AST 222 Problem Set 2

Due on Fri. Feb. 5 at 11:59pm

Please submit via Quercus

Student name: *Student number:*

Problem 1: (2 points)

At the very centre of the Milky Way sits a *supermassive black hole* which is surrounded by a spherical cluster of stars—the central star cluster (sometimes referred to as a nuclear star cluster). We can learn much about the properties of the black hole and its effect on surrounding stars from observing the motions of stars in this region with large telescopes. In this problem, we will gain some understanding of the dynamics of this special place in our Galaxy.
(a) The estimated mass of the supermassive black hole is $4 \times 10^6\, M_\odot$. This mass is essentially concentrated in a point. Considering only the influence of the black hole, write down the rotation curve $\Theta(r)$—the velocity of a circular orbit at different distances—near the black hole in units of km/s as a function of distance in pc.
(b) The mass of the central star cluster within 1 pc from the black hole equals that of the supermassive black hole. Assuming that the density of the central star cluster is spherical and that it depends on radius r as $\rho(r) \propto r^{-1.9}$, what is its density at 1 pc (express your result in $M_\odot/\mathrm{pc}^3$)? That is, what is a in $\rho(r) = a\ (r/1\,\mathrm{pc})^{-1.9}$.

Problem 2: (2 points)

Compute the Oort constants A and B for Keplerian rotation: $\Theta(R) = \Theta_0\ (R/R_0)^{-0.5}$ in terms of Θ_0 and R_0. If Keplerian rotation described the rotation of the Milky Way near the Sun, what would the numerical value of A and B (in $\mathrm{km\,s^{-1}\,kpc^{-1}}$) be and how does this compare to the observed values?

Problem 3: (4 points)

The Sloan Digital Sky Survey (SDSS) is a project that has surveyed a large fraction of the sky and measured spectra for about 2 million stars and galaxies. The host a sky survey that lets you browse the data on in your internet browser:
`http://skyserver.sdss.org/dr16/en/tools/chart/navi.aspx`
In this problem you will study a galaxy cluster, Abell 2255 or 383 in the SDSS. Thier coordinates are shown below:

Cluster Name	RA (deg)	DEC (deg)
Abell 383	42.0288	-3.4922
Abell 2255	258.1292	64.0925

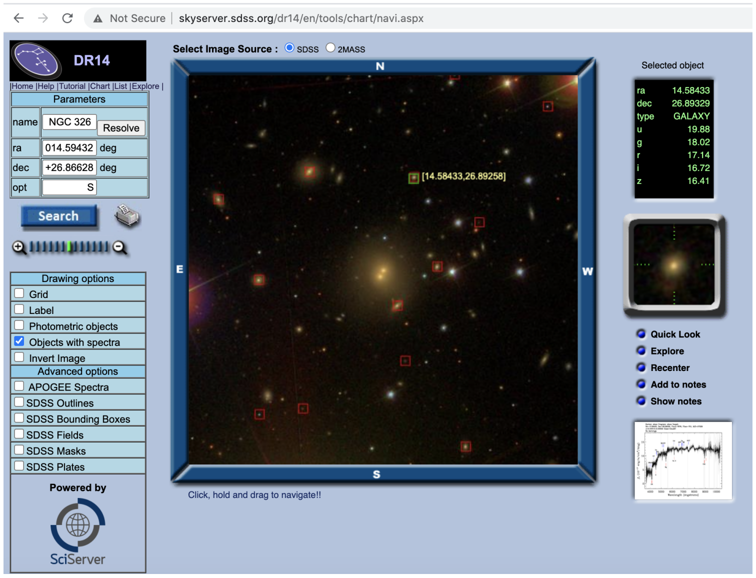

Figure 1: The SDSS Sky Server.

Go to the URL above and navigate to this cluster. (There is a quick video on how to use this navigation tool on Quercus). Select between 5 and 20 galaxies that have spectra from the SDSS sky server and note their redshifts. What color and rough type (spiral or elliptical) are the galaxies? Do you think they are part of the galaxy cluster? Keep looking until you find at least 5 galaxies that are a part of the cluster. These are known as cluster members. Make a table with your results. Include a screenshot of your cluster with the objects you chose indicated. Use the virial theorem and the velocity dispersion of the galaxies in the cluster to calculate the approximate mass of the cluster.

Problem 4: (4 points)

A background source lensed by a point-like lens always produces two observed images (unless when they are exactly lined up, in which case you get an Einstein ring). This means that every star in the sky is lensed by every star between us and the star and if there are N stars in the Milky Way, we should expect to see about $\approx N^2$ images of stars on the sky. Let's see why this does not happen in practice.

(a) Solve the lensing equation for a point-mass lens for the case where $\beta \neq 0$. Because the lensed image's position θ is along the line that connects the lens and the source position β, you can work in one dimension (that is, β and θ are numbers rather than two-dimensional vectors on the sky). That is, give θ as a function of β for a given point-mass. Express your result in terms of the Einstein angle θ_E (your solution should only contain β and θ_E).]

(b) Examine what happens to the solution with the smallest $|\theta|$ as $|\beta|$ increases. What is $|\theta|/\theta_E$ for $\beta = \theta_E, 10\theta_E, 100\theta_E, 1000\theta_E$? Compute θ_E from its definition for a lens star at 5 kpc with $M = 1\ \mathrm{M}_\odot$ and a source star at 10 kpc and use this to express your result for $|\theta|$ in terms of milli-arcseconds as well for this lens-source setup. What is the limit for $\beta \to \infty$?

(c) In reality, stars are not point-sources, but they have a finite angular size given by their

physical radius divided by the distance. When a gravitationally-lensed image computed using the point-source model falls within this angular radius, we cannot trust the solution, because the point-source assumption is broken, and in reality no second image exists. For a $M = 1\,\mathrm{M}_{\odot}$ main-sequence star lens at 5 kpc and a source at 10 kpc, what is β for which the image with the smallest $|\theta|$ lies within the lens star? Express your result both in terms of θ_E and in milli-arcseconds.
(d) Another reason that the second image (that with the smallest $|\theta|$) is typically irrelevant is that its magnifiction is small. Compute the magnifiction of the second image for $\beta = \theta_E, 10\theta_E, 100\theta_E, 1000\theta_E$ for the lens-source setup that we have considered in other parts of this question. What is the limit of the magnifiction of the second image for $\beta \to \infty$?

How long did it take you to do this assignment?