24-677 Project 1

P2

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Given:

$$\frac{d}{dt}\begin{bmatrix} e_1 \\ \dot{e}_1 \\ e_2 \\ \dot{e}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & -\frac{4C_\alpha}{mV_x} & \frac{4C_\alpha}{m} & -\frac{2C_\alpha(l_f-l_r)}{mV_x} \\ 0 & 0 & 0 & 1 \\ 0 & -\frac{2C_\alpha(l_f-l_r)}{I_zV_x} & \frac{2C_\alpha(l_f-l_r)}{I_z} & -\frac{2C_\alpha(l_f^2+l_r^2)}{I_zV_x} \end{bmatrix}\begin{bmatrix} e_1 \\ \dot{e}_1 \\ e_2 \\ \dot{e}_2 \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ \frac{2C_\alpha}{m} & 0 \\ 0 & 0 \\ \frac{2C_\alpha l_f}{I_z} & 0 \end{bmatrix}\begin{bmatrix} \delta \\ F \end{bmatrix}$$

plug in $m = 1888.6$

$C_\alpha = 20000$

$l_f = 1.55$

$l_r = 1.39$

$I_z = 25854$

$$\begin{bmatrix} e_1 \\ \dot{e}_1 \\ e_2 \\ \dot{e}_2 \end{bmatrix} = \underbrace{\begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & \frac{-42.3594}{V_x} & 42.3594 & \frac{-3.3888}{V_x} \\ 0 & 0 & 0 & 1 \\ 0 & \frac{-0.2475}{V_x} & 0.2475 & \frac{-6.7063}{V_x} \end{bmatrix}}_{A} + \underbrace{\begin{bmatrix} 0 & 0 \\ 21.1797 & 0 \\ 0 & 0 \\ 2.3981 & 0 \end{bmatrix}}_{B}$$

where:

$V_x = 2, 5, 8$

Controllability

$$P = [B \quad AB \quad A^2B \quad A^3B]$$

Observability

$$Q = \begin{bmatrix} C \\ CA \\ CA^2 \\ CA^3 \end{bmatrix}$$

Using Matlab to test the controllability for all 3 values, the rank of P, Q are 4.
Therefore, the system is controllable & observable

2. a

the system is more controllable because the σ_n is increasing. Thus the system is more likely to be full rank.

2. b

As a conclusion, the real parts of the poles are moving towards 0 as the speed increase.
Therefore, the system is more unstable as the speed goes up.

Check Q1 - pdf for plots

Performance