CHAPTER 5

Examining Individual Change with Repeated Measures Data

In our previous chapters, the outcome and explanatory variables were only measured on one occasion. One of the limitations of such cross-sectional analyses is that they are not well suited to studying processes that are assumed to be developmental. Because data are collected only at one point in time, they are insufficient to examine possible temporal relationships in a theoretical model. Time is a key factor in understanding how developmental processes unfold. When an outcome is measured several times for an individual, we have a repeated measures design (RMD). A simple example is the pretest and posttest design. For studying developmental processes, however, adding measurement occasions between the pretest and posttest can provide a more thorough examination and often can increase the power of the statistical test used to determine whether a change has taken place (Hox, 2010; Willett, 1989).

Ways to Examine Repeated Observations on Individuals

In the past, analyses of repeated observations on individuals were typically conducted using univariate analysis of variance (ANOVA) or multivariate analysis of variance (MANOVA), depending on the goals of the research and the specific features of the data. There are, however, several limitations of both approaches that arise from assumptions underlying their appropriate use in longitudinal research (e.g., see Hox, 2010; Raudenbush & Bryk, 2002). One primary shortcoming is that both approaches are limited to a subset of research situations involving change within and between individuals, where the timing of the repeated measurements is equidistant, subjects are independently and randomly sampled, and the change is considered as a fixed, rather than randomly varying, parameter between individuals. Most importantly for our purposes, analyses using either approach cannot be extended to include higher level groups such as classrooms or schools since this violates the assumption of sampling independence.

A second limitation concerns the normality of the data. For the univariate ANOVA approach, because differences in means of an outcome are tested over levels of a within-subjects factor (*time*), we must restrict the nature of the residual variances between the repeated measures as well as possible covariances between them (referred to as *sphericity*). The sphericity assumption implies that the within-subject model consists of independent (orthogonal) components. One type of sphericity is *compound symmetry*, which requires the repeated measures variances to be equal and any covariances between them to be the same. Where the assumption is not met, researchers can either use a degrees of freedom (*df*) correction to the *F* tests for hypothesis testing (to guard against Type I errors), or they often use the multivariate (MANOVA) approach since it does not require the sphericity assumption. Repeated measures ANOVA (RM-ANOVA) also requires homogeneity of variance for different levels of between-subjects factors such as treatment and control groups, while MANOVA requires the related assumption of homogeneity of the variance-covariance matrices for dependent variables across between-subjects factors.

A third limitation is the inability to include individuals with partial data in the analysis in either approach. Any individual with missing data on any occasion is eliminated from the analysis through listwise deletion. This can result in a tremendous loss of information about individuals within a longitudinal analysis. Therefore, we suggest having little or no missing data if either approach is to be used. Provided their basic assumptions are met, however, ANOVA and MANOVA remain viable approaches for examining repeated measures data.

Increasingly, however, both the concepts and methods are becoming available that can provide a more rigorous and thorough examination of repeated measures data. One well-known approach, which draws on structural-equation-modeling (SEM) methods, is latent change (or curve) analysis (LCA). In the LCA approach to examining growth, the repeated measures of Y are defined in a covariance matrix, which accommodates a single-level confirmatory factor analysis (CFA); that is, the repeated measures are treated as observed variables that define latent intercept and growth factors. In this way, a time dimension is incorporated into the specification of the latent variables. The SEM specification of individual latent change amounts to a multivariate specification at a single level. Change involving individuals within groups can then be accommodated as a two-level CFA (Muthén & Muthén, 1998–2006).

Repeated measures data with within-subjects and between-subjects factors can also be specified as another type of linear mixed (or random coefficients) model (Laird & Ware, 1982). In this latter approach, change involving individuals can be conceptualized as a two-level analysis, with the repeated measures of Y and the change in their levels over time specified at Level 1 and random variation in the individual intercepts and growth rates examined at Level 2. Differences between individuals (e.g., background or an experimental treatment) can be proposed to explain random variation in individuals' growth parameters at Level 2. Moreover, in the mixed model, the repeated measures can be observed at fixed or varying occasions, and the approach can incorporate missing data on *some* occasions. This can be beneficial if there are subjects who drop out during a longitudinal study. The mixed model can also accommodate time-varying covariates at Level 1. Using a multilevel framework, univariate analyses of individual growth can easily be extended to include more than one growth process or differences in growth due to successive groupings such as classrooms (Level 3) and schools (Level 4). We note, however, that the complexity of adding more analytic levels can begin to challenge available computer memory needed to estimate a proposed model's parameters.

Models with time-ordered relationships definitely offer increased possibilities for studying various types of individual change processes using either the multivariate SEM or univariate two-level mixed model approaches. They encourage researchers to ask a number of different questions of the data: Is there a change in the level of the means over time? If so, what is the shape of the change trajectory? Is the change the same for different groups of individuals?

Considerations in Specifying a Linear Mixed Model

There are a number of considerations to keep in mind in specifying a repeated measures analysis using a linear mixed-modeling approach. First, we should decide whether we are examining one or more growth processes. For our presentation in this chapter, we will use a univariate approach to examine our proposed research questions; that is, we assume there is only one outcome being examined over time. In Chapter 7, we consider a multivariate (e.g., parallel) growth model—one that facilitates the examination of individual changes in reading and math achievement simultaneously. Our primary hypothesis in a univariate RMD concerns whether or not there is a difference in the levels of the means of the dependent variable over time (Raykov & Marcoulides, 2008). If we are able to reject the null hypothesis that the means are the same, then we can assume that some change has taken place.

For investigating individual development in a single outcome, we can test this assumption of equal means using RM-ANOVA; however, as we suggested earlier, we must assume *sphericity*. The sphericity assumption refers to the structure of the repeated measures covariance matrix and stems from the assumption that the repeated observations should be independent and therefore have constant variance and ideally be uncorrelated with each other. This is an important assumption underlying the RM-ANOVA approach (which can be tested using Mauchly's sphericity test) since it affects the calculation of significance levels of the related *F* tests for proposed hypotheses. Mauchly's test provides a test of whether the repeated measures used to define within-subjects growth are represented by a spherical covariance matrix. The test, however, is highly sensitive to even mild departures from the required covariance structure. If sphericity is not met (which is generally the case with real data), there are other options including adjusting the *F* tests in RM-ANOVA for purposes of hypothesis testing or using the multivariate approach. Another alternative is the mixed model approach. A clear advantage of this latter approach is that it provides greater flexibility in identifying a suitable Level 1 covariance structure that captures the nature of the observed relationships between the repeated measures of *Y* in varied longitudinal studies, regardless of time-trend patterns within individuals and among the groups being compared.

Second, after determining an approach to use, we need to consider the expected within-subjects (Level 1) effect for *time*, which concerns individuals' possible change over the *temporal* period of the study. The time effect describes whether individuals are indeed changing over some relevant *interval* of time and by *how much*. This part of the within-subjects model represents the change we would anticipate that each individual would experience over the course of the study (Singer & Willett, 2003). For example, the relevant time interval may be days, weeks, months, or years. In a mixed model, the potential difference in means across measurement occasions can be summarized as a time-related slope. The time slope is generally the most important parameter in the model because it provides a test of whether the outcome means are equal across occasions, as well as information about the *rate* at which individual *i* changes over a particular time interval *t*.

If we find that the means of *Y* are not the same across time, we next would want to investigate further how individuals are changing over time. The analysis of repeated measures facilitates the representation of several different growth trajectories. These can concern, for example, a naturally occurring developmental trend (e.g., students' acquisition of vocabulary between 12 and 36 months of age) or perhaps individuals' reading skills before and after a classroom treatment is introduced. When a categorical variable represents an increasing level of a treatment dosage, age, or a series of successive measurements, the growth trend is often summarized using polynomial curves within individuals (Level 1) because they are very adaptable to a variety of different growth trajectories and can be estimated using standard linear-modeling procedures (Hox, 2010). Most common is a linear growth trend, which assumes that the rate of individual change is the same across each interval of time (a). However, it is not always the case that individuals are changing at the same rate over time. In some cases, adding higher order polynomials to the model for defining the time-related trend can improve prediction.

The polynomial equation is not nonlinear; in fact, mathematically it is linear (Hox, 2010). If there are k measurements, there is a $k - 1$ polynomial that can be used to fit the model. A quadratic trend is interpreted as a change in the rate of change (i.e., accelerating or decelerating) over an interval of time (a^2). A cubic trend (a^3) is *S*-shaped, which describes changes in the rate of change over time such as accelerating, decelerating, and accelerating again. For example, for three measurement occasions, the highest degree term is a^2 (quadratic), while for four measurement occasions, the highest degree term is cubic, or one less than the number of measurement occasions ($k - 1$). Of course, we might prefer to interpret the results of a linear growth model for ease of interpretation, but sometimes a higher order polynomial may describe the data more accurately. Generally, we only include up to the highest significant polynomial in the model. In terms of actually describing the change over time, it becomes increasingly more difficult to interpret models of higher polynomial degrees. There also may be cases where developmental processes cannot be well approximated by using a polynomial function. For example, the logistic curve (which cannot be transformed to a linear model) may be better in approximating a process that is slower in the beginning of the trend, speeds up in the middle, and then slows at the end (representing an *S* curve). In other cases, however, a logistic or exponential function may be well approximated by using a cubic polynomial if there are at least four measurements (Hox, 2010).

It is important to note that the parameters of a higher order polynomial model (e.g., cubic) have no direct interpretation in the growth process over any particular interval, such that interpretation must be made by looking at average plots of the growth or some more typical individual growth curves (Hox, 2010). To illustrate this point, in Figure 5.1 we have plotted the actual growth in an academic outcome *Y* (solid line) over four intervals against what the growth might look like if it were strictly linear, quadratic, or cubic (dotted lines). At the bottom of the figure, we can observe that linear growth rises (or declines) constantly over time. In contrast, quadratic growth tends to rise more quickly (or decline) relative to the constant rate of growth represented in the linear trajectory. We can also notice that the cubic growth curve increases much more rapidly over the four time periods compared to linear or quadratic growth curves. The actual growth trajectory plotted in the figure for *Y* appears to have elements of linear, quadratic, and cubic growth. It appears to change linearly between intervals 1 and 2, to slow between intervals 2 and 3 (quadratic trajectory), and then to accelerate more rapidly between intervals 3 and 4 than between intervals 1 and 2. The cubic polynomial actually combines the effects of four coefficients (i.e., linear, quadratic, and cubic slope coefficients, and the intercept). Different values of each coefficient will move the average trajectory up or down with respect to the horizontal axis (i.e., changing the intercept) and alter the steepness of the cubic curve (changing the cubic coefficient), its slope (changing the linear component), or the curvature of the parabolic element (changing the quadratic coefficient). As we reiterate, when a cubic element is present in a trajectory, it suggests a focus on the whole growth trend and not just any particular interval.

Third, after settling on a reasonable growth trajectory to describe individual development, we can consider possible between-subjects variables that might affect individuals' growth trajectories. A third hypothesis often tested in a RMD is a test of parallelism (Raykov & Marcoulides, 2008); that is, are the trajectories the same for different levels of a factor (e.g., subjects in treatment or control groups)? In repeated measures multilevel designs, it is also important to distinguish within-subjects (Level 1) variables, which change over time, from between-subjects (Level 2) variables, which are considered to be static over the temporal period of the study. For

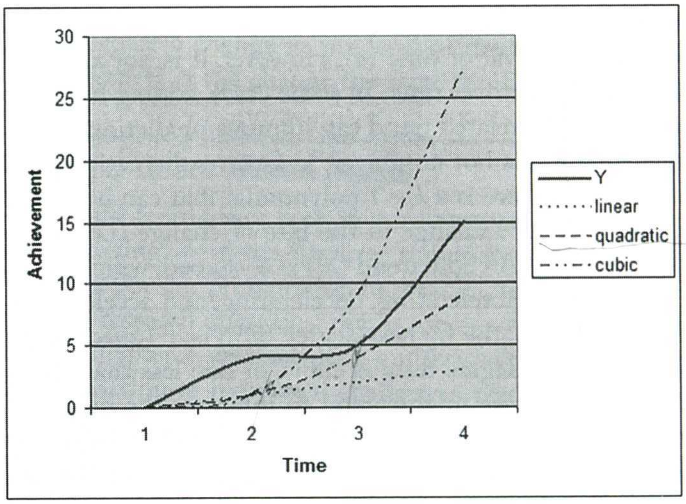


FIGURE 5.1 Examining several different individual growth curves.

example, we can enter motivation as a time-varying covariate at Level 1 that predicts changes in reading scores. In contrast, we could also enter motivation as a between-subjects covariate (i.e., continuous predictor), in which case it would be considered as static over the temporal period of the study. Covariates, which are continuous variables that might affect the rate of growth, can also be added to the model. When covariates are added, the means for each occasion are adjusted for the presence of the covariates.

An Example Study

Consider study where we wish to examine students' growth trajectories in math achievement over time (a within-subjects factor with three levels) and to assess whether their socioeconomic status (SES) background (a between-subjects covariate) and perceptions about their math teacher's effectiveness (a between-subjects factor) are related to different achievement growth patterns.

Research Questions

We may be interested first in whether a change takes place in student math achievement over time. This type of question addresses whether the levels of the means for the outcome are the same or different over the occasions of measurement. The assumption is that if we can reject the null hypothesis of no difference in means across measurement occasions, it implies that a change in individuals has taken place. A second question concerns what the shape of the developmental change might look like for individuals in the study. For example, we might ask whether the rate of individual change per occasion is linear or whether the change might be more complex. Once we have described the shape of individuals' change trajectories over time, we might ask a third question: Are there differences in development between groups of individuals? These differences might be due to an experimental treatment or other types of factors (e.g., background). In this case, we examine whether student growth is related to their individual perceptions about the teaching skill of their teachers. We might also wish to adjust our estimates for the presence of covariates that might affect individuals' development over time. In this example, we use students' SES as a covariate.

The Data

The data used for this study consist of 8,670 secondary students. We will assume that they have been randomly sampled from a larger population of students. The variables used in the example are summarized in Table 5.1.

TABLE 5.1 Data Definition of *ch5growthdata-vertical.sav* (N = 8,670)

Variable	Level[a]	Description	Values	Measurement[1]
id	Individual	Individual student identifier (8,670 students) across three time (test) occasions.	Integer	Ordinal
nschcode	School	School identifier (515 schools).	Integer	Ordinal
Rid	Individual	A within-group level identifier[b] representing a sequential identifier for each student within each school.	1 to 46	Ordinal
Index1	Within Individual	Identifier variable resulting from indexing the math outcomes to create a new identifier with a number sequence (1, 2, 3) corresponding to the three time occasions measuring students' math achievement.	1 = First Time 2 = Second Time 3 = Third Time	Scale

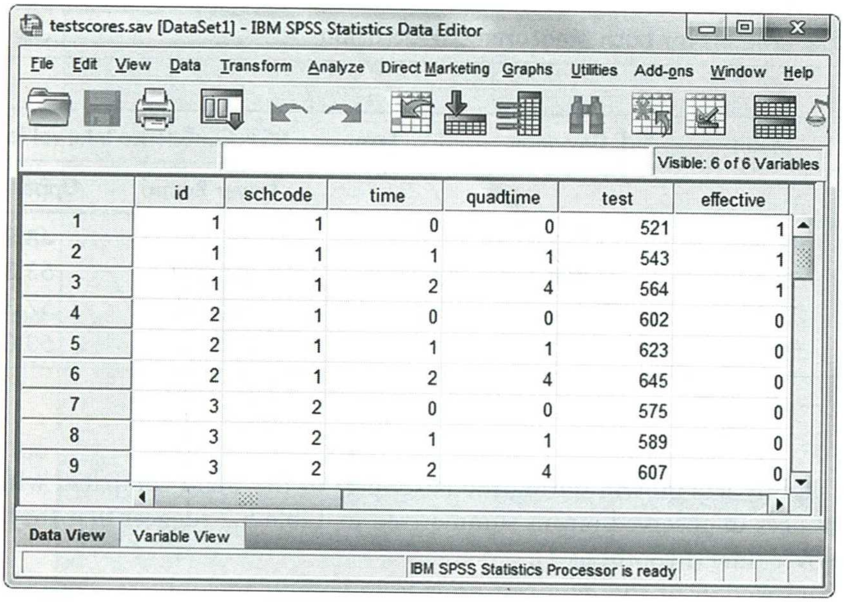
(Continued)

TABLE 5.1 *(Continued)*

Variable	Level[a]	Description	Values	Measurement[1]
time	Within Individual	Variable representing three linear occasions in time measuring students math achievement.	0 = First Time 1 = Second Time 2 = Third Time	Scale
quadtime	Within Individual	Recoded *time* variable from three occasions in time (0, 1, 2) into a "squared" quadratic sequence (0, 1, 4) to capture any changes (acceleration or deceleration) in the rate of change that might occur over the three measurement occasions.	0 = First Time 1 = Second Time 4 = Third Time	Scale
orthtime	Within Individual	Recoded *time* variable from three occasions in time (0, 1, 2) into an orthogonal linear (–1, 0, 1) sequence.	–1 = First Time 0 = Second Time 1 = Third Time	Scale
orthquad	Within Individual	Recoded *time* variable from three occasions in time (0, 1, 2) into an orthogonal quadratic (1, –2, 1) sequence.	1 = First Time –2 = Second Time 1 = Third Time	Scale
test	Within Individual	The dependent variable representing each students' individual scores on the repeated math measurements.	24.35 to 99.99	Scale
effective	Individual	Two-category predictor variable representing teachers' effectiveness in teaching math.	0 = Not Effective 1 = Effective	Nominal
ses	Individual	Predictor interval variable (*z* score) measuring student socioeconomic status composition within the schools.	–2.41 to 1.87	Scale
timenonlin1	Within Individual	Recoded *time* variable from three occasions in time (0, 1, 2) into a time sequence variation that encompasses the whole 3-year period of time (0.00, 0.50, 1.00) measured from 0 to 1.	0.00 = First Time 0.50 = Second Time 1.00 = Third Time	Scale
timenonlin2	Within Individual	Recoded *time* variable from three occasions in time (0, 1, 2) into a time sequence variation representing the whole 3-year period of time (0.00, 0.60, 1.00) measured from 0 to 1.	0.00 = First Time 0.60 = Second Time 1.00 = Third Time	Scale
timenonlin3	Within Individual	Recoded *time* variable from three occasions in time (0, 1, 2) into a time sequence variation representing the whole 3-year period of time (0.00, 0.70, 1.00) measured from 0 to 1.	0.00 = First Time 0.70 = Second Time 1.00 = Third Time	Scale
timenonlin	Within Individual	Recoded *time* variable from three occasions in time (0, 1, 2) into a time sequence variation representing the whole 3-year period of time (0.00, 0.53, 1.00) measured from 0 to 1.	0.00 = First Time 0.53 = Second Time 1.00 = Third Time	Scale

[a] Individual = Level 1; school = Level 2; Within Individual = repeated measures, Level 1; Individual = Level 2; School = Level 3.
[b] Results from ranking student cases (*id*) with the school group identifier (*nschcode*).
[1] Measurement icon settings displayed in subsequent model screenshots may differ from Table 5.1 but will not affect the output.

Using a random-coefficients approach to investigate individual change provides considerably more flexibility than either the univariate ANOVA approach or the multivariate approach, especially in situations where there may be missing data, varying occasions of measurement, and more complex error structures. It is generally useful to spend some time examining the nature of the data initially. To examine individual change using MIXED, the data must first be organized differently. As we noted in Chapter 2, the time-related variable describing the shape of the growth trajectory (e.g., linear or quadratic) is entered into the data set as a variable, with the successive math measurements structured vertically or stacked (i.e., instead of as a horizontal, multivariate



	id	schcode	time	quadtime	test	effective
1	1	1	0	0	521	1
2	1	1	1	1	543	1
3	1	1	2	4	564	1
4	2	1	0	0	602	0
5	2	1	1	1	623	0
6	2	1	2	4	645	0
7	3	2	0	0	575	0
8	3	2	1	1	589	0
9	3	2	2	4	607	0

FIGURE 5.2 Vertical data matrix for repeated measures analysis in IBM SPSS.

set of variables as in MANOVA or SEM) for each subject within the data set $(y_{1i}, y_{2i}, \ldots, y_{ti})'$. As this suggests, the number of lines for each individual is defined by the number of measurement occasions. In this case, therefore, we will have 8,670 × 3, or 26,010 lines, in the database. We can use DATA and RESTRUCTURE menu commands to restructure the data vertically. Figure 5.2 presents data on three subjects in a hypothetical data set.

Closer inspection of the data suggests that there are three observations per individual on the math "test" outcome, and individual and school identifiers, as well as any predictors (e.g., perceived teacher effectiveness), are repeated in the data set for each time interval. The outcome (*test*) represents each individual's scores on the repeated math measurements. We can also see that the repeated observations of *test* are nested within individual identification (*id*) numbers, and student IDs are nested within school identifiers (*schcode*). The grouping variables (*id*, *schcode*) are used to identify each predictor as belonging to a particular level of the data hierarchy. As we noted, often a polynomial function will describe individual growth pretty well. As Figure 5.2 also indicates, the time-related variables (linear and quadratic) are also entered as data. Most often, researchers assume that individuals are changing at a constant rate over time, which can be represented as linear growth, especially over a short period of time, but it may also be the case that subjects are experiencing more complex patterns of growth over time. Since there are three occasions, we can also enter a quadratic component to test for the presence of a change in the rate of growth occurring over time.

Examining the Shape of Students' Growth Trajectories

We will begin by examining descriptive statistics showing the level of the outcome means on each occasion, as summarized in Table 5.2. The table suggests that the average math achievement for the first occasion is 48.632, while for the last occasion it is 57.094, indicating considerable change over time. The table suggests that the grand mean is 52.945, which falls somewhere between the first and second measurement occasions. The grand mean is often not of much interest in examining growth since it just represents the average achievement level across the three measurement occasions. Examining the means more closely, one can see that the change between the first two test means is about 4.5 points, while between the second two means it is about 4.0. This

TABLE 5.2 Means for Each Measurement Occasion

Test	Mean	Std. Deviation	Std. Error	95% Confidence Interval for Mean	
				Lower Bound	Upper Bound
0	48.632	9.713	0.104	48.428	48.837
1	53.107	9.888	0.106	52.899	53.316
2	57.094	9.894	0.106	56.886	57.303
Total	52.945	10.421	0.065	52.818	53.071

suggests slightly less growth during the latter part of the trend compared with the initial part. The differences in observed means summarized in Table 5.2 suggest that they probably are not the same over time (hypothesis 1).

Visual inspection of the data can provide important preliminary clues about the shape of change in math that is taking place among individuals over time. Figure 5.3 provides a plot of the linear growth trajectories of the first 17 subjects in the data set. The plot of these individuals' scores over time suggests that individuals are increasing in their knowledge. Readers will note that the intercepts (i.e., individuals' status at Time 0) appear to vary considerably (i.e., from about 32 to 55) and the steepness of the growth over time also seems to vary within this subset of individuals. Many times, with a few waves of data, a linear model will be adequate to describe individuals' growth. Notice, however, in the graph of these individuals' growth, the linear model does not seem to capture the change over time of all individuals equally well; that is, not all of the individuals' observed scores fall on their predicted growth lines.

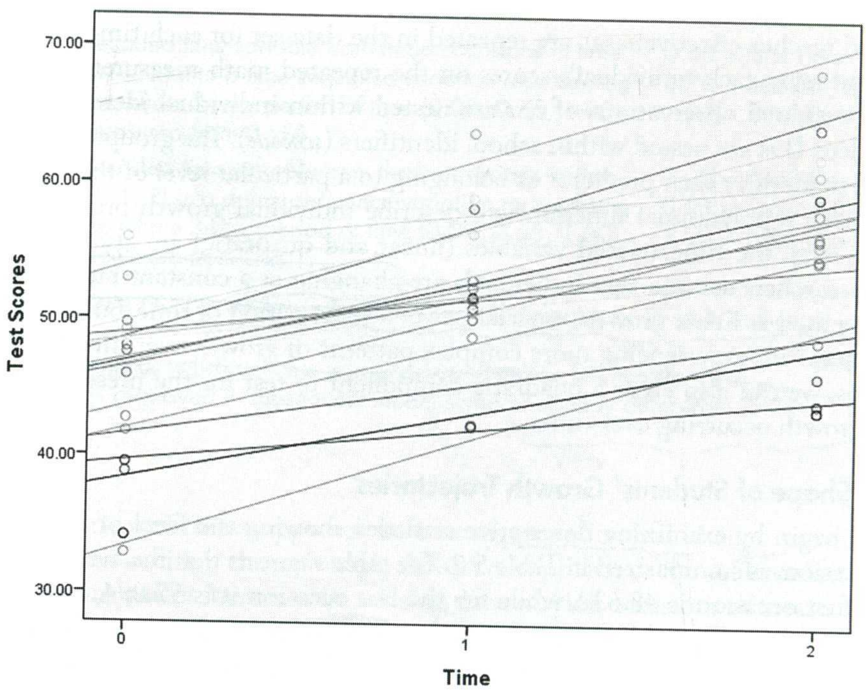


FIGURE 5.3 Individual linear math growth trajectories.

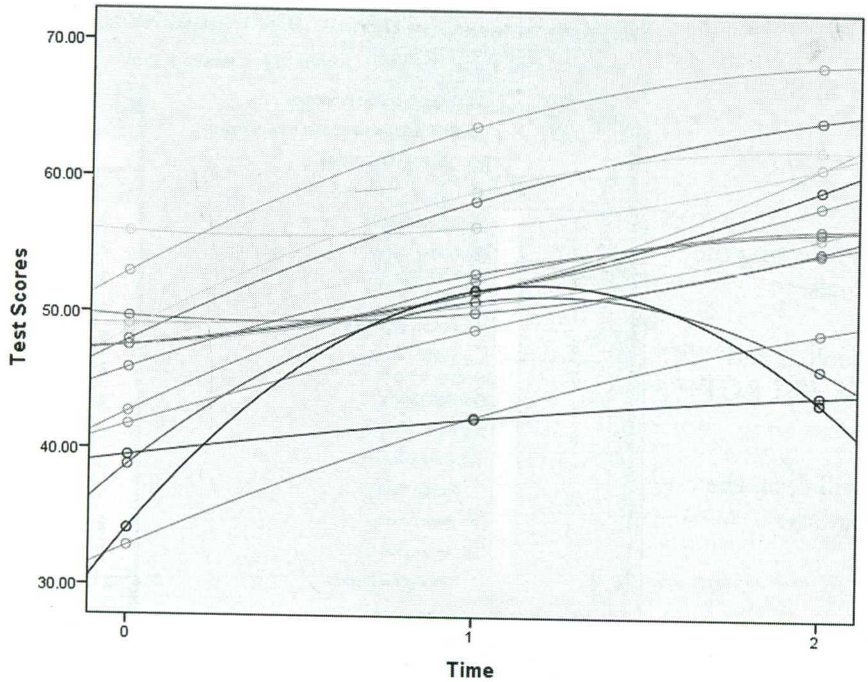


FIGURE 5.4 Individual nonlinear math growth trajectories.

For purposes of contrast, Figure 5.4 is a graph of the same 17 subjects, this time using a quadratic trajectory. With three time points, one can observe that the fit of the curved lines to the data points will be perfect. For some individuals, the plot of their trajectories in the figure suggests that a linear shape might be adequate to describe the growth. For others, however, it appears that their growth might be better described by a curvilinear trajectory. These plots show visually our preliminary interest in determining whether a linear shape, or both linear and quadratic components, would be required to describe the shape of individuals' growth trajectories accurately.

Graphing the Linear and Nonlinear Growth Trajectories with IBM SPSS Menu Commands

We can use the IBM SPSS menu commands to display the information for the subset of individuals in the study, as shown in Figures 5.2, 5.3, and 5.4 in the following series of instructions.

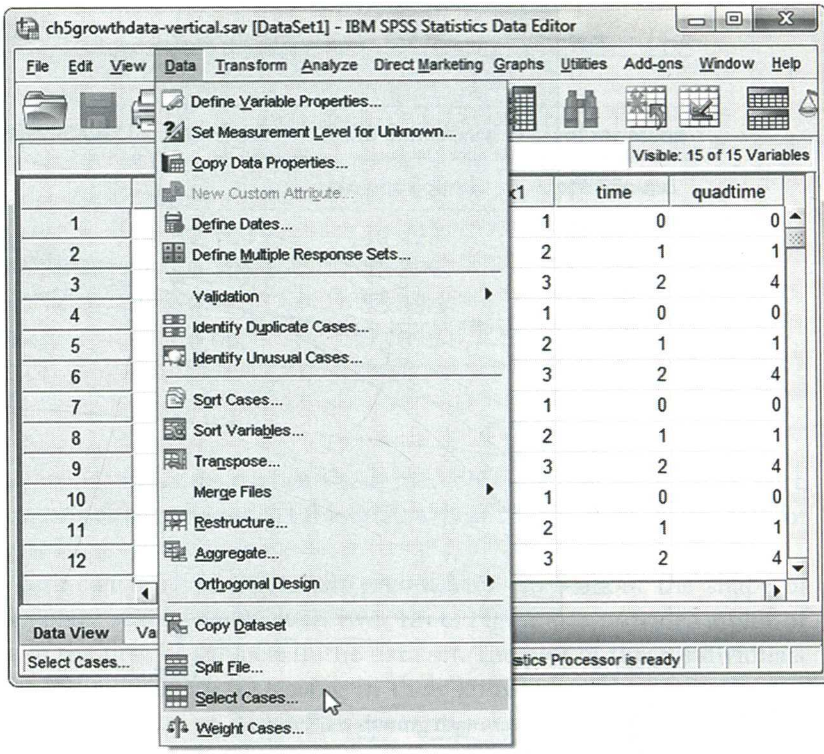
Select Subset of Individuals

Launch the IBM SPSS application and select the *ch5growthdata-vertical.sav* data file.

We will begin by selecting the subset of individuals.

1. Go to the toolbar and select DATA, SELECT CASES.

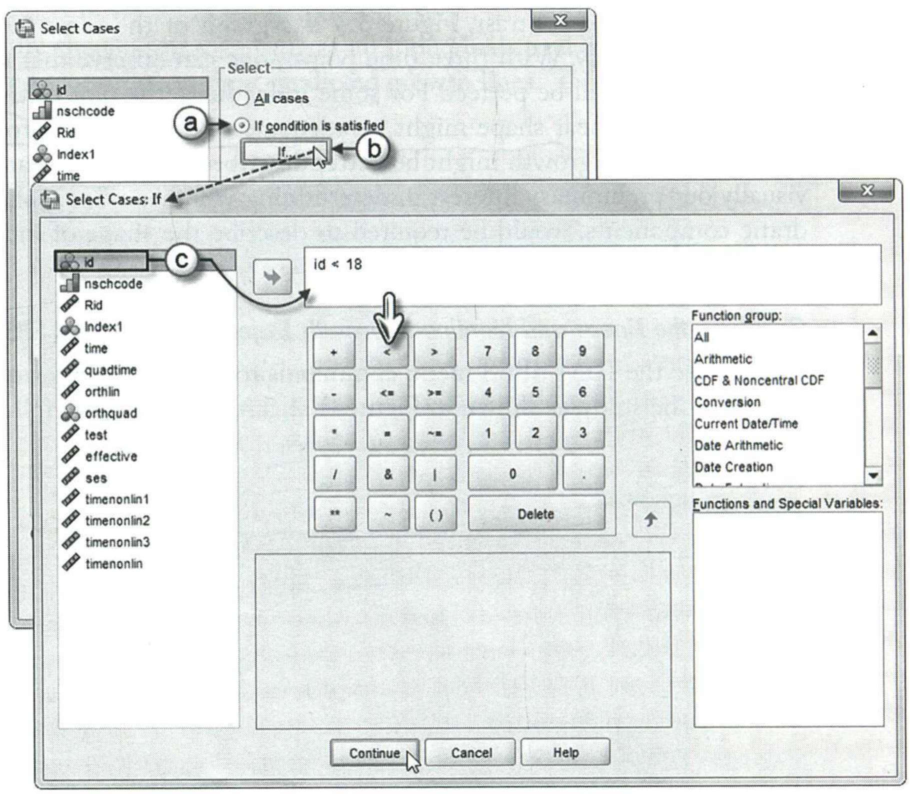
This command will open the *Select Cases* dialog box.



2a. Within the *Select Cases* dialog box, click to select *If condition is satisfied.*

b. Then click the IF button, which will activate the *Select Cases: If* box.

c. Click the variable *id* from the left column listing, and then click the right-arrow button to move *id* into the box.

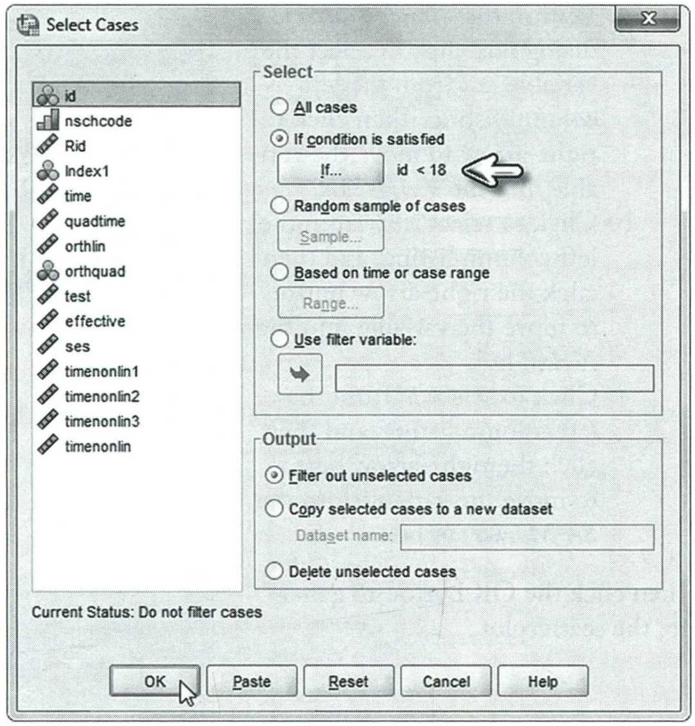


Use the keypad to enter the less than sign (<) followed by the number 18. The resulting command (*id < 18*) instructs IBM SPSS to select only the first 17 cases of the data set.

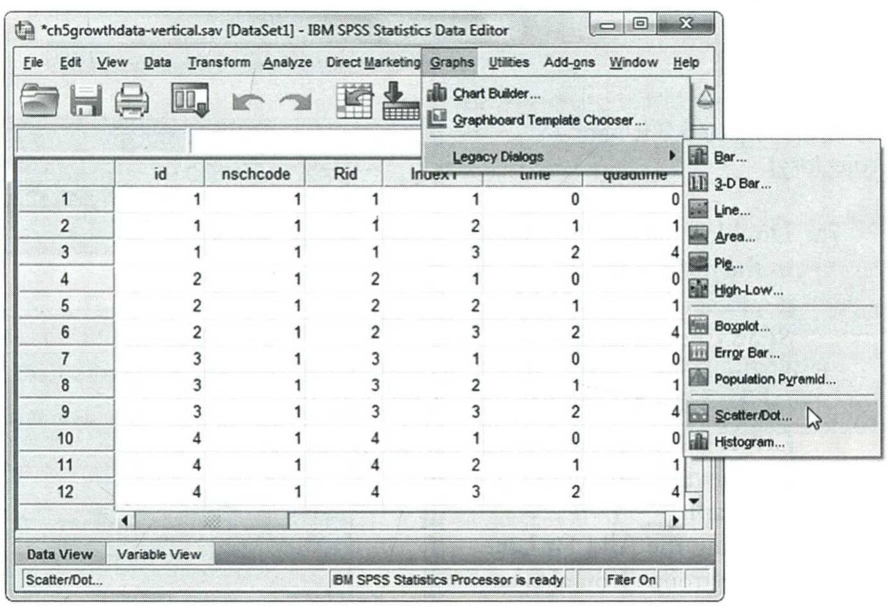
Click the CONTINUE button to return to the *Select Cases* dialog box.

3. Notice that the IF condition statement *id < 18* is listed.

Click the OK button to return to the main menu.

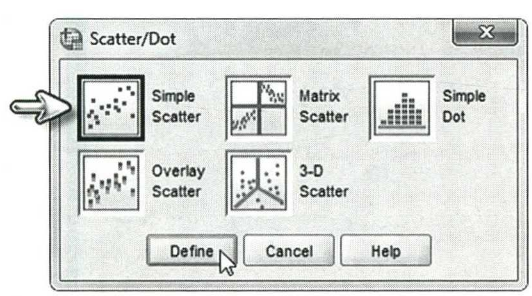


4. To graph the 17 cases, go to the toolbar and select GRAPHS, LEGACY DIALOGS, SCATTER/DOT.



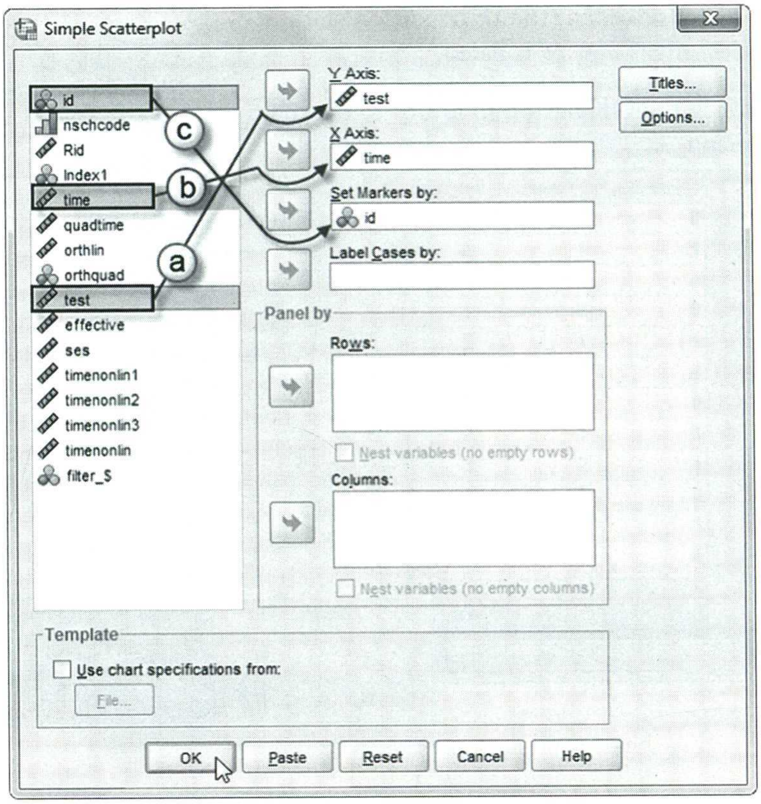
5. Click on the SIMPLE SCATTER icon to select this option among those shown.

Click DEFINE button to open the *Simple Scatterplot* dialog box.



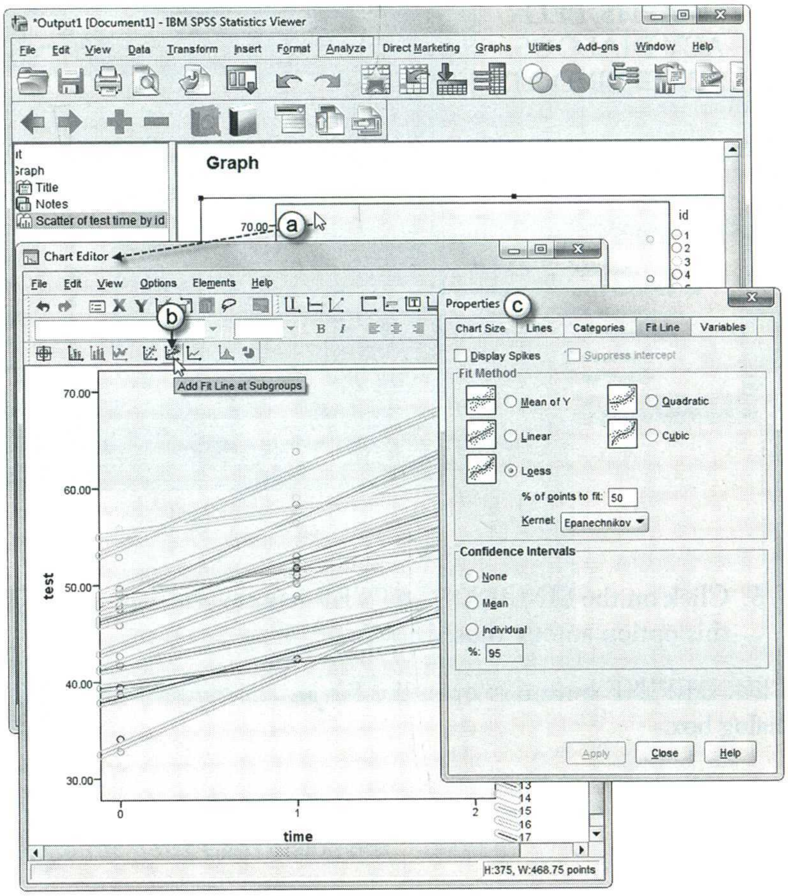
6a. Within the *Simple Scatterplot* dialog box click to select the variable *test* from the left column listing. Then click the right-arrow to move the variable into the *Y Axis* box.
b. Click to select *time* from the left column listing, and then click the right-arrow button to move the variable into the *X Axis* box.
c. Click to select *id* from the left column listing, and then click the right-arrow button to move the variable into the *Set Markers by* box.

Then click the OK button to generate the scatterplot.



Generate Figure 5.3 (Linear Trajectory)

7a. Double-click on the graph in the output to select it and activate the *IBM SPSS Chart Editor.*
b. In the *Chart Editor*, click on the icon ADD FIT LINE OF SUBGROUPS, which will insert lines on the graph.
c. Clicking the Add Fit Line of Subgroups icon also activates the *Properties* box, which provides assorted options.



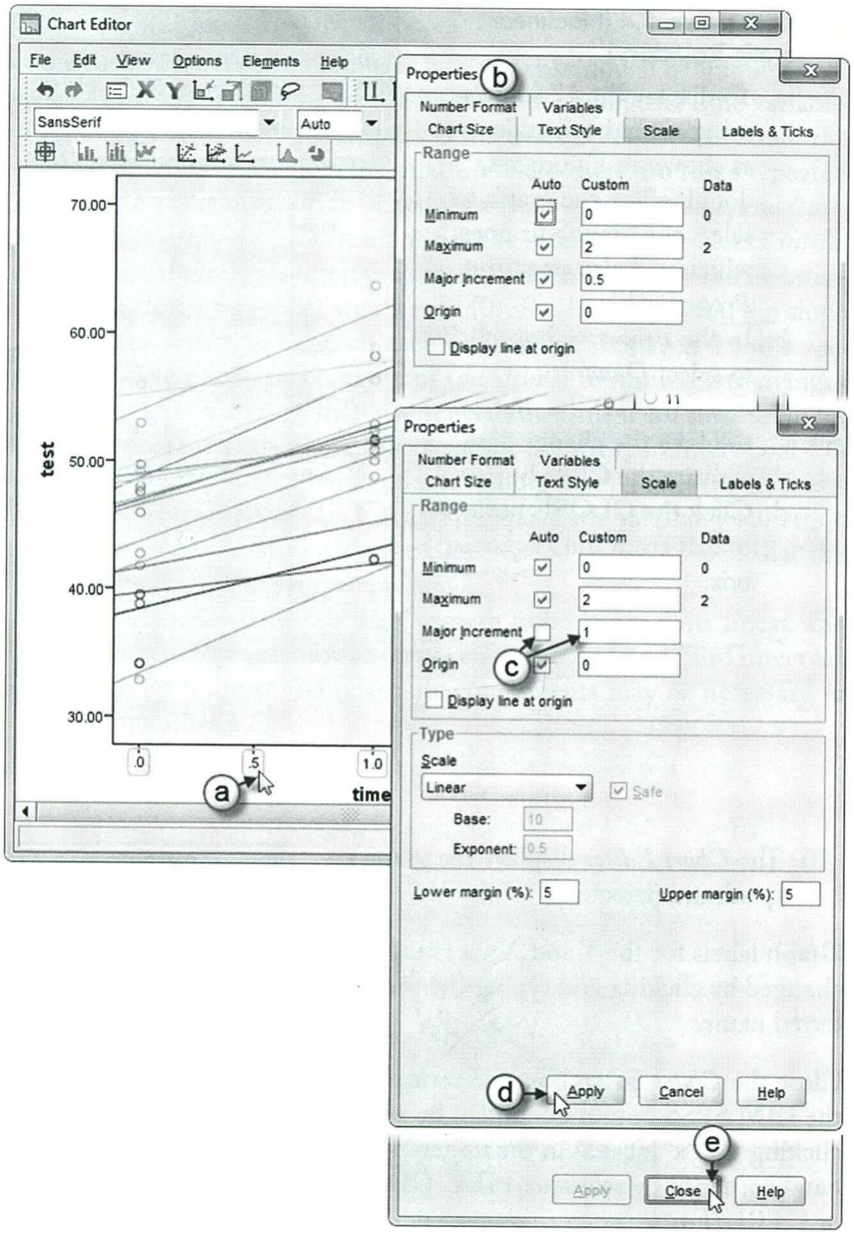
8a. IBM SPSS default settings insert 0.5 increments to the *X* axis representing time. To change the increment, double-click the number to access the *Properties* dialog box.

b. In Properties box, the *Scale* tab displays the *Range* options for changing the increments.

c. Change the *Major Increment* by unchecking the box and then replacing *0.5* with *1*.

d. Click the APPLY button to make the change and activate the Close button.

e. Click the CLOSE button to exit from the *Properties* box.



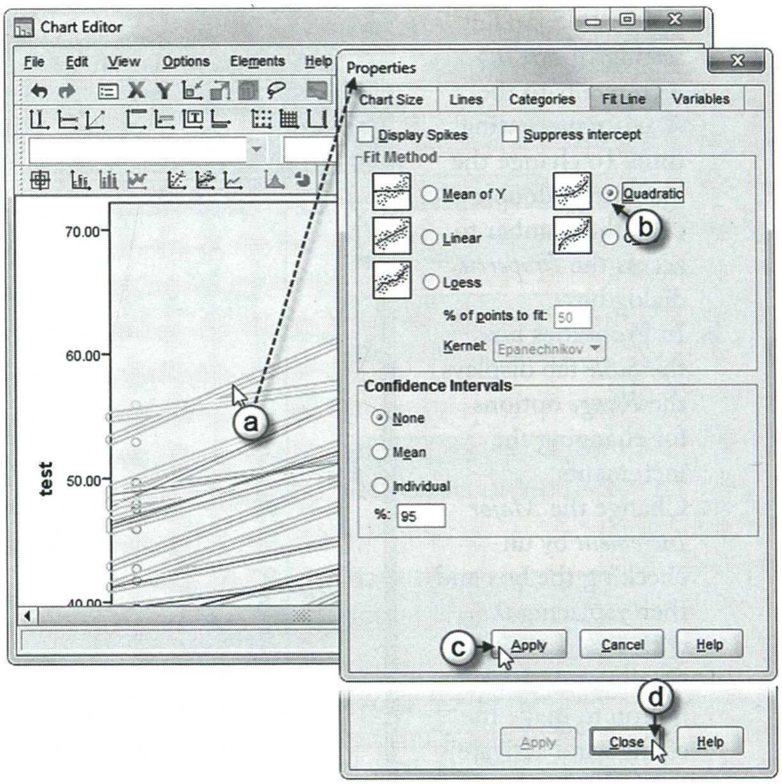
Generate Figure 5.4 (Nonlinear Quadratic Trajectory)

9a. To display the 17 subjects using a quadratic trajectory as shown in Figure 5.4, double-click the graph to select the graph's fit lines, which will also open the *Properties* box.

b. In the *Properties* box, click to select *Quadratic*.

c. Click the APPLY button to make the change and activate the Close button.

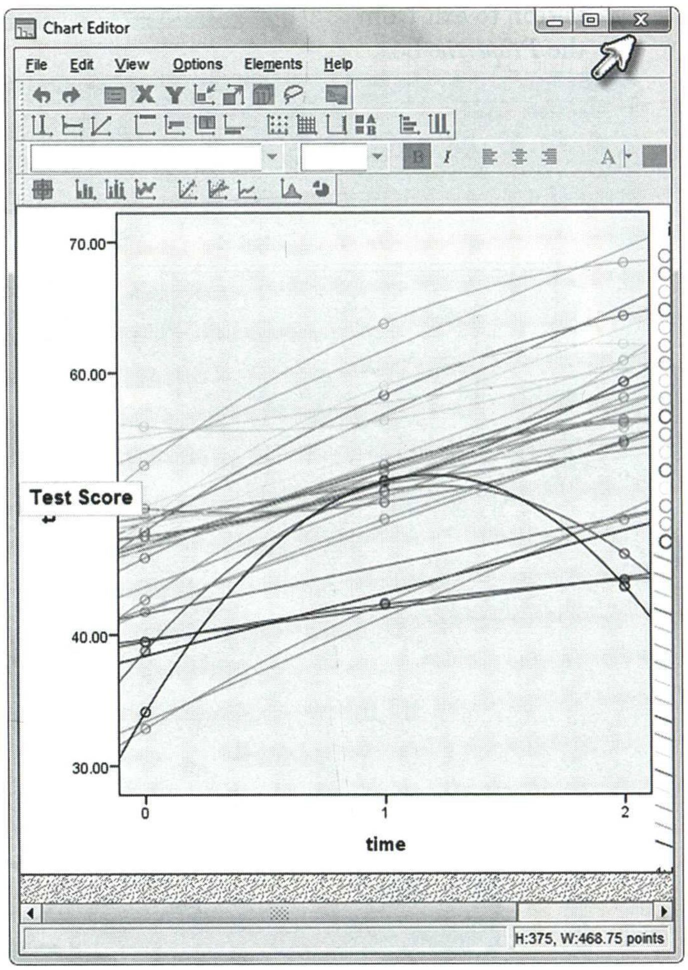
d. Click the CLOSE button to exit from the *Properties* box.



10. The *Chart Editor* displays the graph's quadratic trajectory.

Graph labels for the *Y* and *X* axes may be changed by clicking and typing the preferred name.

Close the *Chart Editor* box, and return to the IBM SPSS output document by either clicking the "x" located in the upper-right-hand corner or by selecting FILE, CLOSE or CTRL+F4.



Coding the Time-Related Variables

There are various ways that the time-related within-subjects factor may be coded in defining possible individual changes over time. Careful thought should be given to coding the time variable, as it can affect the interpretation of the model's parameters (Hox, 2010). We first illustrate the polynomial approach for describing individual growth in math. Where there are three repeated measures, the linear time variable (*time*) is most often coded 0 for year 1, 1 for year 2, and 2 for year 3 (0, 1, 2). This coding pattern is useful because it identifies the intercept as students' *initial* (*year 1*) math achievement level. This approach is often preferred since the intercept can be interpreted as the mean when the predictors in a model are all zero (0). For linear growth, the slope would then be defined as the change occurring between each interval (i.e., between 0 and 1 and between 1 and 2). We can also define a quadratic component (*quadtime*) to capture any changes (acceleration or deceleration) in the rate of change that might occur over the three measurement occasions. We simply "square" the *time* intervals; therefore, *quadtime* is correspondingly coded 0, 1, and 4. If instead we wished to define the intercept as students' ending math achievement status (year 3), we could instead code the linear variable −2, −1, 0, and the quadratic variable would then be −4, −1, 0. In some situations, we might wish to code the time-related variables such that the second measurement (i.e., year 2) represents the intercept.

We can see in Figure 5.5 that the shape of the "average" growth trend is not quite linear. The average trend suggests a slight slowing of the growth rate between the second and third intervals. This suggests that a trajectory with both linear and quadratic components may be necessary to define student growth optimally (hypothesis 2).

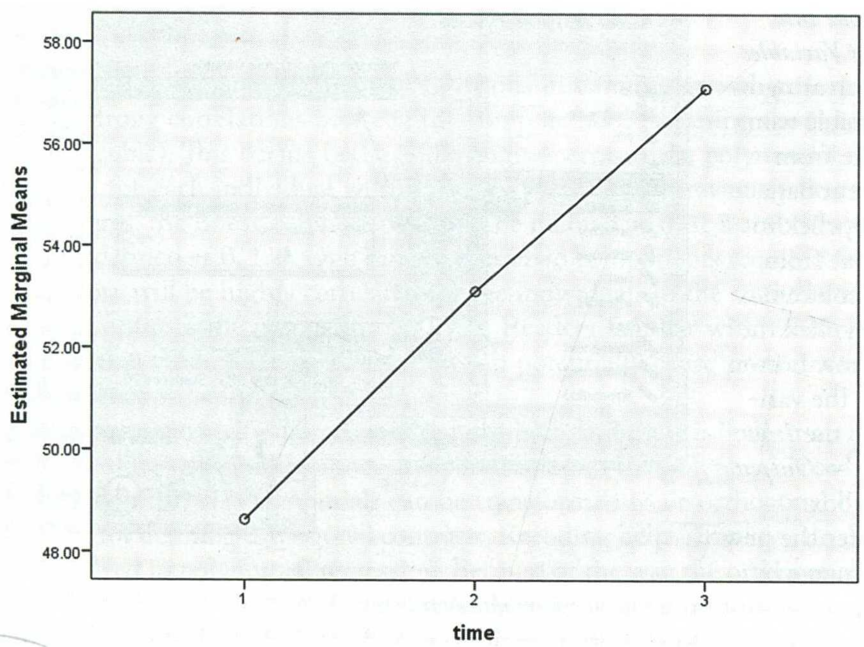
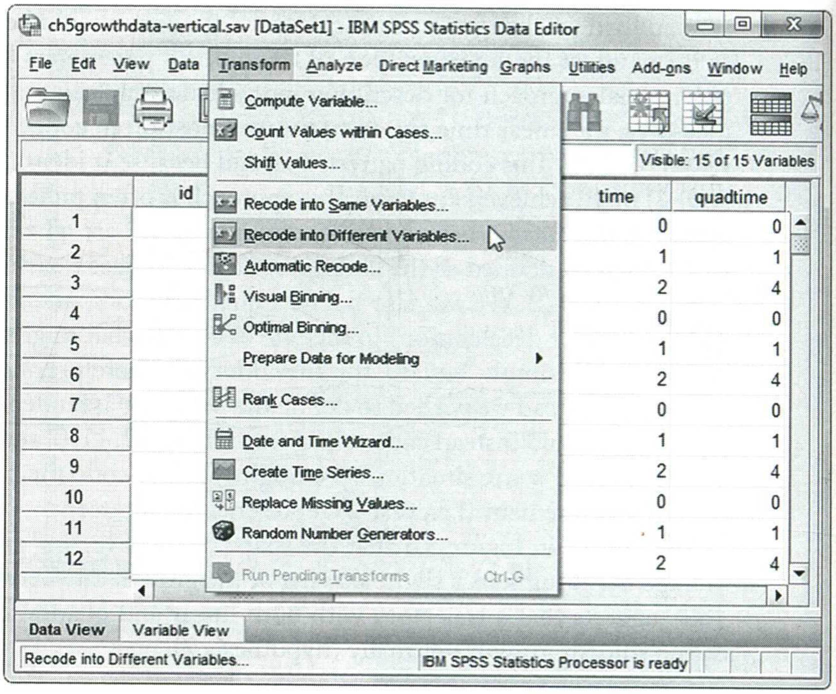


FIGURE 5.5 Curvilinear average math growth trend.

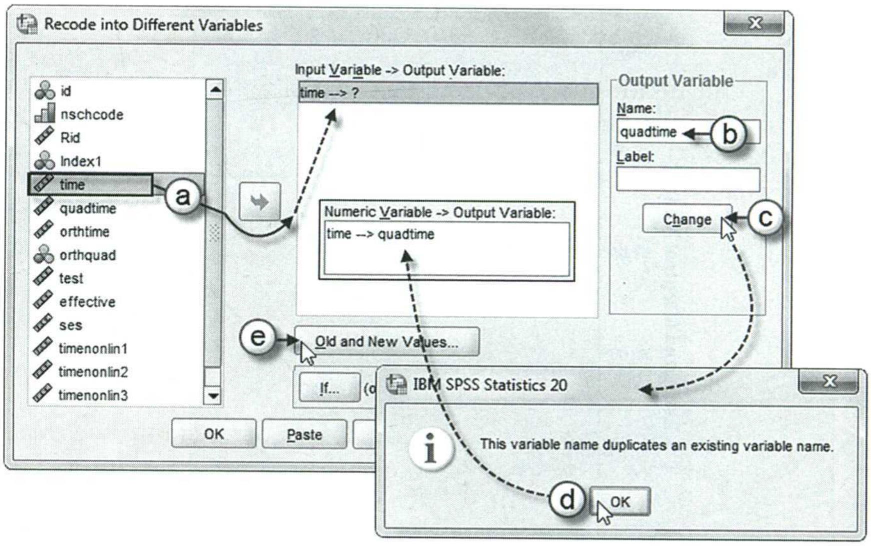
Coding Time Interval Variables (time to quadtime) with IBM SPSS Menu Commands

Note: Continue using *ch5growthdata-vertical.sav*. If continuing from the prior set of graphing instructions (Figures 5.2, 5.3, and 5.4), remove the *Select Cases* conditional setting before proceeding, as the following models will use all cases in the data set. To clear the filter, go to the toolbar, select DATA, SELECT CASES, RESET, OK.



1. Go to the toolbar and select TRANSFORM, RECODE INTO DIFFERENT VARIABLES.

This command will open the *Recode into Different Variables* dialog box.



2a. The *Recode into Different Variables* enables creating a new variable using a variable from the current data set. First, click to select *time* from the left column, and then click the right-arrow button to move the variable into the *Input Variable → Output Variable* box.

b. Now enter the new variable name by typing *quadtime* into the *Output Variable Name* box.

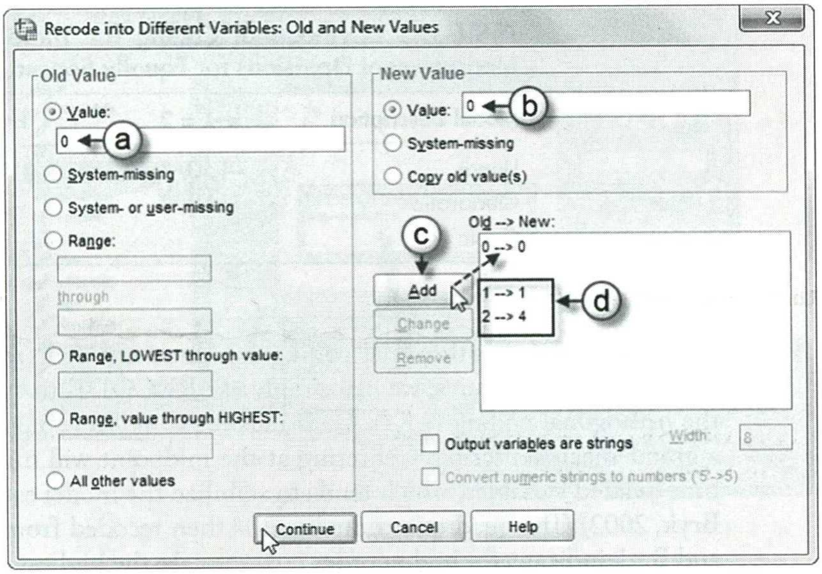
c. Then click the CHANGE button, which will add *quadtime* and complete the RECODE command for *time → quadtime*.

Note: A warning message appears as *quadtime* is an existing variable in the data set.

d. Click OK to continue, which will overwrite the preexisting *quadtime* variable. If you prefer not to overwrite the original variable, rename the output variable (e.g., *quadtime1*).

e. Click the OLD AND NEW VALUES button, which will then display the *Recode into Different Variables: Old and New Values* screen.

3. Within the *Recode into Different Variables: Old and New Values*, we will begin changing the *time* values (0, 1, 2) to reflect *quadtime* (0, 1, 4).
   a. Begin by entering the first value for *time* (*0*) in the *Value* (old) box.
   b. Next, enter the new value (*0*) for *quadtime* in the *Value* (new) box.
   c. Click the button to place the first command *0* → *0* into the *Old* → *New* box.
   d. Repeat steps 3a to 3c to complete the remaining coding changes for *quadtime* values:
      1 → *1*
      2 → *4*



Click the CONTINUE button to return to the *Recode into Different Variables* main dialog box.

Click the OK button to generate the recoded variable *quadtime* and corresponding time values (0, 1, 4).

One disadvantage of using polynomial functions in defining growth trajectories, however, is that there are strong correlations between the components comprising the function (e.g., linear, quadratic, and cubic). This occurs because the components of the polynomial function must be defined as data for each individual within the data set. As we previously described, with three repeated measures, the linear component would be defined as 0, 1, 2 and the quadratic component would be defined as 0, 1, 4. One can see these two components required for defining a curvilinear trajectory will be highly correlated for the individuals in the study. In fact, for the linear and quadratic contrasts the correlation is 0.958. Readers familiar with multiple regression will recognize that this type of strong correlation can potentially cause problems in estimating the model parameters optimally.

multicollinearity

If the occasions are equally spaced, as in our example, and there is little or no missing data, one approach for dealing with multicollinearity is to transform the polynomials to be orthogonal, or uncorrelated (Hox, 2010). Polynomials can be transformed to be orthogonal by recoding using available tables for defining orthogonal contrasts. Recoding helps simplify calculations and interpretations involved in polynomial regression. Because of the way the orthogonal polynomials are defined, the intercept for the transformed model (coded 0) will be centered near the middle of the growth sequence (i.e., representing the grand mean instead of initial status). Readers can consult a table of orthogonal polynomials for other numbers of repeated measures (e.g., Guilford & Frunchter, 1978). Although not required, we can then standardize the orthogonal estimates so that they will be on the same scale of measurement (Hox, 2010). As Hox suggests, even in data situations where the repeated measurements may not be exactly spaced, using orthogonal polynomials will tend to reduce any potential multicollinearity problem. Of course, in situations where the higher order polynomial components are not needed, it would not be necessary to use orthogonal transformation.

TABLE 5.3 Orthogonal Coding for Three and Four Measurement Occasions for Equally Spaced Intervals

Model Description	k–1 = 2	k–1 = 3
Linear	–1, 0, 1	–3, –1, 1, 3
Quadratic	1, –2, 1	1, –1, –1, 1
Cubic		–1, 3, –3, 1

We summarize orthogonal coding for three and four repeated measures in Table 5.3. For three repeated measures, we can simply use RECODE to transform the linear tread (0, 1, 2) to the orthogonal coding (–1, 0, 1), as illustrated in Table 5.3. This simple transformation creates a grand-mean centering. Centering at the midpoint will minimize the correlation between the time-related variables, which tends to stabilize the model estimation procedure (Raudenbush & Bryk, 2002). The quadratic component is then recoded from 0, 1, 4 to 1, –2, 1. As Raudenbush and Bryk indicate, for higher order polynomials, the highest order coefficient will have an invariant interpretation, while the lower order coefficients will have meanings that depend on the centering strategy employed. We reiterate that in defining the growth trajectory using higher order polynomials, we typically only include up to the highest significant component.

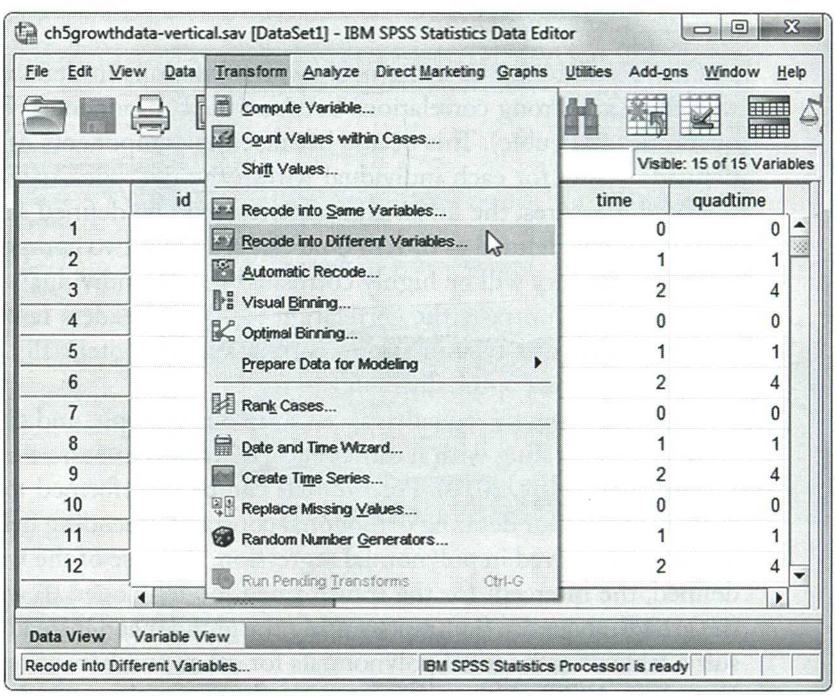
*Coding Time Interval Variables (***time** *to* **orthtime, orthquad***) with IBM SPSS Menu Commands*

Continue using the *ch5growthdata-vertical.sav* data.

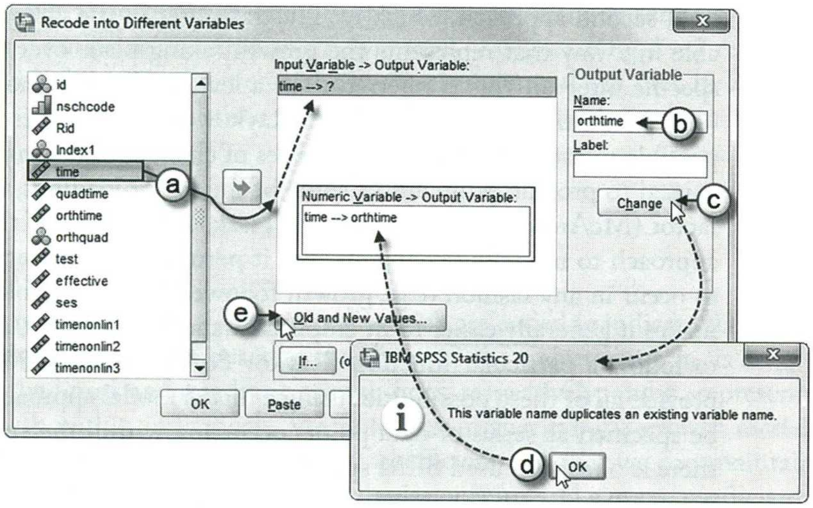
1. Go to the toolbar and select TRANSFORM, RECODE INTO DIFFERENT VARIABLES.

This command will open the *Recode into Different Variables* dialog box.

Note: If continuing from performing the prior coding example (*time* to *quadtime*), click the RESET button before proceeding to clear the default settings.



2a. The *Recode into Different Variables* screen enables creating a new variable using a variable from the current data set. First, click to select *time* from the left column, and then click the right-arrow button to move the variable into the *Input Variable → Output Variable* box.



b. Now enter the new variable name by typing *orthtime* into the *Output Variable, Name* box.

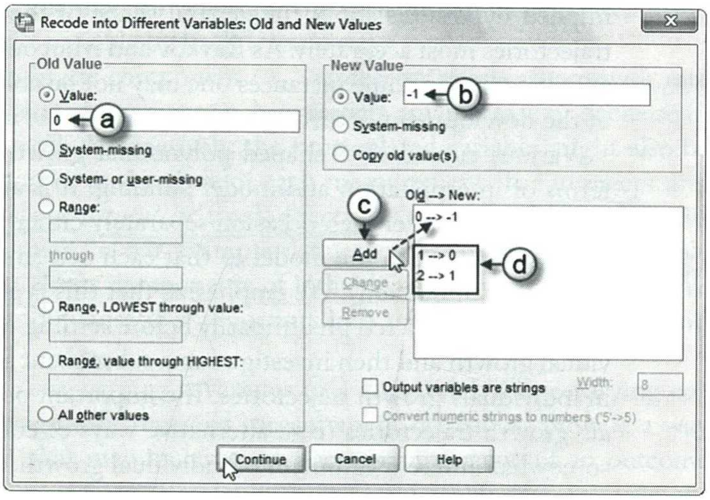
c. Then click the CHANGE button, which will add *orthtime* and complete the RECODE command for *time → orthtime.*

Note: A warning message appears as *orthtime* is an existing variable in the data set.

d. Click OK to continue, which will overwrite the preexisting *orthtime* variable. If you prefer not to overwrite the original variable, rename the output variable (e.g., *orthtime1*).

e. Click the OLD AND NEW VALUES button, which will then display the *Recode into Different Variables: Old and New Values* screen.

3. Within the *Recode into Different Variables: Old and New Values*, we will begin changing the *time* values (0, 1, 2) to reflect *orthtime* (–1, 0, 1).
   a. Begin by entering the first value for *time* (*0*) in the *Value* (old) box.
   b. Next, enter the new value (*-1*) for *orthtime* in the *Value* (new) box.
   c. Click the button to place the first command *0 → -1* into the *Old → New* box.
   d. Repeat steps 3a to 3c to complete the remaining coding changes for *orthtime* values:
      1 → 0
      2 → 1



Click the CONTINUE button to return to the *Recode into Different Variables* main dialog box.

Click the OK button to generate the recoded variable *orthtime* and corresponding time values (–1, 0, 1).

Note: To generate *orthquad* (coded 1, –2, 1), repeat all steps but rename the output variable (*orthquad*) and code the *Old → New* values as follows:

0 → 1
1 → –2
2 → 1

A second approach, which we illustrate later in the chapter, is to code the time-related variable in a way that represents the growth taking place over the whole trend, rather than over a specific interval. This is referred to as a level-and-shape model in the SEM literature on LCA (e.g., McArdle & Anderson, 1990; Raykov & Marcoulides, 2006). In the LCA approach, it is possible to estimate more general types of change, where maximum likelihood (ML) estimation is used to provide estimates of some of the factor loadings, which define the shape (or growth) factor (McArdle, 1988; Meredith & Tisak, 1990). One of the advantages of the latent variable approach to modeling change is that it permits development in the repeated measures variable to occur in any fashion (e.g., growth followed by decline followed by growth). Importantly, this makes it generally easier to fit empirical data than many other models where growth is assumed to follow a particular function (Raykov & Marcoulides, 2006). Although it is not possible to provide all of these possibilities using a mixed model approach, since the repeated measures must be specified as series of data points occurring within the specific temporal period of the study, there is one often used SEM specification that can be adapted. More specifically, since the slope is interpreted as the change in Y for a unit change in X, if we code the first measurement occasion as 0 and the last measurement occasion as 1, we can then describe the slope as the change in Y occurring over the entire trend for a unit change in X. We then use the middle measures to estimate the general *shape* of the trend in between these two endpoints.

If we obtain a graph of the average growth trajectory for the sample or several representative individual trajectories, we can often experiment a little to find an appropriate coefficient for the middle measurement occasion (or occasions). For example, we might start by coding the middle measurement in our example (year 2) as 0.5 if the trend is assumed to be linear, or some other value (e.g., 0.7) if a nonlinear trend is hypothesized. This strategy can of course be adjusted if there are several repeated measures to be specified between the first and last measurement. We can save predicted values from the estimated model using various coding schemes and generate a new graph to see which coding scheme seems to best capture the actual shape of the observed data. We can also examine various model fit indices to determine which coding approach (or implied hypothesis about the general shape of the growth) may capture the individual growth trajectories most accurately. As Raykov and Marcoulides (2008) caution, the downside of this approach is that in some instances one may not be able to obtain a specific quantitative description of the development occurring over the timeframe under consideration.

Various curved or S-shaped polynomial growth trajectories create additional challenges in terms of interpretation and model building. It is also possible to treat the time variable as categorical and model each occasion separately either by using a reference occasion or by eliminating the intercept in the model so that each occasion can be modeled separately (see Hox, 2010, for further discussion). We emphasize that this type of examination of the time-related variable is typically conducted preliminarily before settling on a final set of coefficients that describe individual growth and then investigating a subsequent set of predictors that might explain variability in individuals' growth trajectories. The important point is that various ways of defining individuals' growth trajectories (e.g., alternative ways of coding the time-related variable) may result in somewhat different estimates of individual growth, owing to the underlying assumptions of each type of statistical model.

Specifying the Two-Level Model of Individual Change

After setting up the data set appropriately and considering possible ways to code the time-related variables, we are ready to build a series of models. At Level 1, each person's successive measurements over time are defined by an individual growth trajectory and random error. At Level 2, differences in trajectories between groups of individuals can be examined. Following Raudenbush and Bryk's (2002) notation, we will use two subscripts to describe individuals (i) and occasions of measurement (t). We assume the observed status, Y_{ti}, at time t for individual i is a function of a systematic growth trajectory plus random error. At Level 1, the systematic growth for each individual in reading can be represented as a polynomial of degree P for $k - 1$ repeated measures. In this example, with three measurements, the highest polynomial will then be 2 (3 – 1 = 2), or quadratic, with the Level 1 model at time t for individual i written as

$$Y_{ti} = \pi_{0i} + \pi_{1i}a_{ti} + \pi_{2i}a_{ti}^2 + \varepsilon_{ti} \tag{5.1}$$

where a_{ti} and a^2_{ti} are time-varying variables of interest (e.g., which are coded to indicate the linear and quadratic components hypothesized to describe the shape of the trajectories); π_{0i} is an intercept; π_{1i} and π_{2i} describe the linear and quadratic growth rates, respectively; and ε_{ti} represents variation in estimating growth within individuals. We note that in specifying a growth model, we often use the Greek letter pi to represent the Level 1 coefficients, so that we can maintain the typical beta coefficients to describe between-individual relationships and gamma coefficients to describe between-group relationships. Because we have coded the first repeated measure as 0, the intercept parameter (i.e., the point where the trajectory crosses the Y axis) is interpreted as the child's true score at initial status (or the beginning) of the study. As Singer and Willet (2003) note, the Level 1 model assumes that all the individual change trajectories have the same algebraic form in Equation 5.1, but not every individual has exactly the same trajectory.

The slope parameters (π_{1i} and π_{2i}) represent the predicted change in individuals over a specified time interval. The linear component describes the rate of change per unit of time. A quadratic component can be interpreted as a "change" in the rate of change (e.g., accelerating or decelerating). As shown in Figure 5.5, the linear component of the time-related variable (a) is coded 0, 1, 2 in the data set (referred to as "*time*"), which ensures that the intercept is interpreted as students' *true* initial status (i.e., their corrected achievement level at Time 1). We can also add a quadratic component to the model to test for a change in the rate of growth over time. This component is also coded as a variable (*quadtime*). The interval values for the variable can be generated using COMPUTE and multiplying the time variable by itself (*time*time*).

If this coding scheme is used, the linear component (π_{1i}) represents the yearly growth rate for each child in the study. The quadratic component (π_{2i}) represents any increase or decrease in the rate of change for each time interval. Alternatively, the time-related variable might also be conceptualized as students' age in months at the time of each measurement. The intercept and slope coefficients represent the model's *structural*, or fixed, effects. As mentioned previously, we have also created orthogonal polynomials in our example data in order to remove the sizable correlation between the linear and quadratic components of individuals' growth trajectories. We have added these orthogonal linear and quadratic components into the data set as *orthtime* and *orthquad*, respectively.

We can also specify one or more *time-varying* covariates (X_i) at Level 1. Time-varying covariates (i.e., predictors that also change over repeated measurement occasions) provide a way of accounting for temporal variation that may increase (or decrease) the value of an outcome predicted by the individual's growth trajectory (Raudenbush & Bryk, 2002). For example, we could consider a situation where a variable like motivation, which changes over time, might also affect students' learning in math or reading. Alternatively, we could also consider motivation as a static variable (i.e., individuals' average motivation level), which requires only one value for the covariate. In this latter case, we would then enter the covariate at the between-subjects level (i.e., Level 2). One of the advantages of the time-varying formulation at Level 1, however, is that the effect of motivation level on student achievement can then be modeled as a random parameter at Level 2.

Level 1 Covariance Structure

The other part of the Level 1 model is the *stochastic* part, or the part that describes the variation in measuring each individual i on occasion t. This part of the model implies that there is some error (ε_{ti}) associated with measuring each individual's true growth trajectory. The errors are *unobserved*, which means that we must make some assumptions about their distribution at Level 1 (Singer & Willett, 2003). Often, a simple residual structure is assumed from occasion to occasion and person to person, with each error independently and normally distributed, a mean of 0, and constant variance:

$$\varepsilon_{ti} \sim N(0, \sigma_{\varepsilon}^{2}) \tag{5.2}$$

where ~ means "distributed as," N refers to normal distribution, 0 refers to the mean, and refers to the variance. One way to represent this simplified error structure is as a scaled identity matrix, which provides a single residual variance associated with measurement occasions. With longitudinal data, however, this type of simple Level 1 error structure may have less credibility (Singer & Willett, 2003).

Restrictions about the within-individual residuals over time can be relaxed. This is often necessary because the error covariance matrix for the repeated measures typically will exhibit some correlation between occasions. For example, often the repeated measures will correlate more strongly when they are taken closer together and less strongly as the time interval increases. It is often useful to examine different error structures (e.g., compound symmetry, autocorrelated, or unstructured) in preliminary analyses, depending on the nature of the repeated measurements per subject. The fit of particular covariance structures to the data can then be compared to see which covariance structure provides the best overall choice. IBM SPSS provides a considerable number of choices for the Level 1 residual covariance matrix in a repeated measures model (a complete list of covariance structures can be obtained from the MIXED Commands in the IBM SPSS Help Menu).

Repeated Covariance Dialog Box

The Level 1 covariance matrix can be specified by opening the *Repeated Covariance* dialog box in the Commands menu (Figure 5.6) or with a REPEATED syntax command. There are actually a number of different uses for this dialog box, which we will briefly summarize. The first is the typical growth specification we have just described, where this is used to specify the individual variation around the repeated measures of Y. A second use is when the focus is on defining a multivariate model, for example, when we have several survey items used to define an underlying (or latent) dependent variable such as job satisfaction. The individual items defining the construct can be specified vertically for each individual in the data set using an *index* variable and the *Repeated Covariance* dialog box used to describe variance and covariance relationships between the items, which is similar to representing repeated measures over time. We cover these multivariate situations in more detail in Chapter 7. A third use would be where the researcher has different amounts of data on an outcome for individuals at Level 1. For example, workers might be measured performing a series of work-related tasks. One individual might have one such measure of task performance, while others might have five or six (or even more) measures. This specification amounts to having multiple pieces of data nested within individuals at Level 1, but not incorporating a growth parameter in the sense that individuals are assumed to be changing in performance over time. In this case, the *Repeated Covariance* dialog box is useful in organizing the differing amounts of performance information regarding individuals in the study.

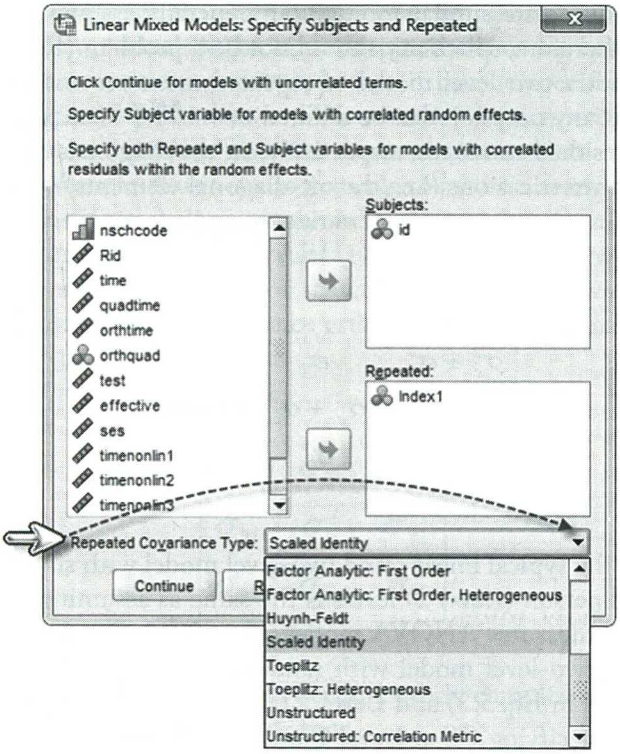


FIGURE 5.6 The *Repeated Covariance Type* dialog box and available covariance structures.

We note that if the *Repeated Covariance* dialog box (or repeated syntax statement) is not used, the default Level 1 matrix will be a scaled identity covariance matrix. As Equation 5.3 indicates, the scaled identity matrix (abbreviated as ID in MIXED syntax) assumes a constant variance across occasions, where σ^2 is the variance, and no covariances between occasions. Therefore, it has only one estimated parameter, as suggested in Equation 5.3. We note that because a covariance matrix is a square matrix, the same elements appear above and below the diagonals. This simplified within-subject error structure may sometimes be sufficient for repeated measures studies of short duration, if the focus is not primarily on defining the relationships between successive measurements:

$$\sigma^2 \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \tag{5.3}$$

Specifying an identity covariance matrix for the repeated measures structure amounts to accepting that Mauchly's test for sphericity holds.

A special form of sphericity is *compound symmetry* (abbreviated CS) or uniformity (Hox, 2010). Conditions for compound symmetry are met if all the variances are equal in the population being sampled and all the covariances (the off-diagonal elements of the covariance matrix) are equal. This means there is one variance and one constant covariance in the Level 1 covariance matrix (or one more parameter to estimate than in Eq. 5.3). If the observed covariances are roughly equal and the variances are similar too, we can generally assume that compound symmetry is not violated and, therefore, sphericity should not be a problem (Raykov & Marcoulides, 2008). It turns out that for a two-level model of repeated measures with random intercept only, the residual variance at any occasion can be defined as $\varepsilon_{ti} \sim N(0,\sigma_\varepsilon^2)$ (i.e., the sum of the occasion-level and person-level residual variances, respectively) in the diagonals of the matrix, and the covariance between any two occasions (i.e., the off-diagonal elements) is $\sigma^2\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$. For this simple type of repeated measures model, the matrix of variances and covariances among occasions would then be defined as follows:

$$\begin{bmatrix} \sigma_\varepsilon^2 + \sigma_{u_0}^2 & \sigma_1 & \sigma_1 \\ \sigma_{u_0}^2 & \sigma_\varepsilon^2 + \sigma_{u_0}^2 & \sigma_1 \\ \sigma_{u_0}^2 & \sigma_{u_0}^2 & \sigma_\varepsilon^2 + \sigma_{u_0}^2 \end{bmatrix} \tag{5.4}$$

This suggests that the typical linear trend two-level model with single residual term at the occasion (Level 1) and person (Level 2) levels is the same as assuming compound symmetry in the univariate repeated measures ANOVA model (Hox, 2010; Raudenbush & Bryk, 2002). In fact, for a simple linear two-level model with random intercept only, specifying an ID covariance matrix at Level 1 (as in Eq. 5.3) and Level 2 (since there is one random effect) will produce an identical model to specifying CS at Level 1 and ID at Level 2.

An alternative Level 1 structure assuming different variances across measurement occasions could also be summarized as a diagonal (DIAG) covariance matrix. This type of covariance matrix assumes heterogeneous variances for each measurement occasion in the diagonals of the matrix and 0s for the off-diagonal elements, which indicates no covariances between occasions:

$$\begin{bmatrix} \sigma_1^2 & 0 & 0 \\ 0 & \sigma_2^2 & 0 \\ 0 & 0 & \sigma_3^2 \end{bmatrix} \tag{5.5}$$

Relative to the scaled identity or compound symmetry covariance matrices, however, the diagonal covariance matrix will have more parameters to estimate for the assumed heterogeneous variances. The limitation of this type of covariance structure is also that it assumes no relationship between measurement occasions (similar to the scaled identity matrix).

Often we find that the repeated measures structures for longitudinal data may have a complex covariance structure. This autoregressive error covariance matrix (AR1) assumes that the Level 1 variance remains constant across occasions but facilitates specifying an autocorrelation coefficient between occasions. The autocorrelation coefficient rho (ρ) represents the correlation between any two adjacent occasions, where $|\rho| \leq 1$. It follows, then, that ρ^2 represents the correlation when there is a skip between occasions. This structure is useful when it is likely that the correlations become weaker as there is longer distance in time between them:

$$\sigma_\varepsilon^2 \begin{bmatrix} 1 & \rho & \rho^2 \\ \rho & 1 & \rho \\ \rho^2 & \rho & 1 \end{bmatrix} \tag{5.6}$$

The autoregressive covariance structure differs from compound symmetry, which assumes that the covariance between residuals is the same despite the time lag between them. One advantage of the autoregressive covariance structure is that it can be specified with only two estimated parameters (i.e., a variance parameter and the correlation parameter). This makes it a relatively simplified covariance structure, but one that does not assume that all covariances between occasions are the same. We note that it is also possible to assume heterogeneity among the occasion variances; that is, the diagonal elements can be replaced with separate variance estimates in the autoregressive structure (abbreviated as ARH1).

In contrast to an autoregressive error structure, a completely *unstructured* (UN) covariance matrix provides separate occasion variance estimates in the diagonals and separate covariances estimated for the off-diagonal elements:

$$\begin{bmatrix} \sigma_1^2 & \sigma_{21} & \sigma_{31} \\ \sigma_{21} & \sigma_2^2 & \sigma_{32} \\ \sigma_{31} & \sigma_{32} & \sigma_3^2 \end{bmatrix} \tag{5.7}$$

This type of error structure, however, can become overly complex when the number of measurement occasions increases beyond three or four.

A general goal is to identify a parsimonious covariance structure that will adequately describe the data, both at Level 1 and Level 2. In this example, we will first assume an identity Level 1 covariance matrix within individuals to examine how much variance in the outcome lies within and between individuals. MIXED provides several fit indices (e.g., AIC and BIC) that can also be used to evaluate various combinations of fixed effects and covariance structures. In simulation studies of growth curve modeling, the AIC has been noted to work well in selecting the true model, provided the sample size is not below 100 individuals (Liu, Rovine, & Molenaar, 2012). We discuss these indices later in this chapter.

More complex covariance structures can also make it more difficult to arrive at a solution that converges (i.e., provides reasonable estimates for all proposed parameters). Often, it is necessary to exercise some type of compromise between the complexity of some covariance structures and the parsimony provided by others in arriving at solution that defines model covariance structures adequately. We illustrate some of these differences in Level 1 covariance matrices subsequently. As Hox (2010) notes, if the focus is primarily on the model's fixed effects, one can often assume a more simplified variance and covariance structure across occasions, as some misspecification in the random part of the model does not generally affect the model's fixed effects (see also Verbeke & Lesaffre, 1997). In some circumstances, however, different user choices regarding the error variance structure at Level 1 may affect the outcome of tests for random effects at higher levels.

Model 1.1: Model with No Predictors

There is some difference of opinion among researchers about what is the proper model to begin with in building a growth model. We can start with a "no-predictors" model if desired. However, when we calculate the intraclass correlation (within individuals = Level 1; between individuals = Level 2) we will only get a rough estimate since the within-individual variance may be different at each measurement occasion. Moreover, this initial estimate of the variance components may not be very accurate. This is because when the variance components are initially estimated, the estimation procedure is assuming *random sampling* at both levels. What this means is that the initial estimate of the variance in the outcome at the between-individual level (Level 2) can ignore possible variation in the between-individual variance component that may be due to additional within-individual variance that actually exists. If this occurs, the initial between-individual estimate must be corrected by the ML estimation procedure as subsequent variables are added (Hox, 2010).

In this first type of null model, for example, we could test whether the grand-mean intercept for math varies across individuals. From Equation 5.1, this is simply defined at Level 1 without the time-related variables as follows:

$$Y_{ti} = \pi_{0i} + \varepsilon_{ti}, \tag{5.8}$$

where π_{0i} is the average achievement across the three occasions, and ε_{ti} represents errors in predicting the average achievement for individuals. Between individuals, we can describe the average growth across occasions as

$$\pi_{0i} = \beta_{00} + u_{0i}, \tag{5.9}$$

where β_{00} is the intercept describing the average initial status mean between individuals, and u_{0i} is the Level 2 random component associated with describing differences in average achievement between individuals. Substituting Equation 5.9 into Equation 5.8, we would arrive at the combined equation

$$Y_{ti} = \beta_{00} + u_{0i} + \varepsilon_{ti}, \tag{5.10}$$

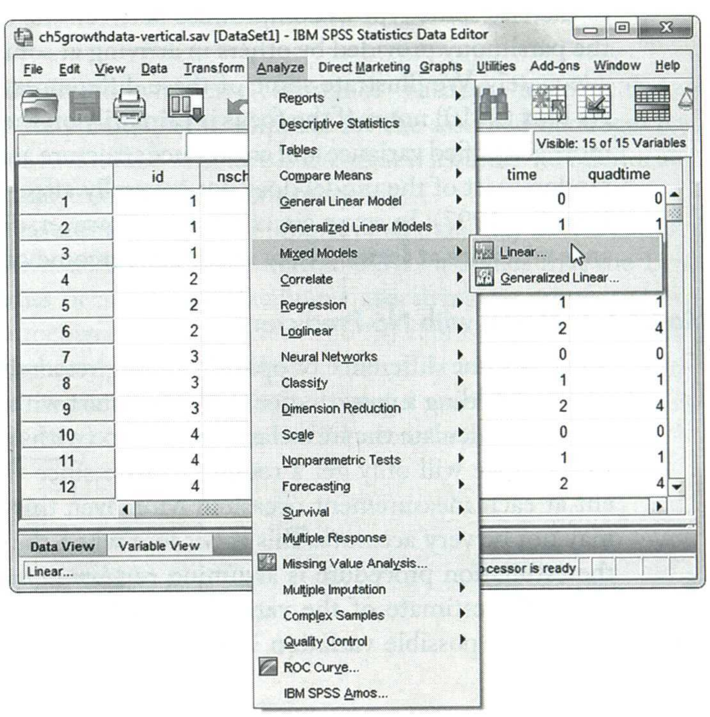
which indicates three parameters to estimate. These include the fixed effect describing average math achievement, the between-individual random variance, and the Level 1 residual variance.

Defining Model 1.1 (Null) with IBM SPSS Menu Commands

Continue using *ch5growthdata-vertical.sav.*

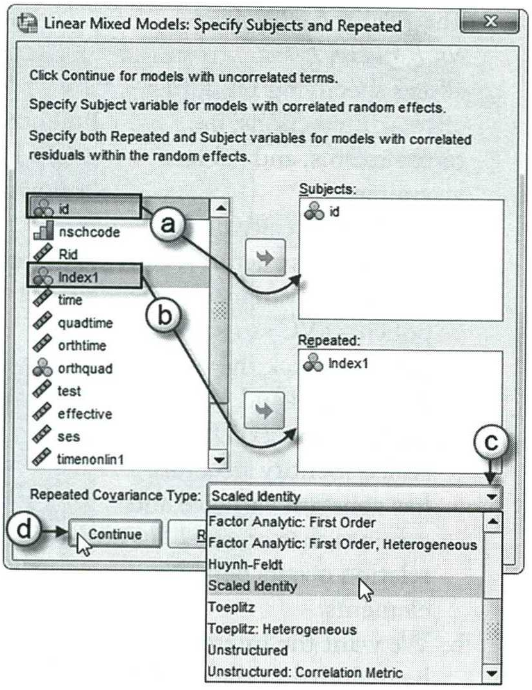
1. Go to the toolbar and select ANALYZE, MIXED MODELS, LINEAR.

This command enables access to the *Linear Mixed Models: Specify Subjects and Repeated* dialog box.



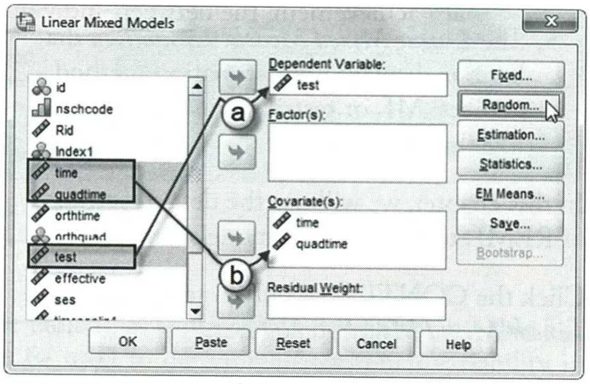
2. The *Linear Mixed Models: Specify Subjects and Repeated* screen displays options for defining variables as subjects, repeated observations, and type of covariance structure in a model.
   a. A subject is an observational unit that may be independent of other subjects. For this model, we will designate *id* (individual identification numbers) as the subject variable. Click to select *id* from the *Variables* column, and then click the right-arrow button to move the variable into the *Subjects* box.
   b. The *Repeated* box allows specifying variables that identify repeated observations. For this model, *Index1* identifies repeated observations over three time periods. Click to select *Index1*, and then click the right-arrow button to move the variable into the *Repeated* box.

   The combination of values for *id* and *Index1* defines a particular student across three time periods.



   c. The *Repeated Covariance Type* specifies a model's covariance structure. For this model, we will use the *Scaled Identity* (ID) covariance type. Click the pull-down menu and select *Scaled Identity.* The ID structure has constant variance and assumes that no correlation occurs between elements.
   d. Click the CONTINUE button to display the *Linear Mixed Models* dialog box.

3. The *Linear Mixed Models* main screen enables specifying the dependent variable, factors, covariates, and access to dialog boxes for defining *Fixed* and *Random* effects, and options for *Estimation*, *Statistics*, *EM Means*, and *Save.*
   a. For this model, we will use math achievement (*test*) as the dependent variable. Click to select the *test* variable from the left column listing. Then click the right-arrow button to transfer *test* into the *Dependent Variable* box.
   b. The null model does not have predictors, but since we will be designating a random effect in step 3, we will need to introduce variable(s) at this point as factors or covariates as a workaround. (Omitting this step will prevent specifying a random effect.)



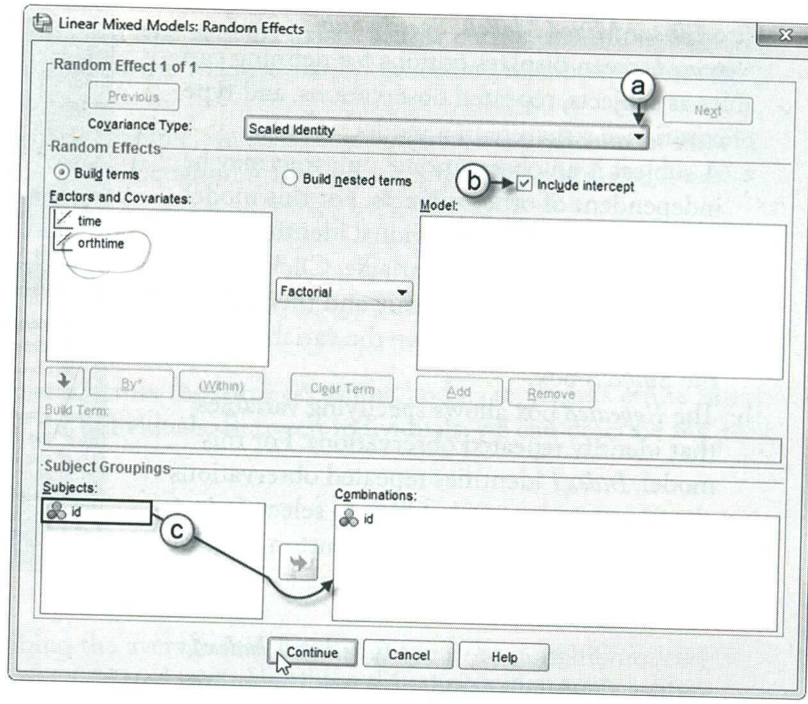
Factors and covariates may be specified in predicting the dependent variable. Factors are categorical predictors that may be numeric or string. Covariates are scale predictors that must be numeric.

We will designate two predictor variables, which will be used later in the upcoming model (Model 5.1A). Locate and click the variables *time* and *quadtime* from the left column listing, and then click the right-arrow button to move the variables into the *Covariate(s)* box.

Since we are not defining effects for the two predictors, we will skip over the FIXED effects button and go to set up the model's random effects.

Click the RANDOM button to access the *Linear Mixed Models: Random Effects* dialog box.

4. The *Linear Mixed Models: Random Effects* screen allows specifying random effects, interactions, intercept terms, and subject groupings.
   a. Begin by specifying the covariance structure from the default variance components (VC) to scaled identity. Click the pull-down menu and select *Scaled Identity* (ID). The scaled identity structure has constant variance and assumes that no correlation occurs between elements.
   b. We want the intercept to be included in the model, so click *Include intercept*.



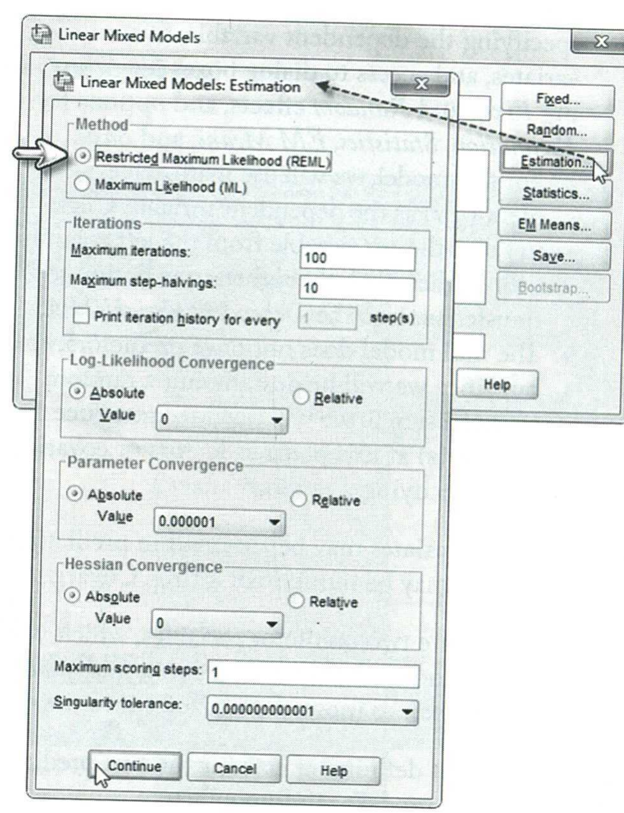
   c. The *Subject Groupings* box displays the *id* variable that was selected as a subject variable in the *Select Subjects and Repeated* dialog box show in step 2a. We will specify *id* as the subject for the random-effects part of this model. Click to select *id*, and then click the right-arrow button to move the variable into the *Combinations* box.

Click the CONTINUE button to return to the *Linear Mixed Models* main dialog box.

5. The *Linear Mixed Models: Estimation* dialog box displays two estimation method choices: ML or restricted maximum likelihood (REML).

In this chapter, we will use the default setting of REML to estimate the models.

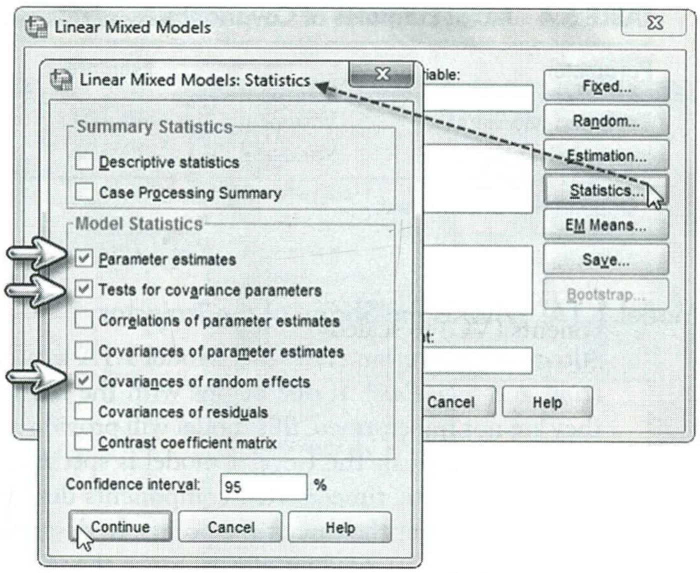
Click the CONTINUE button to return to the *Linear Mixed Models* dialog box.



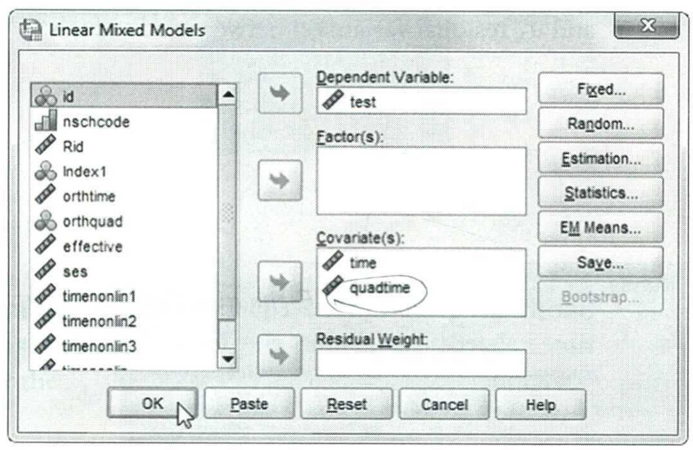
6. In the *Linear Mixed Models* dialog box, click the STATISTICS button to access the *Linear Mixed Models: Statistics* dialog box.

Click and select the following three statistics to be included in the output: *Parameter estimates*, *Tests for covariance parameters*, and *Covariances of random effects*.

Click the CONTINUE button to return to the *Linear Mixed Models* dialog box.



7. Finally, in the *Linear Mixed Models* dialog box, click the OK button to run the model.



Interpreting the Output From Model 1.1 (Null)

The grand mean for achievement is 52.945 (not tabled), as summarized previously in Table 5.2. The variance component table (Table 5.4) can be used to determine how much variability in math achievement is present at each level. At Level 1, the variance ( ) summarizes the population variability in the average individual's achievement estimates around her or his own true growth trajectory (Singer & Willet, 2003). The estimate is 78.101. The Level 2 variance is 30.505 (Wald Z = 33.777, $p < .001$), which suggests there is sufficient variation in intercepts across individuals. The null hypothesis is that the population parameter for the variance is 0 (Singer & Willett, 2003). Keep in mind that the Wald Z statistic provides a two-tailed test and because the null hypothesis is that the population variance is 0, we should use a one-tailed test for variances. As a rough estimate, we can calculate the proportion of variance in math achievement that is between individuals as about 0.281 [30.505/(30.505 +78.101)], or 28.1%.

TABLE 5.4 Initial Estimates of Covariance Parameters[a]

Parameter		Estimate	Std. Error	Wald Z	Sig.
Repeated Measures	Variance	78.101	0.839	93.113	.000
Intercept [subject = id]	Variance	30.505	0.903	33.777	.000

[a] Dependent variable: test.

Model 1.1A: What Is the Shape of the Trajectory?

Alternatively, we can start with Model 1.1A as a model that includes the time-related variables (*time* and *quadtime*). If one begins with the linear component and quadratic components, and they are not transformed, this model will provide a variance component for the intercept at Time 0 (at initial status). The Level 1 model is specified in Equation 5.1, where we have defined linear and quadratic time-related components describing the shape of individual growth in math achievement over the three time points. In describing the shape of the growth trajectory, we can also estimate $k - 1$ random effects. Since there are three time points in this example, we can estimate two random effects. This allows us to estimate a Level 2 randomly varying intercept and either a randomly varying linear component or quadratic component.

In this model, we will initially treat both time-related components as fixed (i.e., with no u_{2i} and u_{2i} residual variances) between individuals:

$$\pi_{1i} = \beta_{10} \tag{5.11}$$

$$\pi_{2i} = \beta_{20}. \tag{5.12}$$

Substituting the Level 2 equations (Eq. 5.10 for the intercept and Eqs. 5.11 and 5.12 for the time-related slopes) into the Level 1 equation (Eq. 5.1) results in the following combined equation:

$$Y_{ti} = \beta_{00} + \beta_{10}a_{ti} + \beta_{20}a_{ti}^2 + u_{0i} + \varepsilon_{ti} \tag{5.13}$$

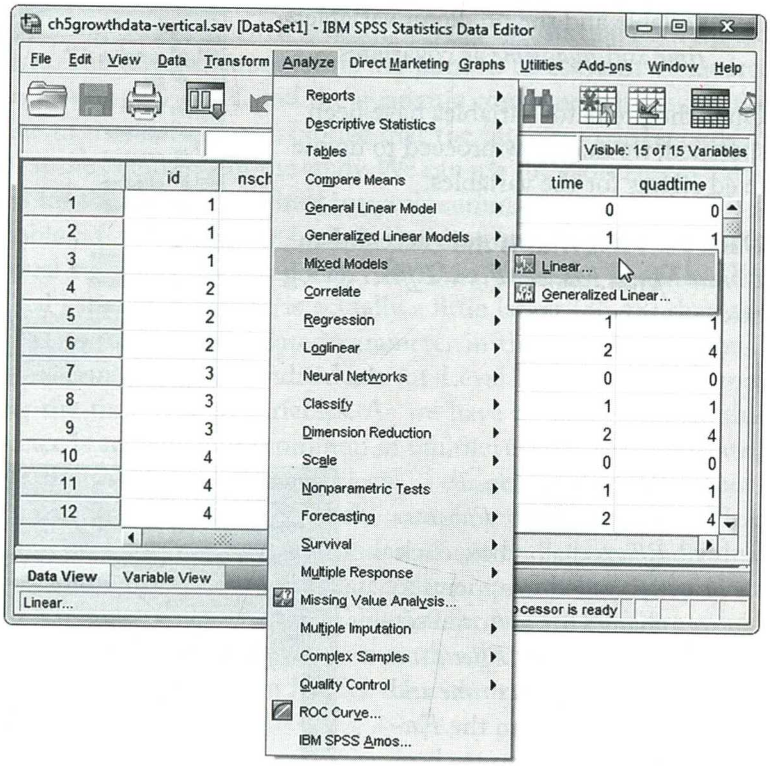
This suggests five parameters to estimate including three fixed effects, one random intercept (u_{0i}), and the Level 1 residual (ε_{ti}).

Defining Model 1.1A with IBM SPSS Menu Commands

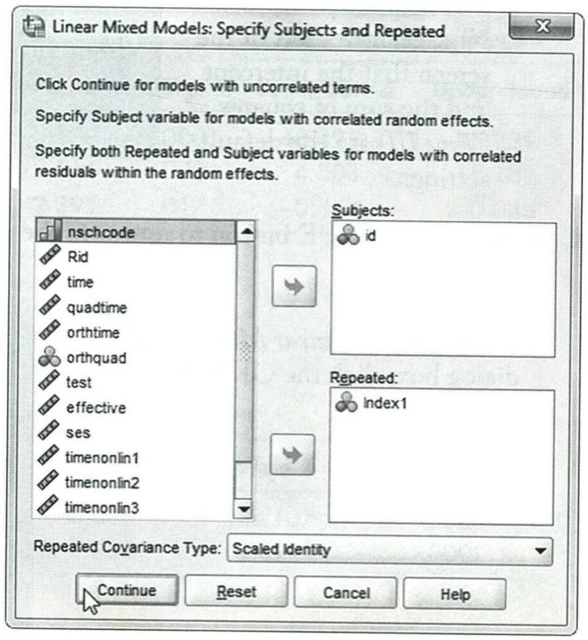
Continue using the *ch5growthdata-vertical.sav* data. Settings will default to those used in Model 1.1 (Null).

1. Go to the toolbar and select ANALYZE, MIXED MODELS, LINEAR.

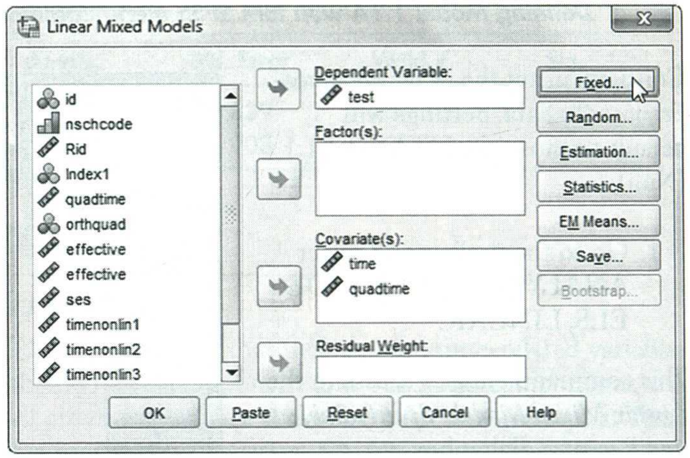
This command enables access to the *Linear Mixed Models: Specify Subjects and Repeated* dialog box.



2. The *Linear Mixed Models: Specify Subjects and Repeated* box displays the default settings from the prior model. We will retain the settings, so click the CONTINUE button to display the *Linear Mixed Models* dialog box.



3. The *Linear Mixed Models* main dialog box displays *test* as the dependent variable and the predictor variables *time* and *quadtime* as covariates.



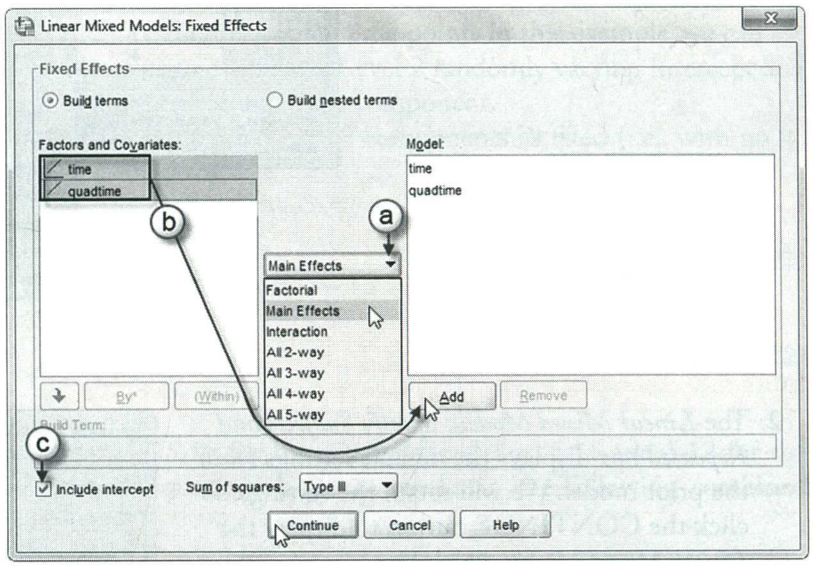
Once the predictor variables have been specified, we may now proceed to define fixed effects for the variables.

Click the FIXED button to access the *Linear Mixed Models: Fixed Effects* dialog box.

4a. Within the *Linear Mixed Models: Fixed Effects* dialog box, click the pull-down menu to change the factorial setting to *Main Effects*.

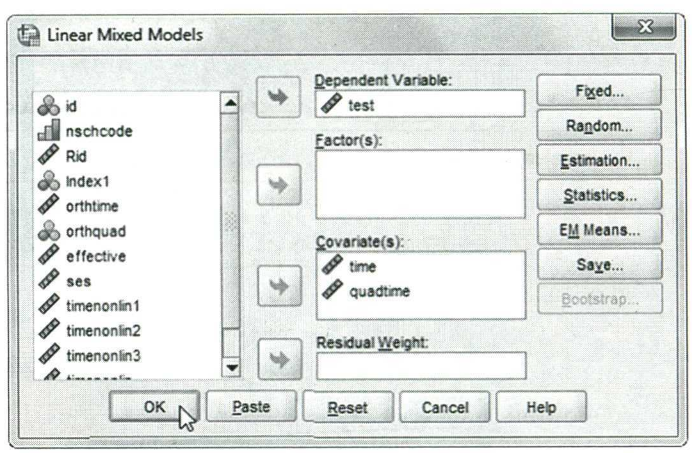
b. Click to select *time* and *quadtime* from the *Factors and Covariates* box, and then click the ADD button to move the variable into the *Model* box.

c. Note on lower left of the screen that the intercept and the sum of squares (*Type III*) are the default settings.



Click the CONTINUE button to return to the *Linear Mixed Models* dialog box.

5. Finally, in the *Linear Mixed Models* dialog box, click the OK button to run the model.



Interpreting the Output From Model 1.1A

The fixed effects are presented in Table 5.5. We can observe that both the linear and quadratic polynomials are significant in explaining student growth in math, which suggests that both should be retained in subsequent analyses. The linear component is the portion of the sum of squares (SS) attributable to the linear regression of Y on X, and the quadratic component measures the additional improvement in fit due to that component. Notice that the intercept corresponds to students' math achievement level at the beginning of the study. We can use the estimates in Table 5.5 to obtain the observed means for the second and third intervals summarized in Table 5.2.

Next, readers will notice in Table 5.6 that when we include the time-related variables in the initial model, the within-individual variance is reduced from 78.10 in Table 5.4 to 60.19. The estimate of the between-individual variance, however, is actually a little larger (36.48) than the initial estimate in Table 5.4 (30.51) with no time-related parameter in the model. This would seem to be explaining "negative" variance between individuals (at Level 2) since the variance component is reduced by adding the time-related variables. As we have noted previously, this problem regarding negative variance is actually quite common in multilevel analyses of repeated measures. It often occurs because the variability among subjects in the repeated measures portion (Level 1) of the outcome is usually much larger than the sampling model assumes (Hox, 2010). This leads to overestimating occasion variance and underestimating between-individual (Level 2) variance in the intercept-only model. If we calculated the proportion of the variance between individuals based on the variance components in Table 5.6 for Model 1.1A, we would find that the between-individual proportion of the variance in math achievement is now 0.377 (36.477/96.664), or almost 38%, rather than 28% from Model 1.1 with no predictors.

TABLE 5.5 Estimates of Fixed Effects[a]

						95% Confidence Interval	
Parameter	**Estimate**	**Std. Error**	***df***	***t***	**Sig.**	**Lower Bound**	**Upper Bound**
Intercept	48.632	0.106	20,242.098	460.579	.000	48.425	48.839
time	4.719	0.212	17,338.000	22.216	.000	4.303	5.135
quadtime	–0.244	0.102	17,338.000	–2.391	.017	–0.444	–0.044

[a] Dependent variable: test.

TABLE 5.6 Estimates of Covariance Parameters Including Time Variables[a]

Parameter		**Estimate**	**Std. Error**	**Wald Z**	**Sig.**
Repeated Measures	Variance	60.187	0.646	93.107	.000
Intercept [subject = id]	Variance	36.477	0.885	41.198	.000

[a] Dependent variable: test.

w/i individual variance

between individual variance

Does the Time-Related Slope Vary Across Groups?

We emphasize there is no "one way" to proceed in developing an initial growth model. We generally favor this latter approach (Model 1.1A), where the analysis begins with focusing on whether the means of the repeated measures differ across time and defining the shape of the growth trajectory. This initial model could include both the linear and quadratic components since both were found to be significantly related to student growth. Therefore, our first model focuses on defining the shape of students' growth trajectories and determining whether the intercept and slopes vary across individuals. We can now test whether the time-related slopes are randomly varying across individuals at Level 2:

$$\pi_{1i} = \beta_{10} + u_{1i}, \tag{5.14}$$

$$\pi_{2i} = \beta_{20} + u_{2i}, \tag{5.15}$$

where the β coefficients represent the intercepts, and the u coefficients represent the variance estimates for the equations. We note that determining which time-related component or components to use in building the Level 2 explanatory models can take a bit of trial and error. It is generally the case that alternative models with various trajectory shapes and covariance structures should be preliminarily investigated in order to arrive at an optimal solution (e.g., see Liu et al., 2012).

As we have noted, when we estimate a polynomial growth model, we need to keep in mind that we are limited in the number of random effects by the number of repeated measures. We can estimate a random effect for the intercept and linear or quadratic term, but not both. If we choose the linear term, this means we will need to assume that the slowing indicated by the quadratic parameter is the same for every person in the sample. Therefore, it is often the choice to consider the quadratic as a fixed effect within individuals in subsequent models to explain differences in students' growth trajectories. Since it does not vary across individuals, we can fix its variability at 0 by removing the random term (u_{2i}), as in Equation 5.12. If we do try to estimate the quadratic effect also, we receive the warning message in Figure 5.7 regarding the model's failure to converge on a solution.

With the quadratic component fixed, through the substitution of Equations 5.10 (intercept) and 5.14 and 5.12 (time-related slopes) into Equation 5.1, we arrive at the single-equation model for examining the fixed and random components without Level 2 predictors:

$$Y_{ti} = \beta_{00} + \beta_{10}time_{ti} + \beta_{20}quadtime_{ti} + u_{1i}time_{ti} + u_{0i} + \varepsilon_{ti}. \tag{5.16}$$

Equation 5.16 suggests that the intercept and linear component (time) are randomly varying across individuals. Because the quadratic component is merely defining the shape of growth

Warnings

Iteration was terminated but convergence has not been achieved. The MIXED procedure continues despite this warning. Subsequent results produced are based on the last iteration. Validity of the model fit is uncertain.

FIGURE 5.7 Warning message.

within individuals, this suggests building the explanatory model for student growth rates on the randomly varying linear component.

Level 2 Covariance Structure

It is also possible to define different covariance structures at successive levels of the proposed model. For Level 2, the dimensionality of the covariance matrix describing the variances and covariances between random effects depends on the number of random effects in the model. In this case, because we are treating the quadratic component as fixed across individuals due to our preliminary examination, we will assume a 2 × 2 *unstructured* covariance matrix of random effects for the intercept (I) and slope (S) at Level 2:

$$\begin{bmatrix} \sigma_I^2 & \sigma_{I,S} \\ \sigma_{I,S} & \sigma_S^2 \end{bmatrix} \tag{5.17}$$

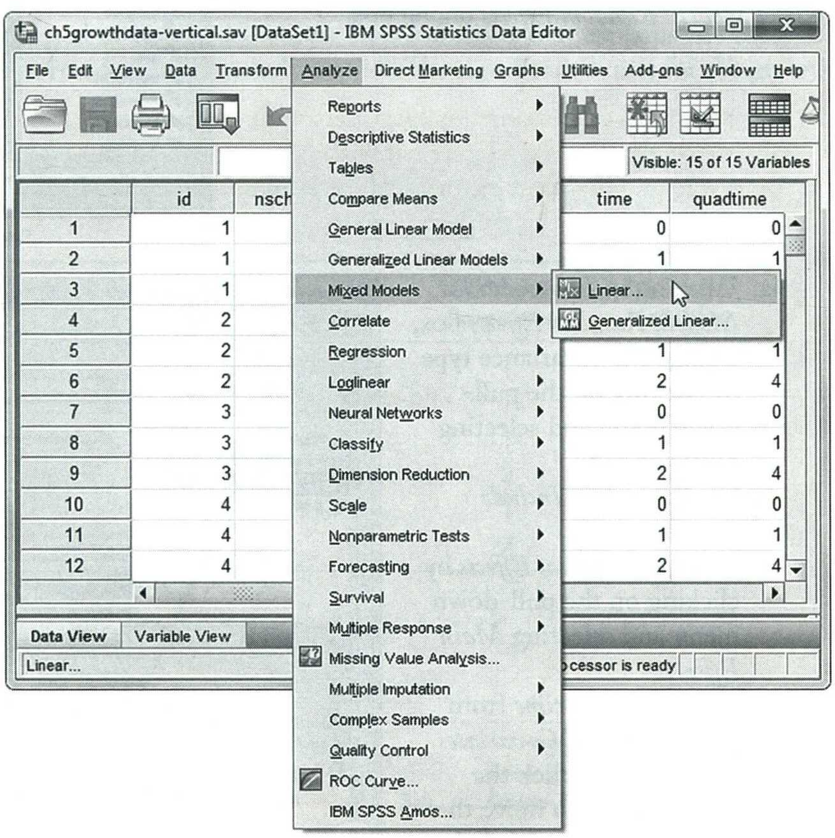
The variances are contained in the diagonals of the matrix, and the covariance is represented by the off-diagonal element. When we combine the covariance parameter estimated in Equation 5.17 with the six parameters defined in Equation 5.16 (i.e., three fixed effects, one random Level 2 intercept, one random Level 2 slope, and the Level 1 residual), we will have a total of seven parameters to estimate in our final version of Model 1.

Defining Model 1.1B with IBM SPSS Menu Commands

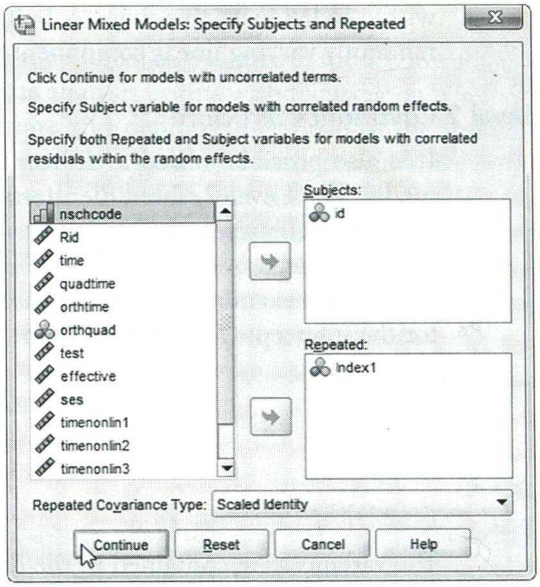
Continue using *ch5growthdata-vertical.sav*. Settings will default to those used in Model 1.1A.

1. Go to the toolbar and select ANALYZE, MIXED MODELS, LINEAR.

This command enables access to the *Linear Mixed Models: Specify Subjects and Repeated* dialog box.



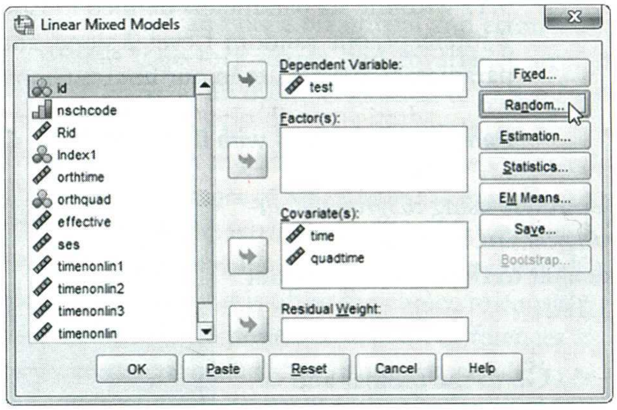
2. The *Linear Mixed Models: Specify Subjects and Repeated* displays the default settings from the prior model. We will retain the settings, so click the CONTINUE button to display the *Linear Mixed Models* dialog box.



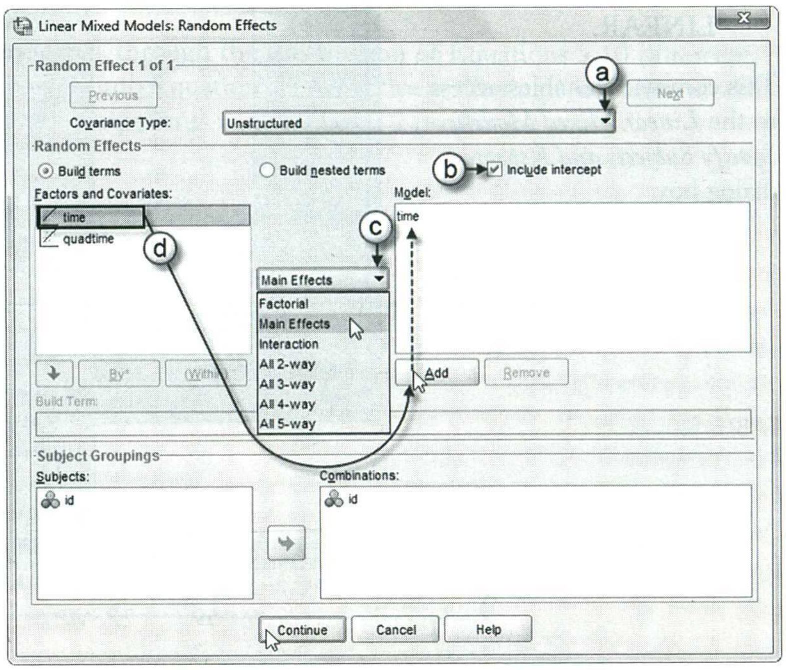
3. The *Linear Mixed Models* main dialog box displays the settings used in the prior model.

We will now add random effects to this model.

Click the RANDOM button to access the *Linear Mixed Models: Random Effects* dialog box.

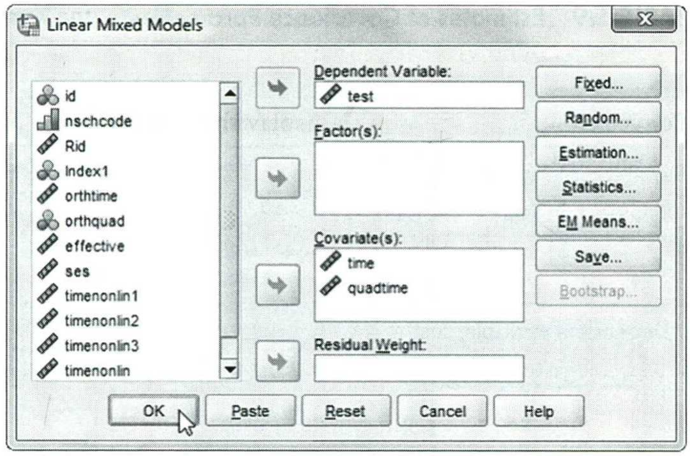


4a. Within the *Linear Mixed Models: Random Effects* box, change the covariance type by clicking on the pull-down menu and selecting *Unstructured.*
b. Click to select *Include intercept.*
c. Change *Factorial Effects* by clicking on the pull-down menu and selecting *Main Effects.*
d. Click to select *time* from the *Factors and Covariates* box, and then click the ADD button to move the variable into the *Model* box.



Click the CONTINUE button to return to the *Linear Mixed Models* dialog box.

5. Finally, in the *Linear Mixed Models* dialog box, click the OK button to run the model.



Interpreting the Output From Model 1.1B

Results of this first model test are presented in the following tables. As in previous chapters, it is often useful to examine the total number of parameters being estimated and the number of random and fixed effects to make sure they correspond with what the analyst might have in mind. The first output (Table 5.7) confirms that there are seven total parameters being estimated, which is consistent with Equations 5.16 and 5.17.

The fixed-effect results in Table 5.8 are summarized as β parameters since they are Level 2 parameters (as suggested in Eq. 5.16). For the true intercept (π_{0i}), the estimate for initial status (β_{00} in Eq. 5.16) is 48.632. This estimate is consistent with estimated mean for Time 0 in Table 5.2. For the linear growth rate (π_{1i}), the estimate (β_{10} in Eq. 5.16) is 4.72 points per year. For the quadratic growth rate (π_{2i}), the estimate (β_{20} in Eq. 5.16) is −0.244. The significance of each fixed effect is tested with a t test (i.e., defined as the ratio of the unstandardized estimate to its standard error). We note that MIXED applies the Satterthwaite (1946) correction for calculating the degrees of freedom used in testing parameter significance, which explains the presence of decimals in the degrees of freedom column in Table 5.8. The significant t tests for the growth

TABLE 5.7 Model Dimension[a]

		Number of Levels	Covariance Structure	Number of Parameters	Subject Variables
	Intercept	1		1	
Fixed Effects	time	1		1	
	quadtime	1		1	
Random Effects	Intercept + time	2	Unstructured	3	id
Repeated Effects	time	3	Identity	1	id
Total		8		7	

[a] Dependent variable: test.

TABLE 5.8 Estimates of Fixed Effects[a]

Parameter	Estimate	Std. Error	*df*	*t*	Sig.	95% Confidence Interval Lower Bound	95% Confidence Interval Upper Bound
Intercept	48.632	0.103	10,597.930	472.847	.000	48.431	48.834
time	4.719	0.206	10,329.959	22.948	.000	4.316	5.122
quadtime	−0.244	0.098	8,669.000	−2.485	.013	−0.436	−0.052

[a] Dependent Variable: test.

TABLE 5.9 Estimates of Covariance Parameters[a]

Parameter		Estimate	Std. Error	Wald Z	Sig.	95% Confidence Interval	
						Lower Bound	Upper Bound
Repeated Measures	Variance	55.720	0.846	65.837	.000	54.085	57.403
Intercept + time	UN (1, 1)	35.992	1.437	25.048	.000	33.283	38.922
[subject = id]	UN (2, 1)	−1.247	0.764	−1.631	.103	−2.745	0.251
	UN (2, 2)	4.467	0.648	6.891	.000	3.361	5.936

[a] Dependent variable: test.

terms suggest that both should be retained in the model and that, on average, individuals' growth rates slow slightly over time.

Next, we can examine the covariance parameters in Table 5.9. The Level 1 estimate is 55.72 (Wald $Z = 65.837, p < .001$). At Level 2, we specified an unstructured covariance matrix, which means that there are variance estimates for the random intercept (UN 1, 1), the random linear slope (UN 2, 2), and an estimate of the covariance between them (UN 2, 1). The table suggests that there is significant variability in the random intercept to be explained between individuals (Wald $Z = 25.048$, $p < .001$). The linear time slope also varies significantly across individuals (Wald $Z = 6.891, p < .001$). The third parameter represents the covariance between the Level 2 initial status and linear growth estimates. We note that the relationship between these two parameters can depend on how the intercept is specified (i.e., with the intercept is defined as initial status, the grand mean, or end status) as well as the presence of other variables in the model (Hox, 2010). In this model, the results suggest that the covariance parameter between the initial status intercept and growth rate is not significantly different from 0 (Wald $Z = -1.631, p > .05$).

Because the covariance can be positive or negative, the two-tailed test Wald Z test implies that we could fix the covariance (UN 2, 1) in subsequent models if we wished to simplify the covariance structure. We also note in passing that researchers have cautioned about the use of these single parameter tests of significance of the model's variance components (Hox, 2010; Raudenbush & Bryk, 2002). The Wald Z test can also perform poorly under multicollinearity problems and in small sample sizes. For small samples, the likelihood ratio test (which can be estimated from MIXED output) tends to be more reliable than the Wald Z test.

Examining Orthogonal Components

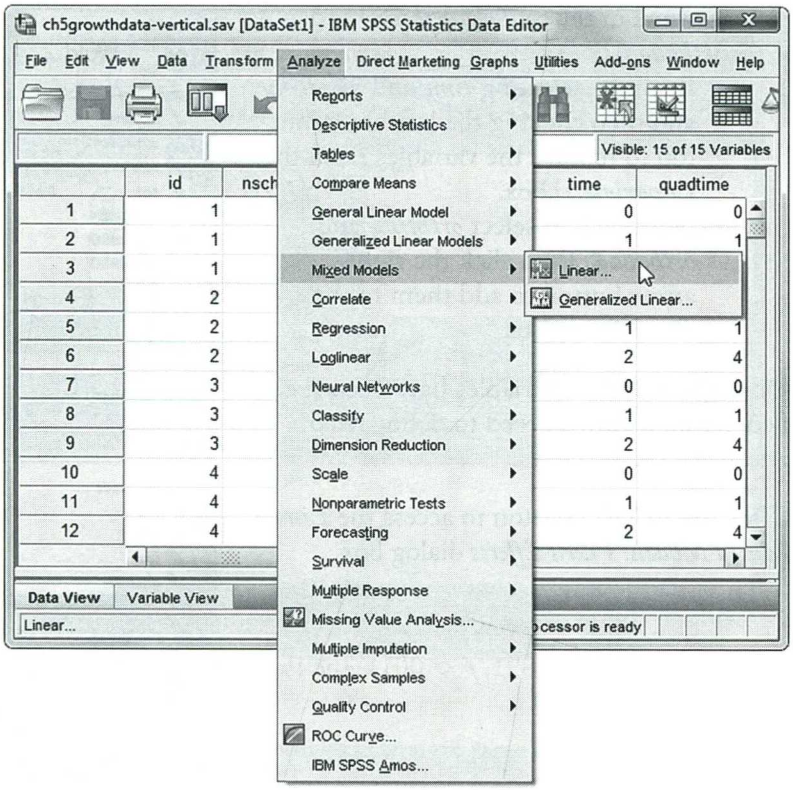
We reiterate that analysts should keep in mind that the polynomial components when untransformed are highly correlated. As we have discussed previously, when repeated measures ANOVA is used, the program automatically transforms the polynomials to be orthogonal. When we use MIXED, the recommendation is that we also transform the coded polynomial components so that they are orthogonal, or we consider other possible ways of coding the time variables. In this next model, we will use the transformed polynomial contrasts for time (*orthtime* and *orthquad*).

Defining Model 1.2 with IBM SPSS Menu Commands

Continue using the *ch5growthdata-vertical.sav* data. Settings will default to those used in Model 1.1B.

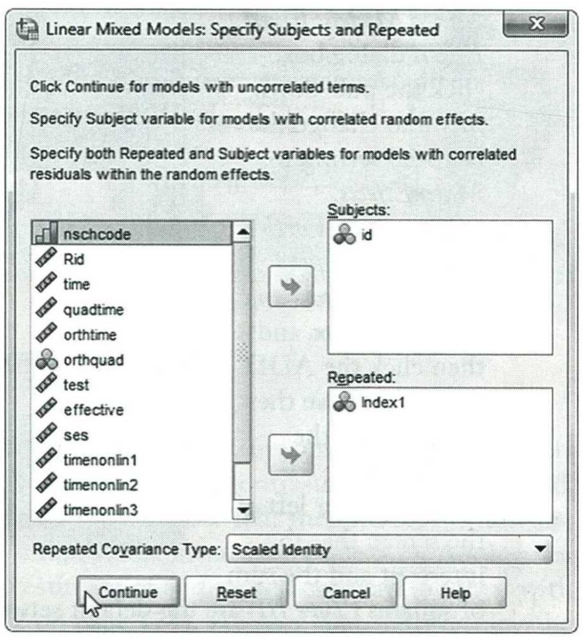
1. Go to the toolbar and select ANALYZE, MIXED MODELS, LINEAR.

This command enables access to the *Linear Mixed Models: Specify Subjects and Repeated* dialog box.

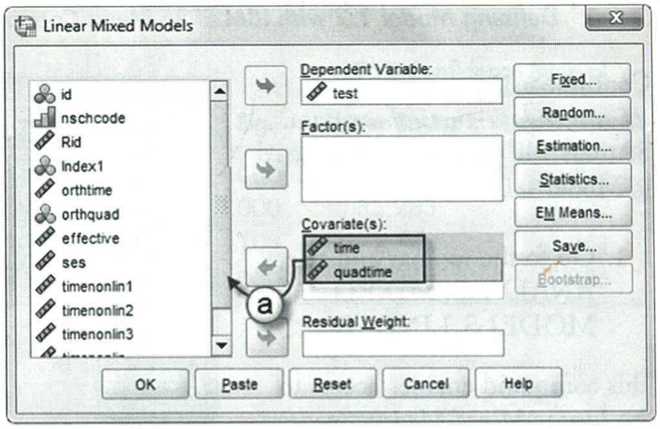


2. The *Linear Mixed Models: Specify Subjects and Repeated* box displays the default settings from the prior model.

We will retain the settings, so click the CONTINUE button to display the *Linear Mixed Models* dialog box.

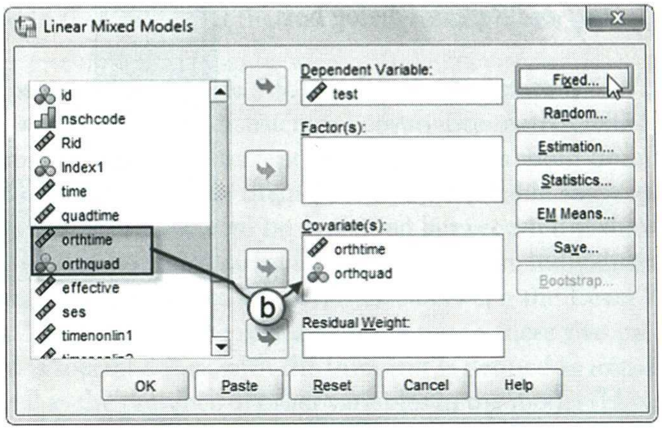


3. We will change the predictors by using the transformed polynomial contrasts for time in the model (*orthtime* and *orthquad*).
   a. Begin by selecting *time* and *quadtime* and then clicking the left-arrow button to remove the variables from the *Covariate(s)* box.
   b. Now click to select *orthtime* and *orthquad*. Then click the right-arrow button to add them to the *Covariate(s)* box.

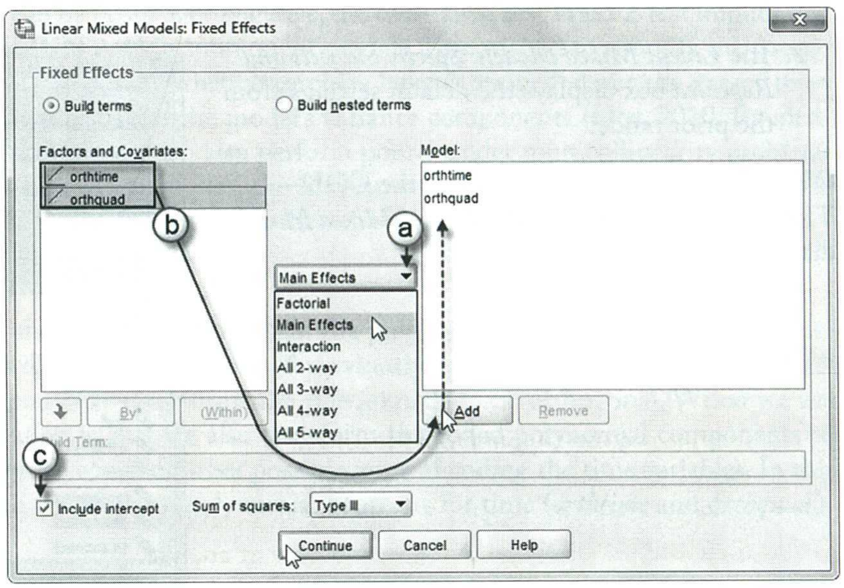


Once the predictor variables have been specified, we may now proceed to define fixed effects for the variables.

Click the FIXED button to access the *Linear Mixed Models: Fixed Effects* dialog box.



4a. Within the *Linear Mixed Models: Fixed Effects* dialog box, click the pull-down menu to change the factorial setting to *Main Effects*.
   b. Click to select *orthtime* and *orthquad* from the *Factors and Covariates* box, and then click the ADD button to move the variable into the *Model* box.
   c. Note on lower left of the screen that the intercept and the sum of squares (*Type III*) are the default settings.

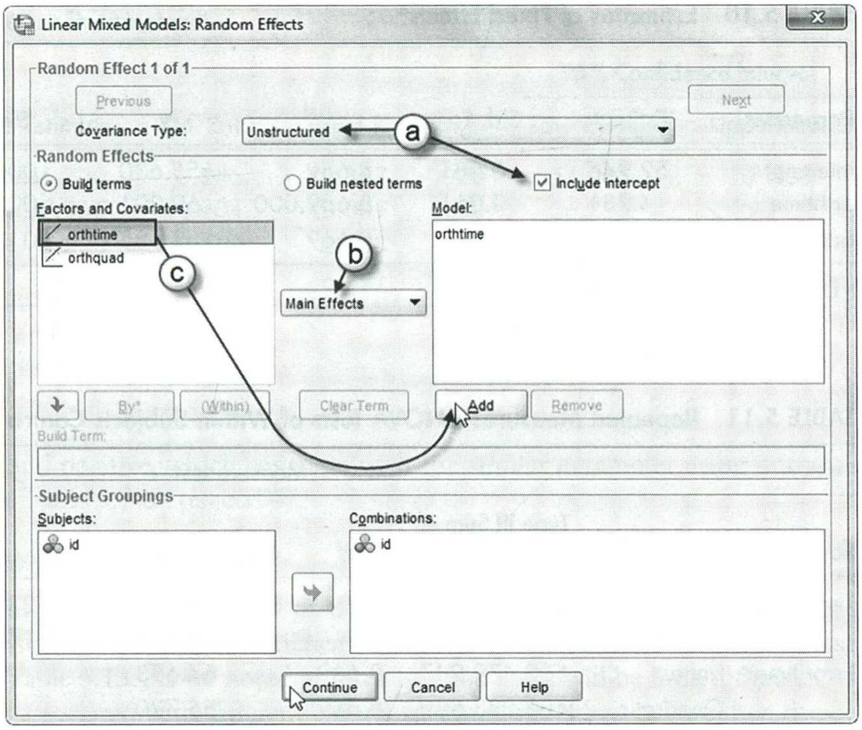


Click the CONTINUE button to return to the *Linear Mixed Models* dialog box.

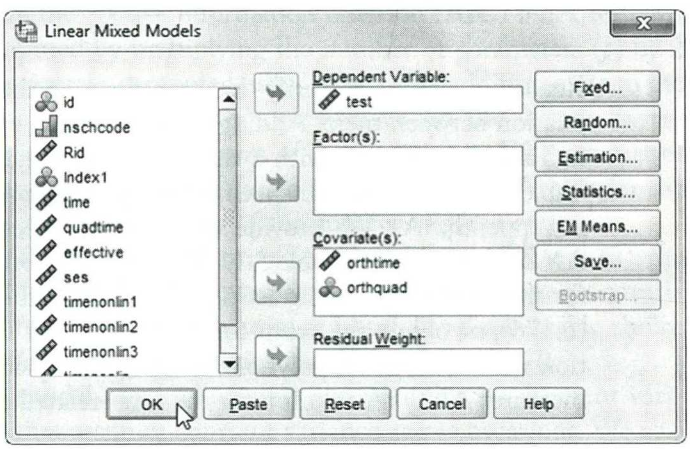
Click the RANDOM button to access the *Linear Mixed Models: Random Effects* dialog box.

5a. Within the *Linear Mixed Models: Random Effects* box, the covariance type and intercept are set to the default settings of the prior model.
b. The *Main Effects* option is preselected.
c. Click to select *orthtime* from the *Factors and Covariates* box, and then click the ADD button to move the variable into the *Model* box.

Click the CONTINUE button to return to the *Linear Mixed Models* dialog box.



6. Finally, in the *Linear Mixed Models* dialog box, click the OK button to run the model.



Interpreting the Output From Model 1.2

When we rerun the model using these transformed polynomials, we obtain the fixed-effects estimates summarized in Table 5.10. We can observe that both contrasts are significant and, therefore, should be retained in subsequent analyses. We can note that the intercept is now the grand mean of the growth trend instead of the initial status mean (see Table 5.2). Adding the growth components to the intercept provides an estimate of ending achievement (57.095) with a slight difference due to rounding (see Table 5.2 on page 174). We note that transforming the polynomials places the interpretation on the overall growth trend rather than change within any particular interval (Hox, 2010).

TABLE 5.10 Estimates of Fixed Effects[a]

Parameter	Estimate	Std. Error	*df*	*t*	Sig.	95% Confidence Interval Lower Bound	95% Confidence Interval Upper Bound
Intercept	52.945	0.081	8,669	655.630	.000	52.786	53.103
orthtime	4.231	0.061	8,669.000	69.291	.000	4.111	4.351
orthquad	–.081	0.033	8,669	–2.485	.013	–0.145	–0.017

[a] Dependent variable: test.

TABLE 5.11 Repeated Measures ANOVA Tests of Within-Subjects Contrasts

Measure:test

Source	Time	Type III Sum of Squares	*df*	Mean Square	*F*	Sig.	Noncent. Parameter	Observed Power[a]
Time	Linear	310,416.221	1	310,416.221	4,801.239	.000	4,801.239	1.000
	Quadratic	344.115	1	344.115	6.176	.013	6.176	0.700
Error(time)	Linear	560,479.947	8,669	64.653				
	Quadratic	483,033.589	8,669	55.720				

[a] Computed using alpha = 0.05.

Readers may wish to verify that transforming the polynomials does succeed in creating no correlation between them. Adding the coefficients will provide the estimate of ending achievement in Table 5.2 (57.095, with a slight difference due to rounding). We can compare the results in Table 5.10 to the similar specifications using repeated measures ANOVA (we include the syntax in Appendix A). We provide the within-subjects contrasts from the same repeated measures ANOVA analysis in Table 5.11. Readers will notice that if we take the square root of the *F* tests for the time-related contrasts (4,801.239) for the linear contrast and for the quadratic contrast (6.176), we obtain the *t* tests for the contrasts in Table 5.10. This suggests that the MIXED solution with orthogonal polynomial linear and quadratic contrasts is consistent with the repeated measures ANOVA solution for the time-related effects.

Specifying the Level 1 Covariance Structure

We also need to consider the nature of the Level 1 (within individuals) covariance matrix in a repeated measures design. For repeated measures ANOVA, Mauchly's sphericity test assumes a specific structure for the repeated measures covariance matrix. We note that the associated test for sphericity was significant (Mauchly's $W = 0.977$, 2 *df*, $p < .001$), which in a similar two-level model (with only random intercept) amounts to not accepting compound symmetry. In MIXED, there is no specific test conducted for the covariance matrix between repeated measures. We can, however, begin by examining whether the Level 1 covariance matrix is consistent with a more simplified covariance structure. We will assume that the Level 1 error covariance matrix is scaled identity (ID) and specify an unstructured (UN) covariance structure for the Level 2 random effects.

In Table 5.12, we present the covariance parameters for the Model 1.2. First, we can see that the linear contrast varies randomly across individuals (Wald $Z = 42.001$, $p < .001$). Second, we can also see that with transformed polynomials the covariance between the intercept and slope

TABLE 5.12 Estimates of Covariance Parameters[a]

Parameter		Estimate	Std. Error	Wald Z	Sig.	95% Confidence Interval	
						Lower Bound	Upper Bound
Repeated Measures	Variance	55.720	0.846	65.837	.000	54.085	57.403
Intercept + orthtime [subject = id]	UN (1, 1)	37.965	0.904	42.001	.000	36.235	39.779
	UN (2, 1)	3.220	0.460	6.993	.000	2.317	4.122
	UN (2, 2)	4.467	0.648	6.891	.000	3.361	5.936

[a] Dependent variable: test.

(UN 2, 1) is significant and positive, which contrasts with the similar parameter using untransformed polynomials (see Table 5.9 on page 204).

Investigating Other Level 1 Covariance Structures

As we suggested previously, we can test the fit of the scaled identity covariance matrix for the repeated measures against other possible covariance structures. We summarize several of these alternative structures in Table 5.13. As we noted previously, often there will be little difference in terms of the fixed effects associated with different covariance structures. How can we then decide which structure to use? For nested models, one way is to conduct a likelihood ratio test using the difference in deviance (–2 times the log of the likelihood [–2LL]) between models. For models that may not be nested, we can compare the Akaike information criterion (AIC) index for each model. The AIC index ($D + 2p$) is computed by multiplying the number of parameters (p) by 2 and adding this product to the deviance statistic, computed using FIML. The addition of $2p$ to the deviance statistic provides a penalty based on the model's complexity. We can also use the Bayesian information criterion (BIC), which is defined as the sum of the deviance and the product of the natural log (ln) of the sample size and the number of parameters [$D + \ln(n)p$]. We prefer the smallest AIC or BIC among models compared, regardless of the number of parameters.

For Model 1, with seven estimated parameters (summarized as Model 1.2 in Table 5.12), the AIC is 189,290.45. After examining Models 2–4, we might select Model 3, with diagonal covariance structure at Level 1 and unstructured covariance structure at Level 2 as a reasonable choice based on available model-fitting information. For this model, the AIC is 189,125.70.

In Table 5.14, we can see that this model more adequately accounts for the different variances between occasions than a model that assumes constant variance across occasions. We also

TABLE 5.13 Comparing Models, AIC Index, and Number of Parameters

Model	Model Description	AIC	Parameters
Model 1	Identity Covariance Matrix, Level 1; Unstructured Matrix, Level 2	189,290.45	7
Model 2	Diagonal Covariance Matrix, Level 1; Diagonal Covariance Matrix, Level 2	189,295.75	8
Model 3	Diagonal Covariance Matrix, Level 1; Unstructured Covariance Matrix, Level 2	189,125.70	9
Model 4	Autoregressive Errors (AR1), Level 1; Diagonal Covariance Matrix, Level 2	189,282.16*	8

*Note: Model did not converge.

TABLE 5.14 Estimates of Covariance Parameters[a]

Parameter		Estimate	Std. Error	Wald Z	Sig.	95% Confidence Interval Lower Bound	95% Confidence Interval Upper Bound
Repeated Measures	Var: [Index1 = 1]	63.583	2.026	31.385	.000	59.734	67.681
	Var: [Index1 = 2]	58.779	1.158	50.756	.000	56.552	61.093
	Var: [Index1 = 3]	35.619	1.980	17.987	.000	31.942	39.720
Intercept + orthtime [subject = id]	UN (1, 1)	38.985	0.946	41.216	.000	37.175	40.884
	UN (2, 1)	7.881	0.610	12.926	.000	6.686	9.076
	UN (2, 2)	7.526	0.964	7.805	.000	5.855	9.675

[a] Dependent variable: test.

investigated a number of other possibilities at Level 1 (unstructured and autoregressive) but found that the model did not converge. We suggest, therefore, that models should generally be judged on various criteria including model fit indices, as well as their substance and sensibility in relation to the study's research purposes.

Defining Other Level 1 Covariance Structures Using IBM SPSS Menu Commands

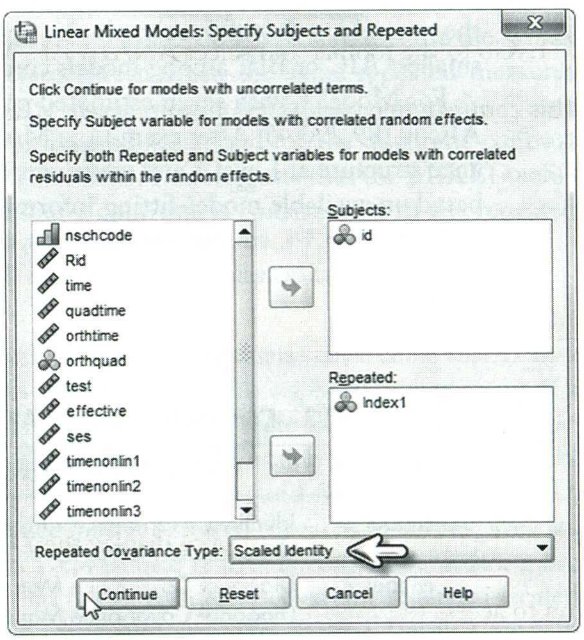
(Examples in this section are based on Model 1.2, with scaled identity covariance structure at Level 1 and unstructured covariance structure at Level 2, but illustrate changes to the Level 1 and Level 2 covariance structures as summarized in Table 5.13.)

Model 1: ID (Level 1), UN (Level 2)

1. Go to the toolbar and select ANALYZE, MIXED MODELS, LINEAR.

This command enables access to the *Linear Mixed Models: Specify Subjects and Repeated* dialog box.

Scaled Identity Covariance Matrix at Level 1



2. The *Linear Mixed Models: Specify Subjects and Repeated* dialog box displays the default setting used for Model 1.2. Note that the *Repeated Covariance Type* function is set as *Scaled Identity*.

The *Scaled Identity* covariance structure has heterogeneous variances and zero correlation between elements (IBM Corporation, 2012).

Click the CONTINUE button to display the *Linear Mixed Models* dialog box. Click the RANDOM button to access the *Linear Mixed Models: Random Effects* dialog box.

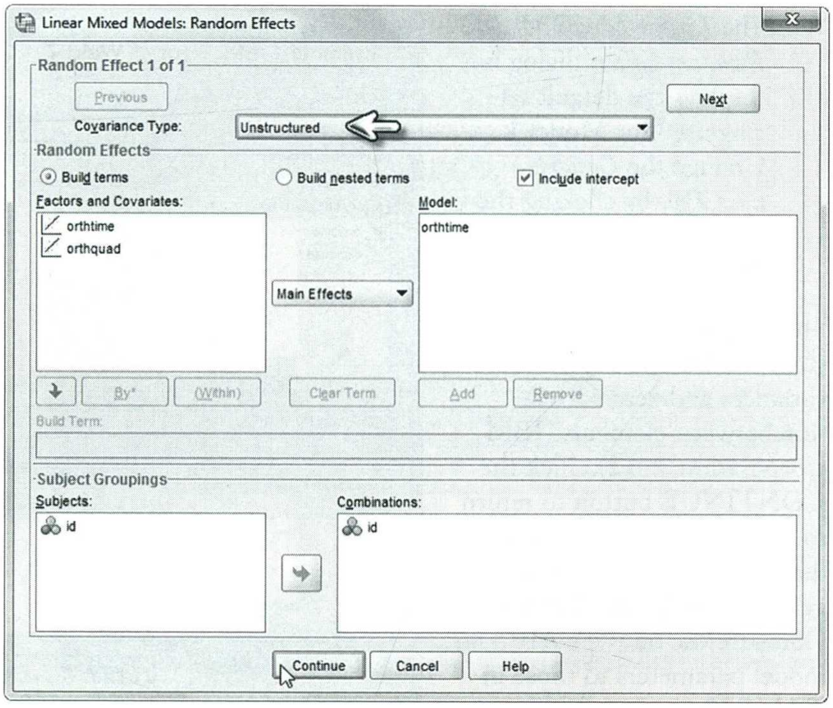
Unstructured Covariance Matrix at Level 2

3. The *Linear Mixed Models: Random Effects* dialog box displays the default setting used for Model 1.2. Note that the *Covariance Type* function is set as *Unstructured.*

Unstructured is a completely general covariance matrix (IBM Corporation, 2012).

Click the CONTINUE button to return to the *Linear Mixed Models* dialog box. Then click OK to generate the Model 1 results. Compare the output's AIC and model parameters to those in Table 5.13.

(Settings default to Model 1.)



Model 2: DIAG (Level 1), DIAG (Level 2)

1. Go to the toolbar and select ANALYZE, MIXED MODELS, LINEAR.

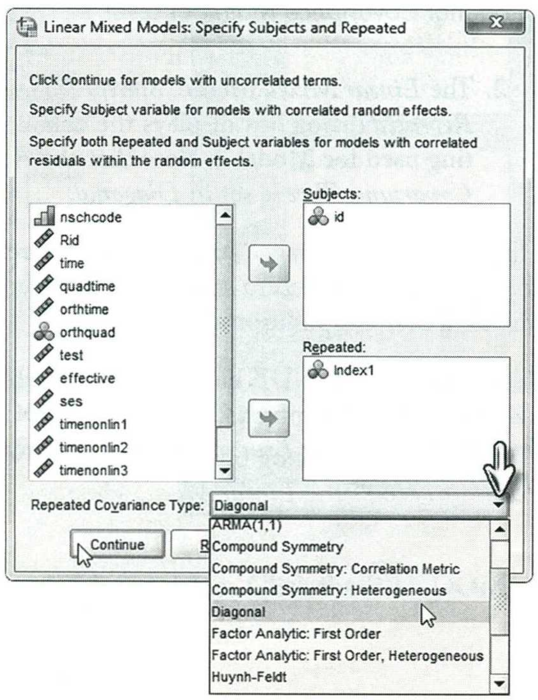
This command enables access to the *Linear Mixed Models: Specify Subjects and Repeated* dialog box.

Diagonal Covariance Matrix at Level 1

2. The *Linear Mixed Models: Specify Subjects and Repeated* dialog box displays the default setting used for Model 1. Change the covariance setting by clicking the pull-down menu to select *Diagonal.*

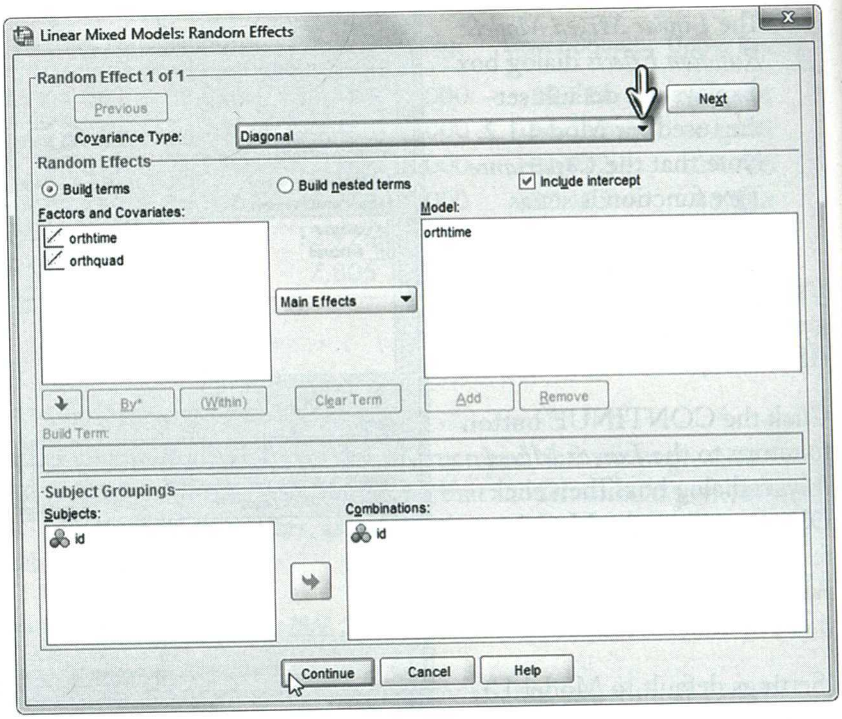
The *Diagonal* covariance structure has heterogeneous variances and zero correlation between elements (IBM Corporation, 2012).

Click the CONTINUE button to display the *Linear Mixed Models* dialog box. Click the RANDOM button to access the *Linear Mixed Models: Random Effects* dialog box.



Diagonal Covariance Matrix at Level 2

3. The *Linear Mixed Models: Random Effects* dialog box displays the default setting used for Model 1. Change the *Covariance Type* by clicking the pull-down menu to select *Diagonal.*



The *Diagonal* covariance structure has heterogeneous variances and zero correlation between elements (IBM Corporation, 2012). Click the CONTINUE button to return to the *Linear Mixed Models* dialog box. Then click OK to generate the Model 2 results. Compare the output's AIC and model parameters to those in Table 5.13.

(Settings default to Model 2.)

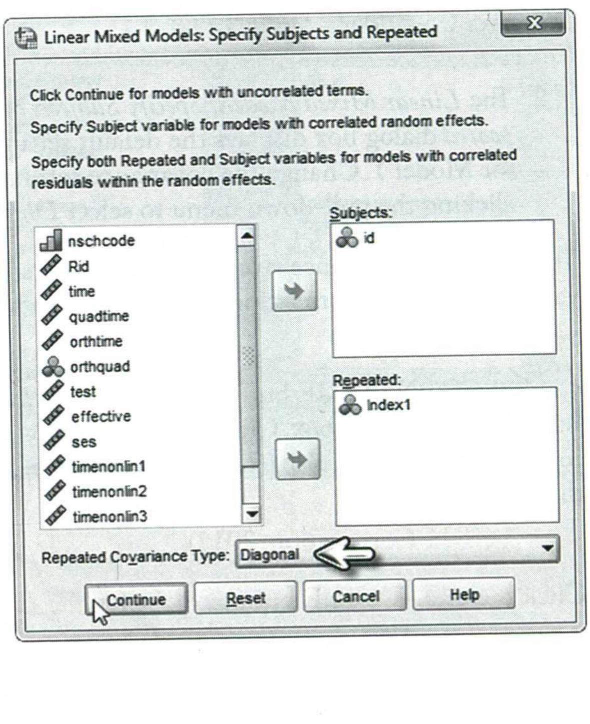
Model 3: DIAG (Level 1), UN (Level 2)

1. Go to the toolbar and select ANALYZE, MIXED MODELS, LINEAR.

This command enables access to the *Linear Mixed Models: Specify Subjects and Repeated* dialog box.

Diagonal Covariance Matrix at Level 1

2. The *Linear Mixed Models: Specify Subjects and Repeated* dialog box displays the default setting used for Model 2. Note that the *Repeated Covariance Type* is set to *Diagonal.*

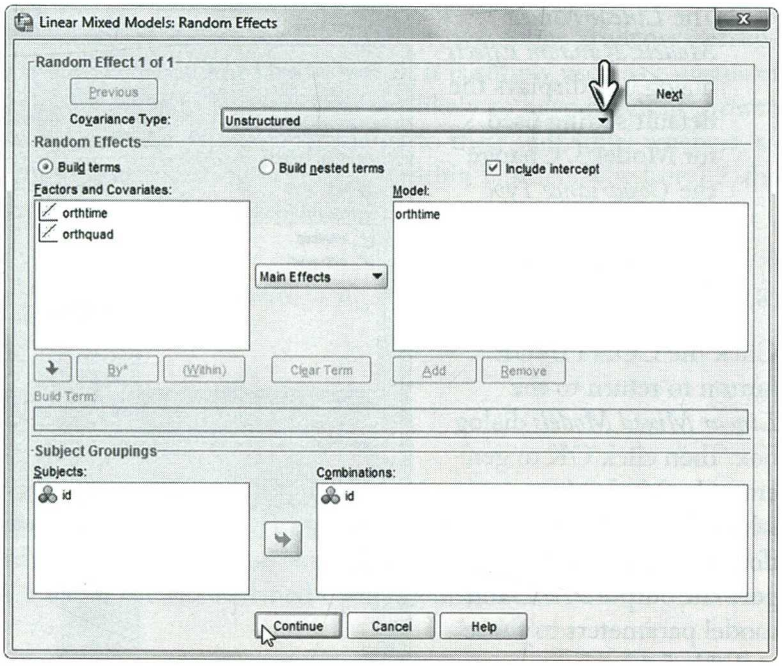


The *Diagonal* covariance structure has heterogeneous variances and zero correlation between elements (IBM Corporation, 2012).

Click the CONTINUE button to display the *Linear Mixed Models* dialog box. Click the RANDOM button to access the *Linear Mixed Models: Random Effects* dialog box.

Unstructured Covariance Matrix at Level 2

3. The *Linear Mixed Models: Random Effects* dialog box displays the default setting used for Model 2. Change the *Covariance Type* by clicking the pull-down menu to select *Unstructured.*



Unstructured is a completely general covariance matrix (IBM Corporation, 2012).

Click the CONTINUE button to return to the *Linear Mixed Models* dialog box. Then click OK to generate the Model 3 results. Compare the output's AIC and model parameters to those in Table 5.13.

(Settings default to Model 3.)

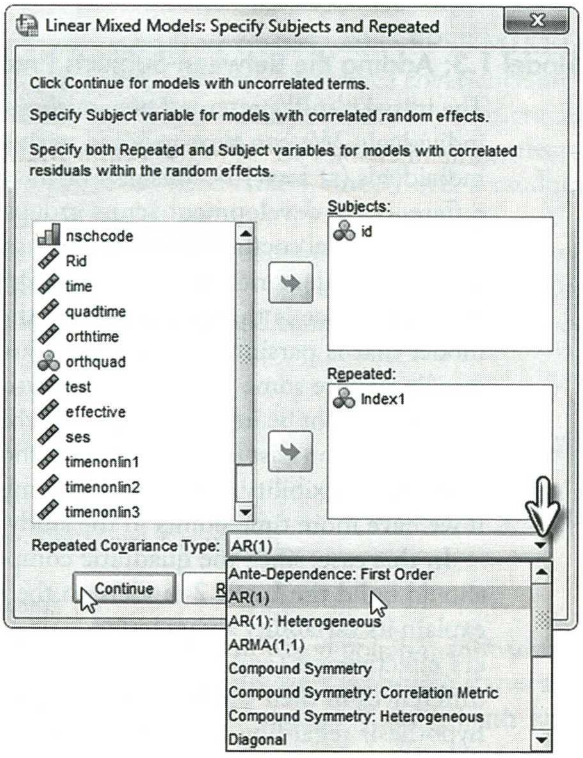
Model 4: AR1 (Level 1), DIAG (Level 2)

1. Go to the toolbar and select ANALYZE, MIXED MODELS, LINEAR.

This command enables access to the *Linear Mixed Models: Specify Subjects and Repeated* dialog box.

Autoregressive Errors (AR1) Covariance Matrix at Level 1

2. The *Linear Mixed Models: Specify Subjects and Repeated* dialog box displays the default setting used for Model 3. Change the covariance setting by clicking the pull-down menu to select *AR(1).*



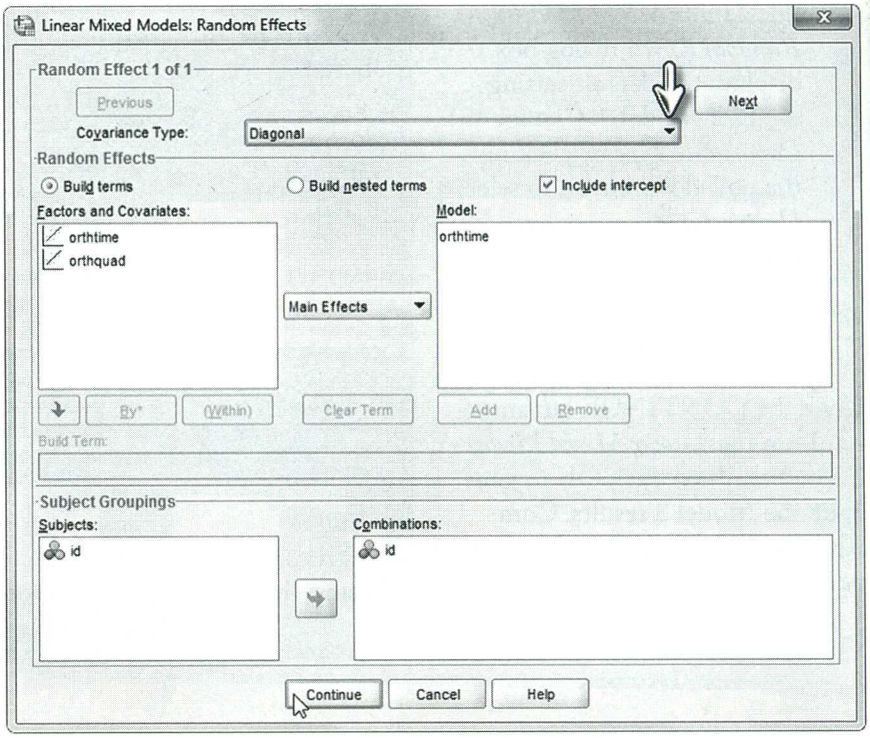
AR(1) is a first-order autoregressive structure with homogenous variances. The correlation between any two elements is equal to rho (ρ) for adjacent elements, ρ^2 for elements that are separated by a third, and so on. We note that ρ is constrained so that $-1 < \rho < 1$ (IBM Corporation, 2012).

Click the CONTINUE button to display the *Linear Mixed Models* dialog box. Click the RANDOM button to access the *Linear Mixed Models: Random Effects* dialog box.

Diagonal Covariance Matrix at Level 2

3. The *Linear Mixed Models: Random Effects* dialog box displays the default setting used for Model 3. Change the *Covariance Type* by clicking the pull-down menu to select *Diagonal.*

Click the CONTINUE button to return to the *Linear Mixed Models* dialog box. Then click OK to generate the Model 4 results, which shows that the model does not converge. Compare the output's AIC and model parameters to those in Table 5.13.



Model 1.3: Adding the Between-Subjects Predictors

The initial results presented previously suggest that intercepts and linear growth rates vary across individuals. We can next proceed with explaining variability in the random parameters across individuals (at Level 2). This part of the analysis addresses the third research question: Are there differences in development across groups of individuals? A first issue to be addressed is which of the polynomial coefficients should be used to build our successive models concerning differences in growth trajectories by student background factors (e.g., gender and socioeconomic status)? The best advice is that it depends on the specifics of the data set. We suggest trying to build a model that is parsimonious but does justice to the particular features of the data set. This will usually require some preliminary analyses. This part of specifying the model can be challenging since it may not be immediately clear which components may be randomly varying and on which polynomial contrast (or contrasts) we should actually build the explanatory model. Of course, we have more flexibility regarding the number of time-related random effects that can be specified if we have more time points in the study.

In this case, since the quadratic component is fixed within individuals, it seems clear that we should build the Level 2 models on the randomly varying linear component; that is, we seek to explain its variability across individuals. We will propose that students' perceptions of their teachers' effectiveness (coded 1 = effective vs. 0 = average or below) and students' SES might explain differences in their math intercepts and linear growth rates. In order to examine the parallelism hypothesis regarding variability in linear growth rates for different student subgroups (i.e., that growth rates are the same for students having effective vs. not effective teachers), or to consider the impact of an interval variable such as SES on student growth rates, we need to create cross-level interaction terms.

Cross-level interactions involve the effects of Level 2 (between-individual) variables such as teacher effectiveness on a Level 1 slope coefficient—that is, students' growth rates. As we have shown in previous chapters, cross-level interaction terms can be created within SPSS MIXED in the menu command format. We reiterate that for this exercise, we define students' perceptions of teacher effectiveness as a between-student factor; that is, it is simply used as a means of identifying subsets of students (e.g., similar to gender) who are likely to have different growth trajectories. In this example, we do not link groups of students to their individual teachers, so we will not consider the more complicated nesting of students within teachers. For Level 2, the following equations can be formulated:

$$\pi_{0i} = \beta_{00} + \beta_{01}ses_i + \beta_{02}effective_i + u_{0i}, \tag{5.18}$$

$$\pi_{1i} = \beta_{10} + \beta_{11}ses_i + \beta_{12}effective_i + u_{1i}, \tag{5.19}$$

where u_{0i} and u_{1i} represent variation associated with estimating the intercept and slope parameters between individuals. The quadratic component is specified as fixed at Level 2 ($\pi_{2i} = \beta_{20}$) as in Equation 5.12. When we substitute the intercept and slope equations (Eqs. 5.18, 5.19, and 5.12) into the Level 1 model (Eq. 5.1 using the transformed components), we obtain the following combined equation:

$$\begin{aligned} Y_{ti} = {} & \beta_{00} + \beta_{01}ses_i + \beta_{02}effective_i + \beta_{10}orthtime_{ti} + \beta_{11}ses_i{}^*orthtime_{ti} + \\ & \beta_{12}effective_i{}^*orthtime_{ti} + \beta_{20}orthquad_{ti} + u_{1i}\; orthtime_{ti}\; u_{0i} + \varepsilon_{ti}. \end{aligned} \tag{5.20}$$

We note that the cross-level interaction terms in Equation 5.20 are created within SPSS MIXED in the menu command format. We reiterate that the Level 1 repeated measures covariance matrix in Model 1.3 is diagonal (as in Eq. 5.5) and the Level 2 covariance matrix is unstructured (Eq. 5.17). The dimensionality of the Level 2 matrix depends on the number of random effects. This makes a total of 13 parameters to estimate (i.e., seven fixed effects, three Level 2 random effects, and three Level 1 residual variances).

If we wished to examine possible differences in the quadratic component (even though we will treat this component as fixed across individuals), we could also add the Level 2 predictors to that portion of the model, but the random component (u_{2i}) would remain 0 (Raudenbush & Bryk, 2002):

$$\pi_{2i} = \beta_{20} + \beta_{21}ses_i + \beta_{22}effective_i \tag{5.21}$$

This equation could then also be substituted into the Level 1 equation.

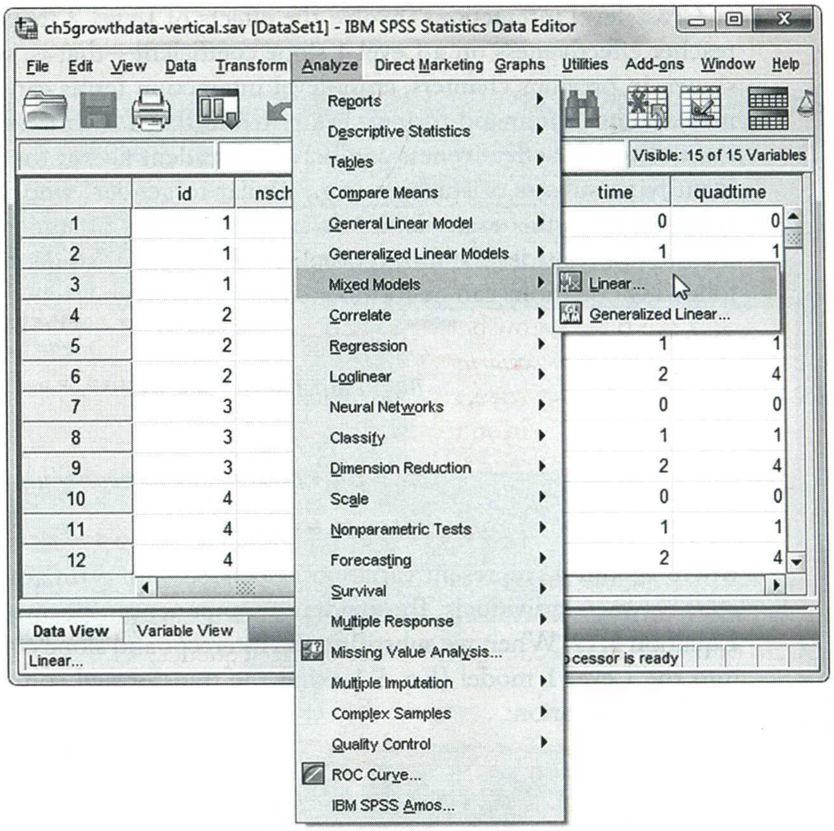
Defining Model 1.3 with IBM SPSS Menu Commands

In this model, we proposed that students' initial status intercept (π_{0i}) will vary across individuals, and this variation in intercepts will in part be explained by students' socioeconomic status and teacher effectiveness. We also proposed that variation in students' average linear growth rates (π_{1i}) will be explained by the same predictors.

Continue using the *ch5growthdata-vertical.sav* data. If continuing from the "Investigating Other Level 1 Covariance Structures" examples, please change settings to those used for Model 1.2 before proceeding.

1. Go to the toolbar and select ANALYZE, MIXED MODELS, LINEAR.

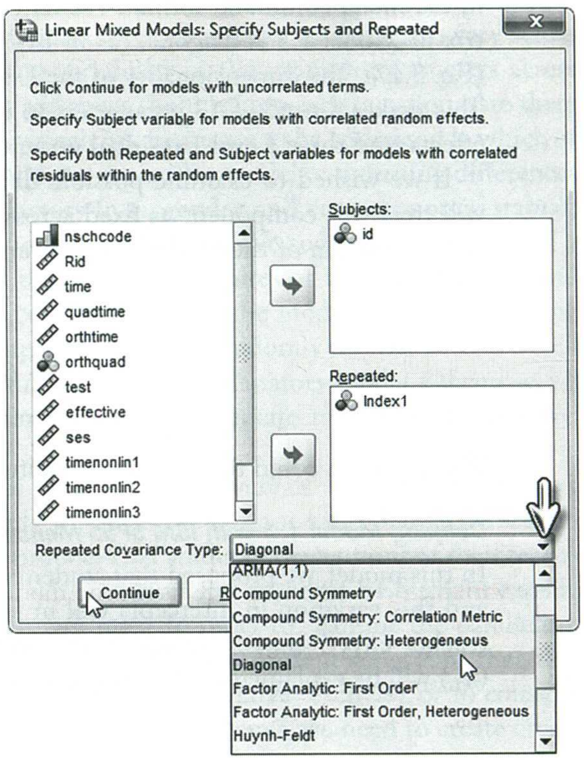
This command enables access to the *Linear Mixed Models: Specify Subjects and Repeated* dialog box.



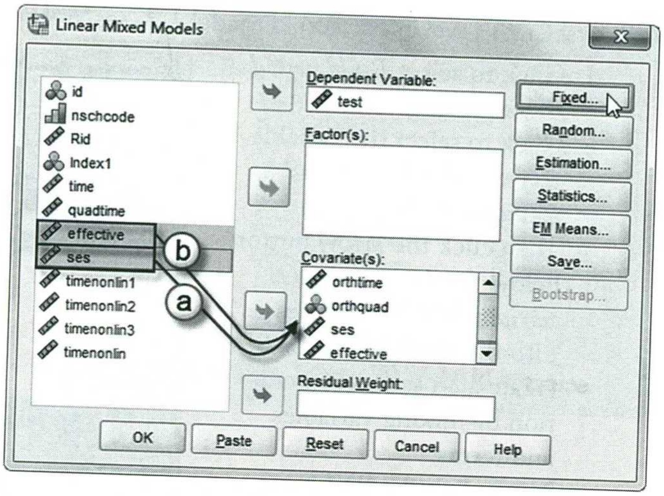
2. The *Linear Mixed Models: Specify Subjects and Repeated* displays the default settings from Model 1.2.

Change the covariance setting by clicking the *Repeated Covariance Type* pull-down menu to select *Diagonal.*

Click the CONTINUE button to display the *Linear Mixed Models* dialog box



3. The *Linear Mixed Models* dialog box displays test in the *Dependent Variable* box with *orthtime* and *orthquad* in the *Covariate(s)* box.
   a. We will add two variables (*ses* and *effective)* into the model. Entering variables in sequential order helps facilitate reading of the output tables. So first click to select *ses*, and then click the right-arrow button to add the variable the *Covariate(s)* box.
   b. Click to select *effective*, and then click the right-arrow button to add the variable the *Covariate(s)* box.

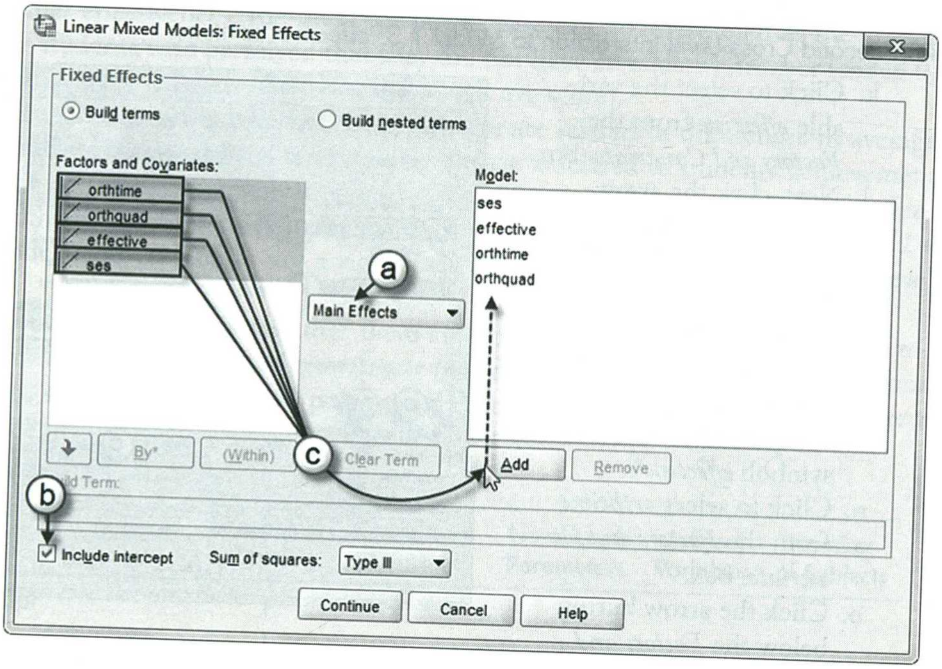


The variable sequence is *orthtime*, *orthquad*, *ses*, and *effective.*

Note: An alternative method is to select both variables simultaneously and then "drag" them into the *Covariate(s)* box. The variables may be rearranged by clicking to select a variable and then dragging it upward or downward to change the order of the sequence.

Click the FIXED button to access the *Linear Mixed Models: Fixed Effects* dialog box.

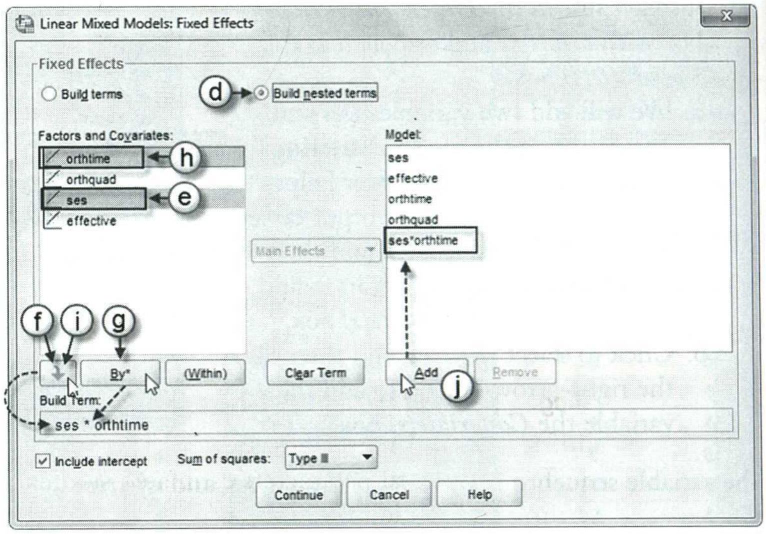
4a. Within the *Linear Mixed Models: Fixed Effects* dialog box, confirm that *Main Effects* is selected.
b. Confirm that *Include intercept* is selected.
c. To facilitate reading the output tables, click to select a variable, and then click the ADD button to move it into the *Model* box in the following order: *ses*, *effective*, *orthtime*, and *orthquad.*



These variables are the "main effects" in the model and will modify the intercept.

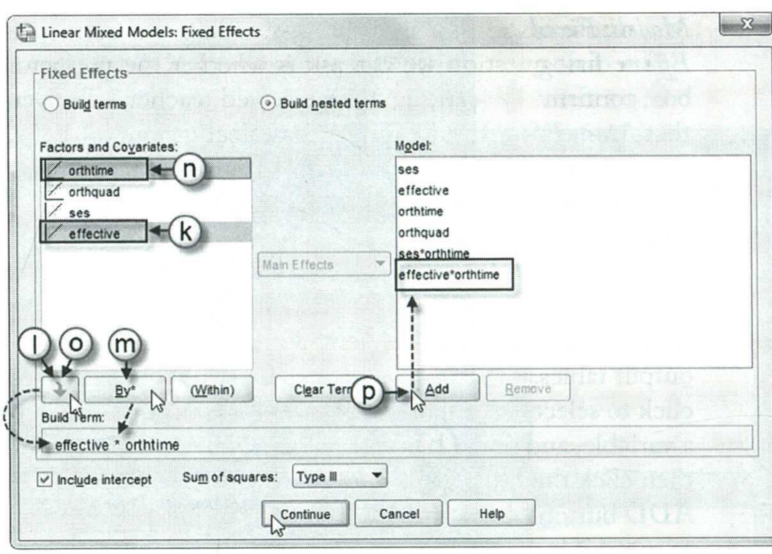
Two cross-level interactions (or nested terms) will be created and added to the model: *ses*orthtime* and *effective*orthtime*. These interactions will tell us if the growth trajectories are parallel for different groups of students.

Add First Cross-Level Interaction to Model 1.3: *ses*orthtime*



d. Click to select *Build nested terms.*
e. Click to select the variable *ses* from the *Factors and Covariates* box.
f. Then click the arrow button below the *Factors and Covariates* box. This moves *ses* into the *Build Term* box to create a cross-level interaction by linking variables and terms.
g. Next, click the BY* button, which will insert the computation command symbol: *ses**.
h. Click to select *orthtime* from the *Factors and Covariates* box.
i. Click the arrow button below the *Factors and Covariates* box to move *orthtime* into the *Build Term* box and complete the interaction term: *ses*orthtime.*
j. Click the ADD button to transfer the interaction into the *Model* box.

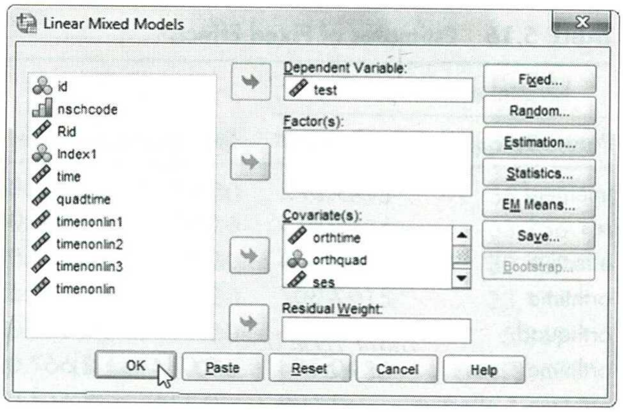
Add Second Cross-Level Interaction to Model 1.3: *effective*orthtime*



k. Click to select the variable *effective* from the *Factors and Covariates* box.
l. Next, click the arrow button below the *Factors and Covariates* box. This moves *effective* into the *Build Term* box.
m. Now click the BY* button, which will insert the computation command symbol: *effective**.
n. Click to select *orthtime* from the *Factors and Covariates* box.
o. Click the arrow button below the *Factors and Covariates* box to move *orthtime* into the *Build Term* box and complete the interaction term: *effective*orthtime.*
p. Click the ADD button to transfer the interaction into the *Model* box.

Click the CONTINUE button to return to the *Linear Mixed Models* dialog box.

5. Finally, in the *Linear Mixed Models* dialog box, click the OK button to run the model.



Interpreting the Output From Model 1.3

Results of the second model test are presented next. Once again, it is useful to examine the random effects and total parameters estimated summarized in Table 5.15. There are seven fixed effects estimated. In addition, there are six variance-covariance parameters to be estimated. At Level 1 (within individuals), there are three variance parameters (i.e., the variances for each occasion). These are listed in the diagonal covariance matrix at Level 1. In addition, there are three random effects at Level 2 (i.e., the intercept, the linear time slope, and the covariance between them). This suggests 13 total parameters to be estimated.

The fixed-effect estimates are summarized in Table 5.16. Students' achievement intercept (β_{00}) is 49.672. This can be described as students' true grand-mean achievement adjusted for SES and perceptions of teacher effectiveness. The intercept in this case can be interpreted as the grand-mean test score for students who perceived that they did not have an effective teacher (coded 0) and whose SES status was 0.00 (since SES was defined as a z-score).

The first question we can ask is whether the predictors are related to differences in average achievement. We can see that perceived teacher effectiveness is related to students' achievement level ($p < .001$). The coefficient for effectiveness ($\beta_{02} = 5.944$) suggests that students with effective teachers would have an estimated grand-mean achievement level of about 55.616 (49.672 + 5.944). In this model, student SES is not a significant predictor of average achievement level ($p > .10$).

The second question we can ask is whether there are differences in student growth rates related to the predictors. The average linear growth rate (*orthtime*) increases significantly over time ($p < .001$). Keep in mind that the actual polynomial contrast coefficients do not have direct

TABLE 5.15 Model Dimension[a]

		Number of Levels	Covariance Structure	Number of Parameters	Subject Variables	Number of Subjects
Fixed Effects	Intercept	1		1		
	ses	1		1		
	effective	1		1		
	orthtime	1		1		
	orthquad	1		1		
	orthtime * ses	1		1		
	orthtime * effective	1		1		
Random Effects	Intercept + orthtimea	2	Unstructured	3	id	
Repeated Effects	Index1	3	Diagonal	3	id	8,670
Total		12		13		

[a] Dependent variable: test.

TABLE 5.16 Estimates of Fixed Effects[a]

Parameter	Estimate	Std. Error	df	t	Sig.	95% Confidence Interval Lower Bound	95% Confidence Interval Upper Bound
Intercept	49.672	0.111	8,701.861	447.802	.000	49.455	49.890
ses	0.074	0.095	8,667.000	0.784	.433	−0.112	0.260
effective	5.944	0.149	8,667.000	39.838	.000	5.652	6.237
orthtime	2.307	0.086	8,795.556	26.693	.000	2.138	2.477
orthquad	−0.081	0.033	8,669.000	−2.485	.013	−0.145	−0.017
orthtime * ses	−0.154	0.074	8,667.000	−2.090	.037	−0.298	−0.010
orthtime * effective	3.508	0.116	8,667.000	30.324	.000	3.281	3.735

[a] Dependent variable: test.

meaning regarding a particular time interval after they have been transformed. Because the linear contrast was specified as a random effect, we are primarily interested in whether it varies between individuals in the study. The variation in the size of the within-individual growth parameter across individuals (UN 2, 2) can be examined by referring to Wald Z test (Wald $Z = 8.547$, $p < .001$) in the variance components table (Table 5.17). The significant test suggests we can reject the null hypothesis that the population growth parameter is 0, and we can infer that growth varies significantly across the population of individuals.

Regarding variables that might explain variability in math growth rates between individuals, we can see that the linear interaction (*orthtime**ses*) is significant at $p < .05$ ($\beta_{11} = -0.154$, $p = .037$). This coefficient can be interpreted as students at higher SES levels demonstrate slightly less growth over time compared with students at the grand mean for SES. The test for students' perceptions of teacher effectiveness is also significant ($\beta_{12} = 3.508, p < .001$). This suggests that students with teachers they perceived as effective have a higher growth rate over time compared to their peers who perceive their teachers to be average or below in effectiveness (since they are coded 0). The quadratic polynomial is also significant, which implies that student growth rates slow slightly over time ($\beta_{20} = -0.081, p < .001$). In this model, we did not propose any cross-level interaction related to explaining the slowing of math growth rates.

After the addition of the predictors, Table 5.17 suggests that there is still significant residual variance in intercepts to be explained (Wald $Z = 37.906, p < .001$). There is also significant residual variance in slopes left to be explained across individuals (Wald $Z = 8.547, p < .001$). The covariance between the intercept and slope is positive (4.159) and also significant ($p < .001$). If desired, we could select an unstructured covariance matrix with a correlation for the slope parameter (UNR). In this case, the correlation between students' initial status (i.e., intercept) and growth of time is 0.27 (not tabled). We reiterate that while this correlation is often of interest in growth models, it can be different depending on how the intercept and slope are defined and the other variables added to the model.

For comparative purposes in Table 5.18, we also provide the repeated measures ANOVA solution that we would obtain for the polynomial contrasts. The relevant output is the tests of within-subject contrasts. That solution also has additional tests of contrasts for the quadratic time-related component and the predictors in the model (i.e., SES and teacher effectiveness). We can see that the linear effect is also significant for individual SES in the ANOVA formulation (but the quadratic effect is not), and the linear and quadratic effects are both significant for *effective* ($p < .001$) We could of course provide this same set of model tests using MIXED by adding the two quadratic contrasts to the fixed-effect model.

TABLE 5.17 Estimates of Covariance Parameters[a]

Parameter		Estimate	Std. Error	Wald Z	Sig.	95% Confidence Interval Lower Bound	95% Confidence Interval Upper Bound
Repeated Measures	Var: [Index1=1]	61.400	2.004	30.641	.000	57.595	65.456
	Var: [Index1=2]	61.448	1.156	53.158	.000	59.224	63.756
	Var: [Index1=3]	27.124	1.727	15.708	.000	23.942	30.729
Intercept + orthtime [subject = id]	UN (1, 1)	31.503	0.831	37.906	.000	29.915	33.174
	UN (2, 1)	4.159	0.566	7.343	.000	3.049	5.269
	UN (2, 2)	7.465	0.873	8.547	.000	5.935	9.389

[a] Dependent variable: test.

TABLE 5.18 Tests of Within-Subjects Contrasts

Measure:test

Source	Time	Type III Sum of Squares	df	Mean Square	F	Sig.	Noncent. Parameter	Observed Power[a]
time	Linear	283,779.090	1	283,779.090	4,795.547	.000	4,795.547	1.000
	Quadratic	637.493	1	637.493	11.551	.001	11.551	0.925
time * ses	Linear	244.668	1	244.668	4.135	.042	4.135	0.529
	Quadratic	1.270	1	1.270	0.023	.879	0.023	0.053
time * effective	Linear	47,359.971	1	47,359.971	800.330	.000	800.330	1.000
	Quadratic	4,712.361	1	4,712.361	85.386	.000	85.386	1.000
Error(time)	Linear	512,874.459	8,667	59.176				
	Quadratic	478,319.939	8,667	55.189				

[a] Computed using alpha = 0.05.

In Table 5.19, we provide a summary of the various models we have tested using untransformed and transformed polynomials. Model 1 summarizes the initial results for time with the polynomial contrasts untransformed. In Model 1, without transformed time-related contrasts, the intercept (48.632) corresponds to student achievement at the beginning of the study. Model 3 is the same model as Model 1 but with orthogonal polynomial contrasts. In Model 3, the intercept (52.945) now corresponds to the grand mean for the trend, which is somewhere between interval 1 and interval 2. We can see that the polynomial components are significant ($p < .05$), again suggesting they should be retained in subsequent analyses. Because the intercept is now the grand mean, however, the time-related estimates no longer correspond to any particular time interval in the study.

Readers can see that the pattern of results is the same despite the strong correlation between the linear and quadratic polynomial components in Model 1. Model 2 in Table 5.19 summarizes the Level 2 effects built only on the randomly varying linear time effect. Model 4 presents those same results but with transformed time-related components. Once again, the pattern of results is almost the same for Models 3 and 4 (with only slight differences in the variance parameters). Model 5 represents the MIXED specification of the repeated measures ANOVA analysis, which is in the last column of the table. Regarding the ANOVA results in the last column, as noted previously, the predictors enter in as interactions with growth over time (i.e., *linear effect*SES*,

TABLE 5.19 Comparing Results of Untransformed Polynomial Models (1–2) and Orthogonal Polynomial Models (3–5) with Repeated Measures ANOVA

Variables	Model 1	Model 2	Model 3	Model 4	Model 5	Within-Subjects ANOVA
Intercept	48.632*	47.284*				
time	4.719*	2.795*				
time quadratic	−0.244*	−0.244*				
SES		0.228				
effective		2.436*				
time*SES		−0.154*				
time*effective		3.508*				
Intercept			52.945*	49.672*	49.741*	NA
orthtime			4.231*	2.307*	2.409*	*
orth quadratic			−0.081*	−0.081*	−0.414*	*
SES				0.074	0.076	NA
effective				5.944*	5.820*	NA
orthtime*SES				−0.154*	−0.152*	*
orth quadratic*effective				3.508*	3.322*	*
orth quadratic*SES					−0.006	
orth quadratic*effective					0.605*	*
AIC	189,123.5	186,832.7	189,125.7	186,834.9	186,758.0	NA

*$p < .05$; NA = parameter is not applicable to the ANOVA model.

*linear effect*teacher effectiveness*, *quadratic effect*SES*, and *quadratic effect*teacher effectiveness*). As shown in the last column, five of the six effects specified are significant, which exactly matches the MIXED results in Model 5, as well as the repeated measures within-subject contrasts summarized in Table 5.18.

Graphing the Results

We can summarize the initial difference in growth trajectories with a graph of different growth rates by teacher effectiveness (Figure 5.8). The trajectories are best interpreted as not parallel over time; that is, the initial observed gap in test learning between students with effective and ineffective teachers widens over time. This graph will look slightly different from the one produced in *Repeated Measures* (MANOVA) since there is no control for student SES in the latter graph. This graph can be produced using the following IBM SPSS commands.

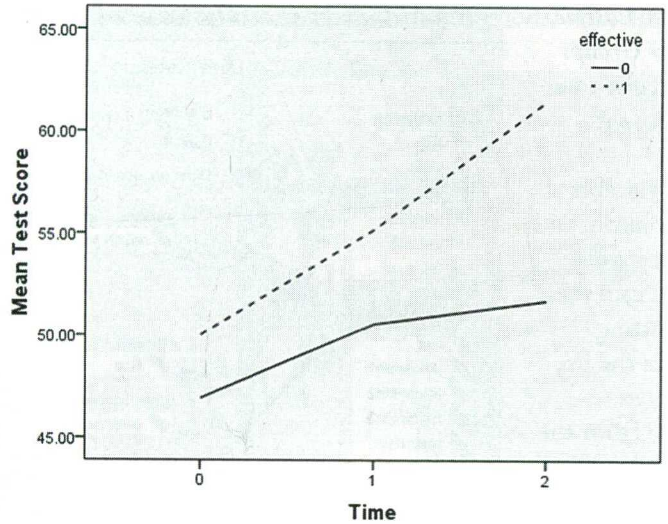
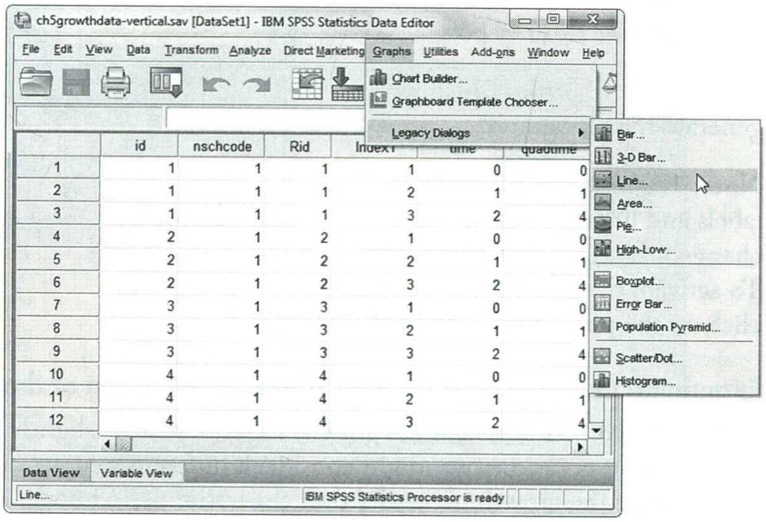


FIGURE 5.8 Individual growth trajectories differentiated by teacher effectiveness.

Graphing the Growth Rate Trajectories with SPSS Menu Commands

1. Go to the toolbar and select GRAPHS, LEGACY DIALOGS, LINE.

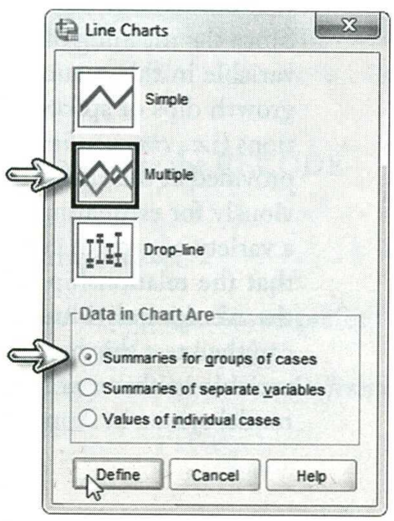
This command will open the *Line Charts* dialog box.



2. In the *Line Charts* dialog box, click to select *Multiple.*

Confirm that *Summaries for groups of cases* is selected.

Click the DEFINE button to continue, which will open the *Define Multiple Line: Summaries for Groups of Cases* dialog box.

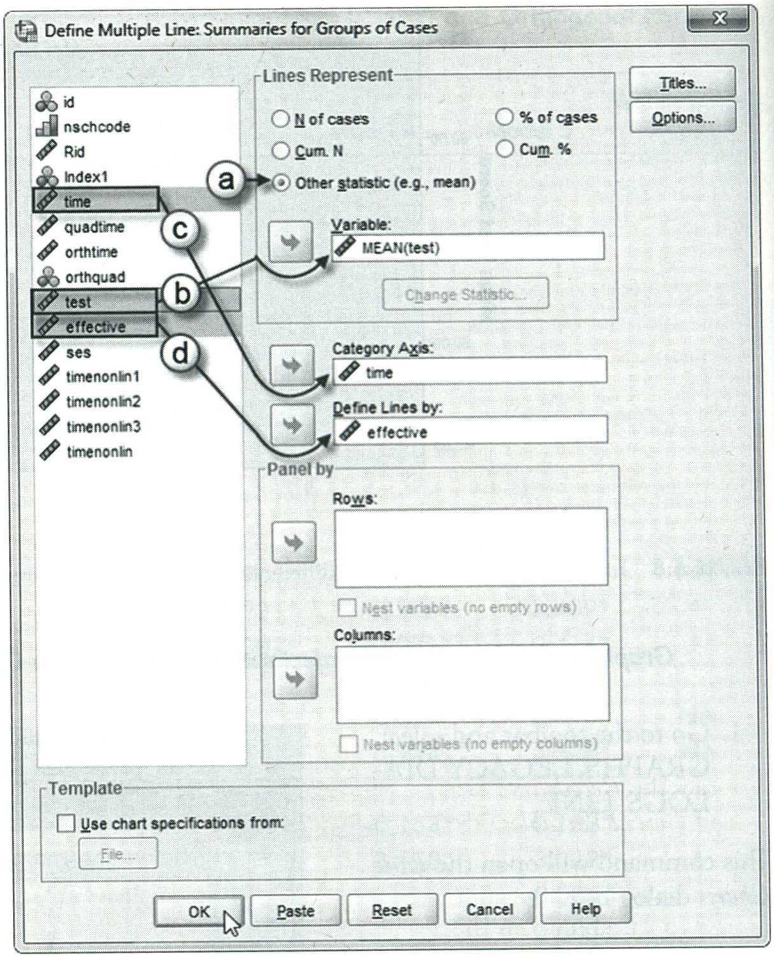


3a. Within the *Define Multiple Line: Summaries for Groups of Cases* dialog box, click to select *Other statistic (e.g., mean)*.

b. Click to select the variable *test* from the left column, and then click the right-arrow button to move *test* into the *Variable* box. The setting *MEAN(test)* defines the use of mean values for *test.*

c. Click to select *time* from the left column. Then click the right-arrow button to move the variable into the *Category Axis* box.

d. Click to select *effective* from the left column. Next, click the right-arrow button to move the variable into the *Define Lines by* box.



Click the OK button, which will generate the plot graph.

Note: The resulting plot graph's labels and lines may be edited or changed through the *Chart Editor*. To activate the *Chart Editor*, double-click on the plot graph in the output (Fig. 5.8).

Examining Growth Using an Alternative Specification of the Time-Related Variable

Since we know that the growth trajectory slows slightly between the second and third measurement occasions, we might consider a different manner of coding the repeated measures to represent the growth occurring over the entire period under consideration. To determine the growth that occurs over the entire trend, we can code the first measurement as 0 and the last one as 1. Since the meaning of the slope is the change in Y for a unit change in X, coding the time-related variable in this manner will capture the growth over the entire trend regardless of whether the growth dips or spikes between the beginning and ending intervals. Steps for coding four variations (i.e., *timenonlin1*, *timenonlin2*, *timenonlin3*, and *timenonlin*) of the time-related variable are provided at the end of this section. This is similar to the level-and-shape approach discussed previously for estimating a latent growth factor. With a little trial and error, it is possible to obtain a variety of growth curves that may match the actual data quite well. More specifically, if we see that the relationship is linear (perhaps by inspecting several individuals' growth trajectories or the average individual trajectory), we could code the middle interval as 0.5 (i.e., 0.0, 0.5, 1.0). We can then test this hypothesized linear formulation against the data. If we specify the time-related variable in this way (*timenonlin1*), we obtain an AIC coefficient of 189,125.5. We provide the model syntax in Appendix A.

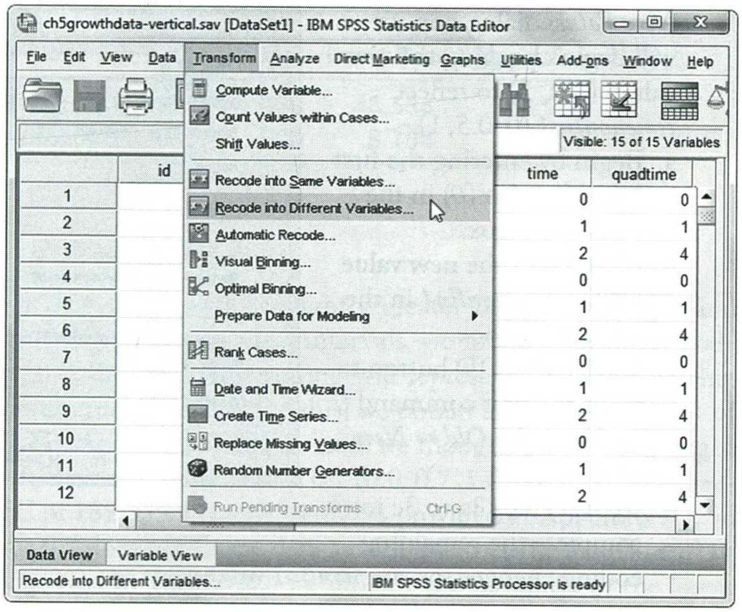
*Coding Time Interval Variables (*time *to* timenonlin *Variations) with IBM SPSS Menu Commands*

Continue using the *ch5growthdata-vertical.sav* data.

1. Go to the toolbar and select TRANSFORM, RECODE INTO DIFFERENT VARIABLES.

This command will open the *Recode into Different Variables* dialog box.

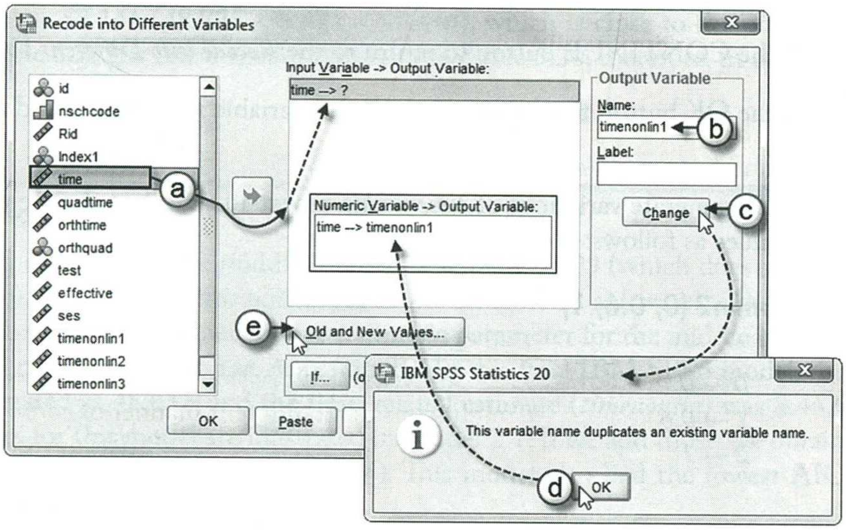
Note: If continuing from performing the prior coding example (*time* to *orthtime* or *orthquad*), click the RESET button before proceeding to clear the default settings.



2a. The *Recode into Different Variables* box enables creating a new variable using a variable from the current data set. First, click to select *time* from the left column, and then click the right-arrow button to move the variable into the *Input Variable → Output Variable* box.

b. Now enter the new variable name by typing *timenonlin1* into the *Output Variable, Name* box.

c. Then click the CHANGE button, which will add *timenonlin1* and complete the RECODE command for *time → timenonlin1*.

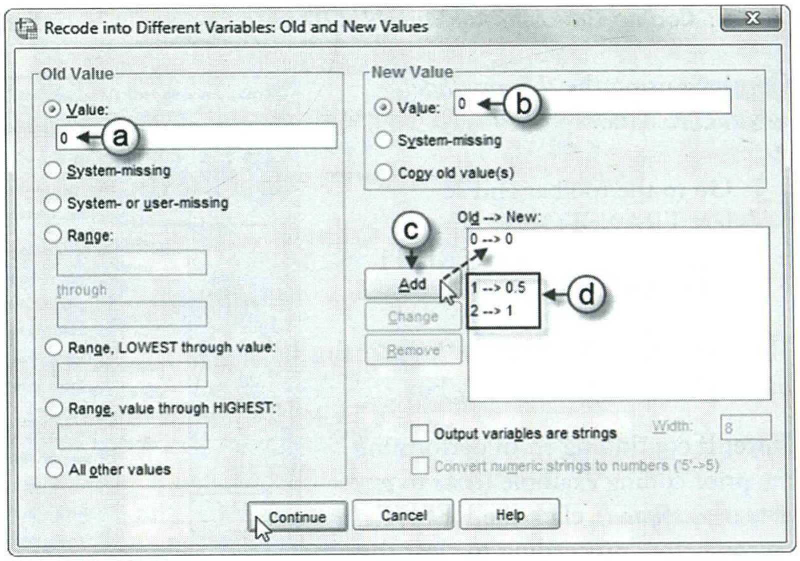


Note: A warning message appears as *timenonlin1* is an existing variable in the data set.

d. Click OK to continue, which will overwrite the preexisting *timenonlin1* variable. If you prefer not to overwrite the original variable, rename the output variable (e.g., *timenonlin1a*).

e. Click the OLD AND NEW VALUES button, which will then display the *Recode into Different Variables: Old and New Values* screen.

3. Within the *Recode into Different Variables: Old and New Values* dialog box, we will begin changing the *time* values (0, 1, 2) to reflect *timenonlin1* (0, 0.5, 1).
   a. Begin by entering the first value for *time* (*0*) in the *Value* (old) box.
   b. Next, enter the new value (*0*) for *timenonlin1* in the *Value* (new) box.
   c. Click the ADD button to place the first command *0 → 0* into the *Old → New* box.
   d. Repeat steps 3a to 3c to complete the remaining coding changes for *timenonlin1* values:



1 → 0.5

2 → 1

Click the CONTINUE button to return to the *Recode into Different Variables* main dialog box.

Click the OK button to generate the recoded variable *timenonlin1* and corresponding time values (0, 0.5, 1).

Note: To generate variations of *timenonlin*, repeat all steps but change the output variable and *Old → New* values as follows:

timenonlin2 (0, 0.6, 1)

0 → 0
1 → 0.6
2 → 1

timenonlin3 (0, 0.7, 1)

0 → 0
1 → 0. 7
2 → 1

timenonlin (0, 0.53, 1)

0 → 0
1 → 0.53
2 → 1

TABLE 5.20 Estimates of Fixed Effects[a]

						95% Confidence Interval	
Parameter	**Estimate**	**Std. Error**	***df***	***t***	**Sig.**	**Lower Bound**	**Upper Bound**
Intercept	48.725	.097	8,669.000	500.287	.000	48.534	48.916
timenonlin1	8.421	.121	8,669.000	69.589	.000	8.184	8.658

[a] Dependent variable: test.

Because we have some evidence that the growth is slightly greater between the first and second intervals and slows between the second and third intervals, we might instead try coding the middle interval as 0.6 (i.e., 0.0, 0.6, 1.0). This coding will represent a slightly slowing trend (*timenonlin2*). When we code the time variable like this, we obtain an AIC coefficient of 189,157.5 (model syntax is provided in Appendix A). If, instead, we thought the slowing might be a bit more extreme, we could code the middle interval as 0.7 (0.0, 0.7, 1.0). In this case (*timenonlin3*), however, we obtain an AIC of 189,335.3 (model syntax is provided in Appendix A). So clearly, coding the middle interval as 0.7 does not fit the data as well as either the first (linear) or second (slightly slowing) coding schemes. In Table 5.20, we provide the estimates of the linear example where we coded the time-related variable as 0.0, 0.5, 1.0. The initial status intercept is estimated to be 48.725, which is a bit higher than the initial intercept of 48.632 in Table 5.2. The growth over the entire trend is estimated as 8.421. In this case, then, we would estimate the ending achievement level as 57.146 (48.725 + 8.421 = 57.146), which is close to the observed intercept of 57.094 in Table 5.2. The linear model estimates in Table 5.20, therefore, fit the data quite well.

Estimating the Final Time-Related Model

We note that the optimal estimate for the middle interval is actually 0.529 (which does suggest a bit of slowing over time). We obtained this optimal estimate using SEM software, as the level-and-shape LCA approach provides the exact, model-estimated parameter for the middle interval of the latent growth factor (see comparison in Appendix B). In this final IBM SPSS model, the initial intercept was estimated as 48.632, and the time-related estimate (*timenonlin*) was 8.462. The key modeling changes for this model are illustrated in Model 2.1. If we add those, we obtain the intercept estimate for Time 3 in Table 5.2 (57.094). This model also had the lowest AIC coefficient compared with the other coding schemes.

Defining Model 2.1 with IBM SPSS Menu Commands

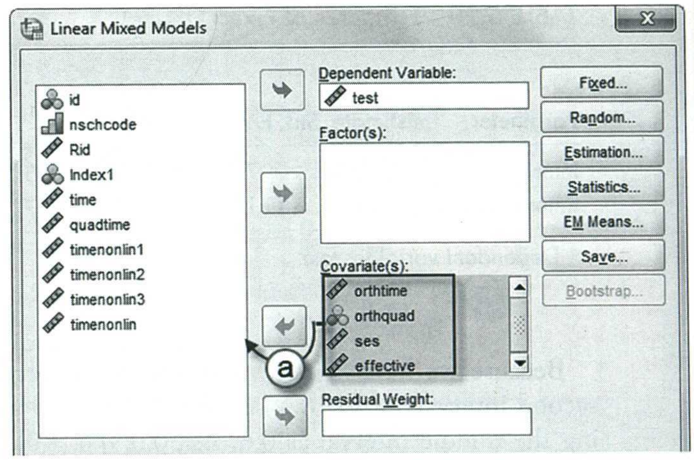
Continue using the *ch5growthdata-vertical.sav* data. Settings default to those used for Model 1.3.

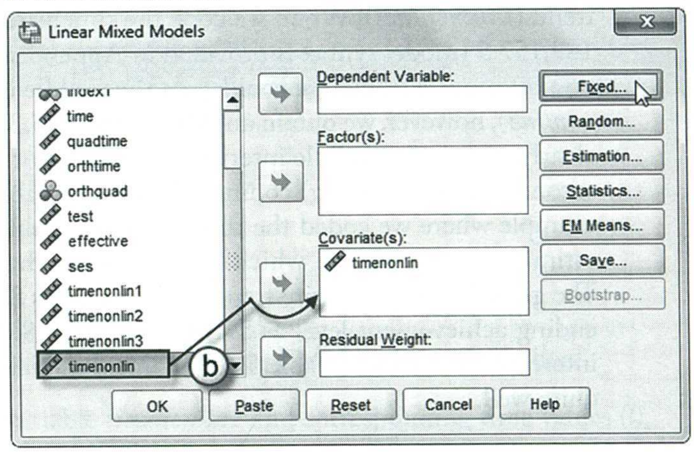
1. Go to the toolbar and select ANALYZE, MIXED MODELS, LINEAR. This command enables access to the *Linear Mixed Models: Specify Subjects and Repeated* dialog box.

2. The *Linear Mixed Models: Specify Subjects and Repeated* box displays the default settings from Model 1.3. Place the variables *id* and *time* within the *Subjects* and *Repeated* boxes. The *Repeated Covariance Type* is specified as *Diagonal*. Click the CONTINUE button to display the *Linear Mixed Models* dialog box.

3. The *Linear Mixed Models* dialog box displays *test* in the *Dependent Variable* box with *orthtime*, *orthquad*, *ses*, and *effective* in the *Covariate(s)* box.
   a. Remove all variables from the *Covariate(s)* box by clicking to select them and then clicking the left-arrow button.
   b. Click to select *timenonlin*, and then click the right-arrow button to move the variable into the *Covariate(s)* box.

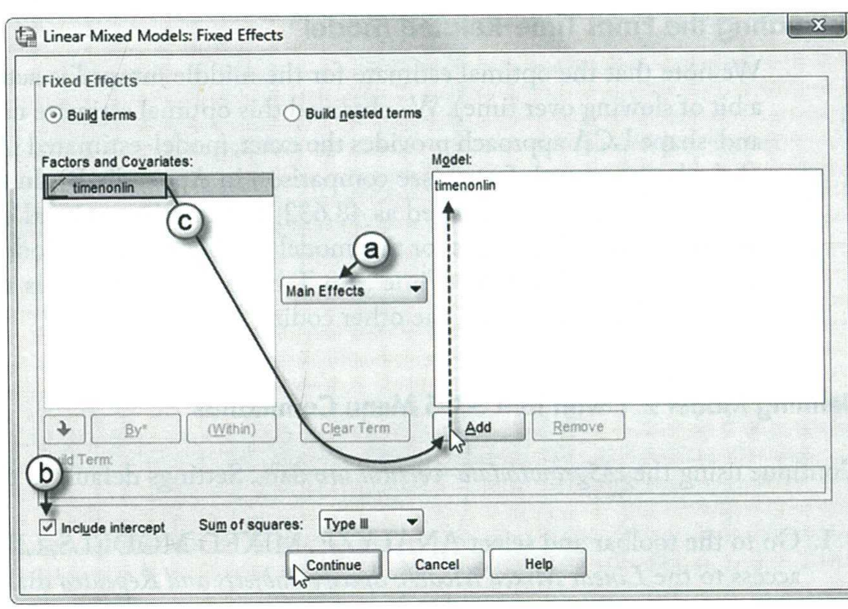
Click the FIXED button to access the *Linear Mixed Models: Fixed Effects* dialog box.





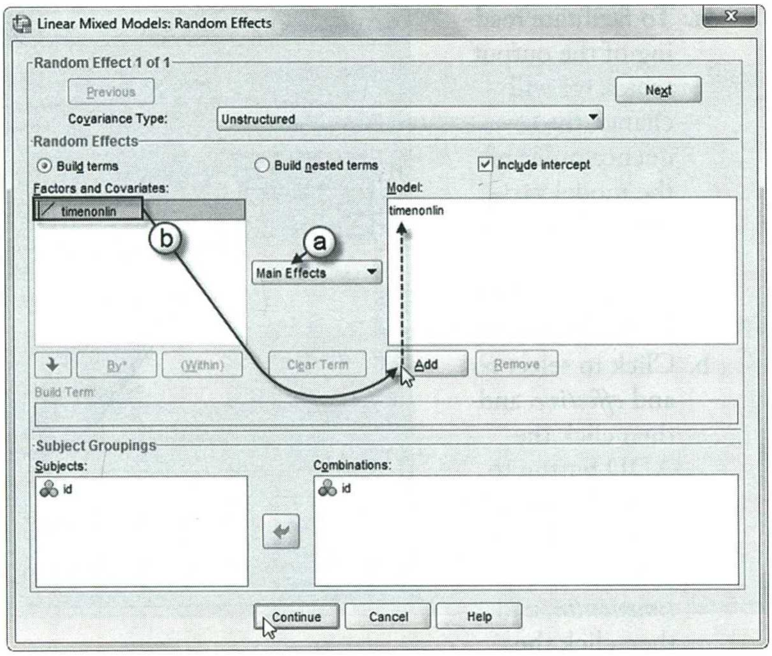
4a. Within the *Linear Mixed Models: Fixed Effects* dialog box, confirm that *Main Effects* is selected.
   b. Confirm that *Include intercept* is selected.
   c. Click to select *timenonlin*, and then click the ADD button to move it into the *Model* box.

Click the CONTINUE button to return to the *Linear Mixed Models* dialog box.



Click the RANDOM button to access the *Linear Mixed Models: Random Effects* dialog box.

5. Within the *Linear Mixed Models: Random Effects* box, the covariance type and intercept are set to the default settings of the prior model.
   a. The *Main Effects* option is preselected.
   b. Click to select *timenonlin* from the *Factors and Covariates* box, and then click the ADD button to move the variable into the *Model* box.



Click the CONTINUE button to return to the *Linear Mixed Models* dialog box.

Finally, in the *Linear Mixed Models* dialog box, click the OK button to run the model.

Adding the Two Predictors

Finally, we add the two between-individual predictors. In this case, we use the optimal estimate for Time 2 (0.529). Key modeling changes are illustrated in Model 2.2.

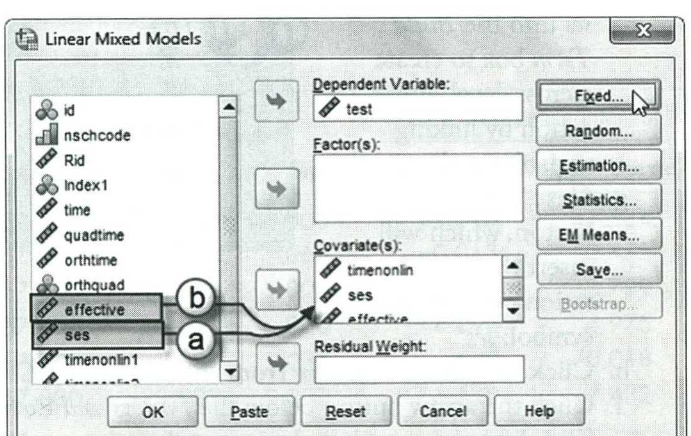
Defining Model 2.2 with IBM SPSS Menu Commands

Continue using the *ch5growthdata-vertical.sav* data. Settings default to those used for Model 2.1.

1. Go to the toolbar and select ANALYZE, MIXED MODELS, LINEAR. This command enables access to the *Linear Mixed Models: Specify Subjects and Repeated* dialog box.

2. The *Linear Mixed Models: Specify Subjects and Repeated* box displays the default settings from Model 2.1. Place the variables *id* and *time* within the *Subjects* and *Repeated* boxes. The *Repeated Covariance Type* is specified as *Diagonal*. Click the CONTINUE button to display the *Linear Mixed Models* dialog box.

3. The *Linear Mixed Models* dialog box displays the default settings for Model 2.1.
   a. Click to select *ses*, and then click the right-arrow button to move the variable into the *Covariate(s)* box.
   b. Click to select *effective*, and then click the right-arrow button to move the variable into the *Covariate(s)* box.

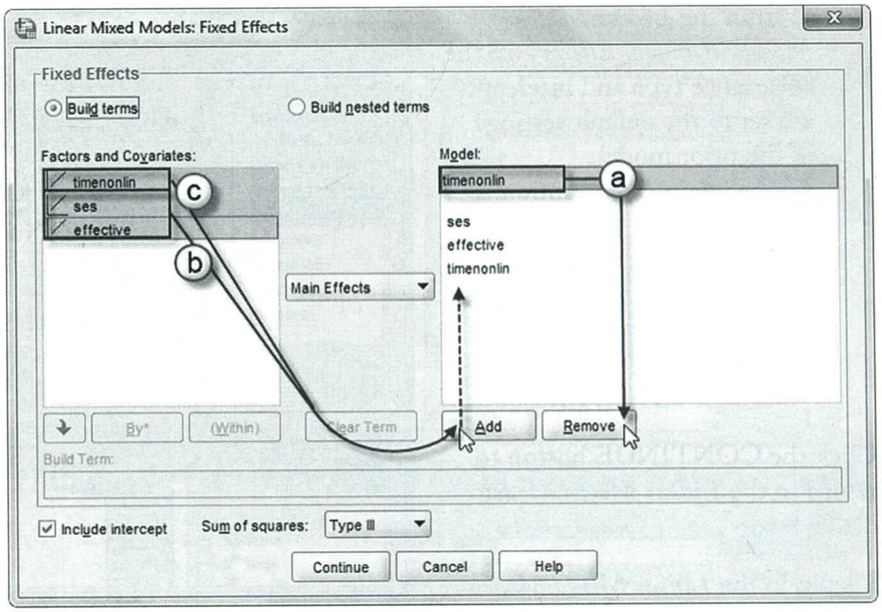


Click the FIXED button to access the *Linear Mixed Models: Fixed Effects* dialog box.

4a. To facilitate reading of the output tables, we will change the sequence order of the model variables. Click to select *timenonlin*, and then click the REMOVE button.

b. Click to select *ses* and *effective*, and then click the ADD button to move the variables into the *Model* box.

c. Click to select *timenonlin*, and then click the ADD button to move it into the *Model* box.



Two cross-level interactions (or nested terms) will be created and added to the model: *ses*timenonlin* and *effective*timenonlin*. These interactions will tell us if the growth trajectories are parallel for different groups of students.

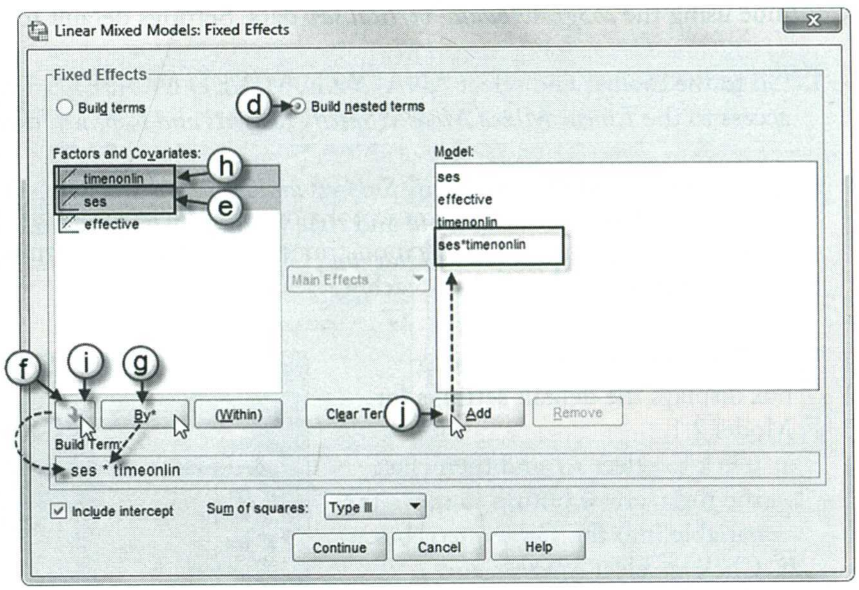
Add First Interaction to Model 2.2: *ses*timenonlin*

d. Click to select *Build nested terms*.

e. Click to select the variable *ses* from the *Factors and Covariates* box.

f. Then click the arrow button below the *Factors and Covariates* box. This moves *ses* into the *Build Term* box to create a cross-level interaction by linking variables and terms.

g. Next, click the BY* button, which will insert the computation command symbol: *ses**.

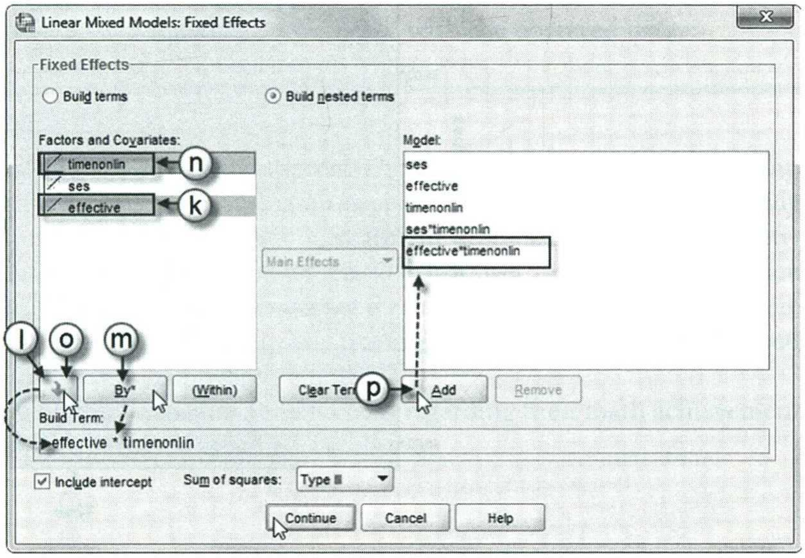


h. Click to select *timenonlin* from the *Factors and Covariates* box.

i. Click the arrow button below the *Factors and Covariates* box to move *timenonlin* into the *Build Term* box and complete the interaction term: *ses*timenonlin*.

j. Click the ADD button to transfer the interaction into the *Model* box.

Add Second Interaction to Model 2.2: *effective*timenonlin*



k. Click to select the variable *effective* from the *Factors and Covariates* box.

l. Next, click the arrow button below the *Factors and Covariates* box. This moves *effective* into the *Build Term* box.

m. Now click the BY* button, which will insert the computation command symbol: *effective**.

n. Click to select *timenonlin* from the *Factors and Covariates* box.

o. Click the arrow button below the *Factors and Covariates* box to move *timenonlin* into the *Build Term* box and complete the interaction term: *effective*timenonlin*.

p. Click the ADD button to transfer the interaction into the *Model* box.

Click the CONTINUE button to return to the *Linear Mixed Models* dialog box. Finally, click OK to run the model.

Interpreting the Output From Model 2.2

The fixed effects for the MIXED solution are presented in Table 5.21. The adjusted initial status intercept is estimated as 47.300. We can see that perceived teacher effectiveness affects the intercept (2.406, $p < .001$). Student SES is again not significantly related to initial status ($p > .05$). Turning to growth, we can see that the *ses*timenonlin* effect is negative and significant (–.308, $p < .05$), once again suggesting that students with higher SES demonstrated less growth over the time of the study. Similar to the polynomial model, we can also note that students with teachers who are rated as effective demonstrated more growth over time (6.976, $p < .001$), compared to their peers who rated their teachers as average or ineffective.

TABLE 5.21 Estimates of Fixed Effects[a]

						95% Confidence Interval	
Parameter	**Estimate**	**Std. Error**	***df***	***t***	**Sig.**	**Lower Bound**	**Upper Bound**
Intercept	47.300	0.145	8,667.000	325.943	.000	47.015	47.584
ses	0.231	0.124	8,667.000	1.856	.063	–0.013	0.475
effective	2.406	0.196	8,667.000	12.302	.000	2.022	2.789
timenonlin	4.637	0.173	8,667.000	26.861	.000	4.298	4.975
ses * timenonlin	–0.308	0.148	8,667.000	–2.084	.037	–0.598	–0.018
effective * timenonlin	6.976	0.233	8,667.000	29.989	.000	6.520	7.432

[a] Dependent variable: test.

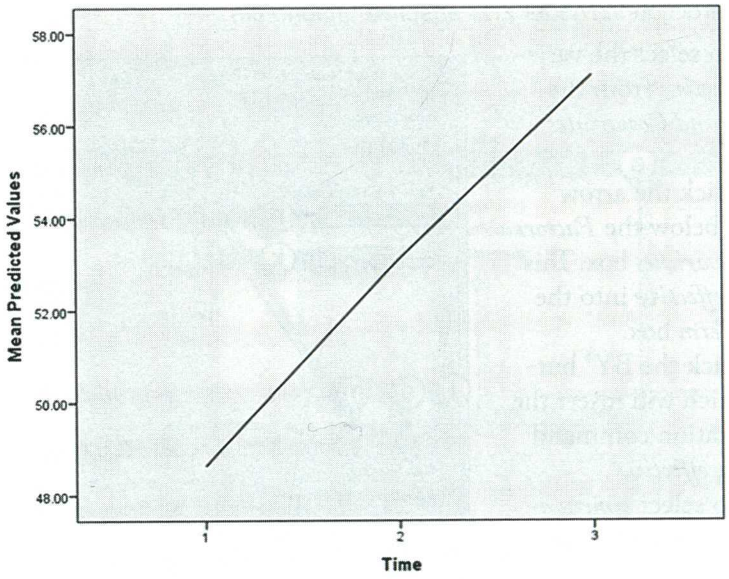


FIGURE 5.9 Mean predicted values from nonlinear growth model.

We can also save mean predicted values from the fixed-effect model in the data set. In Figure 5.9, we can see that the average predicted trajectory appears to slow a bit between the second and third intervals.

We also provide the variance components in Table 5.22. We can see that there is still significant variation in both intercepts and slopes left to be explained between individuals. In this formulation, we note that the covariance between the intercept and slope is significant and negative (–6.917, p = .001). In this case, this negative relationship represents the well-known tendency for students who start with higher achievement to demonstrate less growth over time and vice versa. We again make the point that it is typically the case in growth modeling that when the coding of the growth parameters is changed, the relationship between the intercept and slope also changes.

We also draw attention to the fact that this latter solution using SPSS MIXED produces results almost identical to the level-and-shape specification estimated using Mplus. We note that there are some advantages to defining growth models in this manner. One of the main advantages is that it is easier to build models on one randomly varying time slope parameter, as opposed to sometimes building them on two or more polynomials (depending on the number of

TABLE 5.22 Estimates of Covariance Parameters[a]

						95% Confidence Interval	
Parameter		**Estimate**	**Std. Error**	**Wald *Z***	**Sig.**	**Lower Bound**	**Upper Bound**
Repeated Measures	Var: [Index1 = 1]	61.090	2.107	28.993	.000	57.097	65.363
	Var: [Index1 = 2]	61.343	1.153	53.212	.000	59.125	63.644
	Var: [Index1 = 3]	28.489	1.633	17.450	.000	25.463	31.876
Intercept + timenonlin [subject = id]	UN (1, 1)	30.968	1.943	15.934	.000	27.384	35.022
	UN (2, 1)	–6.917	2.154	–3.211	.001	–11.139	–2.695
	UN (2, 2)	28.798	3.506	8.214	.000	22.685	36.559

[a] Dependent variable: test.

random effects that can be supported by the repeated measures). This alternative to polynomial growth models tends to simplify the model-building process. It does, however, take some work in finding a coding of the time-related variable that is consistent with the observed data.

An Example Experimental Design

Before we leave the individual growth-modeling approach, we provide an example illustrating a comparison of individual growth trajectories using an experimental design. Consider a study to examine whether or not students' participation in a treatment designed to target their deficiencies in solving math problems helps increase their math scores over time. The variables are summarized in Table 5.23. The data in this study consist of 55 middle-school students who were randomly assigned to either a control ($N = 30$) group or a treatment ($N = 25$), where they received individualized instruction with a math tutor over a semester.

The students were measured on four occasions (one pretest) regarding their math achievement during the period of the study. The design is specified as follows:

$$\begin{array}{lllll} R & O_1 & O_2 & O_3 & O_4 \\ R & O_1\,X & O_2 & O_3 & O_4 \end{array}$$

The Rs refer to random assignment. Fixed effects include the treatment (coded = 1) versus control (coded 0), time (coded 0–3 for the four occasions), and the treatment-by-time interaction.

TABLE 5.23 Data Definition of *ch5experimentaldesigndata.sav* (N = 55)

Variable	Level[a]	Description	Values	Measurement
id	Individual	Student identifier (55 students across four time [test] occasions).	Integer	Ordinal
treatment	Individual	Two-category predictor variable representing students assigned for nontreatment (control) or treatment.	0 = No Treatment, (No Tutor), 1 = Treatment (Tutored)	Scale
time	Within Individual	Variable representing four linear occasions in time (0, 1, 2, 3) measuring students math achievement.	0 = First Time, 1 = Second Time, 2 = Third Time, 3 = Fourth Time	Scale
timenonlin	Within Individual	Recoded *time* variable from four occasions in time (0, 1, 2, 3) into a time sequence variation representing the whole four time occasions (0.00, 0.50, 0.70, 1.00) measured from 0 to 1.	0.00 = First Time, 0.50 = Second Time, 0.70 = Third Time, 1.00 = Fourth Time	Scale
math	Within Individual	The dependent variable representing each students individual scores on the repeated math measurements.	523 to 856	Scale
orthtime	Within Individual	Recoded *time* variable from four occasions in time (0, 1, 2, 3) into a different time sequence (–3, –1, 1, 3).	–3 = First Time, –1 = Second Time, 1 = Third Time, 3 = Fourth Time	Scale
orthquad	Within Individual	Recoded *time* variable from four occasions in time (0, 1, 2, 3) into a time sequence variation(1, –1, –1,1).	1 = First Time, –1 = Second Time, –1 = Third Time, 1 = Fourth Time	Scale
orthcubic	Within Individual	Recoded *time* variable from four occasions in time (0, 1, 2, 3) into a time sequence variation (–1, 3, –3,1).	–1 = First Time, 3 = Second Time, –3 = Third Time, 1 = Fourth Time	Scale

[a] Individual = Level 2; Within individual = repeated measures, Level 1.

The initial O_1 coefficients refer to students' initial status achievement. We expect no initial difference between the treatment and control groups. This is captured by the relationship between the treatment variable and the students' initial status intercept at the beginning of the study (where time is coded 0). After implementing the treatment after the initial pretest measurement, we propose that the treatment group will increase more in math achievement over time than the control group. This will be tested by a treatment-by-time interaction; that is, we would expect different growth trends for the control and treatment groups.

We can specify the Level 1 model for individual i measured at time occasion t as follows:

$$Y_{ti} = \pi_{0i} + \pi_{1i} time_{ti} + \varepsilon_{ti}. \tag{5.22}$$

Since the time period for the study is short (i.e., one semester), we might assume a linear model as fitting reasonably well. We note that we could fit the model to a higher order polynomial function. One complication with that approach, however, is that transforming higher order polynomials will shift the intercept to the grand mean. This will obscure the achievement level where each group started before the intervention. We illustrate the two different growth trajectories in Figure 5.10. The figure suggests that the two groups are close before the treatment begins (Time 0). We can see that the treatment growth is closer to a linear growth model over time; however, the control group definitely slows in its progress over the semester.

We first provide results using the model with time specified as linear to describe growth over the short period of time of the study. At Level 2, we assume that the intercept varies between subjects:

$$\pi_{0i} = \beta_{00} + \beta_{01} treatment_i + u_{0i}. \tag{5.23}$$

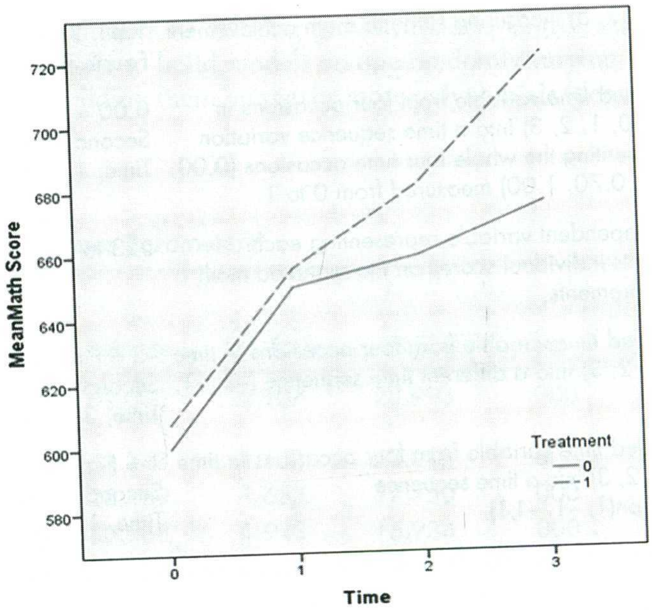


FIGURE 5.10 Examining treatment and control growth trends.

We will also model that the time slope is randomly varying:

$$\pi_{1i} = \beta_{10} + \beta_{11}treatment_i + u_{1i}. \tag{5.24}$$

After substituting the Level 2 equations into the Level 1 equation, the combined model will then be the following:

$$Y_{ti} = \beta_{00} + \beta_{01}treatment_i + \beta_{10}time_{ti} + \beta_{11}time_{ti}{}^{*}treatment_i + u_{1i}time_{ti} + u_{0i} + \varepsilon_{ti}. \tag{5.25}$$

We can see that the key parameter in this simple illustration is the *time*treatment* interaction. This is used to determine if there are different growth trajectories for individuals in the treatment and control groups. We will also specify a diagonal matrix for the residual error structure at Level 1:

$$\begin{bmatrix} \sigma_1^2 & 0 & 0 & 0 \\ 0 & \sigma_2^2 & 0 & 0 \\ 0 & 0 & \sigma_3^2 & 0 \\ 0 & 0 & 0 & \sigma_4^2 \end{bmatrix} \tag{5.26}$$

At Level 2, we will specify an unstructured covariance matrix:

$$\begin{bmatrix} \sigma_{1,1}^2 & \sigma_{1,2} \\ \sigma_{2,1} & \sigma_{2,2}^2 \end{bmatrix} \tag{5.27}$$

We present the fixed-effects estimates in Table 5.24 and syntax to replicate the analysis in Appendix A. We note in passing that we investigated several different covariance structures for the Level 1 repeated measures, but we decided to stay with a diagonal structure, which fit slightly better than the simple scaled identity covariance structure since the within-subject covariance structure was really not the focus of the study. The −2LL for this model with 11 parameters was 2,054.67, and the AIC was 2,068.67.

The fixed-effect estimates suggest that the students in the control group started with a mean score of 609.057. This linear model seems to slightly overestimate the control group's starting achievement in Figure 5.10 (as well as its ending achievement). Over each interval, individuals'

TABLE 5.24 Estimates of Fixed Effects[a]

						95% Confidence Interval	
Parameter	**Estimate**	**Std. Error**	***df***	***t***	**Sig.**	**Lower Bound**	**Upper Bound**
Intercept	609.057	6.636	51.556	91.782	.000	595.738	622.376
time	23.813	2.500	47.331	9.525	.000	18.785	28.842
treatment	0.664	9.843	51.556	0.067	.946	−19.091	20.419
time * treatment	12.996	3.708	47.331	3.505	.001	5.538	20.454

[a] Dependent variable: math.

scores in the control group increased by 23.813 points on average. We can also confirm that there was no difference between the treatment and control groups initially (β = 0.664, p = .946). Over time, however, we can observe that students who received the targeted intervention increased their scores over each interval at a greater rate (12.996 points) than their peers in the control group.

We present the covariance parameter estimates in Table 5.25. We can see that the correlation between initial status and growth was negative (–0.266), suggesting that students who started higher in achievement demonstrated a bit less growth over time (and vice versa), but it was not significant (p > .05).

One problem with this specification, however, is that we do not take in the curvilinear nature of the growth occurring over time. Alternatively, as we mentioned earlier, if we were to treat the time variable as categorical, we could also specify the model to determine the difference between treatment and control group at *each* occasion, instead of assuming a polynomial growth curve over the entire temporal sequence. This model, summarized in Table 5.26, fits the data slightly better than the previous one, using AIC as an index of model fit (2,013.91 to 2,068.67, respectively).

TABLE 5.25 Estimates of Covariance Parameters[a]

Parameter		Estimate	Std. Error	Wald Z	Sig.	95% Confidence Interval	
						Lower Bound	Upper Bound
Repeated Measures	Var: [time = 0]	305.773	188.462	1.622	.105	91.362	1,023.373
	Var: [time = 1]	421.087	104.377	4.034	.000	259.048	684.485
	Var: [time = 2]	150.015	55.704	2.693	.007	72.454	310.604
	Var: [time = 3]	251.289	118.592	2.119	.034	99.646	633.701
Intercept + time [subject = id]	Var(1)	1,099.161	290.476	3.784	.000	654.807	1,845.054
	Var(2)	131.754	37.409	3.522	.000	75.523	229.851
	Corr(2, 1)	–0.266	0.179	–1.480	.139	–0.572	0.106

[a] Dependent variable: math.

TABLE 5.26 Estimates of Fixed Effects[a]

Parameter	Estimate	Std. Error	*df*	*t*	Sig.	95% Confidence Interval	
						Lower Bound	Upper Bound
Intercept	674.900	8.595	94.019	78.524	.000	657.835	691.965
[time = 0]	–75.286	6.832	58.889	–11.019	.000	–88.958	–61.614
[time = 1]	–25.400	6.412	51.339	–3.961	.000	–38.270	–12.530
[time = 2]	–16.167	7.879	74.257	–2.052	.044	–31.864	–0.469
[time = 3]	0.000[b]	0.000	.	.	.	.	.
Treatment	46.220	12.748	94.019	3.626	.000	20.908	71.532
[time = 0] * treatment	–38.685	10.134	58.889	–3.817	.000	–58.964	–18.405
[time = 1] * treatment	–40.480	9.510	51.339	–4.256	.000	–59.569	–21.391
[time = 2] * treatment	–24.033	11.686	74.257	–2.057	.043	–47.317	–0.750
[time = 3] * treatment	0[b]	0	.	.	.	.	.

[a] Dependent variable: math.

[b] This parameter is set to 0 because it is redundant.

In this case, since IBM SPSS uses the last category of *time* as the reference group, we now have the intercept defined as "ending" math achievement status (674.900) for the control group (coded 0). This ending achievement level for the control group is consistent with Figure 5.10. The treatment group would be estimated as 721.120. We can see that students demonstrated considerable (but differing) growth at each time occasion as represented by the *time* effects. The time coefficients refer to how much lower the control group was at each interval preceding the ending status intercept. We can use the coefficients to estimate achievement for each group at any occasion. For example, the control group's achievement at the beginning of the study will be 674.900–75.286, or 599.614. As we might expect, Table 5.26 also suggests that growth for the control group slows considerably over successive intervals. We can also note that at the *end* of the study, there is a positive effect for being in the treatment group (46.220, $p < .001$). We can also see that the amount of difference at each occasion between the treatment and control groups, which is modeled as the *time*treatment* interactions, is significant for each occasion ($p < .05$). In this case, the coefficients for each interaction suggest a differing achievement "advantage" for the treatment group at each of the preceding intervals, rather than assuming a *constant linear* effect over time, as summarized in Table 5.24. The information about the treatment can be used along with the other relevant time-related information to calculate where the treatment group is at any occasion in the study. The pattern of results observed in Table 5.26 reflects the different coding scheme between using initial status in the previous model (see Table 5.24 on page 235, where there was no difference between the two groups) and ending status (Table 5.26), where it is obvious in Figure 5.10 that there is considerable difference in achievement between the treatment and control groups.

We also provide results using an alternative coding for time in Table 5.27. In this case, we defined the time-related variable as 0, 0.5, 0.7, 1.0 (*timenonlin*), in order to capture the change taking place over the whole study. This is one way we can easily deal with the curvilinear shape of the average growth trajectories in each group. We can simply substitute *timenonlin* for *time* in Equation 5.22. This solution has the advantage of having an initial status (602.058) for the control group and illustrates that the treatment growth was not significantly different at the beginning of the study (3.217, $p = .742$). Over the time period studied, the treatment group demonstrated considerably higher growth compared to the control group (31.428, $p < .01$). The covariance parameters were similar to the previous linear growth model in Table 5.25, so we do not reproduce them here. We note also that this model fit better than the first model summarized in Table 5.27 (AIC = 2,048.35 to 2,068.67, respectively), but not as well as the previous model summarized in Table 5.26 (AIC = 2,048.35 to 2,013.91, respectively). Substantively, however, all three models provide the same interpretation regarding the positive effect of the educational treatment under consideration. At the end of the study, for example, we would estimate the math achievement of the control group as 602.058 + 79.072, or 681.130. We would estimate the treatment group's math achievement as 602.058 + 3.217 + 79.072 + 31.428, or 715.775.

These are slightly different from the previous model but certainly consistent with it substantively.

TABLE 5.27 Estimates of Fixed Effects[a]

Parameter	Estimate	Std. Error	df	t	Sig.	95% Confidence Interval	
						Lower Bound	Upper Bound
Intercept	602.058	6.552	50.555	91.885	.000	588.901	615.215
timenonlin	79.072	6.738	47.201	11.735	.000	65.518	92.626
treatment	3.217	9.719	50.555	0.331	.742	16.298	22.732
timenonlin * treatment	31.428	9.995	47.201	3.145	.003	11.324	51.532

[a] Dependent variable: math.

Summary

In this chapter, we presented a basic two-level model for investigating individual change. Longitudinal analysis represents a rapidly growing application of basic multilevel-modeling techniques. In comparison to ANOVA, we suggested that multilevel modeling of growth trajectories is a more flexible approach because of its ability to handle a wide range of data situations (incomplete data and varying occasions of measurement). The model provides considerably more information about students' initial status and growth rates. The individual growth model can easily be extended to include successive grouping structures above the individual level. We investigate models with individual and group components in the next chapter. As we noted previously in this chapter, there are a considerable number of ways to investigate changes in individuals over time. We provided several different coding possibilities throughout the chapter. We encourage readers to consult a number of introductory sources that provide overviews of the assumptions, uses, and programming of longitudinal models (e.g., Duncan, Duncan, & Strycker, 2006; Raudenbush, Bryk, Cheong, & Congdon, 2004; Raykov & Marcoulides, 2008; Singer & Willett, 2003).