Homework3

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Problem 1

$$\mathbf{x} \sim N(\mu, \mathbf{\Sigma})$$

$$\mu = [0, 0, 0]', \mathbf{\Sigma} = \begin{bmatrix} \sigma^2, 0, 0 \\ 0, \sigma^2, 0 \\ 0, 0, \sigma^2 \end{bmatrix}$$

The linear combination of normal distribution is still normal distribution.

$$E(w) = E(\frac{x_1 + x_2 + x_3}{3}) = \frac{1}{3}(\mu_1 + \mu_2 + \mu_3) = 0$$

$$Var(w) = Var(\frac{x_1 + x_2 + x_3}{3}) = \frac{1}{9}(\sigma_1^2 + \sigma_2^2 + \sigma_3^2) = \frac{1}{3}\sigma^2$$

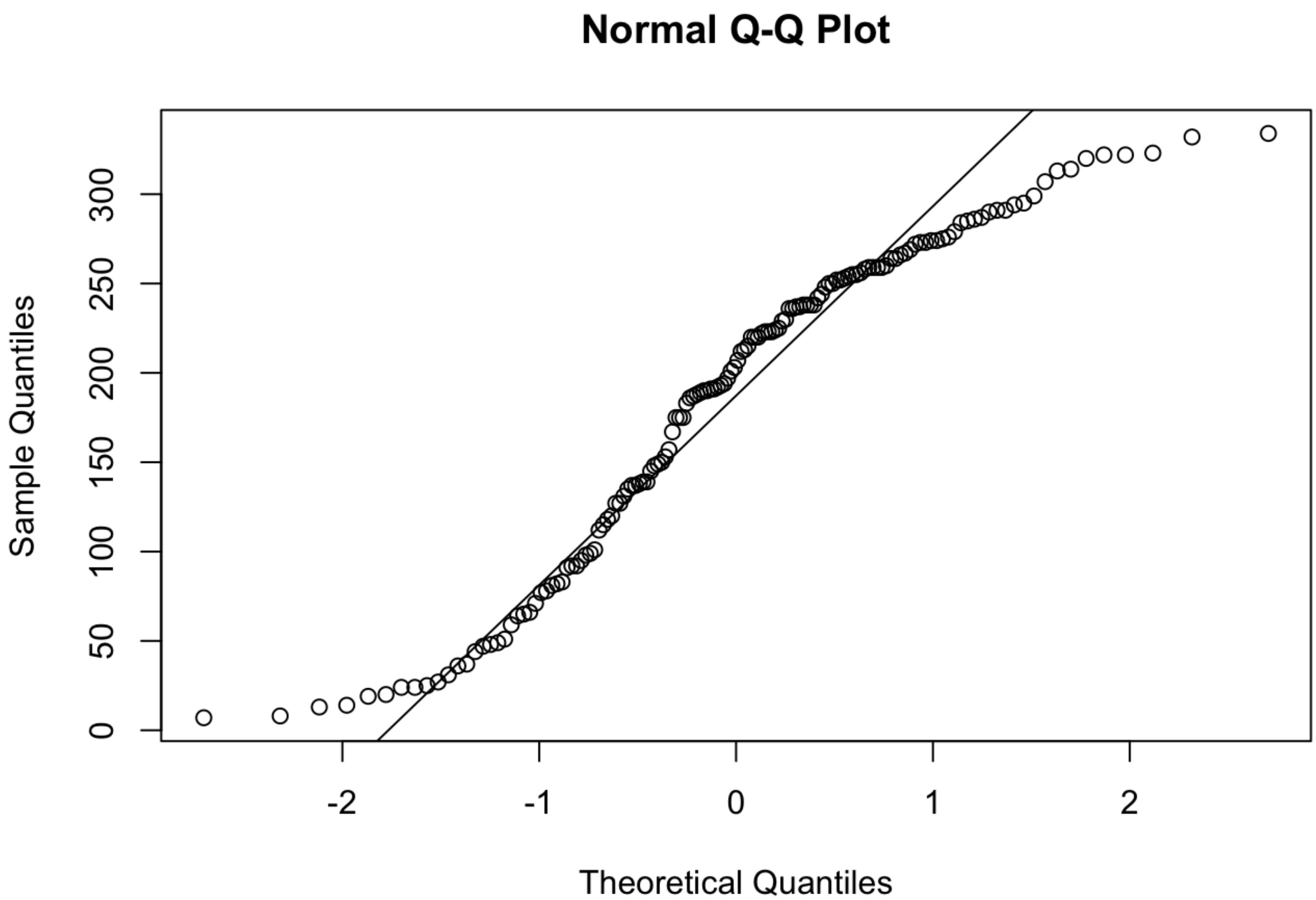
In summary

$$w \sim N(0, \frac{1}{3}\sigma^2)$$

Problem 2

a)

```
data <- airquality
qqnorm(data$Solar.R)
qqline(data$Solar.R)
```



Firstly, the QQ plot indicates that the data is not fully match with the normal distribution.

b)

```
shapiro.test(data$Solar.R)
```

```
##
##  Shapiro-Wilk normality test
##
## data:  data$Solar.R
## W = 0.94183, p-value = 9.492e-06
```

Secondly, Shapiro-Wilks test has very small p-value, which also reject normality.

Problem 3

a)

```
x.bar <- c(2.9,0.9,2.9)
mu.0 <- c(3,1,4)
sigma <- matrix(c(6,1,-2,1,13,4,-2,4,4), 3, 3)
n <- 14
p <- 3
z.obs <- n*t(x.bar - mu.0)%*%solve(sigma)%*%(x.bar - mu.0)
1-pchisq(z.obs,df=p)
```

```
##            [,1]
## [1,] 0.02705413
```

Based on the level of significance $\alpha = 0.01$, the hyphothesis cannot be rejected.

b)

```
z.obs1 <- n*(x.bar[1]-mu.0[1])^2/sigma[1,1]
z.obs2 <- n*(x.bar[2]-mu.0[2])^2/sigma[2,2]
z.obs3 <- n*(x.bar[3]-mu.0[3])^2/sigma[3,3]
1-pchisq(z.obs1,df=1)
```

```
## [1] 0.8785934
```

```
1-pchisq(z.obs2,df=1)
```

```
## [1] 0.917348
```

```
1-pchisq(z.obs3,df=1)
```

```
## [1] 0.03959862
```

Based on the level of significance $\alpha = 0.01$, none of the three individuals hyphothesis' can be rejected.