HUDM 6122
Spring 2020

Homework # 3

Due on 2/12/20 at 3:00 pm

1. Let $\boldsymbol{x}$ be a 3×1 normal random vector with mean $\boldsymbol{\mu} = (0, 0, 0)'$ and covariance matrix $\boldsymbol{\Sigma} = \begin{pmatrix} \sigma^2 & 0 & 0 \\ 0 & \sigma^2 & 0 \\ 0 & 0 & \sigma^2 \end{pmatrix}$

   What is the distribution of $w = \frac{x_1 + x_2 + x_3}{3}$?

   Hint: w is a linear combination of x's.

2. Use the built-in dataset `airquality`.

   a) Construct a Q-Q plot for the solar radiation variable (second column). Add the Q-Q line to the plot. Is it likely that the solar radiation is normally distributed?

   b) Carry out a test for normality with $\alpha = 0.05$. State the conclusion. Does it match with your findings from part a)?

3. Suppose we have a sample of $n = 14$ observations from $N_3(\boldsymbol{\mu}, \boldsymbol{\Sigma})$ population, where we know that $\boldsymbol{\Sigma} = \begin{pmatrix} 6 & 1 & -2 \\ 1 & 13 & 4 \\ -2 & 4 & 4 \end{pmatrix}$, and we obtained $\bar{\boldsymbol{x}} = \begin{pmatrix} 2.9 \\ 0.9 \\ 2.9 \end{pmatrix}$.

   a) Test the multivariate hypothesis:

$$H_0: \boldsymbol{\mu} = \begin{pmatrix} 3 \\ 1 \\ 4 \end{pmatrix} \; vs. \; H_1: \boldsymbol{\mu} \neq \begin{pmatrix} 3 \\ 1 \\ 4 \end{pmatrix}$$

   at $\alpha = 0.01$ level of significance.

   b) Perform the test in a) as three individual z-tests, each at the same level of significance $\alpha = 0.01$. Do the results agree with part a)?