1031

$$\int 5a^2x^6dx = 5a^2\int x^6dx = 5a^2\frac{x^7}{7} = \frac{5}{7}a^2x^7 + C$$

1032

$$\int(6x^2+8x+3)dx = 6\int x^2dx + 8\int xdx + 3\int dx = 6\frac{x^3}{3} + 8\frac{x^2}{2} + 3x = 2x^3+4x^2+3x+C$$

1033

$$\int x(x+a)(x+b)dx = \int x(x^2+xb+ax+ab) = \int(x^3+x^2b+ax^2+xab)dx$$
$$= \int x^3dx + b\int x^2dx + a\int x^2dx + ab\int xdx = \frac{x^4}{4} + b\frac{x^3}{3} + a\frac{x^3}{3} + ab\frac{x^2}{2}$$
$$= \frac{1}{4}x^4 + \frac{1}{3}bx^3 + \frac{1}{3}ax^3 + \frac{1}{2}abx^2 + C$$

1034

$$\int(a+bx^3)^2dx = \int(a^2+2abx^3+b^2x^6)dx = a^2x+2ab\frac{x^4}{4}+b^2\frac{x^6}{7} =$$
$$a^2x + \frac{1}{2}abx^4 + \frac{1}{7}b^2x^6 + C$$

1035

$$\int\sqrt{2px}\,dx$$
$$\left\{\begin{array}{c} 2px = t|' \\ 2pdx = dt \end{array}\right\}$$
$$= \int\sqrt{t}\,\frac{dt}{2p} = \frac{1}{2p}\int t^{\frac{1}{2}}dt = \frac{1}{2p}\frac{2}{3}t^{\frac{3}{2}} = \frac{1}{3p}t^{\frac{3}{2}} = \frac{1}{3p}(2px)^{\frac{3}{2}} = \frac{2}{3}\sqrt{2}\,x\sqrt{px} + C$$

1036

$$\int\frac{dx}{\sqrt[n]{x}} = \int x^{-\frac{1}{n}}dx = \frac{x^{-\frac{1}{n}+\frac{n}{n}}}{-\frac{1}{n}+\frac{n}{n}} = \frac{x^{\frac{n-1}{n}}}{\frac{n-1}{n}} = n\frac{x^{\frac{n-1}{n}}}{n-1} + C$$

1037

$$\int(nx)^{\frac{1-n}{n}}dx$$
$$\left\{\begin{array}{c} nx = u \\ ndx = du \end{array}\right\}$$
$$\int u^{\frac{1-n}{n}}\frac{du}{n} = \frac{1}{n}\int u^{\frac{1-n}{n}}du = \frac{1}{n}\frac{u^{\frac{1-n}{n}+\frac{n}{n}}}{\frac{1-n}{n}+\frac{n}{n}} = \frac{1}{n}\frac{u^{\frac{1}{n}}}{\frac{1}{n}} = u^{\frac{1}{n}} = (nx)^{\frac{1}{n}} + C$$

1038

$$\int(a^{\frac{2}{3}}-x^{\frac{2}{3}})^3dx = \int\left(a^2-3a^{\frac{4}{3}}x^{\frac{2}{3}}+3a^{\frac{2}{3}}x^{\frac{4}{3}}-x^2\right)dx$$
$$= a^2x - 3a^{\frac{4}{3}}\frac{x^{\frac{5}{3}}}{\frac{5}{3}} + 3a^{\frac{2}{3}}\frac{x^{\frac{7}{3}}}{\frac{7}{3}} - \frac{x^3}{3} = a^2x - \frac{9}{5}a^{\frac{4}{3}}x^{\frac{5}{3}} + \frac{9}{7}a^{\frac{2}{3}}x^{\frac{7}{3}} - \frac{1}{3}x^3 + C$$

1039

$$\int(\sqrt{x}+1)(x-\sqrt{x}+1)dx = \int\left(x^{\frac{3}{2}}+1\right)dx = x + \frac{2}{5}x^{\frac{5}{2}} + C$$

1040

$$\int\frac{(x^2+1)(x^2-2)}{\sqrt[3]{x^2}}dx = \int\frac{x^4-x^2-2}{x^{\frac{2}{3}}}dx = \int\left(x^{\frac{10}{3}}-x^{\frac{4}{3}}-2x^{-\frac{2}{3}}\right)dx = \frac{3}{13}x^{\frac{13}{3}} - \frac{3}{7}x^{\frac{7}{3}} - 2\cdot 3x^{\frac{1}{3}}$$
$$= \frac{3}{13}x^{\frac{13}{3}} - \frac{3}{7}x^{\frac{7}{3}} - 6\sqrt[3]{x} + C$$

1041

$$\int\frac{(x^m-x^n)^2}{\sqrt{x}}dx = \int\frac{x^{2m}-2x^{m+n}+x^{2n}}{\sqrt{x}}dx = \int\left(x^{2m}x^{-\frac{1}{2}}-2x^{m+n}x^{-\frac{1}{2}}+x^{2n}x^{-\frac{1}{2}}\right)dx$$
$$= \int\left(x^{2m-\frac{1}{2}}-2x^{m+n-\frac{1}{2}}+x^{2n-\frac{1}{2}}\right)dx = \frac{x^{2m-\frac{1}{2}+1}}{2m-\frac{1}{2}+1} - 2\frac{x^{m+n-\frac{1}{2}+1}}{m+n-\frac{1}{2}+1} + \frac{x^{2n-\frac{1}{2}+1}}{2n-\frac{1}{2}+1} + C$$
$$= \frac{x^{2m+\frac{1}{2}}}{2m+\frac{1}{2}} - 2\frac{x^{m+n+\frac{1}{2}}}{m+n+\frac{1}{2}} + \frac{x^{2n+\frac{1}{2}}}{2n+\frac{1}{2}} + C$$

1042

$$\int\frac{(\sqrt{a}-\sqrt{x})^4}{\sqrt{ax}}dx = \int(ax)^{-\frac{1}{2}}(a-2\sqrt{a}\,\sqrt{x}+x)^2dx = \int(ax)^{-\frac{1}{2}}\left(a^2-4a^{\frac{3}{2}}\sqrt{x}+6ax-4\sqrt{a}\,x^{\frac{3}{2}}+\right.$$
$$= a^{-\frac{1}{2}}\int x^{-\frac{1}{2}}\left(a^2-4a^{\frac{3}{2}}\sqrt{x}+6ax-4\sqrt{a}\,x^{\frac{3}{2}}+x^2\right) = a^{-\frac{1}{2}}\int\left(\frac{1}{\sqrt{x}}a^2-4a^{\frac{3}{2}}+6\sqrt{x}\,a-4x\sqrt{a}\right.$$
$$= a^{-\frac{1}{2}}\left(\frac{x^{-\frac{1}{2}+1}}{-\frac{1}{2}+1}a^2-4a^{\frac{3}{2}}x+6a\frac{2}{3}x^{\frac{3}{2}}-4\frac{x^2}{2}\sqrt{a}+\frac{x^{\frac{5}{2}}}{\frac{5}{2}}\right) = a^{-\frac{1}{2}}\left(2a^2\sqrt{x}-4a^{\frac{3}{2}}x+4ax^{\frac{3}{2}}-2x\right.$$
$$= 2a^{\frac{3}{2}}\sqrt{x} - 4ax + 4\sqrt{a}\,x^{\frac{3}{2}} - 2x^2 + \frac{2}{5\sqrt{a}}x^{\frac{5}{2}} + C$$

1043

$$\int\frac{dx}{x^2+7} = \frac{1}{\sqrt{7}}\arctan\frac{1}{\sqrt{7}}x + C$$

1044

$$\int\frac{dx}{x^2-10} = -\frac{1}{\sqrt{10}}\operatorname{arctanh}\frac{1}{\sqrt{10}}x + C$$

1045

$$\int\frac{dx}{\sqrt{4+x^2}} = \operatorname{arcsinh}\frac{1}{2}x + C$$

1046

$$\int\frac{dx}{\sqrt{8+x^2}} = \operatorname{arcsinh}\frac{1}{4}\sqrt{2}\,x + C$$

1047

$$\int\frac{\sqrt{2+x^2}-\sqrt{2-x^2}}{\sqrt{4-x^2}}dx = \int\frac{\sqrt{2+x^2}-\sqrt{2-x^2}}{\sqrt{2+x^2}\,\sqrt{2-x^2}}dx = \int\frac{1}{\sqrt{2-x^2}}dx - \int\frac{1}{\sqrt{2+x^2}}dx$$
$$= \arcsin\frac{1}{2}\sqrt{2}\,x - \operatorname{arcsinh}\frac{1}{2}\sqrt{2}\,x + C$$

1048*a

$$\int\tan^2x dx = \int\frac{\sin^2x}{\cos^2x}dx = \int\frac{1-\cos^2x}{\cos^2x}dx = \int\frac{1}{\cos^2x}dx - \int dx = \frac{1}{\cos x}\sin x - x + C$$

1048*b

$$\int\tanh^2x dx = \int\frac{\sinh^2}{\cosh^2} = \int\frac{1+\cosh^2x}{\cosh^2} = \int\frac{dx}{\cosh^2} + \int dx = \frac{\sinh x}{\cosh x} + x = \tanh x + x + C$$

1049a

$$\int\cot^2x dx = \int\frac{\cos^2x}{\sin^2x}dx = \int\frac{1-\sin^2x}{\sin^2x}dx = \int\frac{dx}{\sin^2x} - \int dx = -\cot x - x + C$$

1049b

$$\int\coth^2x dx = \int\frac{\cosh^2x}{\sinh^2x}dx = \int\frac{1-\sinh^2x}{\sinh^2x}dx = \int\frac{dx}{\sinh^2x} - \int dx = -\coth x - x + C$$

1050

$$\int 3^xe^xdx = \int(3e)^xdx$$

$$\left\{\begin{array}{c}(3e)^x = u\\ 3^x e^x(1+\ln 3)dx = du\end{array}\right\}$$

$$= \int (3e)^x \tfrac{du}{3^x e^x(1+\ln 3)} = \int \tfrac{du}{(1+\ln 3)} = \tfrac{1}{1+\ln 3} u = \tfrac{(3e)^x}{1+\ln 3} + C$$

1051**

$$\int \tfrac{adx}{a-x} = a\int \tfrac{dx}{a-x}$$

$$\left\{\begin{array}{c}a-x = u|'\\ -dx = du\end{array}\right\}$$

$$= a\int \tfrac{-du}{u} = -a\ln u = -a\ln(a-x) + C$$

1052**

$$\int \tfrac{2x+3}{2x+1}dx = \int \tfrac{2x+1+2}{2x+1}dx = \int dx + \int \tfrac{2}{2x+1}dx = x + \int \tfrac{2}{2x+1}dx$$

$$\left\{\begin{array}{c}2x+1 = u|'\\ 2dx = du\end{array}\right\}$$

$$x + \int \tfrac{du}{u} = x + \ln u = x + \ln(2x+1) + C$$

1054

$$\int \tfrac{xdx}{a+bx} = \tfrac{1}{b}\int \tfrac{bxdx}{a+bx} = \tfrac{1}{b}\int \tfrac{bx+a-a}{a+bx}dx = \tfrac{1}{b}\int \tfrac{bx+a-a}{a+bx}dx = \tfrac{1}{b}\int dx - \tfrac{a}{b}\int \tfrac{dx}{a+bx}$$

$$= \tfrac{x}{b} - \tfrac{a}{b}\int \tfrac{dx}{a+bx}$$

$$\left\{\begin{array}{c}a+bx = u|'\\ bdx = du\end{array}\right\}$$

$$= \tfrac{x}{b} - \tfrac{a}{b}\int \tfrac{\frac{du}{b}}{u} = \tfrac{x}{b} - \tfrac{a}{b^2}\int \tfrac{du}{u} = \tfrac{x}{b} - \tfrac{a}{b^2}\ln u = \tfrac{x}{b} - \tfrac{a}{b^2}\ln(a+bx) + C$$

1055

$$\int \tfrac{ax+b}{\alpha x+\beta} = a\int \tfrac{x+\frac{b}{a}}{\alpha x+\beta}dx = a\int \tfrac{xdx}{\alpha x+\beta} + \int \tfrac{b}{\alpha x+\beta}dx$$

$$= \tfrac{a}{\alpha}\int \tfrac{\alpha x}{\alpha x+\beta}dx + b\int \tfrac{dx}{\alpha x+\beta} = \tfrac{a}{\alpha}\int \tfrac{\alpha x+\beta-\beta}{\alpha x+\beta}dx + b\int \tfrac{dx}{\alpha x+\beta}$$

$$= \tfrac{a}{\alpha}\int dx + \tfrac{a}{\alpha}\int \tfrac{-\beta}{\alpha x+\beta}dx + b\int \tfrac{dx}{\alpha x+\beta} = \tfrac{ax}{\alpha} - \tfrac{a\beta}{\alpha}\int \tfrac{dx}{\alpha x+\beta} + b\int \tfrac{dx}{\alpha x+\beta}$$

$$= \tfrac{ax}{\alpha} + \left(b - \tfrac{a\beta}{\alpha}\right)\int \tfrac{dx}{\alpha x+\beta} = \tfrac{ax}{\alpha} + \tfrac{b\alpha-\beta a}{\alpha}\int \tfrac{dx}{\alpha x+\beta}$$

$$\left\{\begin{array}{c}\alpha x+\beta = u|'\\ \alpha dx = du\end{array}\right\}$$

$$= \tfrac{ax}{\alpha} + \tfrac{b\alpha-\beta a}{\alpha}\int \tfrac{\frac{du}{\alpha}}{u} = \tfrac{ax}{\alpha} + \tfrac{b\alpha-\beta a}{\alpha^2}\int \tfrac{du}{u} = \tfrac{ax}{\alpha} + \tfrac{b\alpha-\beta a}{\alpha^2}\ln u = a\tfrac{x}{\alpha} + \tfrac{b\alpha-\beta a}{\alpha^2}\ln(\alpha x+\beta) + C$$

1056

$$\int \tfrac{x^2+1}{x-1}$$

$$(x^2+1) : (x-1) = x + 1 + \tfrac{2}{(x-1)}$$

$$x^2 - x$$

$$1 + x$$

$$x - 1$$

$$2$$

$$= \int xdx + \int dx + 2\int \tfrac{dx}{(x-1)} = \tfrac{x^2}{2} + x + 2\ln(x-1) + C$$

1057

$$\int \tfrac{x^2+5x+7}{x+3}dx = \int \tfrac{x^2}{x+3}dx + 5\int \tfrac{x}{x+3}dx + 7\int \tfrac{dx}{x+3} = \int \tfrac{x^2+9-9}{x+3}dx + 5\int \tfrac{x+3-3}{x+3}dx + 7\ln(x+3)$$

$$= \int (x-3)dx + 9\int \tfrac{dx}{x+3} + 5\int dx - 15\int \tfrac{dx}{x+3} + 7\ln(x+3)$$

$$= \tfrac{x^2}{2} - 3x + 9\ln(x+3) + 5x - 15\ln(x+3) + 7\ln(x+3)$$

$$= \tfrac{1}{2}x^2 + 2x + \ln(x+3) + C$$

1058

$$\int \tfrac{x^4+x^2+1}{x-1}dx = \int \tfrac{x^4}{x-1}dx + \int \tfrac{x^2}{x-1}dx + \int \tfrac{dx}{x-1} = \int \tfrac{x^4+1-1}{x-1}dx + \int \tfrac{x^2+1-1}{x-1}dx + \ln(x-1)$$

$$= \int \tfrac{x^4-1}{x-1}dx + \ln(x-1) + \int \tfrac{x^2-1}{x-1}dx + \ln(x-1) + \ln(x-1)$$

$$= 3\ln(x-1) + \int \tfrac{(x^2+1)(x+1)(x-1)}{x-1}dx + \int \tfrac{(x+1)(x-1)}{x-1}dx$$

$$= 3\ln(x-1) + \int (x+1)dx + \int (x^2+1)(x+1)dx$$

$$= 3\ln(x-1) + \tfrac{x^2}{2} + x + \int (x^3+x^2+x+1)dx = 3\ln(x-1) + \tfrac{x^2}{2} + x + \tfrac{x^4}{4} + \tfrac{x^3}{3} + \tfrac{x^2}{2} + x$$

$$= \tfrac{1}{4}x^4 + \tfrac{1}{3}x^3 + x^2 + 2x + 3\ln(x-1) + C$$

1059

$$\int \left(a + \tfrac{b}{x-a}\right)^2 dx = \int a^2dx + 2\int \tfrac{ba}{x-a}dx + \int \tfrac{b^2}{(x-a)^2}dx = a^2x + 2ab\ln(x-a) + b^2\int \tfrac{dx}{(x-a)^2}$$

$$\left\{\begin{array}{c}x-a = t|'\\ dx = dt\end{array}\right\}$$

$$= a^2x + 2ab\ln(x-a) + b^2\int \tfrac{dt}{t^2}$$

$$= xa^2 + 2ab\ln(x-a) - \tfrac{b^2}{t} = xa^2 + 2ab\ln(x-a) - \tfrac{b^2}{x-a} + C$$

1060*

$$\int \tfrac{x}{(x+1)^2}dx = \int \tfrac{x+1-1}{(x+1)^2}dx = \int \tfrac{dx}{x+1} - \int \tfrac{dx}{(x+1)^2} = \ln(x+1) - \int \tfrac{dx}{(x+1)^2}$$

$$\left\{\begin{array}{c}x+1 = t|'\\ dx = dt\end{array}\right\}$$

$$= \ln(x+1) - \int \tfrac{dt}{t^2} = \ln(x+1) + \tfrac{1}{t} = \ln(x+1) + \tfrac{1}{x+1} + C$$

1061

$$\int \tfrac{bdy}{\sqrt{1-y}} = b\int \tfrac{dy}{\sqrt{1-y}}$$

$$\left\{\begin{array}{c}1-y = t|'\\ -dy = dt\end{array}\right\}$$

$$= -b\int \tfrac{dt}{\sqrt{t}} = -2b\sqrt{t} = -2b\sqrt{1-y} + C$$

1062

$$\int \sqrt{a-bx}\,dx$$

$$\left\{\begin{array}{l} a-bx = t|' \\ -bdx = dt \end{array}\right\}$$

$$= \int \sqrt{t}\, \frac{dt}{-b} = -\frac{1}{b} \int \sqrt{t}\, dt = -\frac{2}{3b} t^{\frac{3}{2}} = -\frac{2}{3b}(a-bx)^{\frac{3}{2}} + C$$

1063**

$$\int \frac{x}{\sqrt{x^2+1}} dx =$$

$$\left\{\begin{array}{l} x^2+1 = t|' \\ 2xdx = dt \end{array}\right\}$$

$$= \int \frac{x}{\sqrt{t}} \frac{dt}{2x} = \frac{1}{2} \int \frac{dt}{\sqrt{t}} = \sqrt{t} = \sqrt{x^2+1} + C$$

1064

$$\int \frac{\sqrt{x}+\ln x}{x} dx = \int \frac{1}{\sqrt{x}} dx + \int \frac{1}{x} \ln x dx$$

$$\left\{\begin{array}{l} \ln x = t|' \\ \frac{dx}{x} = dt \end{array}\right\}$$

$$= 2\sqrt{x} + \int t dt = 2\sqrt{x} + \frac{1}{2} t^2 = 2\sqrt{x} + \frac{1}{2} \ln^2 x + C$$

1065

$$\int \frac{dx}{3x^2+5} = \frac{1}{3} \int \frac{dx}{x^2+\frac{5}{3}} = \frac{1}{3} * \frac{\sqrt{15}}{5} \arctan \frac{\sqrt{15}}{5} x = \frac{\sqrt{15}}{15} \arctan \frac{\sqrt{15}}{5} x + C$$

1066

$$\int \frac{dx}{7x^2-8} = \frac{1}{7} \int \frac{dx}{x^2-\frac{8}{7}} = \frac{1}{7}\left(-\frac{1}{4}\sqrt{14} \operatorname{arctanh} \frac{1}{4} x \sqrt{14}\right) = -\frac{\sqrt{14}}{28} \operatorname{arctanh} \frac{\sqrt{14}}{4} x + C$$

1067

$$\int \frac{dx}{(a+b)-(a-b)x^2} = \frac{1}{(a-b)} \int \frac{dx}{\frac{(a+b)}{(a-b)}-x^2} = \frac{1}{(a-b)} \frac{1}{\sqrt{\frac{(a+b)}{(a-b)}}} \operatorname{arctanh} \frac{x}{\sqrt{\frac{a+b}{a-b}}}$$

$$= \frac{\sqrt{a-b}}{(a-b)\sqrt{a+b}} \operatorname{arctanh} \frac{x\sqrt{a-b}}{\sqrt{a+b}} \frac{\sqrt{a+b}}{\sqrt{a+b}} = \frac{1}{\sqrt{a^2-b^2}} \operatorname{arctanh} \frac{x\sqrt{a^2-b^2}}{a+b} + C$$

1068

$$\int \frac{x^2}{x^2+2} dx = \int \frac{x^2+2-2}{x^2+2} dx = \int dx - 2\int \frac{dx}{x^2+2} = x - \sqrt{2} \arctan \frac{\sqrt{2}}{2} x + C$$

1069

$$\int \frac{x^3}{a^2-x^2} dx = -\int \frac{-x^3}{(a+x)(a-x)} dx = -\int \frac{-x^3+a^3-a^3}{(a+x)(a-x)} dx = -\left[\int \frac{a^3-x^3}{(a+x)(a-x)} dx - \int \frac{a^3}{a^2-x^2} dx\right]$$

$$= -\left[\int \frac{(a-x)(a^2+ax+x^2)}{(a+x)(a-x)} dx - a^3 \frac{1}{a} \operatorname{arctanh} \frac{x}{a}\right] = -\left[\int \frac{(a^2+ax+x^2)}{(a+x)} - a^3 \frac{1}{a} \operatorname{arctanh} \frac{x}{a}\right]$$

$$= a^3 \frac{1}{a} \operatorname{arctanh} \frac{x}{a} - \int \frac{(a^2+ax+x^2)}{(a+x)} dx = a^3 \frac{1}{a} \operatorname{arctanh} \frac{x}{a} - \int \frac{a^2}{a+x} dx - \int \frac{x(x+a)}{(a+x)} dx$$

$$= a^3 \frac{1}{a} \operatorname{arctanh} \frac{x}{a} - a^2 \ln(a+x) - \int x dx = -\frac{1}{2} x^2 - a^2 \ln(a+x) + a^2 \operatorname{arctanh} \frac{x}{a} + C$$

1070

$$\int \frac{x^2-5x+6}{x^2+4} dx = \int \frac{x^2-5x+4+2}{x^2+4} dx = \int dx - 5\int \frac{xdx}{x^2+4} + 2\int \frac{dx}{x^2+4} =$$

$$x - \frac{5}{2} \ln(x^2+4) + \arctan \frac{1}{2} x + C$$

1071

$$\int \frac{dx}{\sqrt{7+8x^2}} = \frac{1}{\sqrt{8}} \int \frac{dx}{\sqrt{\frac{7}{8}+x^2}} = \frac{1}{2\sqrt{2}} \frac{\sqrt{2}}{\sqrt{2}} \operatorname{arcsinh} \frac{2\sqrt{14}}{7} x = \frac{\sqrt{2}}{4} \operatorname{arcsinh} \frac{2\sqrt{14}}{7} x + C$$

1072

$$\int \frac{dx}{\sqrt{7-5x^2}} = \frac{\sqrt{5}}{5} \int \frac{dx}{\sqrt{\frac{7}{5}-x^2}} = \frac{\sqrt{5}}{5} \arcsin \frac{\sqrt{35}}{7} x + C$$

1073

$$\int \frac{2x-5}{3x^2-2} dx = \frac{1}{3} \int \frac{2x-5}{x^2-\frac{2}{3}} dx = \frac{1}{3}\left[\int \frac{2x}{x^2-\frac{2}{3}} dx - 5\int \frac{dx}{x^2-\frac{2}{3}}\right]$$

$$\left\{\begin{array}{l} x^2-\frac{2}{3} = t|' \\ 2xdx = dt \end{array}\right\}$$

$$= \frac{1}{3}\left[\int \frac{dt}{t} - 5\left(-\frac{\sqrt{6}}{2} \operatorname{arctanh} \frac{\sqrt{6}}{2} x\right)\right] = \frac{1}{3} \ln t + \frac{5}{6}\sqrt{6} \operatorname{arctanh} \frac{1}{2} x\sqrt{6}$$

$$= \frac{1}{3} \ln\left(x^2 - \frac{2}{3}\right) + \frac{5}{6}\sqrt{6} \operatorname{arctanh} \frac{1}{2} x\sqrt{6} + C$$

1074

$$\int \frac{3-2x}{5x^2+7} dx = \frac{1}{5} \int \frac{3-2x}{x^2+\frac{7}{5}} dx = \frac{3}{5} \int \frac{dx}{x^2+\frac{7}{5}} - \frac{1}{5} \int \frac{2xdx}{x^2+\frac{7}{5}} = \frac{3\sqrt{35}}{35} \arctan \frac{\sqrt{35}}{7} x - \frac{1}{5} \int \frac{2xdx}{x^2+\frac{7}{5}}$$

$$\left\{\begin{array}{l} x^2+\frac{7}{5} = t|' \\ 2xdx = dt \end{array}\right\}$$

$$= \frac{3\sqrt{35}}{35} \arctan \frac{\sqrt{35}}{7} x - \frac{1}{5} \int \frac{dt}{t} = \frac{3\sqrt{35}}{35} \arctan \frac{\sqrt{35}}{7} x - \frac{1}{5} \ln t$$

$$= \frac{3\sqrt{35}}{35} \arctan \frac{\sqrt{35}}{7} x - \frac{1}{5} \ln\left(x^2 + \frac{7}{5}\right) + C$$

1075

$$\int \frac{3x+1}{\sqrt{5x^2+1}} dx = \frac{\sqrt{5}}{5} \int \frac{3x+1}{\sqrt{x^2+\frac{1}{5}}} dx = \frac{3\sqrt{5}}{5} \int \frac{x}{\sqrt{x^2+\frac{1}{5}}} dx + \frac{\sqrt{5}}{5} \int \frac{dx}{\sqrt{x^2+\frac{1}{5}}}$$

$$\left\{\begin{array}{l} x^2+\frac{1}{5} = t|' \\ 2xdx = dt \end{array}\right\}$$

$$= \frac{3\sqrt{5}}{5} \int \frac{\frac{dt}{2}}{\sqrt{t}} + \frac{\sqrt{5}}{5} \operatorname{arcsinh} \sqrt{5} x = \frac{3\sqrt{5}}{10} \int \frac{dt}{\sqrt{t}} + \frac{\sqrt{5}}{5} \operatorname{arcsinh} \sqrt{5} x$$

$$= \frac{3\sqrt{5}}{10} * 2\sqrt{t} + \frac{\sqrt{5}}{5} \operatorname{arcsinh} \sqrt{5} x = \frac{3\sqrt{5}}{5} \sqrt{x^2+\frac{1}{5}} + \frac{\sqrt{5}}{5} \operatorname{arcsinh} x\sqrt{5} + C$$

1076

$$\int \frac{x+3}{\sqrt{x^2-4}} dx = \int \frac{xdx}{\sqrt{(x^2-4)}} + 3\int \frac{dx}{\sqrt{(x^2-4)}}$$

$$\left\{\begin{array}{l} x^2-4 = t|' \\ 2xdx = dt \end{array}\right\}$$

$$= \int \frac{\frac{dt}{2}}{\sqrt{t}} + 3\ln\left(x + \sqrt{(x^2-4)}\right) = \frac{1}{2} \int \frac{dt}{\sqrt{t}} + 3\ln\left(x + \sqrt{(x^2-4)}\right)$$

$$= \frac{1}{2} 2\sqrt{t} + 3\ln\left(x + \sqrt{(x^2-4)}\right) = \sqrt{x^2-4} + 3\ln\left(x + \sqrt{(x^2-4)}\right) + C$$

1077

$$\int \frac{xdx}{x^2-5}$$

$$\left\{ \begin{array}{c} x^2 - 5 = t|' \\ 2xdx = dt \end{array} \right\}$$

$$= \int \frac{\frac{dt}{2}}{t} = \frac{1}{2} \int \frac{dt}{t} = \frac{1}{2} \ln t = \frac{1}{2} \ln(x^2 - 5) + C$$

1078

$$\int \frac{xdx}{2x^2+3}$$

$$\left\{ \begin{array}{c} 2x^2 + 3 = t \\ 4xdx = dt \end{array} \right\}$$

$$= \int \frac{\frac{dt}{4}}{t} = \frac{1}{4} \int \frac{dt}{t} = \frac{1}{4} \ln t = \frac{1}{4} \ln(2x^2 + 3) + C$$

1079

$$\int \frac{ax+b}{a^2x^2+b^2} dx \quad (a > 0) = \frac{1}{a} \int \frac{ax+b}{ax^2+\frac{b^2}{a}} dx = \frac{1}{a} \int \frac{ax}{ax^2+\frac{b^2}{a}} dx + \frac{b}{a^2} \int \frac{dx}{x^2+\frac{b^2}{a^2}}$$

$$= \frac{1}{a} \int \frac{ax}{ax^2+\frac{b^2}{a}} dx + \frac{b}{a^2} \int \frac{dx}{x^2+\frac{b^2}{a^2}}$$

$$\left\{ \begin{array}{c} ax^2 + \frac{b^2}{a} = t|' \\ 2axdx = dt \end{array} \right\}$$

$$= \frac{1}{a} \int \frac{\frac{dt}{2}}{t} dx + \frac{b}{a^2} * \frac{a}{b} \arctan x \frac{a}{b} = \frac{1}{2a} \int \frac{dt}{t} + \frac{1}{a} \arctan \frac{a}{b} x$$

$$= \frac{1}{2a} \ln(ax^2 + \frac{b^2}{a}) + \frac{1}{a} \arctan \frac{a}{b} x + C$$

1080

$$\int \frac{xdx}{\sqrt{a^4-x^4}}$$

$$\left\{ \begin{array}{c} x^4 = t^2|^{\frac{1}{2}} \\ x^2 = t|' \\ 2xdx = dt \end{array} \right\}$$

$$= \int \frac{\frac{dt}{2}}{\sqrt{a^4-t^2}} = \frac{1}{2} \int \frac{dt}{\sqrt{a^4-t^2}} = \frac{1}{2} \arctan \frac{t}{\sqrt{(a^4-t^2)}} = \frac{1}{2} \arctan \frac{x^2}{\sqrt{(a^4-x^4)}} + C$$

1081

$$\int \frac{x^2}{1+x^6} dx$$

$$\left\{ \begin{array}{c} x^6 = t^2|^{\frac{1}{2}} \\ x^3 = t|' \\ 3x^2dx = dt \end{array} \right\}$$

$$= \int \frac{\frac{dt}{3}}{1+t^2} = \frac{1}{3} \int \frac{dt}{1+t^2} = \frac{1}{3} \arctan t = \frac{1}{3} \arctan x^3 + C$$

1082

$$\int \frac{x^2dx}{\sqrt{x^6-1}}$$

$$\left\{ \begin{array}{c} x^6 = t^2|^{\frac{1}{2}} \\ x^3 = t|' \\ 3x^2dx = dt \end{array} \right\}$$

$$= \int \frac{\frac{dt}{3}}{\sqrt{t^2-1}} = \frac{1}{3} \int \frac{dt}{\sqrt{t^2-1}} = \frac{1}{3} \ln\left(t + \sqrt{t^2 - 1}\right) = \frac{1}{3} \ln\left(x^3 + \sqrt{(x^6 - 1)}\right) + C$$

1083

$$\int \sqrt{\frac{\arcsin x}{1-x^2}} dx = \left\{ \begin{array}{c} \arcsin x = u|' \\ \frac{dx}{\sqrt{1-x^2}} = du \end{array} \right\} = \int \sqrt{u}\, du = \frac{2}{3} u^{\frac{3}{2}} = \frac{2}{3} \arcsin x^{\frac{3}{2}} + C$$

1084

$$\int \frac{\arctan \frac{x}{2}}{x^2+4}$$

$$\left\{ \begin{array}{c} \arctan \frac{x}{2} = u|' \\ \frac{2dx}{x^2+4} = du \end{array} \right\}$$

$$= \frac{1}{2} \int udu = \frac{u^2}{4} = \frac{1}{4} \arctan^2 \frac{x}{2} + C$$

1085

$$\int \frac{x-\sqrt{\arctan 2x}}{1+4x^2} dx = \int \frac{x}{1+4x^2} dx - \int \frac{\sqrt{\arctan 2x}}{1+4x^2} dx$$

$$\left\{ \begin{array}{c} 1 + 4x^2 = u|' \\ 8xdx = du \end{array} \right\} \left\{ \begin{array}{c} = t \\ \frac{2dx}{1+4x^2} = dt \end{array} \right\}$$

$$= \int \frac{\frac{du}{8}}{u} - \int \sqrt{t}\, \frac{dt}{2} = \frac{1}{8} \ln u - \frac{1}{2} * \frac{2}{3} t^{\frac{3}{2}} = \frac{1}{8} \ln u - \frac{1}{3} t^{\frac{3}{2}} =$$

$$\frac{1}{8} \ln(1 + 4x^2) - \frac{1}{3} \arctan^{\frac{3}{2}} 2x + C$$

1086

$$\int \frac{dx}{\sqrt{(1+x^2) \ln(x+\sqrt{1+x^2})}}$$

$$\left\{ \begin{array}{c} \ln(x + \sqrt{1 + x^2}) = u|' \\ \frac{dx}{\sqrt{(x^2+1)}} = du \end{array} \right\}$$

$$= \int \frac{du}{\sqrt{u}} = 2\sqrt{u} = 2\sqrt{\ln(x + \sqrt{1 + x^2})} + C$$

1087

$$\int ae^{-mx} dx$$

$$\left\{ \begin{array}{c} -mx = u|' \\ -mdx = du \end{array} \right\}$$

$$= \int ae^u(-\frac{du}{m}) = -\frac{a}{m} \int e^u du = -\frac{a}{m} e^u = -\frac{a}{m} e^{-mx} + C$$

1088

$$\int 4^{2-3x} dx =$$

$$\left\{\begin{array}{c} 2-3x = t|' \\ -3dx = dt \end{array}\right\}\left\{\begin{array}{c} 4^t = u|' \\ 4^t\ln 4dt = du \end{array}\right\}$$

$$= \int 4^t \tfrac{dt}{-3} = -\tfrac{1}{3}\int 4^t dt$$

$$= -\tfrac{1}{3}\int 4^t \tfrac{du}{4^t \ln 4} = -\tfrac{1}{3\ln 4}\int du = -\tfrac{1}{3\ln 4}u = -\tfrac{4^t}{3\ln 4} = -\tfrac{1}{3\ln 4}4^{2-3x} + C$$

1089

$$\int(e^t - e^{-t})dt = 2\int \tfrac{e^t - e^{-t}}{2}dt = 2\int \sinh t dt = 2\cosh t + C$$

$$\int(e^t - e^{-t})dt = \int e^t dt - \int e^{-t}dt = e^t + e^{-t} + C$$

1090

$$\int(e^{\frac{x}{a}} + e^{-\frac{x}{a}})^2 dx = \int e^{2\frac{x}{a}}dx + 2\int dx + \int \tfrac{dx}{e^{2\frac{x}{a}}}$$

$$\left\{\begin{array}{c} 2\frac{x}{a} = t|' \\ \frac{2dx}{a} = dt \end{array}\right\}$$

$$= \int e^t \tfrac{a}{2}dt + 2x + \int \tfrac{a}{2}e^{-t}dt = \tfrac{a}{2}\int e^t dt + 2x + \tfrac{a}{2}\int e^{-t}dt$$

$$= \tfrac{1}{2}ae^{2\frac{x}{a}} + 2x - \tfrac{1}{2}ae^{-2\frac{x}{a}} + C$$

1091

$$\int \tfrac{(a^x - b^x)^2}{a^x b^x}dx = \int \tfrac{a^{2x} - 2a^x b^x + b^{2x}}{a^x b^x}dx = \int \tfrac{a^x}{b^x}dx - \int 2dx + \int \tfrac{b^x}{a^x}dx$$

$$= \int \left(\tfrac{a}{b}\right)^x dx - 2x + \int\left(\tfrac{b}{a}\right)^x dx = \frac{\left(\frac{a}{b}\right)^x}{\ln\frac{a}{b}} - 2x + \frac{\left(\frac{b}{a}\right)^x}{\ln\frac{b}{a}} + C$$

1092

$$\int \tfrac{a^{2x}-1}{\sqrt{a^x}}dx = \int a^{2x-\frac{1}{2}x}dx - \int a^{-\frac{1}{2}x}dx = \int a^{\frac{3}{2}x}dx - \int a^{-\frac{1}{2}x}dx = \int a^u \tfrac{2}{3}du - \int a^t(-2)dt$$

$$\left\{\begin{array}{cc} \frac{3}{2}x = u|' & -\frac{1}{2}x = t|' \\ \frac{3}{2}dx = du & -\frac{dx}{2} = dt \end{array}\right\}$$

$$= \tfrac{2}{3\ln a}a^u + \tfrac{2}{\ln a}a^t = \tfrac{2}{\ln a}\left(\tfrac{a^{\frac{3}{2}x}}{3} + a^{-\frac{1}{2}x}\right) + C$$

1093

$$\int(e^{-(x^2+1)})xdx = e^{-x^2-1}xdx$$

$$\left\{\begin{array}{c} -x^2 - 1 = t|' \\ -2xdx = dt \end{array}\right\}$$

$$= \int e^t \tfrac{dt}{-2} = -\tfrac{1}{2}\int e^t dt = -\tfrac{1}{2}e^t$$

$$= -\tfrac{1}{2}e^{-x^2-1} + C$$

1094

$$\int x7^{x^2}dx$$

$$\left\{\begin{array}{c} x^2 = t|' \\ 2xdx = dt \end{array}\right\}$$

$$= \int 7^t \tfrac{dt}{2} = \tfrac{1}{2}\int 7^t dt = \tfrac{1}{2\ln 7}7^t = \tfrac{1}{2\ln 7}7^{x^2} + C$$

1095

$$\int \tfrac{e^{\frac{1}{x}}}{x^2}dx$$

$$\left\{\begin{array}{c} \frac{1}{x} = t|' \\ -\frac{dx}{x^2} = dt \end{array}\right\}$$

$$= -\int e^t dt = -e^t = -e^{\frac{1}{x}} + C$$

1096

$$\int 5^{\sqrt{x}}\tfrac{dx}{\sqrt{x}}$$

$$\left\{\begin{array}{c} \sqrt{x} = t|' \\ \frac{dx}{2\sqrt{x}} = dt \end{array}\right\}$$

$$= \int 5^t 2dt = 2\int 5^t dt = \tfrac{2}{\ln 5}5^t = \tfrac{2}{\ln 5}5^{\sqrt{x}} + C$$

1097

$$\int \tfrac{e^x}{e^x - 1}dx$$

$$\left\{\begin{array}{c} e^x - 1 = t|' \\ e^x dx = dt \end{array}\right\}$$

$$= \int \tfrac{dt}{t} = \ln t = \ln(e^x - 1) + C$$

1098

$$\int e^x\sqrt{a - be^x}\,dx$$

$$\left\{\begin{array}{c} a - be^x = t|' \\ -be^x dx = dt \end{array}\right\}$$

$$= \int \sqrt{t}\tfrac{dt}{-b} = -\tfrac{1}{b}\int \sqrt{t}\,dt = -\tfrac{2}{3b}t^{\frac{3}{2}} = -\tfrac{2}{3b}(a - be^x)^{\frac{3}{2}} + C$$

1099

$$\int(e^{\frac{x}{a}} + 1)^{\frac{1}{3}}e^{\frac{x}{a}}dx$$

$$\left\{\begin{array}{c} e^{\frac{x}{a}} + 1 = t|' \\ \frac{1}{a}e^{\frac{x}{a}}dx = dt \end{array}\right\}$$

$$= \int t^{\frac{1}{3}}adt = a\int t^{\frac{1}{3}}dt = \tfrac{3}{4}at^{\frac{4}{3}} = \tfrac{3}{4}a(e^{\frac{x}{a}} + 1)^{\frac{4}{3}} + C$$

1100*

$$\int \tfrac{dx}{2^x + 3}$$

$$\left\{\begin{array}{c} 2^x + 3 = t|' \\ 2^x\ln 2dx = dt \end{array}\right\}$$

$$= \int \frac{\frac{dt}{2^x\ln 2}}{t} = \frac{1}{\ln 2}\int \frac{dt}{2^x t} = \frac{1}{\ln 2}\int \frac{dt}{(t-3)t} = \frac{1}{\ln 2}\int \frac{dt}{t^2 - 3t} = \frac{1}{\ln 2}\int \frac{dt}{(t-\frac{3}{2})^2 - \frac{9}{4}}$$

$$= \frac{1}{\ln 2} * \frac{1}{\sqrt{\frac{9}{4}}}\operatorname{arctanh}\frac{t-\frac{3}{2}}{\sqrt{\frac{9}{4}}} = \frac{2}{3\ln 2}\operatorname{arctanh}\left(\frac{2}{3}t - 1\right) =$$

$\frac{2}{3\ln 2}\operatorname{arctanh}\left(\frac{2}{3}(2^x+3)-1\right)+C$

1101

$\int \frac{a^x dx}{1+a^{2x}}$

$\left\{\begin{array}{r} a^x = u \\ a^x \ln a dx = du \end{array}\right\}$

$= \int \frac{du}{\ln a(1+u^2)} = \frac{1}{\ln a}\int \frac{du}{1+u^2} = \frac{1}{\ln a}\arctan u = \frac{1}{\ln a}\arctan a^x + C$

1102

$\int \frac{e^{-bx}}{1-e^{-2bx}}dx$

$\left\{\begin{array}{r} e^{-bx} = u \\ -be^{-bx}dx = du \end{array}\right\}$

$= \int \frac{\frac{du}{-b}}{1-u^2} = -\frac{1}{b}\int \frac{du}{1-u^2} = -\frac{1}{b}\operatorname{arctanh} u = -\frac{1}{b}\operatorname{arctanh} e^{-bx} + C$

1103

$\int \frac{e^t}{\sqrt{1-e^{2t}}}dt$

$\left\{\begin{array}{r} e^t = u \\ e^t dt = du \end{array}\right\}$

$= \int \frac{du}{\sqrt{1-u^2}} = \arcsin u = \arcsin e^t + C$

1104

$\int \sin(a+bx)dx$

$\left\{\begin{array}{r} a+bx = z \\ bdx = dz \end{array}\right\}$

$= \int \sin z \frac{dz}{b} = \frac{1}{b}\int \sin z dz = -\frac{1}{b}\cos z = -\frac{1}{b}\cos(a+bx) + C$

1105

$\int \cos \frac{x}{\sqrt{2}}dx$

$\left\{\begin{array}{r} \frac{x}{\sqrt{2}} = z \\ \frac{dx}{\sqrt{2}} = dz \end{array}\right\}$

$= \int \cos z \sqrt{2}\,dz = \sqrt{2}\int \cos z dz = \sqrt{2}\sin z = \sqrt{2}\sin(\frac{x}{\sqrt{2}}) + C$

1106

$\int(\cos ax + \sin ax)^2 dx = \int(\sin^2 ax + 2\sin ax \cos ax + \cos^2 ax)dx = \int(1 + 2\sin ax \cos ax)dx$

$= \int(1+\sin 2ax)dx = x + \int \sin 2ax dx$

$\left\{\begin{array}{r} 2ax = z \\ 2adx = dz \end{array}\right\}$

$= x + \int \sin z \frac{dz}{2a} = x + \frac{1}{2a}\int \sin z dz = x - \frac{1}{2a}\cos z = x - \frac{1}{2a}\cos 2ax + C$

1107

$\int \cos\sqrt{x}\,\frac{dx}{\sqrt{x}}$

$\left\{\begin{array}{r} \sqrt{x} = z \\ \frac{dx}{2\sqrt{x}} = dz \end{array}\right\}$

$= \int \cos z 2dz = 2\int \cos z dz = 2\sin z = 2\sin\sqrt{x} + C$

1108

$\int \sin(\lg x)\frac{dx}{x} = \int \sin(\frac{\ln x}{\ln 2})\frac{dx}{x}$

$\left\{\begin{array}{r} \frac{\ln x}{\ln 2} = z|' \\ \frac{dx}{x\ln 2} = dz \end{array}\right\}$

$= \int \sin(z)\ln 2dz = \ln 2\int \sin z dz$

$= -\ln 2\cos z = -\ln 2\cos\frac{\ln x}{\ln 2} = -\ln 2\cos\lg x + C$

1109

$\int \sin^2 x dx = \int \frac{1-\cos 2x}{2} = \frac{1}{2}\int dx - \frac{1}{2}\int \cos 2x dx$

$\left\{\begin{array}{r} 2x = z|' \\ 2dx = dz \end{array}\right\}$

$= \frac{x}{2} - \frac{1}{2}\int \cos z \frac{1}{2}dz = \frac{x}{2} - \frac{1}{4}\int \cos z dz = \frac{1}{2}x - \frac{1}{4}\sin z = \frac{1}{2}x - \frac{1}{4}\sin 2x + C$

1110*

$\int \cos^2 x dx = \int(\sqrt{\frac{1}{2}(1+\cos 2x)})^2 dx = \int \frac{1}{2}dx + \int \frac{1}{2}\cos 2x dx = \frac{1}{2}x + \frac{1}{2}\int \cos 2x dx$

$\left\{\begin{array}{r} 2x = z|' \\ 2dx = dz \end{array}\right\}$

$= \frac{1}{2}x + \frac{1}{2}\int \cos z \frac{dz}{2} = \frac{1}{2}x + \frac{1}{4}\int \cos z dz = \frac{1}{2}x + \frac{1}{4}\sin z = \frac{1}{2}x + \frac{1}{4}\sin 2x + C$

1111

$\int \sec^2(ax+b)dx = \int \frac{dx}{\cos^2(ax+b)}$

$\left\{\begin{array}{r} ax+b = t \\ adx = dt \end{array}\right\}$

$\int \frac{\frac{dt}{a}}{\cos^2 t} = \frac{1}{a}\int \frac{dt}{\cos^2 t} = \frac{1}{a}\tan t = \frac{1}{a}\tan(ax+b) + C$

1112

$\int \cot^2 ax dx = \int \frac{\cos^2 ax}{\sin^2 ax}dx = \int \frac{1-\sin^2 ax}{\sin^2 ax}dx = \int \frac{dx}{\sin^2 ax} - \int dx$

$\left\{\begin{array}{r} ax = t \\ adx = dt \end{array}\right\}$

$= \int \frac{\frac{dt}{a}}{\sin^2 t} - x = \frac{1}{a}\int \frac{dt}{\sin^2 t} - x = -\frac{1}{a}\cot t - x = -\frac{1}{a}\cot(ax) - x + C$

1113

$$\int \frac{dx}{\sin\frac{x}{a}}$$
$$\left\{ \begin{array}{l} \frac{x}{a} = t \\ \frac{dx}{a} = dt \end{array} \right\}$$
$$\int \frac{adt}{\sin t} = a\int \frac{dt}{\sin t} = a\ln\tan\frac{t}{2} = a\ln\tan\frac{\frac{x}{a}}{2} = a\ln(\tan\frac{x}{2a}) + C$$

1114

$$\int \frac{dx}{3\cos(5x-\frac{\pi}{4})}$$
$$\left\{ \begin{array}{l} 5x - \frac{\pi}{4} = t \\ 5dx = dt \end{array} \right\}$$
$$= \int \frac{\frac{dt}{5}}{3\cos t} = \frac{1}{15}\int \frac{dt}{\cos t} = \frac{1}{15}\ln\tan(\frac{t}{2} + \frac{\pi}{4})$$
$$= \frac{1}{15}\ln(\tan(\frac{1}{2}(5x - \frac{\pi}{4}) + \frac{1}{4}\pi)) = \frac{1}{15}\ln\left(\tan\left(\frac{5}{2}x + \frac{1}{8}\pi\right)\right) + C$$

1115

$$\int \frac{dx}{\sin(ax+b)}$$
$$\left\{ \begin{array}{l} ax + b = t \\ adx = dt \end{array} \right\}$$
$$= \int \frac{\frac{dt}{a}}{\sin t} = \frac{1}{a}\int \frac{dt}{\sin t} = \frac{1}{a}\ln\tan\frac{t}{2} = \frac{1}{a}\ln\tan\frac{ax+b}{2} + C$$

1116

$$\int \frac{xdx}{\cos^2 x^2}$$
$$\left\{ \begin{array}{l} x^2 = t \\ 2xdx = dt \end{array} \right\}$$
$$= \int \frac{\frac{dt}{2}}{\cos^2 t} = \frac{1}{2}\int \frac{dt}{\cos^2 t} = \frac{1}{2}\tan t = \frac{1}{2}\tan x^2 + C$$

1117

$$\int x\sin(1-x^2)dx$$
$$\left\{ \begin{array}{l} 1 - x^2 = t \\ -2xdx = dt \end{array} \right\}$$
$$= -\int \sin t\frac{dt}{2} = -\frac{1}{2}\int \sin tdt = \frac{1}{2}\cos t = \frac{1}{2}\cos(1-x^2) + C$$

1118

$$\int\left(\frac{1}{\sin x\sqrt{2}} - 1\right)^2 dx = \int\left(\frac{1}{\sin^2 x\sqrt{2}} - \frac{2}{\sin x\sqrt{2}} + 1\right)dx = x - 2\int \frac{dx}{\sin x\sqrt{2}} + \int \frac{dx}{\sin^2 x\sqrt{2}}$$
$$\left\{ \begin{array}{l} x\sqrt{2} = t \\ \sqrt{2}\,dx = dt \end{array} \right\}$$
$$= x - 2\int \frac{\frac{dt}{\sqrt{2}}}{\sin t} + \int \frac{\frac{dt}{\sqrt{2}}}{\sin^2 t} == x - \sqrt{2}\int \frac{dt}{\sin t} + \frac{\sqrt{2}}{2}\int \frac{dt}{\sin^2 t} = x - \sqrt{2}\ln\tan\frac{t}{2} - \frac{\sqrt{2}}{2}\cot t$$
$$= x - \sqrt{2}\ln\left(\tan\frac{x\sqrt{2}}{2}\right) - \frac{\sqrt{2}}{2}\cot x\sqrt{2} + C$$

1119

$$\int \tan xdx = \int \frac{\sin x}{\cos x}dx = \int \frac{\sqrt{1-\cos 2x}}{\sqrt{1+\cos 2x}}dx = \int \frac{\sqrt{1-\cos 2x}}{\sqrt{1+\cos 2x}}\frac{\sqrt{1+\cos 2x}}{\sqrt{1+\cos 2x}}dx =$$
$$= \int \frac{\sqrt{1-\cos^2 2x}}{1+\cos 2x}dx = \int \frac{\sqrt{\sin^2 2x}}{1+\cos 2x}dx = \int \frac{\sin 2x}{1+\cos 2x}dx$$
$$\left\{ \begin{array}{l} 1 + \cos 2x = t|' \\ -2\sin 2xdx = dt \end{array} \right\}$$
$$= \int \frac{-\frac{dt}{2}}{t}dx = -\frac{1}{2}\ln t = -\frac{1}{2}\ln(1 + \cos 2x) + C$$

1120

$$\int \cot xdx = \int \frac{\cos x}{\sin x}dx = \int \sqrt{\frac{\frac{1}{2}(1+\cos 2x)}{\frac{1}{2}(1-\cos 2x)}}dx = \int \frac{\sqrt{1+\cos 2x}}{\sqrt{1-\cos 2x}}dx = \int \frac{\sqrt{1+\cos 2x}}{\sqrt{1-\cos 2x}}\frac{\sqrt{1-\cos 2x}}{\sqrt{1-\cos 2x}}dx = \int \frac{\sqrt{1-\cos^2 2x}}{1-\cos 2x}$$
$$\int \frac{\sin 2x}{1-\cos 2x}dx$$
$$\left\{ \begin{array}{l} 1 - \cos 2x = t|' \\ 2\sin 2xdx = dt \end{array} \right\}$$
$$\int \frac{\frac{dt}{2}}{t} = \frac{1}{2}\ln t = \frac{1}{2}\ln(1 - \cos 2x) + C$$

1121

$$\int \cot\frac{x}{a-b}dx$$
$$\left\{ \begin{array}{l} \frac{x}{a-b} = u|' \\ \frac{dx}{a-b} = du \end{array} \right\}$$
$$= \int \cot u(a-b)du = (a-b)\int \cot udu = (a-b)\ln(\sin u) = (a-b)\ln\left(\sin\left(\frac{x}{a-b}\right)\right) + C$$

1122

$$\int \frac{dx}{\tan\frac{x}{5}} = \int \cot\frac{x}{5}dx$$
$$\left\{ \begin{array}{l} \frac{x}{5} = u|' \\ \frac{dx}{5} = du \end{array} \right\}$$
$$= \int \cot u5du = 5\int \frac{\cos u}{\sin u}du$$
$$\left\{ \begin{array}{l} \sin u = t|' \\ \cos udu = dt \end{array} \right\}$$
$$= 5\int \frac{dt}{t} = 5\ln t = -5\ln(\sin u) = 5\ln\sin\frac{x}{5} + C$$

1123

$$\int \tan\sqrt{x}\frac{dx}{\sqrt{x}}$$
$$\left\{ \begin{array}{l} \sqrt{x} = u|' \\ \frac{dx}{2\sqrt{x}} = du \end{array} \right\}$$
$$= 2\int \tan udu = 2\int \tan u = 2\int \frac{\sin u}{\cos u}du$$

$$\left\{ \begin{array}{c} \cos u = t|' \\ -\sin u du = dt \end{array} \right\}$$

$= -2\int \frac{dt}{t} = -2\ln t = -2\ln(\cos u) = -2\ln\cos\sqrt{x} + C$

1124

$\int x\cot(x^2+1)dx$

$$\left\{ \begin{array}{c} x^2+1 = u \\ 2xdx = du \end{array} \right\}$$

$= \int \cot u \frac{du}{2} = \frac{1}{2}\int \cot u du = \frac{1}{2}\ln(\sin u) = \frac{1}{2}\ln\sin(x^2+1) + C$

1125

$\int \frac{dx}{\sin x\cos x} = \int \frac{dx}{\frac{\sin 2x}{2}} = 2\int \frac{dx}{\sin 2x}$

$$\left\{ \begin{array}{c} 2x = u \\ 2dx = du \end{array} \right\}$$

$= 2\int \frac{\frac{du}{2}}{\sin u} = \int \frac{du}{\sin u} = \ln\tan\frac{u}{2} = \ln(\tan x) + C$

1126

$\int \cos\frac{x}{2}\sin\frac{x}{2}dx$

$$\left\{ \begin{array}{c} \frac{x}{2} = u \\ \frac{dx}{2} = du \end{array} \right\}$$

$= 2\int \cos u\sin u du =$

$$\left\{ \begin{array}{c} \sin u = z \\ \cos u du = dz \end{array} \right\}$$

$= 2\int z dz = z^2 = \sin^2 u = \sin^2\frac{x}{2} + C$

1127

$\int \sin^3 6x\cos 6x dx$

$$\left\{ \begin{array}{c} 6x = t \\ 6dx = dt \end{array} \right\}$$

$= \int \sin^3 t\cos t\frac{dt}{6} = \frac{1}{6}\int \sin^3 t\cos t dt$

$$\left\{ \begin{array}{c} \sin t = z \\ \cos t dt = dz \end{array} \right\}$$

$= \frac{1}{6}\int z^3 dz = \frac{1}{24}z^4 = \frac{1}{24}(\sin t)^4 = \frac{1}{24}\sin^4(6x) + C$

1128

$\int \frac{\cos ax}{\sin^5 ax}dx$

$$\left\{ \begin{array}{c} ax = t \\ adx = dt \end{array} \right\}$$

$= \frac{1}{a}\int \frac{\cos t}{\sin^5 t}dt$

$$\left\{ \begin{array}{c} \sin t = z \\ \cos t dt = dz \end{array} \right\}$$

$\frac{1}{a}\int \frac{dz}{z^5} = -\frac{1}{4az^4} = -\frac{1}{4a\sin^4 t} = -\frac{1}{4a\sin^4 ax} + C$

1129

$\int \frac{\sin 3x}{3+\cos 3x}dx$

$$\left\{ \begin{array}{c} 3+\cos 3x = z|' \\ -3\sin 3x dx = dz \end{array} \right\}$$

$= \int \frac{\sin 3x}{z}\frac{dz}{-3\sin 3x} = -\frac{1}{3}\int \frac{dz}{z} = -\frac{1}{3}\ln z = -\frac{1}{3}\ln(3+\cos 3x) + C$

1130

$\int \frac{\sin x\cos x}{\sqrt{\cos^2 x-\sin^2 x}}dx = \int \frac{\frac{\sin 2x}{2}}{\sqrt{\cos 2x}}dx = \frac{1}{2}\int \frac{\sin 2x}{\sqrt{\cos 2x}}dx$

$$\left\{ \begin{array}{c} \cos 2x = t|' \\ -2\sin 2x dx = dt \end{array} \right\}$$

$= \frac{1}{2}\int \frac{\frac{dt}{-2}}{\sqrt{t}} = -\frac{1}{4}\int \frac{dt}{\sqrt{t}} = -\frac{1}{2}\sqrt{t} = -\frac{1}{2}\sqrt{\cos 2x} + C$

1131

$\int \sqrt{1+3\cos^2 x}\sin 2x dx = 2\int \sin x\cos x\sqrt{1+3\cos^2 x}dx$

$$\left\{ \begin{array}{c} \cos x = t|' \\ -\sin x dx = dt \end{array} \right\}$$

$= -2\int t\sqrt{1+3t^2}dt$

$$\left\{ \begin{array}{c} 1+3t^2 = z \\ 6t dt = dz \end{array} \right\}$$

$= -\frac{2}{6}\int \sqrt{z}dz = -\frac{2}{9}z^{\frac{3}{2}} = -\frac{2}{9}(1+3t^2)^{\frac{3}{2}} = -\frac{2}{9}(1+3\cos^2 x)^{\frac{3}{2}} + C$

1132

$\int \tan^3\frac{x}{3}\sec^2\frac{x}{3}dx = \int \frac{\sin^3\frac{x}{3}}{\cos^3\frac{x}{3}}\frac{1}{\cos^2\frac{x}{3}}dx = \int \frac{\sin^3\frac{1}{3}x}{\cos^5\frac{1}{3}x}dx$

$$\left\{ \begin{array}{c} \cos\frac{x}{3} = t|' \\ -\frac{1}{3}\sin\frac{x}{3} = dt \end{array} \right\}$$

$= -3\int \frac{\sin^2\frac{x}{3}dt}{t^5} = -3\int \frac{(1-t^2)}{t^5}dt = -3\int \frac{1}{t^5}dt + 3\int \frac{1}{t^3}dt =$

$3\frac{1}{4t^4} - 3\frac{1}{2t^2} = \frac{3}{4\cos^4\frac{x}{3}} - \frac{3}{2\cos^2\frac{x}{3}} + C$

1133

$$\int \frac{\sqrt{\tan x}}{\cos^2 x} dx$$

$$\left\{ \begin{array}{c} \tan x = t \\ \frac{dx}{\cos^2 x} = dt \end{array} \right\}$$

$$= \int \sqrt{t}\, dt = \frac{2}{3} t^{\frac{3}{2}} = \frac{2}{3} \tan^{\frac{3}{2}} x + C$$

1134

$$a) \int \frac{\cot^{\frac{2}{3}} x}{\sin^2 x} dx = \int \frac{\frac{\cos^{\frac{2}{3}} x}{\sin^{\frac{2}{3}} x}}{\sin^2 x} dx = \int \frac{\cos^{\frac{2}{3}} x}{\sin^{\frac{8}{3}} x} dx$$

$$\left\{ \begin{array}{c} \sin^{\frac{5}{3}} x = t|' \\ \frac{5}{3} \cos^{\frac{2}{3}} x dx = dt \end{array} \right\}$$

$$= \int \frac{\cos^{\frac{2}{3}} x}{\sin^{\frac{8}{3}} x} \frac{dt}{\frac{5}{3} \cos^{\frac{2}{3}} x} = \frac{3}{5} \int \frac{dt}{\sin^{\frac{8}{3}} x} = \frac{3}{5} \int \frac{dt}{\sin^{\frac{5}{3}} x \sin x} = \frac{3}{5} \int \frac{dt}{t \sin x} = \frac{3}{5} \int \frac{dt}{t t^{\frac{3}{5}}} = \frac{3}{5} \int \frac{dt}{t^{\frac{8}{5}}}$$

$$= \frac{3}{5} \left(-\frac{5}{3 t^{\frac{3}{5}}} \right) = -t^{-\frac{3}{5}} = -\left(\sin^{\frac{5}{3}} x \right)^{-\frac{3}{5}} = -\frac{1}{\sin x} + C$$

$$b) \int \frac{\cot^{\frac{2}{3}} x}{\sin^2 x} dx$$

$$\left\{ \begin{array}{c} \cot x = t|' \\ -\frac{dx}{\sin^2 x} = dt \end{array} \right\}$$

$$= -\int t^{\frac{2}{3}} dt = -\frac{3}{5} t^{\frac{5}{3}} = -\frac{3}{5} (\cot x)^{\frac{5}{3}} + C$$

1135

$$\int \frac{1 + \sin 3x}{\cos^2 3x} = \int \frac{dx}{\cos^2 3x} + \int \frac{\sin 3x}{\cos^2 3x}$$

$$\left\{ \begin{array}{c} 3x = t \\ 3dx = dt \end{array} \right\} \left\{ \begin{array}{c} \cos 3x = z \\ -3 \sin 3x dx = dz \end{array} \right\}$$

$$= \int \frac{\frac{dt}{3}}{\cos^2 t} + \int \frac{-\frac{dz}{3}}{z^2} = \frac{1}{3} \int \frac{dt}{\cos^2 t} - \frac{1}{3} \int z^{-2} dz = \frac{1}{3} \cot t + \frac{1}{3z} = \frac{1}{3} \cot 3x + \frac{1}{3 \cos 3x} + C$$

1136

$$\int \frac{(\cos ax + \sin ax)^2}{\sin ax} dx = \int \frac{1 + 2 \sin ax \cos ax}{\sin ax} dx = \int \frac{dx}{\sin ax} + 2 \int \cos ax dx$$

$$\left\{ \begin{array}{c} ax = t \\ adx = dt \end{array} \right\}$$

$$= \frac{1}{a} \int \frac{dt}{\sin t} + \frac{2}{a} \int \cos t dt = \frac{1}{a} \ln \tan \frac{t}{2} + \frac{2}{a} \sin t = \frac{1}{a} \ln \tan \frac{ax}{2} + \frac{2}{a} \sin ax + C$$

1137

$$\int \frac{\csc^2 3x}{b - a \cot 3x} dx = \int \frac{\frac{dx}{\sin^2 3x}}{b - a \cot 3x}$$

$$\left\{ \begin{array}{c} \cot 3x = t \\ -\frac{3dx}{\sin^2 x} = dt \end{array} \right\}$$

$$= \int \frac{-\frac{dt}{3}}{b - at} = -\frac{1}{3} \int \frac{dt}{b - at} = \frac{1}{3a} \ln(b - at) = \frac{1}{3a} \ln(b - a \cot 3x) + C$$

1138

$$\int (2 \sinh 5x - 3 \cosh 5x) dx = 2 \int \sinh 5x dx - 3 \int \cosh 5x dx$$

$$\left\{ \begin{array}{c} 5x = t \\ 5dx = dt \end{array} \right\}$$

$$= \frac{2}{5} \int \sinh t dt - \frac{3}{5} \int \cosh t dt = \frac{2}{5} \cosh t - \frac{3}{5} \sinh t = \frac{2}{5} \cosh 5x - \frac{3}{5} \sinh 5x + C$$

1139

$$\int sh^2 x dx = \int \left(\frac{e^x - e^{-x}}{2} \right)^2 dx = \int \left(\frac{1}{4} e^{2x} - \frac{1}{2} + \frac{1}{4e^{2x}} \right) dx$$

$$\left\{ \begin{array}{c} 2x = t \\ 2dx = dt \end{array} \right\}$$

$$= \frac{1}{4} \int e^t \frac{dt}{2} - \frac{1}{2} x + \frac{1}{4} \int \frac{\frac{dt}{2}}{e^t} = \frac{1}{8} e^t - \frac{1}{2} x - \frac{1}{8e^t} = \frac{1}{8} e^{2x} - \frac{1}{2} x - \frac{1}{8e^{2x}} + C$$

1140

$$\int \frac{dx}{\sinh x} = \int \frac{2}{e^x - e^{-x}} dx = \int \frac{2}{\frac{e^{2x} - 1}{e^x}} dx = \int \frac{2e^x}{e^{2x} - 1} dx =$$

$$\left\{ \begin{array}{c} e^x = t \\ e^x dx = dt \end{array} \right\}$$

$$= \int \frac{2dt}{t^2 - 1} = -2 \operatorname{arctanh} t = -2 \operatorname{arctanh} e^x + C$$

1141

$$\int \frac{dx}{\cosh x} = \int \frac{2}{e^x + e^{-x}} dx = 2 \int \frac{dx}{\frac{e^{2x} + 1}{e^x}} = 2 \int \frac{e^x dx}{e^{2x} + 1}$$

$$\left\{ \begin{array}{c} e^x = t \\ e^x dx = dt \end{array} \right\}$$

$$= 2 \int \frac{dt}{t^2 + 1} = 2 \arctan t = 2 \arctan e^x + C$$

1142

$$\int \frac{dx}{\sinh x \cosh x} = \int \frac{dx}{\sqrt{\frac{1}{2} \left(\cosh \frac{x}{2} - 1 \right)} \sqrt{\frac{1}{2} \left(\cosh \frac{x}{2} + 1 \right)}} = \int \frac{dx}{\sqrt{\frac{1}{4} \left(\cosh^2 \frac{x}{2} - 1 \right)}} = \int \frac{dx}{\frac{1}{2} \sinh \frac{x}{2}} = 2 \int \frac{dx}{\sinh \frac{x}{2}}$$

$$\left\{ \begin{array}{c} \frac{x}{2} = u \\ \frac{dx}{2} = du \end{array} \right\}$$

$$= \int \frac{du}{\sinh u} = -2 \operatorname{arctanh}(e^u) = -2 \operatorname{arctanh}(e^{\frac{x}{2}}) + C$$

1143

$$\int \tanh x dx = \int \frac{\sinh x}{\cosh x} dx$$

$$\left\{ \begin{array}{c} \cosh x = t \\ \sinh x dx = dt \end{array} \right\}$$

$$= \int \frac{dt}{t} = \ln t = \ln \cosh t + C$$

1144

$\int \coth x dx = \int \frac{\cosh x}{\sinh x} dx$

$$\left\{ \begin{array}{c} \sinh x = t \\ \cosh x dx = dt \end{array} \right\}$$

$= \int \frac{dt}{t} = \ln t = \ln \sinh x + C$

1145

$\int x \sqrt[5]{5 - x^2}\, dx$

$$\left\{ \begin{array}{c} 5 - x^2 = t \\ -2x dx = dt \end{array} \right\}$$

$= \int - \sqrt[5]{t}\, \frac{dt}{2} = -\frac{1}{2} \int \sqrt[5]{t}\, dt = -\frac{5}{12} t^{\frac{6}{5}} = -\frac{5}{12} (5 - x^2)^{\frac{6}{5}} + C$

1146

$\int \frac{x^3 - 1}{x^4 - 4x + 1} dx =$

$$\left\{ \begin{array}{c} x^4 - 4x + 1 = t \\ (4x^3 - 4) dx = dt \end{array} \right\}$$

$= \int \frac{\frac{dt}{4}}{t} = \frac{1}{4} \ln t = \frac{1}{4} \ln(x^4 - 4x + 1) + C$

1147

$\int \frac{x^3}{x^8 + 5} dx =$

$$\left\{ \begin{array}{c} x^4 = t \\ 4x^3 dx = dt \end{array} \right\}$$

$= \int \frac{\frac{dt}{4}}{t^2 + 5} = \frac{1}{4} \int \frac{dt}{t^2 + 5} = \frac{\sqrt{5}}{20} \arctan \frac{t\sqrt{5}}{5} = \frac{\sqrt{5}}{20} \arctan \frac{x^4 \sqrt{5}}{5} + C$

1148

$\int x e^{-x^2} dx$

$$\left\{ \begin{array}{c} -x^2 = u \\ -2x dx = du \end{array} \right\}$$

$= \int \frac{e^u}{-2} du = -\frac{1}{2} \int e^u du = -\frac{1}{2} e^u = -\frac{1}{2} e^{-x^2} + C$

1149

$\int \frac{3 - \sqrt{2 + 3x^2}}{2 + 3x^2} dx$

$$\left\{ \begin{array}{c} 2 + 3x^2 = t^2 \\ 3x dx = t dt \end{array} \right\}$$

$= \int \frac{3 - t}{t^2} \frac{t dt}{3x} = \frac{1}{3} \int \frac{3 - t}{t^2 \sqrt{\frac{t^2 - 2}{3}}} t dt = \frac{\sqrt{3}}{3} \int \frac{3 - t}{t\sqrt{t^2 - 2}} dt = \frac{\sqrt{3}}{3} \int \frac{3}{t\sqrt{t^2 - 2}} - \frac{\sqrt{3}}{3} \int \frac{t}{t\sqrt{t^2 - 2}} = \sqrt{3} \int \frac{1}{t\sqrt{t^2 - 2}}$

$$\left\{ \begin{array}{c} t^2 - 2 = z^2 \\ t dt = z dz \end{array} \right\}$$

$= -\frac{1}{3} \sqrt{3} \ln\left(t + \sqrt{t^2 - 2} \right) + \sqrt{3} \int \frac{1}{tz} \frac{z dz}{t} = -\frac{1}{3} \sqrt{3} \ln\left(t + \sqrt{(t^2 - 2)} \right) + \sqrt{3} \int \frac{1}{(z^2 + 2) z} z dz$

$= -\frac{1}{3} \sqrt{3} \ln\left(t + \sqrt{t^2 - 2} \right) + \sqrt{3} \int \frac{1}{z^2 + 2} dz = -\frac{1}{3} \sqrt{3} \ln\left(t + \sqrt{t^2 - 2} \right) + \frac{1}{2} \sqrt{3} \sqrt{2} \arctan \frac{1}{2}$

$= -\frac{1}{3} \sqrt{3} \ln\left(t + \sqrt{t^2 - 2} \right) + \frac{1}{2} \sqrt{3} \sqrt{2} \arctan \frac{\sqrt{2}}{2} \sqrt{t^2 - 2}$

$= -\frac{1}{3} \sqrt{3} \ln\left(\sqrt{2 + 3x^2} + \sqrt{2 + 3x^2 - 2} \right) + \frac{1}{2} \sqrt{3} \sqrt{2} \arctan \frac{\sqrt{2}}{2} \sqrt{2 + 3x^2 - 2}$

$= -\frac{1}{3} \sqrt{3} \ln\left(\sqrt{2 + 3x^2} + \sqrt{3}\, x \right) + \frac{1}{2} \sqrt{3} \sqrt{2} \arctan \frac{1}{2} \sqrt{2} \sqrt{3}\, x + C$

$\int \frac{3 - \sqrt{2 + 3x^2}}{2 + 3x^2} dx = \int \frac{3}{2 + 3x^2} dx - \int \frac{dx}{\sqrt{2 + 3x^2}} = \int \frac{dx}{x^2 + \frac{2}{3}} - \frac{\sqrt{3}}{3} \int \frac{dx}{\sqrt{x^2 + \frac{2}{3}}}$

$= \frac{1}{2} \sqrt{6} \arctan \frac{1}{2} x \sqrt{6} - \left(\frac{\sqrt{3}}{3} \right) \operatorname{arcsinh} \frac{1}{2} x \sqrt{6} =$

$\frac{1}{2} \sqrt{6} \arctan \frac{1}{2} x \sqrt{6} - \frac{1}{3} \sqrt{3} \operatorname{arcsinh} \frac{1}{2} x \sqrt{6} + C$

1150

$\int \frac{(x^3 - 1)}{(x + 1)} dx = \int x^2 dx - \int x dx + \int dx - 2 \int \frac{dx}{x + 1}$

$(x^3 - 1) : (x + 1) = x^2 - x + 1 - \frac{2}{x + 1}$

$x^3 + x^2$

$-1 - x^2$

$-x^2 - x$

$-1 + x$

$x + 1$

-2

$= \frac{1}{3} x^3 - \frac{1}{2} x^2 + x - 2 \ln(x + 1) + C$

1151

$\int \frac{dx}{\sqrt{e^x}} = \int e^{-\frac{1}{2} x} dx$

$$\left\{ \begin{array}{c} -\frac{1}{2} x = u \\ -\frac{dx}{2} = du \end{array} \right\}$$

$= -2 \int e^u du = -2e^u = -2e^{-\frac{1}{2} x} + C$

1152

$\int \frac{1 - \sin x}{x + \cos x} dx$

$$\left\{ \begin{array}{c} x + \cos x = u \\ (1 - \sin x) dx = du \end{array} \right\}$$

$= \int \frac{du}{u} = \ln u = \ln(x + \cos x) + C$

1153

$\int \frac{\tan 3x - \cot 3x}{\sin 3x} dx$

$$\left\{ \begin{array}{c} 3x = u \\ 3dx = du \end{array} \right\}$$

$$= \frac{1}{3}\int \frac{\tan u - \cot u}{\sin u} du = \frac{1}{3}\int \frac{\frac{\sin u}{\cos u} - \frac{\cos u}{\sin u}}{\sin u} du = \frac{1}{3}\int \frac{1}{\cos u} - \frac{1}{3}\int \frac{\cos u}{\sin^2 u}$$

$$\left\{ \begin{array}{c} \sin u = t \\ \cos u du = dt \end{array} \right\}$$

$$= \frac{1}{3}\ln(\frac{1+\sin u}{\cos u}) - \frac{1}{3}\int \frac{dt}{t^2} = \frac{1}{3}\ln\frac{1+\sin u}{\cos u} + \frac{1}{3t} = \frac{1}{3}\ln\frac{1+\sin u}{\cos u} + \frac{1}{3\sin u} =$$
$$\frac{1}{3}\ln\frac{1+\sin 3x}{\cos 3x} + \frac{1}{3\sin 3x} + C$$

1154

$$\int \frac{dx}{x\ln^2 x}$$

$$\left\{ \begin{array}{c} \ln x = t \\ \frac{dx}{x} = dt \end{array} \right\}$$

$$= \int \frac{dt}{t^2} = -\frac{1}{t} = -\frac{1}{\ln x} + C$$

1155

$$\int \frac{\sec^2 x}{\sqrt{\tan^2 x - 2}} dx = \int \frac{\frac{dx}{\cos^2 x}}{\sqrt{\tan^2 x - 2}}$$

$$\left\{ \begin{array}{c} \tan x = t \\ \frac{dx}{\cos^2 x} = dt \end{array} \right\}$$

$$= \int \frac{dt}{\sqrt{t^2-2}} = \ln\left(t + \sqrt{(t^2-2)}\right) = \ln\left(\tan x + \sqrt{(\tan^2 x - 2)}\right) + C$$

1156

$$\int\left(2 + \frac{x}{2x^2+1}\right)\frac{dx}{2x^2+1} = \int \frac{2}{2x^2+1}dx + \int \frac{x}{(2x^2+1)^2}dx$$

$$\left\{ \begin{array}{c} 2x^2 + 1 = t \\ 4xdx = dt \end{array} \right\}$$

$$= \int \frac{dx}{x^2+\frac{1}{2}} + \int \frac{dt}{4t^2} = \sqrt{2}\,\arctan x\sqrt{2} - \frac{1}{4t} = \sqrt{2}\,\arctan x\sqrt{2} - \frac{1}{4(2x^2+1)} + C$$

1157

$$\int a^{\sin x}\cos x dx$$

$$\left\{ \begin{array}{c} \sin x = t \\ \cos x dx = dt \end{array} \right\}$$

$$= \int a^t dt = \frac{1}{\ln a}a^t = \frac{1}{\ln a}a^{\sin x} + C$$

1158

$$\int \frac{x^2}{\sqrt[3]{x^3+1}} dx$$

$$\left\{ \begin{array}{c} x^3 + 1 = u|' \\ 3x^2 dx = du \end{array} \right\}$$

$$= \int \frac{du}{3\sqrt[3]{u}} = \frac{1}{2}\sqrt[3]{u^2} = \frac{1}{2}\sqrt[3]{(x^3+1)^2} + C$$

1159

$$\int \frac{xdx}{\sqrt{1-x^4}}$$

$$\left\{ \begin{array}{c} x^2 = u|' \\ 2xdx = du \end{array} \right\}$$

$$= \int \frac{du}{2\sqrt{1-u^2}} = \frac{1}{2}\arcsin u = \frac{1}{2}\arcsin x^2 + C$$

1160

$$\int \tan^2 ax dx$$

$$\left\{ \begin{array}{c} ax = u|' \\ adx = du \end{array} \right\}$$

$$= \frac{1}{a}\int \tan^2 u du = \frac{1}{a}\int \frac{\sin^2 u}{\cos^2 u} du = \frac{1}{a}\int \frac{1-\cos^2 u}{\cos^2 u} du = \frac{1}{a}\int \frac{du}{\cos^2 u} - \frac{1}{a}\int du = \frac{1}{a}\tan u - \frac{u}{2} =$$
$$\frac{1}{a}\tan ax - \frac{1}{2}ax + C$$

1161

$$\int \sin^2 \frac{x}{2} dx$$

$$\left\{ \begin{array}{c} \frac{x}{2} = t \\ \frac{dx}{2} = dt \end{array} \right\}$$

$$= 2\int \sin^2 t dt = 2\int \frac{1}{2}(1-\cos 2t) = \int(1-\cos 2t) = t - \frac{1}{2}\sin 2t = \frac{x}{2} - \frac{1}{2}\sin x + C$$

1162

$$\int \frac{\sec^2 x dx}{\sqrt{4-\tan^2 x}}$$

$$\left\{ \begin{array}{c} \tan x = t \\ \frac{dx}{\cos^2 x} = dt \end{array} \right\}$$

$$= \int \frac{dt}{\sqrt{4-t^2}} = \arcsin\frac{t}{2} = \arcsin\frac{\tan x}{2} + C$$

1163

$$\int \frac{dx}{\cos\frac{x}{a}}$$

$$\left\{ \begin{array}{c} \frac{x}{a} = t \\ \frac{dx}{a} = dt \end{array} \right\}$$

$$= a\int \frac{dt}{\cos t} = a\ln(\sec t + \tan t) = a\ln\left(\frac{1+\sin\frac{x}{a}}{\cos\frac{x}{a}}\right) + C$$

1164

$$\int \frac{\sqrt[3]{1+\ln x}}{x} dx$$

$$\left\{ \begin{array}{c} 1 + \ln x = t \\ \frac{dx}{x} = dt \end{array} \right\}$$

$$= \int \sqrt[3]{t}\,dt = \frac{3}{4}\sqrt[3]{t^4} = \frac{3}{4}\sqrt[3]{(1+\ln x)^4} + C$$

1165

$$\int \tan\sqrt{x-1}\,\frac{dx}{\sqrt{x-1}}$$
$$\left\{\begin{array}{c}\sqrt{x-1}\ = t\\ \frac{dx}{2\sqrt{x-1}} = dt\end{array}\right\}$$
$$= 2\int \tan t\,dt = -2\ln(\cos t) = -2\ln\left(\cos\sqrt{x-1}\right) + C$$

1166

$$\int \frac{x\,dx}{\sin x^2}$$
$$\left\{\begin{array}{c}x^2 = t\\ 2x\,dx = dt\end{array}\right\}$$
$$= \frac{1}{2}\int\frac{dt}{\sin t} = \frac{1}{2}\ln\left(\frac{1-\cos t}{\sin t}\right) = \frac{1}{2}\ln\left(\frac{1-\cos x^2}{\sin x^2}\right) + C$$

1167

$$\int \frac{e^{\arctan x}+x\ln(1+x^2)+1}{1+x^2}dx = \int\frac{e^{\arctan x}}{1+x^2}dx + \int\frac{x\ln(1+x^2)}{1+x^2}dx + \int\frac{dx}{1+x^2}$$
$$\left\{\begin{array}{c}\arctan x = u\\ \frac{dx}{1+x^2} = du\end{array}\right\}\left\{\begin{array}{c}\ln(1+x^2) = t\\ \frac{2x\,dx}{1+x^2} = dt\end{array}\right\}$$
$$= \int e^u du + \frac{1}{2}\int t\,dt + \arctan x = e^u + \frac{1}{4}t^2 + \arctan x =$$
$$e^{\arctan x} + \frac{1}{4}\ln^2(1+x^2) + \arctan x + C$$

1168

$$\int\frac{\sin x-\cos x}{\sin x+\cos x}dx =$$
$$\left\{\begin{array}{c}\sin x = \frac{2t}{1+t^2}\\ \cos x = \frac{1-t^2}{1+t^2}\\ dx = \frac{2dt}{1+t^2}\end{array}\right\}$$
$$\int\frac{\frac{2t}{1+t^2}-\frac{1-t^2}{1+t^2}}{\frac{2t}{1+t^2}+\frac{1-t^2}{1+t^2}}\,\frac{2dt}{1+t^2} = 2\int\frac{t^2+2t-1}{(2t+1-t^2)(1+t^2)}dt =$$
$$\frac{t^2+2t-1}{(2t+1-t^2)(1+t^2)} = \frac{at+b}{(2t+1-t^2)} + \frac{ct+d}{(1+t^2)}$$
$$t^2+2t-1 = (at+b)(1+t^2) + (ct+d)(2t+1-t^2)$$
$$t^2+2t-1 = at+at^3+b+bt^2+2ct^2+ct-ct^3+2dt+d-dt^2$$
$$t^2+2t-1 = t^3(a-c)+t^2(b+2c-d)+t(a+c+2d)+b+d$$

$a-c = 0$

$b+2c-d = 1$

$a+c+2d = 2$

$b+d = -1$

$\{a = 1, d = 0, c = 1, b = -1\}$

$$\int\frac{t^2+2t-1}{(2t+1-t^2)(1+t^2)}dt = \int\frac{t-1}{(2t+1-t^2)}dt + \int\frac{t}{(1+t^2)}dt$$

$$\left\{\begin{array}{c}2t+1-t^2 = z\\ 2(1-t)dt = dz\end{array}\right\}\left\{\begin{array}{c}1+t^2 = u\\ 2t\,dt = du\end{array}\right\}$$
$$= 2\left(-\int\frac{dz}{2z}+\int\frac{du}{2u}\right) = -\frac{2}{2}\ln z + \frac{2}{2}\ln u$$
$$= -\ln(2t+1-t^2)+\ln(1+t^2) = -\ln\left(2\tan\frac{x}{2}+1-\tan^2\frac{x}{2}\right)+\ln\left(1+\tan^2\frac{x}{2}\right)+C$$
$$\int\frac{\sin x-\cos x}{\sin x+\cos x}dx = \int\frac{\sin x-\cos x}{\sin x+\cos x}\,\frac{\sin x-\cos x}{\sin x-\cos x}dx = \int\frac{\sin^2x-2\sin x\cos x+\cos^2x}{\sin^2x-\cos^2x}dx$$
$$= \int\frac{1-2\sin x\cos x}{\sin^2x-\cos^2x}dx = \int\frac{1-\sin 2x}{-\cos 2x}dx = -\int\frac{1-\sin 2x}{\cos 2x}dx = -\int\frac{dx}{\cos 2x}+\int\tan 2x\,dx$$
$$\left\{\begin{array}{c}2x = t\\ 2dx = dt\end{array}\right\}$$
$$= -\frac{1}{2}\int\frac{dt}{\cos t}+\frac{1}{2}\int\tan t\,dt = -\frac{1}{2}\ln\left(\frac{1+\sin t}{\cos t}\right)-\frac{1}{2}\ln(\cos t) =$$
$$-\frac{1}{2}\ln\left(\frac{1+\sin 2x}{\cos 2x}\right)-\frac{1}{2}\ln(\cos 2x)+C$$

1169

$$\int\frac{\left(1-\sin\frac{x}{\sqrt{2}}\right)^2}{\sin\frac{x}{\sqrt{2}}}dx$$
$$\left\{\begin{array}{c}\frac{x}{\sqrt{2}} = t\\ \frac{dx}{\sqrt{2}} = dt\end{array}\right\}$$
$$= \sqrt{2}\int\frac{(1-\sin t)^2}{\sin t} = \sqrt{2}\int\frac{(1-\sin t)^2}{\sin t} = \sqrt{2}\int\frac{1-2\sin t+\sin^2 t}{\sin t}dt = \sqrt{2}\left(\int\frac{dt}{\sin t}-2\int dt+\int\sin t\right)$$
$$= \sqrt{2}\left(\ln\left(\frac{1-\cos t}{\sin t}\right)-2t-\cos t\right) = \sqrt{2}\left(\ln\left(\frac{1-\cos\frac{x}{\sqrt{2}}}{\sin\frac{x}{\sqrt{2}}}\right)-2\frac{x}{\sqrt{2}}-\cos\frac{x}{\sqrt{2}}\right)+C$$

1170

$$\int\frac{x^2}{x^2-2}dx = \int\frac{x^2-2+2}{x^2-2}dx = \int dx+2\int\frac{dx}{x^2-2} = x-\sqrt{2}\ \mathrm{arctanh}\,\frac{1}{2}x\sqrt{2}+C$$

1171

$$\int\frac{(1+x)^2}{x(1+x^2)}dx = \int\frac{x^2+2x+1}{x(1+x^2)}dx = \int\frac{x}{(1+x^2)}dx+2\int\frac{dx}{(1+x^2)}+\int\frac{dx}{x(1+x^2)}$$
$$= \int\frac{x}{(1+x^2)}dx+2\arctan x+\int\frac{dx}{x(1+x^2)}$$
$$\left\{\begin{array}{c}1+x^2 = t\\ 2x\,dx = dt\end{array}\right\}$$
$$= \int\frac{\frac{dt}{2}}{t}+2\arctan x+\int\frac{dx}{x(1+x^2)} = \frac{1}{2}\ln t+2\arctan x+\int\frac{dx}{x(1+x^2)}$$
$$\frac{1}{x(1+x^2)} = \frac{a}{x}+\frac{bx+c}{(1+x^2)}$$
$$1 = a(1+x^2)+x(bx+c) = a+ax^2+bx^2+xc$$
$$1 = x^2(a+b)+x(c)+a$$
$a+b = 0 \rightarrow$

$\{a = 1\}\{b = -1\}\{c = 0\}$

$$I_1 = \int\frac{a}{x}dx+\int\frac{bx+c}{(1+x^2)}dx = a\ln x+\int\frac{-x}{(1+x^2)}dx = \ln x-\frac{1}{2}\ln(1+x^2)$$
$$I = \frac{1}{2}\ln(1+x^2)+2\arctan x+\ln x-\frac{1}{2}\ln(1+x^2) = 2\arctan x+\ln x+C$$

1172

$$\int e^{\sin^2 x}\sin 2x dx = 2\int e^{\sin^2 x}\sin x\cos x dx$$

$$\left\{\begin{array}{c} \sin^2 x = t \\ 2\sin x\cos x dx = dt \end{array}\right\}$$

$$= \int e^t dt = e^{\sin^2 x} + C$$

1173

$$\int \frac{5-3x}{\sqrt{4-3x^2}}dx = \int \frac{5}{\sqrt{4-3x^2}}dx - \int \frac{3x}{\sqrt{4-3x^2}}dx = \int \frac{5}{\sqrt{4-3x^2}}\,\frac{:\sqrt{3}}{:\sqrt{3}}dx - \int \frac{3x}{\sqrt{4-3x^2}}dx$$

$$\left\{\begin{array}{c} 4-3x^2 = t \\ -6xdx = dt \end{array}\right\}$$

$$= \frac{5}{\sqrt{3}}\int \frac{dx}{\sqrt{\frac{4}{3}-x^2}} - \int \frac{3\frac{dt}{-6}}{\sqrt{t}} = \frac{5}{\sqrt{3}}\arcsin\frac{1}{2}\sqrt{3}x + \frac{1}{2}\int\frac{dt}{\sqrt{t}}$$

$$= \frac{5}{3}\sqrt{3}\arcsin\frac{1}{2}\sqrt{3}x + \sqrt{t} = \frac{5}{3}\sqrt{3}\arcsin\frac{1}{2}\sqrt{3}x + \sqrt{4-3x^2} + C$$

1174

$$\int \frac{dx}{e^x+1}$$

$$\left\{\begin{array}{c} e^x + 1 = t \\ e^x dx = dt \end{array}\right\}$$

$$= \int \frac{dt}{e^x t} = \int \frac{dt}{(t-1)t} = \int \frac{dt}{t^2-t} = \int \frac{dt}{\left(t-\frac{1}{2}\right)^2-\frac{1}{4}}$$

$$\left\{\begin{array}{c} t-\frac{1}{2} = u \\ dt = du \end{array}\right\}$$

$$= \int \frac{du}{u^2-\frac{1}{4}} = \frac{1}{2\sqrt{\frac{1}{4}}}\ln\frac{u-\frac{1}{2}}{u+\frac{1}{2}} = \ln\frac{u-\frac{1}{2}}{u+\frac{1}{2}} = \ln(2u-1)-\ln(2u+1)$$

$$= \ln(2(t-\tfrac{1}{2})-1)-\ln(2(t-\tfrac{1}{2})+1) = \ln(2t-2)-\ln 2t$$

$$= \ln(2(e^x+1)-2)-\ln 2(e^x+1)+C$$

1175

$$\int \frac{dx}{(a+b)+(a+b)x^2} = \frac{1}{(a+b)}\int\frac{dx}{1+x^2} = \frac{1}{a+b}\arctan x + C$$

1176

$$\int \frac{e^x}{\sqrt{e^{2x}-1}}dx$$

$$\left\{\begin{array}{c} e^x = t \\ e^x dx = dt \end{array}\right\}$$

$$= \int \frac{dt}{\sqrt{t^2-1}} = \ln\left(t+\sqrt{t^2-1}\right) = \ln\left(e^x+\sqrt{e^{2x}-1}\right)+C$$

1177

$$\int \frac{dx}{\sin ax\cos ax} = \int \frac{dx}{\frac{\sin 2ax}{2}} = 2\int\frac{dx}{\sin 2ax}$$

$$\left\{\begin{array}{c} 2ax = t \\ 2adx = dt \end{array}\right\}$$

$$= 2\int \frac{\frac{dt}{2a}}{\sin t} = \frac{1}{a}\int\frac{dt}{\sin t} = \frac{1}{a}\ln\frac{1-\cos t}{\sin t} = \frac{1}{a}\ln\frac{1-\cos 2ax}{\sin 2ax} + C$$

1178

$$\int \sin(\tfrac{2\pi t}{T}+\varphi_0)dt = \int \sin(\tfrac{2\pi t}{T}+\varphi_0)dt =$$

$$\left\{\begin{array}{c} \frac{2\pi t}{T}+\varphi_0 = x \\ \frac{2\pi}{T}dt = dx \end{array}\right\}$$

$$= \frac{T}{2\pi}\int\sin x dx = -\frac{1}{2}\frac{T}{\pi}\cos x = -\frac{T}{2\pi}\cos(\tfrac{2\pi t}{T}+\varphi_0)+C$$

1179

$$\int \frac{dx}{x(4-\ln^2 x)}$$

$$\left\{\begin{array}{c} \ln x = t \\ \frac{dx}{x} = dt \end{array}\right\}$$

$$= \int\frac{xdt}{x(4-t^2)} = \int\frac{dt}{4-t^2} = \frac{1}{4}\ln\frac{2+t}{2-t} = \frac{1}{4}\ln\frac{\ln x+2}{2-\ln x} + C$$

1180

$$\int \frac{\arccos\frac{x}{2}}{\sqrt{1-x^2}}dx$$

$$\left\{\begin{array}{c} \arccos\frac{x}{2} = t \\ -\frac{\frac{1}{2}}{\sqrt{1-\left(\frac{x}{2}\right)^2}}dx = -\frac{1}{\sqrt{(4-x^2)}}dx = dt \end{array}\right\}$$

$$= -\int tdt = -\frac{1}{2}t^2 = -\frac{1}{2}\arccos^2\frac{x}{2} + C$$

1181

$$\int e^{-\tan x}\sec^2 x dx = \int e^{-\tan x}\frac{1}{\cos^2 x}dx$$

$$\left\{\begin{array}{c} -\tan x = u \\ -\frac{1}{\cos^2 x}dx = du \end{array}\right\}$$

$$= -\int e^u du = -e^u = -e^{-\tan x} + C$$

1182

$$\int \frac{\sin x\cos x}{\sqrt{2-\sin^4 x}}dx$$

$$\left\{\begin{array}{c} \sin^2 x = t \\ 2\sin x\cos x dx = dt \end{array}\right\}$$

$$= \int\frac{dt}{2\sqrt{2-t^2}} = \frac{1}{2}\int\frac{dt}{\sqrt{2-t^2}} = \frac{1}{2}\arcsin\frac{\sqrt{2}}{2}t = \frac{1}{2}\arcsin\left(\frac{\sqrt{2}}{2}\sin^2 x\right)+C$$

1183

$$\int\frac{dx}{\sin^2 x\cos^2 x} = \int\frac{dx}{\frac{\sin^2 2x}{4}} = 4\int\frac{dx}{\sin^2 2x}$$

$$\left\{\begin{array}{c} 2x = u \\ 2dx = du \end{array}\right\}$$

$$= 4\int \frac{\frac{du}{2}}{\sin^2 u} = 2\int \frac{du}{\sin^2 u} = -2\cot u = -2\cot 2x + C$$

1184

$$\int \frac{\arcsin x + x}{\sqrt{1-x^2}}dx = \int \frac{\arcsin x}{\sqrt{1-x^2}} + \int \frac{x}{\sqrt{1-x^2}}$$

$$\left\{\begin{array}{c} \arcsin x = u \\ \frac{dx}{\sqrt{(1-x^2)}} = du \end{array}\right\}\left\{\begin{array}{c} 1 - x^2 = t \\ -2xdx = dt \end{array}\right\}$$

$$= \int udu - \frac{1}{2}\int \frac{dt}{\sqrt{t}} = \frac{1}{2}u^2 - \sqrt{t} = \frac{1}{2}\arcsin^2 x - \sqrt{1-x^2} + C$$

1185

$$\int \frac{\sec x \tan x}{\sqrt{\sec^2 x + 1}}dx = \int \frac{\frac{1}{\cos x}\frac{\sin x}{\cos x}}{\sqrt{\frac{1}{\cos^2 x}+1}}dx = \int \frac{\frac{\sin x}{\cos^2 x}}{\sqrt{\frac{1+\cos^2 x}{\cos^2 x}}}dx = \int \frac{\frac{\sin x}{\cos x}}{\sqrt{1+\cos^2 x}}dx = \int \frac{\sin x}{\cos x\sqrt{1+\cos^2 x}}dx$$

$$\left\{\begin{array}{c} \sqrt{1+\cos^2 x} = t|^2 \\ 1 + \cos^2 x = t^2 \\ 2\sin x \cos x dx = 2tdt \end{array}\right\}$$

$$= \int \frac{\sin x}{\cos xt}\frac{tdt}{\sin x \cos x} = \int \frac{dt}{\cos^2 x} = \int \frac{dt}{t^2-1} = -\operatorname{arctanh} t = -\operatorname{arctanh}\sqrt{1+\cos^2 x} + C$$

1186

$$\int \frac{\cos 2x}{4+\cos^2 2x}dx$$

$$\left\{\begin{array}{c} 2x = t \\ 2dx = dt \end{array}\right\}$$

$$= \frac{1}{2}\int \frac{\cos t}{4+\cos^2 t} = \frac{1}{2}\int \frac{\cos t}{5-1+\cos^2 t} = \frac{1}{2}\int \frac{\cos t}{5-\sin^2 t}$$

$$\left\{\begin{array}{c} \sin t = u \\ \cos tdt = du \end{array}\right\}$$

$$= \frac{1}{2}\int \frac{du}{5-u^2} = \frac{\sqrt{5}}{10}\operatorname{arctanh}\frac{\sqrt{5}}{5}u = \frac{\sqrt{5}}{10}\operatorname{arctanh}\left(\frac{\sqrt{5}}{5}\sin t\right) =$$
$$\frac{\sqrt{5}}{10}\operatorname{arctanh}\left(\frac{\sqrt{5}}{5}\sin 2x\right) + C$$

1187*

$$\int \frac{dx}{1+\cos^2 x} = \int \frac{dx}{\sin^2 x + \cos^2 x + \cos^2 x}\begin{array}{l}:\cos^2 x \\ :\cos^2 x\end{array} = \int \frac{\frac{dx}{\cos^2 x}}{\tan^2 x + 2} = \int \frac{1}{\cos^2 x(\tan^2 x + 2)}dx$$

$$\left\{\begin{array}{c} \tan x = t \\ \frac{dx}{\cos^2 x} = dt \end{array}\right\}$$

$$= \int \frac{dt}{t^2+2} = \frac{\sqrt{2}}{2}\arctan\frac{t\sqrt{2}}{2} = \frac{\sqrt{2}}{2}\arctan\frac{\tan x\sqrt{2}}{2} + C$$

1188

$$\int \sqrt{\frac{\ln\left(x+\sqrt{x^2+1}\right)}{x^2+1}}\,dx$$

$$\left\{\begin{array}{c} \ln\left(x+\sqrt{x^2+1}\right) = t \\ \frac{1}{\left(x+\sqrt{x^2+1}\right)}\left(1+\frac{2x}{2\sqrt{x^2+1}}\right)dx = dt \\ \frac{1}{\left(x+\sqrt{x^2+1}\right)}\left(\frac{\sqrt{x^2+1}+x}{\sqrt{x^2+1}}\right)dx = dt \\ \frac{1}{\sqrt{x^2+1}}dx = dt \end{array}\right\}$$

$$= \int \sqrt{t}\,dt = \frac{2}{3}t^{\frac{3}{2}} = \frac{2}{3}\ln^{\frac{3}{2}}\left(x+\sqrt{x^2+1}\right) + C$$

1189

$$\int x^2 ch(x^3+3)dx$$

$$\left\{\begin{array}{c} x^3 + 3 = t \\ 3x^2dx = dt \end{array}\right\}$$

$$= \frac{1}{3}\int \cosh tdt = \frac{1}{3}\sinh t = \frac{1}{3}\sinh(x^3+3) + C$$

1190

$$\int \frac{3^{\tanh x}}{\cosh^2 x}dx$$

$$\left\{\begin{array}{c} \tanh x = t \\ \frac{dx}{\cosh^2 x} = dt \end{array}\right\}$$

$$= \int 3^t dt = \frac{1}{\ln 3}3^t = \frac{1}{\ln 3}3^{\tanh x} + C$$

1192

$$\int x(2x+5)^{10}dx$$

$$\left\{\begin{array}{c} 2x + 5 = t \\ 2dx = dt \end{array}\right\}$$

$$= \frac{1}{2}\int t^{10}dt = \frac{1}{22}t^{11} = \frac{1}{22}(2x+5)^{11} + C$$

1191a)

$$\int \frac{dx}{x\sqrt{x^2-2}}$$

$$\left\{\begin{array}{c} \sqrt{x^2-2} = t \\ \frac{xdx}{\sqrt{x^2-2}} = dt \end{array}\right\}$$

$$= \int \frac{dt\frac{\sqrt{x^2-2}}{x}}{x\sqrt{x^2-2}} = \int \frac{dt}{x^2} = \int \frac{dt}{t^2+2} = \frac{\sqrt{2}}{2}\arctan\frac{\sqrt{2}}{2}t$$

$$= \frac{\sqrt{2}}{2}\arctan\frac{\sqrt{2}}{2}\sqrt{x^2-2} + C$$

$$\int \frac{dx}{x\sqrt{x^2-2}}$$

$$\left\{\begin{array}{c} \frac{1}{x} = t \\ -\frac{dx}{x^2} = dt \end{array}\right\}$$

$$= \int \frac{-x^2 dt}{x\sqrt{(\frac{1}{t})^2-2}} = -\int \frac{\frac{1}{t}dt}{\sqrt{\frac{2-t^2}{t^2}}} = -\int \frac{dt}{\sqrt{(2-t^2)}} = -\arcsin\frac{\sqrt{2}}{2}t = -\arcsin\frac{\sqrt{2}}{2x} + C$$

1191b

$$\int \frac{dx}{e^x+1}$$

$$\left\{ \begin{array}{l} x = -\ln t \\ dx = -\frac{dt}{t} \end{array} \right\}$$

$$= \int \frac{-\frac{dt}{t}}{e^{\ln\frac{1}{t}}+1} = -\int \frac{\frac{dt}{t}}{\frac{1}{t}+1} = -\int \frac{dt}{t(\frac{1}{t}+1)} = -\int \frac{dt}{1+t} = -\ln(1+t) = -\ln(1+\frac{1}{e^x}) + C$$

1191c

$$\int x(5x^2-3)^7 dx$$

$$\left\{ \begin{array}{l} 5x^2-3 = t \\ 10xdx = dt \end{array} \right\}$$

$$= \int x(5x^2-3)^7 \frac{dt}{10x} = \frac{1}{10}\int t^7 dt = \frac{1}{80}t^8 = \frac{1}{80}(5x^2-3)^8 + C$$

1191d

$$\int \frac{xdx}{\sqrt{x+1}}$$

$$\left\{ \begin{array}{l} \sqrt{x+1} = t \\ \frac{dx}{2\sqrt{x+1}} = dt \end{array} \right\}$$

$$= 2\int xdt = 2\int (t^2-1)dt = \frac{2}{3}t^3 - 2t = \frac{2}{3}\sqrt{x+1}^3 - 2\sqrt{x+1} + C$$

1191e

$$\int \frac{\cos x dx}{\sqrt{1+\sin^2 x}}$$

$$\left\{ \begin{array}{l} t = \sin x \\ dt = \cos x dx \end{array} \right\}$$

$$= \int \frac{dt}{\sqrt{1+t^2}} = \operatorname{arcsinh} t = \operatorname{arcsinh} \sin x + C$$

1192

$$\int x(2x-5)^{10} dx$$

$$\left\{ \begin{array}{l} 2x-5 = t \quad x = \frac{t+5}{2} \\ 2dx = dt \end{array} \right\}$$

$$= \int \frac{t+5}{2} t^{10} \frac{dt}{2} = \frac{1}{4}\int (t+5)t^{10} dt = \frac{1}{48}t^{12} + \frac{5}{44}t^{11} = \frac{1}{48}(2x-5)^{12} + \frac{5}{44}(2x-5)^{11} + C$$

1193

$$\int \frac{1+x}{1+\sqrt{x}} dx$$

$$\left\{ \begin{array}{l} \sqrt{x} = t \\ \frac{dx}{2\sqrt{x}} = dt \end{array} \right\}$$

$$= \int \frac{1+x}{1+\sqrt{x}} 2\sqrt{x}\, dt = 2\int \frac{1+t^2}{1+t} t dt = 2\int \frac{t+t^3}{1+t} dt$$

$(t^3+t) : (t+1) = t^2 - t + 2 - \frac{2}{(t+1)}$

$t^3 + t^2$

$-t^2 + t$

$-t^2 - t$

$2t$

$2t + 2$

-2

$$= 2\left(\int t^2 dt - \int t dt + 2\int dt - 2\int \frac{dt}{t+1}\right) = \frac{2}{3}t^3 - t^2 + 4t - 4\ln(1+t)$$

$$= \frac{2}{3}\sqrt{x}^3 - \sqrt{x}^2 + 4\sqrt{x} - 4\ln(1+\sqrt{x}) + C$$

1194

$$\int \frac{dx}{x\sqrt{2x+1}}$$

$$\left\{ \begin{array}{ll} \sqrt{2x+1} = t & 2x+1 = t^2 \\ \frac{dx}{\sqrt{2x+1}} = dt & x = \frac{t^2-1}{2} \end{array} \right\}$$

$$= \int \frac{\sqrt{2x+1}}{x\sqrt{2x+1}} dt = \int \frac{dt}{x} = \int \frac{dt}{\frac{t^2-1}{2}} = 2\int \frac{dt}{t^2-1} = -2\operatorname{arctanh} t = -2\operatorname{arctanh}\sqrt{2x+1} + C$$

1195

$$\int \frac{dx}{\sqrt{e^x-1}}$$

$$\left\{ \begin{array}{l} \sqrt{e^x-1} = t \\ \frac{e^x dx}{2\sqrt{e^x-1}} = dt \end{array} \right\}$$

$$= \int \frac{\frac{2\sqrt{e^x-1}}{e^x}dt}{\sqrt{e^x-1}} = 2\int \frac{dt}{e^x} = 2\int \frac{dt}{t^2+1} = 2\arctan t = 2\arctan\sqrt{e^x-1} + C$$

1196

$$\int \frac{\ln 2x dx}{x\ln 4x} = \int \frac{\ln 2x dx}{x(\ln 2x + \ln 2)}$$

$$\left\{ \begin{array}{l} \ln 2x = t \\ \frac{dx}{x} = t \end{array} \right\}$$

$$= \int \frac{txdt}{x(t+\ln 2)} = \int \frac{tdt}{(t+\ln 2)} = \int \frac{t+\ln 2-\ln 2}{(t+\ln 2)} dt$$

$$= \int dt - \ln 2\int \frac{dt}{(t+\ln 2)} = t - \ln 2\ln(t+\ln 2) = \ln 2x - \ln 2\ln(\ln 2x + \ln 2) + C$$

1197

$$\int \frac{(\arcsin x)^2}{\sqrt{1-x^2}} dx$$

$$\left\{ \begin{array}{l} \arcsin x = t \\ \frac{dx}{\sqrt{1-x^2}} = dt \end{array} \right\}$$

$$= \int t^2 dt = \frac{1}{3}t^3 = \frac{1}{3}\arcsin^3 x + C$$

1198

$\int \frac{e^{2x}}{\sqrt{e^x+1}}$

$$\left\{\begin{array}{c} \sqrt{e^x+1} = t \\ \frac{e^x dx}{2\sqrt{e^x+1}} = dt \end{array}\right\}$$

$= \int \frac{e^{2x}}{t}\frac{2\sqrt{e^x+1}}{e^x}dt = 2\int \frac{(t^2-1)t}{t}dt = 2\int(t^2-1)dt = \frac{2}{3}t^3 - 2t =$

$\frac{2}{3}\left(\sqrt{e^x+1}\right)^3 - 2\sqrt{e^x+1} + C$

1199

$\int \frac{\sin^3 x}{\sqrt{\cos x}}dx$

$$\left\{\begin{array}{c} \sqrt{\cos x} = t \\ -\frac{\sin x}{2\sqrt{\cos x}}dx = dt \end{array}\right\}$$

$= -2\int \sin^2 x dt = -2\int(1-\cos^2 x)dt = -2\int\left(1-t^4\right)dt = \frac{2}{5}t^5 - 2t =$

$\frac{2}{5}\sqrt{\cos x}^5 - 2\sqrt{\cos x} + C$

1200*

$\int \frac{dx}{x\sqrt{1+x^2}}$

$$\left\{\begin{array}{c} \sqrt{1+x^2} = t|^2 \\ 1+x^2 = t^2 \\ xdx = tdt \end{array}\right\}$$

$= \int \frac{\frac{tdt}{x}}{xt} = \int \frac{1}{x^2}dt = \int\frac{1}{t^2-1}dt = -\operatorname{arctanh} t = -\operatorname{arctanh}\sqrt{1+x^2} + C$

1201

$\int \frac{x^2}{\sqrt{1-x^2}}dx$

$$\left\{\begin{array}{c} x = \cos t \\ dx = -\sin t dt \end{array}\right\}$$

$= -\int \frac{\cos^2 t}{\sqrt{1-\cos^2 t}}\sin t dt = -\int \frac{\cos^2 t \sin t}{\sin t}dt = -\int\cos^2 t dt = -\int\frac{1+\cos 2t}{2} = -\frac{1}{2} t - \frac{1}{4}\sin 2t$

$= -\frac{1}{2}\arccos x - \frac{1}{4}\sin(2\arccos x) + C$

1202

$\int\frac{x^3}{\sqrt{2-x^2}}dx$

$$\left\{\begin{array}{c} 2-x^2 = \cos t \\ -2xdx = -\sin t dt \end{array}\right\}$$

$= \int\frac{x^3}{\sqrt{\cos t}}\frac{\sin t dt}{2x} = \int\frac{x^2}{\sqrt{\cos t}}\frac{\sin t dt}{2} = \int\frac{2-\cos t}{\sqrt{\cos t}}\frac{\sin t dt}{2} = \int\frac{2\sin t - \cos t\sin t}{2\sqrt{\cos t}}dt$

$= \int\frac{\sin t}{\sqrt{\cos t}}dt - \frac{1}{2}\int\sqrt{\cos t}\sin t dt$

$$\left\{\begin{array}{c} \cos t = u \\ -\sin t dt = du \end{array}\right\}$$

$= -\int\frac{du}{\sqrt{u}} + \frac{1}{2}\int\sqrt{u}\, du = -2\sqrt{u} + \frac{1}{3}u^{\frac{3}{2}} = -2\sqrt{\cos t} + \frac{1}{3}(\cos t)^{\frac{3}{2}}$

$= -2\sqrt{2-x^2} + \frac{1}{3}(2-x^2)^{\frac{3}{2}} + C$

1203

$\int\frac{\sqrt{x^2-a^2}}{x}dx$

$$\left\{\begin{array}{c} x^2-a^2 = \sin^2 t \\ xdx = \sin t\cos t dt \end{array}\right\}$$

$= \int\frac{\sin t}{x}\frac{\sin t\cos t dt}{x} = \int\frac{\sin^2 t\cos t dt}{x^2} = \int\frac{\sin^2 t\cos t dt}{\sin^2 t + a^2}$

$$\left\{\begin{array}{c} \sin t = z \\ \cos t dt = dz \end{array}\right\}$$

$= \int\frac{z^2 dz}{z^2+a^2} = \int dz - a^2\int\frac{dz}{z^2+a^2} = z - a\arctan\frac{z}{a} = \sin t - a\arctan\frac{\sin t}{a}$

$= \sqrt{x^2-a^2} - a\arctan\frac{\sqrt{x^2-a^2}}{a} + C$

1204*

$\int\frac{dx}{x\sqrt{x^2-1}} =$

$$\left\{\begin{array}{c} x = \frac{1}{t} \\ dx = -\frac{dt}{t^2} \end{array}\right\}$$

$= \int\frac{-\frac{dt}{t^2}}{\frac{1}{t}\sqrt{\frac{1}{t^2}-1}} = -\int\frac{\frac{dt}{t^2}}{\frac{1}{t}\sqrt{\frac{1-t^2}{t^2}}} = -\int\frac{\frac{dt}{t}}{\sqrt{\frac{1-t^2}{t^2}}} = -\int\frac{\frac{dt}{t}}{\frac{1}{t}\sqrt{1-t^2}} = -\int\frac{dt}{\sqrt{1-t^2}} = -\arcsin t =$

$-\arcsin\frac{1}{x} + C$

1205

$\int\frac{\sqrt{x^2+1}}{x}dx$

$$\left\{\begin{array}{c} x = \sinh t \\ dx = \cosh t \end{array}\right\}$$

$= \int\frac{\sqrt{\sinh^2 t+1}}{\sinh t}\cosh t = \int\frac{\sqrt{\sinh^2 t+1}}{\sinh t}\cosh t = \int\frac{\cosh^2 t}{\sinh t} = \int\frac{\sinh^2 t+1}{\sinh t} = \int\sinh t + \int\frac{dt}{\sinh t}$

$= \cosh t - 2\operatorname{arctanh}(e^t) = \cosh\operatorname{arcsinh} x - 2\operatorname{arctanh}(e^{\operatorname{arcsinh} x}) + C$

1206*

$\int\frac{dx}{x^2\sqrt{4-x^2}}$

$$\left\{\begin{array}{c} x = 2\sin t \\ dx = 2\cos t dt \end{array}\right\}$$

$\int\frac{2\cos t dt}{(2\sin t)^2\sqrt{4-(2\sin t)^2}} = \int\frac{2\cos t}{4\sin^2 t\sqrt{4-4\sin^2 t}}dt = \int\frac{2\cos t}{4\sin^2 t\sqrt{4-4\sin^2 t}}dt = \int\frac{2\cos t}{8\sin^2 t\sqrt{1-\sin^2 t}}dt = \int\frac{2\cos t}{8\sin^2 t\sqrt{\cos^2 t}}$

$= \int\frac{2\cos t}{8\sin^2 t\cos t}dt = \frac{1}{4}\int\frac{dt}{\sin^2 t} = -\frac{1}{4}\tan t = -\frac{1}{4}\tan(\arcsin\frac{x}{2}) + C$

$$\int \frac{dx}{x^2\sqrt{4-x^2}}$$

$$\left\{ \begin{array}{c} x = \frac{1}{t} \\ dx = -\frac{dt}{t^2} \end{array} \right\}$$

$$\int \frac{-\frac{dt}{t^2}}{\frac{1}{t^2}\sqrt{4-\frac{1}{t^2}}} = -\int \frac{dt}{\sqrt{4-\frac{1}{t^2}}} = -\int \frac{dt}{\frac{1}{t}\sqrt{4t^2-1}} = -\frac{1}{2}\int \frac{t}{\sqrt{t^2-\frac{1}{4}}}dt = -\frac{1}{2}\left(\frac{1}{2}\sqrt{4t^2-1}\right) =$$

$$-\frac{1}{4}\sqrt{4t^2-1}$$

$$= -\frac{1}{4}\sqrt{4\left(\frac{1}{x}\right)^2-1} = -\frac{1}{4x}\sqrt{4-x^2} + C$$

1207

$$\int \sqrt{1-x^2}\,dx = \int \frac{1-x^2}{\sqrt{1-x^2}}dx$$

$$\int \frac{1-x^2}{\sqrt{1-x^2}} = (ax+b)\sqrt{1-x^2} + \int \frac{c}{\sqrt{1-x^2}}|'$$

$$\frac{1-x^2}{\sqrt{1-x^2}} = \left((ax+b)\sqrt{1-x^2}\right)' + \frac{c}{\sqrt{1-x^2}}$$

$$\frac{1-x^2}{\sqrt{1-x^2}} = a\sqrt{1-x^2} + \frac{-2x(ax+b)}{2\sqrt{1-x^2}} + \frac{c}{\sqrt{1-x^2}}|\sqrt{1-x^2}$$

$$1-x^2 = a(1-x^2) - x(ax+b) + c$$

$$1-x^2 = a - 2ax^2 - xb + c$$

$$1-x^2 = x^2(-2a) + x(-b) + a + c$$

$$\{a = \tfrac{1}{2}, b = 0\}$$

$$a + c = 1 \ldots . c = \tfrac{1}{2}$$

$$I = \frac{1}{2}x\sqrt{1-x^2} + \frac{1}{2}\int \frac{dx}{\sqrt{1-x^2}} = \frac{1}{2}x\sqrt{1-x^2} + \frac{1}{2}\arcsin x + C$$

1208

$$\int \frac{dx}{\sqrt{x(1-x)}}$$

$$\left\{ \begin{array}{c} x = \sin^2 t \\ dx = 2\sin t\cos t\,dt \end{array} \right\}$$

$$= \int \frac{2\sin t\cos t}{\sqrt{\sin^2 t(1-\sin^2 t)}}dt = 2\int \frac{\sin t\cos t}{\sin t\sqrt{(1-\sin^2 t)}}dt = 2\int \frac{\sin t\cos t}{\sin t\sqrt{\cos^2 t}}dt = 2\int \frac{\sin t\cos t}{\sin t\cos t}dt = 2\int dt = 2t =$$

$$2\arcsin\sqrt{x} + C$$

1209

$$\int \sqrt{a^2+x^2}\,dx$$

$$\left\{ \begin{array}{c} x = a\sinh t \\ dx = a\cosh t \end{array} \right\}$$

$$= \int \sqrt{a^2+a^2\sinh^2 t}\,a\cosh t\,dt = a^2\int \sqrt{1+\sinh^2 t}\,\cosh t\,dt$$

$$= a^2\int \sqrt{\cosh^2 t}\,\cosh t\,dt = a^2\int \cosh^2 t\,dt = a^2\int \frac{\cosh 2t+1}{2}dt = \frac{a^2}{2}\int \cosh 2t\,dt + \frac{a^2}{2}\int dt$$

$$= \frac{1}{4}a^2\sinh 2t + \frac{1}{2}a^2 t = \frac{1}{4}a^2\sinh\left(2\operatorname{arcsinh}\frac{x}{a}\right) + \frac{1}{2}a^2\operatorname{arcsinh}\frac{x}{a}$$

$$= \frac{1}{2}x\sqrt{(x^2+a^2)} + \frac{1}{2}a^2\operatorname{arcsinh}\frac{x}{a} + C$$

1210

$$\int \frac{x^2}{\sqrt{x^2-a^2}}dx$$

$$\left\{ \begin{array}{c} x = a\cosh t \\ dx = a\sinh t\,dt \end{array} \right\}$$

$$= \int \frac{a^2\cosh^2 t}{\sqrt{a^2\cosh^2 t-a^2}}dt = \int \frac{a^2\cosh^2 t}{a\sqrt{\cosh^2 t-1}}dt = a\int \frac{\cosh^2 t}{\sqrt{\cosh^2 t-1}}dt$$

$$= a\int \frac{1+\sinh^2 t}{\sinh t}dt = a\int \frac{dt}{\sinh t} + a\int \sinh t\,dt = a\cosh t + a\ln\tanh\frac{t}{2}$$

$$= a\cosh\left(\operatorname{arccosh}\frac{x}{a}\right) + a\ln\tanh\frac{\left(\operatorname{arccosh}\frac{x}{a}\right)}{2} = x + a\ln\left(\tanh\left(\frac{1}{2}\operatorname{arccosh}\frac{x}{a}\right)\right) + C$$

1211

$$\int \ln x\,dx$$

$$\left\{ \begin{array}{cc} dx = dv & \ln x = u \\ x = v & \frac{dx}{x} = du \end{array} \right\}$$

$$= x\ln x - \int x\frac{dx}{x} = x\ln x - x + C$$

1212

$$\int \arctan x\,dx$$

$$\left\{ \begin{array}{cc} \arctan x = u & dx = dv \\ \frac{dx}{1+x^2} = du & x = v \end{array} \right\}$$

$$= x\arctan x - \int \frac{x}{1+x^2}dx$$

$$\left\{ \begin{array}{c} 1+x^2 = t \\ 2x\,dx = dt \end{array} \right\}$$

$$= x\arctan x - \int \frac{\frac{dt}{2}}{t} = x\arctan x - \frac{1}{2}\ln t = x\arctan x - \frac{1}{2}\ln(1+x^2) + C$$

1213

$$\int \arcsin x\,dx$$

$$\left\{ \begin{array}{cc} \arcsin x = u & dx = dv \\ \frac{dx}{\sqrt{1-x^2}} = du & x = v \end{array} \right\}$$

$$= x\arcsin x - \int \frac{x}{\sqrt{1-x^2}}dx$$

$$\left\{ \begin{array}{c} 1-x^2 = t \\ -2x\,dx = dt \end{array} \right\}$$

$$= x\arcsin x - \int \frac{\frac{dt}{-2}}{\sqrt{t}} = x\arcsin x + \frac{1}{2}\int \frac{dt}{\sqrt{t}} = x\arcsin x + \sqrt{t} = x\arcsin x + \sqrt{1-x^2} + C$$

1214

$$\int x\sin x\,dx$$

$$\left\{\begin{array}{ll} x=u & \sin x dx = dv \\ dx = du & -\cos x = v \end{array}\right\}$$
$$= -x\cos x - \int -\cos x dx = -x\cos x + \int \cos x dx = -x\cos x + \sin x + C$$

1215

$$\int x\cos 3x dx$$
$$\left\{\begin{array}{l} 3x = t \\ 3dx = dt \end{array}\right\}$$
$$= \frac{1}{9}\int t\cos t dt$$
$$\left\{\begin{array}{ll} t=u & \cos t dt = dv \\ dt = du & \sin t = v \end{array}\right\}$$
$$= \frac{1}{9}\left(t\sin t - \int \sin t dt\right)$$
$$= \frac{1}{9}t\sin t + \frac{1}{9}\cos t = \frac{1}{9}(3x)\sin(3x) + \frac{1}{9}\cos(3x)$$
$$= \frac{1}{3}x\sin 3x + \frac{1}{9}\cos 3x + C$$

1216

$$\int \frac{x}{e^x}dx$$
$$\left\{\begin{array}{ll} x=u & e^{-x}dx = dv \\ dx = du & -e^{-x} = v \end{array}\right\}$$
$$= -xe^{-x} - \int -e^{-x}dx = -xe^{-x} - e^{-x} + C$$

1217

$$\int x2^{-x}dx$$
$$\left\{\begin{array}{ll} x=u & 2^{-x}dx = dv \\ dx = du & -\frac{1}{\ln 2}2^{-x} = v \end{array}\right\}$$
$$= -\frac{1}{\ln 2}x2^{-x} - \int -\frac{1}{\ln 2}2^{-x}dx = -\frac{1}{\ln 2}x2^{-x} + \frac{1}{\ln 2}\int 2^{-x}dx = -\frac{1}{\ln 2}x2^{-x} - \frac{1}{\ln^2 2}2^{-x} + C$$

1218 $**$

$$\int x^2e^{3x}dx$$
$$\left\{\begin{array}{l} 3x = t \\ 3dx = dt \end{array}\right\}$$
$$= \int \frac{t^2}{9}e^t\frac{dt}{3} = \frac{1}{27}\int t^2e^tdt$$
$$\left\{\begin{array}{ll} e^tdt = dv & t^2 = u \\ e^t = v & 2tdt = du \end{array}\right\}$$
$$I = \frac{1}{27}\left(t^2e^t - 2\int e^ttdt\right)$$
$$I_1 = \int e^ttdt$$
$$\left\{\begin{array}{ll} t=u & e^tdt = dv \\ dt = du & e^t = v \end{array}\right\}$$
$$= te^t - \int e^tdt = te^t - e^t$$
$$I = \frac{1}{27}(t^2e^t - 2I_1) = \frac{1}{27}(t^2e^t - 2(te^t - e^t)) = \frac{1}{27}t^2e^t - \frac{2}{27}te^t + \frac{2}{27}e^t$$
$$= \frac{1}{27}(3x)^2e^{(3x)} - \frac{2}{27}(3x)e^{(3x)} + \frac{2}{27}e^{(3x)} = \frac{1}{3}x^2e^{3x} - \frac{2}{9}xe^{3x} + \frac{2}{27}e^{3x} + C$$

1219 $*$

$$\int(x^2-2x+5)e^{-x}dx = \int x^2e^{-x}dx - 2\int xe^{-x}dx + 5\int e^{-x}dx$$
$$\left\{\begin{array}{ll} x^2 = u & e^{-x}dx = dv \\ 2xdx = du & -e^{-x} = v \end{array}\right\}$$
$$I_1 = -e^{-x}x^2 - \int -e^{-x}2xdx = -e^{-x}x^2 + 2\int xe^{-x}dx$$
$$\left\{\begin{array}{ll} x=u & e^{-x}dx = dv \\ dx = du & -e^{-x} = v \end{array}\right\}$$
$$I_{12} = -e^{-x}x - \int -e^{-x}dx = -e^{-x}x + \int e^{-x}dx = -e^{-x}x - e^{-x}$$
$$I_1 = -e^{-x}x^2 + 2(-e^{-x}x - e^{-x}) = -e^{-x}x^2 - 2e^{-x}x - 2e^{-x}$$
$$I_2 = 2\int xe^{-x}dx = 2(-e^{-x}x - e^{-x}) = -2e^{-x}x - 2e^{-x}$$
$$I_3 = -5e^{-x}$$
$$I = I_1 - I_2 + I_3 = -e^{-x}x^2 - 2e^{-x}x - 2e^{-x} - (-2e^{-x}x - 2e^{-x}) - 5e^{-x}$$
$$I = -e^{-x}x^2 - 5e^{-x} + C$$

1220*

$$\int x^3e^{-\frac{x}{3}}$$
$$\left\{\begin{array}{l} -\frac{x}{3} = t \\ -\frac{dx}{3} = dt \end{array}\right\}$$
$$= \int -27t^3e^t(-3dt) = 81\int t^3e^tdt = 81I_1$$
$$I_1 = \int t^3e^tdt$$
$$\left\{\begin{array}{ll} e^tdt = dv & t^3 = u \\ e^t = v & 3t^2dt = du \end{array}\right\}$$
$$= t^3e^t - 3\int e^tt^2dt = t^3e^t - 3I_2$$
$$I_2 = \int e^tt^2dt$$
$$\left\{\begin{array}{ll} e^tdt = dv & t^2 = u \\ e^t = v & 2tdt = du \end{array}\right\}$$
$$I_2 = t^2e^t - 2\int e^ttdt = t^2e^t - 2I_3$$
$$I_3 = \int e^ttdt$$
$$\left\{\begin{array}{ll} e^tdt = dv & t = u \\ e^t = v & dt = du \end{array}\right\}$$
$$= te^t - \int e^tdt = te^t - e^t$$

$I_2 = t^2e^t - 2I_3 = t^2e^t - 2(te^t - e^t) = t^2e^t - 2te^t + 2e^t$

$I_1 = t^3e^t - 3I_2 = t^3e^t - 3(t^2e^t - 2te^t + 2e^t) = t^3e^t - 3t^2e^t + 6te^t - 6e^t$

$I = 81I_1 = 81(t^3e^t - 3t^2e^t + 6te^t - 6e^t)$

$= 81\left(-\frac{x}{3}\right)^3 e^{\left(-\frac{x}{3}\right)} - 243\left(-\frac{x}{3}\right)^2 e^{\left(-\frac{x}{3}\right)} + 486\left(-\frac{x}{3}\right) e^{\left(-\frac{x}{3}\right)} - 486e^{\left(-\frac{x}{3}\right)}$

$= -3x^3e^{-\frac{1}{3}x} - 27x^2e^{-\frac{1}{3}x} - 162xe^{-\frac{1}{3}x} - 486e^{-\frac{1}{3}x} + C$

1221

$\int x\sin x\cos x dx = \frac{1}{2}\int x\sin 2x dx$

$\left\{\begin{array}{c} 2x = t \\ 2dx = dt \end{array}\right\}$

$= \frac{1}{2}\int \frac{t}{2}\sin t\frac{dt}{2} = \frac{1}{8}\int t\sin t dt$

$\left\{\begin{array}{cc} u = t & dv = \sin t dt \\ du = dt & v = -\cos t \end{array}\right\}$

$= \frac{1}{8}\left(-t\cos t - \int -\cos t dt\right) = \frac{1}{8}\left(-t\cos t + \int \cos t dt\right) = -\frac{1}{8}t\cos t + \frac{1}{8}\sin t$

$= -\frac{1}{8}(2x)\cos(2x) + \frac{1}{8}\sin(2x) = -\frac{1}{4}x\cos 2x + \frac{1}{8}\sin 2x + C$

1222*

$\int (x^2 + 5x + 6)\cos 2x dx$

$\left\{\begin{array}{c} 2x = t \\ 2dx = dt \end{array}\right\}$

$= \int\left(\frac{t^2}{4} + \frac{5}{2}t + 6\right)\cos t\frac{dt}{2} = \frac{1}{8}\int t^2\cos t dt + \frac{5}{4}\int t\cos t dt + 3\int \cos t dt$

$= 3\sin t + \frac{1}{8}\int t^2\cos t dt + \frac{5}{4}\int t\cos t dt$

$\left\{\begin{array}{cc} t^2 = u & \cos t dt = dv \\ 2t dt = du & \sin t = v \end{array}\right\}\left\{\begin{array}{cc} t = u & \cos t dt = dv \\ dt = du & \sin t = v \end{array}\right\}$

$I_a = t^2\sin t - 2\int t\sin t dt$

$I_b = t\sin t - \int \sin t dt = t\sin t + \cos t$

$I_{aa} = \int t\sin t dt$

$\left\{\begin{array}{cc} t = u & \sin t dt = dv \\ dt = du & -\cos t = v \end{array}\right\}$

$= -t\cos t - \int -\cos t dt = -t\cos t + \sin t$

$I_a = t^2\sin t - 2(-t\cos t + \sin t) = t^2\sin t + 2t\cos t - 2\sin t$

$I = 3\sin t + \frac{1}{8}I_a + \frac{5}{4}I_b = 3\sin t + \frac{1}{8}(t^2\sin t + 2t\cos t - 2\sin t) + \frac{5}{4}(t\sin t + \cos t) =$
$\frac{11}{4}\sin t + \frac{1}{8}t^2\sin t + \frac{1}{4}t\cos t + \frac{5}{4}t\sin t + \frac{5}{4}\cos t$

$= \frac{11}{4}\sin(2x) + \frac{1}{8}(2x)^2\sin(2x) + \frac{1}{4}(2x)\cos(2x) + \frac{5}{4}(2x)\sin(2x) + \frac{5}{4}\cos(2x)$

$= \frac{11}{4}\sin 2x + \frac{1}{2}x^2\sin 2x + \frac{1}{2}x\cos 2x + \frac{5}{2}x\sin 2x + \frac{5}{4}\cos 2x + C$

1223

$\int x^2\ln x dx$

$\left\{\begin{array}{cc} \ln x = u & x^2dx = dv \\ \frac{dx}{x} = du & \frac{x^3}{3} = v \end{array}\right\}$

$= \frac{x^3}{3}\ln x - \int \frac{x^3}{3}\frac{dx}{x} = \frac{x^3}{3}\ln x - \frac{1}{3}\int x^2dx = \frac{1}{3}x^3\ln x - \frac{1}{9}x^3 + C$

1224

$\int \ln^2 x dx$

$\left\{\begin{array}{cc} \ln^2 x = u & dx = dv \\ \frac{2\ln x}{x}dx = du & x = v \end{array}\right\}$

$= x\ln^2 x - \int x\frac{2\ln x}{x}dx = x\ln^2 x - 2\int \ln x dx$

$I_1 = \int \ln x dx$

$\left\{\begin{array}{cc} \ln x = u & dx = dv \\ \frac{dx}{x} = du & x = v \end{array}\right\}$

$I_1 = x\ln x - \int x\frac{dx}{x} = x\ln x - x$

$I = x\ln^2 x - 2(x\ln x - x) = x\ln^2 x - 2x\ln x + 2x + C$

1225

$\int \frac{\ln x}{x^3}dx$

$\left\{\begin{array}{cc} \ln x = u & x^{-3}dx = dv \\ \frac{dx}{x} = du & \frac{x^{-2}}{-2} = v \end{array}\right\}$

$= \ln x\frac{x^{-2}}{-2} - \int \frac{x^{-2}}{-2}\frac{dx}{x} = -\frac{1}{2}\frac{\ln x}{x^2} + \frac{1}{2}\int \frac{dx}{x^3} = -\frac{1}{2}\frac{\ln x}{x^2} - \frac{1}{4x^2} + C$

1226

$\int \frac{\ln x}{\sqrt{x}}dx$

$\left\{\begin{array}{cc} \ln x = u & \frac{dx}{\sqrt{x}} = dv \\ \frac{dx}{x} = du & 2\sqrt{x} = v \end{array}\right\}$

$= 2\sqrt{x}\ln x - 2\int \sqrt{x}\frac{dx}{x} = 2\sqrt{x}\ln x - 2\int \frac{dx}{\sqrt{x}} = 2\sqrt{x}\ln x - 4\sqrt{x} + C$

1227

$\int x\arctan x dx$

$\left\{\begin{array}{cc} \arctan x = u & x dx = dv \\ \frac{dx}{1+x^2} = du & \frac{x^2}{2} = v \end{array}\right\}$

$= \frac{x^2}{2}\arctan x - \int \frac{x^2}{2}\frac{dx}{1+x^2} = \frac{x^2}{2}\arctan x - \frac{1}{2}\int \frac{x^2}{1+x^2}dx = \frac{x^2}{2}\arctan x - \frac{1}{2}\int \frac{x^2+1-1}{1+x^2}dx$

$= \frac{x^2}{2}\arctan x - \frac{1}{2}\int dx + \frac{1}{2}\int \frac{dx}{1+x^2} = \frac{1}{2}x^2\arctan x - \frac{1}{2}x + \frac{1}{2}\arctan x + C$

1228

$\int x\arcsin x dx$

$$\left\{ \begin{array}{ll} xdx = dv & \arcsin x = u \\ \frac{1}{2}x^2 = v & \frac{dx}{\sqrt{1-x^2}} = du \end{array} \right\}$$

$$= \frac{1}{2}x^2 \arcsin x - \int \frac{1}{2}x^2 \frac{dx}{\sqrt{1-x^2}}$$

$$= \frac{1}{2}x^2 \arcsin x + \frac{1}{2}\int \frac{1-x^2-1}{\sqrt{1-x^2}}dx = \frac{1}{2}x^2 \arcsin x + \frac{1}{2}\int \sqrt{1-x^2}\,dx - \frac{1}{2}\int \frac{dx}{\sqrt{1-x^2}}$$

$$= \frac{1}{2}x^2 \arcsin x + \frac{1}{4}x\sqrt{(1-x^2)} - \frac{1}{4}\arcsin x + C$$

1229

$$\int \ln\left(x + \sqrt{1+x^2}\right)dx$$

$$\left\{ \begin{array}{ll} dx = dv & \ln\left(x+\sqrt{1+x^2}\right) = u \\ x = v & \frac{dx}{\sqrt{1+x^2}} = du \end{array} \right\}$$

$$= x\ln\left(x+\sqrt{1+x^2}\right) - \int x\frac{dx}{\sqrt{1+x^2}} = x\ln\left(x+\sqrt{1+x^2}\right) - \int \frac{x}{\sqrt{1+x^2}}dx$$

$$= x\ln\left(x+\sqrt{1+x^2}\right) - \int \frac{x}{\sqrt{1+x^2}}dx$$

$$\left\{ \begin{array}{l} 1+x^2 = t \\ 2xdx = dt \end{array} \right\}$$

$$= x\ln\left(x+\sqrt{1+x^2}\right) - \int \frac{\frac{dt}{2}}{\sqrt{t}}$$

$$= x\ln\left(x+\sqrt{(1+x^2)}\right) - \sqrt{t} = x\ln\left(x+\sqrt{(1+x^2)}\right) - \sqrt{1+x^2} + C$$

1230

$$\int \frac{xdx}{\sin^2 x}$$

$$\left\{ \begin{array}{ll} x = u & \frac{dx}{\sin^2 x} = dv \\ dx = du & -\cot x = v \end{array} \right\}$$

$$= -x\cot x - \int -\cot x dx$$

$$= -x\cot x + \int \cot x dx = -x\cot x + \ln(\sin x) + C$$

1231

$$I = \int \frac{x\cos x}{\sin^2 x}$$

$$\left\{ \begin{array}{ll} \frac{\cos x}{\sin^2 x}dx = dv & x = u \\ -\frac{1}{\sin x} = v & dx = du \end{array} \right\}$$

$$I_1 = \int \frac{\cos x}{\sin^2 x}$$

$$\left\{ \begin{array}{l} \sin x = t \\ \cos x dx = dt \end{array} \right\}$$

$$= \int \frac{dt}{t^2} = -\frac{1}{t} = -\frac{1}{\sin x}$$

$$I = -x\frac{1}{\sin x} - \int -\frac{1}{\sin x}dx = -\frac{x}{\sin x} + \int \frac{dx}{\sin x} = -\frac{x}{\sin x} + \ln\tan\frac{x}{2} + C$$

1232

$$\int e^x \sin x dx$$

$$\left\{ \begin{array}{ll} e^x dx = dv & \sin x = u \\ e^x = v & \cos x dx = du \end{array} \right\}$$

$$I = e^x \sin x - \int e^x \cos x dx$$

$$I_1 = \int e^x \cos x dx$$

$$\left\{ \begin{array}{ll} e^x dx = dv & \cos x = u \\ e^x = v & -\sin x dx = du \end{array} \right\}$$

$$= e^x \cos x - \int e^x - \sin x dx$$

$$I_1 = e^x \cos x + \int e^x \sin x dx$$

$$I = e^x \sin x - \left(e^x \cos x + \int e^x \sin x dx\right) = e^x \sin x - e^x \cos x - I$$

$$2I = e^x \sin x - e^x \cos x$$

$$I = \frac{e^x \sin x - e^x \cos x}{2} + C$$

$$\int e^x \sin x dx$$

$$\left\{ \begin{array}{ll} e^x = u & \sin x dx = dv \\ e^x dx = du & -\cos x = v \end{array} \right\}$$

$$= -e^x \cos x - \int -\cos x e^x dx$$

$$= -e^x \cos x + \int \cos x e^x dx$$

$$\left\{ \begin{array}{ll} e^x = u & \cos x dx = dv \\ e^x dx = du & \sin x = v \end{array} \right\}$$

$$I = -e^x \cos x + \left(e^x \sin x - \int \sin x e^x dx\right)$$

$$I = -e^x \cos x + e^x \sin x - I$$

$$2I = -e^x \cos x + e^x \sin x$$

$$I = \frac{-e^x \cos x + e^x \sin x}{2} + C$$

1233

$$\int 3^x \cos x dx$$

$$\left\{ \begin{array}{ll} 3^x dx = dv & \cos x = u \\ \frac{3^x}{\ln 3} = v & -\sin x = du \end{array} \right\}$$

$$= \frac{3^x}{\ln 3}\cos x - \int -\sin x \frac{3^x}{\ln 3}dx$$

$$I = \frac{3^x}{\ln 3}\cos x + \frac{1}{\ln 3}\int 3^x \sin x dx$$

$$\left\{ \begin{array}{ll} 3^x dx = dv & \sin x = u \\ \frac{3^x}{\ln 3} = v & \cos x = du \end{array} \right\}$$

$$I = \frac{3^x}{\ln 3}\cos x + \frac{1}{\ln 3}\left(\sin x\frac{3^x}{\ln 3} - \int \frac{3^x}{\ln 3}\cos x\right) = \frac{3^x}{\ln 3}\cos x + \frac{3^x}{\ln^2 3}\sin x - \frac{1}{\ln^2 3}\int 3^x \cos x$$

$$I = \frac{3^x}{\ln 3}\cos x + \frac{3^x}{\ln^2 3}\sin x - \frac{1}{\ln^2 3}I$$

$$I\left(1 + \frac{1}{\ln^2 3}\right) = \frac{3^x}{\ln 3}\cos x + \frac{3^x}{\ln^2 3}\sin x$$

$$I = \frac{\frac{3^x}{\ln 3}\cos x + \frac{3^x}{\ln^2 3}\sin x}{\frac{\ln^2 3+1}{\ln^2 3}} = \frac{\frac{3^x}{\ln 3}\cos x + \frac{3^x}{\ln^2 3}\sin x}{\ln^2 3+1}\ln^2 3 = 3^x\frac{\cos x\ln 3+\sin x}{\ln^2 3+1} + C$$

1234

$$\int e^{ax}\sin bx dx$$

$$\left\{\begin{array}{c} bx = t \\ bdx = dt \end{array}\right\}$$

$$= \int e^{\frac{at}{b}}\sin t\frac{dt}{b} = \frac{1}{b}\int e^{\frac{at}{b}}\sin t dt$$

$$\left\{\begin{array}{cc} e^{\frac{at}{b}} = u & \sin t dt = dv \\ \frac{a}{b}e^{a\frac{t}{b}}dt = du & -\cos t = v \end{array}\right\}$$

$$= \frac{1}{b}\left(-e^{\frac{at}{b}}\cos t + \frac{a}{b}\int e^{a\frac{t}{b}}\cos t dt\right)$$

$$\left\{\begin{array}{cc} e^{\frac{at}{b}} = u & \cos t dt = dv \\ \frac{a}{b}e^{a\frac{t}{b}}dt = du & \sin t = v \end{array}\right\}$$

$$I = \left(-\frac{1}{b}e^{\frac{at}{b}}\cos t + \frac{a}{b^2}\left(e^{\frac{at}{b}}\sin t - a\frac{1}{b}\int e^{a\frac{t}{b}}\sin t dt\right)\right)$$

$$I = \left(-\frac{1}{b}e^{\frac{at}{b}}\cos t + \frac{a}{b^2}\left(e^{\frac{at}{b}}\sin t - aI\right)\right)$$

$$I = \left(-\frac{1}{b}e^{\frac{at}{b}}\cos t + \frac{a}{b^2}e^{\frac{at}{b}}\sin t - \frac{a^2}{b^2}I\right)$$

$$I\left(1 + \frac{a^2}{b^2}\right) = -\frac{1}{b}e^{\frac{at}{b}}\cos t + \frac{a}{b^2}e^{\frac{at}{b}}\sin t$$

$$I = \frac{-\frac{1}{b}e^{\frac{at}{b}}\cos t + \frac{a}{b^2}e^{\frac{at}{b}}\sin t}{\frac{a^2+b^2}{b^2}} = e^{a\frac{t}{b}}\frac{-b\cos t+a\sin t}{a^2+b^2} = e^{a\frac{(bx)}{b}}\frac{-b\cos(bx)+a\sin(bx)}{a^2+b^2}$$

$$I = \frac{-b\cos bx+a\sin bx}{a^2+b^2}e^{ax} + C$$

1235

$$\int \sin(\ln x)dx$$

$$\left\{\begin{array}{c} \ln x = t \\ \frac{dx}{x} = dt \end{array}\right\}$$

$$= \int x\sin t dt = \int e^t\sin t dt$$

$$\left\{\begin{array}{cc} e^t dt = dv & \sin t = u \\ e^t = v & \cos t dt = du \end{array}\right\}$$

$$I = e^t\sin t - \int e^t\cos t dt$$

$$I_1 = \int e^t\cos t dt$$

$$\left\{\begin{array}{cc} e^t dt = dv & \cos t = u \\ e^t = v & -\sin t dt = du \end{array}\right\}$$

$$I_1 = e^t\cos t - \int e^t - \sin t dt = e^t\cos t + \int e^t\sin t dt$$

$$I = e^t\sin t - \left(e^t\cos t + \int e^t\sin t dt\right) = e^t\sin t - e^t\cos t - I$$

$$2I = e^t\sin t - e^t\cos t$$

$$I = \frac{e^t\sin t - e^t\cos t}{2} = \frac{e^{\ln x}\sin\ln x - e^{\ln x}\cos\ln x}{2} = \frac{1}{2}x\sin(\ln x) - \frac{1}{2}x\cos(\ln x) + C$$

1236

$$\int x^3e^{-x^2}dx$$

$$\left\{\begin{array}{c} -x^2 = t \\ -2xdx = dt \end{array}\right\}$$

$$= \int x^3e^t\frac{dt}{-2x} = \frac{1}{2}\int -x^2e^t dt = \frac{1}{2}\int te^t dt$$

$$\left\{\begin{array}{cc} t = u & e^t dt = dv \\ dt = du & e^t = v \end{array}\right\}$$

$$= \frac{1}{2}\left(te^t - \int e^t dt\right) = \frac{1}{2}te^t - \frac{1}{2}e^t$$

$$= \frac{1}{2}(-x^2)e^{(-x^2)} - \frac{1}{2}e^{(-x^2)} = -\frac{1}{2}x^2e^{-x^2} - \frac{1}{2}e^{-x^2} + C$$

1237

$$\int e^{\sqrt{x}}dx$$

$$\left\{\begin{array}{c} \sqrt{x} = t|^2 \\ x = t^2 \\ dx = 2tdt \end{array}\right\}$$

$$= 2\int e^t t dt$$

$$\left\{\begin{array}{cc} t = u & e^t dt = dv \\ dt = du & e^t = v \end{array}\right\}$$

$$= 2\left(te^t - \int e^t dt\right) = 2te^t - 2e^t = 2\sqrt{x}e^{\sqrt{x}} - 2e^{\sqrt{x}} + C$$

1238

$$\int(x^2 - 2x + 3)\ln x dx$$

$$\left\{\begin{array}{cc} (x^2 - 2x + 3)dx = dv & \ln x = u \\ \frac{x^3}{3} - x^2 + 3x = v & \frac{dx}{x} = du \end{array}\right\}$$

$$= \left(\frac{x^3}{3} - x^2 + 3x\right)\ln x - \int\left(\frac{x^3}{3} - x^2 + 3x\right)\left(\frac{dx}{x}\right) = \left(\frac{x^3}{3} - x^2 + 3x\right)\ln x - \int\left(\frac{x^2}{3} - x + 3\right)dx$$

$$= \left(\frac{x^3}{3} - x^2 + 3x\right)\ln x - \frac{x^3}{9} + \frac{x^2}{2} - 3x + C$$

1239

$$\int x\ln\frac{1-x}{1+x}dx$$

$$\left\{\begin{array}{cc} \ln\frac{1-x}{1+x} = u & dv = xdx \\ \frac{2}{x^2-1}dx = du & v = \frac{x^2}{2} \end{array}\right\}$$

$$= \frac{x^2}{2}\ln\frac{1-x}{1+x} - \int\frac{x^2}{2}\frac{2}{x^2-1} = \frac{x^2}{2}\ln\frac{1-x}{1+x} - \int\frac{x^2}{x^2-1} = \frac{x^2}{2}\ln\frac{1-x}{1+x} - \int\frac{x^2-1+1}{x^2-1}$$

$$= \frac{x^2}{2}\ln\frac{1-x}{1+x} - \int dx - \int\frac{dx}{x^2-1} = \frac{1}{2}x^2\ln\frac{1-x}{1+x} - x + \operatorname{arctanh}x + C$$

1240

$$\int\frac{\ln^2 x}{x^2}dx$$

$$\left\{\begin{array}{ll} \ln^2 x = u & dv = \frac{dx}{x^2} \\ \frac{2\ln x}{x}dx = du & v = -\frac{1}{x} \end{array}\right\}$$

$$= -\frac{1}{x}\ln^2 x - \int(-\frac{1}{x})\frac{2\ln x}{x}dx$$

$$= -\frac{1}{x}\ln^2 x + 2\int\frac{\ln x}{x^2}dx$$

$$\left\{\begin{array}{ll} \ln x = u & dv = \frac{dx}{x^2} \\ \frac{dx}{x} = du & v = -\frac{1}{x} \end{array}\right\}$$

$$= -\frac{1}{x}\ln^2 x + 2\left(-\frac{1}{x}\ln x - \int(-\frac{1}{x})\frac{dx}{x}\right)$$

$$= -\frac{1}{x}\ln^2 x + 2\left(-\frac{1}{x}\ln x + \int\frac{dx}{x^2}\right) = -\frac{1}{x}\ln^2 x - \frac{2}{x}\ln x - \frac{2}{x} + C$$

1241

$$\int\frac{\ln(\ln x)}{x}dx$$

$$\left\{\begin{array}{ll} \ln(\ln x) = u & \frac{dx}{x} = dv \\ \frac{dx}{x\ln x} = du & \ln x = v \end{array}\right\}$$

$$= \ln x\ln(\ln x) - \int\ln x\frac{dx}{x\ln x} = \ln x\ln(\ln x) - \int\frac{dx}{x} = \ln x\ln(\ln x) - \ln x + C$$

1242

$$\int x^2\arctan 3x dx$$

$$\left\{\begin{array}{l} 3x = t \\ 3dx = dt \end{array}\right\}$$

$$= \int\frac{t^2}{9}\arctan t\frac{dt}{3} = \frac{1}{27}\int t^2\arctan t dt$$

$$\left\{\begin{array}{ll} \arctan t = u & t^2 dt = dv \\ \frac{dt}{1+t^2} = du & \frac{t^3}{3} = v \end{array}\right\}$$

$$= \frac{1}{27}\left(\frac{t^3}{3}\arctan t - \int\frac{t^3}{3}\frac{dt}{1+t^2}\right)$$

$$= \frac{1}{27}\left(\frac{t^3}{3}\arctan t - \frac{1}{3}\int\frac{t^3}{1+t^2}dt\right) = \frac{1}{81}\left(t^3\arctan t - \int\frac{t^3}{1+t^2}dt\right)$$

$$\left\{\begin{array}{l} 1 + t^2 = z \\ 2tdt = dz \end{array}\right\}$$

$$= \frac{1}{81}\left(t^3\arctan t - \int\frac{t^3}{z}\frac{dz}{2t}\right)$$

$$= \frac{1}{81}\left(t^3\arctan t - \frac{1}{2}\int\frac{t^2}{z}dz\right) = \frac{1}{81}\left(t^3\arctan t - \frac{1}{2}\int\frac{z-1}{z}dz\right)$$

$$= \frac{1}{81}\left(t^3\arctan t - \frac{1}{2}\int dz + \frac{1}{2}\int\frac{dz}{z}\right) = \frac{1}{81}(t^3\arctan t - \frac{1}{2}z + \frac{1}{2}\ln z)$$

$$= \frac{1}{81}(t^3\arctan t - \frac{1}{2}(1+t^2) + \frac{1}{2}\ln(1+t^2))$$

$$= \frac{1}{81}\left((3x)^3\arctan(3x) - \frac{1}{2}\left(1+(3x)^2\right) + \frac{1}{2}\ln\left(1+(3x)^2\right)\right)$$

$$= \frac{1}{81}(27x^3\arctan 3x - \frac{1}{2} - \frac{9}{2}x^2 + \frac{1}{2}\ln(1+9x^2))$$

$$= \frac{1}{3}x^3\arctan 3x - \frac{1}{162} - \frac{1}{18}x^2 + \frac{1}{162}\ln(1+9x^2) + C$$

1243

$$\int x\arctan^2 x dx$$

$$\left\{\begin{array}{ll} \arctan^2 x = u & xdx = dv \\ \frac{2\arctan x}{1+x^2}dx = du & \frac{x^2}{2} = v \end{array}\right\}$$

$$= \frac{x^2}{2}\arctan^2 x - \int\frac{x^2}{2}\frac{2\arctan x}{1+x^2} = \frac{x^2}{2}\arctan^2 x - \int\frac{x^2\arctan x}{1+x^2}$$

$$\left\{\begin{array}{ll} \arctan x = u & \frac{x^2}{1+x^2}dx = dv \\ \frac{dx}{1+x^2} = du & \int\frac{x^2+1-1}{1+x^2}dx = x - \arctan x = v \end{array}\right\}$$

$$= \frac{x^2}{2}\arctan^2 x - \left((x - \arctan x)\arctan x - \int(x - \arctan x)\frac{dx}{1+x^2}\right)$$

$$= \frac{x^2}{2}\arctan^2 x - \left(x\arctan x - \arctan^2 x - \int\frac{x}{1+x^2}dx + \int\frac{\arctan x}{1+x^2}dx\right)$$

$$= \frac{x^2}{2}\arctan^2 x - x\arctan x + \arctan^2 x + I_1 - I_2$$

$$I_1 = \int\frac{x}{1+x^2}dx$$

$$\left\{\begin{array}{l} 1 + x^2 = t \\ 2xdx = dt \end{array}\right\}$$

$$= \frac{1}{2}\int\frac{dt}{t} = \frac{1}{2}\ln t = \frac{1}{2}\ln(1+x^2)$$

$$I_2 = \int\frac{\arctan x}{1+x^2}dx$$

$$\left\{\begin{array}{l} \arctan x = t \\ \frac{dx}{1+x^2} = dt \end{array}\right\}$$

$$= \int tdt = \frac{1}{2}t^2 = \frac{1}{2}\arctan^2 x$$

$$= \frac{x^2}{2}\arctan^2 x - x\arctan x + \arctan^2 x + \frac{1}{2}\ln(1+x^2) - \frac{1}{2}\arctan^2 x$$

$$= \frac{1}{2}x^2\arctan^2 x - x\arctan x + \frac{1}{2}\arctan^2 x + \frac{1}{2}\ln(1+x^2) + C$$

1244

$$I = \int(\arcsin x)^2 dx =$$

$$\left\{\begin{array}{ll} dv = dx & \arcsin^2 x = u \\ v = x & 2\frac{\arcsin x}{\sqrt{(1-x^2)}}dx = du \end{array}\right\}$$

$$I = x\arcsin^2 x - \int x2\frac{\arcsin x}{\sqrt{(1-x^2)}}dx$$

$$I_v = \int x\frac{\arcsin x}{\sqrt{(1-x^2)}}$$

$$\left\{\begin{array}{ll} \arcsin x = u & \frac{x}{\sqrt{(1-x^2)}}dx = dv \\ \frac{1}{\sqrt{1-x^2}}dx = du & -\sqrt{1-x^2} = v \end{array}\right\}$$

$$I_v = uv - \int vdu = -\sqrt{1-x^2}\arcsin x - \int -\sqrt{1-x^2}\frac{1}{\sqrt{1-x^2}}dx = -\sqrt{(1-x^2)}\arcsin x + x$$

$$I_v = \int\frac{x}{\sqrt{(1-x^2)}}$$

$$\left\{\begin{array}{l} 1 - x^2 = t \\ -2xdx = dt \end{array}\right\}$$

$$= \int \frac{-1}{2} \frac{dt}{\sqrt{t}} = -\frac{1}{2} \int t^{-\frac{1}{2}} dt = -\frac{1}{2} * 2t^{\frac{1}{2}} = -\sqrt{t} = -\sqrt{1-x^2}$$

$$I = x \arcsin^2 x - 2(-\sqrt{1-x^2} \arcsin x + x) = x \arcsin^2 x + 2\sqrt{1-x^2} \arcsin x - 2x + C$$

1245

$$\int \frac{\arcsin x}{x^2} dx$$

$$\left\{ \begin{array}{cc} \arcsin x = u & \frac{dx}{x^2=} = dv \\ \frac{dx}{\sqrt{1-x^2}} = du & -\frac{1}{x} = v \end{array} \right\} \left\{ \begin{array}{c} \sqrt{1-x^2} = t \\ 1-x^2 = t^2 \\ -xdx = tdt \end{array} \right\}$$

$$= -\frac{1}{x} \arcsin x - \int -\frac{1}{x} \frac{dx}{\sqrt{1-x^2}} = -\frac{1}{x} \arcsin x + \int \frac{dx}{x\sqrt{1-x^2}}$$

$$I_1 = \int \frac{-\frac{tdt}{x}}{xt} = -\int \frac{\frac{dt}{x}}{x} = -\int \frac{dt}{x^2} = -\int \frac{dt}{(1-t^2)} = -\operatorname{arctanh} t = -\operatorname{arctanh} \sqrt{1-x^2}$$

$$I = -\frac{1}{x} \arcsin x + I_1 = -\frac{1}{x} \arcsin x - \operatorname{arctanh} \sqrt{1-x^2} + C$$

1246

$$\int \frac{\arcsin\sqrt{x}}{\sqrt{1-x}} dx$$

$$\left\{ \begin{array}{c} \sqrt{x} = t \\ x = t^2 \\ dx = 2tdt \end{array} \right\}$$

$$I = 2\int \frac{t \arcsin t}{\sqrt{1-t^2}} dt$$

$$\left\{ \begin{array}{cc} \frac{\arcsin t}{\sqrt{1-t^2}} dt = dv & t = u \\ \frac{1}{2} \arcsin^2 t = v & dt = du \end{array} \right\}$$

$$I = 2\left(t\frac{1}{2} \arcsin^2 t - \int \frac{1}{2} \arcsin^2 t dt\right) = t \arcsin^2 t - \int \arcsin^2 t dt$$

$$I_1 = \int \arcsin^2 t dt$$

$$\left\{ \begin{array}{cc} \arcsin^2 t = u & dt = dv \\ \frac{2\arcsin t}{\sqrt{1-t^2}} dt = du & t = v \end{array} \right\}$$

$$= t \arcsin^2 t - 2\int \frac{t \arcsin t}{\sqrt{1-t^2}} dt$$

$$\left\{ \begin{array}{cc} \arcsin t = u & \frac{tdt}{\sqrt{1-t^2}} = dv \\ \frac{dt}{\sqrt{1-t^2}} = du & v = -\sqrt{1-t^2} \end{array} \right\}$$

$$I_2 = -\sqrt{1-t^2} \arcsin t - \int -\sqrt{1-t^2} \frac{dt}{\sqrt{1-t^2}} = -\arcsin t\sqrt{1-t^2} + \int dt$$

$$I_2 = -\arcsin t\sqrt{1-t^2} + t$$

$$I_1 = t \arcsin^2 t - 2\left(-\arcsin t\sqrt{1-t^2} + t\right) = t \arcsin^2 t + 2\arcsin t\sqrt{1-t^2} - 2t$$

$$I = t \arcsin^2 t - I_1 = t \arcsin^2 t - \left(t \arcsin^2 t + 2\arcsin t\sqrt{1-t^2} - 2t\right)$$

$$I = -2\arcsin t\sqrt{1-t^2} + 2t = -2\arcsin(\sqrt{x})\sqrt{1-(\sqrt{x})^2} + 2\sqrt{x}$$

$$I = -2\sqrt{1-x} \arcsin\sqrt{x} + 2\sqrt{x} + C$$

1247

$$\int x \tan^2 2x dx$$

$$\left\{ \begin{array}{c} 2x = t \\ 2dx = dt \end{array} \right\}$$

$$= \frac{1}{4} \int t \tan^2 t dt$$

$$\left\{ \begin{array}{cc} \tan^2 t = dv & u = t \\ (\tan t - t) = v & du = dt \end{array} \right\}$$

$$I = \frac{1}{4}\left(t(\tan t - t) - \int (\tan t - t) dt\right) = \frac{1}{4}\left(t \tan t - t^2 - \int \tan t dt + \int t dt\right)$$

$$I = \frac{1}{4}(t \tan t - \frac{1}{2} t^2 + \ln(\cos t)) = \frac{1}{4}(2x \tan 2x - 2x^2 + \ln(\cos 2x)) + C$$

1248

$$\int \frac{\sin^2 x}{e^x} dx$$

$$\left\{ \begin{array}{cc} \frac{dx}{e^x} = dv & \sin^2 x = u \\ -e^{-x} = v & 2\sin x \cos x dx = \sin 2x dx = du \end{array} \right\}$$

$$I = -e^{-x} \sin^2 x + \int e^{-x} \sin 2x dx = -e^{-x} \sin^2 x + I_1$$

$$I_1 = \int e^{-x} \sin 2x dx$$

$$\left\{ \begin{array}{cc} \sin 2x = u & \frac{dx}{e^x} = dv \\ 2\cos 2x dx = du & -e^{-x} = v \end{array} \right\}$$

$$= -e^{-x} \sin 2x + 2\int e^{-x} \cos 2x dx$$

$$I_1 = -e^{-x} \sin 2x + 2I_2$$

$$\left\{ \begin{array}{cc} \cos 2x = u & \frac{dx}{e^x} = dv \\ -2\sin 2x dx = du & -e^{-x} = v \end{array} \right\}$$

$$I_2 = -e^{-x} \cos 2x - 2\int e^{-x} \sin 2x dx = -e^{-x} \cos 2x - 2I_1$$

$$I_1 = -e^{-x} \sin 2x + 2(-e^{-x} \cos 2x - 2I_1) =$$

$$- e^{-x} \sin 2x - 2e^{-x} \cos 2x - 4I_1 = \frac{-e^{-x} \sin 2x - 2e^{-x} \cos 2x}{5}$$

$$I = -e^{-x} \sin^2 x + I_1 = -e^{-x} \sin^2 x + \frac{-e^{-x} \sin 2x - 2e^{-x} \cos 2x}{5} + C$$

1249

$$\int \cos^2(\ln x) dx$$

$$\left\{ \begin{array}{cc} \cos^2(\ln x) = u & dx = dv \\ -2\cos \ln x \sin \ln x \frac{dx}{x} = du & x = v \end{array} \right\}$$

$$I = x\cos^2(\ln x) - \int x(-2\cos \ln x \sin \ln x) \frac{dx}{x} = x\cos^2(\ln x) + \int x 2\cos \ln x \sin \ln x \frac{dx}{x}$$

$$I = x\cos^2(\ln x) + \int \sin(2\ln x) dx = x\cos^2(\ln x) + I_1$$

$$\left\{ \begin{array}{cc} \sin(2\ln x) = u & dx = dv \\ \frac{2}{x}\cos(2\ln x) dx = du & x = v \end{array} \right\}$$

$$I_1 = \int \sin(2\ln x) dx = x \sin(2\ln x) - \int x \frac{2}{x} \cos(2\ln x) dx$$

$I_1 = x\sin(2\ln x) - 2\int \cos(2\ln x)dx = x\sin(2\ln x) - 2I_2$

$I_2 = \int \cos(2\ln x)dx$

$\left\{ \begin{array}{cc} \cos(2\ln x) = u & dv = dx \\ -\frac{2}{x}\sin(2\ln x)dx = du & v = x \end{array} \right\}$

$I_2 = x\cos(2\ln x) - \int x(-\frac{2}{x}\sin(2\ln x))dx = x\cos(2\ln x) + 2\int \sin(2\ln x)dx$

$I_2 = x\cos(2\ln x) + 2I_1$

$I_1 = x\sin(2\ln x) - 2(x\cos(2\ln x) + 2I_1) = x\sin(2\ln x) - 2x\cos(2\ln x) - 4I_1$

$I_1 = \frac{x\sin(2\ln x) - 2x\cos(2\ln x)}{5}$

$I = x\cos^2(\ln x) + \frac{x\sin(2\ln x) - 2x\cos(2\ln x)}{5} + C$

1250**

$\int \frac{x^2}{(x^2+1)^2}dx$

$\left\{ \begin{array}{cc} x = u & \frac{xdx}{(x^2+1)^2} = dv \\ dx = du & -\frac{1}{2(1+x^2)} = v \end{array} \right\}$

$= -\frac{1}{2(1+x^2)}x - \int\left(-\frac{1}{2(1+x^2)}\right)dx$

$= -\frac{x}{2(1+x^2)} + \frac{1}{2}\int \frac{1}{1+x^2}dx = -\frac{x}{2+2x^2} + \frac{1}{2}\arctan x + C$

1251*

$\int \frac{dx}{(x^2+a^2)^2} = \frac{1}{a^2}\int \frac{x^2+a^2-x^2}{(x^2+a^2)^2}dx = \frac{1}{a^2}\int \frac{dx}{(x^2+a^2)} - \frac{1}{a^2}\int \frac{x^2}{(x^2+a^2)^2}dx$

$I = \frac{1}{a^3}\arctan\frac{x}{a} - \frac{1}{a^2}\int \frac{x^2}{(x^2+a^2)^2}dx$

$\left\{ \begin{array}{c} x = at \\ dx = adt \end{array} \right\}$

$I = \frac{1}{a^3}\arctan\frac{x}{a} - \frac{1}{a^2}\int \frac{a^2t^2}{(a^2t^2+a^2)^2}adt = \frac{1}{a^3}\arctan\frac{x}{a} - a\int \frac{t^2}{a^4(t^2+1)^2}dt$

$I = \frac{1}{a^3}\arctan\frac{x}{a} - \frac{1}{a^3}\int \frac{t^2}{(t^2+1)^2}dt$

$\left\{ \begin{array}{cc} \frac{t}{(t^2+1)^2}dt = dv & t = u \\ -\frac{1}{2(1+t^2)} = v & dt = du \end{array} \right\}$

$I = \frac{1}{a^3}\arctan\frac{x}{a} - \frac{1}{a^3}\left(-\frac{1}{2(1+t^2)}t + \int \frac{1}{2(1+t^2)}dt\right) = \frac{1}{a^3}\arctan\frac{x}{a} - \frac{1}{2a^3}\left(-\frac{t}{1+t^2} + \int \frac{dt}{1+t^2}\right)$

$I = \frac{1}{a^3}\arctan\frac{x}{a} - \frac{1}{2a^3}\left(-\frac{t}{1+t^2} + \arctan t\right)$

$I = \frac{1}{a^3}\arctan\frac{x}{a} - \frac{1}{2a^3}\left(-\frac{(\frac{x}{a})}{1+(\frac{x}{a})^2} + \arctan(\frac{x}{a})\right) + C$

1252**

$\int \sqrt{a^2 - x^2}\,dx$

$\left\{ \begin{array}{c} x = a\sin t \\ dx = -a\cos t dt \end{array} \right\}$

$= \int \sqrt{a^2 - a^2\sin^2 t}\,(-a\cos t)dt$

$= -a^2\int \sqrt{1-\sin^2 t}\,(\cos t)dt = -a^2\int \cos^2 t dt = -\frac{a^2}{2}\int(1+\cos 2t)dt$

$= -\frac{a^2}{2}t - \frac{a^2}{2}\int \cos 2t dt = -\frac{1}{2}a^2 t - \frac{1}{4}a^2\sin 2t =$

$-\frac{1}{2}a^2\arcsin\frac{x}{a} - \frac{1}{4}a^2\sin(2\arcsin\frac{x}{a}) + C$

1253*

$\int \sqrt{A + x^2}$

$\left\{ \begin{array}{c} x = \sqrt{A}\sinh t \\ dx = \sqrt{A}\cosh t dt \end{array} \right\}$

$= \int \sqrt{A + A\sinh^2 t}\,\sqrt{A}\cosh t dt = A\int \sqrt{1+\sinh^2 t}\cosh t dt = A\int \cosh^2 t dt = \frac{A}{2}\int(\cosh 2t + 1$

$= \frac{A}{4}\sinh 2t + \frac{A}{2}t = \frac{A}{4}\sinh 2\left(\arcsin\frac{x}{\sqrt{A}}\right) + \frac{A}{2}\left(\arcsin\frac{x}{\sqrt{A}}\right) + C$

$\int \sqrt{A + x^2} = \int \frac{A+x^2}{\sqrt{A+x^2}}$

$\int \frac{A+x^2}{\sqrt{A+x^2}} = (ax+b)\sqrt{A+x^2} + c\int \frac{dx}{\sqrt{A+x^2}}|'$

$\frac{A+x^2}{\sqrt{A+x^2}} = a\sqrt{A+x^2} + \frac{2x(ax+b)}{2\sqrt{A+x^2}} + \frac{c}{\sqrt{A+x^2}}|\sqrt{A+x^2}$

$A + x^2 = a(A+x^2) + x(ax+b) + c$

$A + x^2 = aA + 2ax^2 + xb + c$

$aA + c = A$

$2a = 1$

$c = A - aA = A(1-a) = \frac{A}{2}$

$\{b = 0, a = \frac{1}{2}, c = \frac{A}{2}\}$

$I = (\frac{1}{2}x)\sqrt{A+x^2} + \frac{A}{2}\int \frac{dx}{\sqrt{A+x^2}}$

$I = \frac{1}{2}x\sqrt{A+x^2} + \frac{1}{2}A\ln\left(x + \sqrt{A+x^2}\right) + C$

1254*

$\int \frac{x^2}{\sqrt{9-x^2}}dx = -\int \frac{9-x^2-9}{\sqrt{9-x^2}}dx = -\int \sqrt{9-x^2} + 9\int \frac{dx}{\sqrt{9-x^2}} = -\int \sqrt{9-x^2} + 9\arcsin\frac{x}{3}$

$\int \frac{9-x^2}{\sqrt{9-x^2}} = (ax+b)\sqrt{9-x^2} + \int \frac{c}{\sqrt{9-x^2}}dx|'$

$\frac{9-x^2}{\sqrt{9-x^2}} = a\sqrt{9-x^2} - \frac{(ax+b)x}{\sqrt{9-x^2}} + \frac{c}{\sqrt{9-x^2}}|\sqrt{9-x^2}$

$9 - x^2 = a(9-x^2) - (ax+b)x + c$

$9 - x^2 = 9a - 2ax^2 - xb + c$

$9 - x^2 = x^2(-2a) + x(-b) + c + 9a$

$c + 9a = 9\ldots c + \frac{9}{2} = 9$

$\left\{ a = \frac{1}{2}\ ,\ b = 0\ ,\ c = \frac{9}{2} \right\}$

$I_1 = \frac{x}{2}\sqrt{9-x^2} + \frac{9}{2}\int \frac{dx}{\sqrt{9-x^2}} = \frac{x}{2}\sqrt{9-x^2} + \frac{9}{2}\arcsin\frac{1}{3}x$

$I = 9\arcsin\frac{x}{3} - I_1 = 9\arcsin\frac{x}{3} - \left(\frac{x}{2}\sqrt{9-x^2} + \frac{9}{2}\arcsin\frac{x}{3}\right)$

$I = \frac{9}{2}\arcsin\frac{x}{3} - \frac{x}{2}\sqrt{9-x^2} + C$

1255

$\int\frac{dx}{x^2+2x+5} = \int\frac{dx}{x^2+2x+1+4} = \int\frac{dx}{(x+1)^2+4} = \frac{1}{2}\arctan\frac{x+1}{2} + C$

1256

$\int\frac{dx}{x^2+2x} = \int\frac{dx}{x^2+2x+1-1} = \int\frac{dx}{(x+1)^2-1} = -\operatorname{arctanh}(x+1) + C$

1257

$\int\frac{dx}{3x^2-x+1} = \frac{1}{3}\int\frac{dx}{x^2-\frac{1}{3}x+\frac{1}{3}} = \frac{1}{3}\int\frac{dx}{\left(x-\frac{1}{6}\right)^2-\frac{1}{36}+\frac{1}{3}} = \frac{1}{3}\int\frac{dx}{\left(x-\frac{1}{6}\right)^2+\frac{11}{36}} = \frac{2}{\sqrt{11}}\arctan\frac{(6x-1)}{\sqrt{11}} + C$

1258

$\int\frac{xdx}{x^2-7x+13} = \int\frac{xdx}{\left(x-\frac{7}{2}\right)^2-\frac{49}{4}+13} = \int\frac{x-\frac{7}{2}}{\left(x-\frac{7}{2}\right)^2+\frac{3}{4}}dx + \frac{7}{2}\int\frac{dx}{\left(x-\frac{7}{2}\right)^2+\frac{3}{4}}$

$\left\{\begin{array}{c} x-\frac{7}{2} = t \\ dx = dt \end{array}\right\}$

$= \int\frac{t}{t^2+\frac{3}{4}}dt + \frac{7}{2}\int\frac{dx}{\left(x-\frac{7}{2}\right)^2+\frac{3}{4}} = \frac{1}{2}\ln(4t^2+3) + \frac{7}{3}\sqrt{3}\arctan\frac{1}{3}(2x-7)\sqrt{3}$

$= \frac{1}{2}\ln\left(4\left(x-\frac{7}{2}\right)^2+3\right) + \frac{7}{\sqrt{3}}\arctan\frac{2x-7}{\sqrt{3}} =$

$\ln 2 + \frac{1}{2}\ln(x^2-7x+13) + \frac{7}{\sqrt{3}}\arctan\frac{2x-7}{\sqrt{3}} + C$

1259

$\int\frac{3x-2}{x^2-4x+3}dx = \int\frac{3x-2}{(x-2)^2-4+3}dx = \int\frac{3x-2}{(x-2)^2-1}dx = 3\int\frac{xdx}{(x-2)^2-1} - 2\int\frac{dx}{(x-2)^2-1}$

$= 3\int\frac{x-2+2}{(x-2)^2-1}dx - 2\int\frac{dx}{(x-2)^2-1} = 3\int\frac{x-2}{(x-2)^2-1}dx + 6\int\frac{dx}{(x-2)^2-1} - 2\int\frac{dx}{(x-2)^2-1}$

$= 3\int\frac{x-2}{(x-2)^2-1}dx + 4\int\frac{dx}{(x-2)^2-1}$

$\left\{\begin{array}{c} x-2 = t \\ dx = dt \end{array}\right\}\left\{\begin{array}{c} t^2-1 = z \\ 2tdt = dz \end{array}\right\}$

$= 3\int\frac{t}{t^2-1}dt + 4\int\frac{dt}{t^2-1} = 3\int\frac{t}{t^2-1}dt - 4\operatorname{arctanh}t = \frac{3}{2}\int\frac{dz}{z} - 4\operatorname{arctanh}t$

$= \frac{3}{2}\ln z - 4\operatorname{arctanh}t = \frac{3}{2}\ln(t^2-1) - 4\operatorname{arctanh}t =$

$\frac{3}{2}\ln\left((x-2)^2-1\right) - 4\operatorname{arctanh}(x-2)$

$= \frac{3}{2}\ln(x-1) + \frac{3}{2}\ln(x-3) - 4\operatorname{arctanh}(x-2)$

1260

$\int\frac{(x-1)^2}{x^2+3x+4}dx = \int\frac{x^2-2x+1}{x^2+3x+4}dx = \int\frac{x^2+3x+4-2x+1-3x-4}{x^2+3x+4}dx = \int\frac{x^2+3x+4}{x^2+3x+4}dx - \int\frac{5x+3}{x^2+3x+4}$

$= x - \frac{5}{2}\int\frac{2x+\frac{6}{5}}{x^2+3x+4} = x - \frac{5}{2}\int\frac{2x+\frac{6}{5}+3-3}{\left(x+\frac{3}{2}\right)^2-\frac{9}{4}+4} = x - \frac{5}{2}\int\frac{2x+3-\frac{9}{5}}{\left(x+\frac{3}{2}\right)^2+\frac{7}{4}}$

$= x - \frac{5}{2}\int\frac{2x+3}{x^2+3x+4} + \frac{9}{2}\int\frac{dx}{\left(x+\frac{3}{2}\right)^2+\frac{7}{4}}$

$\left\{\begin{array}{cc} x^2+3x+4 = t & x+\frac{3}{2} = z \\ (2x+3)dx = dt & dx = dz \end{array}\right\}$

$= x - \frac{5}{2}\int\frac{dt}{t} + \frac{9}{2}\int\frac{dz}{z^2+\frac{7}{4}} = x - \frac{5}{2}\ln t + \frac{9}{7}\sqrt{7}\arctan\frac{2}{7}z\sqrt{7}$

$= x - \frac{5}{2}\ln(x^2+3x+4) + \frac{9}{\sqrt{7}}\arctan\frac{2x+3}{\sqrt{7}} + C$

1261

$\int\frac{x^2}{x^2-6x+10}dx = \int\frac{x^2-6x+10+6x-10}{x^2-6x+10}dx = \int dx + \int\frac{6x-10}{(x-3)^2-9+10}dx = x + 3\int\frac{2x-6-\frac{10}{3}+6}{(x-3)^2+1}dx$

$= x + 3\int\frac{2x-6}{x^2-6x+10}dx + 8\int\frac{dx}{(x-3)^2+1}$

$\left\{\begin{array}{cc} x^2-6x+10 = t & x-3 = z \\ (2x-6)dx = dt & dx = dz \end{array}\right\}$

$= x + 3\int\frac{dt}{t} + 8\int\frac{dz}{z^2+1} = x + 3\ln t + 8\arctan z$

$= x + 3\ln(x^2-6x+10) + 8\arctan(x-3) + C$

1262

$\int\frac{dx}{\sqrt{3+3x-2x^2}} = \frac{1}{\sqrt{2}}\int\frac{dx}{\sqrt{\frac{3}{2}+\frac{3}{2}x-x^2}} = \frac{1}{\sqrt{2}}\int\frac{dx}{\sqrt{\frac{33}{16}-\left(x-\frac{3}{4}\right)^2}}$

$\left\{\begin{array}{c} x-\frac{3}{4} = t \\ dx = dt \end{array}\right\}$

$= \frac{1}{\sqrt{2}}\int\frac{dt}{\sqrt{\frac{33}{16}-t^2}} = \frac{1}{\sqrt{2}}\arcsin\frac{4t}{\sqrt{33}} = \frac{1}{\sqrt{2}}\arcsin\frac{4x-3}{\sqrt{33}} + C$

1263

$\int\frac{dx}{\sqrt{x-x^2}} = \int\frac{dx}{\sqrt{\frac{1}{4}-\left(x-\frac{1}{2}\right)^2}}$

$\left\{\begin{array}{c} x-\frac{1}{2} = t \\ dx = dt \end{array}\right\}$

$= \int\frac{dt}{\sqrt{\frac{1}{4}-t^2}} = \arcsin 2t = \arcsin(2x-1) + C$

1264

$\int\frac{dx}{\sqrt{x^2+px+q}} = \int\frac{dx}{\sqrt{\left(x+\frac{p}{2}\right)^2-\frac{p^2}{4}+q}} = \int\frac{dx}{\sqrt{\left(x+\frac{p}{2}\right)^2-\frac{p^2}{4}+q}} = \int\frac{dx}{\sqrt{\left(x+\frac{p}{2}\right)^2-\frac{p^2-4q}{4}}}$

$\left\{\begin{array}{c} (x+\frac{p}{2}) = t \\ dx = dt \\ \frac{p^2-4q}{4} = z \end{array}\right\}$

$= \int\frac{dt}{\sqrt{t^2-z}} = \ln\left(t+\sqrt{t^2-z}\right) = \ln\left(x+\frac{p}{2}+\sqrt{\left(x+\frac{p}{2}\right)^2-\frac{p^2-4q}{4}}\right) =$

$\ln\left(\frac{1}{2}p+x+\sqrt{x^2+xp+q}\right) + C$

1265

$\int\frac{3x-6}{\sqrt{x^2-4x+5}}dx = \frac{3}{2}\int\frac{2x-4}{\sqrt{x^2-4x+5}}dx$

$\left\{\begin{array}{c} x^2-4x+5 = t \\ (2x-4)dx = dt \end{array}\right\}$

$$= \frac{3}{2}\int \frac{dt}{\sqrt{t}} = 3\sqrt{t} = 3\sqrt{x^2-4x+5} + C$$

1266

$$\int \frac{2x-8}{\sqrt{1-x-x^2}}dx = \int \frac{2x-8}{\sqrt{1-\left(x+\frac{1}{2}\right)^2+\frac{1}{4}}}dx = 2\int \frac{x-4}{\sqrt{\frac{5}{4}-\left(x+\frac{1}{2}\right)^2}}dx = 2\int \frac{x+\frac{1}{2}-4-\frac{1}{2}}{\sqrt{\frac{5}{4}-\left(x+\frac{1}{2}\right)^2}}dx$$

$$= 2\int \frac{x+\frac{1}{2}}{\sqrt{\frac{5}{4}-\left(x+\frac{1}{2}\right)^2}}dx - 9\int \frac{dx}{\sqrt{\frac{5}{4}-\left(x+\frac{1}{2}\right)^2}}$$

$$\left\{\begin{matrix} \frac{5}{4}-(x+\frac{1}{2})^2 = t & x+\frac{1}{2} = z \\ -2(x+\frac{1}{2})dx = dt & dx = dz \end{matrix}\right\}$$

$$= -\int \frac{dt}{\sqrt{t}} - 9\int \frac{dz}{\sqrt{\frac{5}{4}-z^2}} = -2\sqrt{t} - 9\arcsin\frac{2}{5}\sqrt{5}z = -2\sqrt{1-x-x^2} - 9\arcsin\frac{2x+1}{\sqrt{5}} + C$$

1267

$$\int \frac{x}{\sqrt{5x^2-2x+1}}dx = \frac{1}{10}\int \frac{10x-2+2}{\sqrt{x^2-\frac{2}{5}x+\frac{1}{5}}}dx = \frac{1}{10}\int \frac{10x-2}{\sqrt{5x^2-2x+1}}dx + \frac{1}{5\sqrt{5}}\int \frac{dx}{\sqrt{\left(x-\frac{1}{5}\right)^2-\frac{1}{25}+\frac{1}{5}}}$$

$$\left\{\begin{matrix} 5x^2-2x+1 = t & x-\frac{1}{5} = z \\ (10x-2)dx = dt & dx = dz \end{matrix}\right\}$$

$$= \frac{1}{10}\int \frac{dt}{\sqrt{t}} + \frac{1}{5\sqrt{5}}\int \frac{dx}{\sqrt{\left(x-\frac{1}{5}\right)^2+\frac{4}{25}}}$$

$$= \frac{1}{10}2t^{\frac{1}{2}} + \frac{1}{5\sqrt{5}}\int \frac{dz}{\sqrt{z^2+\frac{4}{25}}} = \frac{\sqrt{t}}{5} + \frac{\sqrt{5}}{25}\operatorname{arcsinh}\frac{5}{2}z = \frac{\sqrt{5x^2-2x+1}}{5} + \frac{\sqrt{5}}{25}\operatorname{arcsinh}\frac{5x-1}{2} + C$$

1268

$$\int \frac{dx}{x\sqrt{1-x^2}}$$

$$\left\{\begin{matrix} \sqrt{1-x^2} = t \\ 1-x^2 = t^2 \\ -xdx = tdt \end{matrix}\right\}$$

$$\int \frac{-\frac{tdt}{x}}{xt} = -\int \frac{tdt}{x^2 t} = -\int \frac{dt}{1-t^2} = -\operatorname{arctanh}t = -\operatorname{arctanh}\sqrt{1-x^2} + C$$

1269

$$\int \frac{dx}{x\sqrt{x^2+x-1}}$$

$$\left\{\begin{matrix} x = \frac{1}{t} \\ dx = -\frac{dt}{t^2} \end{matrix}\right\}$$

$$\int \frac{-\frac{dt}{t^2}}{\frac{1}{t}\sqrt{\frac{1}{t^2}+\frac{1}{t}-1}} = \int \frac{-\frac{dt}{t}}{\sqrt{\frac{1}{t^2}+\frac{1}{t}-1}} = \int \frac{-\frac{dt}{t}}{\sqrt{\frac{1+t-t^2}{t^2}}} = -\int \frac{\frac{dt}{t}}{\frac{1}{t}\sqrt{1+t-t^2}} = -\int \frac{dt}{\sqrt{1+t-t^2}} = -\int \frac{dt}{\sqrt{1-\left(t-\frac{1}{2}\right)^2+\frac{1}{4}}} =$$

$$\left\{\begin{matrix} t-\frac{1}{2} = z \\ dt = dz \end{matrix}\right\}$$

$$= -\int \frac{dz}{\sqrt{\frac{5}{4}-z^2}} = -\arcsin\frac{2}{\sqrt{5}}z = -\arcsin\frac{2}{\sqrt{5}}(t-\frac{1}{2})$$

$$= -\arcsin\frac{2}{\sqrt{5}}(\frac{1}{x}-\frac{1}{2}) = -\arcsin\frac{2}{\sqrt{5}}(\frac{2-x}{2x}) = -\arcsin\frac{2-x}{x\sqrt{5}} + C$$

1270

$$\int \frac{dx}{(x-1)\sqrt{x^2-2}}$$

$$\left\{\begin{matrix} x-1 = t \\ dx = dt \end{matrix}\right\}$$

$$= \int \frac{dt}{t\sqrt{(t+1)^2-2}} = \int \frac{dt}{t\sqrt{t^2+2t-1}}$$

$$\left\{\begin{matrix} t = \frac{1}{z} \\ dt = -\frac{dz}{z^2} \end{matrix}\right\}$$

$$= \int \frac{-\frac{dz}{z^2}}{\frac{1}{z}\sqrt{\frac{1}{z^2}+\frac{2}{z}-1}} = \int \frac{-\frac{dz}{z}}{\sqrt{\frac{1+2z-z^2}{z^2}}} = -\int \frac{dz}{\sqrt{1+2z-z^2}} = -\int \frac{dz}{\sqrt{2-(z-1)^2}} = -\arcsin\frac{z-1}{\sqrt{2}}$$

$$= -\arcsin\frac{\frac{1}{t}-1}{\sqrt{2}} = -\arcsin\frac{\frac{1}{x-1}-1}{\sqrt{2}} = -\arcsin\frac{\frac{1-x+1}{x-1}}{\sqrt{2}} = -\arcsin\frac{\frac{2-x}{x-1}}{\sqrt{2}}$$

$$= -\arcsin\frac{2-x}{\sqrt{2}(x-1)} + C$$

1271

$$\int \frac{dx}{(x+1)\sqrt{x^2+2x}} = \int \frac{dx}{(x+1)\sqrt{(x+1)^2-1}}$$

$$\left\{\begin{matrix} x+1 = t \\ dx = dt \end{matrix}\right\}$$

$$= \int \frac{dt}{t\sqrt{t^2-1}}$$

$$\left\{\begin{matrix} t = \frac{1}{z} \\ dt = -\frac{dz}{z^2} \end{matrix}\right\}$$

$$= -\int \frac{\frac{dz}{z^2}}{\frac{1}{z}\sqrt{(\frac{1}{z})^2-1}} = -\int \frac{\frac{dz}{z}}{\sqrt{(\frac{1}{z})^2-1}} = -\int \frac{\frac{dz}{z}}{\sqrt{\frac{1-z^2}{z^2}}} = -\int \frac{dz}{\sqrt{1-z^2}} = -\arcsin z$$

$$= -\arcsin\frac{1}{t} = -\arcsin\frac{1}{x+1} + C$$

1272

$$\int \sqrt{x^2+2x+5}\,dx = \int \sqrt{x^2+2x+1+4}\,dx = \int \sqrt{(x+1)^2+4}\,dx$$

$$\left\{\begin{matrix} x+1 = t \\ dx = dt \end{matrix}\right\}$$

$$= \int \sqrt{t^2+4}\,dt$$

$$\left\{\begin{matrix} t = 2\sinh z \\ dt = 2\cosh z dz \end{matrix}\right\}$$

$$= \int \sqrt{4\sinh^2 z+4}\,2\cosh z dz = 4\int \sqrt{\sinh^2 z+1}\,\cosh z dz = 4\int \cosh^2 z dz$$

$$= 4\int \frac{1}{2}(1+\cosh 2z)dz = 2\int (1+\cosh 2z)dz = 2\int dz + 2\int \cosh 2z dz$$

$$\left\{\begin{array}{c} 2z = u \\ 2dz = du \end{array}\right\}$$

$= 2z + \int \cosh u\,du = 2z + \sinh u = 2z + \sinh 2z$

$= 2\operatorname{arcsinh}\frac{t}{2} + \sinh(2\operatorname{arcsinh}\frac{t}{2}) = 2\operatorname{arcsinh}\frac{x+1}{2} + \sinh(2\operatorname{arcsinh}\frac{x+1}{2}) + C$

1273

$\int \sqrt{x-x^2}\,dx = \int \sqrt{\frac14 - (x-\frac12)^2}\,dx$

$$\left\{\begin{array}{c} x-\frac12 = t \\ dx = dt \end{array}\right\}$$

$\int \sqrt{\frac14 - t^2}\,dt = \int \frac{\frac14 - t^2}{\sqrt{\frac14-t^2}}dt$

$\int \frac{\frac14-t^2}{\sqrt{\frac14-t^2}}dt = (at+b)\sqrt{\frac14-t^2} + \int\frac{c}{\sqrt{\frac14-t^2}}dt|'$

$\frac{\frac14-t^2}{\sqrt{\frac14-t^2}} = a\sqrt{\frac14-t^2} + \frac{-2t(at+b)}{2\sqrt{\frac14-t^2}} + \frac{c}{\sqrt{\frac14-t^2}} \,|\, \sqrt{\frac14-t^2}$

$\frac14 - t^2 = a(\frac14-t^2) - t(at+b) + c$

$\frac14 - t^2 = t^2(-2a) + t(-b) + c + \frac14 a$

$\{a=\frac12, b=0\}$

$.c + \frac14 a = \frac14 \ldots . c + \frac18 = \frac14$

$\{c = \frac18\}$

$I = \frac t2\sqrt{\frac14-t^2} + \frac18\int\frac{dt}{\sqrt{\frac14-t^2}} = \frac14 t\sqrt{1-4t^2} + \frac18\arcsin 2t$

$= \frac14(x-\frac12)\sqrt{1-4(x-\frac12)^2} + \frac18\arcsin 2(x-\frac12)$

$= \frac14(2x-1)\sqrt{x-x^2} + \frac18\arcsin(2x-1)+C$

1274

$\int\sqrt{2-x-x^2}\,dx = \int\sqrt{2-\frac14+\frac14-x-x^2}\,dx = \int\sqrt{\frac94-(x+\frac12)^2}$

$$\left\{\begin{array}{c} x+\frac12 = t \\ dx = dt \end{array}\right\}$$

$= \int\sqrt{\frac94-t^2}\,dt = \frac14 t\sqrt{9-4t^2} + \frac98\arcsin\frac23 t$

$= \frac14(x+\frac12)\sqrt{9-4(x+\frac12)^2} + \frac98\arcsin\frac23(x+\frac12)$

$= \frac12 x\sqrt{2-x^2-x} + \frac14\sqrt{2-x^2-x} + \frac98\arcsin\left(\frac23x+\frac13\right)+C$

1275

$\int\frac{xdx}{x^4-4x^2+3} = \int\frac{xdx}{x^4-4x^2+4-1} = \int\frac{xdx}{(x^2-2)^2-1}$

$$\left\{\begin{array}{c} x^2-2 = t \\ 2xdx = dt \end{array}\right\}$$

$= \frac12\int\frac{dt}{t^2-1} = -\frac12\operatorname{arctanh} t = -\frac12\operatorname{arctanh}(x^2-2)+C$

1276

$\int\frac{\cos x}{\sin^2x-6\sin x+12}dx = \int\frac{\cos x}{(\sin x-3)^2+3}dx$

$$\left\{\begin{array}{c} \sin x-3 = t \\ \cos x dx = dt \end{array}\right\}$$

$= \int\frac{dt}{t^2+3} = \frac1{\sqrt3}\arctan\frac t{\sqrt3} = \frac1{\sqrt3}\arctan\frac{\sin x-3}{\sqrt3}+C$

1277

$\int\frac{e^x}{\sqrt{1+e^x+e^{2x}}}dx$

$$\left\{\begin{array}{c} e^x = t \\ e^x dx = dt \end{array}\right\}$$

$= \int\frac{dt}{\sqrt{1+t+t^2}} = \int\frac{dt}{\sqrt{1+(t+\frac12)^2-\frac14}} = \int\frac{dt}{\sqrt{\frac34+(t+\frac12)^2}} = \operatorname{arcsinh}\frac{2t+1}{\sqrt3} = \operatorname{arcsinh}\frac{2e^x+1}{\sqrt3}+C$

1278

$\int\frac{\sin x dx}{\sqrt{\cos^2x+4\cos x+1}} = \int\frac{\sin x}{\sqrt{(\cos x+2)^2-3}}dx =$

$$\left\{\begin{array}{c} \cos x+2 = t \\ -\sin x dx = dt \end{array}\right\}$$

$= -\int\frac{dt}{\sqrt{t^2-3}} = -\ln\left(t+\sqrt{t^2-3}\right) = -\ln\left(\cos x+2+\sqrt{\cos^2x+4\cos x+1}\right)+C$

1279

$I = \int\frac{\ln x dx}{x\sqrt{1-4\ln x-\ln^2x}} = \int\frac{\ln x dx}{x\sqrt{-(\ln x+2)^2+4+1}} = \int\frac{\ln x dx}{x\sqrt{5-(\ln x+2)^2}}$

$$\left\{\begin{array}{c} \ln x = u \\ \frac{dx}{x} = du \end{array}\right\}$$

$I = \int\frac{udu}{\sqrt{5-(u+2)^2}}$

$$\left\{\begin{array}{c} u+2 = z \\ du = dz \end{array}\right\}$$

$= \int\frac{z-2}{\sqrt{5-z^2}}dz = \int\frac{zdz}{\sqrt{5-z^2}} - 2\int\frac{dz}{\sqrt{5-z^2}}$

$I_1 = \int\frac{zdz}{\sqrt{5-z^2}} \left\{\begin{array}{c} 5-z^2 = t \\ -2zdz = dt \end{array}\right\} = \int\frac{\frac{dt}{-2}}{\sqrt t} = -\sqrt t = -\sqrt{5-z^2}$

$I_1 = -\sqrt{5-(u+2)^2} = -\sqrt{1-u^2-4u} = -\sqrt{1-\ln^2x-4\ln x}$

$I_2 = \int\frac{dz}{\sqrt{5-z^2}} = \arcsin\frac z{\sqrt5} = \arcsin\frac{u+2}{\sqrt5} = \arcsin\frac{\ln x+2}{\sqrt5}$

$I = I_1 - 2I_2 = -\sqrt{1-\ln^2x-4\ln x} - 2\arcsin\frac{\ln x+2}{\sqrt5}+C$

1280

$I = \int\frac{dx}{(x+a)(x+b)}$

$\frac{1}{(x+a)(x+b)} = \frac{A}{(x+a)} + \frac{B}{(x+b)}$

$1 = A(x+b) + B(x+a)$

$1 = Ax + Ab + Bx + Ba$

$1 = x(A+B) + Ab + Ba$

$A + B = 0$

$Ab + Ba = 1$

$\ldots.$

$Ba - Bb = 1$

$B(a-b) = 1$

$\left\{B = \frac{1}{a-b}, A = \frac{1}{b-a}\right\}$

$I = \int \frac{\frac{1}{b-a}}{(x+a)} + \int \frac{\frac{1}{a-b}}{(x+b)} = \frac{1}{b-a}\int \frac{dx}{x+a} + \frac{1}{a-b}\int \frac{dx}{x+b}$

$I = \frac{1}{b-a}\ln(x+a) + \frac{1}{a-b}\ln(x+b) + C$

1281

$I = \int \frac{x^2-5x+9}{x^2-5x+6}dx = \int dx + 3\int \frac{dx}{x^2-5x+6} = x + 3\int \frac{dx}{(x-2)(x-3)}$

$\frac{1}{(x-2)(x-3)} = \frac{a}{x-2} + \frac{b}{x-3} | (x-2)(x-3)$

$1 = a(x-3) + b(x-2) = ax - 3a + bx - 2b = x(a+b) - 3a - 2b$

$a + b = 0 \ldots a = -b$

$-3a - 2b = 1$

$-3(-b) - 2b = 1$

$\{b = 1, a = -1\}$

$I = x + \int \frac{-1}{x-2} + \int \frac{1}{x-3} = x - 3\ln(x-2) + 3\ln(x-3) + C$

1282

$\int \frac{dx}{(x-1)(x+2)(x+3)} = \frac{1}{12}\int \frac{dx}{x-1} - \frac{1}{3}\int \frac{dx}{x+2} + \frac{1}{4}\int \frac{dx}{x+3}$

$= \frac{1}{12}\ln(x-1) - \frac{1}{3}\ln(x+2) + \frac{1}{4}\ln(x+3)$

$\frac{1}{(x-1)(x+2)(x+3)} = \frac{a}{(x-1)} + \frac{b}{(x+2)} + \frac{c}{(x+3)} | (x-1)(x+2)(x+3)$

$1 = a(x+2)(x+3) + b(x-1)(x+3) + c(x-1)(x+2)$

$1 = ax^2 + 5ax + 6a + bx^2 + 2bx - 3b + cx^2 + cx - 2c$

$1 = x^2(a+b+c) + x(5a+2b+c) + 6a - 3b - 2c$

$a + b + c = 0$

$5a + 2b + c = 0$

$6a - 3b - 2c = 1$

$\left\{b = -\frac{1}{3}, a = \frac{1}{12}, c = \frac{1}{4}\right\}$

1283

$\int \frac{2x^2+41x-91}{(x-1)(x+3)(x-4)}dx$

$\frac{2x^2+41x-91}{(x-1)(x+3)(x-4)} = \frac{a}{x-1} + \frac{b}{x+3} + \frac{c}{x-4} | (x-1)(x+3)(x-4)$

$2x^2 + 41x - 91 = a(x+3)(x-4) + b(x-1)(x-4) + c(x-1)(x+3)$

$2x^2 + 41x - 91 = ax^2 - ax - 12a + bx^2 - 5bx + 4b + cx^2 + 2cx - 3c$

$2x^2 + 41x - 91 = x^2(a+b+c) + x(-a-5b+2c) - 12a + 4b - 3c$

$2 = a + b + c$

$41 = -a - 5b + 2c$

$-91 = -12a + 4b - 3c$

$43 = -4b + 3c\left\{c = \frac{43}{3} + \frac{4}{3}b\right\}$

$-12a + 4b - 3\left(\frac{43}{3} + \frac{4}{3}b\right) = -12a - 43 = -91$

$4 + b + \frac{43}{3} + \frac{4}{3}b = \frac{55}{3} + \frac{7}{3}b = 2$

$2 = 4 - 7 + c = -3 + c$

$\{b = -7, c = 5, a = 4\}$

$I = 4\int \frac{dx}{x-1} - 7\int \frac{dx}{x+3} + 5\int \frac{dx}{x-4} = 4\ln(x-1) - 7\ln(x+3) + 5\ln(x-4) + C$

1284

$\int \frac{5x^3+2}{x^3-5x^2+4x}dx = \int \frac{5x^3+2}{x(x^2-5x+4)}dx = \int \frac{5x^3+2}{x(x-1)(x-4)}dx = 5\int \frac{x^3+\frac{2}{5}}{x(x-1)(x-4)}dx$

$= 5\int \frac{x^3-5x^2+4x+\frac{2}{5}-(-5x^2+4x)}{x(x-1)(x-4)}dx = 5\int dx + \int \frac{25x^2-20x+2}{x(x-1)(x-4)}$

$\frac{25x^2-20x+2}{x(x-1)(x-4)} = \frac{a}{x} + \frac{b}{x-1} + \frac{c}{x-4} | x(x-1)(x-4)$

$25x^2 - 20x + 2 = a(x^2 - 5x + 4) + bx(x-4) + cx(x-1)$

$25x^2 - 20x + 2 = ax^2 - 5ax + 4a + bx^2 - 4bx + cx^2 - cx$

$25x^2 - 20x + 2 = x^2(a+b+c) + x(-5a-4b-c) + 4a$

$25 = a + b + c$

$-20 = -5a - 4b - c$

$2 = 4a$

$\{a = \frac{1}{2}\}$

$-4a - 3b = -4(\frac{1}{2}) - 3b = -2 - 3b = 5$

$\left\{b = -\frac{7}{3}\right\}$

$\frac{1}{2} - \frac{7}{3} + c = 25$

$\left\{c = \frac{161}{6}\right\}$

$I_1 = \frac{1}{2}\int \frac{dx}{x} - \frac{7}{3}\int \frac{dx}{x-1} + \frac{161}{6}\int \frac{dx}{x-4} = \frac{1}{2}\ln x - \frac{7}{3}\ln(x-1) + \frac{161}{6}\ln(x-4)$

$I = 5x + I_1 = 5x + \frac{1}{2}\ln x - \frac{7}{3}\ln(x-1) + \frac{161}{6}\ln(x-4) + C$

1285

$\int \frac{dx}{x(x+1)^2}$

$\left\{\begin{array}{c} x = \frac{1}{t} \\ dx = -\frac{dt}{t^2} \end{array}\right\}$

$\int \frac{-\frac{dt}{t^2}}{\frac{1}{t}(\frac{1}{t}+1)^2} = -\int \frac{dt}{t(\frac{1+t}{t})^2} = -\int \frac{t}{(1+t)^2}dt = -\int \frac{t+1-1}{(1+t)^2}dt$

$= -\int \frac{dt}{(1+t)} + \int \frac{dt}{(1+t)^2} = -\ln(1+t) - \frac{1}{1+t} = -\ln(\frac{x+1}{x}) - \frac{x}{x+1} + C$

1286

$\int \frac{x^3-1}{4x^3-x}dx = \frac{1}{4}\int \frac{4x^3-4}{4x^3-x}dx = \frac{1}{4}\int \frac{4x^3-x+x-4}{4x^3-x}dx = \frac{1}{4}\int dx + \frac{1}{4}\int \frac{x-4}{4x^3-x}dx$

$= \frac{x}{4} + \frac{1}{4}\int \frac{x-4}{x(4x^2-1)}dx = \frac{x}{4} + \frac{1}{4}\int \frac{x-4}{x(2x-1)(2x+1)}dx$

$\frac{x-4}{x(2x-1)(2x+1)} = \frac{a}{x} + \frac{b}{(2x-1)} + \frac{c}{(2x+1)} | x(2x-1)(2x+1)$

$x-4=a(2x-1)(2x+1)+bx(2x+1)+cx(2x-1)$

$x-4=4ax^2-a+2bx^2+bx+2cx^2-cx$

$x-4=x^2(4a+2b+2c)+x(b-c)-a$

$\{a=4\}$

$4a+2b+2c=0$

$b-c=1$

$16+2(1+c)+2c=0$

$\{c=-\frac{9}{2}\}$

$b=1+c=1-\frac{9}{2}=-\frac{7}{2}$

$I=\int\frac{4}{x}dx+\int\frac{-\frac{7}{2}}{(2x-1)}dx+\int\frac{-\frac{9}{2}}{(2x+1)}dx=4\ln x-\frac{7}{4}\ln(2x-1)-\frac{9}{4}\ln(2x+1)+C$

1287

$I=\int\frac{x^4-6x^3+12x^2+6}{x^3-6x^2+12x-8}dx=\int\frac{x^4-6x^3+12x^2+6}{(x-2)^3}dx$

$(x^4-6x^3+12x^2+6):(x^3-6x^2+12x-8)=x+\frac{6+8x}{x^3-6x^2+12x-8}$

$x^4-6x^3+12x^2-8x$

$6+8x$

$I=\int xdx+2\int\frac{4x+3}{(x-2)^3}dx=\frac{x^2}{2}+2\int\frac{4x-8+3+8}{(x-2)^3}dx=\frac{x^2}{2}+8\int\frac{x-2}{(x-2)^3}dx+22\int\frac{dx}{(x-2)^3}$

$=\frac{x^2}{2}+8\int\frac{dx}{(x-2)^2}+22\int\frac{dx}{(x-2)^3}=\frac{x^2}{2}-\frac{8}{x-2}-\frac{11}{(x-2)^2}+C$

1288

$I=\int\frac{5x^2+6x+9}{(x-3)^2(x+1)^2}dx$

$I=\int\frac{5x^2+6x+9}{(x^2-2x-3)^2}dx=\frac{ax+b}{(x^2-2x-3)}+\int\frac{cx+d}{(x^2-2x-3)}dx|'$

$\frac{5x^2+6x+9}{(x^2-2x-3)^2}=\left(\frac{ax+b}{(x^2-2x-3)}\right)'+\frac{cx+d}{(x^2-2x-3)}$

$\frac{5x^2+6x+9}{(x^2-2x-3)^2}=\frac{a(x^2-2x-3)-(2x-2)(ax+b)}{(x^2-2x-3)^2}+\frac{cx+d}{(x^2-2x-3)}|(x^2-2x-3)^2$

$5x^2+6x+9=a(x^2-2x-3)-(2x-2)(ax+b)+(cx+d)(x^2-2x-3)$

$5x^2+6x+9=-ax^2-3a-2xb+2b+cx^3-2cx^2-3cx+dx^2-2dx-3d$

$5x^2+6x+9=x^3(c)+x^2(-a-2c+d)+x(-2b-3c-2d)-3a+2b-3d$

$\{c=0\}$

$-a-2c+d=5=d-a\ldots d=5+a$

$-2b-3c-2d=6=-2d-2b$

$9=-3a+2b-3d$

$15=-2d-2b-3a+2b-3d=-5d-3a$

$-5(5+a)-3a=-8a-25=15$

$\{a=-5,d=0\}$

$6=-2b\ldots\{b=-3\}$

$I=\int\frac{5x^2+6x+9}{(x^2-2x-3)^2}dx=\frac{-5x-3}{(x^2-2x-3)}+\int\frac{0}{(x^2-2x-3)}dx=\frac{-5x-3}{x^2-2x-3}+C$

1289

$\int\frac{x^2-8x+7}{(x^2-3x-10)^2}dx=\int\frac{x^2-8x+7}{(x+2)^2(x-5)^2}dx$

$\frac{x^2-8x+7}{(x+2)^2(x-5)^2}=\frac{a}{(x+2)}+\frac{b}{(x+2)^2}+\frac{c}{(x-5)}+\frac{d}{(x-5)^2}|(x+2)^2(x-5)^2$

$x^2-8x+7=ax^3-8ax^2+5ax+50a+bx^2-10bx+25b+cx^3-cx^2-16cx-20c+dx^2$

$x^2-8x+7=x^3(a+c)+x^2(-8a+b-c+d)+x(5a-10b-16c+4d)+(50a+25b-20c$

$a+c=0$

$-8a+b-c+d=1$

$5a-10b-16c+4d=-8$

$50a+25b-20c+4d=7$

$\{a=-\frac{30}{343},c=\frac{30}{343},b=\frac{27}{49},d=-\frac{8}{49}\}$

$I=\int\frac{-\frac{30}{343}dx}{(x+2)}+\int\frac{\frac{27}{49}dx}{(x+2)^2}+\int\frac{\frac{30}{343}dx}{(x-5)}+\int\frac{-\frac{8}{49}dx}{(x-5)^2}=-\frac{27}{49(x+2)}-\frac{30}{343}\ln(x+2)+\frac{8}{49(x-5)}+\frac{30}{343}\ln($

1290

$\int\frac{2x-3}{(x^2-3x+2)^3}dx$

$\left\{\begin{array}{l}x^2-3x+2=t\\(2x-3)dx=dt\end{array}\right\}$

$=\int\frac{dt}{t^3}=-\frac{1}{2t^2}=-\frac{1}{2((x^2-3x+2))^2}+C$

1291

$\int\frac{x^3+x+1}{x(x^2+1)}dx=\int\frac{x^3+x+1}{x^3+x}dx=\int dx+\int\frac{dx}{x(x^2+1)}$

$\frac{1}{x(x^2+1)}=\frac{a}{x}+\frac{bx+c}{x^2+1}|x(x^2+1)$

$1=ax^2+a+bx^2+cx$

$a+b=0$

$\{c=0\}\{a=1\}\{b=-1\}$

$I_1=\int\frac{dx}{x}-\int\frac{x}{x^2+1}dx=\ln x-\frac{1}{2}\ln(x^2+1)$

$I=x+I_1=x+\ln x-\frac{1}{2}\ln(x^2+1)+C$

1292

$\int\frac{x^4}{x^4-1}dx=\int\frac{x^4-1+1}{x^4-1}dx=\int dx=x+\int\frac{dx}{x^4-1}=x+I_1$

$\frac{1}{(x^2-1)(x^2+1)}=\frac{ax+b}{x^2-1}+\frac{cx+d}{x^2+1}|(x^2-1)(x^2+1)$

$1=ax^3+ax+bx^2+b+cx^3-cx+dx^2-d$

$1=x^3(a+c)+x^2(b+d)+x(a-c)+(b-d)$

$a+c=0 \qquad a=-c$

$b+d=0 \qquad b=-d$

$a-c=0 \qquad -2c=0 \qquad \{c=0\}\{a=0\}$

$b-d=1 \qquad -2d=1 \qquad \{d=-\frac{1}{2}\}\{b=\frac{1}{2}\}$

$I_1=\int\frac{\frac{1}{2}}{x^2-1}dx+\int\frac{-\frac{1}{2}}{x^2+1}dx=-\frac{1}{2}\operatorname{arctanh}x-\frac{1}{2}\arctan x$

$I=x-\frac{1}{2}\operatorname{arctanh}x-\frac{1}{2}\arctan x+C$

1293

$\int\frac{dx}{(x^2-4x+3)(x^2+4x+5)}=\int\frac{dx}{(x-3)(x-1)(x^2+4x+5)}$

$$x_{12} = \frac{4 \pm \sqrt{16-12}}{2}$$
$$x_1 = \frac{4+2}{2} = 3$$
$$x_2 = \frac{4-2}{2} = 1$$
$$\frac{1}{(x-3)(x-1)(x^2+4x+5)} = \frac{a}{x-3} + \frac{b}{x-1} + \frac{cx+d}{x^2+4x+5} | (x-3)(x-1)(x^2+4x+5)$$
$$1 = ax^3 + 3ax^2 + ax - 5a + bx^3 + bx^2 - 7bx - 15b + cx^3 - 4cx^2 + 3cx + dx^2 - 4dx + 3d$$
$$1 = x^3(a+b+c) + x^2(3a+b-4c+d) + x(a-7b+3c-4d) - 5a - 15b + 3d$$
$$\begin{bmatrix} 1 & 1 & 1 & 0 & 0 \\ 3 & 1 & -4 & 1 & 0 \\ 1 & -7 & 3 & -4 & 0 \\ -5 & -15 & 0 & 3 & 1 \end{bmatrix} \begin{array}{ccc} -3 & -1 & 5 \\ \leftarrow & & \\ & \leftarrow & \\ & & \leftarrow \end{array} = \begin{bmatrix} 1 & 1 & 1 & 0 & 0 \\ 0 & -2 & -7 & 1 & 0 \\ 0 & -8 & 2 & -4 & 0 \\ 0 & -10 & 5 & 3 & 1 \end{bmatrix} \begin{array}{l} \\ \\ :4 \\ :-5 \end{array}$$
$$= \begin{bmatrix} 1 & 1 & 1 & 0 & 0 \\ 0 & -2 & -7 & 1 & 0 \\ 0 & -2 & \frac{1}{2} & -1 & 0 \\ 0 & 2 & -1 & -\frac{3}{5} & -\frac{1}{5} \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 & 0 & 0 \\ 0 & -2 & -7 & 1 & 0 \\ 0 & -2 & \frac{1}{2} & -1 & 0 \\ 0 & 0 & -\frac{1}{2} & -1-\frac{3}{5} & -\frac{1}{5} \end{bmatrix}$$
$$= \begin{bmatrix} 1 & 1 & 1 & 0 & 0 \\ 0 & -2 & -7 & 1 & 0 \\ 0 & -2 & \frac{1}{2} & -1 & 0 \\ 0 & 0 & -\frac{1}{2} & -\frac{8}{5} & -\frac{1}{5} \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 & 0 & 0 \\ 0 & -2 & -7 & 1 & 0 \\ 0 & 2 & -\frac{1}{2} & 1 & 0 \\ 0 & 0 & -\frac{1}{2} & -\frac{8}{5} & -\frac{1}{5} \end{bmatrix}$$
$$= \begin{bmatrix} 1 & 1 & 1 & 0 & 0 \\ 0 & -2 & -7 & 1 & 0 \\ 0 & 0 & -7-\frac{1}{2} & 2 & 0 \\ 0 & 0 & -\frac{1}{2} & -\frac{8}{5} & -\frac{1}{5} \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 & 0 & 0 \\ 0 & -2 & -7 & 1 & 0 \\ 0 & 0 & -\frac{15}{2} & 2 & 0 \\ 0 & 0 & -\frac{1}{2} & -\frac{8}{5} & -\frac{1}{5} \end{bmatrix} \begin{array}{l} \\ \\ \\ -15 \end{array}$$
$$= \begin{bmatrix} 1 & 1 & 1 & 0 & 0 \\ 0 & -2 & -7 & 1 & 0 \\ 0 & 0 & -\frac{15}{2} & 2 & 0 \\ 0 & 0 & \frac{15}{2} & \frac{8*15}{5} & 3 \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 & 0 & 0 \\ 0 & -2 & -7 & 1 & 0 \\ 0 & 0 & -\frac{15}{2} & 2 & 0 \\ 0 & 0 & \frac{15}{2} & 24 & 3 \end{bmatrix} =$$
$$\begin{bmatrix} 1 & 1 & 1 & 0 & 0 \\ 0 & -2 & -7 & 1 & 0 \\ 0 & 0 & -\frac{15}{2} & 2 & 0 \\ 0 & 0 & 0 & 26 & 3 \end{bmatrix}$$
$$\left\{ d = \frac{3}{26}, b = -\frac{1}{20}, a = \frac{1}{52}, c = \frac{2}{65} \right\}$$
$$I = \frac{1}{52} \int \frac{dx}{x-3} - \frac{1}{20} \int \frac{dx}{x-1} + \int \frac{\frac{2}{65}x + \frac{3}{26}}{x^2+4x+5} dx$$
$$I = \frac{1}{52} \ln(x-3) - \frac{1}{20} \ln(x-1) + \frac{1}{65} \int \frac{2x + \left(\frac{3}{26}\right)65}{x^2+4x+5} dx$$
$$I = \frac{1}{52} \ln(x-3) - \frac{1}{20} \ln(x-1) + \frac{1}{65} \int \frac{2x + \frac{15}{2}}{x^2+4x+5} dx$$
$$I_1 = \frac{1}{65} \int \frac{2x+4+\frac{7}{2}}{x^2+4x+5} = \frac{1}{65} \int \frac{2x+4}{x^2+4x+5} + \frac{7}{2} \frac{1}{65} \int \frac{dx}{x^2+4x+5}$$
$$\left\{ \begin{array}{c} x^2+4x+5 = t \\ (2x+4)dx = dt \end{array} \right\} \left\{ \begin{array}{c} x+2 = z \\ dx = dz \end{array} \right\}$$
$$I_1 = \frac{1}{65} \int \frac{dt}{t} + \frac{7}{2} \frac{1}{65} \int \frac{dx}{(x+2)^2+1} = \frac{1}{65} \ln t + \frac{7}{130} \int \frac{dz}{z^2+1}$$
$$I_1 = \frac{1}{65} \ln t + \frac{7}{130} \arctan z = \frac{1}{65} \ln(x^2+4x+5) + \frac{7}{130} \arctan(x+2)$$
$$I = \frac{1}{52} \ln(x-3) - \frac{1}{20} \ln(x-1) + \frac{1}{65} \ln(x^2+4x+5) + \frac{7}{130} \arctan(x+2) + C$$

1294

$$\int \frac{dx}{x^3+1} = \int \frac{dx}{(x+1)(x^2-x+1)}$$
$$\frac{1}{(x+1)(x^2-x+1)} = \frac{a}{x+1} + \frac{bx+c}{x^2-x+1} | (x+1)(x^2-x+1)$$
$$1 = ax^2 + bx^2 - ax + bx + cx + c + a$$
$$1 = x^2(a+b) + x(-a+b+c) + c + a$$
$$a + b = 0$$
$$-a + b + c = 0$$
$$c + a = 1$$
$$\begin{bmatrix} 1 & 1 & 0 & 0 \\ -1 & 1 & 1 & 0 \\ 1 & 0 & 1 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & 2 & 1 & 0 \\ 0 & 1 & 2 & 1 \end{bmatrix} \begin{array}{l} \\ \\ (-2) \end{array}$$
$$= \begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & 2 & 1 & 0 \\ 0 & -2 & -4 & -2 \end{bmatrix} = \begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & 2 & 1 & 0 \\ 0 & 0 & -3 & -2 \end{bmatrix}$$
$$\left\{ b = -\frac{1}{3}, c = \frac{2}{3}, a = \frac{1}{3} \right\}$$
$$I = \int \frac{\frac{1}{3}}{x+1} dx + \int \frac{-\frac{1}{3}x + \frac{2}{3}}{x^2-x+1} dx = \frac{1}{3} \ln(x+1) - \frac{1}{3} \int \frac{x-2}{x^2-x+1}$$
$$I = \frac{1}{3} \ln(x+1) - \frac{1}{6} \int \frac{2x-4}{x^2-x+1}$$
$$I = \frac{1}{3} \ln(x+1) - \frac{1}{6} \int \frac{2x-1}{x^2-x+1} + \frac{1}{2} \int \frac{dx}{x^2-x+1}$$
$$I = \frac{1}{3} \ln(x+1) - \frac{1}{6} \ln(x^2-x+1) + \frac{1}{2} \int \frac{dx}{\left(x-\frac{1}{2}\right)^2 - \frac{1}{4} + 1}$$
$$I = \frac{1}{3} \ln(x+1) - \frac{1}{6} \ln(x^2-x+1) + \frac{1}{2} \int \frac{dx}{\left(x-\frac{1}{2}\right)^2 + \frac{3}{4}}$$
$$I = \frac{1}{3} \ln(x+1) - \frac{1}{6} \ln(x^2-x+1) + \frac{\sqrt{3}}{3} \arctan \frac{2x-1}{\sqrt{3}} + C$$

1295

$$\int \frac{dx}{x^4+1} = \int \frac{dx}{(x^2+1)^2 - 2x^2 + 1 - 1} = \int \frac{dx}{(x^2+1)^2 - 2x^2} = \int \frac{dx}{\left(x^2 - \sqrt{2}x + 1\right)\left(x^2 + \sqrt{2}x + 1\right)}$$
$$\frac{1}{\left(x^2 - \sqrt{2}x + 1\right)\left(x^2 + \sqrt{2}x + 1\right)} = \frac{ax+b}{\left(x^2 - \sqrt{2}x + 1\right)} + \frac{cx+d}{\left(x^2 + \sqrt{2}x + 1\right)}$$
$$1 = (ax+b)\left(x^2 + \sqrt{2}x + 1\right) + (cx+d)\left(x^2 - \sqrt{2}x + 1\right)$$
$$1 = ax^3 + ax + a\sqrt{2}x^2 + bx^2 + b + b\sqrt{2}x + cx^3 + cx - c\sqrt{2}x^2 + dx^2 + d - d\sqrt{2}x$$

$$1 = x^3(a+c) + x^2\left(a\sqrt{2} + b - c\sqrt{2} + d\right) + x\left(a + b\sqrt{2} + c - d\sqrt{2}\right) + (b+d)$$

$$a + c = 0$$
$$a\sqrt{2} + b - c\sqrt{2} + d = 0$$
$$a + b\sqrt{2} + c - d\sqrt{2} = 0$$
$$b + d = 1$$

$$\begin{bmatrix} 1 & 0 & 1 & 0 & 0 \\ \sqrt{2} & 1 & -\sqrt{2} & 1 & 0 \\ 1 & \sqrt{2} & 1 & -\sqrt{2} & 0 \\ 0 & 1 & 0 & 1 & 1 \end{bmatrix} \begin{array}{c} :-\sqrt{2} \\ (-1) \end{array} \sim \begin{bmatrix} 1 & 0 & 1 & 0 & 0 \\ -1 & -\frac{\sqrt{2}}{2} & 1 & -\frac{\sqrt{2}}{2} & 0 \\ -1 & -\sqrt{2} & -1 & \sqrt{2} & 0 \\ 0 & 1 & 0 & 1 & 1 \end{bmatrix} \sim$$

$$\begin{bmatrix} 1 & 0 & 1 & 0 & 0 \\ 0 & -\frac{\sqrt{2}}{2} & 2 & -\frac{\sqrt{2}}{2} & 0 \\ 0 & -\sqrt{2} & 0 & \sqrt{2} & 0 \\ 0 & 1 & 0 & 1 & 1 \end{bmatrix} \begin{array}{c} -\frac{2}{\sqrt{2}} \\ :\sqrt{2} \\ (-1) \end{array} \sim \begin{bmatrix} 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & -\frac{4}{\sqrt{2}} & 1 & 0 \\ 0 & -1 & 0 & 1 & 0 \\ 0 & -1 & 0 & -1 & -1 \end{bmatrix} \sim$$

$$\begin{bmatrix} 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & -\frac{4}{\sqrt{2}} & 1 & 0 \\ 0 & 0 & -\frac{4}{\sqrt{2}} & 2 & 0 \\ 0 & 0 & -\frac{4}{\sqrt{2}} & 0 & -1 \end{bmatrix} \sim \begin{bmatrix} 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & -\frac{4}{\sqrt{2}} & 1 & 0 \\ 0 & 0 & -\frac{4}{\sqrt{2}} & 2 & 0 \\ 0 & 0 & 0 & 2 & 1 \end{bmatrix}$$

$$\left\{d = \tfrac{1}{2}, c = \tfrac{\sqrt{2}}{4}, b = \tfrac{1}{2}, a = -\tfrac{\sqrt{2}}{4}\right\}$$

$$I = \int \frac{-\frac{\sqrt{2}}{4}x + \frac{1}{2}}{\left(x^2 - \sqrt{2}x + 1\right)} + \int \frac{\frac{\sqrt{2}}{4}x + \frac{1}{2}}{\left(x^2 + \sqrt{2}x + 1\right)} = \frac{1}{4}\int \frac{-\sqrt{2}x + 2}{\left(x^2 - \sqrt{2}x + 1\right)} + \frac{1}{4}\int \frac{\sqrt{2}x + 2}{\left(x^2 + \sqrt{2}x + 1\right)}$$

$$I = \frac{-\sqrt{2}}{4}\int \frac{x - \sqrt{2}}{\left(x^2 - \sqrt{2}x + 1\right)} + \frac{\sqrt{2}}{4}\int \frac{x + \sqrt{2}}{\left(x^2 + \sqrt{2}x + 1\right)} = \frac{-\sqrt{2}}{8}\int \frac{2x - 2\sqrt{2}}{\left(x^2 - \sqrt{2}x + 1\right)} + \frac{\sqrt{2}}{8}\int \frac{2x + 2\sqrt{2}}{\left(x^2 + \sqrt{2}x + 1\right)}$$

$$I = \frac{-\sqrt{2}}{8}\int \frac{2x - \sqrt{2} - \sqrt{2}}{x^2 - \sqrt{2}x + 1} + \frac{\sqrt{2}}{8}\int \frac{2x + \sqrt{2} + \sqrt{2}}{x^2 + \sqrt{2}x + 1}$$

$$\left\{ \begin{array}{c} x^2 - \sqrt{2}x + 1 = t \\ \left(2x - \sqrt{2}\right)dx = dt \end{array} \right\} \left\{ \begin{array}{c} x^2 + \sqrt{2}x + 1 = z \\ \left(2x + \sqrt{2}\right)dx = dz \end{array} \right\}$$

$$I = \frac{-\sqrt{2}}{8}\int \frac{dt}{t} + \frac{\sqrt{2}}{8}\int \frac{dz}{z} + \frac{-\sqrt{2}}{8}\int \frac{-\sqrt{2}}{x^2 - \sqrt{2}x + 1} + \frac{\sqrt{2}}{8}\int \frac{+\sqrt{2}}{x^2 + \sqrt{2}x + 1}$$

$$I = -\frac{1}{8}\sqrt{2}\ln t + \frac{1}{8}\sqrt{2}\ln z + \frac{2}{8}\int \frac{dx}{x^2 - \sqrt{2}x + 1} + \frac{2}{8}\int \frac{dx}{x^2 + \sqrt{2}x + 1}$$

$$I = -\frac{1}{8}\sqrt{2}\ln t + \frac{1}{8}\sqrt{2}\ln z + \frac{1}{4}\int \frac{dx}{\left(x - \frac{\sqrt{2}}{2}\right)^2 - \frac{1}{2} + 1} + \frac{1}{4}\int = \frac{dx}{\left(x + \frac{\sqrt{2}}{2}\right)^2 - \frac{1}{2} + 1}$$

$$I = -\frac{\sqrt{2}}{8}\ln t + \frac{\sqrt{2}}{8}\ln z - \frac{\sqrt{2}}{4}\arctan\left(-\sqrt{2}x + 1\right) + \frac{\sqrt{2}}{4}\arctan\left(\sqrt{2}x + 1\right)$$

$$I = -\frac{\sqrt{2}}{8}\ln\left(x^2 - \sqrt{2}x + 1\right) + \frac{\sqrt{2}}{8}\ln\left(x^2 + \sqrt{2}x + 1\right) - \frac{\sqrt{2}}{4}\arctan\left(-\sqrt{2}x + 1\right) + \frac{\sqrt{2}}{4}\arctan$$

1296

$$\int \frac{dx}{x^4 + x^2 + 1} = \int \frac{(x^2 - 1)}{(x^4 + x^2 + 1)(x^2 - 1)}dx = \int \frac{x^2 - 1}{x^6 - 1}dx = \int \frac{x^2 - 1}{(x^3 - 1)(x^3 + 1)}dx$$

$$\frac{x^2 - 1}{(x^3 - 1)(x^3 + 1)} = \frac{ax^2 + bx + c}{x^3 - 1} + \frac{dx^2 + ex + f}{x^3 + 1} \mid (x^3 - 1)(x^3 + 1)$$

$$x^2 - 1 = (ax^2 + bx + c)(x^3 + 1) + (dx^2 + ex + f)(x^3 - 1)$$
$$x^2 - 1 = ax^5 + ax^2 + bx^4 + bx + cx^3 + c + dx^5 - dx^2 + ex^4 - ex + fx^3 - f$$
$$x^2 - 1 = x^5(a + d) + x^4(b + e) + x^3(c + f) + x^2(a - d) + x(b - e) + c - f$$
$$a + d = 0$$
$$b + e = 0$$
$$c + f = 0$$
$$a - d = 1$$
$$b - e = 0$$
$$c - f = -1$$

$$\begin{bmatrix} 1 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & -1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & -1 & -1 \end{bmatrix} \rightarrow gauss \rightarrow \begin{bmatrix} 1 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -2 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & -2 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -2 & -1 \end{bmatrix}$$

$$\{f = \tfrac{1}{2} \qquad e = 0 \qquad d = -\tfrac{1}{2}\}$$
$$c + \tfrac{1}{2} = 0 \qquad \{c = -\tfrac{1}{2}\} \qquad \{b = 0\}$$
$$a - \tfrac{1}{2} = 0 \qquad \{a = \tfrac{1}{2}\}$$

$$I = \int \frac{\frac{1}{2}x^2 - \frac{1}{2}}{x^3 - 1} + \int \frac{-\frac{1}{2}x^2 + \frac{1}{2}}{x^3 + 1} = \frac{1}{2}\int \frac{x^2 - 1}{x^3 - 1} - \frac{1}{2}\int \frac{x^2 - 1}{x^3 + 1} = \frac{1}{2}\int \frac{x^2 - 1}{(x - 1)(x^2 + x + 1)}dx - \frac{1}{2}\int \frac{x^2 - 1}{(x + 1)(x^2 - x + 1)}dx$$

$$I = \frac{1}{2}\int \frac{x + 1}{x^2 + x + 1}dx - \frac{1}{2}\int \frac{x^2 - 1}{x^3 + 1}dx = \int \frac{2x + 1 + 1}{x^2 + x + 1}dx - \frac{1}{2}\int \frac{(x - 1)(x + 1)}{(x + 1)(x^2 - x + 1)}dx$$

$$I = \int \frac{2x + 1}{x^2 + x + 1}dx + \int \frac{dx}{x^2 + x + 1} - \int \frac{2x - 1 - 1}{x^2 - x + 1}dx$$

$$I = \int \frac{dt}{t} + \int \frac{dx}{\left(x + \frac{1}{2}\right)^2 - \frac{1}{4} + 1} - \int \frac{2x - 1}{x^2 - x + 1}dx + \int \frac{dx}{x^2 - x + 1}$$

$$I = \ln t + \int \frac{dx}{\left(x + \frac{1}{2}\right)^2 + \frac{3}{4}} - \int \frac{dz}{z} + \int \frac{dx}{\left(x - \frac{1}{2}\right)^2 - \frac{1}{4} + 1}$$

$$I = \ln(x^2 + x + 1) + \frac{2\sqrt{3}}{3}\arctan\frac{(2x + 1)\sqrt{3}}{3} - \ln z + \int \frac{dx}{\left(x - \frac{1}{2}\right)^2 + \frac{3}{4}}$$

$$I = \ln(x^2 + x + 1) + \frac{2\sqrt{3}}{3}\arctan\frac{(2x + 1)\sqrt{3}}{3} - \ln(x^2 - x + 1) + \frac{2\sqrt{3}}{3}\arctan\frac{(2x - 1)\sqrt{3}}{3}$$

$$I = \ln\frac{x^2 + x + 1}{x^2 - x + 1} + \frac{2\sqrt{3}}{3}\arctan\frac{2x + 1}{\sqrt{3}} + \frac{2\sqrt{3}}{3}\arctan\frac{2x - 1}{\sqrt{3}} + C$$

?????????

1297

$$\int \frac{dx}{(1 + x^2)^2}$$

$$\left\{ \begin{array}{c} x = \tan t \\ dx = (1 + \tan^2 t)dt \end{array} \right\}$$

$$= \int \frac{(1 + \tan^2 t)dt}{(1 + \tan^2 t)^2} = \int \frac{dt}{(1 + \tan^2 t)} = \int \frac{dt}{1 + \frac{\sin^2 t}{\cos^2 t}} = \int \frac{dt}{\frac{\cos^2 t + \sin^2 t}{\cos^2 t}} = \int \frac{\cos^2 t dt}{\cos^2 t + \sin^2 t} = \int \frac{\cos^2 t dt}{1} = \int \cos^2 t dt$$

$$= \int \frac{1}{2}(1 + \cos 2t) = \frac{1}{2}\int dt + \frac{1}{2}\int \cos 2t dt = \frac{1}{2}t + \frac{1}{4}\sin 2t = \frac{1}{2}\arctan x + \frac{1}{4}\sin(2\arctan x$$

1298

$$I = \int \frac{3x+5}{(x^2+2x+2)}dx = \int \frac{3x+5}{\left((x+1)^2+1\right)}dx = 3\int \frac{x}{\left((x+1)^2+1\right)}dx + 5\int \frac{dx}{\left((x+1)^2+1\right)} = 3\int \frac{x}{\left((x+1)^2+1\right)}dx +$$

$$I = 3\int \frac{x+1-1}{\left((x+1)^2+1\right)}dx + 5\arctan(x+1) = 3\int \frac{x+1}{\left((x+1)^2+1\right)}dx - 3\int \frac{dx}{\left((x+1)^2+1\right)} + 5\arctan(x+1)$$

$$I = 3\int \frac{x+1}{\left((x+1)^2+1\right)}dx - 3\arctan(x+1) + 5\arctan(x+1) = 3\int \frac{x+1}{\left((x+1)^2+1\right)}dx + 2\arctan(x+1$$

$$I_1 = \left\{ \begin{array}{c} x+1 = t \\ dx = dt \end{array} \right\} = 3\int \frac{t}{t^2+1}dt = \left\{ \begin{array}{c} t^2+1 = u \\ 2tdt = du \end{array} \right\} = 3\int \frac{du}{2u} = \frac{3}{2}\ln u =$$

$$\frac{3}{2}\ln(t^2+1)$$

$$I_1 = \frac{3}{2}\ln\left((x+1)^2+1\right) + C$$

$$I = \frac{3}{2}\ln\left((x+1)^2+1\right) + 2\arctan(x+1) + C$$

1299

$$\int \frac{dx}{(x+1)(x^2+x+1)^2}$$

$- - - - - - - - -$

$$\frac{1}{(x+1)(x^2+x+1)^2} = \frac{a}{x+1} + \frac{bx+c}{(x^2+x+1)^2} + \frac{dx+e}{(x^2+x+1)} | (x+1)(x^2+x+1)^2$$

$$1 = a(x^2+x+1)^2 + (bx+c)(x+1) + (dx+e)(x+1)(x^2+x+1)$$

$$1 = ax^4 + 2ax^3 + 3ax^2 + 2ax + a + bx^2 + bx + cx + c + dx^4 + 2dx^3 + 2dx^2 + dx + ex^3 + 2e$$

$$x^2 + 2ex + e$$

$$1 = x^4(a+d) + x^3(2a+e+2d) + x^2(3a+b+2d+2e) + x(2a+b+c+d+2e) + a + c +$$

$$a + d = 0$$

$$2a + e + 2d = 0$$

$$3a + b + 2d + 2e = 0$$

$$2a + b + c + d + 2e = 0$$

$$a + c + e = 1$$

$$\begin{bmatrix} 1 & 0 & 0 & 1 & 0 & 0 \\ 2 & 0 & 0 & 2 & 1 & 0 \\ 3 & 1 & 0 & 2 & 2 & 0 \\ 2 & 1 & 1 & 1 & 2 & 0 \\ 1 & 0 & 1 & 0 & 1 & 1 \end{bmatrix} \rightarrow \begin{array}{c} Gaussova \\ eliminacija \end{array} \rightarrow \begin{bmatrix} 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & -1 & 2 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 1 & 1 \\ 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix}$$

$$\{e = 0\}\{d = -1\}\{c = 0\}$$

$$b - d + 2e = 0$$

$$b + 1 = 0$$

$$\{b = -1\}\{a = 1\}$$

$- - - - - - - - - - -$

$$\int \frac{dx}{(x+1)(x^2+x+1)^2} = \int \frac{dx}{x+1} + \int \frac{-xdx}{(x^2+x+1)^2} + \int \frac{-xdx}{x^2+x+1}$$

$$= \ln(x+1) - \int \frac{dx}{x^2+x+1} - \int \frac{dx}{(x^2+x+1)^2}$$

$$= \ln(x+1) - \int \frac{dx}{x^2+x+1+\frac{1}{4}-\frac{1}{4}} - \int \frac{dx}{(x^2+x+1)^2}$$

$$I = \ln(x+1) - I_1 - I_2$$

$$I_1 = \int \frac{dx}{x^2+x+1+\frac{1}{4}-\frac{1}{4}} = \int \frac{dx}{\left(x+\frac{1}{2}\right)^2+\frac{3}{4}} =$$

$$I_1 = \left\{ \begin{array}{c} x+\frac{1}{2} = t \\ dx = dt \end{array} \right\} =$$

$$I_1 = \int \frac{dt}{t^2+\frac{3}{4}} == \frac{2}{\sqrt{3}}\arctan\left(\frac{2}{\sqrt{3}}t\right) = \frac{2}{\sqrt{3}}\arctan\left(\frac{2}{\sqrt{3}}(x+\frac{1}{2})\right) = \frac{2}{\sqrt{3}}\arctan\left(\frac{2x+1}{\sqrt{3}}\right)$$

$$I_2 = \int \frac{dx}{(x^2+x+1)^2} = \int \frac{dx}{\left(\left(x+\frac{1}{2}\right)^2+\frac{3}{4}\right)^2}$$

$$I_2 = \left\{ \begin{array}{c} x+\frac{1}{2} = t \\ dx = dt \end{array} \right\}$$

$$I_2 = \int \frac{dt}{\left(t^2+\frac{3}{4}\right)^2}$$

$$I_2 = \left\{ \begin{array}{cc} u = \frac{1}{\left(t^2+\frac{3}{4}\right)^2} & dt = dv \\ -\frac{256t}{(4t^2+3)^3}dt = du & t = v \end{array} \right\}$$

$$I_2 = \frac{t}{\left(t^2+\frac{3}{4}\right)^2} - \int\left[-\frac{256t^2}{(4t^2+3)^3}\right]dt = \frac{t}{\left(t^2+\frac{3}{4}\right)^2} + 64\int \frac{4t^2+3-3}{(4t^2+3)^3}dt$$

$$I_2 = \frac{t}{\left(t^2+\frac{3}{4}\right)^2} + 64\int \frac{dt}{(4t^2+3)^2} - 3\int \frac{dt}{\left(t^2+\frac{3}{4}\right)^3} = \frac{t}{\left(t^2+\frac{3}{4}\right)^2} + \int \frac{dt}{\left(t^2+\frac{3}{4}\right)^2} - 3\int \frac{dt}{\left(t^2+\frac{3}{4}\right)^3}$$

$I_2 = \frac{t}{\left(t^2+\frac{3}{4}\right)^2} + I_2 - 3\int \frac{dt}{\left(t^2+\frac{3}{4}\right)^3}$.... odusta san