AST 222 Problem Set 3 Chris Compierchio

Question 1:

a) $m_{DM} = 10 GeV/c^2$, m_X = mass of xenon nucleus $= 122.3\ GeV/c^2$
m_G = mass of Germanium $= 67.7\ GeV/c^2$

For xenon, we have: $\sigma_{DM,N} \propto \frac{(10)^2(122.3)^2}{(10+122.3)^2} = 2.716 \times 10^{-52}\ kg^2$ or $85.45\ GeV^2/c^4$

For Germanium, we have: $\sigma_{DM,N} \propto \frac{(10)^2(67.7)^2}{(10+67.7)^2} = 2.413 \times 10^{-52}\ kg^2$ or $75.92\ GeV^2/c^4$

b) If m_{DM} is constant, we can see from part A that $\sigma_{DM,N}$ increases if m_N increases and decreases if m_N decreases. This makes sense because in the numerator, we have some mass raised to the power of 4 and in the denominator, we have some mass squared, and so the numerator determines the proportionality. This always results in the direct proportionality explained above.

c) If we assume $m_N \gg m_{DM}$, we can say $(m_{DM} + m_N) \approx m_N$, so the proportionality becomes:

$$\sigma_{DM,N} \approx \frac{m_{DM}^2 m_N^2}{(m_N^2} = m_{DM}^2$$

So the cross-section is about m_{DM}^2.

d) Using a similar approach to part C, we can approximate the cross-section as

$$\sigma_{DM,N} \approx \frac{M_{DM}^2 M_N^2}{M_{DM}^2} = m_N^2$$

e.) Ideally, if we were to detect dark matter, we would want the chance of detecting a dark matter particle to be as high as possible. Given the relationships described above, we would want to use a very massive nucleus to have a higher proportionality of having it collide with a dark matter particle.

Question 2:

a.) To calculate number density, we first need the number of black holes. If each black hole is 1 million $M_\odot$ and the total amount of dark matter has a mass of 2×10^{54} kg, the ratio between these gives # of black holes:

$$N_{BH} = \frac{2\times10^{54}\,kg}{1{,}000{,}000\,M_\odot} = \frac{1.0056\times10^{24}\,M_\odot}{1{,}000{,}000\,M_\odot} \simeq 1.0056\times10^{18} \text{ Black holes.}$$

Dividing the number of black holes by the volume of the universe gives the number density:

$$n = \frac{\#\,\text{black holes}}{4\times10^{80}\,m^3} = \frac{1.0056\times10^{18}}{1.3615\times10^{13}\,Mpc^3} = 73449.97/Mpc^3$$

b.) The Local Group (our local cluster) has a diameter of 3 MPC, so if we assume spherical symmetry, we can estimate its volume to be:

$$V = \tfrac{4}{3}\pi(1.5)^3$$
$$V \simeq 14.14\ MPC^3 \rightarrow (14.14)(73449.97) = 1.038\times10^6 \text{ BH's}$$

Therefore we can find about 1.038×10^6 black holes in our local cluster.

These would most likely be observable from earth because we can't actually see dark matter. It is possible, however, that we could observe their gravitational effects on other galactic objects.

Question 3:

a.) If we assume circular orbits, we know $\frac{V^2}{R} = \frac{GM}{R^2} \Rightarrow V = \sqrt{\frac{GM}{R}}$, so

$$V \propto \frac{1}{R^{1/2}}$$

and therefore Plot B shows the relationship for planets orbiting the sun. This is governed by the inverse square law in Newton's Law of Gravitation.

b.) Plot A would best describe this situation as stars far from the galactic center move extremely fast. This is due to some gravitational pull acting on these stars, which we speculate to be some dark matter structure (halo). We showed this relationship in Assignment 1, Question 5 where the rotation curve for dark matter follows the proportionality $V \propto \sqrt{4\pi G}$