Capital Asset Pricing Model (CAPM)

Statistical Methods in Finance

Capital Asset Pricing Model

- **Capital Market Line**. The line (on σ-μ plane) that forms the efficient frontier is known as the capital market line (CML). It satisfies the following equation:

$$\mu = \mu_f + \frac{\mu_M - \mu_f}{\sigma_M}\sigma.$$

- **Sharpe Ratio**. The slope of CML,

$$\frac{\mu_M - \mu_f}{\sigma_M},$$

is called the Sharpe ratio, which measures the excess return (in excess of the risk-free return) over risk (standard deviation).

Capital Asset Pricing Model

- **Security Market Line**. The market portfolio has a special relationship with any individual asset or fund. Let R_i be an asset (or fund) with mean return μ_i. Let $\sigma_{i,M} = Cov(R_i, R_M)$, its covariance with the tangent portfolio. Define $\beta_i = \sigma_{i,M}/\sigma_M^2$. Then the following equation holds

$$\mu_i = \mu_f + \beta_i(\mu_M - \mu_f).$$

  By viewing μ_i as a function of β_i, it is a straight line with intercept μ_f and slope $\mu_M - \mu_f$ and is known as the security market line (SML). In other words, every (β_i, μ_i) lines in the SML.

Capital Asset Pricing Model

Example (Luenberger, 98) An oil drilling company is currently priced at \$875 a share. Suppose that the expected share price after one year is \$1000 and that the standard deviation of the return is $\sigma_{oil} = 40\%$. Furthermore, the risk-free rate is 10% and the market portfolio has an expected return 17% with a standard deviation 12%.

Capital Asset Pricing Model

Example (continued)

- **Evaluation** From the above specification, we know that investing in the oil drilling company will result in an expected return

$$\mu_{oil} = \frac{1000 - 875}{875} = 14\%.$$

On the other hand, at σ =40%, the CML implies the expected return

$$\mu = 0.10 + \frac{0.17 - 0.10}{0.12} \times 0.40 = 33\%.$$

Thus, the expected return of the oil drilling venture is well below the CML so that the investment itself does not appear to be a good one.

Example (continued)

- **Pricing** The SML may be used as a pricing tool. Suppose that instead of \$875 the fair price P_0 for the oil drilling company is to be determined. Suppose we know that its beta is 0.6. The SML implies

$$\frac{1000 - P_0}{P_0} = 0.1 + 0.6 \times (0.17 - 0.10)$$

or

$$P_0 = \frac{1000}{1 + 0.1 + 0.6 \times (0.17 - 0.10)} = 876.$$

So the price is right, even though itself is not a good investment.