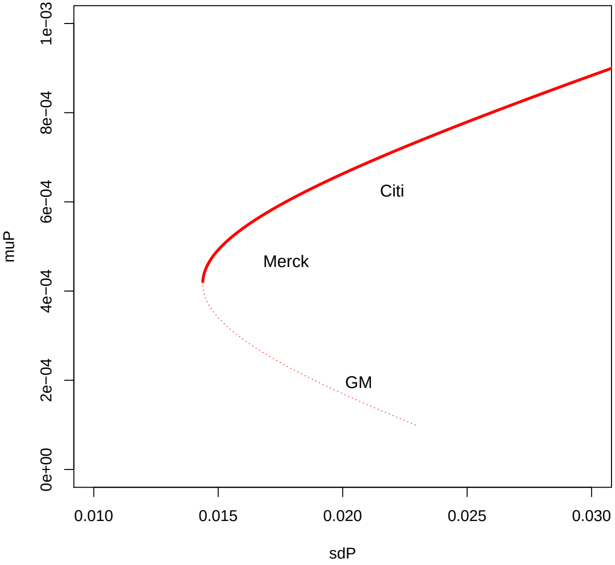
Examples for EF and CAPM

Statistical Methods in Finance

Efficient Frontier

- "Stock_Bond.csv" contains the adjusted closing prices of 10 stocks from Jan 2, 1987 to Sep 1, 2006. We only consider General Motor, Merck and Citigroup to form our portfolio.
- From the adjusted closing prices, we compute their log returns, and the corresponding mean vector and covariance matrix.
- With the estimated mean vector and covariance matrix, we then construct a portfolio from the three stocks to minimize the portfolio variance while keeping the expected return at a target level.
- For a given portfolio return, the portfolio variance is quadratic in the weights, with the following two linear constraints
  - $\sum$(weights) $= 1$.
  - $\sum$(weights x individual returns) $= \mu$ (target level).
- The optimal solutions are plotted, and the red solid line represents the efficient frontier (the part of the curve above its leftmost point).

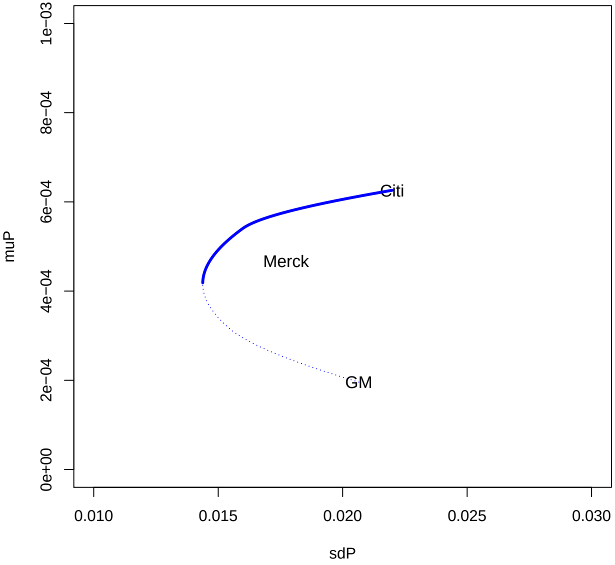
Efficient Frontier



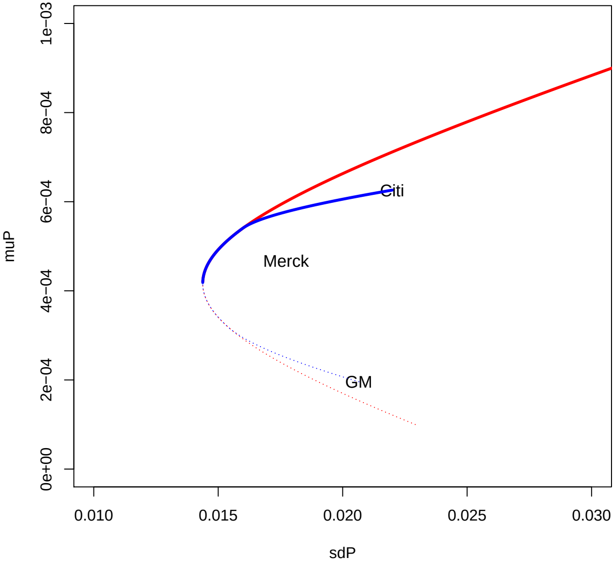
Efficient Frontier

- Now suppose that short selling is not allowed.
- i.e., we impose three additional constraints: all weights ≥ 0.
- This is again a quadratic programming problem with linear constraints.
- The next figure plots the solutions, and the blue solid line represents the efficient frontier in this short-sale prohibited case.

Efficient Frontier



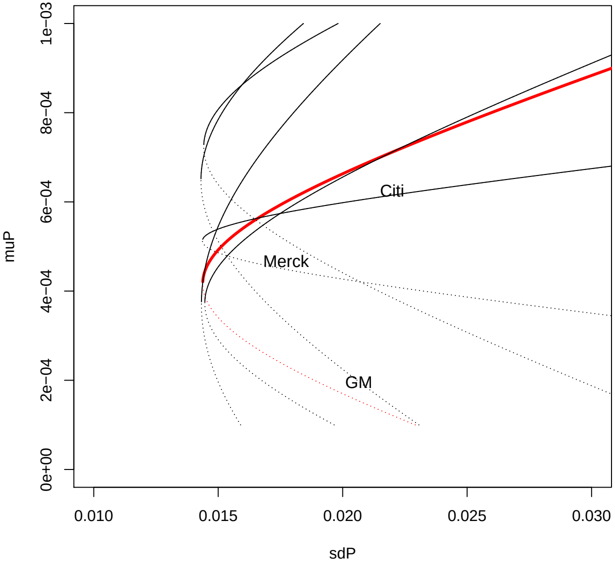
Efficient Frontier



Efficient Frontier

- Note that in our construction of the efficient frontier relies on the estimated mean vector and covariance matrix, rather than their population values which we do not know.
- To assess the effect of replacing the population mean vector and covariance matrix by their estimators, we need to conduct sensitivity analysis.
- Here, we generate a sample of log returns from a multivariate normal with the estimated mean vector and covariance matrix.
- Using this generated sample, we calculate the sample mean vector and covariance matrix and construct the efficient frontier based on the new figures.
- We repeat this process 5 times and get five "efficient frontiers".
- Results: Pattern of substantial deviation from the "true one" (red curve).

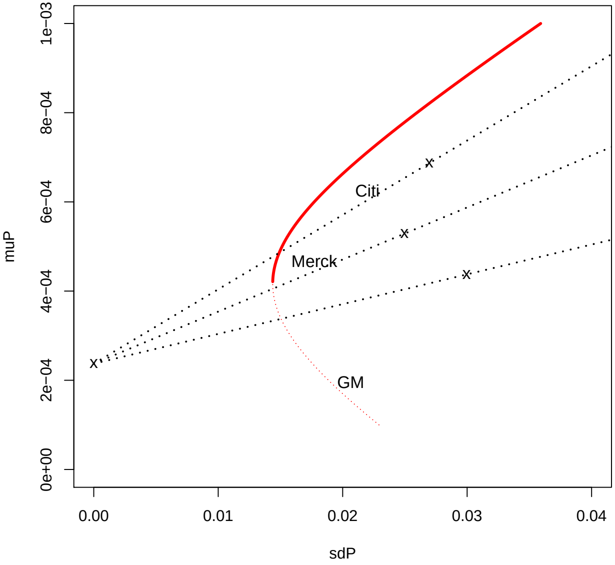
Efficient Frontier



CAPM

- Now suppose in addition to the three risky assets, a risk-free one is added to the market.
- Any straight line connecting the risk-free asset to a risky asset/portfolio represents all possible combinations between the portfolio and the risk-free asset.

CAPM

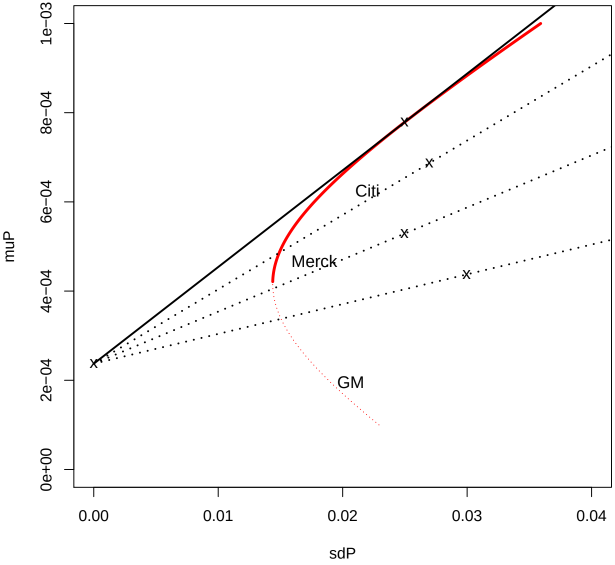


muP
1e−03
8e−04
6e−04
4e−04
2e−04
0e+00
0.00
0.01
0.02
0.03
0.04
sdP
Citi
Merck
GM

CAPM

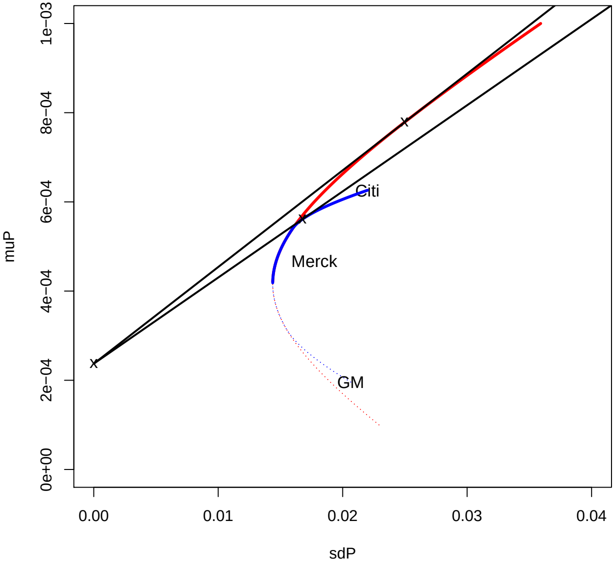
- As we vary the risky asset, we get different straight lines, all of which start at the same point (for the risk-free asset).
- The line having the maximum slope corresponds to the efficient frontier.
- It is the tangent line to the set of feasible region for the market without the risk-free asset.
- The tangent portfolio is also known as the market portfolio in an efficient market.

CAPM



muP
1e−03
8e−04
6e−04
4e−04
2e−04
0e+00
0.00
0.01
0.02
0.03
0.04
sdP
Citi
Merck
GM

CAPM



1e−03
8e−04
6e−04
4e−04
2e−04
0e+00
muP
0.00
0.01
0.02
0.03
0.04
sdP
Citi
Merck
GM

CAPM

- Let R, μ_f and R_M be the return of a risky asset, the risk-free rate and the return of the market portfolio respectively.
- It follows that, in an efficient market,

$$R - \mu_f = \beta(R_M - \mu_f) + \varepsilon$$

  where $\beta = Cov(R, R_M)/Var(R_M)$.
- As an illustration, we again use GM, Merck and Citi as examples. We use S&P 500 as the market benchmark and 1 year Treasury rate as the risk-free rate.
- We regress $R - \mu_f$ on $R_M - \mu_f$ for each of the three stocks, with and without the intercept term.

Fit without intercept:

- GM

```
          Estimate Std. Error t value Pr(>|t|)
mkreturn     0.985      0.032    30.8   <2e-16 ***
```

- Merck

```
          Estimate Std. Error t value Pr(>|t|)
mkreturn    0.7833     0.0296    26.5   <2e-16 ***
```

- Citi

```
          Estimate Std. Error t value Pr(>|t|)
mkreturn    1.2643     0.0313    40.4   <2e-16 ***
```

Fit with intercept:

- GM

```
              Estimate Std. Error t value Pr(>|t|)
(Intercept) -0.01537    0.00027   -56.9   <2e-16 ***
mkreturn     1.02946    0.02491    41.3   <2e-16 ***
```

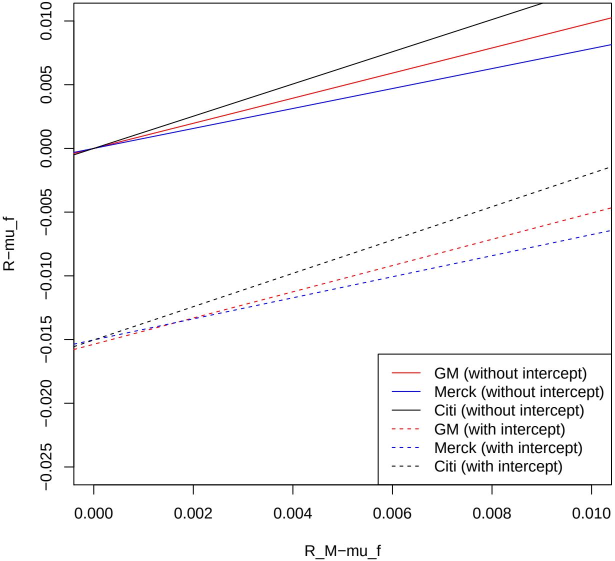
- Merck

```
               Estimate Std. Error t value Pr(>|t|)
(Intercept) -0.015029   0.000239   -62.8   <2e-16 ***
mkreturn     0.826351   0.022073    37.4   <2e-16 ***
```

- Citi

```
               Estimate Std. Error t value Pr(>|t|)
(Intercept) -0.015033   0.000264   -57.0   <2e-16 ***
mkreturn     1.307332   0.024339    53.7   <2e-16 ***
```

CAPM



R−mu_f
R_M−mu_f
0.010
0.005
0.000
−0.005
−0.010
−0.015
−0.020
−0.025
0.000
0.002
0.004
0.006
0.008
0.010
GM (without intercept)
Merck (without intercept)
Citi (without intercept)
GM (with intercept)
Merck (with intercept)
Citi (with intercept)