Hw. 4 Chris Badolato

1 What is the sum of an arithmetic Sequence with 100 terms with the first term equal to 17 and a common difference of 4.

$$\sum_{i=0}^{99} (17 + 4i) \qquad S = \frac{n(n+1)}{2}$$

$$\sum_{i=0}^{99} 17 + 4\sum_{i=1} i \qquad i = \frac{n(n+1)}{2}$$

$$i = \frac{99(100)}{2}$$

$$1700 + 4(4{,}950)$$

$$\sum_{i=0}^{99} (17+4i) = 21{,}500$$

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2 Let $a_1, a_2 \dots a_{65}$ be an arithmetic sequence, such that $a_{22} = 46$ $a_{35} = 267$ find $\sum_{i=1}^{65} a_i$

$a_{22} = a + (n-1)d$

$46 = a + (22-1)d$

$46 = a + 21 \cdot d$

$a = 46 - 21d$

$a_{35} = a + (n-1)d$

$267 = a + (35-1)d$

$a = 267 - 34d$

d is difference between the terms

$35 - 22 = 13 = d$

$267 - 46 = \frac{221}{13} = 17$ $a_1 = 17$

$n = 65$

$d = 13$

$S_n = \frac{n}{2}(2(a_1) + (n-1)d)$

$S_{65} = \frac{n}{2}(2(17) + (65-1)13)$

$S_{65} = \frac{65(34 + 64(13))}{2}$

$S_{65} = \frac{65(866)}{2}$

$S_{65} = 28{,}145$

3 What is the sum of an infinite geometric sequence with a first term of 7 and a common ratio of $\frac{2}{5}$

$$S = \frac{a_1}{1-r} = \frac{7}{1-\frac{2}{5}} = \frac{7}{\frac{3}{5}}$$

$$S = \frac{7(5)}{3} = \frac{35}{3}$$

4 geometric to a_{15} $a_5 = 48$ $a_9 = 768$ find $\sum_{i=1}^{15} a_i$

$r = \frac{a_n}{a_{n-1}}$ $a_n = r a_{n-1}$

$a_5 = 48$ $48 = a_1 r^{5-1}$ $768 = a_1 r^{9-1}$

$a_9 = 768$ $48 = a_1 r^4$ $a_1 = \frac{48}{r^4}$

$768 = a_1 r^8$

so $768 = \frac{48}{r^4} r^8$ $768 = 48 r^4$

$\frac{768}{48}$ $16 = r^4$

$r = 2$

$48 = a_1 (2)^4$ $a_1 = 3$

$$S_n = 3\left(\frac{1-2^{15}}{1-2}\right) = \left(\frac{1-32{,}768}{-1}\right)3$$

$$= (32767)3$$

$$\sum_{i=1}^{15} a_i = 98{,}301$$

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6 By noticing that $\sum_{i=1}^{n} (i+1)^2 - i^2 = \sum_{i=1}^{n} (i+1)^2 - \sum_{i=1}^{n} i^2$
noticing the telescope nature of the sum on the right determine $\sum_{i=1}^{n} (2i+1)$

$$\sum_{i=1}^{n} 2i+1$$
$$= \sum_{i=1}^{n} 2i + \sum_{i=1}^{n} 1$$
$$= 2\left(\frac{n(n+1)}{2}\right) + n$$
$$= n(n+1) + n$$

5 Determine the following sum in terms of n. $\sum_{i=1}^{n} (i(i+1)(i+2))$

$$\left(\frac{n(n+1)}{2}\right)\left(\frac{n(n+1)}{2} + n\right)\left(\frac{n(n+1)}{2} + 2n\right)$$
$$= \frac{n(n+1)}{2}\left(\frac{n(n+1)}{2} + \frac{2n}{2}\right)\left(\frac{n(n+1)}{2} + \frac{4n}{2}\right)$$
$$= \frac{n(n+1)}{2}\left(\frac{n(n+1)+2n}{2}\right)\left(\frac{n(n+1)+4n}{2}\right)$$
$$= \frac{n(n+1((n+1)+2)(n+1+4)}{2}$$
$$= \frac{n(n+1)(n+3)(n+5)}{2}$$

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7 what is $\sum_{i=1}^{\infty} \left(\frac{3}{5}\right)^{i-1}$?

$$\sum_{K=1}^{\infty} Kx^{K-1}, \quad |x|<1$$

$$\sum_{K=1}^{\infty} Kx^{K-1} = \frac{1}{(1-x)^2}$$

$$= \frac{1}{\left(1-\frac{3}{5}\right)^2} \quad \frac{1}{\left(\frac{2}{5}\right)^2} = \frac{25}{4}$$

$$= \frac{25}{4}$$

8 $\begin{bmatrix} 1 & 3 & 4 \\ 6 & 2 & -3 \end{bmatrix} \begin{bmatrix} 2 & 3 \\ -1 & 1 \\ 4 & -3 \end{bmatrix} \begin{bmatrix} (1)(2)+(3)(-1)+(4 \cdot 4) & 1(3)+(3)(1)+(4)(3) \\ (6)(2)+(2(-1))+(-3)(4) & 6(3)+(2)(1)+(3)(3) \end{bmatrix}$

$$= \begin{bmatrix} 15 & -6 \\ -2 & 29 \end{bmatrix}$$

8 $\begin{bmatrix} 1 & 3 & 4 \\ 6 & 2 & -3 \end{bmatrix} \begin{bmatrix} 2 & 3 \\ -1 & 1 \\ 4 & 3 \end{bmatrix} \begin{bmatrix} (1)(2)+(1)(-1)+(4\cdot 4) & (1)(3)+(3)(1)+(4)(3) \\ (6)(2)+2(-1)+-3(4) & 6(3)+(2)(1)+(-3)(3) \end{bmatrix}$

$\begin{bmatrix} 17 & 18 \\ -2 & 11 \end{bmatrix}$

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9 Let $M_1 = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \end{bmatrix}$, $M_2 \begin{bmatrix} 0 & 1 & 1 \\ 1 & 1 & 0 \end{bmatrix}$, and $m_3 \begin{bmatrix} 0 & 1 \\ 1 & 1 \\ 1 & 0 \end{bmatrix}$

Calculate the a) join of m_1 and m_2
B) meet of M_1 and M_2 and
C) Boolean product of M_1 and M_3

$\underset{M_1}{\begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \end{bmatrix}} \vee \begin{bmatrix} 0 & 1 & 1 \\ 1 & 1 & 0 \end{bmatrix} = \begin{bmatrix} 1 \vee 0 & 0 \vee 1 & 1 \vee 1 \\ 0 \vee 1 & 1 \vee 1 & 1 \vee 0 \end{bmatrix}$

join M_2

$= \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$

$\begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \end{bmatrix} \wedge \begin{bmatrix} 0 & 1 & 1 \\ 1 & 1 & 0 \end{bmatrix} = \begin{bmatrix} 1 \wedge 0 & 0 \wedge 1 & 1 \wedge 1 \\ 0 \wedge 1 & 1 \wedge 1 & 1 \wedge 0 \end{bmatrix}$

M_1 meet M_2

$= \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}$

Boolean product of M_1 and m_3

$\begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \end{bmatrix} \times \begin{bmatrix} 0 & 1 \\ 1 & 1 \\ 1 & 0 \end{bmatrix} = \begin{bmatrix} (1 \wedge 0) \vee (0 \wedge 1) \vee (1 \wedge 1) & (1 \wedge 1) \vee (0 \wedge 1) \vee (1 \wedge 0) \\ (0 \wedge 0) \vee (1 \wedge 1) \vee (1 \wedge 1) & (0 \wedge 1) \vee (1 \wedge 1) \vee (1 \wedge 0) \end{bmatrix}$

$= \begin{bmatrix} 0 \vee 0 \vee 1 & 1 \vee 0 \vee 1 \\ 0 \vee 1 \vee 1 & 0 \vee 1 \vee 0 \end{bmatrix}$

$\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$

10 express the system of equations as a multiplication

$$3x + 4y + 5z = 16$$
$$2x - 5y + 11z = 3$$
$$x + 6y + 2z = 15$$

$$\begin{bmatrix} 3 & 4 & 5 \\ 2 & -5 & 11 \\ 1 & 6 & 2 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 16 \\ 3 \\ 15 \end{bmatrix}$$

11 using induction prove that

$$\sum_{i=1}^{n} (i(i!)) = (n+1)! - 1$$

B.C. $n = 1$ LHS $\sum_{i=1}^{n} (1)(1)! = 1$ ✓

RHS $= (1+1)! - 1 = 2 - 1 = 1$ ✓

I.H. $\sum_{i=1}^{k} i = (k(k!))$

I.S. $n = k+1$ $\sum_{i}^{n+1} i = ((k+2)! - 1$

$$\sum_{i}^{k+1} i(i!) = (k+1)(k+1)! + (k+1)! - 1 = (k+2)(k+1)! - 1$$
$$(k+1)!((k+1)+1) - 1$$
$$(k+1)!(k+2) - 1 = k+2\ k+1! - 1$$

12 $\sum_{i=1}^{n} \frac{1}{i}$ prove $\sum_{i=1}^{n} H_i = (n+1)H_n - n$

b.c. $n = 1$ LHS $= H = 1$ ✓

RHS $= (1+1)H_1 - 1$ $2(1) - 1 = 1$ ✓

i.h. $\sum_{i=1}^{k} H_i = (k+1)H_k - k$

i.s. prove for $n = k+1$ $\sum_{i=1}^{k+1} H_i = (k+2)H_{k+1} - k+1$

$$\sum_{i=1}^{k+1} H_i = \sum_{i=1}^{k} H_i + H_{k+1}$$

$H_{k+1} = H_k + \frac{1}{k+1}$

$$= (k+1)H_k - k + H_{k+1}$$

$$= (k+1)\left(H_{k+1} - \frac{1}{k+1}\right) - k + H_{k+1}$$

$$= (k+1)H_{k+1} - 1 - k + H_{k+1}$$

$$= (k+2)H_{k+1} - k+1$$ ✓

13 using Induction prove $21 \mid 4^{n+1} + 5^{2n-1}$

$n=1$ $\quad 21 \mid 1$ ✓

$21 \mid 4^2 + 5^1 \qquad 21 \mid 21$ ✓

i.H. $n=k \qquad 21 \mid 4^{k+1} + 5^{2k-1} = 21c$

I.S. $n=k+1 \qquad 21 \mid 4^{k+2} + 5^{2k+1}$

$$4^{k+2} + 5^{2k+1} = 4 \cdot 4^{k+1} + 5^2 \cdot 5^{2k-1}$$
$$= 4 \cdot 4^{k+1} + ((4+21) \cdot 5^{2k-1})$$
$$= 4 \cdot (4^{k+1} + 5^{2k-1}) + 21 \cdot 5^{2k-1}$$
$$= 4(21c) + 21(5^{2k-1}),$$
$$= 21(4c + 5^{2k+1})$$

c is an integer and k is a positive int, it follows that $4c + 5^{2k+1}$ is divisable. $21 \mid 4^{k+2} + 5^{2k+1}$

14 prove $\begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}^n = \begin{bmatrix} 1 & n \\ 0 & 1 \end{bmatrix}$ for all positive ints n.

$n=1$ LHS $= \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}^1 = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$ ✓

RHS $= \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$ ✓

Stmt is true for $n=1$

i.h. for arbitrary non-negative ints $n=k$

$$\begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}^k = \begin{bmatrix} 1 & k \\ 0 & 1 \end{bmatrix}$$

inductive step prove $n=k+1$

$$\begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}^{k+1} = \begin{bmatrix} 1 & k+1 \\ 0 & 1 \end{bmatrix}, \quad \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}^{k+1} = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}^{k}\begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & k \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} \text{ using i.H.}$$

$$= \begin{bmatrix} (1)(1) + k\cdot 0 & 1\cdot 1 + k\cdot 1 \\ 0\cdot 1 + 1\cdot 0 & 0\cdot 1 + 1\cdot 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & k+1 \\ 0 & 1 \end{bmatrix} ✓$$

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15 using Mathematical Induction, Prove that $\sum_{i=1} i^2 < n^3$, for all positive ints $n \geq 2$.

B.C.

$n \geq 2$ $n=3$ LHS $i^2 = 9$ ✓

RHS $n^3 = 27$ ✓

$i^2 < n^3$

I.H. $n = k$

$\sum_{i=1} i^2 < k^3$

$n = k+1$

I.S. $\sum_{i=1} i^2 < (k+1)^3$

$\sum_{i=1}^{k} i^2 < k^3 + 3k^2 + 3k + 1$

$\sum_{i=1}^{k} i^2 + (k+1)$

$= k^3 + (k+1)^2 < k^3 + 3k^2 + 3k + 1$