Psychological Test and Assessment Modeling, Volume 52, 2010 (1), 8-28

Hierarchical mixtures of diagnostic models

Matthias von Davier[1]

Abstract

Psychological testing aims at making inferences about individual differences or the estimation of distributions of psychological constructs in groups of interest. However, a test instrument's relationship to the construct, the actual variable of interest, may change across subpopulations, or the instrument's measurement accuracy is not the same across subpopulations.

This paper introduces an extension of the mixture distribution general diagnostic model (GDM) that allows studying the population dependency of multidimensional latent trait models across observed and latent populations. Note that so-called diagnostic models do not aim at diagnosing individual test takers in the sense of a clinical diagnosis, or an extended case-based examination using multiple test instruments. The term cognitive diagnosis was coined following the development of models that attempt to identify (diagnose?) more than a single skill dimension. The GDM is a general modeling framework for confirmatory multidimensional item response models and includes well-known models such as item response theory (IRT), latent class analysis (LCA), and located latent class models as special cases. The hierarchical extensions of the GDM presented in this paper enable one to check the impact of clustered data, such as data from students with different native language background taking an English language test, on the structural parameter estimates of the GDM. Moreover, the hierarchical version of the GDM allows the examination of differences in skill distributions across these clusters.

Key words: Latent class analysis, hierarchical models, item response models, diagnostic models, logistic models

[1] *Correspondence concerning this article should be addressed to:* Matthias von Davier, PhD, Principal Research Scientist, Statistical and Psychometric Theory and Practice, ETS, MS 02-T, Princeton, NJ 08541, USA; email: mvondavier@ets.org

1. Introduction

The assessment of measurement invariance in testing applications is central to statements about the validity of observed test results. Psychological testing aims at valid inferences about individual differences or the estimation of distributions of psychological constructs in groups of interest. However, a test instrument's relationship to the construct, the actual variable of interest, may change across subpopulations, or the measurement accuracy of the test is not the same across subpopulations. In those cases, the validity of a test is threatened by the fact that additional information is needed to make valid inferences, i.e., information about variables that are seemingly not part of the construct.

Most researchers in the field will agree that the assumed direction of causality is that differences in the psychological variable of interest cause the observed differences in response behavior among examinees: A more intelligent person will, on average, produce more correct responses on tests of cognitive ability, but the reverse direction does not necessarily hold: A person with more correct responses can be one that copied responses from a neighbor, or one that by mistake received a sheet with correct responses together with the test. Almost nobody, except maybe the lucky test taker, will argue that an increase in the number of correct responses due to these factors makes him (or her) a more intelligent person.

The assumed causal direction from psychological construct to test behavior is, in naïve applications, taken to be the sole agent at work: No thought is given to the possibility that there may be factors other than the variable of interest that are interfering with the test results. This is especially true for psychological and educational tests that leave the realm of research and scientific inquiry and enter applied or non-scientific use of tests. In these cases, a fixed scoring rule is applied that produces a test score from responses based on a previously defined rule, without regard that the score obtained may be fallible, or might be affected by contingent variables other than the construct of interest. The mechanical process of scoring a test can be done with any set of responses, for example responses by a sample of examinees that were not part of the population the test was developed for, or by the proverbial chimp hitting the keyboard in a computer-administered test. If there are factors that distinguish the population the test was designed for from the population the new sample was drawn from, unexpected results can occur: A sample of students who takes a math test first thing in the morning may show very different results compared to a sample of students that just finished 6 hours of other exams. In this case, we would identify the two populations as *fatigued* versus *non-fatigued* test takers. Effects of fatigue on test performance have been studied for almost a century now (Thorndike, 1914). A test that was built for high school students who intend to go to college may work well to assess skill differences between these test takers. However, if all students who finish high school are tested, those who do not intend to go on to college and university may already be considered as a different population. This is true because a college admissions test is most likely of less consequence for those students who do not intend to go to college, so that it can be expected that these test takers may not be as motivated as those who will depend on a good result to get into the college they want to. Nunnally (1967) names a host of contingent variables as potential factors influencing test

results, and thus moderating the relationship between the proficiency – the target of inference – and the observed success in a given test. In tests of language proficiency it was also found that test taker characteristics do contribute to observed response behavior. Bachman (1990) reports that characteristics such as cultural background, gender, cognitive abilities, gender and age, among other things, can have an impact on test results.

Unfortunately, the analyst faced with the data can often not see whether a test taker is motivated, is faking, has been cheating, or fell into a random response mode due to time running out (speededness; see for example Yamamoto & Everson, 1997) when only looking at the total score of correctly solved items. Also, the test administrator cannot really expect truthful responses when asking test takers whether they were faking good, or cheating, or in whether they in other ways tried to change their responses from what they would have been. In a foreign language comprehension test, the test agency cannot ask whether the responses are not so much based on learning the language over many years in school but rather are in part the memorized results of rote-learning in crash courses preparing inexperienced foreign language speakers. In these cases, a more in-depth analysis of responses may reveal that test takers are indeed different with respect to more than what the test was intended to measure, the variable of interest.

In this paper, a class of models is introduced that enables the researcher to investigate whether a sample of test takers can be identified as one where more than the intended factors are at work. Examples of models that incorporate different relationships between the variables involved depending on a grouping variable are numerous. Hierarchical linear models allow random intercepts and random slopes (Raudenbush & Bryk, 1992). Selection models (Heckman, 1979) assume that a regression cannot be observed for parts of the sample. Hybrid models (Yamamoto, 1989; von Davier 1994) assume that in some subpopulation there is systematic co-variation between latent trait and observed response variables, whereas in other subpopulations, there is no such relationship. Multiple group models (e.g. Bock & Zimowski, 1997) assume that the same item response model with different sets of parameters holds in different groups. The mixed Rasch model (Rost, 1990; von Davier & Rost, 1995) assumes that the Rasch model holds, with different parameters, in different subpopulations. More general IRT models have been extended to mixture distribution models (Mislevy & Verhelst, 1990, Kelderman & Macready, 1990, von Davier & Yamamoto, 2004).

The modeling framework used in this paper allows specification of a discrete mixture model with a hierarchical component. This hierarchical component of the model allows assessing the composition of observed groups with respect to a proficiency or personality variable, the primary variable of interest, as well as with respect to the potential existence of a contingent variable. In addition, the model enables an assessment of the distribution of the contingent variable in the observed clusters (e.g. language groups, classes, schools) of the sample. More specifically, the hierarchical extensions presented in this paper enable one to check the impact of the clustering of observed data, such as data for students within schools in large scale educational surveys, on the structural parameter estimates of the model. Moreover, the hierarchical version of the general diagnostic model (GDM) allows the study of differences in skill distributions across the clusters of the sample.

2. The general diagnostic model

Diagnostic models typically assume a multivariate, but discrete, latent variable that represents the absence or presence, or more gradual levels, of multiple skills. Note that these so-called diagnostic models do not aim at diagnosing individual test takers in the sense of a clinical diagnosis, nor do they aim at an extended case-based examination using multiple test instruments with the goal of identifying a specific syndrome. The term cognitive diagnosis was coined following the development of models that attempt to identify (diagnose?) more than a single skill dimension. Diagnostic models, also sometimes called diagnostic classification models (Rupp & Templin, 2008), are nothing else but latent structure models with a discrete, multivariate latent variable (von Davier, 2009). While it is important to distinguish diagnostic models from diagnostic test batteries, there are at least some commonalities. Diagnostic tests aim at the identification of the presence or absence, or the level, of indicators of clinical or educational relevance, while diagnostic models provide a framework of specifying statistical descriptions for the identification of binary (or binary and ordinal, as is the case in the GDM) profiles of skills. These skill profiles have to be inferred through model assumptions with respect to how the observed data relate to the unobserved skill profile. The absence or presence of skills is commonly represented by a Bernoulli (0/1) random variable in the model. Given that the number of skills represented in the model is larger than in unidimensional models (obviously greater than 2, but smaller than 14 skills in most cases), the latent distribution of skill profiles needs some specification of the relationship between skills in order to avoid the estimation of up to 2^{14}-1 = 16,383 separate skill-pattern probabilities. The GDM (von Davier, 2005a) allows ordinal skill levels and different forms of skill dependencies to be specified so that more gradual differences between examinees can be modeled in this framework.

This section introduces the GDM for dichotomous and partial credit data and binary as well as ordinal latent skill variables. Then the mixture distribution GDM (MGDM) will be introduced. The differences between the components of a discrete mixture model are a reflection of differential relationships between the construct of interest and the observed data. Third, an extension that allows utilizing information about the structure will be introduced. This hierarchical mixture GDM (HGDM) allows an assessment of the dependency of the mixture components on the cluster structure of a hierarchically organized sample. Finally, examples of applications of the HGDM in large-scale data analysis will be presented.

Assume an I-dimensional categorical random variable $\vec{x} = (x_1,...,x_I)$ with $x_i \in \{0,...,m_i\}$ for $i \in \{1,...,I\}$, referred to as a response vector in the following. Further assume that there are N independent and identically distributed (i.i.d.) realizations $\vec{x}_1,...,\vec{x}_N$ of this random variable $\vec{x}$, so that x_{ni} denotes the i-th component of the n-th realization $\vec{x}_n$. In addition, assume that there are N unobserved realizations of a K-dimensional categorical variable, $\vec{a} = (a_1,...,a_K)$, so that the vector

$$(\vec{x}_n, \vec{a}_n) = (x_{n1},...,x_{nI}, a_{n1},...,a_{nK})$$

exists for all $n \in \{1,...,N\}$. The data structure

$$(X, A) = \left((\vec{x}_n, \vec{a}_n)\right)_{n=1,..,N}$$

is referred to as the complete data, and $\left(\vec{x}_n\right)_{n=1,..,N}$ is referred to as the observed data matrix. Denote $\left(\vec{a}_n\right)_{n=1,..,N}$ as the latent skill or attribute patterns, which is the unobserved target of inference.

Let $P(\vec{a}) = P\left(\vec{A} = (a_1,...,a_K)\right) > 0$ for all $\vec{a}$ denote the non-vanishing discrete count density of $\vec{a}$. Assume that the conditional discrete count density $P(x_1,...,x_I \mid \vec{a})$ exists for all $\vec{a}$. Then the probability of a response vector $\vec{x}$ can be written as

$$P(\vec{x}) = \sum_{\vec{a}} P(\vec{a}) P(x_1,...,x_I \mid \vec{a}).$$

1. Conditional Independence

So far, no assumptions have been made about the specific form of the conditional distribution of $\vec{x}$ given $\vec{a}$, other than that $P(x_1,...,x_I \mid \vec{a})$ exists. For the general diagnostic model, local independence (LI) of the components x_i given $\vec{a}$ is assumed, which yields

$$P(x_1,...,x_I \mid \vec{a}) = \prod_{i=1}^{I} p_i(x = x_i \mid \vec{a})$$

so that the probability $p_i(x = x_i \mid \vec{a})$ is the one component left to be specified to arrive at a model for $P(\vec{x})$.

2. Logistic Model Specification

Logistic models have widespread applications and apart from early disputes about the merits of probit versus logit models (Berkson as cited in Cramer, 2003) have secured a prominent position among models for categorical data. The general diagnostic model is also specified as model with a logistic link between an argument, which depends on the random variables involved and some real valued parameters, and the probability of the observed response.

Using the above definitions, the GDM is defined as follows. Let

$$Q = \left(q_{ik}\right), i = 1,...,I, k = 1,...,K$$

be a binary $I \times K$ matrix, that is $q_{ik} \in \{0,1\}$. Let

$$\left(\gamma_{ikx}\right), i = 1,...,I, k = 1,...,K, x = 1,...,m_i$$

be a cube of real valued parameters, and let β_{ix} for $i = 1,...,I$ and $x \in \{0,...,m_i\}$ be real valued parameters. Then define

$$p_i(x \mid \vec{a}) = \frac{\exp\left(\beta_{ix} + \sum_k \gamma_{ikx} h(q_{ik}, a_k)\right)}{1 + \sum_{y=1}^{m_i} \exp\left(\beta_{iy} + \sum_k \gamma_{iky} h(q_{ik}, a_k)\right)}.$$

It is often convenient to constrain the γ_{ikx} somewhat and to specify the real valued function $h(q_{ik}, a_k)$ and the a_k in a way that allows emulation of models frequently used in educational measurement and psychometrics. It is convenient to use $h(q_{ik}, a_k) = q_{ik} a_k$, and $\gamma_{ikx} = x\gamma_{ik}$, which defines the general diagnostic model for partial credit data (Muraki, 1992).

Von Davier (2005a,b) has shown that this model already contains several models from the areas of item response theory (IRT; Lord & Novick, 1968), latent-class analysis (Lazarsfeld & Henry, 1968), multiple classification latent-class models (Goodman, 1974; Haberman, 1979; Maris, 1999) and diagnostic models (see, for example, von Davier, DiBello, & Yamamoto, 2006).

3. Mixture general diagnostic models

Von Davier (2008b) introduced the discrete mixture distribution version of the GDM, referred to as the MGDM. In discrete mixture models for item response data (Mislevy & Verhelst, 1990; Rost, 1990; for an overview, see von Davier & Rost, 2006), the probability of an observation $\vec{x}$ depends on the unobserved latent trait in the case of the GDMs, $\vec{a}$, and on a subpopulation indicator g, which is also unobserved. The rationale for mixture distribution models is that observations from different subpopulations may either differ in their distribution of skills or in their approach to the items (e.g., in terms of strategies employed) or both. A discrete mixture distribution in the setup of random variables as introduced above includes an unobserved grouping indicator g_n for $n = 1,...,N$. The complete data for examinee n then becomes $(x_n, \vec{a}_n, g_n)$, of which only $\vec{x}_n$ is observed in mixture distribution models. In multiple group models, $(\vec{x}_n, g_n)$ is observed.

The conditional independence assumption has to be modified to account for differences between groups, that is

$$P(\vec{x} \mid \vec{a}, g) = P(x_1,...,x_I \mid \vec{a}, g) = \prod_{i=1}^{I} p_i(x = x_i \mid \vec{a}, g).$$

Moreover, assume that the conditional probability of the components x_i of $\vec{x}$ depends on nothing but $\vec{a}$ and g, that is,

$$P(\vec{x} \mid \vec{a}, g, z) = \prod_{i=1}^{I} p_i(x = x_i \mid \vec{a}, g) = P(\vec{x} \mid \vec{a}, g) \qquad (1)$$

for any random variable z. In mixture models, when the g_n are not observed, the marginal probability of a response vector $\vec{x}$ needs to be found, that is,

$$P(\vec{x}) = \sum_g \pi_g P(\vec{x} \mid g), \tag{2}$$

where $P(\vec{x} \mid g) = \sum_{\vec{a}} p(\vec{a} \mid g) P(\vec{x} \mid \vec{a}, g)$. The $\pi_g = P(G = g)$ are referred to as mixing proportions, or class sizes. The class-specific probability of a response vector $\vec{x}$ given skill pattern $\vec{a}$ in class g is then

$$P(\vec{x} \mid \vec{a}, g) = \prod_{i=1}^{I} P(x_i \mid \vec{a}, g) = \prod_{i=1}^{I} \left[\frac{\exp\left(\beta_{ixg} + \sum_k x_i \gamma_{ikg} q_{ik} a_k\right)}{1 + \sum_y \exp\left(\beta_{iyg} + \sum_k y \gamma_{ikg} q_{ik} a_k\right)} \right]. \tag{3}$$

with class-specific item difficulties β_{ixg}. The γ_{ikg} are the slope parameters relating skill k to item i in class g.

Note that mixture models and multiple group models are two extremes, for mixtures models no g_n is observed, while for multiple group models all g_n are observed. Von Davier and Yamamoto (2004) pointed this out and described an extension of the GPCM that includes the mixture GPCM, multiple-group GPCM, and partially observed grouping GPCM, where the g_n information is missing only for a portion of the sample.

One important special case of the MGDM is a model that assumes measurement invariance across populations, which is expressed in the equality of $p(\vec{x} \mid \vec{a}, g)$ across groups, or, more formally:

$$P(x_i \mid \vec{a}, g) = p(x_i \mid \vec{a}, c) \text{ for all } i \in \{1, ..., I\} \text{ and all } g, c \in \{1, ..., G\}.$$

This assumption allows one to write the model equation without the group index g in the conditional response probabilities, so that

$$P(\vec{x}) = \sum_g \pi_g P(\vec{x} \mid g) = \sum_g \pi_g \sum_{\vec{a}} p(\vec{a} \mid g) \prod_{i=1}^{I} P(x_i \mid \vec{a}). \tag{4}$$

Note that the differences between groups are only present in the $p(\vec{a} \mid g)$, so that the skill distribution is the only component with a condition on g in the above equation. The next section introduces hierarchical GDM based on mixture distribution versions of the GDM.

4. Hierarchical general diagnostic models

Hierarchical models introduce an additional structure, often referred to as a cluster variable, in the modeling of observed variables to account for correlations in the data. These are attributed to the complex structure of the environment in which the data are observed. More concretely, one standard example for clustered data is the responses to educational

assessments sampled from students within schools or classrooms. As a rather sloppy explanation, it seems plausible to assume that students within schools are more similar than students across schools (even though the amount to which this statement is true may depend on the educational system). Hierarchical models have been developed for linear models (e.g., Bryk & Raudenbush, 1992; Goldstein, 1987) as well as for Rasch-type models (e.g., Kamata & Cheong, 2006).

For the developments presented here, the extension of the LCA to a hierarchical model (e.g., Vermunt, 2003, 2004) is of importance. In addition to the latent class or grouping variable g, the hierarchical extension of the LCA assumes that each observation n is characterized by an outcome s_n on a clustering variable s. The clusters identified by this outcome may be schools, classrooms, or other sampling units representing the hierarchical structure of the data collection. As Vermunt outlined, the (unobserved) group membership g_n is thought of as an individual classification variable; for two examinees $n \neq m$ there may be two different group memberships, that is, both $g_n = g_m$ and $g_n \neq g_m$ are permissible even if they belong to the same cluster (i.e., $s_n = s_m$).

Moreover, it is assumed that the skill distribution depends only on the group indicator g and no other variable, that is,

$$P(\vec{a} \mid g, z) = P(\vec{a} \mid g) \tag{5}$$

for any random variable z. More specifically, for the clustering variable s,

$$P(g) = \sum_{s=1}^{S} p(s) P(g \mid s).$$

With Equation 5,

$$P(\vec{a} \mid s) = \sum_{g} P(g \mid s) P(\vec{a} \mid g),$$

since

$$P(g \mid s) P(\vec{a} \mid g) = p(g \mid s) P(\vec{a} \mid g, s) = P(\vec{a}, g \mid s).$$

As above for the MGDM, assume that the observed responses $\vec{x}$ depend on the skill pattern $\vec{a}$ and the group index g only. Then

$$P(\vec{x} \mid g, s) = \sum_{\vec{a}} p(\vec{a} \mid g, s) P(\vec{x} \mid \vec{a}, g, s) = \sum_{\vec{a}} p(\vec{a} \mid g) P(\vec{x} \mid \vec{a}, g) = P(\vec{x} \mid g)$$

with Equations 1 and 5. Then the marginal distribution of a response pattern $\vec{x}$ in the hierarchical GDM (HGDM) is given by

$$P(\vec{x}) = \sum_{s} p(s) \sum_{g} P(g \mid s) \sum_{\vec{a}} P(\vec{a} \mid g) P(\vec{x} \mid \vec{a}, g), \tag{6}$$

where, as before in the MGDM, the $p(\vec{a} \mid g)$ denote the distribution of the skill patterns in group g, and the $p(\vec{x} \mid \vec{a}, g)$ denote the distribution of the response vector $\vec{x}$ conditional on skill pattern $\vec{a}$ and group g. A hierarchical GDM that assumes measurement invariance across clusters and across groups is defined by

$$P(\vec{x}) = \sum_s p(s) \sum_g P(g \mid s) \sum_{\vec{a}} P(\vec{a} \mid g) P(\vec{x} \mid \vec{a}), \tag{7}$$

with conditional response probabilities $p(\vec{x} \mid \vec{a}) = \prod_i p(x_i \mid \vec{a})$ that do not depend on cluster or group variables.

The increase in complexity of hierarchical GDMs over nonhierarchical versions lies in the fact that the group distribution $P(g \mid s)$ depends on the cluster variable s. If effects of the group membership is considered a fixed effect, this increases the number of group or class size parameters depending on the number of clusters $\#\{s : s \in S\}$. If the groups are considered to be random draws from a population, the group effect $P(g \mid s)$ can be modeled as a random effect that follows a Dirichlet distribution. The estimation of item parameter $\beta_{ix(g)}$ and $\gamma_{ik(g)}$ as well as the estimation of the conditional probabilities of skill patterns given group $P(\vec{a} \mid g)$ and other quantities involved is outlined in the next section.

5. Estimation of hierarchical general diagnostic models

The case of fitting models with cluster-dependent response probabilities $P(\vec{x} \mid \vec{a}, s)$ will not be discussed here. The reason is that a model in which both the skill distributions and the probability of correct responses depend on the cluster variable does not allow attribution of the variation of observed responses across clusters to differences in skill distributions. Such a model would essentially assume that items have different difficulty in different clusters. Even though this is a very empathic view of the world, this does not allow drawing any conclusions involving cluster differences other than clusters are different. Apart from that, the fact that most applications of hierarchical models offer only moderate sample sizes within clusters makes the estimation of a multitude of cluster-specific parameters infeasible.

The estimation of GDMs and MGDMs has been outlined in von Davier (2008a, 2008b). This approach is extended here to the estimation of HGDMs. The expectation-maximization (EM) algorithm has been shown to be a suitable one for this kind of estimation problems (Vermunt, 2003), so that other, more computationally costly methods are not necessary. For the most part, researchers will be concerned with fitting less highly parameterized versions of the HGDM, such as the models given in Equations 6 and 7.

The *mdltm* software (von Davier, 2005) enables one to estimate MGDMs and HGDMs according to Equations 6 and 7. The extensions to enable estimation of these models were recently implemented in *mdltm* based on the research presented in this paper.

Since the data are structured hierarchically, the first step is to define the complete data for the case of the HGDM. Let S denote the number of clusters in the sample, and let N_s denote the number of examinees in cluster s, for $s = 1,...,S$. Then

- let x_{ins} denote the i-th response of the n-th examinee in cluster s and let $\vec{x}_{ns}$ denote the complete observed response vector of examinee n in cluster s
- let a_{kns} denote the k-th skill of examinee n in cluster s and let $\vec{a}_{ns}$ denote the skill pattern of examinee n in cluster s
- let g_{ns} denote the group membership of examinee n in cluster s

Note that only the x_{ins} are observed, as are the cluster sizes N_s and the number of clusters S. The a_{kns} and g_{ns} are unobserved and have to be inferred by making model assumptions and calculating posterior probabilities such as $P(g \mid s)$ and $P(\vec{a}, g \mid \vec{x}, s)$.

1. Marginal calculations in hierarchical general diagnostic models

For the complete data (i.e, the observed data $\vec{x}$ in conjunction with the unobserved skill profiles $\vec{a}$ and group membership g), the marginal likelihood is

$$L = \prod_{s=1}^{S} \prod_{n=1}^{N_s} P(\vec{x}_{ns}, \vec{a}_{ns}, g_{ns}; s),$$

that is, a sum over cluster-specific distributions of the complete data. With the above assumptions,

$$L = \prod_{s=1}^{S} \prod_{n=1}^{N_s} P(\vec{x}_{ns} \mid \vec{a}_{ns}, g_{ns}) p(\vec{a}_{ns} \mid g_{ns}) p(g_{ns} \mid s),$$

which equals

$$L = L_{\vec{x}} \times L_{\vec{a}} \times L_g,$$

with

$$L_{\vec{x}} \times L_{\vec{a}} \times L_g = \left(\prod_{s=1}^{S} \prod_{n=1}^{N_s} P(\vec{x}_{ns} \mid \vec{a}_{ns}, g_{ns}) \right) \left(\prod_{s=1}^{S} \prod_{n=1}^{N_s} p(\vec{a}_{ns} \mid g_{ns}) \right) \left(\prod_{s=1}^{S} \prod_{n=1}^{N_s} p(g_{ns} \mid s) \right).$$

Note that these components may be rearranged and rewritten as

$$L_{\vec{x}} = \prod_{s=1}^{S} \prod_{n=1}^{N_s} \prod_{i=1}^{I} P(x_{ins} \mid \vec{a}_{ns}, g_{ns}) = \prod_{g} \prod_{\vec{a}} \prod_{i} \prod_{x} P(X_i = x \mid \vec{a}, g)^{n_i(x, \vec{a}, g)},$$

with $n(x_i, i, \vec{a}, g) = \sum_s n(x_i, i, \vec{a}, g, s)$ is the frequency of category x_i responses on item i for examinees with skill pattern $\vec{a}$ in group g. Also,

$$L_{\vec{a}} = \prod_{s=1}^{S} \prod_{n=1}^{N_s} p(\vec{a}_{ns} \mid g_{ns}) = \prod_{\vec{a}} \prod_{g} p(\vec{a} \mid g)^{n(\vec{a};g)},$$

where $n(\vec{a};g)$ is the frequency of skill pattern $\vec{a}$ in group g. Finally,

$$L_g = \prod_{s=1}^{S} \prod_{n=1}^{N_s} p(g_{ns} \mid s) = \prod_{s} \prod_{g} p(g \mid s)^{n(g;s)}$$

holds. The $n(g;s)$ represent the frequency of group membership in g in cluster s.

2. Estimation of cluster-skill distributions with the EM algorithm

Since unobserved latent variables are involved, the EM algorithm (Dempster, Laird, & Rubin, 1977) is a convenient choice for estimating GDMs (von Davier, in press-a) as well as MGDMs (von Davier, in press-b) and HGDMs. The EM algorithm cycles through the generation of expected values and the maximization of parameters given these preliminary expectations until convergence is reached. For details on this algorithm, refer to McLachlan and Krishnan (2000). For the HGDM, there are three different types of expected values to be generated in the E-step:

1. $\hat{n}_i(x,\vec{a},g) = \sum_s \sum_n 1\{x_{ins} = x\} P(\vec{a},g \mid \vec{x}_{ns},s)$ is the expected frequency of response x to item i for examinees with skill pattern $\vec{a}$ in group g, estimated across clusters and across examinees within clusters
2. $\hat{n}(\vec{a},g) = \sum_s \sum_n P(\vec{a},g \mid \vec{x}_{ns},s)$ is the expected frequency of skill pattern $\vec{a}$ and group g, estimated across clusters and across examinees within clusters
3. $\hat{n}(g;s) = \sum_n P(g \mid \vec{x}_{ns},s)$ is the expected frequency of group g in cluster s, estimated across examinees in that cluster

For the first and second type of the required expected counts, this involves estimating

$$P(\vec{a},g \mid \vec{x},s) = \frac{P(\vec{x},s,\vec{a},g)}{\sum_g P(\vec{x},s,g)} = \frac{P(\vec{x} \mid \vec{a},g)p(\vec{a} \mid g)p(g \mid s)}{\sum_g P(\vec{x},s,g)},$$

with

$$P(\vec{x},s,g) = \sum_{\vec{a}} P(\vec{x},s,\vec{a},g) = \sum_{\vec{a}} P(\vec{x} \mid \vec{a},g)p(\vec{a} \mid g)p(g \mid s)$$

for each response pattern $\vec{x}_{ns}$, for $s = 1,...,S$ and $n = 1,...,N_s$. For the third type of expected count, use

$$p(g \mid x,s) = \sum_{\vec{a}} P(\vec{a},g \mid \vec{x},s),$$

which is equivalent to

$$p(g \mid \vec{x}, s) = \frac{P(\vec{x}, s, g)}{\sum_g P(\vec{x}, s, g)} = \frac{\sum_{\vec{a}} P(\vec{x} \mid \vec{a}, g) p(\vec{a} \mid g) p(g \mid s)}{\sum_g \left[\sum_{\vec{a}} P(\vec{x} \mid \vec{a}, g) p(\vec{a} \mid g) p(g \mid s) \right]}.$$

This last probability then allows one to estimate the class membership g given both the observed responses $\vec{x}$ and the known cluster membership s. The utility of the clustering variable may be evaluated in terms of increase of the maximum a posteriori probabilities $p(g \mid \vec{x}, s)$ over $p(g \mid \vec{x})$. If the clustering variable s is informative for the classification g, a noticeable increase of the maximum posterior probabilities should be observed. The improvement should also be seen in terms of the marginal log-likelihood if s is informative for g. The cluster group sizes $P(g \mid s)$ for $g = 1, ..., G$ can be assumed to follow a Dirichlet distribution with parameters $np_1, ..., np_G$. Maximum likelihood estimation of the parameters involved can be carried out following Ronning (1989) or Narayanan (1991).

6. An application to language testing data

Simulated data have advantages, such as the truth (i.e., the set of generating values) is known and comparisons of different levels of model complexity and misspecification can be made on the basis of known deviations from the true model. The disadvantage is that simulated data are by origin artificial, so that the impact of model assumptions on model-data fit can only be studied under often less than realistic settings. The accuracy of parameter recovery using simulated data has been studied with quite satisfactory results for the GDM by von Davier (2005, 2008) using flat item response data with no missing values, and by Xu and von Davier (2006) for sparse matrix samples of item responses as collected in national and international surveys of educational outcomes.

The current exposition focuses on the comparison of results based on two administration of a test of English language proficiency (TELP). The target of inference is the stability of estimates of English language reading and listening skills relating to clustering variables given by language group. The analyses carried out are independent scaling runs of two TELP administrations for which Q-matrices were produced. Von Davier (2005, 2008) pointed out that the GDM applied to TELP data resulted in highly correlated skill variables, and found that a two-dimensional, two-parameter logistic (2PL) IRT model across reading and listening domains provided a more parsimonious data description. However, the eight-skill model across reading and listening domains was the subject of further investigation by TELP experts, so that this model is adopted for the analyses with the hierarchical GDM.

In a first step, the HGDM was compared to the GDM without hierarchical extension, both adopting the same Q-matrix based on eight mastery/nonmastery skills for the February and November administrations of the TELP. The HGDM was estimated according to Equation 7. In other words, measurement invariance was assumed across mixture components so that only the skill distribution could vary across clusters and the response probabilities $P(\vec{x} \mid \vec{a})$ depended on the skill profile only, not on cluster s or mixture

component g. Table 1 shows the skill correlations for the February administration, as well as the marginal skill mastery probabilities for the GDM. Table 2 shows the same information for the November administration.

The correlations range between 0.67 and 0.86 for skills of the same domain (i.e., among the four reading or four listening skills) and are slightly lower across the domains as expected. For correlations between one of the four reading skills and one of the four listening skills, the range is 0.56 to 0.77. These are still substantial correlations, which is due to the fact that overall reading and listening domains themselves are highly corre-

Table 1:
Skill Correlations and Marginal Probabilities of Skill Mastery for the February Administration Based on the Nonhierarchical Eight-Skill General Diagnostic Model Across 76 Items Assuming Four Listening and Four Reading Skills

	Skill 1	Skill 2	Skill 3	Skill 4	Skill 5	Skill 6	Skill 7	Skill 8
Skill 1	1.00	0.76	0.73	0.80	0.75	0.60	0.69	0.57
Skill 2		1.00	0.83	0.81	0.65	0.64	0.67	0.58
Skill 3			1.00	0.75	0.68	0.69	0.70	0.63
Skill 4				1.00	0.61	0.55	0.58	0.45
Skill 5					1.00	0.79	0.76	0.66
Skill 6						1.00	0.86	0.80
Skill 7							1.00	0.80
Skill 8								1.00
P(master)	0.63	0.61	0.57	0.69	0.54	0.46	0.49	0.39

Table 2:
Skill Correlations and Marginal Probabilities of Skill Mastery for the November Administration Based on the Nonhierarchical Eight-Skill General Diagnostic Model Across 76 Items Assuming Four Listening and Four Reading Skills

	Skill 1	Skill 2	Skill 3	Skill 4	Skill 5	Skill 6	Skill 7	Skill 8
Skill 1	1.00	0.79	0.81	0.67	0.62	0.62	0.57	0.59
Skill 2		1.00	0.86	0.68	0.60	0.61	0.56	0.59
Skill 3			1.00	0.70	0.64	0.63	0.58	0.60
Skill 4				1.00	0.71	0.67	0.77	0.67
Skill 5					1.00	0.82	0.78	0.80
Skill 6						1.00	0.85	0.72
Skill 7							1.00	0.86
Skill 8								1.00
P(master)	0.63	0.62	0.62	0.44	0.48	0.47	0.40	0.43

lated. A two-dimensional 2PL IRT model estimated with the *mdltm* software (von Davier, 2005) results in estimated correlations between the reading and listening domains of 0.81 and 0.85 for the two administrations.

When estimating the HGDM for the two administrations, the resulting statistics differ from those from the GDM in two ways. First, there are two skill distributions $P(\vec{a} \mid c)$ estimated, one for each of two mixture components $c = 1$ and $c = 2$, representing the largest of the between-cluster differences (here language group) that can be expected. Then cluster-skill distributions are formed by a proportion $P(c \mid s)$ modeled as a random effect following a Dirichlet distribution. This effect represents the probability of belonging to each of the mixture component skill distributions. The parameters of the Dirichlet distribution were estimated using the procedures described in Ronning (1989).

The log likelihood for the eight-skill GDM and HGDM are reported in Table 3 together with the number of estimated parameters and the average log likelihood per observation. Note that the November administration included a larger number of language groups, some of which were of rather small size. This led to a larger increase in the number of estimated parameters from GDM to HGDM for the November administration than for the February administration.

The average likelihood per response pattern is improved by a small amount when including the language group as clustering variable. However, compared to the gain by assuming the GDM rather than independence of all observed variables, the gain in going from GDM to HGDM seems quite small. For comparisons, the log-likelihood, parameters, and average-response pattern likelihoods are also presented for the two-dimensional 2PL/GPCM, which are estimated as a nonhierarchical model (2PL2) and a hierarchical model (H2PL2), and are also given in the table. As von Davier (2005, 2008) reported, the

Table 3:
Log Likelihood and Number of Parameters for the Eight-Skill General Diagnostic Model and Hierarchical General Diagnostic Model for Both Administrations

	Log likelihood	Parameters	Average log-likelihood
Independence			-43.24
FEB GDM	-164435.2	194	-38.83
FEB HGDM	-163883.0	196	-38.70
FEB 2PL2	-160799.2	160	-37.97
FEB H2PL2	-160297.3	162	-37.85
Independence			-41.92
NOV GDM	-196009.8	195	-37.44
NOV HGDM	-195480.1	197	-37.33
NOV 2PL2	-191431.6	160	-36.56
NOV H2PL2	-190905.6	162	-36.47

two-dimensional 2PL IRT model is a more parsimonious description of the TELP data than the eight-skill model, a result that holds up for both the February and the November administrations. The eight-skill model, however, is the focus of an ongoing methods comparison by TELP researchers, so it is adopted for subsequent comparisons between GDM and HGDM here without any comparisons to the two-dimensional 2PL/GPCM model.

Table 4 shows the two resulting marginal skill distributions for the February administration, and Table 5 shows the same information for the November administration. For both administrations, the mixture component $C1$ shows much lower mastery probabilities than component $C2$. The mixture component $C2$ is characterized by high probabilities of mastery of all eight skills for both administrations. The marginal sizes of the two components $\pi_{C2,Feb}$ and $\pi_{C2,Nov}$ for the two administrations differ somewhat; there is about 42 % in the high proficiency class in November, whereas there is about 51 % in February.

The two mixture components $C1$ and $C2$ represent the largest possible differences between clusters (language groups) in the sample, since each cluster receives an estimate of a proportion $P(C2 \mid s)$ – and with that, implicitly, $P(C1 \mid s) = 1 - P(C2 \mid s)$ – of members estimated to belong in the high versus low proficiency components $C2$ and $C1$. Since the mastery probabilities of all skills are much higher in $C2$ compared to $C1$ for both administrations, this proportion can be interpreted as the proportion of examinees in each language group who are highly proficient with respect to the assessment items re-

Table 4:

Marginal Skill Distributions for the Two Mixture Components $C1$ and $C2$ in the February Eight-Skill Hierarchical General Diagnostic Model With Skill Mastery Probabilities Given and Marginal Sizes of the Mixture Components Are $\pi_{C1} = 0.49$ and $\pi_{C2} = 0.51$

	Skill 1	Skill 2	Skill 3	Skill 4	Skill 5	Skill 6	Skill 7	Skill 8
P(mastery|C1)	0.33	0.28	0.16	0.38	0.15	0.05	0.13	0.06
P(mastery|C2)	0.92	0.93	0.96	0.97	0.94	0.85	0.85	0.72

Table 5:

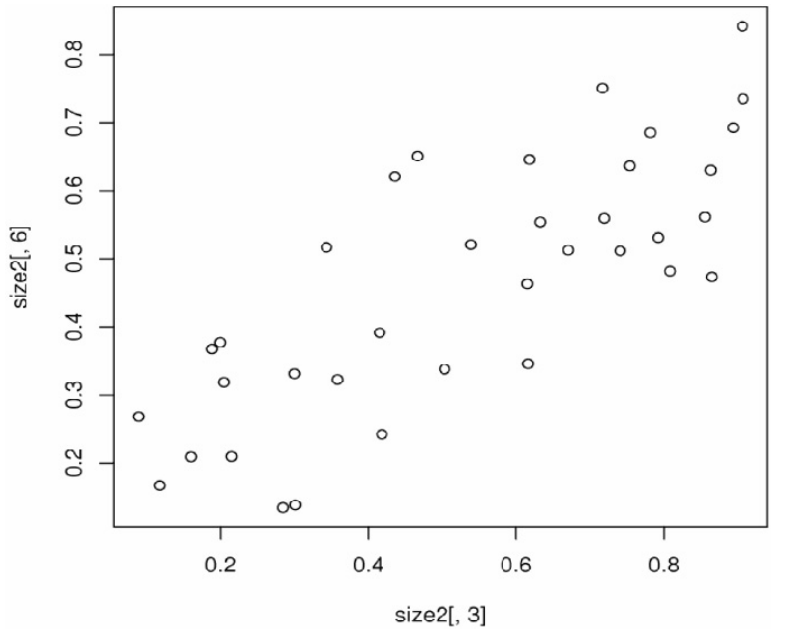
Marginal Skill Distributions for the Two Mixture Components $C1$ and $C2$ in the November Eight-Skill Hierarchical General Diagnostic Model With Skill Mastery Probabilities Given and Marginal Sizes of the Mixture Components Are $\pi_{C1} = 0.58$ and $\pi_{C2} = 0.42$

	Skill 1	Skill 2	Skill 3	Skill 4	Skill 5	Skill 6	Skill 7	Skill 8
P(mastery|C1)	0.40	0.38	0.38	0.16	0.16	0.11	0.04	0.09
P(mastery|C2)	0.97	0.95	0.94	0.90	0.93	0.98	0.93	0.91

flected in the skill definitions. These proportions can be studied across administrations, so that the variation (or the lack thereof) of the proportion of highly proficient students in the language groups becomes a target of inference. This target delivers information about how well-aligned the English language test is for the different language groups represented in the sample.

Figure 1 shows the proportion of students falling in the high performing class for the November and February administrations. The table contains only those language groups for which at least 10 students were observed for each administration of the test. It can be seen that the class sizes vary across administrations but are relatively stable when languages are compared. For example, the proportion of students with a Chinese (CHI) language background is smaller than the proportion of students with a French (FRE) language background (see the appendix for the language-specific class sizes). The correlation between the two high proficient class-size estimates across 37 countries is 0.787. When a weighted correlation (with weights defined as the geometric mean of the two language-group-specific sample sizes, one for each administration) across all 116 language groups is calculated, the correlation between the class-size estimates is 0.89.

Figure 1:
Plot of the high proficiency class-size correspondence across two administrations of the English language test based on 37 language groups for which sample sizes exceeded 10 in both administrations.



The consistency of the language-group-proportion estimates and the substantial correlation of these estimates across the two administrations are evident from Figure 1. For estimates of the skill-mastery probabilities of language groups, the $P(C2 \mid s)$, and the mixture-component skill probabilities can be combined, resulting in

$$P(\vec{a} \mid s) = \sum_{c=C1}^{C2} P(\vec{a} \mid c) P(c \mid s)$$

for the language-group-specific skill distribution. As an illustration, the marginal skill mastery probabilities for the November and February administrations have been calculated for the CHI and Spanish (SPA) language groups. Table 6 shows the language-group-specific marginal skill mastery probabilities for CHI and SPA for the two administrations. It can be seen that the skill mastery probabilities range between 0.54 and 0.69 for the listening skills in the Spanish language sample and between 0.40 and 0.58 for the Chinese language sample for the November administration. For the reading skills, the mastery probabilities range between 0.32 and 0.41 for the Chinese language sample and between 0.49 and 0.55 for the Spanish language sample.

It is important to note that the language-group proportions as well as the estimates of skill-mastery probabilities will vary somewhat across administrations, even though the ordering of language-group-specific mastery estimates may stay stable. The estimates presented here are based on 4 + 4 skills with high correlations within the reading and listening domains as well as across domains. Therefore, a similar analysis may be tried with a model that joins the four postulated skills per domain into one overarching dimension by estimating a two-dimensional model instead. However, for the purpose of providing statistics on skill mastery for ongoing language testing research, it was necessary in

Table 6:
Language-Group-Specific Mastery Probabilities Exemplified Using the November and February Administrations Based on Mixing Components and the Chinese and Spanish Language Groups

CHI and SPA in Nov	Skill 1	Skill 2	Skill 3	Skill 4	Skill 5	Skill 6	Skill 7	Skill 8
C1: 0.68 (CHI), 0.49 (SPA)	0.40	0.38	0.38	0.16	0.16	0.11	0.04	0.09
C2: 0.32 (CHI), 0.51 (SPA)	0.97	0.95	0.94	0.90	0.93	0.98	0.93	0.91
P(SKILL|CHI)	0.58	0.56	0.56	0.40	0.41	0.39	0.32	0.35
P(SKILL|SPA	0.69	0.67	0.67	0.54	0.55	0.55	0.49	0.51

CHI and SPA in Feb	Skill 1	Skill 2	Skill 3	Skill 4	Skill 5	Skill 6	Skill 7	Skill 8
C1: 0.79 (CHI), 0.33 (SPA)	0.33	0.28	0.16	0.38	0.15	0.05	0.13	0.06
C2: 0.21 (CHI), 0.67 (SPA)	0.92	0.93	0.96	0.97	0.94	0.85	0.85	0.72
P(SKILL|CHI)	0.45	0.42	0.33	0.50	0.32	0.22	0.28	0.20
P(SKILL|SPA)	0.63	0.61	0.57	0.68	0.55	0.46	0.50	0.40

the current study to use the expert-generated eight-skill matrix. As a result, the language-group-specific profiles of skill mastery will, due to the nature of the highly correlated skills, mostly reflect overall differences in the proficiency level of the applicant samples across language groups.

7. Conclusions

This paper introduces an extension of the GDM (von Davier, 2005), the hierarchical general diagnostic model (HGDM), and shows the effect of clustering through a comparison of results from two administrations of the English language assessment when estimating language-group-specific proficiencies. The HGDM provides reliable estimates of proportions of high proficiency across language groups. The correlation of the estimates is 0.78 for the 37 largest language groups not weighted by sample size, and it increases to 0.89 when all language groups that are present in both administrations are weighted according to their pooled sample size.

If the clustering is informative as it seems to be in the case presented here, the prediction of proficiency can potentially be improved, as seen in the slight increase of average log-likelihood (see Table 3). The clustering, or language-group membership in the analyses presented here, acts as ancillary information, so that the fit of the HGDM to the observed cognitive item responses can be compared to models without a clustering variable. The results presented here indicate that a mixture of different class-specific skill distributions is a useful tool in conjunction with cluster-specific mixing proportions to model the dependency of skill distribution on a clustering variable. The approach estimates conditional skill distributions across the whole sample representing different expected skill profiles in unknown subpopulations of a mixture distribution. The cluster-specific mixing proportions then estimate the composition of the clusters – here language groups – based on the assumption that the mixture-distribution subpopulations are represented in varying levels across clusters. In this example, the mixture components turned out to be ordered proficiency classes, due to the nature of the eight skills applied, which are known to be substantially correlated.

The estimated proportions, more specifically the variance of these proportions across clusters, and the consistency of identified proportions across administrations can provide valuable information about the sources of proficiency variation in hierarchically organized data. The HGDM provides a tool to study such variations in the context of item response models, latent class models, and diagnostic models for profile scoring.

References

Bachman, L. (1990). *Fundamental considerations in language testing.* Oxford University Press.

Bryk, A. S., & Raudenbush, S. W. (1992). *Hierarchical linear models in social and behavioral research: Applications and data analysis methods* (1st ed.). Newbury Park, CA: Sage Publications.

Cramer, J. S. (2003). *Logit models from economics and other fields.* Cambridge, UK: Cambridge University Press.

Fox, J. P., & Glas, C. A. W. (2001). Bayesian estimation of a multi-level IRT model using Gibbs sampling. *Psychometrika, 66*, 271-288.

Goldstein, H. (1987). *Hierarchical models in educational and social research.* London: Griffin.

Goodman, L. A. (1974). Exploratory latent structure analysis using both identifiable and unidentifiable models. *Biometrika, 61*, 215-231.

Haberman, S. J. (1979). *Qualitative data analysis: Vol. 2. New developments.* New York: Academic Press, 1979.

Heinen, T. (1996). *Latent class and discrete latent trait models, similarities and differences.* Thousand Oaks, CA: Sage Publications.

Kamata, A., & Cheong, Y. F. (2007). Hierarchical Rasch models. In M. von Davier & C. H. Carstensen (Eds.), *Multivariate and mixture distribution Rasch models – Extensions and applications* (pp. 217-232). New York: Springer.

Kunnan, A. J. (1995). *Test Taker Characteristics and Test Performance: A Structural Modeling Approach.* Cambridge, U.K.: CUP.

Lazarsfeld, P. F., & Henry, N. W. (1968). *Latent structure analysis.* Boston: Houghton Mifflin.

Lord, F. M., & Novick, M. R. (1968). *Statistical theories of mental test scores.* Reading, MA: Addison-Wesley.

Maris, E. (1999). *Estimating multiple classification latent class models.* Psychometrika, *64*(2), 187-212.

McLachlan, G., & Krishnan, T. (2000). *The EM-algorithm and extensions.* New York: Wiley.

Mislevy, R. J., & Verhelst, N. D. (1990). Modeling item responses when different subjects employ different solution strategies. *Psychometrika, 55*(2), 195-215.

Muraki, E. (1992). A generalized partial credit model: Application of an EM algorithm. *Applied Psychological Measurement, 16*(2), 159-176.

Narayanan, A. (1991). Algorithm as 266: Maximum likelihood estimation of the parameters of the Dirichlet distribution. *Applied Statistics, 40*(2), 365-374.

Ronning, G. (1989). Maximum-likelihood estimation of Dirichlet distributions. *Journal of Statistical Computation and Simulation, 32*, 215-221.

Rost, J. (1990). Rasch models in latent classes: An integration of two approaches to item analysis. *Applied Psychological Measurement, 14*, 271-282.

Rupp, A. A., & Templin, J. L. (2008). Unique characteristics of diagnostic classification models: A comprehensive review of the current state-of-the-art. *Measurement, 6*(4), 219-262.

Vermunt, J. K. (2003). Hierarchical latent class models. *Sociological Methodology, 33*, 213-239.

Vermunt, J. K. (2004). An EM-algorithm for the estimation of parametric and nonparametric hierarchical nonlinear models. *Statistica Neerlandica, 58*(2), 220-233.

von Davier, M. (2005a). *mdltm: Software for the general diagnostic model and for estimating mixtures of multidimensional discrete latent traits models* [Computer software]. Princeton, NJ: ETS.

von Davier, M. (2005b). *A general diagnostic model applied to language testing data.* ETS Research Report RR-05-16

von Davier, M. (2008). A general diagnostic model applied to language testing data. *British Journal of Mathematical and Statistical Psychology, 61*(2), 287-307.

von Davier, M. (2008b). *The Mixture General Diagnostic Model.* In G. R. Hancock & K. M. Samuelson (Eds.), Advances in Latent Variable Mixture Models. Information Age Publishing.

von Davier, M. (2009). Some Notes on the Reinvention of Latent Structure Models as Diagnostic Classification Models. *Measurement: Interdisciplinary Research & Perspectives, 7*(1),67-74.

von Davier, M., DiBello, L., & Yamamoto, K. (2008). Reporting Test Outcomes Using Models for Cognitive Diagnosis. Chapter 7. In J. Hartig, E. Klieme & D. Leutner (Eds.), *Assessment of competencies in educational contexts* (pp. 151-176). Hogrefe & Huber Publishers.

von Davier, M., & Rost, J. (2006). Mixture distribution item response models. In C. R. Rao & S. Sinharay (Eds.), *Handbook of statistics, Vol. 26: Psychometrics* (pp. 643-768). Amsterdam: Elsevier.

von Davier, M., & Yamamoto, K. (2004). Partially observed mixtures of IRT models: An extension of the generalized partial credit model. *Applied Psychological Measurement, 28*(6), 389-406.

von Davier, M., & Yamamoto, K. (2007). Mixture distribution Rasch models and hybrid Rasch models. In M. von Davier & C. H. Carstensen (Eds.), *Multivariate and mixture distribution Rasch models – Extensions and Applications* (pp. 99-116). New York: Springer.

Xu, X., & von Davier, M. (2006). *Cognitive diagnosis for NAEP proficiency data* (ETS Research Rep. No. RR-06-08). Princeton: ETS.

Appendix

Proportions of High Proficiency Class Membership by Country for the February and November Administration of the English language test for 37 Language Groups for Which Sample Sizes Exceeded 10 in Both Administrations

Lang.	N(FEB)	P(C2|FEB)	N(NOV)	P(C2|NOV)
CHI	609	0.2046	657	0.3185
VIE	33	0.0883	92	0.2681
KOR	604	0.2148	832	0.2094
RUM	32	0.8560	35	0.5611
FRE	467	0.7818	357	0.6850
URD	29	0.3433	43	0.5167
GER	458	0.9067	433	0.8412
POL	27	0.8082	58	0.4816
ITA	378	0.6154	331	0.4629
IND	27	0.1886	45	0.3673
SPA	294	0.6712	483	0.5125
TAM	21	0.6182	26	0.6456
JPN	245	0.2847	410	0.1344
BEN	19	0.4665	19	0.6505
ARA	119	0.3010	187	0.1382
BUL	19	0.7412	19	0.5115
TGL	82	0.5033	111	0.3377
HEB	19	0.8938	28	0.6925
RUS	74	0.7192	136	0.5592
MAL	18	0.8636	25	0.6301
TEL	60	0.6326	43	0.5539
UKR	15	0.4151	15	0.3913
POR	59	0.7922	73	0.5308
ALB	14	0.4363	18	0.6206
ENG	58	0.5389	66	0.5206
CZE	13	0.6163	13	0.3456
THA	48	0.1178	91	0.1669
IBO	12	0.8651	13	0.4731
HIN	48	0.7168	70	0.7504
PAN	11	0.3585	15	0.3225
TUR	48	0.3000	76	0.3310
N/A	11	0.7544	20	0.6363
FAS	43	0.4183	58	0.2420
YOR	10	0.9075	11	0.7347
GUJ	37	0.1997	40	0.3770
AMH	10	0.1595	23	0.2089