Python 3 AST221

2a.) The hydrostatice equilibrium formula is approximately $dP = -\frac{GM\rho}{R^2}dr$ so $P = \int dP = \frac{GM\rho}{R}$ where $\rho = \frac{M}{R^3}$ so $P \propto \frac{M^2}{R^4}$

3.a.) Using the ideal gas Law $P = \frac{\rho K T}{\mu m_H}$, and subbing it in the equilibrium formula, we get:

$$\frac{\rho K T}{\mu m_H} = \frac{GM\rho}{R}$$

$$\Rightarrow \frac{KT}{\mu m_H} = \frac{GM}{R} \quad \text{so } T_c \propto \frac{M}{R}$$

4.a.) Density is in units Kg/m^3, so:

$$\rho \propto \frac{M}{R^3}$$

6.) Given that $R \propto M^{2/3}$ from my slope, we can sub this proportionality in for R in the proportionalities found in 2a, 3a and 4a:

$$P_c \propto \frac{M^2}{R^4} \Rightarrow P_c \propto \frac{M^{4/3}}{M^{8/3}} \Rightarrow P_c \propto \frac{1}{M^{4/3}}$$

$$T_c \propto \frac{M}{R} \Rightarrow T_c \propto \frac{M}{M^{2/3}} \Rightarrow T_c \propto M^{1/3}$$

$$\rho \propto \frac{M}{R^3} \Rightarrow \rho \propto \frac{M}{M^2} \Rightarrow \rho \propto \frac{1}{M}$$

7.) Using $\bar{\rho}_\odot$, $\mu = 0.62$ and $T \sim 1.44 \times 10^7$ K like in 10.2, we find the ideal gas pressure to be:

$$P_i = \frac{\bar{\rho}_\odot K T}{\mu m_H}$$

$$P_i = \frac{(1410)(1.38 \times 10^{-23})(1.44 \times 10^7)}{(0.62)(1.673 \times 10^{-27})}$$

$$P_i = 2.7 \times 10^{14} \text{ N/m}^2$$

The radiation pressure using the same temperature is:

$$P_r = \frac{1}{3} a T^4$$

$$P_r = \frac{1}{3}(7.5658 \times 10^{-16})(1.44 \times 10^7)^4$$

$$P_r = 3.63 \times 10^{12} \text{ N/m}^2$$

Find the ratio between P_r and the total pressure gives:

$$\frac{P_r}{P_r + P_i} = \frac{3.63 \times 10^{12}}{2.7363 \times 10^{14}} = 0.013 = 1.3\%$$

Therefore, the radiation pressure is only 1.3% of the total pressure making it almost negligable.

We can say P_r becomes important when $P_r \geq P_g$, therefore

$$\frac{aT^4}{3} \geq \frac{\bar{\rho}_\odot K T}{\mu m_H}$$

$$\frac{aT^3}{3} \geq \frac{\rho_\odot K}{\mu m_H}$$

Subbing in scaling relations for T_c with M, we get:

$$\frac{a(M^{1/3})^3}{3} \geq \frac{\rho_\odot K}{\mu m_H}$$

$$M \geq \frac{3 \rho_\odot K}{a \mu m_H}$$

$$M \geq \frac{3(1410)(1.38 \times 10^{-23})}{(7.5658 \times 10^{-16})(0.62)(1.673 \times 10^{-27})} = 7.44 \times 10^{22} \text{ kg}$$