H.W. 7 Chris Badolato

1 Let R_1 and R_2 be relations on a set $A = \{1,2,3,4\}$

Let $R1 = \{(1,1)(1,2)(2,1)(3,4)(4,4)$

$R2 = \{(1,2)(2,3)(3,4)\}$ ← Not

a) Determine whether or Not R_1 is Reflexive, irreflexive Symmetric, anti-symmetric and transative or Not.

Reflexive?

Not reflexive, missing (2,2) and (3,3)

irreflexive?

Is irreflexive Because we are missing (2,2) (3,3).

Symmetric?

Is not symmetric we need (4,3) for it to be

Anti-symetric?

Not Antisymmetric because we have (1,2) and (2,1). but its okay to have (4,4) and (1,1).

Transative? $(a,b) \in R$ $(b,c) \in R \Rightarrow (a,c) \in R$

Not transative because we don't have (4,3)

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16. $R_2 = \{(1,2)(2,3)(3,4)\}$

Reflexive?

Not reflexive, does not contain $(1,1)(2,2)(3,3)$ or $(4,4)$

Irreflexive?

Is irreflexive because $(a,a)(b,b)(c,c)(d,d) \notin R_2$

Symmetric?

No, R must contain $(2,1)(3,2)$ and $(4,3)$

Anti-symmetric?

Not antisymmetric

Transative?

Not transative, we need $(2,1)(3,2)(4,3)$

C $R_1 \circ R_2$?

$R_1 \circ R_2 = \{(1,1),(2,4)(3,4)\}$

D $R_2 \circ R_1 = \{(1,2)(1,3)(2,2)\}$

e $R_1 \cup R_2 = \{(1,1)(1,2)(2,1)(2,3)(3,4)(4,4)\}$

F $R_1 \cap R_2 = \{(1,2)(3.4)\}$

g for reflexive we add $(2,2)(3,3)$. for symmetric we add $(3,2)(4,3)$.

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2 Let R be a relation on Z^+ defined as

$R = \{(a,b) \mid \exists c \in Z^+$ s.T. $c^2 = a^2 + b^2$

determine (with proof) whether or not R is reflexive, irreflexive symmetric or Anti symmetric or transative or not.

Reflexive? No $(a,a) \in R$ $(1,1)$ $1^2 + 1^2 = 1$

$a^2 + a^2 = c^2$ $\quad 2a^2 = c^2$

$c = \sqrt{2a^2}$ is not an integer, therefore it is Irreflexive.

Symmetric? if $(a,b) \in R$ then $(b,a) \in R$

yes because

$a^2 + b^2 = c^2 \equiv c^2 = a^2 + b^2$

Swap a and b and our answer is the same.

finally it is not transative because

if $a = 4$, $b = 3$, $c = 5$, $d = 12$ $e = 13$

$4^2 + 3^2 = 5^2$ ✓ $(5^2 \mid 12^2)$

13^2 is true but $(4^2 \mid 13^2) \neq 13^2$

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3 Let $b(n)$ equal the number of bits in the binary representation of pos ints n. prove that R is an equivalent relation
List all the members of classes $[2]$, $[12]$, $[27]$.
How many elements are in $[2^n]$ for any Z^+ K in terms of K

$R = \{(x, y) \mid b(x) = b(y)$

If $b(x) = b(y)$ then $b(y) = b(x)$ meaning it is symmetric by definition

$b(x) = b(x)$ therefore it is also reflexive.

If $b(x) = b(y)$ and $b(y) = b(z)$ then $b(x) = b(z)$ is transative

therefore this is a equivalance relation

4 $R = \{(a, b) \mid |a-b| \leq 5$

- Is reflexive because $|a-a| = 0$ which is always ≤ 5
- Symmetric because if $a = 9$ and $b = 9$ $|9-9| \leq 5$ ✓
- It's NOT transative though because

if $a = 30$ $b = 25$, $c = 20$

$|30 - 25| \leq 5$ ✓

$|25 - 20| \leq 5$ ✓

but $|30 - 20| \leq 5$ X

therefore its not transative.

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5. $R = \{((a,b),(c,d)) \mid a+b \geq c+d\}$.
Is R a partial-order relation? Why or why not?

$S = \{((a,b);(c,d))\}$
Let R be the relation defined on $Z^+ \times Z^+$
$R = \{(a,b),(c,d) \mid a+b \geq c+d\}$
R is not anti symmetric
$(1,4)\, R\, (2,3)$ $\quad 1+4 \geq 2+3$
and $2+3 \geq 1+4$ but $(1,4) \neq (2,3)$
R is not anti-symmetric or a partially order set.

S is reflexive since $(a,b)\, s\, (a,b)$
since $a+b = a+b$ ✓

S is symmetric $(a,b)\, S\, (c,d)$
$\Rightarrow a+b = c+d$
$\Rightarrow c+d = a+b$
$(c,d)\, S\, (a,b)$
So S is symmetric

Transative?
if $(a,b)\, S (c,d)$ and $(c,d) S (a,b)$
$\Rightarrow a+b = c+d$ and $c+d = e+f$
$\Rightarrow a+b = e+f$
$(a,b)\, S\, (e,f)$
So S is transitive

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6 How many Anti-symmetric relations on the set $A = \{1, 2, 3, 4, 5, 6\}$ contain the ordered pairs $(2,2)$ $(3,4)$ $(5,6)$

$(1,1)(1,2)(1,3)(1,4)(1,5)(1,6)$

$(2,2)(2,3)(2,4)(2,5)(2,6)$

$(3,3)(3,4)(3,5)(3,6)$ 21 total.

$(4,4)(4,5)(4,6)$

$(5,5)(5,6)$

$(6,6)$

7 Let $f(x) = x^2 + 4x - 32$ domain $x \in [-\infty, -2]$ prove f is injective; what is the Range of f?

if $f(a) = f(b)$ then $a = b$

two different x's are not mapped to the same f value therefore it is injective.

$(x^2 + 4x + 4) - 32 - 4$

$(x+2)^2 - 36$

$(x+2)^2 = 36$

$x + 2 = 2 \pm 6$

$x = 8 \quad x = -4$

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8 Let A be a set of 10 elements and B be a set of 15 elements How many functions can be defined with the Domain of A and Co-domain of B?

$|A| \leq |B|$ would show that these sets are injective

function definition says that

$\forall a \in A$ there exists only 1 value such that $b \in B$.

$f(a) = b$.

A = Domain

B = Co-domain

we need to match every element in set A to set B. there are 15 possibilities in B there are 15^{10} possibilities for the 10 elements in set A.

9) $f(x) = 3x^3 + 2x - 7$ $g(x) = 4x - 5$ find $f(g(x))$ and $g(f(x))$

$$f(g(x)) = 3(4x-5)^3 + 2(4x-5) - 7$$
$$= 3[64x^3 - 125 - 60x(4x-5)] + (8x-10) - 7$$
$$= 192x^3 - 375 - 720x^2 + 900x + 8x - 10 - 7$$
$$= 192x^3 - 720x^2 + 908x - 392$$

$$g(f(x)) = 4(3x^3 + 2x - 7) - 5$$
$$= 12x^3 + 8x - 28 - 5$$
$$= 12x^3 + 8x - 33$$

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10 find f^{-1} of $x^2 + 4x - 32$

$$x = y^2 + 4y - 32$$

$$x = (y^2 + 4y + 4) - 32 - 4$$

$$x + 36 = (y+2)^2$$

$$-2 \pm \sqrt{x+36} = y$$

$$y = -2 - \sqrt{x+36}$$ Negative because domain: $\{-\infty, -2\}$

11 Let $f(x) = 3x + 5$, $f^n(x)$ to be the function f composed of itself n times $f^3 = f(f(f(x)))$

prove $f^n(x) = 3^n x + \frac{5}{2}(3^n + 1)$

for all positive int n.

$$f^1(x) = 3^1(3x+5) + \frac{5}{2}(3^1+1)$$

$$= 3(3x+5) + \frac{5}{2}(4)$$

$$9x + 15 + 10$$

$$= 9x + 25$$

$$f^2(x) = 3^2(3x+5) + \frac{5}{2}(3^2+1)$$

$$= 9(3x+5) + \frac{5}{2}(9+1)$$

$$= 27x + 45 + 25$$

$$= 27x + 70$$