Hw 5 Chris Badolato

1 Consider an ant that is walking on a Cartesian grid starting at (0,0) ending at (20,12). The ant always chooses to walk exactly one unit either up or to the right (towards his destination) whenever he arrives at a lattice point. (A Lattice point with int coordinates) thus, from (0,0) he either walks to (1,0) or (0,1) If he is not allowed to go to points (10,5) and (12,8), how many different paths can he take on his walk?

$\frac{32!}{20!\,12!}$ total ways to get there

we can represent this as a perm of a string with 20 R's and 12 U's

ways through (10,5)

$\frac{15!}{10!\,5!} \times \frac{17!}{10!\,7!}$ using a permutation then multiplying the number or ways to get from (10,5) to (20,12)

ways through (12,8)

$\frac{20!}{12!\,8!} \times \frac{12}{8!\,4!}$ ways from (12,8) to (20,12)

without going through (10,5) or (12,8)

$$\frac{32!}{20!\,12!} - \left(\frac{20!}{12!\,8!} \times \frac{12!}{8!\,4!}\right) - \left(\frac{15!}{10!\,5!}\right)\left(\frac{17!}{10!\,7!}\right)$$

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2 permutations of "HILLARY CLINTON"

a) how many total permutaition are there?

$$\frac{14!}{2!\,3!\,2!}$$ 14 letter 2, 3, 2 repeats

B) How many perm, Start and end with vowels?

$$5 \cdot \frac{14!}{2!\,3!} + 2 \cdot \frac{14!}{2!\,3!\,2!}$$

C) How many perm do not have Consecutive vowels?

_ H _ L _ L _ R _ Y _ C _ L _ N _ T _ N _

11 spaces 4 vowels

$\binom{11}{4}$ vowels $= \frac{4!}{2!}$ NoN-vowels $= \frac{10!}{2!\,3!}$

total: $\binom{11}{4} \cdot \frac{4!}{2!} \cdot \frac{10!}{2!\,3!}$

D) 1, the letters can only be put in Alpha order one way.

e) RANT Can be thought of as one character so there are 11 total Letters.

$$\frac{11!}{2!\,3!}$$

3 A class contains 25 girls and 22 boys. for all parts of this question each boy and girl are distinguishable from one another.

A) How many ways can a committee of one boy and one girl be choosen

$(25)(22)$ each boy you can choose 25 girls.

B) How many ways can a committee of five

$\binom{47}{5} = \frac{47!}{5!\,42!}$ Because you would be choosing 5 from the total 47.

C) How many ways can a committee of four girls and three boys be choosen.

$\binom{25}{4} \cdot \binom{22}{3} = \frac{25!}{4!\,21!} \cdot \frac{22!}{3!\,19!}$

Choose 4 girls from 25 and 3 boys from 22.

D) How many ways can a committee of six students be choosen such that all the students are the same sex.

$\binom{25}{6} + \binom{22}{6} = \frac{25!}{6!\,19!} + \frac{22!}{6!\,16!}$

e) How many ways can the boys and girls form a line where no two boys are next to eachother.

_G_G_G_G_G_G_G_G_G_G_G_G_G_G_G_G_G_G
_G_G_G_G_G_G_G_

$\binom{26}{22} \cdot \binom{25}{25} = \frac{26!}{22!\,4!} \cdot \frac{25!}{25!} = \frac{26!}{22!\cdot 4!}$

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3 continued.

f) How many committees of seven students contain at least two girls?

$$\binom{25}{2} \cdot \binom{25-2+22}{5} = \frac{25!}{2!\,23!} \cdot \frac{45!}{5!\,40!}$$

4) D is greater than or equal to 5.

$$a + b + c + d' + e + f = 20$$

$$\binom{25}{5}$$

A is greater than or equal to 5

$$a' + b + c + d' + e + f = 15$$

$$\binom{20}{5}$$

B is greater than or equal to 7

$$a + b' + c + d' + e + f = 13$$

$$\binom{18}{5}$$

Combo of all requirements

$$a' + b' + c + d' + e + f = 8$$

$$\binom{13}{5} \qquad \binom{25}{5} - \binom{20}{5} - \binom{18}{5} + \binom{13}{5}$$

By subtracting the second and third terms we are left with the cases when $a \le 4$ and $b \le 6$.

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5 $a+b+c+d+e+f+g \leq 30$

add slack for the inequality.

$a+b+c+d+e+f+g+\text{slack}=30$

$$\binom{37}{7}$$

6 The total number of ways a request can be sent to a server m^n

m is the number of servers and n is customers.

${}_mP_n$ is the number of ways that a request is sent to a server if he is not able to recieve a request.

therefore the number of ways in which there is atleast one collision is the total − the ways with no collision

$$m^n - {}_mP_n$$

7 Inclusion exelusion

$$|A \cup B \cup C| = |A| + |B| + |C| - |A \cap B| - |A \cap C| - |B \cap C| + |A \cap B \cap C|$$

A is the numbers divisible by 2. B is divisible by 5. C by 11.

So/ we have to remove any double counting which is why we subtract the 4th 5th and 6th terms

$$\left\lfloor \frac{10^7}{2} \right\rfloor + \left\lfloor \frac{10^7}{5} \right\rfloor + \left\lfloor \frac{10^7}{11} \right\rfloor - \left\lfloor \frac{10^7}{10} \right\rfloor - \left\lfloor \frac{10^7}{22} \right\rfloor - \left\lfloor \frac{10^7}{55} \right\rfloor + \left\lfloor \frac{10^7}{110} \right\rfloor$$

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8 3 A's, 5 B's, 7 C's

_A_A_A_C_C_C_C_C_C_C_

11 gaps to place B.

$$\binom{11}{5}$$

The remaining spaces need to be filled with the A's and C's

$$\binom{10}{3}$$

So we get

$$\binom{11}{5} \cdot \binom{10}{3}$$

9 This approach is undercounting Because it is only paying attention to the first letter but disregaurds the second one.

10 How many positive integer solutions does the equation $a+b+c=100$ have if $a<b<c$?

there are $\binom{99}{2}$ number of triples that add to 100.

There are 49 triples such that $a=b$.

So/

$0<x<50$ $a=b=x$ and $c=100-2x$

also 49 such that $a=c$ or $b=c$.

and none that $a=b=c$ so

$3 \cdot 49 = 147$.

theres 6 ways to permute a,b,c therefore we divide by 6 to throw out triples in the wrong order.

$$\frac{\binom{99}{2} - 147}{6}$$