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An Empirical Comparison of Multiple Imputation Methods for Categorical Data

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ABSTRACT

Multiple imputation is a common approach for dealing with missing values in statistical databases. The imputer fills in missing values with draws from predictive models estimated from the observed data, resulting in multiple, completed versions of the database. Researchers have developed a variety of default routines to implement multiple imputation; however, there has been limited research comparing the performance of these methods, particularly for categorical data. We use simulation studies to compare repeated sampling properties of three default multiple imputation methods for categorical data, including chained equations using generalized linear models, chained equations using classification and regression trees, and a fully Bayesian joint distribution based on Dirichlet process mixture models. We base the simulations on categorical data from the American Community Survey. In the circumstances of this study, the results suggest that default chained equations approaches based on generalized linear models are dominated by the default regression tree and Bayesian mixture model approaches. They also suggest competing advantages for the regression tree and Bayesian mixture model approaches, making both reasonable default engines for multiple imputation of categorical data. Supplementary material for this article is available online.

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1. Introduction

Nearly all sample surveys and censuses suffer from item nonresponse, for example, individuals do not answer some questions. It is well-known that statistical analyses based on only the complete cases (with all variables observed) or available cases (with all variables observed for the specific analysis) can be problematic. At best, such analyses are inefficient, as they sacrifice information from partially observed responses. At worst, they result in biased inferences when there are systematic differences between the observed data and the missing data (Rubin 1976). See Little and Rubin (2002) for additional discussion of the drawbacks of complete/available case analysis.

A common approach to handling item nonresponse is multiple imputation (MI) (Rubin 1987). In MI, the analyst creates multiple, completed datasets by replacing the missing values with draws from (posterior) predictive distributions estimated with the observed data. The analyst estimates quantities of interest using each completed dataset, and combines the estimates using methods developed by Rubin (1987). This process incorporates the additional uncertainty from the missing data in inferences. The completed datasets also can be released as public use files. For reviews of MI, see Rubin (1996), Schafer (1997), Barnard and Meng (1999), Reiter and Raghunathan (2007), and Harel and Zhou (2007).

When implementing MI, most analysts adopt one of two general classes of strategies: joint modeling (JM) and fully conditional specification (FCS) (van Buuren 2007). In the JM strategy, the analyst specifies a joint distribution for all variables in the data. Imputations are sampled from the implied conditional distributions of the variables with missing data, given all other variables. The JM strategy is appealing, in that it accords with the theory in Rubin (1987). In practice, however, specifying accurate joint distributions for large numbers of variables can be challenging. Indeed, most popular JM approaches — such as AMELIA (Honaker, King, and Blackwell 2011), proc MI in SAS (Yang 2011), and the routine "norm" in R (Schafer 1997) — make a simplifying assumption that the data follow multivariate Gaussian distributions, even for categorical variables. Another option for categorical data is to use a log-linear model; this is implemented in the R package "cat" (Schafer 1997).

In the FCS strategy, the analyst directly specifies and samples from the univariate distributions of each variable conditional on all other variables, without first forming a proper joint distribution. The most popular FCS approach is known as multiple imputation by chained equations (MICE), which uses generalized linear models for each conditional distribution (Raghunathan et al. 2001; Royston and White 2011; Su et al. 2011; van Buuren and Groothuis-Oudshoorn 2011). The main strength of MICE lies in its simplicity and flexibility — one can tailor the predictive model for individual variables, for example, use a logistic regression for binary variables and a linear regression for continuous variables. However, MICE lacks theoretical basis in that the specified univariate conditional distributions may not be compatible (Arnold and Press 1989; Gelman and Speed 1993), that is, the set of the conditional distributions may not correspond to any joint distribution. Therefore, MICE defines a potentially incompatible Gibbs sampler (Li et al. 2014).

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Supplementary materials for this article are available online. Please go to www.tandfonline.com/r/TAS.

Despite this theoretical drawback, simulation-based research suggests that MICE performs well in practice (Brand 1999; van Buuren, Boshuizen, and Knook 1999; Raghunathan et al. 2001; Rubin 2003; van Buuren et al. 2006; van Buuren 2007, to name a few). Software packages implementing MICE include "IVEware" in SAS (Raghunathan, Solenberger, and Hoewyk 2002), "mice" (van Buuren and Groothuis-Oudshoorn 2011; van Buuren 2012) and "mi" in R (Su et al. 2011), and "mi" and "ice" in STATA (Royston and White 2011).

When the data include a large number of exclusively categorical variables, analysts implementing MI face a challenging task. When using log-linear models for a JM approach, the space of possible models is enormous due to the large number of potential interaction effects, making it difficult to select an imputation model. Similar model selection issues plague MICE approaches, for example, when specifying a multinomial logistic regression model for a multi-valued variable with missing data. Many analysts default to including main effects only; indeed, this is the default option in most popular MICE packages.

Motivated by these shortcomings, several authors have developed more flexible, default MI engines for both strategies. For JM imputation, Si and Reiter (2013) and Manrique-Vallier and Reiter (2014a, 2014b) use Dirichlet process mixture of products of multinomial distributions (DPMPM), which are nonparametric Bayesian versions of the latent class models used by Vermunt et al. (2008). The DPMPM imputation routines are implemented in the R software package, "NPBayesImpute" (Manrique-Vallier et al. 2014). For FCS imputation, Burgette and Reiter (2010) and van Buuren (2012) use classification and regression trees (CART). MI via trees is available as an option in "mice" in R. Independently, these techniques have shown promise as general-purpose routines for MI, suggesting that they may be preferable to default applications of MICE. However, to our knowledge, these two techniques have never been compared in simulation contexts with large-dimensional, genuine categorical data, nor have they been compared to default implementations of chained equations with generalized linear models. These lack of comparisons make it difficult, if not impossible, to assess the relative merits of each procedure.

In this article, we compare these three MI methods for categorical data using repeated sampling simulations. Using hypothetical populations comprising data from the American Community Survey (ACS), we find that the DPMPM and CART imputation engines (which we label with MI-DPM and MI-CART, respectively) result in better repeated sampling performance than the standard chained equations via generalized linear models (which we label with MI-GLM). The results suggest competing advantages for the MI-CART and MI-DPM approaches, depending on the sample size and amount of missing data. Both procedures are sensible default choices for MI of categorical data.

The remainder of this article is organized as follows. In Section 2, we review the three methods for MI with categorical data. In Section 3, we describe the simulation design and results with ACS data. In Section 4, we conclude with general lessons learned and some suggestions for practical implementation.

2. Multiple Imputation for Categorical Data

We first introduce notation for describing MI in the context of categorical data. Let $Y_{ij} \in \{1, \ldots, D_j\}$ be the value of variable j for individual i, where $j = 1, \ldots, p$ and $i = 1, \ldots, n$. For each individual i, let $\mathbf{Y}_i = (Y_{i1}, \ldots, Y_{ip})$. Let $\mathbf{Y} = (\mathbf{Y}_1, \ldots, \mathbf{Y}_n)$ be the $n \times p$ matrix comprising the data for all records included in the sample. We write $\mathbf{Y} = (\mathbf{Y}_{\text{obs}}, \mathbf{Y}_{\text{mis}})$, where $\mathbf{Y}_{\text{obs}}$ and $\mathbf{Y}_{\text{mis}}$ are respectively the observed and missing parts of $\mathbf{Y}$. We write $\mathbf{Y}_{\text{mis}} = (\mathbf{Y}_{\text{mis}1}, \ldots, \mathbf{Y}_{\text{mis}p})$, where $\mathbf{Y}_{\text{mis}j}$ represents all missing values for variable j, where $j = 1, \ldots, p$. Similarly, we write $\mathbf{Y}_{\text{obs}} = (\mathbf{Y}_{\text{obs}1}, \ldots, \mathbf{Y}_{\text{obs}p})$ for the corresponding observed data.

In MI, the analyst generates values of $\mathbf{Y}_{\text{mis}}$ using models estimated with $\mathbf{Y}_{\text{obs}}$. This results in a completed dataset $\mathbf{Y}^{(l)}$. The analyst draws L completed datasets, $\{\mathbf{Y}^{(l)} : l = 1, \ldots, L\}$, that are available for dissemination or analysis.

2.1. Overview of MI-GLM

In MI-GLM, we generate imputations from a sequence of predictive distributions derived from univariate generalized linear models. The approach proceeds as follows. We first specify an order for imputing the variables. For example, the "mice" package in R by default imputes the variables in the order that they appear in the data matrix (although a different order can be specified by the user), and the "IVEWare" package imputes variables in increasing order of the number of missing cases. Suppose that $r_0 \leq p$ variables have missing values, and that $r_1 = p - r_0$ variables are fully complete. Let the ordered list of variables be defined as $(Y_{(1)}, \ldots, Y_{(p)})$, where any variable with no missing values is put at the end of the list. We next fill in initial imputations at the missing values. These can be obtained from random draws from the observed marginal distributions for each variable with missing data (the "mice" package in R does this). Alternatively, for each variable j with missing values, we can sample from conditional distributions, $(Y_{(j)} \mid Y_{(1)}, \ldots, Y_{(j-1)}, Y_{(r_0+1)}, \ldots, Y_{(p)})$, where each model is estimated on the available cases for that model.

With this initial set of completed data, we now use an iterative process akin to a Gibbs sampler to update the imputations. At each iteration t of the updating, we estimate the predictive model, $(Y_{(1)} \mid \mathbf{Y}_{\text{obs}(1)}, \{\mathbf{Y}\}_{(\mathbf{k})}^{(\mathbf{t-1})} : \mathbf{k} > \mathbf{1}\})$, where $\mathbf{Y}_{(k)}^{(t-1)}$ includes the set of observed and imputed values for variable k at iteration $t-1$. We replace $\mathbf{Y}_{\text{mis}(1)}^{(t-1)}$ with draws from this conditional distribution, $\mathbf{Y}_{\text{mis}(1)}^{(t)}$. We repeat this process for each variable j with missing data, estimating and imputing from each predictive model, $(Y_{(j)} \mid \mathbf{Y}_{\text{obs}(j)}, \{\mathbf{Y}_{(k)}^{(t)} : k < j\}, \{\mathbf{Y}_{(k)}^{(t-1)} : k > j\})$. We repeat the cycle $t > 1$ times. The values at the final iteration are used to create the completed dataset, $\mathbf{Y}^{(l)} = (\mathbf{Y}_{\text{obs}}, \mathbf{Y}_{\text{mis}}^{(l)})$. The entire process is replicated L times to create the full set of multiple imputations.

For categorical data, most implementations of MI-GLM use logistic regressions for binary variables, multinomial logistic regressions for unordered variables, and cumulative logistic regressions for ordered variables. The default specification of the predictor functions is to include main effects only for all

$p-1$ variables. It is possible to remove predictors from and add interactions to certain univariate conditionals using special commands; for "mice" in R, the predictor Matrix argument can be used to add or remove predictors and the passive imputation option can be used to add interactions. For $Y_{(j)}$ corresponding to unordered categorical variables with large numbers of levels, multinomial logistic regressions can have large numbers of parameters, which can result in unstable coefficient estimates with high standard errors.

2.2. Overview of MI-CART

CART for categorical data (Breiman et al. 1984), originally developed for classification, can be modified to be an imputation engine. For any outcome $Y_{(j)}$, CART recursively partitions the predictor space in a way that can be effectively represented by a tree structure, with leaves corresponding to the subsets of units. The values of $Y_{(j)}$ in each leaf can be conceived as draws from the conditional distribution of $Y_{(j)}$ for the set of predictor values that satisfy the partitioning criteria that define that leaf. To illustrate, Figure 1 displays a fictional regression tree for an outcome variable and two predictors, gender (male or female) and race/ethnicity (African-American, Caucasian, or Hispanic). To approximate the conditional distribution of $Y_{(j)}$ for a particular gender and race/ethnicity combination, one uses the values in the corresponding leaf. For example, to approximate the distribution of $Y_{(j)}$ for female Caucasians, one uses the observed values of the outcome in leaf L3.

MI-CART operates like MI-GLM, except that CART models are used in place of logistic regressions. Initial imputations can be obtained by running CART on the available cases for $(Y_{(j)} \mid Y_{(1)}, \ldots, Y_{(j-1)}, Y_{(r_0+1)}, \ldots, Y_{(p)})$. For any value of $(Y_{i(1)}, \ldots, Y_{i(j-1)}, Y_{i(r_0+1)}, \ldots, Y_{i(p)})$, including values for previously initialized variables, we sample the initial imputation by dropping down the tree for $Y_{(j)}$ until finding the appropriate leaf, and sample from the values in that leaf. Burgette and Reiter (2010) suggested an additional step before sampling values within the leaves, namely to use Bayesian bootstraps within each leaf (holding constant the number of observations in each leaf) to create the pools from which to sample. The Bayesian bootstrap incorporates uncertainty about the distributions in each leaf. As with MI-GLM, the process is cycled $t > 1$ times, resulting in a completed dataset. We note, however, that this imputation routine does not incorporate uncertainty about the split points in the trees.

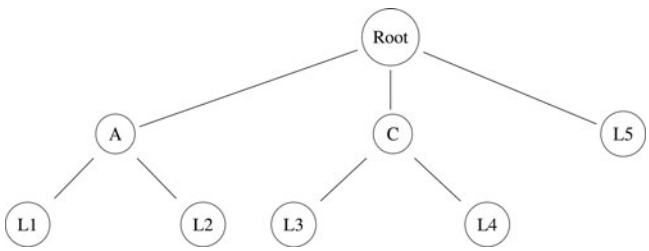


Figure 1. Illustration of the tree structure in CART. A: African-Americans; C: Caucasian; H: Hispanic; M: male; F: female. Leaf L1 contains female African-Americans; leaf L2 contains male African-Americans; leaf L3 contains female Caucasians; leaf L4 contains male Caucasians; and leaf L5 contains Hispanics of both genders.

CART algorithms, and hence MI-CART, can struggle with categorical predictors with many levels. For example, a categorical predictor with 32 levels results in over two billion potential partitions, making it computationally difficult to run the algorithm in sensible time with standard computers.

2.3. Overview of MI-DPM

The MI-DPM procedure assumes that the distribution of the categorical data can be characterized using a latent class model. We assume that all combinations of variables are possible a priori—that is, there are no structural zeros (Si and Reiter 2013). Each individual belongs to one of $K < \infty$ latent classes. Within each class, all variables follow independent multinomial distributions. To express this as a formal probability model, let $z_i \in (1, \ldots, K)$ represent the latent class of individual i; $\pi_k = \Pr(z_i = k)$, for $k = 1, \ldots, K$, be the probability of being in latent class k; $\lambda_{kjy} = \Pr(Y_{ij} = y \mid z_i = k)$ be the probability that variable Y_{ij} takes on the value y for records in latent class k. Also, let $\boldsymbol{\pi} = (\pi_1, \ldots, \pi_K)$, and let $\boldsymbol{\lambda} = \{\lambda_{kjy} : k = 1, \ldots, K;\ j = 1, \ldots, p;\ y = 1, \ldots, D_j\}$. We have

$$Y_{ij} \mid z_i, \boldsymbol{\lambda} \overset{\text{indep}}{\sim} \text{Discrete}_{1:D_j}(\lambda_{z_i j1}, \ldots, \lambda_{z_i jD_j}) \quad \text{for all } i \text{ and } j \tag{1}$$

$$z_i \mid \boldsymbol{\pi} \overset{\text{iid}}{\sim} \text{Discrete}_{1:K}(\pi_1, \ldots, \pi_K) \quad \text{for all } i. \tag{2}$$

We can express the marginal probability of any quantity by averaging over the latent classes, for example,

$$Pr(Y_{i1} = y_1, \ldots, Y_{ip} = y_p \mid \boldsymbol{\lambda}, \boldsymbol{\pi}) = \sum_{k=1}^{K} \pi_k \prod_{j=1}^{p} \lambda_{kjy_j}. \tag{3}$$

Although variables are independent within classes, averaging over classes can result in dependence among the variables. Dunson and Xing (2009) showed that, with large enough K, this model is consistent for any joint probability distribution.

For prior distributions, Si and Reiter (2013) and Manrique-Vallier and Reiter (2014b) used a truncated version of the stick-breaking representation of the Dirichlet process proposed by Dunson and Xing (2009),

$$(\lambda_{kj1}, \ldots, \lambda_{kjD_j}) \overset{\text{indep}}{\sim} \text{Dirichlet}(\mathbf{1}_{D_j}) \tag{4}$$

$$\pi_k = V_k \prod_{h<k} (1 - V_h) \tag{5}$$

$$V_k \overset{\text{iid}}{\sim} \text{Beta}(1, \alpha) \text{ for } k = 1, \ldots, K-1;\ V_K = 1 \tag{6}$$

$$\alpha \sim \text{Gamma}(0.25, 0.25). \tag{7}$$

This prior distribution facilitates efficient computation while allowing the data to dominate the posterior distribution.

To set K, we recommend the approach in Si and Reiter (2013) and Manrique-Vallier and Reiter (2014b). The analyst starts with a modest value, say $K = 20$. In each iteration of the MCMC, the analyst can compute the number of classes that have at least one individual. If the number of occupied classes reaches K across any of the iterations, then the analyst should increase K and repeat. As long as K is large enough, the estimated posterior

distribution is largely insensitive to the value of K. The iterative method of selecting K primarily serves to improve computational efficiency, as it avoids estimating parameters for many empty (and essentially irrelevant) classes.

Posterior inferences are obtained via a Gibbs sampler. Missing values are handled within the sampler. Given a draw of the parameters and observed data, one samples a value of the latent class indicator for the record using (2). Given a draw of the latent class indicator, one samples values for the missing items using independent draws from (1). The independence of variables within the latent classes facilitates computation and imputation in datasets with many categorical variables. The MI-DPM approach can be adapted for categorical data with structural zeros (Manrique-Vallier and Reiter 2014a), and it is not limited to a maximum number of levels. Because it relies on a coherent joint model, the MI-DPM engine can be the building block for handling more complicated missing data situations, including nonignorable nonresponse (Si, Reiter, and Hillygus 2015) and measurement error (Manrique-Vallier and Reiter 2017).

3. Repeated Sampling Evaluations

We base the simulation studies on data from the public use microdata files from the 2012 ACS, available for download from the United States Bureau of the Census (*http://www2.census.gov/acs2012_1yr/pums/*). As brief background, the ACS is sent to about 1 in 38 households in the United States, with the goal of enabling estimation of population demographics and housing characteristics at small geographies, for example, at census tracts. For each sampled household, the survey includes questions about the individuals living in the household (e.g., their ages, races, incomes) and about the characteristics of the housing unit (e.g., number of bedrooms, presence of running water or not, presence of a telephone line or not). The public use data include about 1.5 million housing units.

To construct the population data for the simulation, we treat the housing unit as the unit of analysis. We disregard data on individual people but keep ACS summaries of family compositions that can be viewed as household-level variables (e.g., presence of children or not, number of workers in household). We remove all identification variables (serial number, region, state, area code and division), variables corresponding to allocation flags (binary variables indicating if the corresponding variables have been imputed), variables corresponding to replicate survey weights, and all continuous variables. To simplify comparisons, we eliminate structural zeros—that is, impossible combinations of categories like a married 12-year old—by removing variables or levels. We also delete about 830,000 units corresponding to vacant houses and households with single occupants, again to avoid structural zeros. The final data comprise 671,153 housing units and 35 categorical variables. We treat these as a population from which to sample. We treat all variables as unordered categorical variables to simplify imputation modeling and analysis. The data comprise 11 binary variables (seven of which have marginal probabilities very close to one or zero), 17 variables with three to nine levels, and 7 variables with ten or more levels. The variables are described in Table 1 in the supplementary material.

We simulate the processes of random sampling and nonresponse repeatedly, so as to examine frequentist properties of the three imputation procedures. We take samples of size $n \in \{1000, 10{,}000\}$ and randomly blank either 30% or 45% of the values of each variable independently across variables. This results in data that are missing completely at random (MCAR), which is the most favorable scenario for all MI procedures. We also create a missing at random (MAR) scenario with $n = 10{,}000$ and a 30% missingness rate. In each sample, we use MI-GLM, MI-CART, and MI-DPM to create three sets of $L = 10$ completed datasets. We repeat the process 200 times, each time generating new samples and new missingness patterns. We note that the missing data mechanism tends to result in datasets with 10 or fewer cases with all variables measured, making complete case analysis untenable.

To implement both MI-GLM and MI-CART, we use the "mice" package in R (van Buuren and Groothuis-Oudshoorn 2014). For MI-GLM, we use the default choices for binary and nominal variables with two or more levels: logistic regressions with main effects for all variables. For MI-CART (the "cart" option in "mice"), we use the default arguments for fitting the trees, for example, at least four observations in any terminal node. We run both MI-GLM and MI-CART for $t = 10$ cycles as suggested by van Buuren and Groothuis-Oudshoorn (2011). To implement the DPMPM, we use the "NPBayesImpute" package in R developed by Manrique-Vallier et al. (2014). We set the number of latent classes $K = 35$, which appears sufficiently large based on tuning with initial runs. We run each MCMC chain for 10,000 iterations using the first 2000 as burn-in.

3.1. Performance Measures

We evaluate the MI methods using sets of marginal probabilities, bivariate probabilities, and trivariate probabilities. We consider only estimands that satisfy $np > 10$ and $n(1-p) > 10$, where p is the probability in the population, to eliminate estimands where the central limit theorem is not likely to hold even with no missing data. For each estimand, we use each set of L completed datasets to compute point and interval estimates via the methods of Rubin (1987). As a brief review, let q be the completed-data point estimator of some estimand Q, and let u be the estimator of variance associated with q. For $l = 1, \ldots, L$, let q_l and u_l be the values of q and u in completed dataset $\mathbf{Y}^{(l)}$. We use $\bar{q} = \sum_{l=1}^{L} q_l/L$ as the point estimate of Q. We use $T = (1 + 1/L)b + \bar{u}$ as the estimated variance of $\bar{q}$, where $b = \sum_{l=1}^{L}(q_l - \bar{q})^2/(L-1)$ and $\bar{u} = \sum_{l=1}^{L} u_l/L$. We form 95% confidence intervals using $(\bar{q} - Q) \sim t_v(0, T)$, where t_v is a t-distribution with $v = (L-1)(1 + \bar{u}/[(1 + 1/L)b])^2$ degrees of freedom. Multiple imputation inferences require that (i) the central limit theorem applies for the complete-data estimate, and (ii) the sampling distribution of the complete-data point estimates is approximately normal. When p is near zero or one, these conditions may not hold, which can result in unreliable inferences including intervals that include zero or one.

For each estimand, we compare the three MI procedures by two metrics. We also compute these metrics for estimates

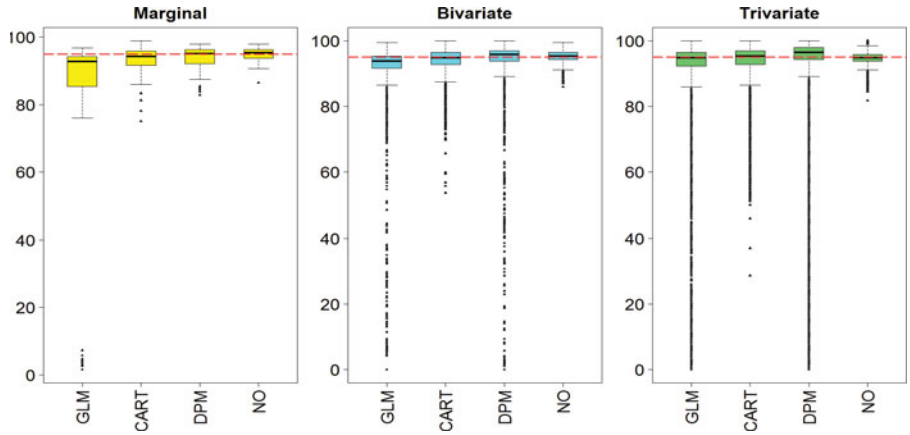


Figure 2. Simulated coverage rates for MI-GLM, MI-CART, MI-DPM, and the premissing data intervals when $n = 10{,}000$ with 30% values MCAR. We exclude seven variables with more than 10 levels, resulting in $p = 28$ variables for imputation and analysis.

based on the sampled data before introduction of missing values. First, we compute the proportion of the two hundred 95% confidence intervals that contain the corresponding Q from the full ACS data. Second, we compute the relative mean squared error (Rel.MSE) for $\bar{q}$, defined as

$$\text{Rel.MSE} = \frac{\sum_{h=1}^{200}(\bar{q}^{(h)} - Q)^2}{\sum_{h=1}^{200}(\hat{q}^{(h)} - Q)^2}, \tag{8}$$

where $\bar{q}^{(h)}$ is the value of $\bar{q}$ in simulation h, and $\hat{q}^{(h)}$ is the estimate of Q from the sampled data before introduction of missing values in simulation h.

3.2. Summary of Results

In our initial experiments, MI-GLM crashed when the data include more than one nominal variable with more than ten categories. This results from problems estimating the multinomial logistic regressions with many outcome levels. Therefore, we consider two types of simulations. In the first type, we remove the seven variables with more than 10 categories, leaving 28 variables, to allow comparisons across all three procedures. In the second type, we include the seven variables and compare only MI-CART and MI-DPM. We summarize results using graphical displays and tables. In all plots, MI-GLM is abbreviated as GLM, MI-CART as CART, MI-DPM as DPM, and the premissing data results as NO. Median coverage rates across all estimands for each scenario are available in the supplementary material. In all tables, the entries are summaries of the Rel.MSEs combined over all relevant probabilities, for example, the 25th, 50th, and 75th percentiles of the Rel.MSEs for all marginal probabilities. For all results, Monte Carlo standard errors for the coverage rates and the reported summaries of the Rel.MSEs are sufficiently small to rule out chance error as explanation for apparent differences in the results.

3.2.1. Simulations with n = 10,000 and 30% MCAR

We first create scenarios with $n = 10{,}000$ using the MCAR mechanism with the 30% missingness rate. Including all 28 variables, we have 98 marginal probabilities, 3343 bivariate probabilities, and 58,778 trivariate probabilities. Typically, to create $L = 10$ completed datasets with a standard desktop computer, MI-GLM runs for about 1 hour and 54 minutes, MI-CART runs for about 45 minutes, and MI-DPM runs for about 50 minutes.

Figure 2 displays the simulated coverage rates of the 95% confidence intervals based on the 28 variables. For most estimands, all three MI methods result in reasonable coverage rates. All three distributions are skewed to the left, particularly for the bivariate and trivariate probabilities. Overall among the three MI procedures, MI-GLM tends to result in the most coverage rates far from the nominal 95% level. MI-CART offers the fewest extremely low coverage rates. MI-DPM tends to result in the highest coverage rates, although it also has many low rates for bivariate and trivariate probabilities. Across all MI methods, the lowest coverage rates tend to be associated with seven binary variables with marginal probabilities very close to one in the population. These variables include questions about the presence of bathtubs, refrigerators, running water, sinks, stoves, telephones and flush toilets in the households.

Table 1 displays the values of Rel.MSE for all $\bar{q}$. The MI-GLM procedure tends to result in the most inaccurate point estimates, and the MI-CART procedure tends to result in the most accurate point estimates. Detailed investigations indicate that the differences in performance reflect differences in biases more than differences in variances. The largest values in the distributions (e.g., the maxima in Table 1) usually involve the seven variables with low probabilities in the population.

As a sensitivity analysis, we also remove the seven variables with probabilities near one, and perform an independent

Table 1. Distributions of relative mean squared errors when $n = 10{,}000$ and 30% values MCAR. We exclude seven variables with more than 10 levels, resulting in $p = 28$ variables for analysis between MI-GLM, MI-CART, and MI-DPM.

	Marginal			Bivariate			Trivariate		
	GLM	CART	DPM	GLM	CART	DPM	GLM	CART	DPM
Min.	1.0	1.0	1.0	1.0	0.6	0.6	0.6	0.6	0.4
1st Qu.	1.4	1.2	1.3	1.4	1.2	1.2	1.3	1.1	1.2
Median	1.6	1.4	1.5	1.6	1.4	1.4	1.5	1.3	1.4
3rd Qu.	2.6	1.5	1.9	2.0	1.6	1.7	1.9	1.6	1.6
Max.	27,670	27,550	6.7	39,530	38,530	188.4	49,040	47,150	202.6

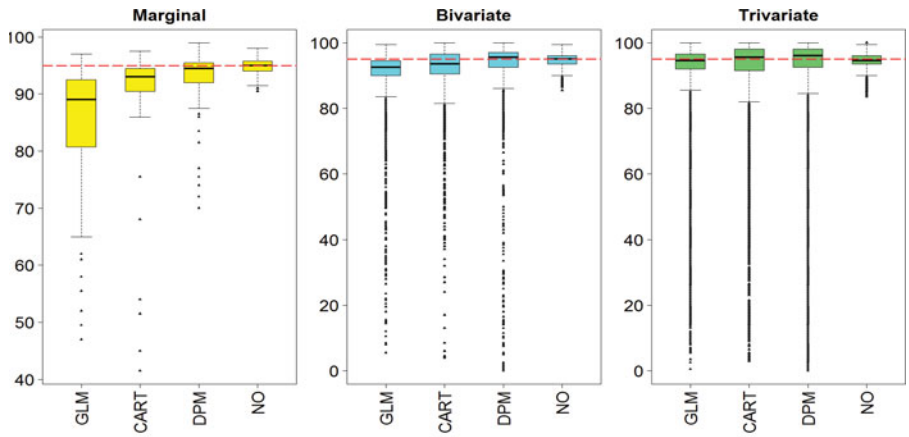
simulation of 200 runs on the remaining 21 variables. We are left with 83 marginal probabilities, 2590 bivariate probabilities and 37,216 trivariate probabilities. The relevant figure and table are presented in the online supplementary material. In short, the overall patterns are similar to those in Figure 2 and Table 1. Removing these seven variables also removes most of the extremely low coverage rates for MI-GLM, making it more competitive with MI-CART although MI-CART continues to result in slightly better coverage rates overall. MI-DPM yields median coverage rates around or slightly above 95%; however, it continues to have longer left tails than the other methods for bivariate and trivariate probabilities. Removing the seven variables also removes the extremely large Rel.MSE values seen in Table 1. The lowest coverage rates and largest Rel.MSEs continue to be associated with probabilities closest to zero or one, which is generally the case in all the simulation scenarios we considered.

As a final sensitivity analysis, we add in the seven variables with more than ten categories, and perform an independent set of 200 simulations comparing MI-CART and MI-DPM only. We continue to exclude the variables with probabilities near one. As a result, the comparison focuses on 28 variables with 177 marginal probabilities, 9049 bivariate probabilities and 18,0218 trivariate probabilities. To generate $L = 10$ completed datasets, typically MI-CART takes about 2 hours and 45 minutes, and MI-DPM takes about 55 minutes. The relevant figure and table are presented in the online supplement. In short, including the variables with 10 levels does not fundamentally change the conclusions about MI-CART and MI-DPM.

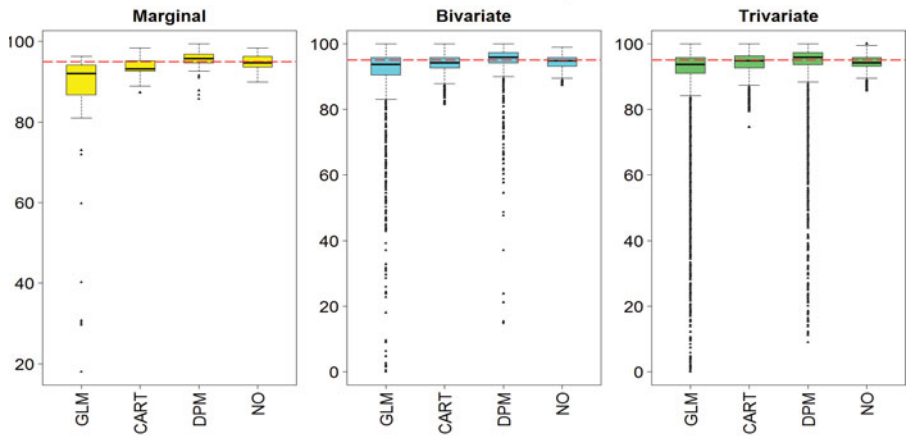
3.2.2. Simulations With Other MCAR Scenarios

We next consider two other MCAR scenarios to examine whether or not overall conclusions change when (i) increasing the rate of missingness and (ii) decreasing the sample size. For both scenarios, we use only the $p = 21$ variables described in the first sensitivity analysis of Section 3.2.1, that is, we discard the variables with more than 10 levels and the variables associated with marginal probabilities very close to zero or one.

To examine the role of missingness rate, we increase from 30% to 45% of values MCAR independently for each variable. We continue to use $n = 10{,}000$. The results are presented in the top panels of Figure 3 and Table 2. Compared to results with the 30% missing data rate, we see increased numbers of low coverage rates for all three MI methods. This results from reduced numbers of observations on which to estimate the imputation models. MI-DPM offers the highest coverage rates for marginal probabilities. The distributions of coverage rates for the three methods are similar for bivariate and trivariate probabilities, with again MI-DPM tending to yield higher coverage rates. As before, MI-CART tends to offer the smallest values of Rel.MSE, with MI-DPM a close second. MI-GLM tends to have the largest Rel.MSEs.



(a) Results with 45% missing data



(b) Results with $n = 1000$

Figure 3. Simulated coverage rates for MI-GLM, MI-CART, MI-DPM, and the premissing data intervals for other MCAR scenarios with $p = 21$ variables. Top panel is for scenario with $n = 10{,}000$ and a 45% missing data rate, and bottom panel is for scenario with $n = 1000$ and a 30% missing data rate.

Table 2. Distributions of relative mean squared errors for MI-GLM, MI-CART, and MI-DPM for other MCAR scenarios with $p = 21$ variables. Top panel is for scenario with $n = 10,000$ and a 45% missing data rate, and bottom panel is for scenario with $n = 1000$ and a 30% missing data rate.

	Marginal			Bivariate			Trivariate		
	GLM	CART	DPM	GLM	CART	DPM	GLM	CART	DPM
				Results with 45% missing data					
Min.	1.2	1.1	1.0	1.0	0.7	0.5	0.6	0.5	0.4
1st Qu.	1.8	1.4	1.4	1.8	1.4	1.5	1.6	1.3	1.4
Median	2.4	1.7	1.8	2.3	1.8	1.8	2.0	1.7	1.8
3rd Qu.	3.2	2.0	2.1	3.1	2.3	2.4	2.6	2.3	2.5
Max.	15.1	7.1	17.2	86.8	44.8	367.0	90.5	119.8	342.8
				Results with $n = 1000$					
Min.	1.0	1.0	1.0	0.9	0.8	0.7	0.9	0.7	0.5
1st Qu.	1.4	1.1	1.2	1.4	1.1	1.1	1.3	1.1	1.1
Median	1.9	1.3	1.3	1.6	1.3	1.3	1.5	1.2	1.2
3rd Qu.	2.9	1.4	1.5	2.1	1.4	1.4	1.8	1.4	1.4
Max.	27.8	14.2	3.5	47.0	21.1	20.4	66.2	21.5	31.6

We also consider a simulation with 10% of values MCAR independently for each variable. Not surprisingly, with the low missingness rate, for the most part the Rel.MSEs are similar across the three methods, although MI-GLM still tends to have the largest Rel.MSEs overall. All three methods have higher coverage rates than in the previous scenarios. MI-DPM continues to yield higher coverage rates for most estimands, but also longer left tails, than MI-CART and MI-GLM, particularly for bivariate and trivariate probabilities. The relevant figure and table are presented in the online supplementary material.

To examine the role of sample size, we return to the 30% missingness rate and set $n = 1000$. As evident in the bottom panel of Figure 3, MI-GLM tends to result in the most coverage rates below the nominal 95% level and the most extremely low rates. For most estimands, MI-DPM results in the highest coverage rates with most of the density concentrated above 95%, although the distribution has a long left tail as in previous scenarios. For marginal and bivariate probabilities, MI-CART coverage rates tend to concentrate below 95% although with a much shorter left tail than the other methods. As evident in Table 2, the MI-GLM procedure still tends to result in the least accurate point estimates, and the MI-CART and MI-DPM procedures give reasonably similar performance. The large values of Rel.MSE in Table 2 again correspond to low probability events in the population.

Table 3. Distributions of relative mean squared errors for MI-GLM, MI-CART, MI-DPM, and the premissing data intervals when $n = 10,000$ and 30% values MAR. We exclude seven variables with more than 10 variables and seven variables with marginal probabilities near one, resulting in $p = 21$ variables for imputation and analysis.

	Marginal			Bivariate			Trivariate		
	GLM	CART	DPM	GLM	CART	DPM	GLM	CART	DPM
Min.	1.0	1.0	1.0	0.9	0.8	0.6	0.6	0.6	0.5
1st Qu.	1.2	1.1	1.1	1.3	1.1	1.1	1.1	1.1	1.1
Median	1.5	1.3	1.4	1.5	1.3	1.4	1.4	1.2	1.3
3rd Qu.	1.8	1.5	2.1	1.8	1.5	1.8	1.6	1.4	1.7
Max.	4.5	2.5	9.4	28.2	19.1	144.0	43.6	41.8	195.4

3.2.3. Simulation With MAR Scenario

Often MAR is a more plausible assumption than MCAR in practice. We therefore design a simulation scenario with MAR, setting $n = 10,000$ and using the 21 variables described previously. We set two variables, household type (HHT) which has 5 levels and tenure of property/home (TEN) which has 4 levels, to be always fully observed. Among households with HHT $= 1$, in each sample we randomly and independently blank 15% of values in each of the remaining 19 variables; the corresponding rates for HHT $\in (2, 3, 4, 5)$ are 35%, 50%, 50% and 30%, respectively. Similarly, for TEN $= (1, 2, 3, 4)$ we use missing item rates of 40%, 15%, 30% and 5%, respectively. This results in around 40% missing values spread across the 19 variables. We select TEN and HHT as the fully complete variables because they have statistically significant associations with most of the other 19 variables.

The results are displayed in Figure 4 and Table 3. The patterns and conclusions from the MAR scenario are similar to those from the corresponding MCAR scenario. Overall, MI-CART tends to result in the most coverage rates concentrated around 95% and fewest very low rates, and the most accurate point estimates. For most bivariate and trivariate estimands, MI-DPM still results in coverage rates above 95%, but it has the longest lower tail, sometimes reaching very low rates. MI-GLM results in coverage rates for marginal and bivariate probabilities that are concentrated slightly below 95%, but its lower tail is comparable to that of MI-CART. The large values of Rel.MSE in Table 3 correspond to low probability events in the population.

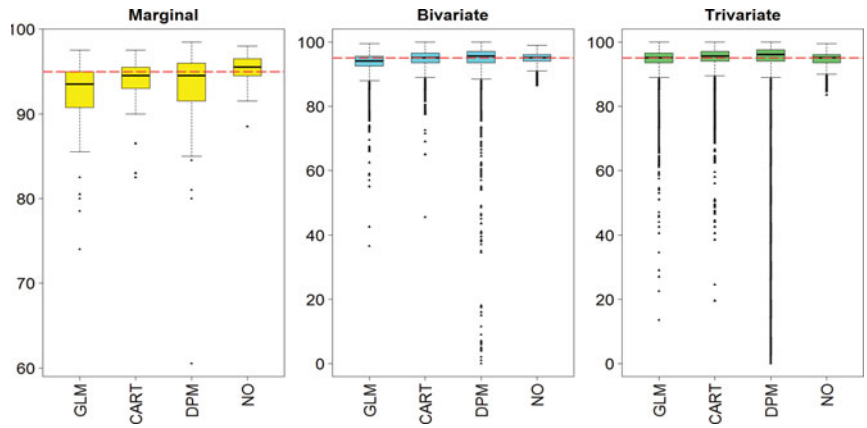


Figure 4. Simulated coverage rates for MI-GLM, MI-CART, MI-DPM, and the premissing data intervals when $n = 10,000$ and 30% values MAR. We exclude seven variables with more than 10 variables and seven variables with marginal probabilities near one, resulting in $p = 21$ variables for imputation and analysis.

4. Concluding Remarks

The simulation results suggest several general conclusions about the three MI procedures for categorical data. First, default applications of MI-GLM with main effects, which are arguably the most common implementation for multiple imputation, appear to be inferior to MI-CART and MI-DPM overall. These latter procedures automatically find and model important dependence structures that are missed with default applications of MI-GLM. Of course, one could use more complicated predictor functions in the generalized linear models, but with high dimensional variables selecting appropriate sets of interaction effects to include in the conditional models is daunting. Second, it is difficult to identify a clear winner between MI-CART and MI-DPM. Across the simulations, the median coverage rates for MI-DPM tend to be larger than the median coverage rates for MI-CART, although both tend to be close to 95%. On the other hand, MI-CART tends to result in fewer very low rates than MI-DPM does, and it tends to result in smaller relative mean squared errors. Therefore, analysts concerned with getting at least nominal coverage rates for most estimands, but potentially at the expense of some very low rates, may prefer MI-DPM. Analysts willing to accept slightly lower coverage rates for most estimands with potentially lower risk of very low rates, in addition to smaller mean squared errors, may prefer MI-CART. Third, the results suggest that one might favor MI-DPM over MI-CART in situations where one needs to lean on the model more heavily—the scenarios with smaller sample size and higher missing data rates in our simulations—and favor MI-CART otherwise. Intuitively, by design the joint model underlying the MI-DPM engine tends to shrink probability estimates from low count cells toward those for higher count cells. In modest-sized samples or with high rates of missing data, this can improve accuracy. The effects of this shrinkage generally decrease with sample size. Thus, we conjecture that MI-DPM and MI-CART would give similar results for much larger n; indeed, we found this to be the case in a small number of simulation runs with $n = 100{,}000$. Of course, when the sample size is too small, MI-DPM and MI-CART can lack sufficient data to estimate complex relationships accurately, highlighting the importance of checking the quality of imputations, for example, with predictive checks as in Burgette and Reiter (2010), for any method.

As with any simulation study, the conclusions suggested by these results may not generalize to all other settings. The simulations were based on relatively simple missing data mechanisms, default applications of imputation strategies that were not tuned specifically to the data at hand, and only a subset of the (huge space of) possible estimands. For example, it may be that MI-GLM is most effective among the three imputation procedures when the parameters of interest are a particular GLM that is also used in one of the conditional specifications, although this is not guaranteed to be the case when the specified GLM poorly describes the relationships in the data.

We considered only data comprising nominal categorical variables. Of course, many datasets also include ordinal and continuous variables. The MI-CART procedure can be easily applied for such mixed data types, but obviously MI-DPM cannot. One needs a different joint model, such as a general location model (Schafer 1997) or the mixture model of Murray and Reiter (2017). Thus, one should not generalize findings here to mixed data types. Additionally, we did not use the ordinal nature of the data when fitting MI-GLM. Models that do so, like cumulative logit models (Agresti 2013), can be more effective than multinomial logistic regressions, provided that their underlying assumptions (e.g., proportional odds) are sensible for the data at hand. Proportional odds assumptions do not always match the data distribution, for example, when the ordinal variable has most mass at the lowest and highest values of the variable, so that careful model checking is warranted before using them in MI-GLM in lieu of the weaker multinomial logit assumptions.

The simulation results suggest some useful practical steps to improve MI. First, the performance of the procedures suffered when variables with probabilities nearly equal to one (or zero) are included in the models. Additionally, occasionally MI-CART, and less frequently MI-GLM and MI-DPM, generated completed datasets with $b = 0$ for these variables, so that the multiple imputation variance estimator and degrees of freedom broke down. Accepting such decreased performance seems unnecessary, since imputation from marginal distributions are likely to work just as effectively for these variables. We suggest that analysts remove such variables before imputation. CART initially suffered the most for probabilities close to zero or one before restricting analyses only to quantities with $np > 10$ and $n(1 - p) > 10$. Imposing that restriction and removing those seven variables improved the performance of CART. Second, we recommend not considering multinomial regression modeling, as is done in default applications of MI-GLM, when some multinomial variables with missing values have many levels. We could not get default versions of MI-GLM even to fit in such circumstances. It may be possible to collapse categories or use fewer predictors in the models, although this may sacrifice model quality. This could be problematic when such variables are central to a secondary analysis. Third, the lack of a clear winner suggests benefits for assessing the quality of an imputation method for data akin to those at hand; for example, using repeated sampling simulations with other, representative data sources or, when large enough, the complete cases. Such evaluations are not routinely used in applications, although examples exist (e.g., Ezzati-Rice et al. 1995; Raghunathan and Rubin 1997; Schafer et al. 1998; Tang et al. 2005; van Buuren et al. 2006).

Supplementary Materials

The supplementary material contains a table describing the variables used in the simulation, median coverage rates across all scenarios, and figures for the sensitivity checks in Section 3.

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