Robotics 16-748: Problem Set #3

Due **October 26, 11:59 PM**

Problem #1 (30 Points) In this problem you'll implement the energy-based swing-up and LQR balancing controllers for the cart-pole system. For all parts set each of the system parameters and gravity all equal to 1.

a) **(15 points)** Compute the system's linearization, either by hand or using a symbolic solver, about the point $x^* = (0, \pi, 0, 0)^T$, $u^* = 0$. Is the linearized system controllable? Next, derive an infinite horizon LQR controller using the linearized model and implement the controller for the nonlinear system. Explore the basin of attraction for the system by trying different initial configurations around $x = (0, \pi, 0, 0)^T$. Use these trials to generate an estimate of the region of attraction for the controller. Assume that $Q = I$ and $R = 1$. Display your estimate by showing the set of initial conditions from which your controller stabilizes the system about $x^* = (0, \pi, 0, 0)^T$.

b) **(15 points)** Use the energy-based swing up controller discussed in class to swing the pendulum into the basin of attraction of your LQR controller around $x^* = (0, \pi, 0, 0)$ from the initial condition $x = (0, 0, 0, 0)$ and then switch controllers such that the system stabilizes around the unstable equilibrium (you will need to add a small perturbation to your initial conditions to "jump start your controller"). Submit the code for your controller and a plot of the phase spaces defined over $(x, \dot{x})$ and $(\theta, \dot{\theta})$.

Problem #2: (20 points) In this problem you will implement a single shooting algorithm for the simple pendulum with actuator constraints using Matlab's *fmincon* function. Define a cost function such that

$$J = \sum_{t=0}^{T} u^T u$$

where $T = 200 * \text{dt}$ and $dt = 0.025$, i.e., the cost is the norm squared value of the control over 200 individual time steps. Define the parameter set over which you will optimize to be the value of the control input u in each of the 200 windows. Initialize each of the values in the control sequence to zero. Then define linear equality and inequality constraints to be empty sets, i.e.,

$$\begin{aligned} A &= [] \\ b &= [] \\ Aeq &= [] \\ beq &= []. \end{aligned}$$

Next, define a function that specifies a set of nonlinear equality ceq and inequality c constraints. The function should be defined such that $[c, ceq] = func(\alpha)$, where $c = []$ (i.e., c is the empty set) and there are two equality constraints: 1) the system's final position is constrained to be $\theta(T) = \pi$ and 2) the system's final velocity is constrained to be $\dot{\theta}(T) = 0$. You will use the system's dynamics (with the parameter values $m = l = g = 1$) and Euler integration to calculate these values. What are the minimum upper and lower bounds for which you can get the optimization to converge starting from an initial condition of $x(0) = [0, 0]$? Plot your solutions for the state and control and turn in your code.

Use the following options for *fmincon*:

$$\begin{aligned} options = &optimoptions(@fmincon,'TolFun', 0.00000001,' MaxIter', 10000, ...\\ &'MaxFunEvals', 100000,' Display',' iter', ...\\ &'DiffMinChange', 0.001,' Algorithm',' sqp'); \end{aligned}$$

Problem #3: (20 Points) In this problem you will implement the Hermite-Simpson direct collocation algorithm we discussed in class for the cart-pole system. The parameter set over which you will optimize will contain both the state as well as controls for the system defined at a discrete number of points in time. Select the number of windows to be $n = 50$ and use a time step of $dt = 0.1$. Define the linear constraints to be empty sets and constrain the system's final state to be $[x(T), \theta(T), \dot{X}(T), \dot{\theta}(T)] = [0, \pi, 0, 0]$. Constrain the system's initial state to $[x(0), \theta(0), \dot{x}(0), \dot{\theta}(0)] = [0, 0, 0, 0]$. Next, define constraints that enforce the system's dynamics at collocation points defined to be midway between each of the time steps where the states and controls are parameterized. More specifically, include defects

$$\Delta_k = (x(t_k) - x(t_{k+1})) + \frac{dt}{6}(f(x(t_k), u(t_k)) + 4f(x(t_{c,k}), u(t_{c,k})) + f(x(t_{k+1}), u(t_{k+1}))),$$

in the nonlinear constraints, where c refers to the collocation point associated with the k-th interval. Set the upper and lower bounds on the control to be $|u| \leq 5$. Define the upper and lower bounds on the state to be ∞ and $-\infty$, respectively. Use the same objective function as that from Problem #2. Plot your solutions for the state and control and turn in your code.

Problem #4: (10 Points) Design a feedback-stabilizing controller around the open-loop state and control trajectories determined by your single shooting method in Question 2. Note that the open-loop state trajectory is derived by applying the open-loop control signal solved for by the shooting method. Assume that the terminal cost is quadratic and is defined in terms of $Q_f = [1, 0; 0, 1]$. Show that you converge to the open loop optimal trajectories starting from an initial condition $x(0) = [0.5, 0]$. Set you $Q = [1, 0; 0, 1]$ and $R = 1$. For the controller, assume that you do not have any torque limits. Plot your solutions for the state and control and turn in your code.