Computer Organization

Homework 1.

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1. Transform the following numbers from one base to another.

Decimal to binary.

• 10

10 |2
0 5 |2
1 2 |2
0 1

=> 1010

• 1369 = 10101011001

1369 |2
1 684 |2
0 342 |2
0 171 |2
1 85 |2
1 42 |2
0 21 |2
1 10 |2
0 5 |2
1 2 |2
0 1

• 9234876 = 100011001110100110111100

9234876 |2
0 4617438 |2
0 2308719 |2
1 1154359 |2
1 577179

577179 |2
1 288589 |2
1 144294 |2
0 72147 |2
1 36073 |2
1 18036

18036 |2
0 9018 |2
0 4509 |2
1 2254 |2
0 1127 |2
1 563

563 |2
1 281 |2
1 140 |2
0 70 |2
0 35 |2
1 17 |2
1 8 |2
0 4 |2
0 2 |2
0 1

• 49263.749 = 10111011111011010010000101

49263749 |2
1 24631874 |2
0 12315937 |2
1 6157968 |2
0 3078984 |2
0 1539492 |2
0 769746

769746 |2
0 384873 |2
1 192436 |2
0 96218 |2
0 48109 |2
1 24054 |2
0 12027 |2
1 6013 |2
1 3006

3006 |2
0 1503 |2
1 751 |2
1 375 |2
1 187 |2
1 93 |2
1 46 |2
0 23 |2
1 11 |2
1 5 |2
1 2 |2
0 1

Decimal to binary using 2's complement.

Use the minimum number of bits required to express the number.

- −20

20 | 2
0 10 | 2
0 5 | 2
1 2 | 2
0 1

20 → $(10100)_2$ → en 6 bits $(010100)_2$

20 (1's) → 101011
\+ 1
101100

−20 → 101100 2's comp

- −1025

1025 | 2
1 512 | 2
0 256 | 2
0 128 | 2
0 64 | 2
0 32 | 2
0 16 | 2
0 8 | 2
0 4 | 2
0 2 | 2
0 1

1025 = 10000000001 → 010000000001 (12 bits)

1025 (1's) → 101111111110
\+ 1
101111111111

−1025 (2's) → 101111111111

- −3925

3925 | 2
1 1962 | 2
0 981 | 2
1 490 | 2
0 245 | 2
1 122 | 2
0 61 | 2
1 30 | 2
0 15 | 2
1 7

7 | 2
1 3 | 2
1 1

3925 → $(111101010101)_2$
→ $(0111101010101)_2$ (13 bits)

3925 (1's) → 1000010101010
\+ 1
1000010101011

−3925 (2's) → 1000010101011

- −104596

104596 | 2
0 52298 | 2
0 26149 | 2
1 13074 | 2
0 6537 | 2
1 3268 | 2
0 1634 | 2
0 817 | 2
1 408

408 | 2
0 204 | 2
0 102 | 2
0 51 | 2
1 25 | 2
1 12 | 2
0 6 | 2
0 3 | 2
1 1

104596 → $(011001100010010100)_2$ (18 bits)

104596 (1's) → 100110011101101011
\+ 1
100110011101101100

−104596 (2's) → 100110011101101100

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Unsigned binary to hex

Use the long and the short methods.

• 1100111101010110011011101101100000101001

Método largo. Bin → Dec

$1\cdot2^0+0\cdot2^1+0\cdot2^2+1\cdot2^3+0\cdot2^4+1\cdot2^5+0\cdot2^6+0\cdot2^7+0\cdot2^8+0\cdot2^9+0\cdot2^{10}+1\cdot2^{11}+1\cdot2^{12}+0\cdot2^{13}+1\cdot2^{14}+1\cdot2^{15}$
$+0\cdot2^{16}+2^{17}+2^{18}+2^{19}+0\cdot2^{20}+1\cdot2^{21}+2^{22}+0\cdot2^{23}+0\cdot2^{24}+2^{25}+2^{26}+0\cdot2^{27}+2^{28}+0\cdot2^{29}+2^{30}+0\cdot2^{31}$
$+2^{32}+2^{33}+2^{34}+2^{35}+0\cdot2^{36}+0\cdot2^{37}+2^{38}+2^{39} = 890508335145$

Dec → Hex

890508335145 | 16
9 55656770946 | 16
2 3478548184 | 16
8 217409261 | 16
D 13588078 | 16
E 849254 | 16
6 53078 | 16
6 3317

3317 | 16
5 207 | 16
F C

$(CF566ED829)_{16}$

Método corto

1100	1111	0101	0110	0110	1110	1101	1000	0010	1001
12 → C	F	5	6	6	14 → E	13 → D	8	2	9

$\Rightarrow (CF566ED829)_{16}$

• 1000011110001110001110001110001111110011

Método largo. Bin → Dec

$2^0+2^1+0\cdot2^2+0\cdot2^3+2^4+2^5+2^6+2^7+2^8+2^9+0\cdot2^{10}+0\cdot2^{11}+0\cdot2^{12}+2^{13}+2^{14}+2^{15}+0\cdot2^{16}+0\cdot2^{17}+0\cdot2^{18}+2^{19}+2^{20}+$
$2^{21}+0\cdot2^{22}+0\cdot2^{23}+0\cdot2^{24}+2^{25}+2^{26}+2^{27}+0\cdot2^{28}+0\cdot2^{29}+0\cdot2^{30}+2^{31}+2^{32}+2^{33}+2^{34}+0\cdot2^{35}+0\cdot2^{36}+0\cdot2^{37}$
$+0\cdot2^{38}+2^{39}=582206678003$

Dec → Hex

582206678003 | 16
3 36387917375 | 16
F 2274244835 | 16
3 142140302

$142140302 \div 16$

E $8883768 \div 16$

8 $555235 \div 16$

3 $34702 \div 16$

E $2168 \div 16$

8 $135 \div 16$

7 8

$\Rightarrow (878E38E3F3)_{16}$

Método Corto

$$\underbrace{1000}_{8}\quad \underbrace{0111}_{7}\quad \underbrace{1000}_{8}\quad \underbrace{1110}_{14 \to E}\quad \underbrace{0011}_{3}\quad \underbrace{1000}_{8}\quad \underbrace{1110}_{14 \to E}\quad \underbrace{0011}_{3}\quad \underbrace{1111}_{15 \to F}\quad \underbrace{0011}_{3}$$

$\rightarrow (878E38E3F3)_{16}$

- 1010110101011100011001010100101010101010

Método largo. Bin -> Dec

$$0\cdot2^0+2^1+0\cdot2^2+2^3+0\cdot2^4+2^5+0\cdot2^6+2^7+0\cdot2^8+2^9+0\cdot2^{10}+2^{11}+0\cdot2^{12}+0\cdot2^{13}+2^{14}+0\cdot2^{15}+2^{16}+0\cdot2^{17}+2^{18}$$
$$+0\cdot2^{19}+0\cdot2^{20}+2^{21}+2^{22}+0\cdot2^{23}+0\cdot2^{24}+0\cdot2^{25}+2^{26}+2^{27}+2^{28}+0\cdot2^{29}+2^{30}+0\cdot2^{31}+2^{32}+0\cdot2^{33}+2^{34}+2^{35}$$
$$+0\cdot2^{36}+2^{37}+0\cdot2^{38}+2^{39}=744579484330$$

Dec -> Hex

$744579484330 \div 16$

A $46536217770 \div 16$

A $2908513610 \div 16$

A $181782100 \div 16$

4 $11361381 \div 16$

5 $710086 \div 16$

6 $44380 \div 16$

C $2773 \div 16$

5 $173 \div 16$

D A

$\rightarrow (AD5C654AAA)_{16}$

Método Corto.

$$\underbrace{1010}_{A}\quad \underbrace{1101}_{D}\quad \underbrace{0101}_{5}\quad \underbrace{1100}_{C}\quad \underbrace{0110}_{6}\quad \underbrace{0101}_{5}\quad \underbrace{0100}_{4}\quad \underbrace{1010}_{A}\quad \underbrace{1010}_{A}\quad \underbrace{1010}_{A}$$

- 1010001010101010101010101011111111000000

<u>Método largo.</u> Bin to Dec

$$0\cdot2^0 + 0\cdot2^1 + 0\cdot2^2 + 0\cdot2^3 + 0\cdot2^4 + 0\cdot2^5 + 2^6 + 2^7 + 2^8 + 2^9 + 2^{10} + 2^{11} + 2^{12} + 2^{13} + 0\cdot2^{14} + 2^{15} + 0\cdot2^{16} + 2^{17} + 0\cdot2^{18}$$
$$+ 2^{19} + 0\cdot2^{20} + 2^{21} + 0\cdot2^{22} + 2^{23} + 0\cdot2^{24} + 2^{25} + 0\cdot2^{26} + 2^{27} + 0\cdot2^{28} + 2^{29} + 0\cdot2^{30} + 2^{31} + 0\cdot2^{32} + 2^{33} + 0\cdot2^{34} + 0\cdot2^{35} +$$
$$0\cdot2^{36} + 2^{37} + 0\cdot2^{38} + 2^{39} = 698648018880$$

Dec → Hex

698648018880	16	0
43665501180	16	C
2729093823	16	F
170568363	16	B
10660522	16	A
666282	16	A
41642	16	A
2602	16	A
162	16	2
		A

$\Rightarrow (A2AAAABFC0)_{16}$

<u>Método Corto</u>

1010	0010	1010	1010	1010	1010	1011	1111	1100	0000
A	2	A	A	A	10 → A	11 → B	15 → F	12 → C	0

$\Rightarrow (A2AAAABFC0)_{16}$

Signed binary to Octal.
Use the long and short methods.

- 11111100000111110000000111010101011

Long method. Bin → Dec. Negative Number.

(1's comp) → 000000111110000001111111000101010100
$+ \quad 1$

$\underbrace{}$ 000000111110000001111111000101010101 → (2's comp)

$$000000\underbrace{111110000001111111000101010101}_{\text{magnitude}}$$

$$2^0 + 0\cdot2^1 + 2^2 + 0\cdot2^3 + 2^4 + 0\cdot2^5 + 2^6 + 0\cdot2^7 + 0\cdot2^8 + 0\cdot2^9 + 2^{10} + 2^{11} + 2^{12} + 2^{13} + 2^{14} + 2^{15} + 2^{16} + 0\cdot2^{17} + 0\cdot2^{18}$$
$$+ 0\cdot2^{19} + 0\cdot2^{20} + 0\cdot2^{21} + 2^{22} + 2^{23} + 2^{24} + 2^{25} + 2^{26} = 130153557$$

Dec → Oct

1301535557 | 8
5 16269194 | 8
2 2033649 | 8
1 254206 | 8
6 31775 | 8
7 3971 | 8
3 496 | 8
0 62 | 8
6 7

$\rightarrow (-760376125)_8$

Métado Corto

$\underbrace{111}_{7}\ \underbrace{110}_{6}\ \underbrace{000}_{0}\ \underbrace{011}_{3}\ \underbrace{111}_{7}\ \underbrace{110}_{6}\ \underbrace{001}_{1}\ \underbrace{010}_{2}\ \underbrace{101}_{5}$

$\rightarrow (-760376125)_8$

• 01010101010111111111111110000000

Positive number.

Long method. Bin → Dec

$$0\cdot2^0+0\cdot2^1+0\cdot2^2+0\cdot2^3+0\cdot2^4+0\cdot2^5+0\cdot2^6+2^7+2^8+2^9+2^{10}+2^{11}+2^{12}+2^{13}+2^{14}+2^{15}+2^{16}+2^{17}+2^{18}$$
$$+2^{19}+2^{20}+2^{21}+0\cdot2^{22}+2^{23}+0\cdot2^{24}+2^{25}+0\cdot2^{26}+2^{27}+0\cdot2^{28}+2^{29}+0\cdot2^{30}+2^{31}=2864709504$$

Dec → Oct

2864709504 | 8
0 358088688 | 8
0 44761086 | 8
6 5595135 | 8
7 699391 | 8
7 87423 | 8
7 10927 | 8
7 1365 | 8
5 170 | 8
2 21 | 8
5 2

$\rightarrow (25257777600)_8$

Short Method

Added

0 ↓ + : $\underbrace{010}_{2}\ \underbrace{101}_{5}\ \underbrace{010}_{2}\ \underbrace{101}_{5}\ \underbrace{111}_{7}\ \underbrace{111}_{7}\ \underbrace{111}_{7}\ \underbrace{111}_{7}\ \underbrace{110}_{6}\ \underbrace{000}_{0}\ \underbrace{000}_{0}$

$\rightarrow (25257777600)_8$

• 111000 111000000111 111111 000000101010

Long Method. Bin→Dec. Negative number

(1's comp) → 000111000 111 111 000 000000 111 11 01 01 01

$$\begin{array}{r} 000111000\,111\,111\,000\,000000\,111\,11\,01\,01\,01 \\ +\quad 1 \\ \hline \underbrace{000\,111\,000\,111111\,000000\,000\,111\,11\,01\,01\,10}_{\text{magnitude}} \end{array}$$

$$0\cdot2^0 + 2^1 + 2^2 + 0\cdot2^3 + 2^4 + 0\cdot2^5 + 2^6 + 2^7 + 2^8 + 2^9 + 2^{10} + 0\cdot2^{11} + 0\cdot2^{12} + 0\cdot12^{13} + 0\cdot12^{14} + 0\cdot12^{15} + 0\cdot12^{16} + 0\cdot2^{17} + 0\cdot2^{18} + 2^{19} + 2^{20} + 2^{21} + 2^{22} + 2^{23} + 2^{24} + 0\cdot2^{25} + 0\cdot2^{25} + 0\cdot2^{26} + 2^{27} + 2^{28} + 2^{29} = 1912080342.$$

Dec → Oct

1912080342	8	6
239010042	8	2
29876255	8	7
3734531	8	3
466816	8	0
58352	8	0
7294	8	6
911	8	7
113	8	1
14	8	6
1		

→ $(-16176003726)_8$

Short Method

$$\underbrace{001}_{1}\ \underbrace{110}_{6}\ \underbrace{001}_{1}\ \underbrace{111}_{7}\ \underbrace{110}_{6}\ \underbrace{000}_{0}\ \underbrace{000}_{0}\ \underbrace{011}_{3}\ \underbrace{111}_{7}\ \underbrace{010}_{2}\ \underbrace{110}_{6}$$

→ $(-16176003726)_8$

• 1010 10101010 10 000010 10 1010 10 10 111111 000

Long Method. Bin → Dec. Negative Number

(1's) comp → 01010101 01 01 111101 010101 01 01 00000 111

$$\begin{array}{r} 01010101\,01\,01\,111101\,010101\,01\,01\,00000\,111 \\ +\quad 1 \\ \hline \underbrace{01010101\,01\,01\,111101\,010101\,01\,0\,000\,1\,000}_{\text{Magnitude}} \end{array}$$

$$0\cdot2^0 + 0\cdot2^1 + 0\cdot2^2 + 2^3 + 0\cdot2^4 + 0\cdot2^5 + 0\cdot2^6 + 0\cdot2^7 + 2^8 + 0\cdot2^9 + 2^{10} + 0\cdot2^{11} + 2^{12} + 0\cdot2^{13} + 2^{12} + 0\cdot2^{13} + 2^{14} + 0\cdot2^{15} + 2^{16} + 2^{17} + 2^{18} + 2^{19} + 2^{20} + 0\cdot2^{21} + 2^{22} + 0\cdot2^{23} + 2^{24} + 0\cdot2^{25} + 2^{26} + 0\cdot2^{27} + 2^{28} + 0\cdot2^{29} + 2^{30} = 5729244424$$

Dec → Oct

5729244424	8	0
716155553	8	1
89519444	8	4
11189930	8	2
1398741	8	5
174842	8	2
21855	8	7
2731	8	3
341	8	5
42	8	2
5		

→ $(-52537252410)_8$

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Short Method

$$\underbrace{101}_{5}\ \underbrace{010}_{2}\ \underbrace{101}_{5}\ \underbrace{011}_{3}\ \underbrace{111}_{7}\ \ \underbrace{010}_{2}\ \underbrace{101}_{5}\ \underbrace{010}_{2}\ \underbrace{100}_{4}\ \underbrace{001}_{1}\ \underbrace{000}_{0}$$

$\rightarrow (-52537252410)_8$

2. Boolean Circuits.

Draw the boolean circuit and make the thruth table for the following:

- Multiplication of two binary numbers of length' 2 bits.

- Podemos tener los números $A: A_1 A_0$ y $B: B_1 B_0$
- Cuando multiplicamos 2 números binarios de 2 bits el máximo valor posible a obtener es un número binario de 4 bits.
- Ahora consideremos:

$A \times B \rightarrow$		A_1	A_0
	$\times$	B_1	B_0
	Carry1	A_1B_0	A_0B_0
Carry2	A_1B_1	A_0B_1	
Carry2	$A_1B_1 +$ Carry1	$A_1B_0 + A_0B_1$	A_0B_0
	Carry2	Carry1	

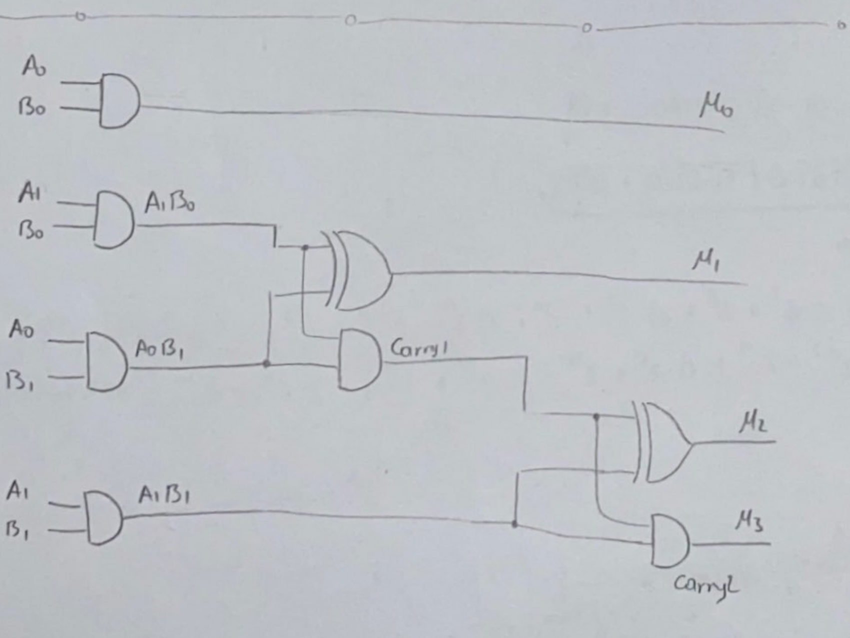
$A \times B = M$

$M = M_3 M_2 M_1 M_0$

$\Rightarrow$ $\boxed{\begin{aligned} M_0 &= A_0B_0 \\ M_1 &= A_1B_0 + A_0B_1 \\ M_2 &= A_1B_1 + C_1 \\ M_3 &= C_2 \end{aligned}}$

$\rightarrow$ Carry1 $= C_1$

$\rightarrow$ Carry2 $= C_2$



A_1	A_0	B_1	B_0	C_1	C_2	M_3	M_2	M_1	M_0
0	0	0	0	0	0	0	0	0	0
0	0	0	1	0	0	0	0	0	0
0	0	1	0	0	0	0	0	0	0
0	0	1	1	0	0	0	0	0	0
0	1	0	0	0	0	0	0	0	0
0	1	0	1	0	0	0	0	0	1
0	1	1	0	0	0	0	0	1	0
0	1	1	1	0	0	0	0	1	1
1	0	0	0	0	0	0	0	0	0
1	0	0	1	0	0	0	0	1	0
1	0	1	0	0	0	0	1	0	0
1	0	1	1	0	0	0	1	1	0
1	1	0	0	0	0	0	0	0	0
1	1	0	1	0	0	0	0	1	1
1	1	1	0	0	0	0	1	1	0
1	1	1	1	1	1	1	0	0	1

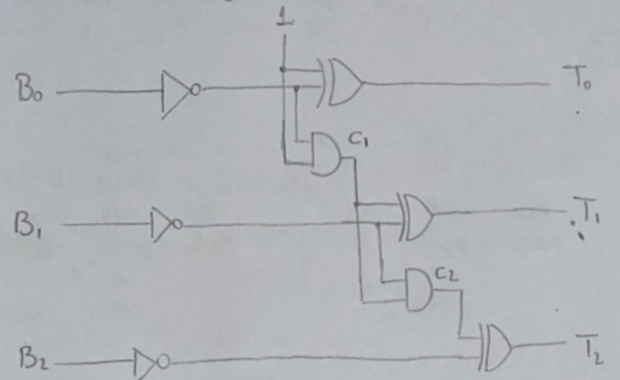
• Two's complement for a binary number of length 3 bits.

Primero hay que recordar como se realiza el complemento de 2.

i.e → 101 → (1's) → 010

$$\begin{array}{r} 010 \\ +\quad 1 \\ \hline 011 \end{array} \quad (2's)$$

B respresents binary number. $B = B_2 B_1 B_0$. T represents 2's comp. of B. $T = T_2 T_1 T_0$



Truth table

B_2	B_1	B_0	C_1	C_2	T_2	T_1	T_0
0	0	0	0	0	0	0	0
0	0	1	0	0	1	1	1
0	1	0	0	0	1	1	0
0	1	1	0	0	1	0	1
1	0	0	1	0	1	0	0
1	0	1	1	0	0	1	1
1	1	0	1	1	0	1	0
1	1	1	1	1	0	0	1

3. Do the following multiplications in binary.

- Use the minimum number of bits required.

- −5 × 8.

Only magnitude. 5×8 = 40 $\xrightarrow{\text{to bin}}$ 101000 (6 bits) → 0101000 (7 bits) $\xrightarrow{(2's)\ comp}$ 1010111 + 1 = $\boxed{1011000}$ (−40) (7 bits)

$\boxed{11011000}$ (8 bits)

5 → 0101 $\xrightarrow{(2's)}$ 1010 + 1 = 1011 → −5 | 8 → 1000

PD 0000, MD 1011, MR 1000, MX 0, CR 0100 → PD 0000, MD 1011, MR 0100, MX 0, CR 0011 → PD 0000, MD 1011, MR 0010, MX 0, CR 0010 → PD 0000, MD 1011, MR 0001, MX 0, CR 0001

PD = PD − MD: 0000 − 0101 = 0101

PD 0101, MD 1011, MR 0001, MX 0, CR 001 → PD 0010, MD, MR 1000, MX 1, CR 000

PD = PD + MD: 0010 + 1011 = 1101

PD 1101, MR 1000

⇒ 11011000

11011000 → (1's com) → 00100111 + 1 = 00101000 → (2's comp) → 40 ⇒ 11011000 = −40

- 11 × (−10)

11 = 01011

10 = 01010 ; −10 = 10110

PD 00000, MD 10110, MR 01011, MX 0, CR 00101

PD = PD − MD: 00000 − 01010 = 01010

PD 01010, MD 10110, MR 01011, MX 0, CR 00101 → PD 00101, MR 00101, MX 1, CR 00100 → PD 00010, MR 10010, MX 1, CR 00011

PD = PD + MD: 00010 + 10110 = 11000

PD 11000, MR 10010, MX 1, CR 00011 → PD 11100, MR 01001, MX 0, CR 00010

PD = PD − MD: 11100 − 01010 = *00110

PD 00110, MR 01001, MX 0, CR 00010 → PD 00011, MR 00100, MX 1, CR 00001

PD = PD + MD: 00011 + 10110 = 11001

PD 11001, MR 00100, MX 1, CR 00001 → PD 11100, MR 10010, MX 0, CR 00000

⇒ $\boxed{1110010010}$ = −110

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• (2×3) = 6

2 = 010 (MR)
3 = 011 (MD) → (2's) 100
                     +1
                     101

MD 011

PD	MR	MX
000	010	0
	CR 011	

→

PD	MR	MX
000	001	0
	CR 010	

PD = PD − MD
 000
 101
 101

PD	MR	MX
101	001	0
	CR 010	

→

PD	MR	MX
110	100	1
	CR 001	

PD = PD + MD
 110
 011
 $\cancel{1}$001

PD	MR	MX
001	100	1
	CR 001	

→

PD	MR	MX
000	110	0
	CR 000	

⇒ $\boxed{000110}$ = 6

• (−4)×(−8) = 32 → 100000

4 → 00100 → (1's comp) 11011
                          +1
                        11100 → (2's)

8 → 01000 → (1's) 10111
                     +1
                  11000

MD 11100

PD	MR	MX
00000	11000	0
	CR 00101	

→

PD	MR	MX
00000	01100	
	CR 00100	

→

PD	MR	MX
00000	00110	0
	CR 00011	

→

PD	MR	MX
00000	00011	0
	CR 00010	

PD = PD − MD
 00000
 00100
 00100

PD	MR	MR
00100	00011	0
	CR 00010	

→

PD	MR	MX
00010	00001	1
	CR 00001	

→

PD	MR	MX
00001	00000	1
	CR 00000	

$\boxed{0000100000}$ = 32

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