STAT GU4261/GR5261 Statistical Methods in Finance

Spring 2018

Homeowork 2 Suggested Solution

Due date: 8 Feb 2017 (Thu)

(P.491-492) Exercise 1 in textbook:

(a) $0.023w + 0.045(1 - w) = 0.03$ implies $w = 15/22$. Invest $15/22$ amount of money in A and remaining to B.

(b) $6w^2 + 11(1-w)^2 + 2w(1-w)(\sqrt{6})(\sqrt{11})(0.17) = 5.5$ gives $w = 0.94$ or $w = 0.41$. The expected return is largest when $w = 0.41$ (invest 0.41 amount of money in A).

(P.491-492) Exercise 2 in textbook:

$r_p = (1-w)r_f + wr_T$. Then $\sigma_p^2 = w^2\sigma_T^2$. This gives $w = \pm\frac{5}{7}$. Weights for risk-free asset, asset C and asset D are $(2/7, 5/7 \cdot 0.65, 5/7 \cdot 0.35) = (2/7, 13/28, 1/4)$ or $(12/7, -5/7 \cdot 0.65, -5/7 \cdot 0.35) = (12/7, -13/28, -1/4)$.

(P.491-492) Exercise 3 in textbook:

(a)

$$\begin{aligned} w &= \frac{75(300)}{75(300) + 115(100)} = 0.6618 \\ 1 - w &= 0.3382. \end{aligned}$$

(b)

$$w_j = \frac{P_j n_j}{\sum_{i=1}^N P_i n_i}.$$

(P.491-492) Exercise 4 in textbook:

Let P_{t-1} be the value of the portfolio and $P_{i,t-1}$ be the price of stock i at $t-1$. At $t-1$, you invested $w_i P_{t-1}$ amount of money in stock i and have $\frac{w_i P_{t-1}}{P_{i,t-1}}$ number of stock i. Hence, the value of portfolio at t is

$$P_t = \sum_{i=1}^N \frac{w_i P_{t-1}}{P_{i,t-1}} P_{i,t}.$$

Net return of the portfolio is

$$\mathcal{R}_P^{\text{Net}} = \frac{P_t - P_{t-1}}{P_{t-1}} = \sum_{i=1}^N w_i \frac{P_{i,t}}{P_{i,t-1}} - 1 = \sum_{i=1}^N w_i \frac{P_{i,t} - P_{i,t-1}}{P_{i,t-1}} = \sum_{i=1}^N w_i \mathcal{R}_i^{\text{Net}}.$$

Gross return of the portfolio is

$$\mathcal{R}_P^{\text{Gross}} = \frac{P_t}{P_{t-1}} = \sum_{i=1}^N w_i \frac{P_{i,t}}{P_{i,t-1}} = \sum_{i=1}^N w_i \mathcal{R}_i^{\text{Gross}}.$$

Log return of the portfolio is

$$\mathcal{R}_P^{\text{Log}} = \log \frac{P_t}{P_{t-1}} = \log \sum_{i=1}^{N} w_i \frac{P_{i,t}}{P_{i,t-1}} > \sum_{i=1}^{N} w_i \log \frac{P_{i,t}}{P_{i,t-1}} = \sum_{i=1}^{N} w_i \log \mathcal{R}_i^{\text{Log}},$$

where the strict inequality holds by Jensen's inequality for nonnegative weights and $\frac{P_{i,t}}{P_{i,t-1}}$'s are not all equal. That means in general the equality does not hold for log return.

<u>(P.491-492) Exercise in textbook:</u>

(a) $\begin{pmatrix} 1 & 0.35 \\ 0.35 & 1. \end{pmatrix}$.

(b) $r_P = wr_1 + (1-w)r_2$. $w = \frac{100\cdot 200}{100\cdot 200 + 125\cdot 100} = 8/13$.
$\mu_P = \frac{8}{13} \cdot 0.001 + \frac{5}{13} \cdot 0.0015 = 0.00119$.
$\sigma_P = \sqrt{w^2\sigma_1^2 + (1-w)^2\sigma_2^2 + 2\rho w(1-w)\sigma_1\sigma_2} = 0.02786$.