

GR5261/GU4261 Statistical Methods in Finance  
Homework 1 (due on Feb 1, 2018; online submission only)

Questions from the textbook (Ruppert/Matteson, 2015)

Page 15: Problems 9, 11, 12

Pages 16-17: Exercises 1, 3, 10

Pages 40-43: Exercises 1, 3, 8, 11, 12, 16, 22

Other questions:

- (1) If  $X$  is a continuous random variable with a strictly increasing distribution function  $F$ , find the distribution of  $U = F(X)$ .
- (2) Let  $X$  have a normal distribution with mean  $\mu$  and variance  $\sigma^2$  and let  $Y = e^X$ .  $Y$  is said to have a lognormal distribution with parameters  $\mu$  and  $\sigma^2$  (since  $X = \log Y$  has a normal distribution).
  - (a) Find the density  $f_Y$ . (Hint: compute  $F_Y(y) = \mathbb{P}(Y \leq y)$ ).
  - (b) Find the mean and the variance of  $Y$ . (Hint: if  $X \sim \mathcal{N}(\mu, \sigma^2)$ , then  $\mathbb{E}(e^{tX}) = e^{\mu t + \frac{1}{2}\sigma^2 t^2}$ ).
- (3) Find the largest eigenvalue of the following matrices:
  - (i)  $\begin{pmatrix} 1 & \rho \\ \rho & 1 \end{pmatrix}$ .
  - (ii)  $\begin{pmatrix} 1 & \rho & 0 \\ \rho & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$ .
- (4) Suppose that  $X$  and  $Y$  are independent and identically distributed (iid) exponentially distributed random variables with  $\mathbb{P}(X \geq t) = \mathbb{P}(Y \geq t) = e^{-t}$ . Are  $X + Y$  and  $X - Y$  uncorrelated? Are  $X + Y$  and  $X - Y$  independent? Explain your answer.