Spring 2018 STAT5261 Midterm Practice Questions

No Submission is required. Solution will be posted later.

1 Fixed Income Securities

1. Suppose that the yearly compounded rate is 3%. There is a 5-year coupon bond with face value (PAR) \$1000 and yearly coupon payment \$30. Suppose that this coupon bond may default and the default time X follows a geometric distribution: $P(X = t) = (1 - p)^{t-1}p$, $t = 1, 2, ...$ and $p = 0.05$. Find the expected present value of the coupon. (if the default happens at year 1, no coupon or the premium will be paid; if the default happens at year 2, only the coupon \$30 for year 1 will be paid...)

2. Suzanne paid \$800 for a 1-year zero-coupon bond with face value (PAR) \$1000. The current 1-year interest rate (yearly compounded) is $r = 5\%$. Suppose there is a 10% probability that the bond will default and the recovery rate is 0 (i.e. no recovery).

   (a) For Suzanne's purchase, the (1-year) net return is a random variable due to the possibility of default. Compute this (1-year) net return and find its mean and standard deviation.

   (b) Suppose that Suzanne's portfolio contains, in addition to the above bond (A), another 1-year zero-coupon bond (B) for which she also paid \$800. Suppose that bond B has face value (PAR) of \$1000 and that its default probability is 10%. If the two default events are mutually exclusive, find the mean and standard deviation of the (1-year) net return of this portfolio.

3. Suppose that the forward rate is $r(t) = 0.03 + 0.001t + 0.0002t^2$

   (a) What is the 5-year spot rate?

   (b) What is the price of a zero-coupon bond that matures in 5 years?

4. Suppose that the continuous forward rate is $r(t) = 0.04 + 0.001t$ when a 8-year zero coupon bond is purchased. Six months later the forward rate is $r(t) = 0.03 + 0.0013t$ and the bond is sold. What is the return?

2 Portfolio Theory

1. Suppose that the market consists of two risky assets. The mean and standard deviation of the return for asset 1 are $\mu_1 = 10\%$ and $\sigma_1 = 20\%$, respectively and those for asset 2 are $\mu_2 = 4\%$ and $\sigma_2 = 10\%$.

   (a) Assume that the two returns are uncorrelated. Find the minimum variance point, (σ_{mv}, μ_{mv}), on the $\mu - \sigma$ diagram for this market.

   (b) What if the correlation between the two returns is $\rho = 0.3$ or $\rho = -0.3$?

2. Suppose that a market consists of one risk-free asset with rate of return $\mu_f = 1\%$, and two risky assets with means and standard deviations of returns being $\mu_1 = 3\%$, $\mu_2 = 6\%$, $\sigma_1 = 20\%$ and $\sigma_2 = 30\%$.

   (a) Find the Sharpe ratios of the tangency portfolios for different $\rho = 0.2, 0.4, 0.8$ and write down the equations for the capital market lines.

   (b) Suppose instead we have $\sigma_1 = 40\%$ and $\sigma_2 = 60\%$. How do Sharpe ratios change and the portfolio weights change corresponding to the changes in σ's?

3. Suppose that the risk-free interest rate is 0.023, that the expected return on the market portfolio is $\mu_M = 0.10$, and that the volatility of the market portfolio is $\sigma_M = 0.12$.

   (a) What is the expected return on an efficient portfolio with $\sigma_R = 0.05$?

   (b) Stock A returns have a covariance of 0.004 with market returns. What is the beta of Stock A?

   (c) Stock B has beta equal to 1.5 and $\sigma_\epsilon = 0.08$. Stock C has beta equal to 1.8 and $\sigma_\epsilon = 0.10$. What is the expected return of a portfolio that is one-half Stock B and one-half Stock C? What is the volatility of a portfolio that is one-half Stock B and one-half Stock C? Assume that the ϵ's of Stocks B and C are independent.

3 Rank Correlation and Copula

1. U is a uniform random variable on the interval $[0, 1]$, V is a Bernoulli random variable with $P(V = 1) = p$, where $0 \leq p \leq 1$. Suppose U and V are independent, Compute Kendall's τ, Spearman's ρ and Pearson's correlation coefficient for $(U, U + V)$ as a function of p.

2. Let (X, Y) has the bivariate distribution $F_\theta(x, y)$ of the form

$$F_\theta(x, y) = 1 - e^{-x} - e^{-y} - e^{-(x+y+\theta xy)}$$

   if $x \geq 0, y \geq 0$ and $F_\theta(x, y) = 0$ otherwise. Here $\theta \in [0, 1]$ is a parameter.

   (a) What are the marginal distributions of X and Y?

   (b) What is the copula function associated with (X, Y)?

3. Suppose that we have two bonds A and B. Denote by T_A and T_B their respective default times (in year). Suppose that T_A follows exponential distribution with hazard $\lambda_A = 0.03$ (i.e. $P(T_A \geq t) = e^{-\lambda_A t}$) and T_B follows exponential distribution with hazard $\lambda_B = 0.02$. Suppose that jointly they satisfy the Gumbel copula with $\alpha = 3$. Find the probabilities that

   (a) Both will default by the end of the second year;

   (b) At least one will default by the end of the second year.

   (c) Only Bond A defaults by the end of the second year.

4 Risk Management

1. Suppose that $\mathcal{L}$ is the loss over 1 year. Suppose that $\mathcal{L}$ follows a double exponential distribution, whose density $f(\cdot)$ has the form

$$f(x) = \frac{1}{Z}\exp(-\lambda|x|), \quad x \in \mathbb{R}$$

where Z is the normalizing constant and $\lambda > 0$ is a parameter. Find

   (a) VaR(0.05) as a function of λ

   (b) ES(0.05) as a function of λ.

2. Suppose the yearly returns of stock A, B and C have a multivariate normal distribution with mean $(0.03, 0.04, 0.05)$ and covariance matrix

$$\begin{pmatrix} 0.4 & 0.3 & -0.1 \\ 0.3 & 0.5 & 0.2 \\ -0.1 & 0.2 & 0.8 \end{pmatrix}.$$

Suppose we have a portfolio that invest $\$50,000$, $\$100,000$ and $\$200,000$ to stock A, B and C respectively. Compute the following quantities for the portfolio:

   (a) VaR(0.05).

   (b) ES(0.05).

3. For a loss random variable L, recall that expected shortfall is defined as

$$\mathrm{ES}(\alpha) := \frac{\int_0^\alpha \mathrm{VaR}(s)ds}{\alpha}.$$

Another risk measure, the conditional tail expectation (CTE) is defined as

$$\mathrm{CTE}(\alpha) = \mathbb{E}(L|L > \mathrm{VaR}(\alpha)). \tag{1}$$

Show that $\mathrm{ES}(\alpha) = \mathrm{CTE}(\alpha)$ when L is a continuous random variable. Note that (1) can be used to derive the expected shortfall formula when the loss follows a normal distribution (see HW6).