24-677 Project 1

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P1

$$S_1 = \begin{bmatrix} y \\ \dot{y} \\ \psi \\ \dot{\psi} \end{bmatrix} \qquad \dot{S_1} = \begin{bmatrix} \dot{y} \\ \ddot{y} \\ \dot{\psi} \\ \ddot{\psi} \end{bmatrix} \qquad S_2 = \begin{bmatrix} x \\ \dot{x} \end{bmatrix} \qquad \dot{S_2} = \begin{bmatrix} \dot{x} \\ \ddot{x} \end{bmatrix}$$

$$\dot{S_1} = \begin{bmatrix} f_1 \\ f_2 \\ f_3 \\ f_4 \end{bmatrix} = \begin{bmatrix} \dot{y} \\ -\psi\dot{x} + \frac{2C_a}{m}\left(\cos\delta\left(\delta - \frac{\dot{y}+l_f\dot{\psi}}{\dot{x}}\right) - \frac{\dot{y}-l_r\dot{\psi}}{\dot{x}}\right) \\ \dot{\psi} \\ \frac{2\cdot l_f C_a}{I_z}\left(\delta - \frac{\dot{y}+l_f\dot{\psi}}{\dot{x}}\right) - \frac{2\,l_r C_a}{I_z}\left(-\frac{\dot{y}-l_r\dot{\psi}}{\dot{x}}\right) \end{bmatrix}$$

$$\dot{S_2} = \begin{bmatrix} \dot{x} \\ \dot{\psi}\,\dot{y} + \frac{1}{m}(F - f\cdot mg) \end{bmatrix}$$

while $u = \begin{bmatrix} \delta \\ F \end{bmatrix}$

to linearize S_1

$$\begin{bmatrix} \dot{y} \\ \ddot{y} \\ \dot{\psi} \\ \ddot{\psi} \end{bmatrix} = \begin{bmatrix} \frac{\partial f_1}{\partial y} & \frac{\partial f_1}{\partial \dot{y}} & \frac{\partial f_1}{\partial \psi} & \frac{\partial f_1}{\partial \dot{\psi}} \\ \frac{\partial f_2}{\partial y} & \frac{\partial f_2}{\partial \dot{y}} & \frac{\partial f_2}{\partial \psi} & \frac{\partial f_2}{\partial \dot{\psi}} \\ \frac{\partial f_3}{\partial y} & \frac{\partial f_3}{\partial \dot{y}} & \frac{\partial f_3}{\partial \psi} & \frac{\partial f_3}{\partial \dot{\psi}} \\ \frac{\partial f_4}{\partial y} & \frac{\partial f_4}{\partial \dot{y}} & \frac{\partial f_4}{\partial \psi} & \frac{\partial f_4}{\partial \dot{\psi}} \end{bmatrix}_{S_1}$$

$$= \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & A_{22} & 0 & A_{24} \\ 0 & 0 & 0 & 1 \\ 0 & A_{42} & 0 & A_{44} \end{bmatrix} \begin{bmatrix} y \\ \dot{y} \\ \psi \\ \dot{\psi} \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ B_{21} & 0 \\ 0 & 0 \\ B_{41} & 0 \end{bmatrix} \begin{bmatrix} \delta \\ F \end{bmatrix}$$

$$A_{22} = \frac{-2\, Ca\,(\cos\delta + 1)}{m \cdot \dot{x}}$$

$$A_{24} = -\dot{x} + \frac{2\, Ca\,(-\cos\delta\, lf + lr)}{m \cdot \dot{x}}$$

$$A_{42} = \frac{-2\, lf \cdot Ca}{Iz \cdot \dot{x}} + \frac{2 \cdot lr \cdot Ca}{Iz \cdot \dot{x}}$$

$$A_{44} = -\frac{2\, lf^2 \cdot Ca}{Iz \cdot \dot{x}} - \frac{2\, lr^2 \cdot Ca}{Iz \cdot \dot{x}}$$

$$B_{21} = \frac{2\, Ca\,(-\sin\delta)\left(\delta - \frac{lf\dot{x} + \dot{y}}{\dot{x}}\right) + \cos\delta)}{m}$$

$$B_{41} = -\frac{2C_a(l_f^2 + l_r^2)}{I_z \dot{x}}$$

To linearize S_2

$$\begin{bmatrix} \dot{x} \\ \ddot{x} \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} x \\ \dot{x} \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 0 & \frac{1}{m} \end{bmatrix} \begin{bmatrix} \delta \\ F \end{bmatrix}$$

Equilibrium point: $\begin{bmatrix} \dot{y} = 0 \\ \dot{\psi} = 0 \\ \delta = 0 \end{bmatrix}$

$m = 1888.6\ kg$
$l_r = 1.39\ m$
$l_f = 1.55\ m$
$C_a = 20000\ N$
$I_z = 2585.4\ kg \cdot m^2$

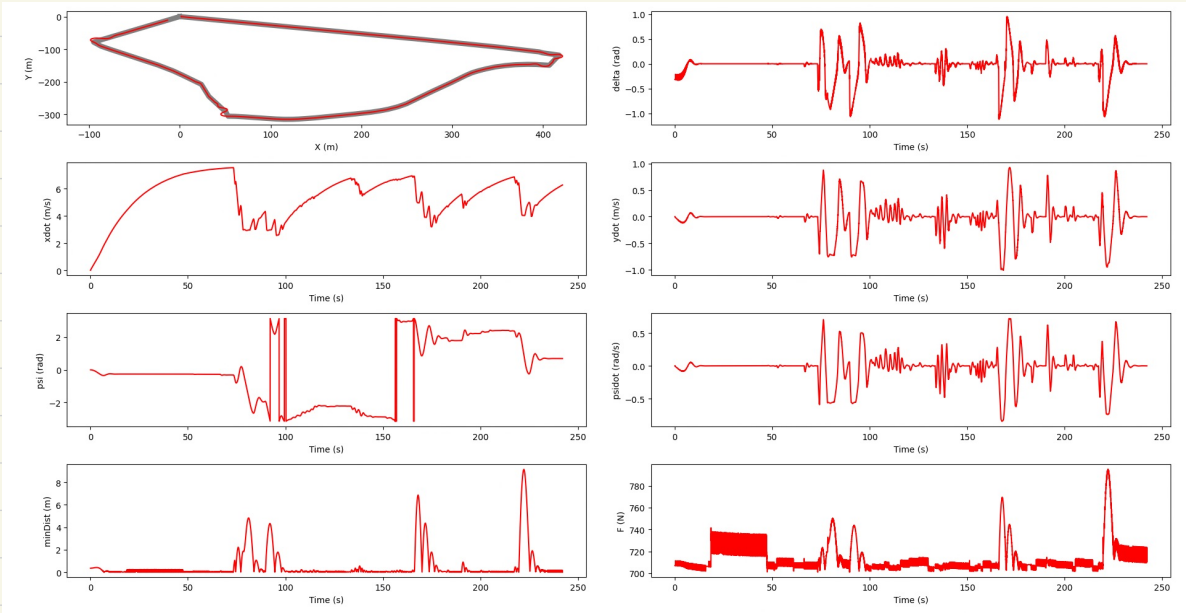
Plug in

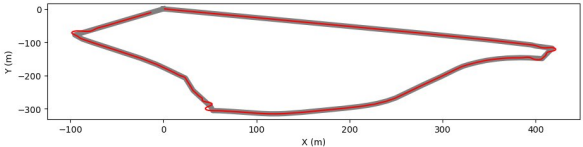
$$\dot{S_1} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & \frac{-42.36}{\dot{x}} & 0 & -\dot{x} + \frac{-3.39}{\dot{x}} \\ 0 & 0 & 0 & 1 \\ 0 & \frac{-0.25}{\dot{x}} & 0 & \frac{-6.71}{\dot{x}} \end{bmatrix} S_1 + \begin{bmatrix} 0 & 0 \\ 21.18 & 0 \\ 0 & 0 \\ 6.71 & 0 \end{bmatrix} U$$

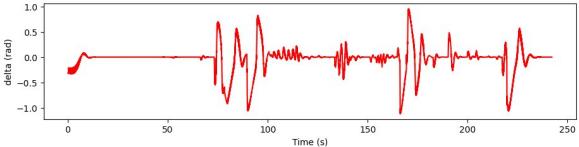
$$\dot{S_2} = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} S_2 + \begin{bmatrix} 0 & 0 \\ 0 & 5.24 \times 10^{-4} \end{bmatrix} U$$

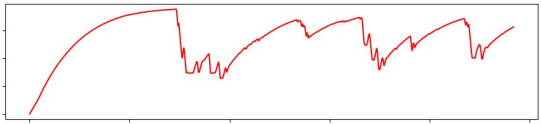
Exercise 2

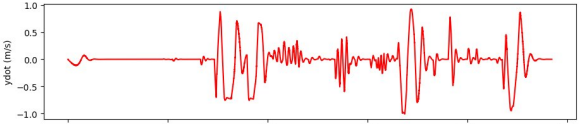
Performance Plot

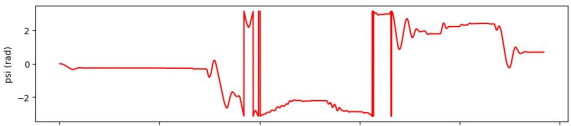


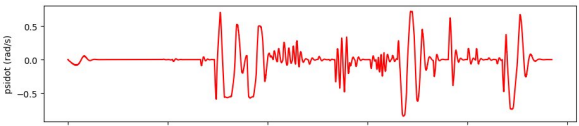

Y (m)
X (m)

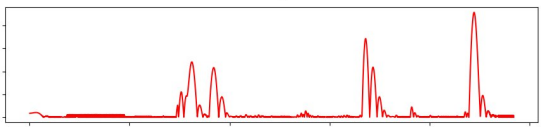

delta (rad)
Time (s)

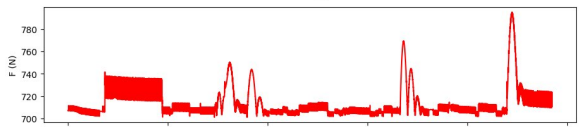

xdot (m/s)
Time (s)


ydot (m/s)
Time (s)


psi (rad)
Time (s)


psidot (rad/s)
Time (s)


minDist (m)
Time (s)


F (N)
Time (s)