

PUBLIC KEY CRYPTOGRAPHY - Computer Science

Lab 2 (Weeks 3-4)

All programs will be written in versions of C or Python with commented code.

Topics: ciphers, congruences, pseudoprimality.

- Implement one of the following algorithms, which will be assigned to you during the labs:

1. Belaso cipher.
2. Hill cipher (for $m = 2$).
3. Permutation cipher.
4. Algorithm for solving systems of congruences.
5. Algorithm for computing the value of Euler's function for natural numbers. For a given value v and a given bound b , list all natural numbers less than b which have v as the value of Euler's function.
6. The sieve of Eratosthenes algorithm for generating all prime numbers less than a given bound.
7. Algorithm for determining all Carmichael numbers less than a given bound.
8. Algorithm for determining all bases b with respect to which a composite odd number n is pseudoprime. Use the repeated squaring modular exponentiation method.
Let $n \in \mathbb{N}$ be odd composite and let $b \in \mathbb{Z}$ with $(b, n) = 1$. Then n is called *pseudoprime to b* if $b^{n-1} \equiv 1 \pmod{n}$.
9. Algorithm for determining all bases b with respect to which a composite odd number n is strong pseudoprime. Use the repeated squaring modular exponentiation method.
Let $n \in \mathbb{N}$ be odd composite and write $n - 1 = 2^s t$ for some odd t . Let $b \in \mathbb{Z}$ with $(b, n) = 1$. Then n is called *strong pseudoprime to the base b* if either $b^t \equiv 1 \pmod{n}$ or $\exists 0 \leq j < s : b^{2^j t} \equiv -1 \pmod{n}$.
10. Algorithm for determining all generators of the cyclic group $(\mathbb{Z}_n, +)$, where $n \geq 2$ is a natural number.
A *generator* of $(\mathbb{Z}_n, +)$ is an element $\hat{g} \in \mathbb{Z}_n$ such that for every $\hat{x} \in \mathbb{Z}_n$ there is $k \in \{0, 1, \dots, n-1\}$ such that $\hat{x} = k\hat{g}$.

Points

- **0.5 points** if handed in by Week 5 (odd week groups) or Week 6 (even week groups).
- **0.25 points** if handed in by Week 7 (odd week groups) or Week 8 (even week groups).

Note: *Each student will keep her/his semigroup for the lab throughout the semester! Taking and presenting labs in weeks with a changed parity may only be done in exceptional cases, if the teaching assistant agrees with it and if time allows.*