PHY254 Homework Problems #2, Fall 2020

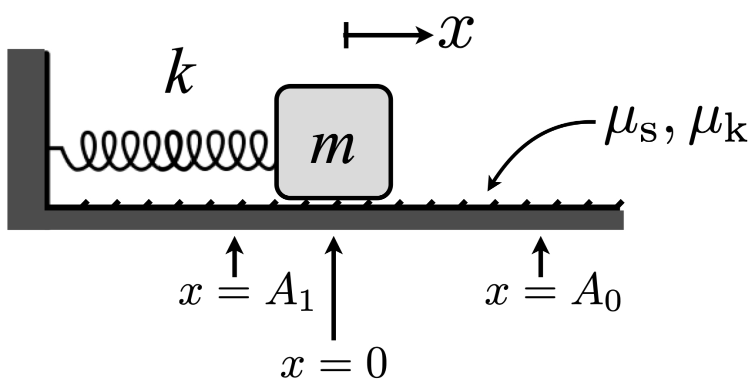
Due Wednesday October 28th at 11:59pm, sharp.

CORRECTION in red to part (b) of question 1.

Thanks to a sharp student for pointing out this error.

1. **Damped spring with ordinary Coulomb friction.**

   The mass and spring combination shown below is sliding on a rough surface with the normal "Coulomb" kind of friction (*i.e.* **not** the linear, v dependent damping forces we have been concentrating on in class). The surface has a coefficient of static friction μ_s and a coefficient of kinetic friction μ_k and, as usual, $\mu_s > \mu_k$. The block has mass m and the spring constant is k. As usual, a useful parameter is $\omega_0^2 = k/m$.



   (a) Suppose you pull the mass to the right and release it from rest. You find there is a limiting value of $x = A_0 > 0$ below which the mass just sticks and does not move. For $x > A_0$, it starts sliding when you release it from rest. Find A_0.

   (b) Now you pull the mass a tiny bit farther than the limiting A_0 so that it just starts to move when you release it from rest. You find that the mass slides to the left and then sticks again the moment it stops moving, having never reversed direction. Suppose it stops and re-sticks at position $x = A_1$. Find an expression for A_1. Show that $A_1 < 0$ (*i.e.* it is to the left of the position $x = 0$ for which the spring is not stretched or compressed) if $\mu_s > 2\mu_k$.

   Hint: In your expression for A_1, you can check the special case $\mu_k = 0$, where you can guess that $A_1 = -A_0$.

(c) Show that at position A_1, the forces are such that the mass never comes unstuck again, assuming it was released at the limiting A_0, and that $\mu_\mathrm{s} > \mu_\mathrm{k}$.

2. **Maximum speed of undamped vs. critically damped motion:** Morin exercise 4.28, page 125. This is one of the textbook's clever double star questions.

   Make sure you have read and fully understand the comments in the footnote. Refer to Morin chapter 1 for a discussion of dimensional analysis.

   Here is a scan of the question and the footnote:

   4.28. **Ratio of maxima** **

   A mass on the end of a spring is released from rest at position x_0. The experiment is repeated, but now with the system immersed in a fluid that causes the motion to be critically damped. Show that the maximum speed of the mass in the first case is e times the maximum speed in the second case.[7]

   [7] The fact that the maximum speeds differ by a fixed numerical factor follows from dimensional analysis, which tells us that the maximum speed in the first case must be proportional to ωx_0. And since $\gamma = \omega$ in the critical-damping case, the damping doesn't introduce a new parameter, so the maximum speed has no choice but to again be proportional to ωx_0. But showing that the maximum speeds differ by the nice factor of e requires a calculation.

3. **An oscillator with a realistic potential energy, integrated with Python.**

   The *Lennard-Jones potential* models the radial potential energy of vibrating diatomic molecules. The "LJ" or "6-12" potential function is

$$V(r) = \epsilon\left[\left(\frac{r_\mathrm{m}}{r}\right)^{12} - 2\left(\frac{r_\mathrm{m}}{r}\right)^{6}\right], \tag{1}$$

   where $\epsilon > 0$ is a constant with units of energy and r_m is a length. The potential has a minimum at $r = r_\mathrm{m}$, where $r > 0$ represents the radial separation between two atoms. The minimum has a depth of $-\epsilon$. $V(r)$ goes to zero as $r \to \infty$ and diverges as $r \to 0$.

   You can look at this Wikipedia page for more information.

   (a) Calculate the force radial $F(r)$ associated with this potential energy. Where is it attractive and where is it repulsive? Where is the stable equilibrium point and why is it stable?

For the following questions, turn in your answers with selected plots with your name in the title. Do not turn in your code.

(b) Choosing parameters $\epsilon = 1$ and $r_{\mathrm{m}} = 1$ (in some suitable atomic-scale units), write a python script to plot the potential in the range $0.8 \le r \le 3.0$, and its associated force (in these units). For $V(r)$, you should get a plot that looks roughly like the one on the Wikipedia page.

(c) Modify the script `lennard_jones_template.py` to calculate the motion under the force you found above. Set up the script to calculate the velocity and $v(t)$ and position $r(t)$, using an Euler-Cromer time stepping scheme. Assume the mass of the atom $m = 1$ in some atomic units.

Now run the script with initial conditions $r_0 = 1.02$, $v_0 = 0$. Make a plot showing the position and velocity for about 10 periods of this motion. You should observe that the motion is oscillatory. Does it look approximately sinusoidal? Estimate the period by looking at graphical output or by computationally finding twice the average time between crossings of $r = 1$. What period do you get?

Note: by choosing $\epsilon = 1$, $m = 1$ and $r_{\mathrm{m}} = 1$, we have effectively chosen "atomic time units" where each unit of time is given by $r_{\mathrm{m}}\sqrt{m/\epsilon}$. Your answer will naturally emerge in these units.

(d) Let $x = r - 1$ in these atomic units. Now the minimum is located at $x = 0$. Expand $V(x)$ about $V(0)$ up to second order in x. Use this calculation to find the period T of small oscillations and compare your analytic answer to the numerical result found in the previous part.

(e) Now decrease the initial displacement r_0 toward smaller r, for example try $r_0 = \{0.95,\ 0.90,\ 0.85\}$. Describe what happens as r_0 decreases; include selected plots. Go back to your plot of the potential in part 3b and indicate the locations of all the various initial conditions you tried. Can you explain this behavior and relate it to molecular physics?