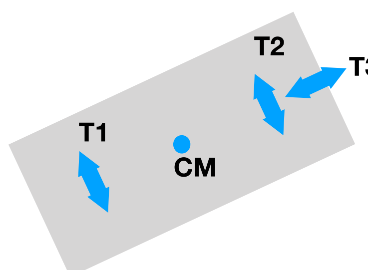
Robotics 16-748: Problem Set #1

Due **September 24th**

Problem #1 a) **(10 points)** The system shown below has three thrustors T1, T2, and T3. The system can move freely in the plane. Assume that there is a body frame fixed to the system's center of mass (CM) that is principally aligned with the main body axes. The orientation of the body with respect to an inertial frame is defined as θ. Thrustors T1 and T2 each produce forces in the directions shown, located $1m$ away from the center of mass. The forces produced by T2 and T3 are centered at the same point. Is this system underactuated?



b) **(10 points)** A planar system with two links that each have equal mass and are the same length are connected by a single rotational joint that is directly actuated. The system is assumed to be in a vacuum. The physical "no penetration" constraints of the system cannot be violated. The orientations of each link with respect to an inertial frame are defined as θ_1 and θ_2. The system's body frame is assumed to be rigidly attached to the first link (i.e., $\theta = \theta_1$). If we define the system's configuration space to be the space of planar rotations of the system's body frame, θ, is the system underactuated?

Problem #2 Consider the simple pendulum with dynamics

$$ml^2\ddot{\theta} + b\dot{\theta} + mgl\sin(\theta) = u. \tag{1}$$

Submit Matlab code as well as plots for each part of this question.

a) **(15 points)** Determine the value of any fixed points. Next, numerically calculate the basin of attraction of any stable fixed points by creating an evenly spaced mesh ($\delta x = 0.1$) over the state space (over the range $x \in \{-2\pi, 2\pi\}$ and range $\dot{x} = \{-2\frac{g}{l}, 2\frac{g}{l}\}$ where $m = 1$, $l = 1$, and $g = 9.8$) for the following parameter sets: $\{b = u = 0\}, \{b = 0.25, u = 0\}, \{b = 0.25, u = \frac{g}{2l}\}$.

b) **(15 points)** Using $m = 1$, $l = 1$, and $g = 9.8$, show the phase plot for the pendulum where $b = 0.25$ and the control u "inverts" the forces due to gravity. Plot the phase trajectory starting from the initial condition $(\theta, \dot{\theta}) = (\pi/4, 0)$? How much total power is required to execute this trajectory?

c) **(15 points)** Implement a feedback linearizing controller that stabilizes the system about the "unstable equilibrium" at $(\theta, \dot{\theta}) = (\pi, 0)$ starting from the initial condition $(\theta, \dot{\theta}) = (\pi/4, 0)$. How much total power is required to implement this trajectory?

Problem #3 Consider the double integrator system:

$$\ddot{x} = u. \tag{2}$$

Submit code as well as plots for each part of this question.

a) **(15 points)** Determine the minimum-time policy for this system by deriving first the relation $x = c + 1/2\dot{x}^2$, with the assumption $|u| \leq 1$, starting from the initial condition $x = 1$, $\dot{x} = 2$.

b) **(10 points)** Use MATLAB's **lqr** function to determine the infinite horizon optimal feedback policy for the brick with $Q = 0.25I$, where I is the identity matrix, and $R = 5I$. Plot the phase trajectory starting from the initial condition $x = 1$, $\dot{x} = 2$

c) **(10 points)** Compare the time it takes the minimum-time policy and the LQR policy to get within $|.05|$ of the goal (in both x and $\dot{x}$, i.e., within the radius in state space). How does the time change as Q in the LQR policy is increased for constant R? How does the time change as R is decreased while holding Q fixed? When $Q = 100I$ and $R = 10I$ how long does it take the LQR solution to converge in comparison to the "time-optimal policy?" Explain why or why not this result makes sense.