Statistical Method in Finance

Home Work 4 yc3356 YI CHEN

(1) (a) $\begin{pmatrix} 1 & \rho \\ \rho & 1 \end{pmatrix} \Rightarrow AX=\lambda X \Rightarrow (A-\lambda E)X=0 \Rightarrow \det(A-\lambda E)=0 \Rightarrow \det\begin{pmatrix} 1-\lambda & \rho \\ \rho & 1-\lambda \end{pmatrix}=0 \Rightarrow (1-\lambda)^2-\rho^2=0 \Rightarrow \begin{cases} \lambda_1=\rho+1 \\ \lambda_2=1-\rho \end{cases}$

$\Rightarrow \begin{cases} \text{for } \lambda_1=\rho+1, \text{ the eigenvector is } (1,1)^T \\ \text{for } \lambda_2=1-\rho, \text{ the eigenvector is } (-1,1)^T \end{cases} \Rightarrow Q=\begin{pmatrix} 1 & -1 \\ 1 & 1 \end{pmatrix}$

$\Rightarrow$ if $\begin{cases} \rho \geq 0 : \lambda_1 \geq \lambda_2 \Rightarrow \text{proportation} = \frac{\rho+1}{2} \\ \rho \leq 0 : \lambda_1 \leq \lambda_2 \Rightarrow \text{proportation} = \frac{1-\rho}{2} \end{cases}$

(b) $\begin{pmatrix} 1 & \rho & 0 \\ \rho & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \Rightarrow AX=\lambda X \Rightarrow (A-\lambda E)X=0 \Rightarrow \det(A-\lambda E)=0 \Rightarrow \det\begin{pmatrix} 1 & \rho & 0 \\ \rho & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}=0 \Rightarrow (1-\lambda)^3-\rho^2(1-\lambda)=0$

$\Rightarrow \begin{cases} \lambda_1=1+\rho \\ \lambda_2=1 \\ \lambda_3=1-\rho \end{cases} \Rightarrow \begin{cases} \text{for } \lambda_1=1+\rho, \text{ eignvector}: (1,1,0)^T \\ \text{for } \lambda_2=1, \text{ eignvector}: (0,0,1)^T \\ \text{for } \lambda_3=1-\rho, \text{ eignvector}: (1,-1,0)^T \end{cases}$

$\Rightarrow \begin{cases} \text{if } \rho \geq 0, \ \lambda_1 \geq \lambda_2 \geq \lambda_3 \Rightarrow \text{proportation} = \frac{1+\rho}{3} \\ \text{if } \rho \leq 0, \ \lambda_1 \leq \lambda_2 \leq \lambda_3 \Rightarrow \text{proportation} = \frac{1-\rho}{3} \end{cases}$

(3) $\mu_f = 10\%$, $\mu_m = 17\%$, $E_m = 12\%$, $P_t = 1000\$$

(a) $\beta = 0.5 \Rightarrow \frac{1000-P_0}{P_0} = \mu_f + \beta(\mu_m - \mu_f) \Rightarrow \frac{1000-P_0}{P_0} = 0.1 + 0.5 \times (0.17 - 0.1)$

$\Rightarrow P_0 = 881$

(b) $\beta = 0.7 \Rightarrow \frac{1000-P_0}{P_0} = \mu_f + \beta(\mu_m - \mu_f) \Rightarrow \frac{1000-P_0}{P_0} = 0.1 + 0.7 \times (0.17 - 0.1)$

$\Rightarrow P_0 = 870$