Ω

O

$\Theta \leftarrow O$

$\quad \leftarrow \Omega$

$$f(n) \in O(g(n)) \overset{def}{\Longleftrightarrow} \exists\, n_0 \in \mathbb{N}^*,\ c \in \mathbb{R}_+^*$$
$$\forall\, n \geq n_0,\ f(n) \leq c \cdot g(n)$$

$$\overset{T}{\Longleftrightarrow} \lim_{n\to\infty} \frac{f(n)}{g(n)} < \infty$$

1

$\log_2 n$

$n \leftarrow$ liniar

$n^2 \leftarrow$ quadratic

$\dots$ polynomial

$2^n,\ n!,\ n^n \leftarrow$ exponential

$$\lim_{n\to\infty} \frac{\overset{f}{a n^2 + b}}{\underset{g}{n^2}} = a$$

$f(n) \in O(g(n))$

$\in \Theta(g(n))$

$\in \Omega(g(n))$

$f(n) = an^2 + b$

$g(n) = n^3$

$f(n) \in O(g(n))$

$\notin \Theta(g(n))$

$\notin \Omega(g(n))$

$\log_2 n \in O(n)$

$\notin \Theta(n)$

$2^{2n} = 2^n * 2^n$

$$\lim_{n \to \infty} \frac{2^{2n}}{2^n} = \lim_{n \to \infty} 2^n = \infty$$

$\left.\begin{matrix} \in \\ = \end{matrix}\right\}$ same thing

$$\underbrace{O(f_1(n))}_{h_1(n)} + \underbrace{O(f_2(n))}_{h_2(n)} \overset{?}{=} O(g(n))$$

$h_1(n) + h_2(n) \in O(g(n))$

$$\Theta(n) + \Theta(n^2) \overset{?}{=} \Theta(n^2)$$

$\underbrace{\leq n \qquad \leq n^2}_{+}$

$\leq n^2 \in O(n^2)$

$\underbrace{\geq 0 \qquad \geq n^2}_{+}$

$2n^2 \in \Omega(n^2)$

$\in \Theta(n^2)$

$$(n+m)^2 = n^2+m^2+2nm \le m^2+n^2+2(m^2+n^2)$$
$$= 3(m^2+n^2)$$

$$1+2+\ldots+n = \frac{n(n+1)}{2}$$

$$1^2+2^2+\ldots+n^2 = \frac{n(n+1)(2n+1)}{6}$$

Stirling's approximation : $\log_2 n! = \Theta(n \log_2 n)$

3.

$\boxed{S1:}$ $T(n) = n \cdot \log_2 n$

BC = WC = AC = Total : $T(n) \in \Theta(n \log_2 n)$

$\boxed{S_2}$ $T(n) = \log_2 1 + \log_2 2 + \ldots \log_2 n = \sum_{i=1}^{n} \log_2 i$

$$= \log_2 n! \in \Theta(n \log_2 n)$$

BC = WC = AC = Total: $T(n) \in \Theta(n \log_2 n)$

$\boxed{S_3}$ $T(n) = n \in \Theta(n)$

$\boxed{S_4}$ AC: $\sum_{I \in D} P(I) \cdot E(I)$

Case	No. of iteration (no. of steps)	Description
1	1	$x_1 = a$
2	2	$x_2 = a$ & $x_1 \neq a$
3	3	$x_3 = a$ & $a \notin \{x_1, x_2\}$
...	...	...
n	n	$x_n = a$ & $a \notin \{x_1, x_2, \dots, x_{n-1}\}$
$n+1$	n	$a \in \{x_1, x_2, \dots, x_n\}$

Assume that all the cases have the same probability

$$P = \frac{1}{n+1}$$

AC: $$T(n) = P \cdot 1 + P \cdot 2 + P \cdot 3 + \dots + P \cdot n + P \cdot n =$$

$$= \frac{1}{n+1} \sum_{i=1}^{n} i + \frac{1}{n+1} \cdot n$$

$$= \frac{1}{\cancel{n+1}} \cdot \frac{n\cancel{(n+1)}}{2} + \frac{1}{n+1} \cdot n \in \Theta(n)$$

$\boxed{S5}$ $S = \sum_{i=1}^{n} x_i$ $x_i \in \mathbb{N}$

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k = 0
for i ← 1,n execute
    for j ← 1,n execute
        k ← k+1
    ■
■
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a) $x_i \in \mathbb{N}^* \Rightarrow T(n) = S$

b) $x_i = 0 \quad \forall\, i \in \{1, \dots, n\}$

$\Rightarrow T(n) = n \in \Theta(n)$

c) $x_i \in \mathbb{N}$

$T(n) \in \Theta(\max(s, n))$

$\Theta(s+n)$

$\boxed{56}$ AC: → inner while

Fixed $i \Rightarrow T_{im} = (n, i)$

Nr.	Steps	Values for j
1	1	i
2	2	$i, i+1$
...	...	...
$n-i+1$	$n-i+1$	$i, i+1, \dots, n$

(Assumption ....): $p = \frac{1}{n-i+1}$

$$T_{im}(n,i) = \frac{1}{n-i+1}\left(1+2+\dots+n-i+1\right) = \frac{n-i+2}{2}$$

AC: → outter while

Nr.:	Steps:	Values for i	
1	1	1	$T_{im}(n,1)$
2	2	1, 2	$T_{im}(n,1) + T_{im}(n,2)$
...	...	...	...
$n-1$	$n-1$	$1, 2, \dots, n-1$	$T_{im}(n,1) + T_{im}(n,2) + \dots + T(n, n-1)$

$n-2$

$n-1$

1

(Assumption ...): $p = \frac{1}{n-1}$

$$T_{outter}(n) = p \cdot \Big[(n-1)\, T_{in}(n,1) + n-2\, T_{in}(n,2) + \dots +$$

$$+ T_{in}(n, n-1) \Big] = \frac{1}{n-1} \sum_{i=1}^{n-1} (n-i) \cdot \frac{n-i+2}{2} \in \Theta(n^2)$$