Lab #1: Introductory Exercise - Oscillations of a Hoop

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Abstract:

The main purpose of this lab was to find the acceleration due to gravity of four different metal rings varying in diameter and mass. Each ring was placed on a stand with a knife support where they were each forced into oscillatory motion. The time for 50 oscillations was recorded for each ring to find their periods of oscillation. Using the period of the four rings, their acceleration due to gravity was found. As expected, the accelerations varied depending on the masses and radii.

Introduction:

The purpose of this experiment was to measure the acceleration due to gravity of four different rings varying in mass and diameter. This was done by using a knife support on a retort stand so that each ring could rest on the knife support and undergo oscillatory motion. Using a stopwatch, the period of each ring's motion was calculated. From there, the radius of each ring was measured using a ruler and a tape measure in order to help calculate their moments of inertia. Since the pivot of each ring was not located at the center of mass, the parallel axis theorem was used to calculate the moments of inertia.
The moments of inertia were then used to calculate their accelerations due to gravity.

Equipment:

- Four Different Sized Metal Rings - the four rings being analyzed
- Retort Stand/Knife Support - used to support the rings in motion
- Ruler - used to measure the diameter/radius of each ring
- Measuring Tape - used to measure the diameter/radius of the largest ring
- StopWatch - used to time the period of each ring

Experimental Procedure:

The first step in the experiment was to determine if the four rings were round. This was done by measuring both the inner and outer diameter of the ring being analyzed, then slightly rotating it and measuring again. This was done three times on each ring. If each measured diameter was equal, the ring would be considered round, otherwise, it would be considered warped and therefore some error may occur in the measurements and calculations. Since three measurements were taken, an average of the three diameters per ring was recorded and then halved in order to accurately measure the radius of each. After this, the mass of each ring, which was written on the outer face of the rings, was also recorded.

With the measurements recorded, the experiment began by placing the small ring on the knife support. The ring was pushed slightly on one of its sides so that it began oscillating. While fifty oscillations were being counted out, a stopwatch was used to record how long those fifty oscillations took. This was done three times per ring and an average of the times was taken to provide more accurate measurements. The average times were then divided by fifty in order to find the time it took for one oscillation to occur. This is the period of motion of each ring.

From here, the moment of inertia of each ring was calculated using their masses and radii. Since each ring oscillated from a pivot located at the top of their inner radii, the parallel axis theorem was used to determine their moments of inertia. This will be shown in the calculations.

The final step was to calculate the acceleration due to gravity of each ring. Using the formula for the period of a physical pendulum (shown in the calculations) and the recorded values (period, moment of inertia, mass, inner radius), the acceleration due to gravity was calculated for each ring.

Results:

Table 1 was used to record the inner and outer radius of each ring for all three trials, along with the average diameter. The average diameter of each ring was done by adding all trials for a specific ring and dividing by 3. These average diameters were then halved to get the average radii. The mass of each ring was also recorded in this table.

Table 1: Ring Measurements

	Ring 1	**Ring 2**	**Ring 3**	**Ring 4**
Trial 1 (diameter ($\pm\ 0.02$ **cm))**	d_{out} = 5.90 cm d_{in} = 5.70cm	d_{out} = 11.4cm d_{in} = 11.0cm	d_{out} = 21.8cm d_{in} = 21.2cm	d_{out} = 48.0cm d_{in} = 47.9cm
Trial 2 ($\pm\ 0.02$ **cm)**	d_{out} = 5.90cm d_{in} = 5.70cm	d_{out} = 11.4cm d_{in} = 11.0cm	d_{out} = 21.7cm d_{in} = 21.1cm	d_{out} = 48.0cm d_{in} = 47.9cm
Trial 3 ($\pm\ 0.02$ **cm)**	d_{out} = 5.80cm d_{in} = 5.60cm	d_{out} = 11.4cm d_{in} = 11.0cm	d_{out} = 21.8cm d_{in} = 21.2cm	d_{out} = 48.0cm d_{in} = 47.9cm
Average Diameter ($\pm\ 0.012$ **cm)**	d_{out} = 5.870cm d_{in} = 5.670cm	d_{out} = 11.40cm d_{in} = 11.00cm	d_{out} = 21.77cm d_{in} = 21.17cm	d_{out} = 47.97cm d_{in} = 47.87cm
Average Radius ($\pm\ 0.006$ **cm)**	r_{out} = 2.935cm r_{in} = 2.835cm	r_{out} = 5.700cm r_{in} = 5.500cm	r_{out} = 10.88cm r_{in} = 10.58cm	r_{out} = 23.98cm r_{in} = 23.94cm
Mass(kg)	0.01575kg	0.05368kg	0.16402kg	0.15557kg

*Note: one extra significant digit was carried after performing calculations in order to maintain accuracy.

The following equations were used to calculate the average diameter and radius of each ring respectively in Table 1:

$$Average\ Diameter\ =\ \frac{Trial\ 1 + Trial\ 2 + Trial\ 3}{3} \qquad Average\ Radius\ =\ \frac{Average\ Diameter}{2}$$

The uncertainty in the average diameter was calculated as follows:

$$u_{avg} = \frac{u_x}{\sqrt{N}}$$

where u_x is the uncertainty in each diameter measurement (0.02cm) and N is the number of uncertainty values (3). Therefore:

$$u_{avg} = \frac{0.02}{\sqrt{3}} = 0.012$$

The uncertainty in the average radius was simply $u_{avg}/2$.

Table 2 was used to record the times each ring took to oscillate 50 times for three separate trials. The average was then taken to find the average time of each ring using the same method as the diameter. The period of each ring's motion was also calculated and recorded in Table 2.

Table 2: Times and Periods of Oscillation

	Ring 1	**Ring 2**	**Ring 3**	**Ring 4**
Trial 1 (time ($\pm$ 0.3 s))	23.65	33.03	45.75	68.34
Trial 2 ($\pm$ 0.3 s)	24.62	33.03	45.75	68.53
Trial 3 ($\pm$ 0.3 s)	23.71	32.94	45.69	68.25
Average Time ($\pm$ 0.17 s)	23.993	32.977	45.730	68.373
Period ($\pm$ 0.0034 s)	0.47996	0.65954	0.91460	1.3675

*Note: the $\pm$ 0.3 comes from the average human reaction time.

The following Equations were used to calculate the average times and period, respectively, in Table 2:

$$Average\ Time = \frac{Trial\ 1 + Trial\ 2 + Trial\ 3}{3} \qquad Period\ (T) = \frac{Average\ Time}{50}$$

The uncertainties in the average time and period were calculated the same way as the radius and diameter in Table 1, except the average time was divided by 50 for the period uncertainty.

After the measurements were recorded, the moments of inertia of each ring were calculated using the parallel axis theorem since the pivot was not located at the center of mass of each ring. The distance from the center of mass to the pivot of each ring was the average inner radius of each ring recorded in Table 1. Both the inner and outer radii were taken into account for the calculations to maintain accurate results, therefore the following equation was simplified and used in calculations:

$$I = I_{com} + md^2$$

$$I = \frac{1}{2}m(r^2{}_{in} + r^2{}_{out}) + mr^2{}_{in}$$
$$I = \frac{1}{2}m(3r^2{}_{in} + r^2{}_{out})$$

For example, the moment of inertia for Ring 1 is:

$$I = \frac{1}{2}m(3r^2{}_{in} + r^2{}_{out})$$
$$I = \frac{1}{2}(0.1575)(3(0.02835)^2 + (0.02935)^2)$$
$$I = 2.313x10^{-5} kg\cdot m^2$$

*Note: the radii used were converted to m by dividing the values in Table 1 by 100.

The moments of inertia for each ring are recorded in Table 3 below.

Table 3: Moments of Inertia for Each Ring

Ring	**Moment of Inertia ($kg\cdot m^2$)**
1	$2.313x10^{-5}$
2	$3.3078x10^{-4}$
3	$3.725x10^{-3}$
4	$1.785x10^{-2}$

The last step was to calculate the acceleration due to gravity of each ring (g). The formula for the period of a physical pendulum was used and rearranged to solve for g.

$$T = 2\pi\sqrt{\frac{I}{mgr_{in}}} \quad \Rightarrow \quad g = \frac{4\pi^2 I}{T^2 mr_{in}}$$

For example, the acceleration due to gravity of Ring 1 is:

$$g = \frac{4\pi^2 I}{T^2 mr_{in}}$$
$$g = \frac{4\pi^2(2.313x10^{-5})}{(0.47996)^2(0.01575)(0.02835)}$$
$$g = 8.881 m/s^2$$

The uncertainty in the gravity values was found by combining uncertainty rules and using the formula:

$$u_g = 4\pi^2\sqrt{(\frac{2u_T}{T})^2 + (\frac{u_d}{d_{out}})^2}$$

For example, the uncertainty for ring 1 is:

$$u_g = 4\pi^2\sqrt{(\frac{2(.0034)}{0.47996})^2 + (\frac{0.012}{5.870})^2} = 0.56$$

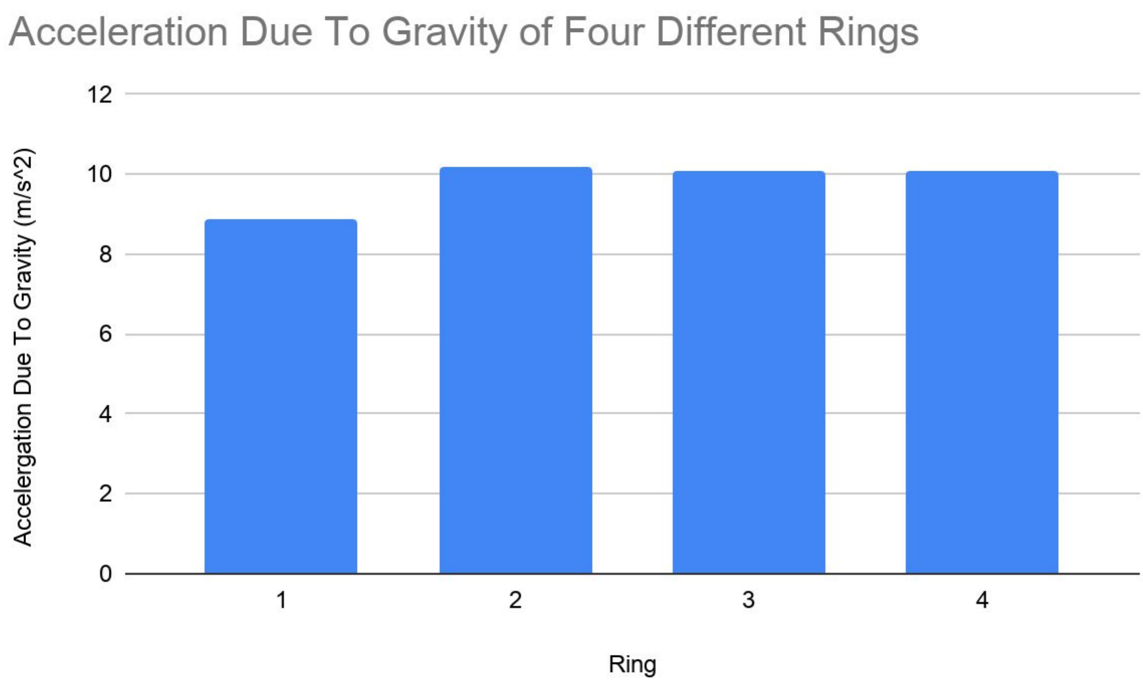
The accelerations due to gravity were recorded in Table 4 and Plot 1 below. These values are the final results of the lab.

Table 4: Accelerations Due To Gravity of Each Ring

Ring	**Acceleration Due To Gravity (m/s^2)**
1	8.88 $\pm$ 0.56
2	10.2 $\pm$ 0.41
3	10.1 $\pm$ 0.29
4	10.1 $\pm$ 0.20

*Note: These are the final results. Significant figures were reduced back down to three in order to match the original measurements recorded in Table 1.

Plot 1: Acceleration Values



The average of these values (calculated the same way as before) gives:

$$g = 9.82 \pm 0.73\ m/s^2$$

Discussion:

As expected, most of the values for the accelerations due to gravity were close to the 9.8m/s^2 with the average being almost exactly that. The individual values of g for each ring were not exactly 9.8 and there are several factors that may have led to those uncertainties. Firstly, it was mentioned above that the radii of each ring were not only measured multiple times to calculate an average, but to also determine if the rings were perfectly round. Table 1 shows that Rings 1, 3 and 4 were not perfectly round as the measured radii varied throughout the trials. The result of this is that a warped ring will have an altered mass distribution and therefore a different center of mass than what would be expected. This means there could have been a slight error in the moment of inertia calculations as it was assumed the center of mass was in the center of each ring.

Secondly, it is important to note the error in the measurements. When measuring with the ruler and tape measure, the resolution is 1mm and therefore the observer can only accurately measure to the nearest millimeter, hence the uncertainty in Table 1. The stopwatch was also operated by hand and therefore reaction time (0.3s on average) would have added time to the recorded values, hence the uncertainty in Table 2. The rings were also in contact with the knife support throughout their oscillations, so although very small, the friction between the equipment would have definitely altered the expected outcomes.

Conclusion:

Overall the results of the lab were as expected. An increase in mass and radius in the rings lead to an increase in the acceleration due to gravity of the rings.