Robotics 16-748: Problem Set #2

Due **October 8th, 11:59 PM**

Problem #1: For the sliding block example with time-invariant linear dynamics

$$\dot{x} = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} x + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u, \tag{1}$$

and initial condition $x(0) = [0, 0]^T$ consider deriving finite-time, optimal quadratic regulator solutions for each of the following cases. In each case, use the desired terminal cost $\phi(x(T)) = (x(T) - x_d)^T Q_f (x(T) - x_d)$, where $x_d = [10, 0]^T$ and Q_f and T are defined below. Turn in plots of the state trajectories and control signal for a) - d).

a) **(5 points)** $Q = I$, $R = 1$, $Q_f = 0 \times I$, and $T = 4$.
b) **(5 points)** $Q = I$, $R = 1$, $Q_f = 1 \times I$, and $T = 8$.
c) **(5 points)** $Q = I$, $R = 1$, $Q_f = 10 \times I$, and $T = 3$.
d) **(5 points)** $Q = I$, $R = 1$, $Q_f = 100 \times I$, and $T = 2$.
e) **(5 points)** Do any of the trajectories actually "make it all the way" to the desired set point? Explain your answer.

Problem #2: Grid World (15 Points) For a 15×15 grid, define the goal state s^* to be the cell at location $(7, 8)$. Define the action space to be *move up, move down, stay in place, move right,* and *move left.* Assume that the one step cost is defined by

$$g(s, a) = \begin{cases} 1 & s \neq s^* \\ 0 & s = s^* \end{cases} \tag{2}$$

Turn in code that sequentially plots (insert the Matlab "pause()" command appropriately in your code) the optimal value function as well as optimal policy as you step backward in time, i.e., perform value iteration for this problem similarly to what you saw in class.

Problem #3 (30 Points) For the simple pendulum with dynamics

$$\ddot{\theta} + \dot{\theta} + \sin\theta = u, \tag{3}$$

discretize the system's state space and use bilinear interpolation (look at link to "Planning Algorithms" in the class readings) to derive the optimal value function and optimal policy for this system using the instantaneous cost is $g(x, u) = (x - x_d)^T Q (x - x_d) + Ru^2$, with $Q = I$, $R = 1$, $Q_f = 0 \cdot I$, and $x_d = (\pi, 0)$. Assume the input is constrained s.t., $|u| \leq 2$. Discretize the action space using $\delta u = 0.5$. Discretize the state space over the region $\dot{x} \in [-3\pi/2, 3\pi/2]$ and $x = [-\pi, 2\pi]$ using $\delta x = \pi/6$. Use Euler integration to define you transition function for each state and action pair (dt = 0.05). If a state transition goes outside of this region assume that your transition is onto the nearest boundary point. Plot the value function and optimal policy after running twenty iterations.