R-lab: Returns

Statistical Methods in Finance

Monte Carlo Simulation

- To approximate the probability of an event A, we run the experiment a large number of times (say 1000) and count how many times A has occurred.
- If it has occurred 100 times, then an estimate of $P(A) = 100/1000$.
- 1000 is called the number of replications or iterations in this simulation and 100 is the frequency of A.

Example from Textbook

- A hedge fund used \$50,000 of its own capital and borrowed \$950,000 to purchase \$1,000,000 of a stock.
- The daily log-returns are independent normally distributed with mean $0.05/253$ and standard deviation $0.23/\sqrt{253}$ (253 trading days in a year).
- Estimate the probability that the stock will go below \$950,000 in the first 45 days.

Procedure

- We need to generate 45 daily log-returns from this distribution. This is done by using the R-function

  ```
  r=rnorm(45, 0.05/253, 0.23/sqrt(253))
  ```

- The daily log-prices are given by

  ```
  logPrice = log(1e6) + cumsum(r)
  ```

- The minimum log-price over these 45 days is

  ```
  minlogP= min(logPrice)
  ```

- Let $A = \{\text{Price went below } 950000\}$ and note that event A will occur if and only if minlogP $\leq$ log(950000). The following command returns 1 if A has occurred and 0 otherwise.

  ```
  as.numeric[minlogP < log(950000)]
  ```

- Repeat these steps a large number of times and count how many "1" was returned.

Full Program

```
set.seed(2018)   # for reproducibility
niter = 1e5   # number of replications
below = rep(0,niter)   # set up storage
for (i in 1:niter)
{
r = rnorm(45,mean=.05/253,sd=.23/sqrt(253))
logPrice = log(1e6) + cumsum(r)
minlogP = min(logPrice)
below[i] = as.numeric(minlogP < log(950000))
}
mean(below)   # estimate of the probability
[1] 0.64023
```

Further Problems

- Suppose the hedge fund will sell the stock if the value of the stock rises to at least \$1,100,000 or falls below \$950,000 **at the end of one of the first 100 trading days**. Otherwise, it will sell after 100 trading days if the value has stayed in between.
- The same technique can be generalized to compute:
- What is the probability that the hedge fund will make a profit of at least \$100,000?
- What is the probability the hedge fund will suffer a loss?
- What is the expected profit from this trading strategy?
- What is the expected return? Note that only \$50,000 was invested.

R code

```
set.seed(2018)   # for reproducibility
niter = 1e5
sellTime = rep(0,niter)
profit = rep(0,niter)
for (i in 1:niter)
{
r = rnorm(100,mean=.05/253,sd=.23/sqrt(253))
logPrice = log(1e6) + cumsum(r)

# find out on which days the stock was sold
# note that the minimum of a null set is taken to Inf in R
SellForLossTime = min((1:100)[(logPrice < log(950000))])
SellForProfitTime =  min((1:100)[(logPrice > log(1100000))])
sellTime[i] = min(SellForLossTime,SellForProfitTime,100)

profit[i] = exp(logPrice[sellTime[i]]) - 1000000
}
```

R code

```
mean(profit >= 100000)   # prob of profit>100000
[1] 0.38956

mean(profit <= 0)   # prob of loss
[1] 0.59262

mean(profit)   # expected profit
[1] 9287.024

mean(profit/(50000*sellTime))   # expected daily return
[1] -0.02205996
```