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1 In alpha testing a new software package, a software engineer finds that the number of defects per 100 lines of code is a random variable X with a probability distribution

x	1	2	3	4
$pr(x = X)$	0.5	0.3	0.15	.05

Find a) $E(x)$ and $var(x)$

$E(x) = \sum_{x \in x} x \cdot P(x)$

$var(x) = \sum_{x \in x} (x - E(x))^2 \cdot P(x)$

$E(x) = 1(.5) + 2(.3) + 3(.15) + 4(.05)$

$\frac{1}{2} \quad \frac{3}{5} + \frac{9}{20} + .2 \quad \frac{1}{5}$

$\frac{10}{20} + \frac{12}{20} + \frac{9}{20} + \frac{4}{20} \quad \frac{35}{20} = \frac{7}{4} = E(x)$

$\left(1 - \frac{7}{4}\right)^2 \cdot (.5) + \left(2 - \frac{7}{4}\right)^2 \cdot (.3) + \left(3 - \frac{7}{4}\right)^2 (.15)$
$+ \left(4 - \frac{7}{4}\right)^2 (.05)$

$0.28125 + 0.01875 + 0.234375 + 0.253125$

$v(x) = 0.7875$ or $\frac{63}{80}$

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2 A lottery allows a player to choose 6 out of 60 values the goal of the lottery is to breah even making its cash prizes equal the amount of money spent. Each ticket costs 1$ to buy and players recieve winnings if they match 3, 4, 5, 6. If the Payout for matching 3 numbers is $10, 4 is 100 5 is 10K what does the payout for matching all 6 numbers need to be.

$p(n) \to$ prob of getting n numbers correct

n	3	4	5	6
$p(n)$	$\frac{\binom{6}{3} \cdot \binom{54}{3}}{\binom{60}{6}}$	$\frac{\binom{6}{4} \cdot \binom{54}{2}}{\binom{60}{6}}$	$\frac{\binom{6}{5} \cdot \binom{54}{1}}{\binom{60}{6}}$	$\frac{\binom{6}{6}\binom{54}{0}}{\binom{60}{6}}$

$E(x) = 1 \quad E(x) = ((10) \cdot p(3)) + (100 \cdot p(4)) + 10{,}000\, p(5) + y \cdot p(6)$

$$\frac{60!}{(60-6)!\,6!} = \frac{60!}{54!\,6!}$$

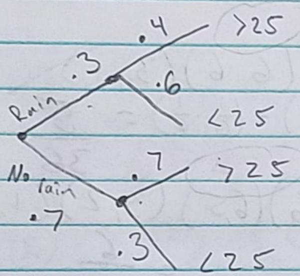
$1 = .099 + .043 + .065 + y(1.99 \times 10^{-8})$

$y = 39{,}700{,}640.98$

payout for 6 correct numbers is

39,700,640.98 $

3 The probability that it rains during a summer day in a certain town is 0.3. In this town, the probability that the daily max temp exceeds 25 degrees C is 0.4 when it rains and .7 when it does not rain. given the max daily temp exceeds 25°C on a particular summer day, find probability that it rained on that day.



$$= \frac{(.3 \times .4)}{(.3)(.4) + (.7)(.7)}$$

$$= \frac{.12}{.61}$$

$$P = 0.196721311.$$

4 Derek has a 32% chance of making exactly 1 out of 2 free throws. what is his chance of making a single freethrow?

$p(1-p) + p(1-p)$, makes or misses shot onfirst or second try.

$$p(1-p) + p(1-p) = .32$$

$$2p(1-p) = .32 = p(1-p) = .16$$

$$= p^2 - p + .16 = 0$$

So $\boxed{p = .8 \text{ or } p = .2}$

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5 In Arups game of dice, you roll a fair pair of six-sided dice and record the total
If the total is 2, 4, or 12 you win. If its 5, or 11 you lose, allother Cases (3, 6, 7, 8, 9, 10) you roll again. If the sum of the second roll exceeds the sum of the first roll you win, otherwise you lose. What is the probability of winning Arups Game of dice?

winning (1,1) (1, 3) (2,2) (6,6) (3,1)

$$\left(\frac{1}{6}\cdot\frac{1}{6}\right)+\left(\frac{1}{6}\cdot\frac{1}{6}\right)+\left(\frac{1}{6}\cdot\frac{1}{6}\right)+\left(\frac{1}{6}\cdot\frac{1}{6}\right)+\left(\frac{1}{6}\cdot\frac{1}{6}\right)=\frac{5}{36}$$

lose if (2,3) (4,1) (5,6) (6,5) (3,2) (1,4)

$$=\frac{6}{36}$$

$$p(3 \text{ then } 2): \quad p(3)\cdot(1-(P(2)+P(3)))=\frac{2}{36}\cdot\frac{33}{36}$$

$$p(5)=\frac{4}{36},\quad P(6)=\frac{5}{36},\quad p(7)=\frac{6}{36}$$

$$p(8)=\frac{5}{36},\quad p(9)=\frac{4}{36},\quad P(10)=\frac{3}{36}$$

$$\left(\frac{2}{36}\cdot\frac{33}{36}\right)+\sum_{i=6}^{10}\left(P(i)\cdot\left(1-\sum_{j=5}^{i}P(j)\right)\right)$$

$$=\left(\frac{2}{36}\cdot\frac{33}{36}\right)+\left(\frac{5}{36}\cdot\frac{27}{36}\right)+\left(\frac{6}{36}\cdot\frac{21}{36}\right)+\left(\frac{5}{36}\cdot\frac{16}{36}\right)$$

$$+\left(\frac{4}{36}\cdot\frac{12}{36}\right)+\left(\frac{3}{36}\cdot\frac{9}{36}\right)=\frac{482}{1296}$$

$$P(\text{winning})=\frac{5}{36}+\frac{482}{1296}=\frac{662}{1296}=\boxed{.511}$$

6 Kellog makes 27 cereal types you are Sent a random box six months in a row what is the probability you recieve a repeated box?

No repeats $\frac{27}{27} \cdot \frac{26}{27} \cdot \frac{25}{27} \cdot \frac{24}{27} \cdot \frac{23}{27} \cdot \frac{22}{27} = .5501$

prob of at least one repeat (1 − p(no repeat))

$1 - .5501 = \boxed{.4499}$

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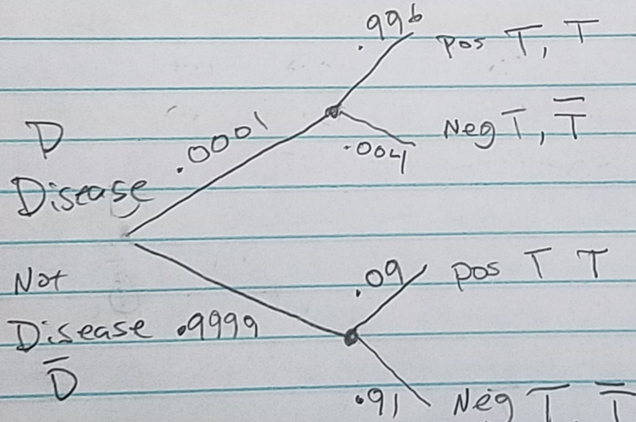
7 Suppose that one person in 10,000 people has a rare genetic disease. There is an excellent test for the disease; 99.6% of the people with the disease test positive and only 0.09% of the people who don't have it test positive, what is the probability that someone who tests positive has the disease? What is the probability that someone who test negative does not have the disease?



$$P(D \mid T) = \frac{P(D \cap T)}{P(T)} = \frac{.0000996}{.0900906} = \boxed{.0011}$$

$$P(\bar{D} \mid \bar{T}) = \frac{P(\bar{D} \cap \bar{T})}{P(\bar{T})} = \frac{.909909}{.9099094}$$

$$\boxed{= .9999996}$$

8 Suppose we flip a coin until we get the same results two times in a row. What is the total number of times we are expected to flip the coin?

$t =$ expected throws to heads

$t = \frac{1}{2} + \frac{1}{2}(1+t) = \frac{1}{2}t = 1 \quad t = 2$

$R =$ throw 2 consecutive heads

$E = \frac{1}{2}(t+1) + \frac{1}{2}(t+1+E)$

$\frac{1}{2}E = t+1$

$E = 2t+2$

$E = 6$

for two heads we just need to divide by 2 so our expected throws is just $\boxed{3.}$

9 Suppose E and F are events in a sample space and $p(E) = \frac{2}{3}$, $P(F) = \frac{3}{4}$ and $P(F|E) = \frac{5}{8}$

Find $p(E|F)$

$p(F|E) = \frac{p(F \cap E)}{p(E)}$ $\quad$ $P(E|F) = \frac{P(F \cap E)}{P(F)}$

$\frac{5}{8} = \frac{P(F \cap E)}{2/3}$ $\quad$ $P(E|F) = \frac{5/12}{3/4}$

$P(F \cap E) = \frac{5}{12}$ $\quad$ $p(E|F) = \frac{5}{9}$

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10 what is the probability that each player has a hand containing an ace when each of four players recieves 13 cards from the standard deck deck of 52?
(No two players will have the same card.)

There are $\binom{52}{4}$ ways to order the aces, and there are 13^4 ways to arrange the cards in 4 sets for each containing at least one ace would be

$$\boxed{\frac{13^4}{\binom{52}{4}} = 0.1055}$$