(1) $R_{tr}(\hat{\beta}) = \frac{1}{N}\sum_{i=1}^{N}(y_i - \hat{\beta}^T x_i)^2 \Rightarrow E(R_{tr}(\hat{\beta})) = \frac{n-p-1}{n}\sigma^2$, where $\hat{\beta}^T$ based on train data.

method 2 $R_{te}(\hat{\beta}) = \frac{1}{M}\sum_{i=1}^{M}(\tilde{y}_i - \hat{\beta}^T \tilde{x}_i)^2$

$= \frac{1}{M}\sum_{i=1}^{M}\{(\beta^T\tilde{x}_i + \varepsilon_i) - [(X^TX)^{-1}X^TY]^T\tilde{x}_i\}^2$, where β is the true β

$= \frac{1}{M}\sum_{i=1}^{M}\{(\beta^T\tilde{x}_i + \varepsilon_i) - [(X^TX)^{-1}X^T(X\beta + \varepsilon)]^T\tilde{x}_i\}^2$

$= \frac{1}{M}\sum_{i=1}^{M}\{(\beta^T\tilde{x}_i + \varepsilon_i) - [(\beta^TX^T + \varepsilon^T)X(X^TX)^{-1}]\tilde{x}_i\}^2$

$= \frac{1}{M}\sum_{i=1}^{M}\{\beta^T\tilde{x}_i + \varepsilon_i - (\beta^TX^TX(X^TX)^{-1} + \varepsilon^TX(X^TX)^{-1})\tilde{x}_i\}^2$

$= \frac{1}{M}\sum_{i=1}^{M}\{\beta^T\tilde{x}_i + \varepsilon_i - \beta^TX^TX(X^TX)^{-1}\tilde{x}_i + \varepsilon^TX(X^TX)^{-1}\tilde{x}_i\}^2$

$= \frac{1}{M}\sum_{i=1}^{M}\{\varepsilon_i - \varepsilon^TX(X^TX)^{-1}\tilde{x}_i\}^2$

$= \frac{1}{M}\sum_{i=1}^{M}\{\varepsilon_i^2 - 2\varepsilon_i\varepsilon^TX(X^TX)^{-1}\tilde{x}_i + (\varepsilon^TX(X^TX)^{-1}\tilde{x}_i)^2\}$

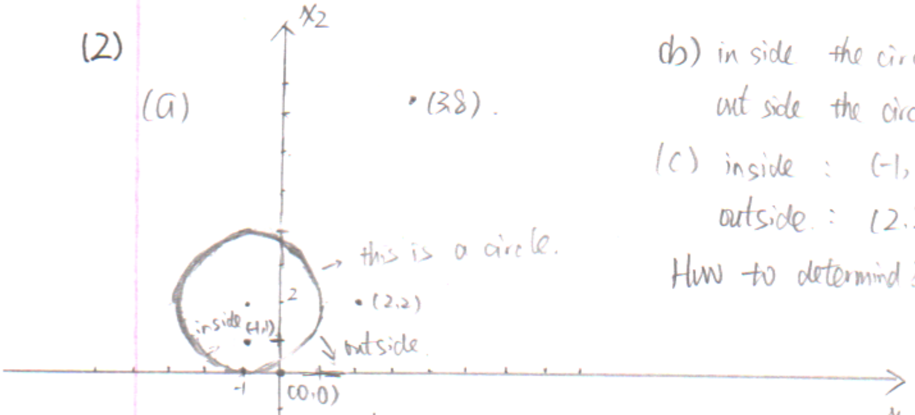
$\Rightarrow E(R_{te}(\hat{\beta})) = \frac{1}{M}\sum_{i=1}^{M}[E(\varepsilon_i^2) - 2E(\varepsilon_i\varepsilon^TX(X^TX)^{-1}\tilde{x}_i) + E(\varepsilon^TX(X^TX)^{-1}\tilde{x}_i)^2]$

$= \frac{1}{M}\sum_{i=1}^{M}[\sigma^2 + E(\varepsilon^TX(X^TX)^{-1}\tilde{x}_i)^2] \geq \sigma^2 \geq \frac{n-p-1}{n}\sigma^2$.

$\Rightarrow E(R_{te}(\hat{\beta})) \geq E(R_{tr}(\hat{\beta}))$

(2)

(a)



(b) inside the circle: $(1+x_1)^2 + (2-x_2)^2 \leq 4$ (red)

outside the circle: $(1+x_1)^2 + (2-x_2)^2 \geq 4$ (blue)

(c) inside: (-1,1)

outside: (2,2), (3,8), (0,0).

How to determind? 1. look at the plot
2. calculate, plug in values $(x_1+1)^2 + (2-x_2)^2$ ≤ 4 red, ≥ 4 blue.

(d) $(1+x_1)^2 + (2-x_2)^2 = 4 \Leftrightarrow x_1^2 + 2x_1 + x_2^2 - 4x_2 + 1 = 0$.

Thus, it's linear based on x_1^2, x_2^2, x_1, x_2