KOLOKVIJ 2 NUMERIČKA ANALIZA

ZADACI

1.

① Odredi vrijednosti $\alpha, \beta \in \mathbb{R}$ tako da integracijska formula

$$\frac{1}{b-a}\int_a^b f(x)\,dx \simeq \alpha f(a) + f\left(\frac{a+b}{2}\right) + \beta f(b) \quad / \cdot (b-a)$$

bude što većeg stupnja egzaktnosti.

$$\int_a^b f(x)\,dx \simeq \underbrace{\alpha(b-a)}_{\omega_0} f(a) + \underbrace{(b-a)}_{\omega_1} \cdot f\left(\frac{a+b}{2}\right) + \underbrace{\beta(b-a)}_{\omega_2} f(b)$$

$$b-a = \int_a^b x^0\,dx = \omega_0 \cdot 1 + \omega_1 \cdot 1 + \omega_2 \cdot 1$$

$$\frac{b^2-a^2}{2} = \int_a^b x^1\,dx = \omega_0 \cdot a + \omega_1 \cdot \frac{a+b}{2} + \omega_2 \cdot b$$

Dovoljno je 2 obzirom da tražimo α i β

① $\cancel{b-a} = \alpha(\cancel{b-a}) + (\cancel{b-a}) + \beta(\cancel{b-a}) \Rightarrow 0 = \alpha + \beta \Rightarrow \beta = -\alpha$

② $\frac{(\cancel{b-a})(b+a)}{2} = \alpha a(\cancel{b-a}) + (\cancel{b-a}) \cdot \frac{a+b}{2} + \beta b(\cancel{b-a})$

$$\frac{b+a}{\cancel{2}} = \alpha a + \frac{a+b}{\cancel{2}} - \alpha b \Rightarrow \alpha(a-b) = 0$$

$$\boxed{\begin{aligned}\alpha &= 0\\ \beta &= 0\end{aligned}}$$

Znači $\frac{1}{b-a}\int_a^b f(x)\,dx \simeq f\left(\frac{a+b}{2}\right) \quad / \cdot (b-a)$

$$\int_a^b f(x)\,dx \simeq (b-a)\, f\left(\frac{a+b}{2}\right)$$

dobijemo midpoint

2. **Fali samo sa desne strane di je dX = 0 i a*pi=0 odma ispod toga**

2) Odredite čvor X_0 u težinskoj Gaussovoj integracijs formuli s jednim čvorom oblika:

$$\int_{-1}^{1} \frac{1}{\sqrt{1-x^2}} f(x)\,dx \simeq \omega_0 f(x_0)$$

UPUTA Odredite polinom $\omega_1(x) = a + x$ (čija je onda nul traženi X_0) tako da s obzirom na težinu W (kao u integ formuli) na $[-1,1]$ bude okomit na sve polinome manjeg stu u ovom slučaju sve polinome stupnja nula.

$W(x) = \frac{1}{\sqrt{1-x^2}}$ $\qquad \partial\omega_1 = 1$ $\quad$ ω_1 mora biti okomit na sve polinome stupn

$\longrightarrow P_0(x) = 1$

$$\int_{-1}^{1} W(x)\cdot \omega_1(x)\cdot P_0(x)\,dX = 0$$

$$\int_{-1}^{1} \frac{1\cdot(a+x)}{\sqrt{1-x^2}}\,dx = 0 \xRightarrow{\text{RAZDVAJAMO NA 2}} \underbrace{\frac{1}{2}\int_{-1}^{1}\frac{2x\,dx}{\sqrt{1-x^2}}}_{} + a\int_{-1}^{1}\frac{1}{\sqrt{1-x^2}}\,dx$$

$t = 1-x^2$

$dt = -2x\,dx$

x	-1	1
t	0	0

$$\Rightarrow \underbrace{-\frac{1}{2}\int_0^0 \frac{dt}{\sqrt{t}}}_{\text{ovo je nula}} + a\cdot \arcsin x\Big|_{-1}^{1} = a\Big(\underbrace{\frac{\pi}{2}}_{\arcsin 1} - \Big(\underbrace{-\frac{\pi}{2}}_{\arcsin(-1)}\Big)\Big) = a\pi =$$

$$\boxed{a = 0}$$

$\omega_1(x) = \overset{0}{a} + X \longrightarrow X_0 = -a$

$\omega_1(x) = X$

$X_0 = 0$

3.

3) Odredite $c \in \mathbb{R} \setminus \{0\}$ tako da iteracija $x_{n+1} = x_n + c(\ln x_n - 1)$ konvergira k točnom rješenju jednadžbe $\underbrace{x = x + c(\ln x - 1)}$

$$c(\ln x - 1) = 0$$
$$\ln x = 1$$
$$\ln x = \ln e^1$$
$$\boxed{x = e}$$

$$g(x) = x + c(\ln x - 1)$$

$$g'(x) = 1 + \frac{c}{x}$$

$$\left|1 + \frac{c}{x}\right| < 1$$

$$-1 < 1 + \frac{c}{x} < 1$$

$$-2 < \frac{c}{x} < 0$$

Dva su slučaja:

a) $\frac{c}{x} < 0$

$c < 0$

b) $\frac{c}{x} > -2$ za $x > 0$

$c > -2x$

$c > -2e$

Znači konvergira na intervalu, tj gledo uvijek presjek naravno:

$$c \in \langle -2e, 0 \rangle$$