HUDM5124

Assignment 1 Exercises on metric spaces and distance:

1. Consider the following set of axioms of a metric space (note this is not the minimal axiomatization, but it is clear):

   a. symmetry ($d_{xy} = d_{yx}$)

   b. positivity ($d_{xy} > 0$)

   c. minimality ($d_{xx} = 0$)

   d. the triangle inequality ($d_{xy} + d_{yz} \geq d_{xz}$)

For the Rothkopf Morse code data, AND Ekman's data on similarity of color chips, discuss which of these axioms seem to be satisfied and which do not. Note that you will have to "translate" these axioms as appropriate for similarity data; one way to do this is to assume that your similarities will be translated into dissimilarities via a linear transformation: for example, what do positivity and minimality together imply about similarities?

2. Prove that the triangle inequality generalizes to more than three points; i.e. that for points x,y,z,w, $d(x,w) \leq d(x,y) + d(y,z) + d(z,w)$

3. Imagine a flat landscape with circular lakes of varying sizes scattered through it. You cannot travel through the lakes, only over land. Would the metric axioms hold for distances in this space? You can offer formal proofs, or merely arguments.