Statistical Method in Finance

Homework 06 YI CHEN

(1) return $\sim N(0.002, 0.016)$ Loss = −Return

$P(L-\mu > \sigma) = P(\mu - R \leq -\sigma) = P(\frac{R-\mu}{\sigma} > 1)$

Since $\frac{R-\mu}{\sigma} \sim N(0,1)$ we can get $P(\frac{R-\mu}{\sigma} > 1) = 15.86\%$

(2)(a) $R \sim N(0.002, 0.016)$ $S = 1000000$

$VaR(0.1) = S\times(\sigma \times Z_{0.1} - \mu) = (0.016 \times 1.281552 - 0.002) \times 1000000 = 18504.83$

(b) $R \sim N(0.1, 0.2)$ $S = 1000$

$VaR(0.05) = S\times(\sigma \times Z_{0.05} - \mu) = (0.2 \times 1.644854 - 0.1) \times 1000 = 228.9707$

(3)(a) $\frac{R-\mu}{\sigma} \sim t(2)$, where $\mu = 0.002$, $\sigma = 0.016$ $S = 1000$

$VaR(0.1) = S\times(\sigma \times t_{0.1}(2) - \mu) = 1000 \times (0.016 \times 1.885618 - 0.002) = 28.16989$

(b) $\frac{R-\mu}{\sigma} \sim t(5)$, where $\mu = 0.002$, $\sigma = 0.016$, $S = 1000$

$VaR(0.1) = S\times(\sigma \times t_{0.1}(5) - \mu) = 1000 \times (0.016 \times 1.475884 - 0.002) = 21.61414$

(4) $S_A = 500$, $S_B = 1000$ $R_A \sim N(0.01, 0.05)$ $R_B \sim N(0.005, 0.01)$

(a) when they are independent: $\mu = \sum_{i=1}^{2} w_i \mu_i = \frac{1}{3}\times 0.01 + \frac{2}{3} \times 0.005 = 0.0066667$

$\sigma^2 = \sum_{i=1}^{2}\sum_{j=1}^{2} w_i w_j \sigma_{ij} = \frac{1}{3}\times\frac{1}{3}\times(0.05)^2 + \frac{2}{3}\times\frac{2}{3}\times(0.01)^2 = 0.00032222 \Rightarrow \sigma = 0.01795$

$VaR(0.05) = S\times(\sigma \times Z_{0.05} - \mu) = (500 + 1000) \times (0.01795 \times 1.644854 - 0.006667)$

$= 34.28904$

(b) when the correlation is 0.3 $\Rightarrow \rho_{xy} = \frac{Cov(X,Y)}{\sqrt{Var(X)}\sqrt{Var(Y)}} \Rightarrow Cov(A,B) = 0.3 \times 0.05 \times 0.01 = 0.00015$

$\mu = \frac{1}{3}\times 0.01 + \frac{2}{3}\times 0.005 = 0.006667$

$\sigma^2 = (\frac{1}{3})^2\times(0.05)^2 + (\frac{2}{3})^2(0.01)^2 + 2\times(\frac{1}{3})\times(\frac{2}{3})\times(0.00015) = 0.00038889 \Rightarrow \sigma = 0.0197027$

$VaR(0.05) = S\times(\sigma \times Z_{0.05} - \mu) = (500 + 1000) \times (0.0197027 \times 1.64485 - 0.006667)$

$= 38.61543$

(c) when the correlation is −0.3 $\Rightarrow Cov(A,B) = -0.3 \times 0.05 \times 0.01 = -0.00015$

$\mu = 0.006667$ $\sigma^2 = (\frac{1}{3})^2\times(0.05)^2 + (\frac{2}{3})^2\times(0.01)^2 - 2\times(\frac{1}{3})\times(\frac{2}{3})\times(0.00015) = 0.00025556$

$\sigma = 0.01598611$

$VaR(0.05) = S\times(\sigma \times Z_{0.05} - \mu) = (500 + 1000) \times (0.01598611 \times 1.64485 - 0.006667)$

$= 29.4422$

5/(a) $F(x) = \int_{-\infty}^{-1} f(x)\,dx + \int_{-1}^{+\infty} f(x)\,dx = \int_{-\infty}^{-1} -\frac{1}{z}\frac{x+1}{(x^2+1)^2}dx + \int_{-1}^{+\infty}\frac{1}{z}\frac{x+1}{(x^2+1)^2}dx$

$= -\frac{1}{z}\int_{-\infty}^{-1}\left(\frac{x}{x^2+1} + \frac{1}{x^2+1}\right)dx + \frac{1}{z}\int_{-1}^{+\infty}\left(\frac{x}{(x^2+1)^2} + \frac{1}{(x^2+1)^2}\right)dx$

$= \frac{1}{2z}\left[\frac{1}{x^2+1}\right]_{+\infty}^{-1} - \frac{1}{2z}\left[\arctan(x) + \frac{x}{1+x^2}\right]_{-\infty}^{-1} - \frac{1}{2z}\left[\frac{1}{x^2+1}\right]_{-1}^{+\infty} + \frac{1}{2z}\left[\arctan(x) + \frac{x}{1+x^2}\right]_{-1}^{+\infty}$

$= \frac{1}{2z}\cdot\frac{1}{2} - \frac{1}{2z}\left(\frac{\pi}{4} - \frac{1}{2}\right) - \frac{1}{2z}\left(-\frac{1}{2}\right) + \frac{1}{2z}\left(\frac{3\pi}{4} + \frac{1}{2}\right) = 1$

$\therefore\ z = 1 + \frac{\pi}{4}$

(b) $\int_a^{\infty}\frac{1}{1+\pi/4}\frac{x+1}{(x^2+1)^2}dx = 0.05$

$\frac{1}{2}\left[-\frac{1}{x^2+1} + \arctan(x) + \frac{x}{x^2+1}\right]_a^{\infty} = 0.05\left(1+\frac{\pi}{4}\right)$

$\frac{a-1}{a^2+1} + \arctan(a) - \frac{19\pi}{40} + \frac{1}{20} = 0$

$\therefore\ a = 2.463713$

$\therefore\ VaR(0.05) = 2.463713$

6/(a) Let $L \sim N(0,1)$ Let ϕ be the pdf and Φ be the cdf

$ES_\alpha(L) = \frac{1}{1-\alpha}\int_{\Phi^{-1}(\alpha)}^{\infty} L\,d\Phi(L) = \frac{1}{1-\alpha}\int_{\Phi^{-1}(\alpha)}^{\infty} L\,\phi(L)\,dL$

$= \frac{1}{1-\alpha}\int_{\Phi^{-1}(\alpha)}^{\infty} L\frac{1}{\sqrt{2\pi}}e^{-L^2/2}\,dL$

$= \frac{1}{1-\alpha}\left[-\frac{1}{\sqrt{2\pi}}e^{-L^2/2}\right]_{\Phi^{-1}(\alpha)}^{\infty} = \frac{1}{1-\alpha}\left[-\phi(L)\right]_{\Phi^{-1}(\alpha)}^{\infty} = \frac{\phi(\Phi^{-1}(\alpha))}{1-\alpha}.$

Let $L' \sim N(\mu, \sigma^2)$

$ES_\alpha(L') = E(L' \mid L' > VaR_\alpha(L')) = E(\mu+\sigma L \mid \mu+\sigma L \geq VaR_\alpha(\mu+\sigma L))$

$= E(\mu+\sigma L \mid L \geq VaR_\alpha(L)) = \mu + \sigma ES_\alpha(L)$

$= \mu + \sigma\frac{\phi(\Phi^{-1}(\alpha))}{1-\alpha}$

(b) $ES(0.05) = 100000 \times \left(-0.04 + \frac{0.18\times\phi(z_{0.05})}{0.05}\right) = 33128.83$

(c) $ES(0.05) = (5000+5000)\times\left(\left(\frac{1}{2}\right)\times 0.04 + \left(\frac{1}{2}\right)\times 0.04 + \left(\frac{1}{2}\right)^2\times 0.18^2\times 2 + 0.18^2\times\left(\frac{1}{2}\right)^2\times 0.2\right)$

$= 24759.87.$