COSC 462 Fall 2024 : Homework 2

Date: October 7, 2024 **Total points: 55 points** **Due: 11:59 PM, October 14, 2024**

Problem 1: (5 points) Describe the communication cost model used in class for complexity analyses of all parallel algorithms. Be sure to describe all the assumptions underlying the cost model.

Problem 2: Assume that a pool of 4 processors $\{p_0, p_1, p_2, p_3\}$ is available.
(i) (5 points) If p_0 wants the value of a local variable a to be known to all the 4 processors, which collective communication call needs to be executed?

(ii) (5 points) If p_0 wants to distribute a locally defined array $[d_0, d_1, d_2, d_3]$ amongst the 4 processors such that d_0 is sent to p_0, d_1 is sent to p_1, d_2 is sent to p_2 and d_3 is sent to p_3, which collective call needs to be executed.

Problem 3:
(i) (5 points) Give an example of a bitonic sequence of 8 integers.

(ii) (5 points) Sort this bitonic sequence of 8 integers using bitonic splits. Show <u>*each*</u> bitonic splitting step during the sorting algorithm clearly.

Cost Optimality
The **cost** of solving a problem on a parallel system is defined as the product of the parallel runtime and the number of processing elements used. In other words, it is the sum of the time that each processing element spends solving the problem. The cost of solving a problem on a single processing element is the execution time of the fastest known sequential algorithm. A parallel algorithm is said to be **cost-optimal** if the cost of solving the parallel algorithm has the same asymptotic growth (in Θ terms) as a function of the input size as the fastest known sequential algorithm on a single processor. Since efficiency is the ratio of sequential cost to parallel cost, a cost-optimal parallel system has an efficiency of $\Theta(1)$ (that is, a constant).

4. Matrix-Vector Multiplication

(i) (5 points) The parallel complexity of a matrix-vector algorithm can be shown to be:

$$T(n,p) = O\left(\frac{n^2}{p} + \tau \log p + \mu n\right)$$

where the symbols have their usual meanings. What is the *cost* of this parallel algorithm where cost is defined as above.

(ii) (10 points) *Cost-optimality* of this algorithm will be achieved when (a) $p = O(\sqrt{n})$, (b) $p = O(n)$ (c), $p = O(n^2)$, or (d) $p = O(n^3)$. Choose the correct answer from the four given options and analytically justify your choice.

5. Matrix-Matrix Multiplication

(i) (5 points) The parallel complexity of a simple matrix-matrix algorithm can be shown to be:

$$T(n,p) = O\left(\frac{n^3}{p} + \tau \log p + 2\mu \frac{n^2}{\sqrt{p}}\right)$$

where the symbols once againhave their usual meanings. What is the *cost* of this parallel algorithm where cost is again defined as above.

(ii) (10 points) *Cost-optimality* of this algorithm will be achieved when (a) $p = O(\sqrt{n})$, (b) $p = O(n)$ (c), $p = O(n^2)$, or (d) $p = O(n^3)$. Choose the correct answer from the four given options and analytically justify your choice.

Note that you will need to know the complexities of the fastest sequential algorithms for matrix-vector and matrix-matrix multiplications, both of which were covered in class.