

ZADACI ZA NUMERIČKU KOLOKVIJ 1!

1. Bez pivotiranja nađite LU faktORIZACIJU matrice

$$A = \begin{bmatrix} 2 & -1 & 1 \\ 3 & 3 & 9 \\ 3 & 3 & 5 \end{bmatrix}.$$

① $A = \begin{bmatrix} 2 & -1 & 1 \\ 3 & 3 & 9 \\ 3 & 3 & 5 \end{bmatrix} \xrightarrow{\cdot (-\frac{3}{2})} = \begin{bmatrix} 2 & -1 & 1 \\ 0 & \frac{9}{2} & \frac{15}{2} \\ 0 & \frac{9}{2} & \frac{7}{2} \end{bmatrix} \xrightarrow{\cdot (-1)} = \begin{bmatrix} 2 & -1 & 1 \\ 0 & \frac{9}{2} & \frac{15}{2} \\ 0 & 0 & -4 \end{bmatrix}$

$L = \begin{bmatrix} 1 & 0 & 0 \\ \frac{3}{2} & 1 & 0 \\ \frac{3}{2} & 1 & 1 \end{bmatrix}$

$LU = \begin{bmatrix} 1 & 0 & 0 \\ \frac{3}{2} & 1 & 0 \\ \frac{3}{2} & 1 & 1 \end{bmatrix} \begin{bmatrix} 2 & -1 & 1 \\ 0 & \frac{9}{2} & \frac{15}{2} \\ 0 & 0 & -4 \end{bmatrix}$

2. Odredite LU faktORIZACIJU matrice

$$A = \begin{bmatrix} 0 & 2 & 3 \\ 1 & 1 & -1 \\ 0 & -1 & 1 \end{bmatrix}.$$

Rješenje:

$$LU = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & -1/2 & 1 \end{bmatrix} \begin{bmatrix} 1 & 1 & -1 \\ 0 & 2 & 3 \\ 0 & 0 & 5/2 \end{bmatrix}.$$

② $A = \begin{bmatrix} 0 & 2 & 3 \\ 1 & 1 & -1 \\ 0 & -1 & 1 \end{bmatrix} \xrightarrow{\cdot (-\frac{1}{2})} = \begin{bmatrix} 1 & -1 & -1 \\ 0 & 2 & 3 \\ 0 & -1 & 1 \end{bmatrix} \xrightarrow{\cdot \frac{1}{2}} = \begin{bmatrix} 1 & -1 & -1 \\ 0 & 1 & \frac{3}{2} \\ 0 & 0 & \frac{5}{2} \end{bmatrix}$

$L = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 3 \\ 0 & -1/2 & 1 \end{bmatrix}$

3. Odredite potpuni kubični splajn za podatke

x	0	1	2
y	0	1	2

uz dodatne uvjete $s'(0) = s'(2) = 1$.

3) $[0, 1]$

$$s_1(0) = 0 \rightarrow d_1 = 0$$

$$s_1(1) = 1 \rightarrow a_1 + b_1 + c_1 = 1 \Rightarrow a_1 + b_1 = 0$$

$$s_1'(0) = 1 \rightarrow c_1 = 1$$

$$s_1'(1) = s_2'(1) \rightarrow 3a_1 + 2b_1 + c_1 = 3a_2 + 2b_2 + c_2$$

$$s_1''(1) = s_2''(1) \rightarrow 6a_1 + 2b_1 + 6a_2 + 2b_2 = 0$$

$[1, 2]$

$$s_2(1) = 1 \rightarrow a_2 + b_2 + c_2 + d_2 = 1 \Rightarrow$$

$$s_2(2) = 2 \rightarrow 8a_2 + 4b_2 + 2c_2 + d_2 = 2$$

$$s_2'(2) = 1 \rightarrow 12a_2 + 4b_2 + c_2 = 1$$

$$s_1''(1) = s_2''(1) \rightarrow 6a_1 + 2b_1 = 6a_2 + 2b_2$$

$$a_1 + b_1 = 0 \Rightarrow b_1 = -a_1$$

$$3a_1 + 2b_1 + 1 = 3a_2 + 2b_2 + c_2 \rightarrow 3a_1 - 2a_1 + 1 = 3a_2 + 2b_2 + c_2$$

$$\rightarrow a_1 + 1 = 3a_2 + 2b_2 + c_2$$

$$a_2 + b_2 + c_2 + d_2 = 1$$

$$8a_2 + 4b_2 + 2c_2 + d_2 = 2$$

$$12a_2 + 4b_2 + c_2 = 1$$

$$3a_1 + b_1 = 3a_2 + b_2 \rightarrow 2a_1 = 3a_2 + b_2 \Rightarrow a_1 = \frac{3a_2 + b_2}{2}$$

$$\begin{aligned}
3a_2 + b_2 + 2 &= 6a_2 + 2b_2 + c_2 \\
3a_2 + b_2 + c_2 &= 2 \Rightarrow b_2 = 2 - 3a_2 - c_2 \\
a_2 + b_2 + c_2 + d_2 &= 1 \\
8a_2 + 4b_2 + 2c_2 + d_2 &= 2 \\
12a_2 + 4b_2 + c_2 &= 1 \\
\hline
a_2 + 2 - 3a_2 - c_2 + c_2 + d_2 &= 1 \\
8a_2 + 8 - 12a_2 - 4c_2 + 2c_2 + d_2 &= 2 \\
12a_2 + &
\end{aligned}$$

Još do kraja riješiti ovih 6 jednačbi sa 6

nepoznanica

4. Odredite vrijednosti a, b, c, d tako da funkcija

$$s(x) = \begin{cases} -x^3 + 2x + 1, & 0 \leq x \leq 1 \\ a(x-1)^3 + b(x-1)^2 + c(x-1) + d, & 1 \leq x \leq 2 \end{cases}$$

bude prirodni kubični splajn.

Rješenje: $a = 1, b = -3, c = -1, d = 2$.

$$④ \quad S_1(1) = S_2(1)$$

$$-x^3 + 2x + 1 = a(x-1)^3 + b(x-1)^2 + c(x-1) + d$$

$$-1 + 2 + 1 = 0 + d$$

$$\underline{d = 2}$$

$$S_1'(x) = -3x^2 + 2$$

$$S_2'(x) = 3a(x-1)^2 + 2b(x-1) + c$$

$$S_1'(1) = S_2'(1)$$

$$-3 + 2 = c \quad \underline{c = -1}$$

$$S_1''(x) = -6x$$

$$S_2''(x) = 6a(x-1) + 2b$$

$$S_1''(1) = S_2''(1)$$

$$-6 = 2b \quad \underline{b = -3}$$

$$S''(2) = 0$$

$$6a + 2b = 0$$

$$6a - 6 = 0$$

$$\underline{a = 1}$$