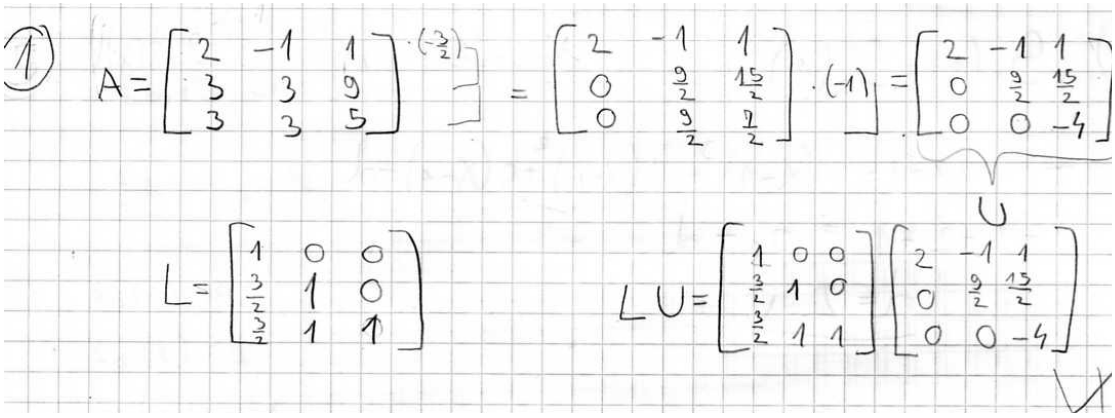
ZADACI ZA NUMERIČKU KOLOKVIJ 1!

1. Bez pivotiranja nađite LU faktorizaciju matrice

$$A = \begin{bmatrix} 2 & -1 & 1 \\ 3 & 3 & 9 \\ 3 & 3 & 5 \end{bmatrix}.$$

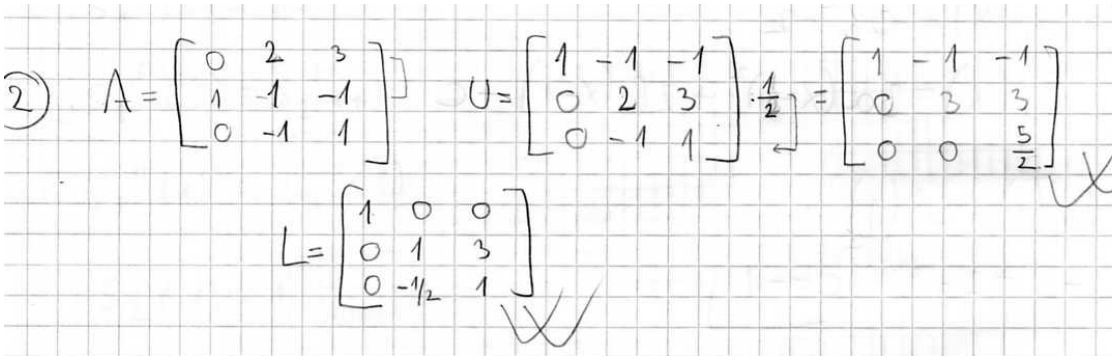


2. Odredite LU faktorizaciju matrice

$$A = \begin{bmatrix} 0 & 2 & 3 \\ 1 & 1 & -1 \\ 0 & -1 & 1 \end{bmatrix}.$$

Rješenje:

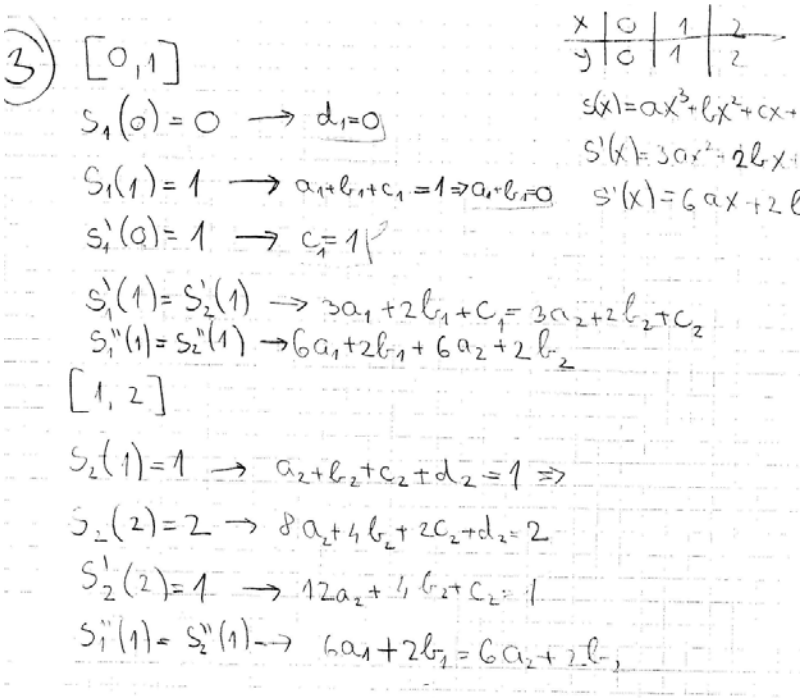
$$LU = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & -1/2 & 1 \end{bmatrix} \begin{bmatrix} 1 & 1 & -1 \\ 0 & 2 & 3 \\ 0 & 0 & 5/2 \end{bmatrix}.$$

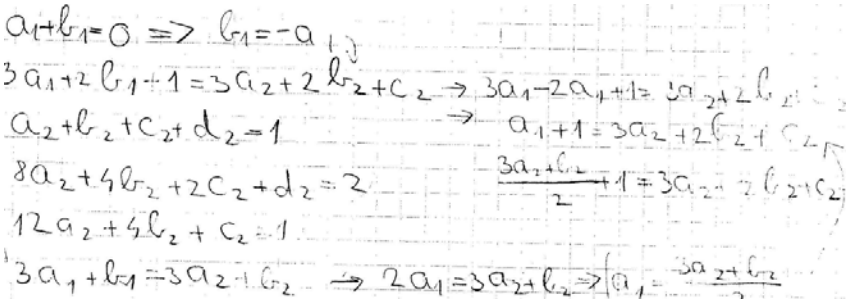


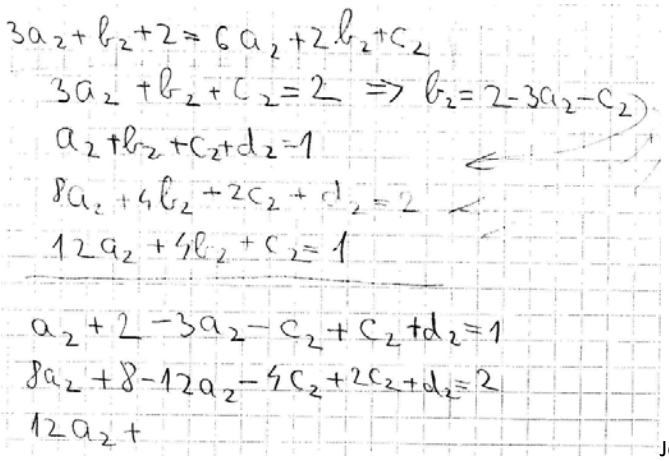
3. Odredite potpuni kubični splajn za podatke

x	0	1	2
y	0	1	2

uz dodatne uvjete $s'(0) = s'(2) = 1$.







Još do kraja riješit ovih 6 jednadzbi sa 6 nepoznanica

4. Odredite vrijednosti a, b, c, d tako da funkcija

$$s(x)=\begin{cases} -x^3+2x+1, & 0\le x\le 1 \\ a(x-1)^3+b(x-1)^2+c(x-1)+d, & 1\le x\le 2 \end{cases}$$

bude prirodni kubični splajn.
Rješenje: $a=1,\ b=-3,\ c=-1,\ d=2$.

④ $S_1(1) = S_2(1)$

$-x^3+2x+1 = a(x-1)^3 + b(x-1)^2 + c(x-1) + d$

$-1+2+1 = 0 + d$

$d = 2$

$S_1'(x) = -3x^2+2$

$S_2'(x) = 3a(x-1)^2 + 2b(x-1) + c$

$S_1'(1) = S_2'(1)$

$-3+2 = c \quad c = -1$

$S_1''(x) = -6x$

$S_2''(x) = 6a(x-1) + 2b$

$S_1''(1) = S_2''(1)$

$-6 = 2b \quad b = -3$

$S''(2) = 0$

$6a+2b = 0$

$6a-6 = 0$

$a = 1$