Student name: AST221H

Student #: # Assignment 2 Due: Oct. 8, 2020

You may work together with other students to solve these problem sets, but all solutions must be written and submitted independently. Submit your assignment as a single .pdf file following the instructions on Quercus. ***Part marks only will be given for solutions with no explanation. Show your work, including intermediate steps and diagrams if necessary!*** *Check the syllabus for reading recommendations. Careful with units!*

Problem 1: Tides

A star of mass m and radius r approaches a black hole of mass M to within a distance $d >> r$.

1. Express, in terms of m, r, and M, the distance d at which the Newtonian radial force exerted by the black hole on the star equals the gravitational binding force of the star, and hence the star will be torn apart. This is the tidal disruption distance.

2. Find the black hole mass M above which the tidal disruption distance, d, is smaller than the Schwarzchild radius of the black hole, and evaluate it for a star with $m = M_\odot$ and $r = R_\odot$. Black holes with masses above this value can swallow Sun-like stars whole, without first tidally shredding them! (The Schwarzchild radius is given by $R_S = \frac{2GM}{c^2}$.)

3. Derive a Newtonian expression for the *tangential* tidal force exerted inward on the star, in terms of m, r, M, and d, again under the approximation that $d >> r$. The combined effects of the radial tidal force in 1) and the tangential force will lead to 'spaghettification' of the star, or other objects that approach the black hole to within the disruption distance.

   Hint: Remember that the star is in a ***radial*** *gravitational field, and hence there is a tangential component to the gravitational force exerted on regions of the star that are off the axis defined by the black hole and the centre of the star. The tangential component can be found by noting that the small angle between the axis and the edge of the star is* $\sim r/d$.

Problem 2: Energy generation on the horizontal branch

At the start of the horizontal branch phase of a 1 $M_\odot$ star's lifetime, about 10% of the original stellar mass is in the form of helium nuclei ($m_{He} = 6.647 \times 10^{-24}$ g). While on the horizontal branch, the star powers itself via the "triple-α process", in which three helium nuclei are converted into one carbon nucleus ($m_C = 1.993 \times 10^{-23}$ g).

1. How much energy is released per triple-α reaction?
2. What is the total energy released by fusing this amount of helium into carbon via the triple-α process?
3. While on the horizontal branch, the star's luminosity is $L = 100L_\odot$. If all of this luminosity is provided by the triple-α process in the stellar core, how long will the horizontal branch phase last?
4. Is the assumption in part 3) (that all of this luminosity is fueled by the triple-α process in the stellar core) valid? Why or why not?