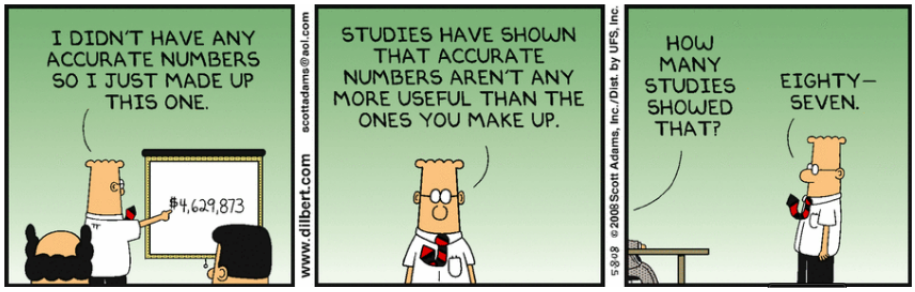
HUDM 5123 - Linear Models and Experimental Design
Mid-term Exam

1 Exam Rules

Don't cheat. If you don't know the answer, take your best guess. 40 points total.



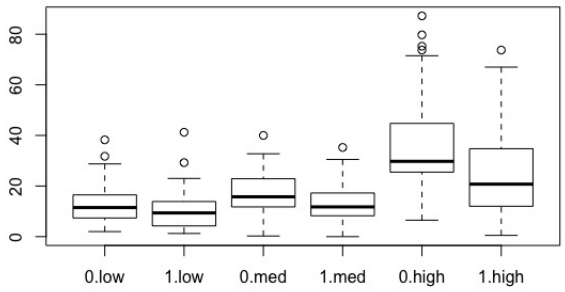
2 True/False (1 pt each) No Tiffs!

1. _____ The degrees of freedom associated with the regression model

$$Y_i = \beta_0 + \beta_1 X_{i1} + \beta_2 X_{i2} + \beta_3 X_{i1} \times X_{i2} + \epsilon_i$$

   fit to a sample of $n = 212$ observations is 209.

2. _____ Group sample variances for a 2×3 two-way ANOVA design are as follows: 61.2, 58.4, 69.6, 65.8, 404.2, 322.7; boxplots by cell of the design are displayed in the plot below.



   Based on the sample variances and boxplots, it is safe to conclude that the constant residual variance assumption is tenable here.

3. _____ A researcher planned to test four complex contrast comparisons. The four p-values, ordered from smallest to largest, were .002, .015, .210, and .685. Based on the Holm-Bonferroni method to hold the familywise error rate to .05, two of the four tests are determined to be significant.

4. _____ For a linear model of the form

$$Y_i = \beta_0 + \beta_1 X_{i1} + \beta_2 X_{i2} + \cdots + \beta_p X_{ip} + \epsilon_i,$$

the statistical assumptions encoded in the statement $\epsilon_i \overset{iid}{\sim} N(0, \sigma^2_\epsilon)$ are (a) constant residual variance, (b) normally distributed residuals, (c) correct model specification (linearity), and (d) independence of errors.

5. _____ The use of methods to control false discovery rate, as opposed to familywise error rate, when conducting multiple comparisons, is most appropriate when the number of multiple comparisons to be conducted is relatively small.

6. _____ The formula for the incremental F test for comparing nested regression models is as follows:

$$F = \frac{(SSResid_R - SSResid_F)/(df_R - df_F)}{SSResid_R/df_R}.$$

3 Multiple Choice (1 pt each)

The multiple choice questions deal with the acupuncture data set we have analyzed in class and in lab. The data come from a randomized experiment to study the efficacy of acupuncture for treating headaches. The data set includes 301 cases, 140 control (no acupuncture) and 161 treated (acupuncture). Participants were randomly assigned to treatment group. Variables in the data set include age in years, sex (0 = male, 1 = female), migraine status (0 = no diagnosis of migraine, 1 = diagnosis of migraine), chronicity (years of headache disorder at baseline), acupuncturist (ID number for acupuncture provider), group (0 = control, 1 = treatment), pk1 (headache severity at baseline), and the outcome variable pk5 (headache severity at 1 year). R output from the one-way ANOVA to test the effect of acupuncture is as follows:

```
Response: pk5
            Sum Sq  Df F value    Pr(>F)
(Intercept) 111517   1 474.083 < 2.2e-16 ***
group         2783   1  11.833 0.0006648 ***
Residuals    70333 299
```

7. Which of the following is the full model used to produce the one-way ANOVA output above?

   (a) $\text{pk5}_i = \beta_0 + \epsilon_i$

   (b) $\text{pk5}_i = \beta_0 + \beta_1 \text{group}_i + \epsilon_i$

   (c) $\text{pk5}_i = \beta_0 + \beta_1 \text{group}_{i1} + \beta_2 \text{group}_{i2} + \epsilon_i$

   (d) $\text{pk5}_i = \beta_0 + \beta_1 \text{group}_i + \text{pk1}_i + \epsilon_i$

   (e) $\text{pk5}_i = \beta_0 + \beta_1 \text{group}_{i1} + \beta_2 \text{group}_{i2} + \text{pk1}_i + \epsilon_i$

8. The parameter β_0 in the full model for one-way ANOVA may be correctly interpreted as

   (a) the predicted value for a control group member at the mean of pk1.

   (b) the predicted value for a treatment group member at the mean of pk1.

   (c) the grand mean of the treatment and control groups.

   (d) the mean of the treatment group.

   (e) the mean of the control group.

9. The degrees of freedom for the full and reduced models for the one-way ANOVA (output displayed above) are, respectively,

   (a) 1 & 299

   (b) 299 & 1

   (c) 300 & 299

   (d) 299 & 300

   (e) 300 & 301

The researchers followed up with a one-way ANCOVA, controlling for headache severity at baseline (assume any assumptions required for the validity of the test are satisfied). The output:

```
Response: pk5
            Sum Sq  Df  F value    Pr(>F)
(Intercept)    106   1   0.9109 0.3406440
group         1568   1  13.4269 0.0002935 ***
pk1          35534   1 304.2869 < 2.2e-16 ***
Residuals    34799 298
```

10. The F statistic value for the group effect changed from 11.83 in the one-way ANOVA to 13.43 in the one-way ANCOVA. The change in group sum of squares (GroupSS), residual sum of squares (ResidSS), and the residual degrees of freedom (RDF) all played a role in the change of the F statistic value. Pick the option below that describes the effect each had on the F statistic, where $\uparrow$ implies the change caused the F statistic to increase, and $\downarrow$ implies the change caused the F statistic to decrease. For example, GroupSS $\uparrow$ means that the change in GroupSS caused the F statistic to increase.

    (a) GroupSS $\uparrow$, ResidSS $\uparrow$, RDF $\uparrow$

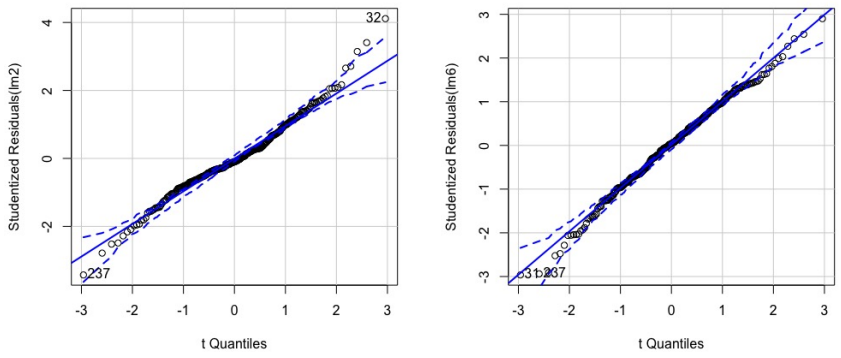
    (b) GroupSS $\downarrow$, ResidSS $\uparrow$, RDF $\uparrow$

    (c) GroupSS $\downarrow$, ResidSS $\uparrow$, RDF $\downarrow$

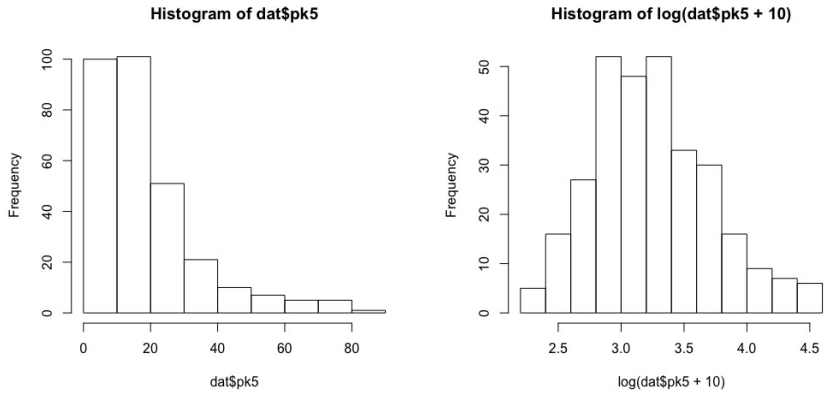
    (d) GroupSS $\downarrow$, ResidSS $\downarrow$, RDF $\uparrow$

    (e) GroupSS $\uparrow$, ResidSS $\downarrow$, RDF $\downarrow$

11. The QQ plot below on the left is based on the residuals from the one-way ANCOVA model. The QQ plot below on the right is based on the residuals from the one-way ANCOVA model after adding 10 and taking a log transformation of the outcome variable, pk5.



The histogram below on the left is of the outcome, pk5. The histogram below on the right is of log(pk5 + 10).



The graphical evidence (QQ plots and histograms) suggest that

(a) the constant residual variance assumption is violated with the untransformed data but tenable with the transformed data.

(b) the constant residual variance assumption is tenable with the untransformed data but violated with the transformed data.

(c) the residual normality assumption is violated with the untransformed data but tenable with the transformed data.

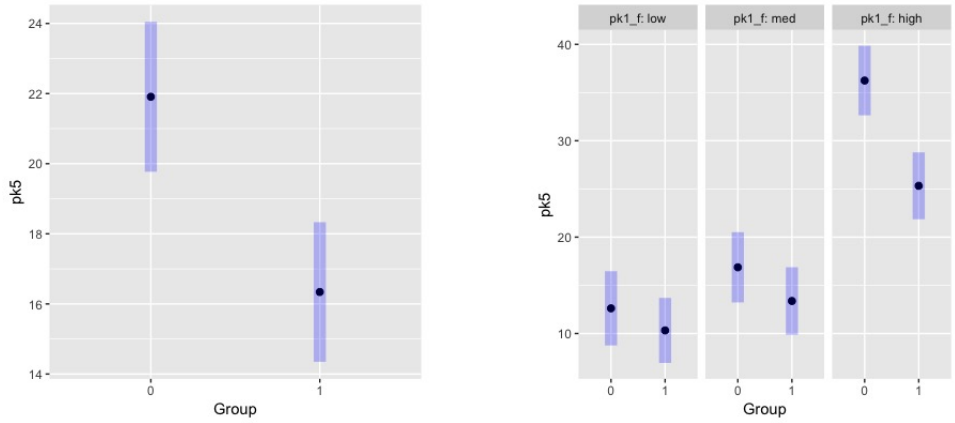
(d) the residual normality assumption is tenable with the untransformed data but violated with the transformed data.

(e) the independence of residuals assumption is violated with both the untransformed and transformed data.

12. A three-category variable was created based on low, medium, and high values of the pretest variable, pk1. Output from the two-way ANOVA, with factors based on group and the three-category pretest variable, called pk1_f, is shown below.

```
Response: pk5
            Sum Sq  Df  F value    Pr(>F)
(Intercept) 109314   1 663.6596 < 2.2e-16 ***
group         2317   1  14.0660 0.0002124 ***
pk1_f        21145   2  64.1882 < 2.2e-16 ***
group:pk1_f   1097   2   3.3312 0.0370988 *
Residuals    48591 295
```

Group means, averaged over the three categories of pretest score, are shown in the plot on the left below. Group means, conditional on the three categories of pretest (low, med, high), are shown in the plot on the right below.



Given the results of the two-way ANOVA, which of the two mean plots is most appropriate, and why?

(a) The plot on the left is most appropriate because it graphically displays the significant main effect of group.

(b) The plot on the left is most appropriate because the two-way interaction is not significant here, so it is reasonable to average over the levels of the pretest factor.

(c) The plot on the right is most appropriate because it graphically displays the significant main effect of the pretest factor.

(d) The plot on the right is most appropriate because the two-way interaction is significant here, so it is reasonable to condition on the levels of the pretest factor.

(e) Neither plot is appropriate based on the two-way ANOVA results. The significant intercept implies that group averages do not make sense here.

13. Suppose a member of the research team hypothesized during the planning stages (i.e., *a priori*) of the experiment that there would be pairwise differences between low, medium and high pretest groups separately in the control and treatment groups. Which strategy for handling multiple comparisons would control the familywise error rate, while also providing optimal power to detect true differences?

    (a) Scheffé's Critical Value

    (b) Tukey's HSD

    (c) Bonferroni Adjustment

    (d) Holm-Bonferroni Adjustment

    (e) Shaffer's Planned Post-Omnibus

14. Suppose that after the experiment had been run and data had been analyzed, a member of the research team wondered (i.e, *post hoc*) if there might be pairwise differences between low, medium and high pretest groups separately in the control and treatment groups. Which strategy for handling multiple comparisons would control the familywise error rate, while also providing optimal power to detect true differences?

    (a) Scheffé's Critical Value

    (b) Tukey's HSD

    (c) Bonferroni Adjustment

    (d) Holm-Bonferroni Adjustment

    (e) Shaffer's Planned Post-Omnibus

15. The group means for the outcome variable (pk5) for control and acupuncture groups are, respectively, 22.3 and 16.4. After controlling for baseline (pk1) via ANCOVA, the adjusted group means are, respectively, 21.5 and 16.9. The adjusted group mean for the control group is attained from the model-predicted value from the ANCOVA full model with

    (a) the group indicator set to 0, and pk1 set to its mean.

    (b) the group indicator set to 1, and pk1 set to its mean.

    (c) the group indicator set to 0, and pk1 set to 0.

    (d) the group indicator set to 1, and pk1 set to 0.

    (e) the group indicator set to 0, and pk1 set to the intercept.

4 Free Response

16. (5 pts) List five properties of this table that are not aligned with APA style guidelines.

Variable	Estimate	SE	p-value
Intercept	114.29211	1.50	3.1e-14
WSES14	0.04733	.020	0.011
pg_Educ	0.03901	.033	0.475

Table 1: Regression coefficients

(a)

(b)

(c)

(d)

(e)

17. (6 pts) The correlation matrix for the acupuncture data is shown below, rounded to two decimal places. Comment on the following three aspects of the matrix by offering an explanation for each.

```
           id   age   sex  migr  chrn acpnst group   pk1   pk5
id       1.00 -0.03 -0.06  0.05 -0.02   0.01  0.00  0.00  0.01
age     -0.03  1.00 -0.02 -0.06  0.47  -0.06  0.01 -0.03  0.09
sex     -0.06 -0.02  1.00  0.09 -0.05  -0.06 -0.04 -0.02 -0.05
migr     0.05 -0.06  0.09  1.00  0.12   0.07  0.00  0.03 -0.03
chrn    -0.02  0.47 -0.05  0.12  1.00  -0.06 -0.02  0.08  0.13
acpnst   0.01 -0.06 -0.06  0.07 -0.06   1.00 -0.01  0.04  0.12
group    0.00  0.01 -0.04  0.00 -0.02  -0.01  1.00 -0.07 -0.20
pk1      0.00 -0.03 -0.02  0.03  0.08   0.04 -0.07  1.00  0.71
pk5      0.01  0.09 -0.05 -0.03  0.13   0.12 -0.20  0.71  1.00
```

(a) The correlations between group and the other pre-treatment covariates are close to zero. Why is this expected?

(b) Which pairwise correlation is informative regarding pk1's suitability as a covariate and what do you conclude about its suitability based on the value of that pairwise correlation?

(c) Which pairwise correlation would allow you to predict that the sum of squares for group will decrease after controlling for pk1, and why?

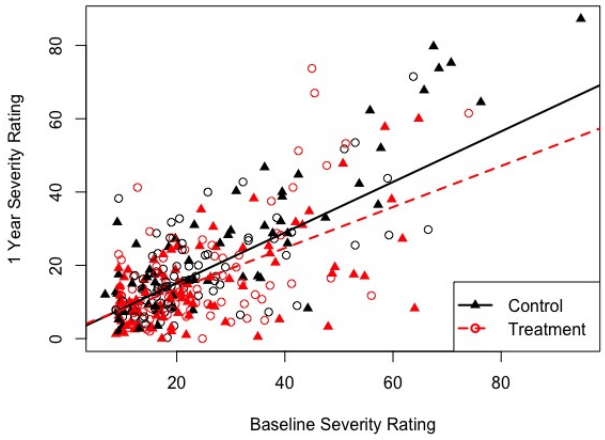
18. (5 pts) Summarize the result of the one-way ANOVA test of the acupuncture treatment (output was given on p. 2) in a sentence in APA format. Be sure to note the null hypothesis, whether it was rejected or not, the α level, the F statistic and the p-value.

19. (9 pts) A more detailed inspection reveals that the continuous pretest variable interacts with the treatment variable such that the effect of the acupuncture treatment varies depending on the baseline headache severity. The summary of the full model that includes the pretest and its interaction with treatment is as follows.

```
Call:
lm(formula = pk5 ~ group + pk1 + pk1:group, data = dat)

Coefficients:
            Estimate Std. Error t value Pr(>|t|)
(Intercept)  1.35480    1.19641   1.132  0.25838
group1       0.95996    1.19641   0.802  0.42298
pk1          0.69430    0.04015  17.291  < 2e-16 ***
group1:pk1  -0.12741    0.04015  -3.173  0.00167 **
```

The scatterplot with group-specific regression lines:



Write a brief mini-report in which you describe how to interpret the group effect based on the output given above. The best answers will condition on two specific values of pk1 and use the resultant equations to demonstrate how the effect of group depends on the value of pk1.