10 Latent Class/Profile Analysis

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10.1 Introduction to Mixture Models

The literature on latent class/profile analysis (LCA) dates back to the seminal works of Lazarsfeld and Henry (1968) and Goodman (1974); it reflects an extraordinarily flexible technique that has been utilized in a variety of fields. In medicine, LCA has been used extensively for a variety of purposes including: estimating the quality of procedures used for diagnosing specific diseases when a gold standard is not available (e.g., Albert, McShane, Shih, & the U.S. National Cancer Institute Bladder Tumor Market Network 2001; Formann & Kohlmann, 1996), identifying disease subtypes (e.g. Huh et al., 2011), and analyzing and interpreting diagnostic agreement (e.g., Uebersax & Grove, 1990). Outside of medicine, LCA has been applied for such disparate purposes as modeling jury verdicts (e.g., Gelfand, & Solomon, 1974), investigating social capital (Owen & Videras, 2009), and research on market segmentation (Wedel & Kamakura, 2001). This technique is not as popular in the fields of education and psychology, but there are applications to academic cheating (Dayton & Scheers, 1997), psychiatry (e.g., Kendler, Karkowski, & Walsh, 1998), and the validation of cognitive theory (Jansen & van der Maas, 1997), to name a few.

LCA draws parallels to many commonly used techniques including item response theory and factor analysis. This chapter begins by highlighting the similarities and differences between LCA and other latent variable models. This is followed by a brief discussion of how LCA is distinct from a related method, cluster analysis. The methodological focus of this chapter is on the assumptions underlying LCA, how to perform and interpret the output from a LCA in Mplus, and how to define the appropriate number of classes in an exploratory analysis. Recommendations are also provided for how to use the posterior probabilities of class membership in other models.

This chapter does not provide a discussion of the mathematical foundations of LCA. For that, readers should consult McCutcheon (1987), Dayton (1999), and Heinen (1996), as well as the seminal works by Clogg and Goodman (1984), Dayton and Macready (1976), and Goodman (1974).

10.2 General Description of Latent Class Analysis and Comparison to Similar Techniques

Latent class analysis and cluster analysis are both techniques for classifying objects into mutually exclusive groups such that objects within groups have similar values on a set of manifest variables. The objects that are the focus of these analyses can be individuals (e.g., Reboussin, Song, Shrestha, Lohman, & Wolfson, 2006), countries (e.g., Pierce & Osmond, 1999), or other tangible entities (e.g., media reports: Niederkrotenthaler et al., 2010). These techniques share a similar goal and model (an example of which is shown

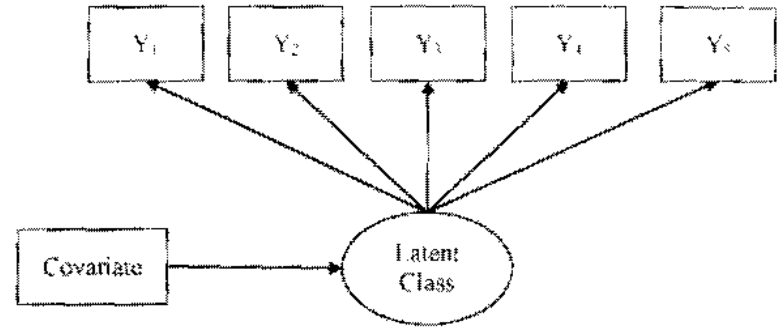

Figure 10.1 LCA model.

in Figure 10.1) with a categorical latent variable predicting manifest outcomes (denoted as Ys). They differ in the manner in which they achieve that goal and in their underlying assumptions.

A nonhierarchical cluster analysis would generally be considered an exploratory *numerical* technique for summarizing data. The goal of traditional methods, such a K-means clustering, is to maximize the degree of association between objects in the same cluster, and minimize the associations between objects in different clusters. Put differently, we could say that variability within clusters should be minimized, and between-cluster variability should be maximized. Clustering strategies such as this assign individual objects to a single group.

LCA is a *statistical* technique based on the concept of likelihood. The fact that LCA is a statistical technique may be what leads some to think of it as an "improved" cluster analysis (Francis, 2008). The assumption underlying LCA is that observed variables are (conditionally) independent assuming knowledge of the latent structure. This is equivalent to saying that the observed variables are not directly related but rather that they are indirectly related through the latent class variable. In LCA, membership in a latent class is probabilistic rather than deterministic. Suppose that we had three latent classes of individuals: habitual liars, situational liars, and those who always tell the truth. Using a LCA, we can get probabilities for each person as to his or her membership in each class. Beyond that, given that a person responded in the affirmative to a question asking whether he or she had ever stretched the truth on a job application, we could also obtain the probability that the person is a habitual liar.

LCA is often compared to other latent variable techniques. The following tables help to highlight some of the differences between the techniques, but not all. Table 10.1, which is adapted from Bartholomew (1987), illustrates that latent profile analysis is similar to LCA but with continuous, rather than categorical, manifest variables. LCA is often viewed as the being analogous to factor analysis in that both use measured variables to gain information about latent variables and have the assumption that the observed variables are conditionally independent given the factor structure. Both techniques also can be practiced in exploratory and confirmatory manners. In each case, the exploratory technique involves choosing between a variety of different models with different numbers of latent classes (or factors). Both confirmatory analyses test models that had been specified previously; in the case of LCA, researchers will have ideas regarding the number of classes and restrictions on the conditional probabilities that can be tested. As is clear from the Table 10.1, factor analysis has continuous manifest and latent variables in contrast to LCA's categorical

Table 10.1 Classification of latent variable methods.

		Manifest Variables	
		Continuous	*Categorical*
Latent Variables	*Continuous*	Factor Analysis	Latent Trait Analysis
	Categorical	Latent Profile Analysis	Latent Class Analysis

Table 10.2 Latent variable hybrids: Overview of old and new models.

	Continuous Latent Variables	*Categorical Latent Variables*	*Hybrids*
Cross-sectional Models	Factor Analysis, Structural Equation Modeling	Latent Class Analysis	Factor Mixture Analysis
Longitudinal Models	Growth Analysis (random effects)	Latent Transition Analysis, Latent Class Growth Analysis	Growth Mixture Analysis

ones. Perhaps the key difference between these two techniques is that the focus in factor analysis is on the structure of the variables, whereas in LCA, it is the structure of the cases that is more important. LCA is closely related to latent trait analysis, or item response theory as it is termed in educational and psychological measurement. The chief difference between these strategies is that the latter has continuous latent variables as opposed to categorical latent variables in LCA. Lindsay, Clogg, and Grego (1991) showed that LCA can be viewed as a generalization of models such as the Rasch model and that both techniques yield the same item parameters estimates under certain conditions.

Table 10.2, adapted from Muthén (2007), situates LCA analysis within a larger literature of both cross-sectional and longitudinal designs, and acknowledges that new, hybrid models allow for more complex designs than were previously possible. We include this table to highlight the fact that latent class analysis is the basis for more complex models, including latent transition analysis, latent class growth analysis, and factor mixture analysis.

10.3 Comparison of Latent Class/Profile Analysis to Profile Analysis

Another technique that seems to share characteristics with LCA is profile analysis (PA). PA is a multivariate version of within-subjects designs in analysis of variance, including both repeated measures and mixed designs. In this case, the word *profile* can refer to a set of scores from a test battery (such as the Artistic, Social, and Realistic scales of the Strong Interest Inventory [CPP, Inc.]), or different measures of the same trait (e.g., measuring Verbal and Nonverbal IQ with the [WAIS: Pearson Assessments] and Wechsler Intelligence Scale for Children [WISC: Pearson Assessments]. Those sets of scores are the dependent variables that are plotted for different groups; the question to be investigated is whether those profiles are the same for the groups.

At the coarsest level, one could answer this question by simply investigating the between-groups main effect averaging over the dependent variables. This only answers questions related to whether the groups are, on average, scoring differently overall. Given that profile analysis uses plots of the data, this univariate test is said to examine whether there are "equal levels" across the observations. A second relevant question is whether the profiles for the groups are the same; that is, are the plots of data "parallel"? This is a test of the interaction of the repeated measures and between-subjects factors. When that interaction is not statistically significant, the profiles are parallel. If the test for parallelness is not rejected, then a test of the "flatness" of the profiles is examined by looking at the main effect across the dependent variables. These questions of whether the lines are parallel and flat require multivariate tests and, as such, can be examined using multivariate analysis of variance (MANCOVA).

The preceding paragraphs on PA have focused on similarities or differences between groups that are known a priori. It is possible to use PA to classify individuals into latent groups, and it is that technique, known as latent profile analysis (LPA), that shares its function with LCA. As Table 10.1 highlights, the major difference between the two techniques is that the manifest variables are continuous in LPA and categorical in LCA. An example of LPA from the field of criminology (Vaughn, DeLisi, Beaver, & Howard, 2008) used 15 measures of criminal behaviors to arrive at a solution that yielded four categories of offenders. Other applications of LPA include the development of typologies for achievement goal orientation (Pastor, Barron, Miller, & Davis, 2007), subtypes of adolescent depression (Herman, Ostrander, Walkup, Silva, & March, 2007), and neighborhood profiles in relation to adolescent physical activity (Norman et al., 2010).

10.4 Sample Size Requirements and Assumptions in LCA

There are no rules of thumb for the sample size necessary for a latent class analysis, perhaps because sample size depends on many different issues related to the latent classes and the items themselves. One of the most important considerations is the size of the classes; when small classes exist, a larger sample size is necessary. The number of classes also has an impact in that when more classes are present, a larger sample size is necessary. A final consideration regarding the classes is that a higher degree of separation between the latent classes means that smaller sample sizes may be acceptable, all other things being equal.

The characteristics of the manifest variables can also have an impact on sample size. Generally, we prefer items whose responses show large variability. For dichotomous variables, that means that when the probabilities of endorsing items are close to either boundary, we will need a larger sample size to compensate. The relationship between items and the classes is also a consideration. Items that have the same probabilities of endorsement for multiple classes would be less sensitive to class differences; again, a larger sample size would be necessary in this situation. Another variable-specific concern is that items with less reliability necessitate larger samples due to a greater degree of measurement error.

The final consideration, vis-à-vis the manifest variables, is how many to include in the model. Obviously, more items allow us to estimate models that are more complex in structure or that have more latent classes. Given that, the researcher's natural tendency is to always want to use more items. However, one must consider that in LCA, when M dichotomous manifest variables are investigated there are 2^M possible unique response vectors. That means that when we have only 10 manifest variables, there are 1,024 possible response patterns. Unless the sample size is extremely large, the contingency table containing those response frequencies will contain large numbers of empty cells.

Given that, convergence may become problematic, and goodness-of-fit tests will not be applicable.

In addition to considerations involving latent classes and manifest variables, one must also reflect on the complexity of the latent class model when deciding on the necessary sample size. In general, models that are more complex will require larger sample sizes. However, models that impose restrictions on the permissible response vectors or force parameters to be equal across classes could be estimated with smaller samples.

Beyond issues of sample size, the researcher must also consider whether the data can be expected to meet the assumptions underlying LCA. The most important assumption in LCA is that of local independence, which means that manifest variables should be independent within each class. One simple way to check this assumption is by examining the standardized bivariate residuals; significant residuals indicate that local independence does not hold (Vermunt & Magidson, 2008). Dependencies among residuals may be modeled; however, this is not common and should not be done to improve model fit (Flaherty, 2007).

Another assumption in LCA is that the model is just identified or over identified; if this is not the case, an infinite number of solutions will exist. Typically, underidentification is associated with the situation in which there are more parameters to be estimated than degrees of freedom, and this is main source of underidentification in LCA. A basic LC model with two manifest variables will be unidentified no matter the number of rating levels (Goodman, 1974). Likewise, a three-class model with three manifest variables will also be unidentified since the number of degrees of freedom is less than the number of parameters to be estimated. Other models with polytomous data that have specific combinations of variables and classes may also be unidentified. McCutcheon (1987) indentified one such problematic combination. Underidentifcation may also occur due to unforeseen structures in the data. For example, an unrestricted three-class model with four binary manifest variables will be unidentified even though it has 15 degrees of freedom and only 14 parameters to estimate. For more information on the topic of model identifiability, go to John Uebersax's website (http://john-uebersax.com/stat/faq.htm#ident).

In latent class analysis, there are few, if any, assumptions with regard to the nature of the manifest variables used. These may be either dichotomous or polytomous. When the data are either binary or nominal, the analyses can be run without any constraints. However, for ordered-category or Likert-scale data, constraints will help to ensure that the results are consistent with the ordering of the responses. See Clogg (1979) for an example of equality constraints for handling this sort of data. Latent profile analysis (Lazarfeld & Henry, 1968) is the version of LCA suitable for continuous manifest variables. The lone type of data that is not appropriate for LCA is rank-order data. LCA does not make any distributional assumptions or assume linearity or homogeneity.

10.5 Issues in LCA

One issue in the estimation of latent class models, either using maximum likelihood or Bayesian estimation, is that the solution may find a local, rather than global, maximum. According to Uebersax (2009), "Local maximum solutions were generally different enough from the global maximum solution to significantly affect substantive interpretation of results" (Conclusions section, para. 1). To ensure this is not an issue, the researcher should execute the LCA program several times with different starting values each time; this is illustrated later in this chapter. If these replications arrive at

the same result, the researcher has evidence that a global maximum has been reached. This evidence is strengthened when a stringent convergence criterion (1 E-8 or lower) is used for those runs. Evidence that a local maximum has been reached could be that parameter estimates approach 0 or 1. When more estimates approach those boundaries, there is greater concern that a local maximum has been found (Uebersax, 2009). To reduce the possibility of local maxima being found, researchers may want to consider limiting the number of latent classes in the model. They may also want to model the latent classes as gradations of a unidimensional trait. The constraints on the conditional response probabilities needed to estimate that model make local maxima less likely.

10.6 Exploratory Latent Class Analysis

As with any statistical analyses, researchers must first be clear about whether they are operating in an exploratory or confirmatory mode; this decision has an impact on all that follow it. Typically, one begins an exploratory LCA by investigating a one-class solution and comparing the fit of that model to ones that include additional classes. Models are compared to each other using a variety of fit statistics as well as indications of model usefulness. Measures of fit typically include chi-square statistics, information criteria, and standardized residuals. Indicators of model usefulness include entropy and information from classification tables; however, these must be used in conjunction with substantive interpretations and judgments regarding the meaningfulness of the size of the latent classes. Just as in factor analysis, determining the best fitting model tends to be quite subjective even with all of the quantitative data available regarding fit and usefulness.

Typically, the goodness of fit of a model to the observed data is examined using the Pearson and likelihood ratio chi-square tests; however, those statistics are often problematic in LCA. The first issue with them is that both distributions are not well approximated by the chi-square distribution when there is a sparse data set; as a result, hypothesis testing can either be too liberal or too conservative (Collins, Fidler, Wugalter, & Wong, 1993). Often in statistics, we compare two nested models using the difference in the likelihood ratio statistics with degrees of freedom calculated as the difference in the degrees of freedom (*dfs*) for the models; this difference statistic has a theoretical chi-squared distribution. This test is not appropriate for comparing LC models that only differ in the number of latent classes. Although these would appear to be nested, they do not meet all of the assumptions necessary for the difference likelihood ratio to have a chi-squared distribution (Uebersax, 2009). Note that if the models were different in terms of some equality constraints on the parameters, the difference test would be appropriate.

In addition to examining the absolute fit of a model, researchers in an exploratory mode will compare alternate models using information criterion. The Akaike Information Criterion (AIC; Akiake, 1987), the Bayesian Information Criterion (BIC; Schwarz, 1978) and the sample-size adjusted BIC (aBIC; Sclove, 1987) all appear in Mplus output by default. Because the AIC does not include sample size, it lacks asymptotic consistency (Bozdogan, 1987); therefore, BIC and aBIC are favored over it as indicators of model fit. Comparative fit can also be investigated using the (Vuong)-Lo-Mendell-Rubin likelihood ratio test (VLMR), the adjusted Lo-Mendell-Rubin (aLMR) test (Lo, Mendell, & Rubin, 2001), and the bootstrapped likelihood ratio test (BLRT: MacLachlan & Peel, 2000). These three tests use approximations to the likelihood ratio distribution, which makes them usable with LCA with different numbers of classes. For all three, the null hypothesis is that the c–1 class model fits the data; therefore, a nonsignificant result for a four-class model would indicate that the three-class model fits the data.

Muthén and Asparouhov (2006) recommend that researchers examine the standardized Pearson residuals as another source of information regarding model fit. As has already been discussed, with LCA models many cells have expected frequencies that approach zero. Typically, however, there are a number of response patterns that are prevalent. Examination of the standardized residuals of these more commonly occurring response patterns can be one source of information. Additionally, Skrondal and Rabe-Hesketh (2004) suggest that the standardized residuals from both the individual items and from pairs of items should be examined. In all cases, the standardized residuals can compared to a standard normal distribution, with residuals greater than 1.96 indicating poor fit.

The information regarding the class counts is also useful in determining the number of classes to retain. If one of the classes had an extremely small count, the researcher might opt for a simpler model with one fewer latent class. The definition of *extremely small* depends on both sample size and substantive considerations. When the sample size is very large, small proportions may still represent a large number of people; therefore, the researcher might be more likely to retain that class. Likewise, when assignment in the classes is substantively meaningful, researchers might also be compelled to keep extremely small classes.

Classification quality (i.e., relative entropy or entropy in Mplus) and the average latent class probabilities for most likely class membership provide additional information regarding the quality of class assignment. Celeux and Soromenho (1996) state that entropy values approaching 1.0 indicate that there is a clear delineation between the classes. Note that lower entropy values may produce acceptable parameter estimates, but it will be difficult to identify the classes to which people belong.

Estimated conditional probabilities approaching the boundaries (0 or 1) may indicate that the data may have been overfit. As a result, this may be more evidence in favor of simplifying or restricting the model. Researchers should be aware this situation creates computational issues for the parameter estimates that call into question the confidence intervals and significance tests (Galindo-Garre & Vermunt, 2006).

10.6.1 Performing a Latent Class/Profile Analysis in Mplus

In this section, we provide step-by-step instructions for how to run a simple latent class analysis using Mplus, Sixth Edition (Muthén & Muthén, 2010). To facilitate our explanation, we use an example dataset consisting of 10 dichotomously scored items designed to measure adolescents' perception of their parents' support for different responses to conflict (Orpinas, Murray, & Kelder, 1999). The data used in this example are composed of responses from 775 middle school (i.e., sixth grade) students participating in a longitudinal study of adolescent social development. The measure is made up of two scales: the first five items measure parental support for aggressive responses to conflict, and the final five items measure parental support for nonaggressive responses to conflict. For all questions, the stem is "Does your parent tell you these things about fighting?" followed by an example of an aggressive or nonaggressive response to conflict. Responses are scored 0 for no and 1 for yes. Table 10.3 provides the text of the 10 items.

Table 10.3 Items included in example measure.

Number	*Item*	*Scale*
1	If someone hits you, hit them back.	Parental Support for Aggressive Responses
2	If someone calls you names, hit them.	
3	If someone calls you names, call them names back.	
4	If someone asks you to fight, hit them first.	
5	If you can't solve the problem by talking, it is best to solve it through fighting.	
6	If someone calls you names, ignore them.	Parental Support for Nonaggressive Responses
7	If someone asks you to fight, you should try to talk your way out of a fight.	
8	You should think the problem through, calm yourself, and then talk the problem out with your friend.	
9	If another student asks you to fight, you should tell a teacher or someone older.	
10	No matter what, fighting is not good; there are other ways to solve problems.	

10.6.2 Mplus Input

The Mplus code for a two-class LCA with our data is shown in the following. After the code is a discussion of the parts of the code that are particularly applicable to LCA.

```
Title: Parental Support: 2 Class
Data: file is LCA.dat;
    format is 10f1.0;
Variable:
    Names are v1-v10;
    usevariables = v1-v10;
    classes = c(2);
    categorical = v1-v10;
    missing are blank;
Analysis:
    type = mixture;
    lrtstarts = 2 1 50 15;
Plot: type is plot3;
    series is v1 (1) v2 (2) v3 (3) v4 (4) v5 (5) v6 (6) v7 (7) v8 (8) v9 (9) v10 (10);
```

```
Savedata:
    file is save-lca-2.txt;
    save is cprob;
    format is free;
Output: tech10 tech11 tech14;
```

As usual, the **Analysis** command provides information about the type of analysis called for. The type statement indicates that we want to conduct a mixture analysis. This, in conjunction with the statements indicating that there are classes and categorical variables, specifies that an LCA is being run. The **lrtstarts** statement specifies the number of starting values and optimizations to use during the bootstrapped likelihood ratio test. The first two numbers specify the $k - 1$ class model. In this case, we have specified two random sets of starting values in the initial stage and one optimization in the final stage. The second two numbers specify the k class model. Here, we have specified 50 random sets of starting values in the initial stage and 15 optimizations in the final stage. Note that this is an increase over the default values of 20 and 5, respectively, and is necessitated because a "failure to replicate" warning. As discussed earlier, the use of these multiple starting values should help ameliorate the issue of identifying local, instead of global, maxima.

The **Plot** command provides information about the plot we would like provided. The **type** statement indicates the type of plot that is requested. In this case, plot3 gives us a graph of the conditional probabilities of item response given latent class. The series statement specifies the variables to be included in the graph and a label for each of the x-axis values. In this case, we include all 10 items and label them 1 through 10 on the x-axis. We include a graph of the conditional probabilities because this information is helpful in terms of interpreting the latent classes and making decisions about the number of classes to be retained. See Samuelsen and Dayton (2010) for an additional example of the use of this type of graph.

The **Savedata** command requests that the program saves the original data file along with additional information obtained from running the analysis. The **save** statement specifies the new statistics that we would like to preserve from the analysis. In this case, the **cprob** statement specifies that we would like to save the class probabilities (i.e., the probabilities of each respondent belonging to each class) and the assigned latent class. This class membership information might be useful in subsequent analysis performed in Mplus or another statistical software package.

The **Output** command requests additional options beyond the default Mplus output which includes AIC, BIC, and adjusted BIC. In this case, we opted to request additional fit indices by calling for Tech10, Tech11, and Tech14. The Tech10 option reports the standardized residuals for the items individually (Univariate), for pairs of two items (Bivariate) and for the response patterns. It is also possible to check the number and frequency of the observed response patterns using this command. Tech11 provides the Vuong-Lo-Mendell-Rubin and the Lo-Mendell-Rubin adjusted likelihood ratio tests (LMR: Lo, Mendell, & Rubin, 2001). Tech14 provides a bootstrapped likelihood ratio test (BLRT: McLachlan & Peel, 2000). Although Nylund, Asparouhov, and Muthén (2007) seem to indicate that there are few advantages of the LMR over the BLRT, we called for both to support our discussion of the differences between these likelihood ratio tests.

10.6.3 Mplus Output

The first part of the analysis output includes descriptive statistics and other information about the dataset and analysis procedure. The section titled "Univariate Proportions and Counts for Categorical Variables" provides descriptive statistics regarding item response. For all of the variables included in the latent class analysis, Mplus reports the percentage and the number of respondents selecting each response option. Note that Mplus has renamed our response options "Category 1" and "Category 2." In our case, Category 1 corresponds to "No" and Category 2 corresponds to "Yes." For Variable 1, 409 (53.1%) students responded no to the question, "Does your parent tell you if someone hits you, you should hit back?" and 361 (46.9%) responded yes. As always, it is a good idea to check this output against the basic descriptive statistics that you will have already obtained in a basic statistical software program (e.g., SPSS Version 21, IBM Corporation, 2012) to make sure the data were read in correctly.

UNIVARIATE PROPORTIONS AND COUNTS FOR CATEGORICAL VARIABLES (V1)

Category 1	0.531	409.000
Category 2	0.469	361.000

The next major section in the Mplus output pertains to the fit of the model. From this section, we obtain log-likelihood, information criteria (AIC, BIC, and aBIC), and chi-square tests (Pearson and Likelihood ratio). We also acquire the values for the additional fit indices that were requested—the Lo-Mendell-Rubin likelihood ratio test (and its adjusted test) and the bootstrapped likelihood ratio test. To find these results, one must scroll down to the end of our output; however, we report these here for completeness.

In an exploratory LCA, one compares a single-class solution to a series of more complex models. Following are the results of the latent class analyses of our data with the number of classes ranging from 1 to 5. To increase the number of latent classes, we simply change the number in the **classes** statement in the **Variable** command in our input. For example, to request a three-class solution, the input line will read: classes = c(3). The rest of the input statements remain the same. Summarized fit information for these models are shown in Tables 10.4 and 10.5.

MODEL FIT INFORMATION	
Number of Free Parameters	21
Loglikelihood	
H0 Value	-3254.472
H0 Scaling Correction Factor for MLR	1.046
Information Criteria	
Akaike (AIC)	6550.945
Bayesian (BIC)	6648.655
Sample-Size Adjusted BIC	
$(n^* = (n + 2) / 24)$	6581.970

Chi-Square Test of Model Fit for the Binary and Ordered Categorical (Ordinal) Outcomes**

Pearson Chi-Square	
Value	1173.206
Degrees of Freedom	994
P-Value	0.0001
Likelihood Ratio Chi-Square	
Value	686.919
Degrees of Freedom	994
P-Value	1.0000
TECHNICAL 11 OUTPUT	
Random Starts Specifications for the k-1 Class Analysis Model	
Number of initial stage random starts	10
Number of final stage optimizations	2
VUONG-LO-MENDELL-RUBIN LIKELIHOOD RATIO TEST FOR 1 (H0) VERSUS 2 CLASSES	
H0 Loglikelihood Value	-3693.033
2 Times the Loglikelihood Difference	877.121
Difference in the Number of Parameters	11
Mean	10.497
Standard Deviation	15.444
P-Value	0.0000
LO-MENDELL-RUBIN ADJUSTED LRT TEST	
Value	865.297
P-Value	0.0000
TECHNICAL 14 OUTPUT	
PARAMETRIC BOOTSTRAPPED LIKELIHOOD RATIO TEST FOR 1 (H0) VERSUS 2 CLASSES	
H0 Loglikelihood Value	-3693.033
2 Times the Loglikelihood Difference	877.121
Difference in the Number of Parameters	11
Approximate P-Value	0.0000
Successful Bootstrap Draws	5

Based on the BIC and adjusted BIC the four-class solution is preferred since it has the lowest values for these statistics. However, examination of the Lo-Mendell-Rubin likelihood ratio tests (i.e., VLMR and aLMR) seems to yield somewhat contradictory information. The p values associated with the LMR tests and the BLRT can be used to help one decide whether the $c - 1$model should be retained. Research has shown that the values for the LMR tests tend to alternate from statistical significance to non-significance and back, while the BLRT does not appear to be quite so fickle (Nylund et al., 2007). Nylund

Table 10.4 Chi-square based fit indices for latent class analysis.

Class	*Log-likelihood*	*Pearson (df)*	*p-value*	*Likelihood ratio*	*p-value*
1	−3,693.033	2,288.288 (983)	< 0.0000	679.337	1.0000
2	−3,254.472	1,173.206 (994)	0.0001	686.919	1.0000
3	−3,211.324	967.614 (984)	0.6392	600.338	1.0000
4	−3,168.907	765.843 (972)	1.0000	506.121	1.0000
5	−3,154.493	743.393 (960)	1.0000	480.077	1.0000

Table 10.5 Additional fit indices for latent class analysis.

Class	*Information Criteria*			*Likelihood Ratio Tests p values*		
	AIC	*BIC*	*aBIC*	*VLMR*	*aLMR*	*Bootstrap*
1	7,406.065	7,452.594	7,420.839	n/a	n/a	n/a
2	6,550.945	6,648.655	6,581.970	0.0000	0.0000	0.00
3	6,486.648	6,635.540	6,533.924	0.1670	0.1712	0.00
4	6,423.815	$\boxed{6,623.888}$	$\boxed{6,487.342}$	0.0162	0.0170	0.00
5	6,416.987	6,668.241	6,496.765	0.0471	0.0490	0.07

Note: AIC = Akaike Information Criterion; BIC = Bayesian Information Criterion; aBIC = sample-size adjusted BIC; VLMR = Vuong-Lo-Mendell Rubin likelihood ratio test; aLMR = adjusted Lo-Mendell-Rubin test.

et al. suggest that, based on their experience, "preliminary results suggest the first time the p value of the LMR is nonsignificant might be a good indication to stop increasing the number of classes" (p. 563). Our data do evidence that bouncing pattern for the LMR tests and indicate that three-class solution does not significantly improve on the fit of the two-class solution.

The results of the BLRT, on the other hand, suggest that the four-class solution should be retained. The BLRT has been shown to outperform other methods in determining the correct number of latent classes (Nylund et al., 2007). Because the BLRT, the BIC, and the aBIC (i.e., the top three methods based on Nylund et al.) all point toward the four-class solution, we can feel relatively confident in this solution. However, because of the inconsistency in the results of the LMR test, and in order to demonstrate other techniques for exploring the fit of the models, we now turn to the standardized residuals to gather more evidence about the fit of the two- and four-class models.

The output from Tech10 includes the standardized residuals for the response frequencies. Since we have over 200 unique response frequencies for our dataset, in Table 10.6 we show only a subset of these from both the two- and four-class solution. Table 10.7 includes a subset of the bivariate model fit statistics. We have not included any output regarding univariate model fit because all items yielded nonsignificant results for both the two- and four-class solutions. The residuals from both the response pattern frequencies and the bivariate model fit statistics indicate that the two-class model does not fit the data

Table 10.6 Response pattern frequencies and chi-square contributions.

Response Pattern	*Frequency*		*Standardized Residuals (z-score)*	*Chi-square Pearson*	*Contribution Log-likelihood*
	Observed	*Estimated*			
2-Class Solution					
193	2.00	0.47	2.23	4.99	5.70
194	1.00	0.11	2.75	7.58	4.45
195	249.00	213.25	2.89	5.95	70.59
196	66.00	94.49	−3.13	8.59	−48.20
197	1.00	5.73	−1.98	3.90	−3.30
4-Class Solution					
193	2.00	0.55	1.96	3.85	5.02
194	1.00	0.19	1.88	3.55	3.31
195	249.00	241.32	0.60	0.20	9.00
196	66.00	71.42	−0.67	0.41	−11.26
197	1.00	1.57	−0.45	0.20	−0.71

as well as the four-class model does. Of particular interest is that the standardized bivariate residuals for the two-class model showed statistically significant results for 14 of the 45 pairs. We take this as an indication that local independence does not hold. The four-class model, however, yielded only two significant results. Although no rules of thumb exist regarding the number of results that may be statistically significant and still meet the local independence assumption, we take consider our outcome to be acceptable.

The recommended fit indices and the standardized residuals suggest that four classes best represent the data. Another way to compare the two- and four-class solutions relates to how individuals are classified into groups. Mplus provides output regarding the number of people (i.e., class count) and proportion of individuals assigned to each class. This information, which is summarized in Table 10.8, is provided based on several different methods: the estimated model, the estimated posterior probabilities, and the classification of individuals given their most likely latent class. The first two methods assign class membership probabilistically; therefore, fractions of individuals will be assigned to each class (i.e., Class 1 has 255.97226 members). Using the method based on most likely membership, individuals are wholly assigned to only one class—the class in which they most likely belong. Using this method, 225 respondents were assigned to Class 1. The fact that all three methods yield essentially the same result provides us evidence that the two classes in this solution are distinct. The four-class model evidences some disagreement between the parameter estimates obtained via the different models. As an example, consider Latent Class 3, which has estimates of 388.62303 and 422 from the different models. This would seem to indicate that the four classes are not readily distinguishable.

The inferences drawn from the class counts regarding the delineation between the classes is echoed in the entropy values for the two solutions. For the two-class model, entropy equaled 0.789; that value for the four-class model was 0.711. Note that entropy is not a measure of fit and was not designed as a model selection tool (Ramaswamy, Desarbo, Reibstein, & Robinson, 1993); instead, it provides some sense of model usefulness. If the

Table 10.7 Bivariate model fit information.

Variable	*Variable*	*Estimated Probabilities*		
		H1	*H0*	*Standardized Residual (z-score)*
2-Class Model				
V1	V2			
Category 1	Category 1	0.519	0.507	0.691
Category 1	Category 2	0.012	0.024	−2.236
Category 2	Category 1	0.401	0.415	−0.815
Category 2	Category 2	0.068	0.054	1.765
Bivariate Pearson Chi-Square				8.450
Bivariate Log-Likelihood Chi-Square				9.367
V1	V3			
Category 1	Category 1	0.480	0.452	1.571
Category 1	Category 2	0.051	0.079	−2.881
Category 2	Category 1	0.262	0.291	−1.758
Category 2	Category 2	0.207	0.178	2.080
Bivariate Pearson Chi-Square				14.740
Bivariate Log-Likelihood Chi-Square				15.711
4-Class Model				
V1	V2			
Category 1	Category 1	0.519	0.517	0.135
Category 1	Category 2	0.012	0.014	−0.463
Category 2	Category 1	0.401	0.405	−0.237
Category 2	Category 2	0.068	0.065	0.418
Bivariate Pearson Chi-Square				0.417
Bivariate Log-Likelihood Chi-Square				0.425
V1	V3			
Category 1	Category 1	0.480	0.479	0.027
Category 1	Category 2	0.051	0.051	0.000
Category 2	Category 1	0.262	0.263	−0.057
Category 2	Category 2	0.207	0.206	0.028
Bivariate Pearson Chi-Square				0.003
Bivariate Log-Likelihood Chi-Square				0.003

Note. V1 = item 1; V2 = item 2; V3 = item 3; H1 refers to observed probabilities; H0 refers to IRT model predictions.

Table 10.8 Final class counts and proportions for the latent classes.

	Latent Classes	*Class Counts*	*Proportions*
2-Class Model			
Based on the Estimated Model	1	255.97226	0.33029
	2	519.02774	0.66971
Based on Estimated Posterior Probabilities	1	255.97226	0.33029
	2	519.02774	0.66971
Based on Their Most Likely Latent Class Membership	1	255	0.32903
	2	520	0.67097
4-Class Model			
Based on the Estimated Model	1	120.58387	0.15559
	2	184.28104	0.23778
	3	388.62303	0.51045
	4	81.51207	0.10528
Based on Their Most Likely Latent Class Membership	1	103	0.13290
	2	165	0.21290
	3	422	0.54452
	4	85	0.10968

Table 10.9 Average latent class probabilities for most likely latent class membership (row) by latent class (column).

Solution	*Class*	*Class 1*	*Class 2*	*Class 3*	*Class 4*
2-Class	1	0.918	0.082	—	—
	2	0.042	0.958	—	—
4-Class	1	0.825	0.088	0.040	0.047
	2	0.087	0.782	0.086	0.045
	3	0.027	0.096	0.877	0.000
	4	0.114	0.070	0.000	0.815

researchers' purpose is to identify homogeneous groups of subjects who are distinct from one another, then a low entropy value would indicate that a model is not helpful for that purpose. Further, entropy, as a single number, provides no information on which classes are problematic. For that, more nuanced, information one can examine the average latent class probabilities for most likely latent class membership, shown in Table 10.9. When the magnitudes of the off-diagonal cells for a latent class are large, there is some evidence that class is less distinguishable. There are no specific cutoffs for when the magnitudes of those cells are problematic, perhaps because those may vary from domain to domain. In this case, for the four-class solution, LC2 could be cause for concern based on the off-diagonal

Table 10.10 Results in probability scale.

Class	*Item 1*	*Item 2*	*Item 3*	*Item 4*
Latent Class 1				
Category 1 (NO)	0.198	0.029	6.843	0.000
Category 2 (YES)	0.802	0.029	27.736	0.000
Latent Class 2				
Category 1 (NO)	0.695	0.027	26.099	0.000
Category 2 (YES)	0.305	0.027	11.445	0.000

values of 0.087 and 0.086 for Classes 1 and 3, respectively. LC4, with an off-diagonal value of 0.114 for Class 1, also piques our interest.

At this point, there still seems to be evidence in favor of both the two- and four-class solutions. The BLRT, BIC, aBIC, and standardized residuals favor the four-class model. The LMR statistics indicate that the two-class model best fits the data, and the entropy/classification indicators provide evidence that there is more separation between classes with that model. In the absence of a substantive rationale for choosing one or the other, we continue to use the output to assist in our final decision. (We discuss the role of theory in making decisions regarding LCA in a subsequent section.)

The next section in the output reports the model results. These results help to describe what it means for a person to be classified in one class instead of another. We present the results for the two-class solution first, shown in Table 10.10. The section titled "Results in Probability Scale" reports the conditional probabilities of item response for each latent class (along with the standard errors, the z-statistics, and the p values associated with those probabilities). This section allows us to begin to understand how our classes differ. For example, the probability of responding yes to Question 1 is 80.2% for a student in Class 1. The comparable probability for this question is only 30.5% for a student in Class 2. Because this question refers to an aggressive response to conflict, we can tentatively begin to define Class 1 as a group whose parents are more supportive of aggressive responses to conflict.

One can continue to look at each result in an item-by-item fashion. However, a helpful way to get an overall snapshot of the results is to examine the plots that we requested. These plots will give a visual display of the differences between latent classes. By using the "Plot" command on the menu bar, we can access the graphs we requested. We click "Plots," "View Plots," "Sample Proportions," and "Probability of One Category: 2" to obtain the following graph. Figure 10.2 plots the probability of a yes response given each latent class.

From this graph, we see how Class 1 and Class 2 differ across all items. For Class 2, the probability of a yes response to the first five items (measuring aggressive responses) is low, while the probability of a yes response to the last five items (measuring nonaggressive responses) is high. Therefore, Class 2 represents a group whose parents communicate support for nonaggressive solutions and do not advocate aggressive responses to conflict. The pattern for Class 1 is quite different. Overall, these students report more parental support for aggressive responses; however, the responses that are supported are those that are proportional to original action. Items 1 and 3, which both represent proportional retaliation (if someone hits you/calls you names, hit them/call them names back) are endorsed by those in latent Class 1. There is lower support for Items 2 (if someone calls you names, hit them) and 4 (if someone asks you to fight, hit them first). In Class 1, there appears to be tepid support for nonaggressive responses to conflict.

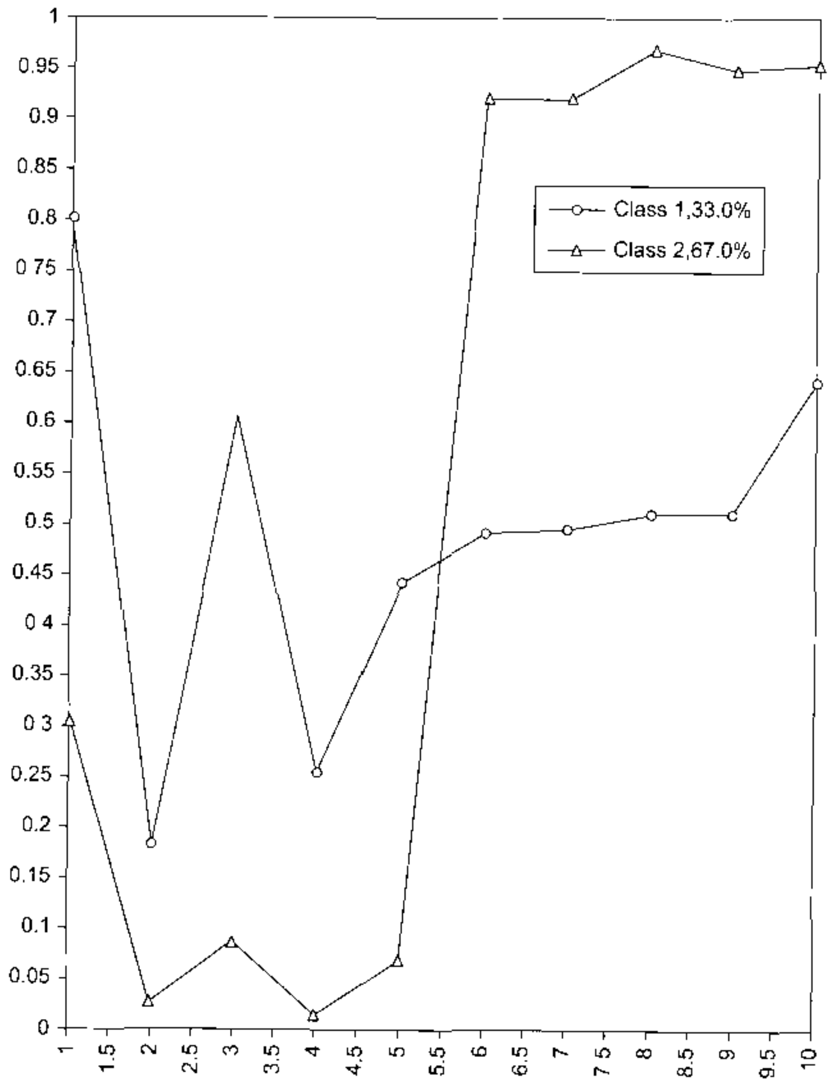

Figure 10.2 Two-class solution.

The graph of the four-class solution is provided in Figure 10.3. Several features are noteworthy. First, Class 3 in the four-class solution appears to be a group of students whose parents are supportive of nonaggressive solutions and not supportive of aggressive responses. This class is similar to Class 2 from the two-class solution. (Note that the ordering of latent classes as assigned by Mplus is arbitrary. Therefore, Class 2 in the two-class solution may be labeled differently in a solution with a different number of classes.) In the four-class solution, Class 2 reports not only high parental support for nonaggression, but also high support for aggressive retaliation. Classes 1 and 4 appear to have more complex pattern regarding the types of actions their parents support. Each latent class has a clearly different pattern of responses across the 10 items, and each class appears large enough to warrant inclusion in the model.

In some ways, the four-class solution appears to reduce to the two-class solution. Class 3 in the four-class solution is quite similar to the less aggressive class (i.e., Class 2) in the

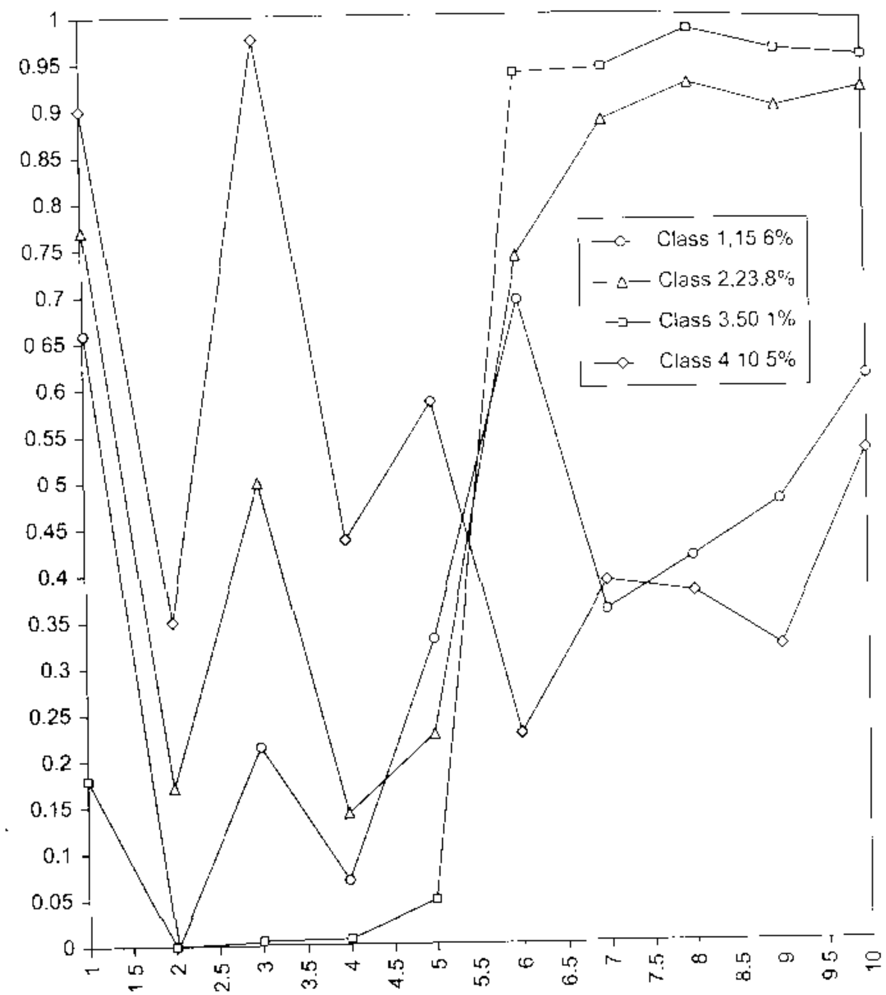

Figure 10.3 Four-class solution.

two-class solution, and Class 4 in the four-class solution is similar to the more aggressive class (i.e., Class 1) in the two-class solution. Class 2 in the four-class solution is somewhat similar to the more aggressive class in the two-class solution in the first five items, but is more similar to the less aggressive class for the second five items. Class 1 in the four class solution appears to be relatively different from either of the classes in the two-class solution.

10.6.4 Selecting the Final Model

Relying solely on the statistical and descriptive evidence presented thus far, our recommendation is to retain the four-class solution. The three recommended fit indices and an examination of the standardized residuals all forwarded a four-class solution. Further, the four classes all appear to be qualitatively different in terms of patterns of response (i.e., not just different levels of the same pattern) and of reasonable size. However, it would be unwise to make a final decision regarding the number of classes to retain without referring to substantive theory. It is beyond the scope of this article to evaluate the latent class solutions of our example from a theoretical framework, but theory should play an important role in evaluating and interpreting the latent class solutions. Ideally, the best-fitting solution also has a solid theoretical grounding. However, it may be the case that a good-fitting solution is theoretically implausible; in that case, the solution may need to be discarded. LCA is often used as an exploratory technique, and all of the usual caveats for exploratory analysis apply. Replication of results over multiple samples lends credibility to solutions; without such replication, results should be considered tentative.

10.7 Other LCA Applications and Extensions

Although this chapter has focused on exploratory LCA, we end by briefly summarizing some other applications and extensions to LCA. First, the issue of using covariates to predict latent class membership are addressed. Then overviews of confirmatory LCA, multilevel LCA, and multiple-groups LCA are provided. Note that we are attempting to shine a light into a dark room rather than fully illuminate it; however, we do provide current references for readers who prefer total illumination.

10.7.1 Covariates

Using covariates to predict latent class membership can help one to understand the nature of the latent classes. One way to go about this is to use the latent class regression model (Dayton & Macready, 1988) that estimates the effect of covariates as part of the latent class model. One problem with this procedure is that the inclusion of the covariate in the model can change the formation and interpretation of the classes (Clark & Muthén, 2009). An alternative to this procedure would seem to be to first estimate the predicted posterior class membership probabilities from a standard LCA and then use those as the outcome variables with covariates as the predictors. However, Bolck, Croon, and Hagenaars (2004) found the coefficient estimates from this stepwise procedure to be biased. Researchers might also be tempted to assign individuals to latent classes based on probabilities of membership and use that as a categorical outcome variable. This strategy is not recommended because, by treating those with a 99% probability as equal to someone with a 51% probability of membership, incorrect standard errors and potentially incorrect inferences may result. Clark and Muthén (2009) note that researchers who want to use this deterministic method should only do so if entropy exceeds 0.80, meaning the classes are quite distinct.

10.7.2 Confirmatory Latent Class Analysis

LCA may also be applied in a confirmatory manner. Prior research or clinical observation can provide researchers with theories regarding the latent classes underlying their data. These theories drive restrictions that are placed on the LC parameters. Three types of possible restrictions are (a) equality restrictions, (b) deterministic restrictions, and (c) inequality restrictions (McCutcheon, 2002). We briefly describe each type of restriction in the context of our example dataset and provide the Mplus code necessary for the associated analyses.

Equality restrictions are one type of constraint that researchers may place on their data. These restrictions are employed when latent classes are hypothesized to have the same

probability of endorsing an item. Although latent classes should be qualitatively different, in some cases, there may be a theoretical reason why certain classes will respond to an item in the same way. In the preceding exploratory four-class LCA, for Item 7, we obtained a response probability of .359 for Class 1 and a response probability of .389 for Class 4. Assume for the sake of illustration that, in addition to this sample-based finding, we have a strong theoretical reason for assuming these two classes have an equal probability of endorsing that item. In a future confirmatory LCA, we may wish to impose an equality restriction on Item 7 for Class 1 and Class 4.

In Mplus, this is accomplished by constraining the threshold for that item to equality across the two classes. The relationship between the probability of endorsing an item (P) and the threshold (τ) is (Muthén, 2001):

$$P = \frac{1}{1+e^{-\tau}} \qquad (1)$$

Large positive thresholds correspond to a low probability of item endorsement and large negative thresholds correspond to a high probability of item endorsement. To constrain the thresholds to equality, we use the MODEL command in Mplus. The following code can be inserted into our existing syntax between the ANALYSIS and the OUTPUT commands:

```
Model:
    %overall%
    %c#1%
    [v7$1*-.5] (1);
    %c#4%
    [v7$1*-.5] (1);
```

In this code, the threshold for item 7 (i.e., v7$1) is given a starting value (designated by an asterisk) –.5 of for Classes 1 (i.e., %c#1%) and 4 (i.e., %c#4%). This starting value was chosen because it corresponds to a probability of item endorsement of .377, which is the mean of probabilities of the two classes obtained from our exploratory analysis. The equality constraints are imposed by the (1) statement, and can be checked in the Mplus output. In our case, the estimated threshold for Class 1 and Class 4 was –.520 in both cases, which corresponds to a probability of endorsement of .373. Output should be reviewed carefully; in our case, what had been Class 1 in earlier data analysis became Class 4, and vise versa.

The next type of restriction is deterministic restrictions. Deterministic restrictions may be used to set item response probabilities to a hypothesized value. Theory may dictate that it is desirable to establish a latent class with a set response probability for an item, typically 0 or 1. For instance, in an item on drug use, there may be a theoretical justification for establishing an "abstainers class" with a response probability of 0. In our example, Class 4 has very high probability of endorsement for Item 3, providing some evidence for setting that response probability to 1 in future confirmatory studies. These constraints are also accomplished in the MODEL command. The threshold is set to –15 to correspond to a response probability value of essentially 1, as demonstrated by Finch and Bronk (2011).

The following code can be inserted into our existing syntax between the ANALYSIS and the OUTPUT commands:

```
Model:
    %overall%
    %c#4%
    [v3$1@-15] (1);
```

In this code, the threshold for Item 3 (i.e., v3$1) is set (designated by the @ symbol) to −15 for Class 4 (i.e., %c#4%).

Finally, inequality restrictions may be imposed. These may be employed when there is a hypothesized ordering of the latent classes with regard to the probability of endorsing an item. Theory may dictate that there is a clear ordering of classes on a particular item. For instance, in our example, there is a large separation between the classes on Item 3, where Class 4 > Class 2 > Class 1 > Class 3. If there was a compelling theoretical reason to do so, we could establish inequality restrictions in a future study to confirm this pattern. MODEL and MODEL CONSTRAINT commands are required.

```
Model:
    %overall%
    %c#1%
    [v3$1] (p1);
    %c#2%
    [v3$1] (p2);
    %c#3%
    [v3$1] (p3);
    %c#4%
    [v3$1] (p4);
Model Constraint:
    p4>p2;
    p2>p1;
    p1>p3;
```

In this code, the threshold for Item 3 is given a label for each latent class (i.e., p1, p2, p3, p4), and the inequality restrictions are imposed in the MODEL CONSTRAINT command.

The examples described earlier are intended to be illustrative of some of the options available in confirmatory LCA; other configurations are available. We do not interpret the output of any of the preceding examples because these constraints were selected for illustrative purposes only and are not based on theory. For a more comprehensive discussion of exploratory LCA including Mplus syntax, we recommend Finch and Bronk (2011).

10.7.3 Multilevel and Multigroup LCA

In many applications within the social sciences, the assumption that observations are independent is not tenable due to nested data structures. For example, students may be nested

within teachers or schools, people could be nested within cities or regions, or doctors might be nested within areas of specialization. Multilevel LCA (MLCA; Vermunt, 2003, 2008; Asparouhov & Muthén, 2008) has been developed to examine latent classes within this framework. Using this technique, and incorporating both student- and community-level predictors of smoking typologies, Henry and Muthén (2010) investigated a data structure in which adolescent females were nested within rural communities across the United States. The multilevel nature of the design allowed them to make statements regarding the probabilities that a student would be classified as a heavy smoker (as an example) given the characteristics of the community in which she resided. This sort of location-specific information could be useful in the development of community-based smoking cessation interventions.

Researchers sometimes wish to test whether measurement is invariant across some individual characteristic variable; this can be done using multigroup LCA. Kankaras, Moors, and Vermunt (2010) explain that this is more than merely estimating measurement model parameters freely and then estimating them a second time with the parameters constrained to be equal across groups. The rather simplistic notion of invariance as a dichotomy is replaced by a richer focus on degrees of homogeneity across classes. From Kankaras et al.'s perspective, "the objective of researching measurement invariance is to find the model with the lowest level of inequivalence possible that fits the data well" (p. 369). That means that we start with the simple question of whether the different groups require the same number of latent classes, and move from there to increasingly complex models.

What seems clear from this brief overview of advanced latent class techniques is that LCA is becoming more flexible and, as a result, applicable in a wider variety of applications. That, along with the fact that LCA provides a rigorous, statistical method of investigating the qualitative differences between subjects, makes LCA an extremely powerful tool for researchers.

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