5^1 = Res^{lock}_q (C_1, C_2) = \underset{(2)}{r} \vee \underset{(5)}{\neg p}$$

$$C_6^1 = Res^{lock}_p (C_3, C_4) = \underset{(7)}{\neg r}$$

$$S^2 = \{ Res^{lock}(C_i, C_j) \mid C_i \in S^1, C_j \in S^0 \cup S^1 \} =$$

$$C_7^1 = Res^{lock}_r (C_5^1, C_6^1) = \underset{(5)}{\neg p} \qquad = \{ C_7^1 \}$$

$$S^3 = \{ Res^{lock}(C_i, C_j) \mid C_i \in S^2, C_j \in S^0 \cup S^1 \cup S^2 \} =$$

$$C_8^1 = Res^{lock}_p (C_7^1, C_4) = \square \qquad = \{ C_8^1 = \square \}$$

$\square \in S^3$, so S is inconsistent.

7. Check the consistency using lock resolution

$$S_5 = \{\neg p \vee \neg q \vee \neg r,\ q \vee \neg r,\ p \vee r\}$$

a) indexing 1: $C_1 = {}_{(2)}\neg p \vee {}_{(1)}\neg q \vee {}_{(3)}\neg r$

$C_2 = {}_{(1)}q \vee {}_{(5)}\neg r$

$C_3 = {}_{(6)}p \vee {}_{(7)}r$

$$S_5^0 = S_5$$

$$S_5^1 = \{ Res^{lock}(C_i, C_j) \mid C_i \in S_5^0,\ C_j \in S_5^0 \} = \{C_4\}$$

$$C_4 = Res^{lock}_{\underline{q}}(C_1, C_2) = {}_{(2)}\neg p \vee {}_{(3)}\neg r$$

$$S_5^2 = \{ Res^{lock}(C_i, C_j) \mid C_i \in S_5^1,\ C_j \in S_5^0 \cup S_5^1 \} = \{C_5\}$$

$$C_5 = Res^{lock}(C_4, C_3) = {}_{(7)}r \vee {}_{(3)}\neg r \equiv T$$

(tautology)

Ex. $C_{10} = {}_{(1)}p \vee {}_{(2)}q$

$C_{11} = {}_{(3)}\neg p \vee {}_{(4)}p$

$Res^{lock}_{p}(C_{10}, C_{11}) = {}_{(2)}q \vee {}_{(4)}p$

we keep tautology bcs. this helps us change the relative order

$S_5^3 = \{ Res^{lock}(C_i, C_j) \mid C_i \in S_5^2, C_j \in S_5^0 \cup S_5^1 \cup S_5^2 \} =$

$= \emptyset \Rightarrow S$ is consistent

b) indexing 2:

$C_1 = \underset{(1)}{\neg p} \vee \underset{(2)}{\neg q} \vee \underset{(3)}{\neg r} \qquad C_2 = \underset{(6)}{q} \vee \underset{(7)}{\neg r} \qquad C_3 = \underset{(5)}{p} \vee \underset{(4)}{r}$

$S^1 = \emptyset \Rightarrow S$ is consistent.

1. Transform the following formulas into prenex, Skolem and clausal normal forms.

$U_5 = (\forall x)(\exists y)(\exists z)\, Q(z, y) \wedge (\exists u)(P(x, u) \to (\exists z) R(u, z))$

$C = P(x) \vee \neg Q(y)$

(T1) $U \equiv U^P$

(T2) U is inconsistent
iff U^P — '' —
iff U^S — '' —
iff U^C — '' —

$A \to B \equiv \neg A \vee B$

replace →

$$\equiv \quad (\forall x)(\exists y)\big((\exists z)\, Q(z,y) \wedge (\exists u)(\neg P(x,u) \vee (\exists z) R(u,z))\big)$$

$z \to t$

rename the bound variables and extract the quantifiers

$$(\forall x)(\exists y)\big[(\exists z)\, Q(z,y) \wedge (\exists u)(\exists t)(\neg P(x,u) \vee R(t,z))\big]$$

Extract quantifiers:

$$(\forall x)\, A(x) \vee B(y) \equiv (\forall x)(A(x) \vee B(y))$$

$$U_5 \equiv U_5^{P_1} = (\forall x)(\exists y)(\exists z)(\exists u)(\exists t)\big[Q(z,y) \wedge (\neg P(x,u) \vee R(t,z))\big]$$

$$U_5 \equiv U_5^{P_2} = \underbrace{(\forall x)(\exists y)(\exists u)(\exists t)(\exists z)}_{\text{prefix}}\ \underbrace{\big[Q(z,y) \wedge (\neg P(x,u) \vee R(t,z))\big]}_{\text{matrix}}$$

⇒ prenex forms.

Skolem forms:

$U_5^{P_1}$ provides $U_5^{S_1} = (\forall x)\big[Q(g(x), f(x)) \wedge (\neg P(x, h(x)) \vee R(i(x), g(x)))\big]$

$y \leftarrow f(x),\ z \leftarrow g(x)$

$u \leftarrow h(x),\ t \leftarrow i(x)$

f, g, h, i Skolem functions

clausal form:

$U_5^{C_1} = \big[Q(g(x), f(x)) \wedge (\neg P(x, h(x)) \vee R(i(x), g(x)))\big]$

$U^P = (\exists x)(\forall y)(\exists z)(\forall t)(\exists u)(P(x,y,z) \wedge Q(t,u))$ - diff. example

$U^S = (\forall y)(\forall t)[P(a,y,f(y)) \wedge Q(t,g(y,t))]$

$[x \leftarrow a, z \leftarrow f(y), u \leftarrow g(y,t)]$

a - Skolem const.

f, g - Skolem functions.

$U^C = [P(a,y,f(y)) \wedge Q(t,g(y,t))]$