0.A: Math background

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1. The inner product of vectors $y = \begin{bmatrix} 1 \\ 3 \end{bmatrix}$ and $z = \begin{bmatrix} 2 \\ 3 \end{bmatrix}$, sometimes written $y^T z$ is $(1)(2) + (3)(3) = 11$.

2. The product of matrix $x = \begin{bmatrix} 2 & 4 \\ 1 & 3 \end{bmatrix}$ and y is

$$\begin{bmatrix} 2 & 4 \\ 1 & 3 \end{bmatrix} \begin{bmatrix} 1 \\ 3 \end{bmatrix} \begin{bmatrix} 14 \\ 10 \end{bmatrix}$$

3. matrices are invertible when their determinants are non-zero. The determinant of matrix X is $6 - 4 = 2$. Matrix X is invertible.
To compute the inverse of X
1) calculate matrix of minors $\begin{bmatrix} 3 & 1 \\ 4 & 2 \end{bmatrix}$
2) matrix of cofactors $\begin{bmatrix} 3 & -1 \\ -4 & 2 \end{bmatrix}$
3) adjucate $\begin{bmatrix} 3 & -4 \\ -1 & 2 \end{bmatrix}$
4) multiply by 1/ determinant
$\frac{1}{2} \begin{bmatrix} 3 & -4 \\ -1 & 2 \end{bmatrix} = \begin{bmatrix} 3/2 & -2 \\ -1/2 & 1 \end{bmatrix}$

The inverse of matrix X is $\begin{bmatrix} 3/2 & -2 \\ -1/2 & 1 \end{bmatrix}$

Calculus

1. $y = x^3 + x - 5$
$\frac{dy}{dx} = 3x^2 + 1$

2. $y = x\sin(z)e^{-x}$
$\frac{dy}{dx} = xce^{-x}$ $\quad c = \sin(z)$
$1ce^{-x} + (-1)e^{-x}xc$
$\sin(z)e^{-x} - x\sin(z)e^{-x}$

Probability + Statistics

1. Sample of data $S = \{1, 1, 0, 1, 0\}$. The sample mean is $\frac{\sum_{i=1}^{|S|} x_i}{|S|} = \frac{3}{5}$

2. Sample variance is $\frac{\sum_{i=1}^{|S|} (x_i - \bar{x})}{|S| - 1}$

$\frac{3(1 - 3/5)^2 + 2(0 - 3/5)^2}{4}$

$= \frac{3(4/25) + 2(9/25)}{4}$

$= \frac{30/25}{4} = \cancel{30}^{15}/\cancel{25} \cdot 1/\cancel{4}^{2}$ $\quad 15/50 = 3/10$ is the sample variance.

3. The sample data was generated by flipping a coin 5 times where 0 denotes tails + 1 denotes heads. This describes a Binomial distribution.
The probability mass function of a Binomial Random variable is $P(X = k) = \binom{n}{k} p^k (1-p)^{n-k}$
The probability for observing 2 heads is
$P(X = 2) = \binom{5}{2}(.5)^2(.5)^3 = (10)(.25)(.125) = .3125$

4. The probability, θ, that maximizes the probability of sample S is the sample mean / sample size, or 3/5.

5. a. $P(z = T \text{ and } y = b) = .1$
b. $P(z = T \mid y = b) = \frac{P(z = T \wedge y = b)}{P(y = b)} = \frac{.1}{.1 + .15} = .4$

Big O-notation

1. $\ln(n) = O(\lg(n))$ and $\lg(n) = O(\ln(n))$ because all logs regardless of their base are related by a constant factor.
2. $3^n = O(n^{10})$ because as n approaches infinity exponential growth exceeds power growth.
3. $3^n = O(2^n)$ and $2^n = O(3^n)$ because as n approaches infinity the difference b/wn the two functions is negligible.