Pylab 3 - Curve Fitting With Radioactive Decay

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Abstract:

The purpose of this lab was to plot the data from a decaying radioactive isotope and use the curvefit() function in python to see whether it was best to use a linear model function or an exponential model function. It turns out a linear model function was more accurate in regards to the calculations of the half-life of the isotope.

Introduction:

As mentioned in the abstract, two different model functions were compared with the data collected from a radioactive isotope. The model functions were created in python using the curvefit() function and the half-lives were calculated using the half-life formulas $I(t) = I_0 e^{-\frac{t}{\tau}}$ and $I(t) = I_0 \frac{1}{2}^{t/t_{1/2}}$.

Equipment:

- Metastable Barium (Ba-137m) - the radioactive isotope being analyzed
- Acid - Washes out the Ba-137m from the generator it was prepared in
- Geiger Counter - Measures the rate of gamma emissions

Procedure:

First, the Barium isotope was set to decay as the Geiger counter detected its emissions. After that, the data collected (number of counts and samples) was inputted into a python program and the mean background radiation was subtracted from the actual counts to gain more accurate data. The counts were then converted to rates by dividing by the time interval between samples (20s). The data was plotted twice; once as a linear function and once as an exponential function. Both a linear model function and an exponential model function were then used to get the curve fit, theoretical plot and error bars were also plotted. After that, the half-life values from each plot were calculated and compared to the expected value.

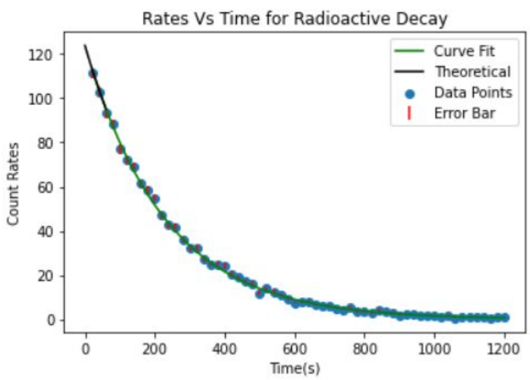
Results:

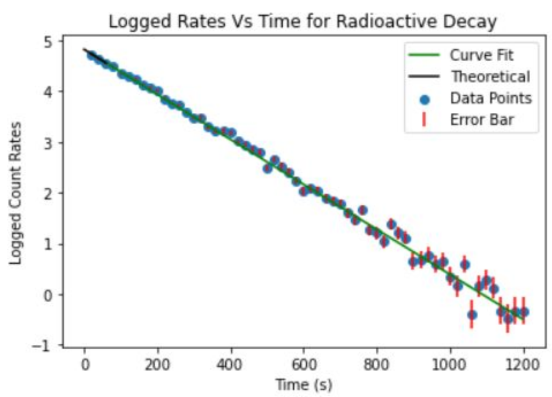
The rates of each count were calculated using the following formula:

$$R = \frac{N}{\Delta t} \pm \frac{\sqrt{N}}{\Delta t}$$

Where N = the number of counts and Δt is the time interval (20s).

The plots of rates vs samples were created, one using an exponential model function and the other using a linear model function (logged):





The half-lives were calculated and converted to minutes using the following formula:

$$t_{1/2} = \left|\frac{log(2)}{m}\right| * 20/60 \pm \sqrt{\frac{var(t_{1/2})}{t_{1/2}}}$$

Where m is the slope of the curve fit function and var($t_{1/2}$) refers to the p_cov[0,0] value found using the curve_fit() function.

The half-life in the linear plot was 2.6073min ± 0.0004min and the half-life using the exponential plot was 2.6418min ± 0.0002min.

The chi-squared value was then calculated using the formula:

$$\chi^2 = \frac{1}{N-n} \sum_{i=1}^{N} \left(\frac{y_i - y(x_i)}{\sigma_i}\right)^2$$

Where: N = number of observations
n = number of parameters
y_i = rates
x_i = samples
σ_i = the error in the rate values

Discussion:

The linear model function gave a more accurate result than the exponential function. This may mean that the linear curve fits are generally more accurate than the non-linear curve fits.

Both of my results for half-life were close to the expected value of 2.6min, however, the exponential plot did not agree with it. This could be due to the accuracy of the data calculated. The data did not produce a perfect exponential decay as the Geiger detector sometimes picked up more emissions in later samples than in earlier samples. This means there were slight increases in emissions as opposed to the expected constant exponential decay.

Both of my chi-squared values were also slightly greater than 1. This means my error variance in my data may have been underestimated. I believe this to be true as the curve_fit() functions seem to create plots almost in perfect alignment with the data. This caused an overestimation in the accuracy of the results.

Questions:

1. (Section 5) The function using the linear model function gave a result closer to the expected value. It is hard to tell directly from the graphs whether this is obvious mainly due to how accurate both model functions were.
2. (Section 6.1) The half-life values found were 2.6073min ± 0.0004min for the linear plot and 2.6418min ± 0.0002min for the exponential plot. The linear plot is in range

with the expected value of 2.6min, however, the exponential value is not, although it is very close.

3. (Section 6.2) The chi-squared values for the linear plot and the exponential plot were 1.38 and 1.61 respectively. These values are both slightly greater than 1 which indicates an incomplete fit or an underestimated error variance. I think it is an underestimated error variance that caused these values as my curves seem to be very accurate.