

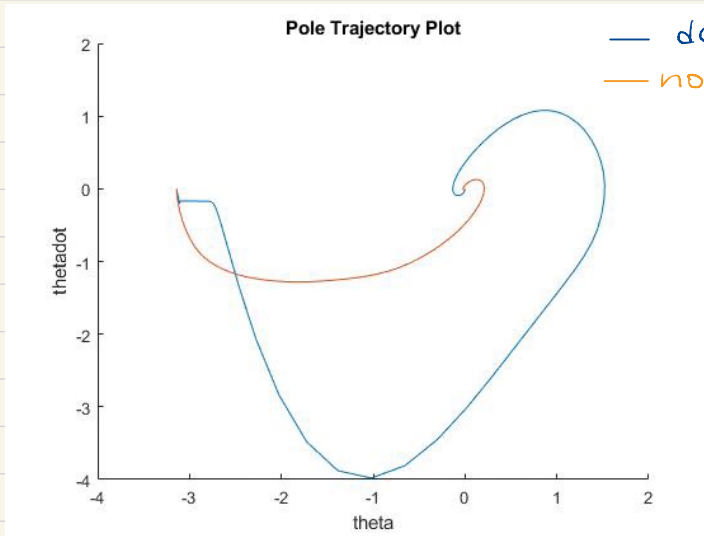
16-748 Problem set 4

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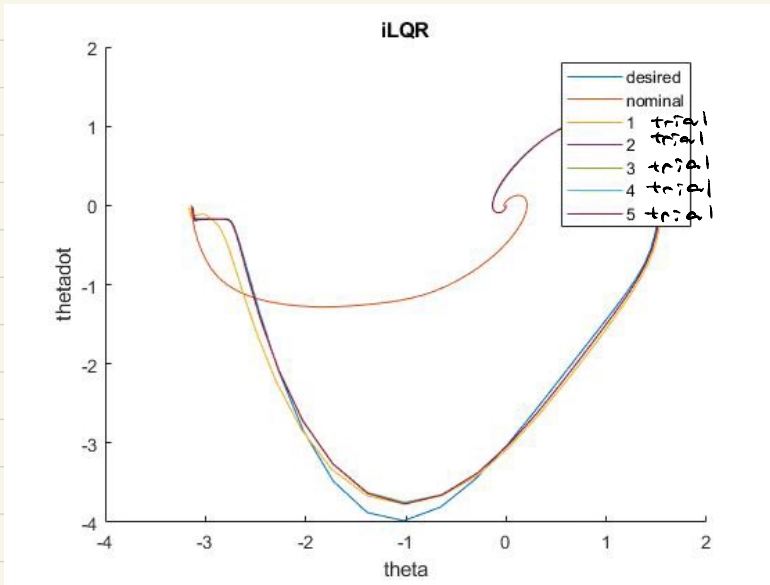
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Problem 1.

a) b)

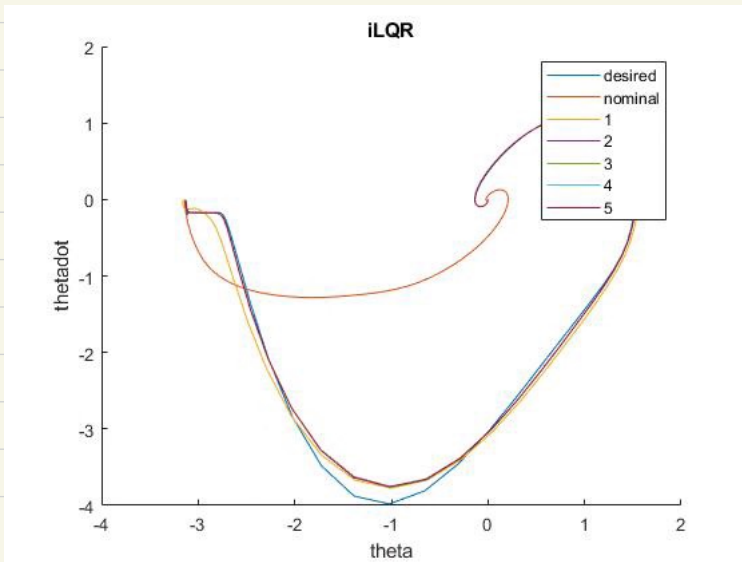


c)

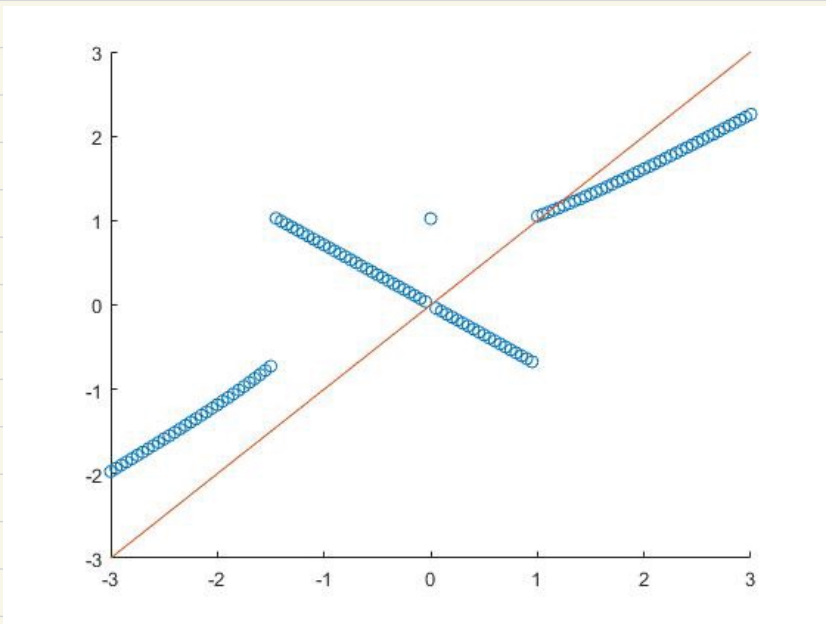


d)

$$u_d = 0$$



2 a)



2b)

forward walking occurs when $\dot{\theta}(0^+) > \omega_1$

$$\omega_1 = \sqrt{2 \frac{g}{l} (1 - \cos(\alpha - \alpha))}$$

the system has enough kinetic energy to go forward

when $\dot{\theta} < \omega_2$ it will rotate in the opposite direction

$$\text{where } \omega_2 = -\sqrt{2 \frac{g}{l} (1 - \cos(\alpha + \alpha))}$$

when $\omega_1 < \dot{\theta} < \omega_2$, collision

$$\dot{\theta}_{n+1}^+ = -\dot{\theta}_n^+ \cos(2\alpha)$$

$$\dot{\theta}_{n+1}^+ = \begin{cases} \cos(2\alpha) \sqrt{(\dot{\theta}_n^+)^2 + 4 \frac{g}{l} \sin(\alpha) \sin(\alpha)} & \dot{\theta}_n^+ \geq \omega_1 \\ -\dot{\theta}_n^+ \cos(2\alpha) & \omega_2 < \dot{\theta}_n^+ < \omega_1 \\ -\cos(2\alpha) \sqrt{(\dot{\theta}_n^+)^2 - 4 \frac{g}{l} \sin(\alpha) \sin(\alpha)} & \dot{\theta}_n^+ \leq \omega_2 \end{cases}$$

\uparrow
 $P(x)$

$$\therefore \frac{\partial P}{\partial \dot{\theta}_n} = \begin{cases} \cos(2\alpha) \sqrt{\dot{\theta}^2 + \frac{4g \sin(\alpha) \sin(\alpha)}{l}} & \dot{\theta}_n^+ \geq \omega_1 \\ -\cos(2\alpha) & \omega_2 < \dot{\theta}_n^+ < \omega_1 \\ -\cos(2\alpha) \sqrt{\dot{\theta}^2 - \frac{4g \sin(\alpha) \sin(\alpha)}{l}} & \dot{\theta}_n^+ \leq \omega_2 \end{cases}$$



∴ newton's method

$$x_{i+1} = x_i - \frac{x P(x)}{2 x P(x)}$$

$$x P(x) = x - P(x)$$

$$dx P(x) = I - \frac{\partial P(x)}{\partial x}$$

$$\uparrow$$
$$\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$$

Plug into matlab

fixpoints are $[-0.3127 \quad 1.0950]$

$$\begin{bmatrix} -0.3127 & 0 \end{bmatrix}$$

↑ $\theta = 0$

doesn't count

the eigenvalues of $dx P(x)$ are
 0.5000 & 1.000 ≤ 1

thus it is stable I.S.L