

GR5261/GU4261 Statistical Methods in Finance
Homework 4 (due on Feb 22, 2018; online submission only)

- (1) For each of the following two covariance matrices, perform the principal component decomposition, i.e. calculate its eigenvalues and corresponding eigenvectors. What proportion of the total variance does the first principal component explain in each case?

(a) $\begin{pmatrix} 1 & \rho \\ \rho & 1 \end{pmatrix};$

(b) $\begin{pmatrix} 1 & \rho & 0 \\ \rho & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}.$

Hint: You could refer to Homework 1 for reference.

- (2) The file “FamaFrench_mon_69_98.txt” contains monthly figures (in %) of the market portfolio excess return (Mkt.RF), SMB, HML and the risk-free rate (RF) from Jan 1969 to Dec 1998 (i.e. 360 observations). On the other hand, the data set “CRSPmon” in the “Ecdat” R-package includes monthly returns of General Electric (GE), IBM and Mobil over the same period. You may use the following two lines of codes for access to the “CRSPmon” data.

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library("Ecdat")  
data(CRSPmon)
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- (a) Perform principal component analysis (PCA) on the monthly returns of GE, IBM and Mobil. Report the following quantities.
- (i) The proportion of total variance explained by the first principal component.
 - (ii) The loading vectors (the eigenvectors).
 - (iii) The principal components for the first two observations.
- (b) Now, for each of following cases, fit the Fama-French three factor model on GE, IBM and Mobil respectively. Comment on any difference between the two cases.
- (i) Using data in the first 180 months only;
 - (ii) Using data in the last 180 months only.

Hint: You may refer to “fama-french-3f.pdf” and “FF3.R” for an illustration.

- (3) As in the oil drilling example (see “capm-oil-drilling-example.pdf” in folder “Lecture 4”), suppose we have the same market parameters: $\mu_f = 10\%$, $\mu_M = 17\%$ and $\sigma_M = 12\%$. Suppose further that the expected share price of the oil drilling company after one year is also \$1000/share.

- (a) If the oil drilling company has a beta of 0.5, what is its fair price according to the SML?
- (b) If its beta is 0.7, what should be its fair price then?