

Simulating and estimating a state-space random walk model in R and TMB

Christopher L. Cahill

Quantitative Fisheries Center

Michigan State University

Tutorial available at:

<https://github.com/ChrisFishCahill/MSC/tree/master/rw>

Random walk in R and TMB

Goal:

- estimate/simulate a state-space model

Steps:

- A bit of math
- Simulate fake data in R
- Build estimation model in TMB
- Pray that it converges, weep if it does not (how to assess?)
- How to deal with missing data

Math for a simple state-space random walk

$$\lambda_i = \lambda_{i-1} + \eta_i$$

$$Y_i = \lambda_i + \varepsilon_i$$

where $i = 1 \dots 43$, $\eta_i \sim N(0, \sigma_{rw}^2)$,

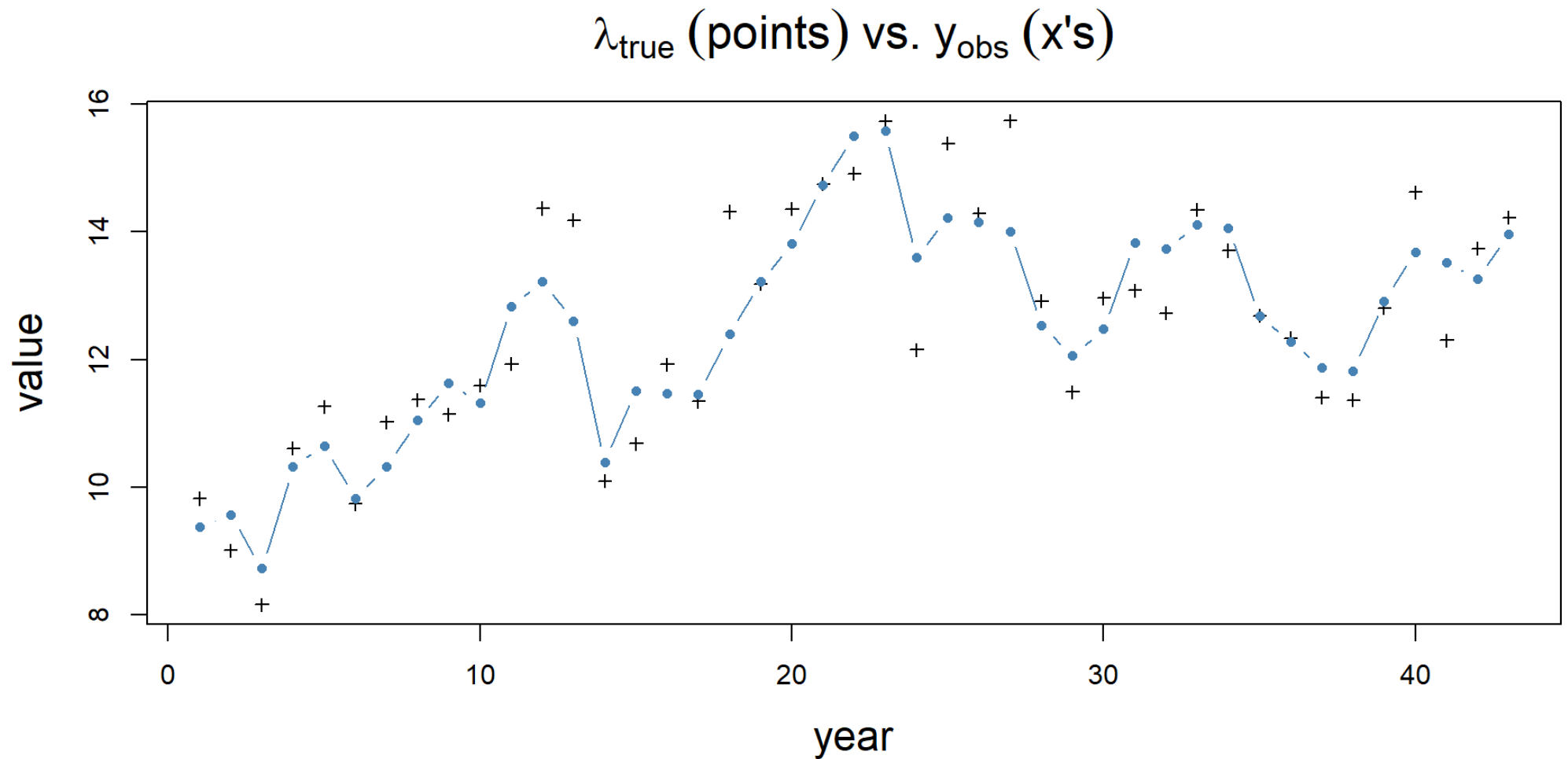
and $\varepsilon_i \sim N(0, \sigma_{obs}^2)$ all independent.

R code for simulating a random walk:

rw.R

```
1 # set leading parameters
2 years = 1:43
3 lam0 = 10 # initial value
4 sd_rw = 1 # stdev of process
5 sd_obs = 0.8 # stdev of observations
6 lams = rep(NA, length(years)) # true lambdas
7 y_obs = rep(NA, length(years)) # observed data
8 set.seed(1) # ensure "random" data are same
9
10 lams[1] = rnorm(1, lam0, sd_rw) # initialize the stochastic process
11
12 # do the random walk, add in process error:
13 for(i in 2:length(years)){
14   lams[i] = rnorm(1, lams[i-1], sd_rw)
15 }
16
17 # add observation error to true (latent) process:
18 y_obs = rnorm(length(lams), lams, sd_obs)
```

Visualizing the simulated data



TMB code for estimating a random walk:

rw.cpp

```
1 #include <TMB.hpp>
2 template<class Type>
3 Type objective_function<Type>::operator() ()
4 {
5     DATA_VECTOR(y_obs);           // observed data
6     PARAMETER(ln_sd_rw);           // log(sd) process
7     PARAMETER(ln_sd_obs);          // log(sd) observation
8     PARAMETER(lam0);               // initial state
9     PARAMETER_VECTOR(lams);        // random effects
10
11     int n_year = y_obs.size();
12     Type sd_rw = exp(ln_sd_rw);
13     Type sd_obs = exp(ln_sd_obs);
14
15     // random effects:
16     jnll -= dnorm(lams(0), lam0, sd_rw, true); // pr(initial state)
17     for(int i=1; i<n_year; i++){
18         jnll -= dnorm(lams(i), lams(i-1), sd_rw, true); // pr(subsequent states)
19     }
20
21     // likelihood:
22     for(int i=0; i<n_year; i++){
23         jnll -= dnorm(y_obs(i), lams(i), sd_obs, true); // pr(observations)
24     }
25
26     return jnll; // joint neg log likelihood
27 }
```

Pulling it all together in `rw.R`

```
1 library(TMB)
2 compile("rw.cpp")
[1] 0
3
4 # create dynamically linked library:
5 dyn.load(dynlib("rw"))
6
7 # create a tagged data list:
8 data <- list(y_obs = y_obs)
9
10 # create a tagged parameter list w/ start values:
11 parameters <- list(ln_sd_rw = 0,
12                   ln_sd_obs = 0,
13                   lam0 = 0,
14                   lams = rep(0, length(data$y_obs)))
15
16 # create objective function based on template:
17 obj <- MakeADFun(data, parameters, random = "lams", DLL= "rw")
```

Pulling it all together cont'd

```
1 obj$fn() # return the objective f(x) value
```

Optimizing tape... Done

iter: 1 value: 124.1655 mgc: 15.7428 ustep: 1

iter: 2 mgc: 8.881784e-15

[1] 105.1815

attr(,"logarithm")

[1] TRUE

Pulling it all together cont'd

```
1 obj$gr() # examine par gradients  
iter: 1   mgc: 8.881784e-15  
Matching hessian patterns... Done  
outer mgc: 34.28911  
[1] -34.28911 -12.98452 -5.88800
```

Run the optimization:

```
1 opt = nlminb(obj$par, obj$fn, obj$gr)
iter: 1  mgc: 8.881784e-15
iter: 1  mgc: 8.881784e-15
outer mgc: 34.28911
iter: 1  value: 144.2216 mgc: 1.358261 ustep: 1
iter: 2  mgc: 9.298118e-16
iter: 1  mgc: 9.298118e-16
outer mgc: 13.72374
iter: 1  value: 117.857 mgc: 3.022377 ustep: 1
iter: 2  mgc: 1.970646e-15
iter: 1  mgc: 1.970646e-15
outer mgc: 15.32134
iter: 1  value: 117.4061 mgc: 7.513776 ustep: 1
iter: 2  mgc: 3.941292e-15
iter: 1  value: 112.2716 mgc: 0.7330823 ustep: 1
iter: 2  mgc: 2.220446e-15
iter: 1  mgc: 2.220446e-15
```

Get standard deviations

```
1 sdr = sdreport(obj)
iter: 1  mgc: 8.21565e-15
outer mgc: 3.136849e-05
iter: 1  value: 82.22413 mgc: 0.003330541 ustep: 1
iter: 2  mgc: 6.439294e-15
outer mgc: 0.01252672
iter: 1  value: 82.16155 mgc: 0.003337209 ustep: 1
iter: 2  mgc: 8.715251e-15
outer mgc: 0.01259331
iter: 1  value: 82.20457 mgc: 0.003330541 ustep: 1
iter: 2  mgc: 8.65974e-15
outer mgc: 0.04996776
iter: 1  value: 82.18115 mgc: 0.003337209 ustep: 1
iter: 2  mgc: 7.438494e-15
outer mgc: 0.05006597
iter: 1  value: 82.19284 mgc: 0.002730473 ustep: 1
iter: 2  mgc: 6.661222e-15
```

Get standard deviations

```
1 print(sdr)

sdreport(.) result
              Estimate Std. Error
ln_sd_rw    -0.50223741  0.3639872
ln_sd_obs    0.07582131  0.1690006
lam0         9.57015318  0.9388089
Maximum gradient component: 3.136849e-05
```

- Note: may need to apply a bias correction
- see `?sdreport`
- **Important:** are the SDs too big?

Are diagnostics consistent with convergence?

```
1 final_gradient = obj$gr(opt$par)
iter: 1   mgc: 8.21565e-15
outer mgc: 3.136849e-05

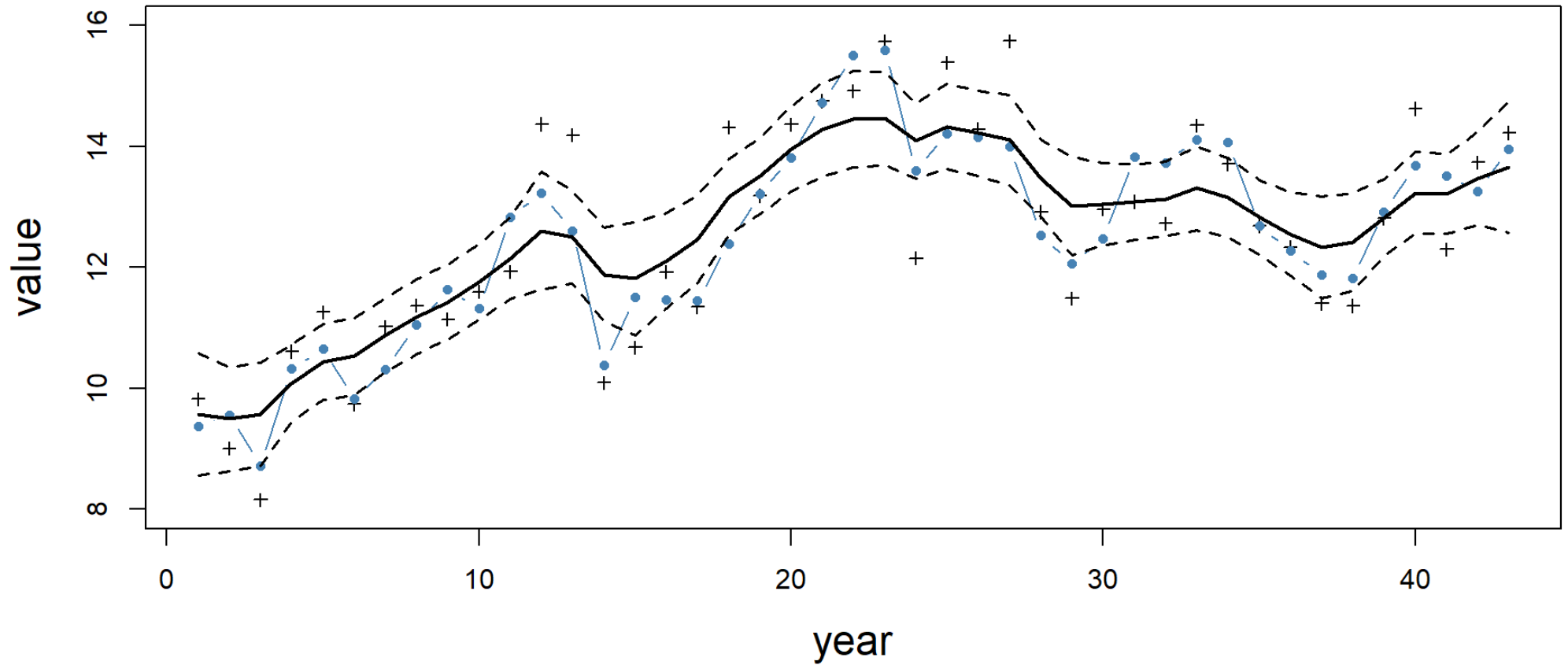
1 if (any(abs(final_gradient) > 0.001) || sdr$pdHess == FALSE) {
2   message("Model did not converge: check results")
3 } else {
4   message("Model diagnostics consistent with convergence")
5 }

[1] "Model fit consistent with convergence"
```

- Why are these reasonable things to check?

Fits vs. simulated data

λ_{true} (points) vs. y_{obs} (x's) vs. λ_{est} (black)



How did we plot those results?

- First, look at the structure of the `sdreport()` output

```
1 str(sdr)
List of 10
 $ value          : num(0)
 $ sd             : num(0)
 $ cov            : logi[0 , 0 ]
 $ par.fixed      : Named num [1:3] -0.5022 0.0758 9.5702
 ..- attr(*, "names")= chr [1:3] "ln_sd_rw" "ln_sd_obs" "lam0"
 $ cov.fixed      : num [1:3, 1:3] 0.1325 -0.0336 -0.0494 -0.0336 0.0286 ...
 ..- attr(*, "dimnames")=List of 2
 .. ..$ : chr [1:3] "ln_sd_rw" "ln_sd_obs" "lam0"
 .. ..$ : chr [1:3] "ln_sd_rw" "ln_sd_obs" "lam0"
 $ pdHess         : logi TRUE
 $ gradient.fixed : num [1:3] -7.01e-07 -3.14e-05 -4.76e-06
 $ par.random     : Named num [1:43] 9.57 9.49 9.57 10.09 10.44 ...
 ..- attr(*, "names")= chr [1:43] "lams" "lams" "lams" "lams" ...
 $ diag.cov.random: num [1:43] 0.515 0.442 0.435 0.323 0.319 ...
 > -----
```

How did we plot those results?

- How do we get 95% confidence intervals?

```
1 mle = sdr$par.random
2 upper_ci = mle + 1.96 * sdr$diag.cov.random
3 lower_ci = mle - 1.96 * sdr$diag.cov.random
```


Break

Part II: dealing with missing data

- Two minor changes to our `rw.cpp` file allows us to deal with missing data

Dealing with missing data

- First, we write a custom function to detect NA values and place it at the top of the `rw.cpp`:

```
1  #include <TMB.hpp>
2  template<class Type>
3  // write a custom function to deal with NAs:
4  bool is_NA(Type x){
5      return R_IsNA(asDouble(x));
6  }
7
8  template<class Type>
9  Type objective_function<Type>::operator() ()
10 {
11     DATA_VECTOR(y_obs);           // observed data
12     PARAMETER(ln_sd_rw);           // log(sd) process
13     PARAMETER(ln_sd_obs);          // log(sd) observation
14     PARAMETER(lam0);               // initial state
15     ...
```

Dealing with missing data

- Next, we correct the likelihood by adding an if-statement:

```
1  ...
2  // random effects:
3  jnll -= dnorm(lams(0), lam0, sd_rw, true);           // pr(initial state)
4  for(int i=1; i<n_year; i++){
5      jnll -= dnorm(lams(i), lams(i-1), sd_rw, true);   // pr(subsequent states)
6  }
7
8  // likelihood:
9  for(int i=0; i<n_year; i++){
10     if(!is_NA(y_obs(i))){
11         jnll -= dnorm(y_obs(i), lams(i), sd_obs, true); // pr(observations)
12     }
13 }
14 ...
```

- Note these aren't the true line numbers in `rw.cpp`

Remove some data and try it out

```
1 compile("rw.cpp")
```

Warning: 4 external pointers will be removed

Note: Library 'rw.dll' was unloaded.

```
[1] 0
```

```
1 # create dynamically linked library:
2 dyn.load(dynlib("rw"))
3
4 # make some funky data:
5 y_obs2 = y_obs
6 y_obs2[31:35] = NA
7
8 # create a tagged data list:
9 data <- list(y_obs = y_obs2)
10
11 # create a tagged parameter list w/ start values:
12 parameters <- list(ln_sd_rw = 0,
13                   ln_sd_obs = 0,
14                   lam0 = 0,
15                   lams = rep(0, length(data$y_obs)))
16 )
17
```

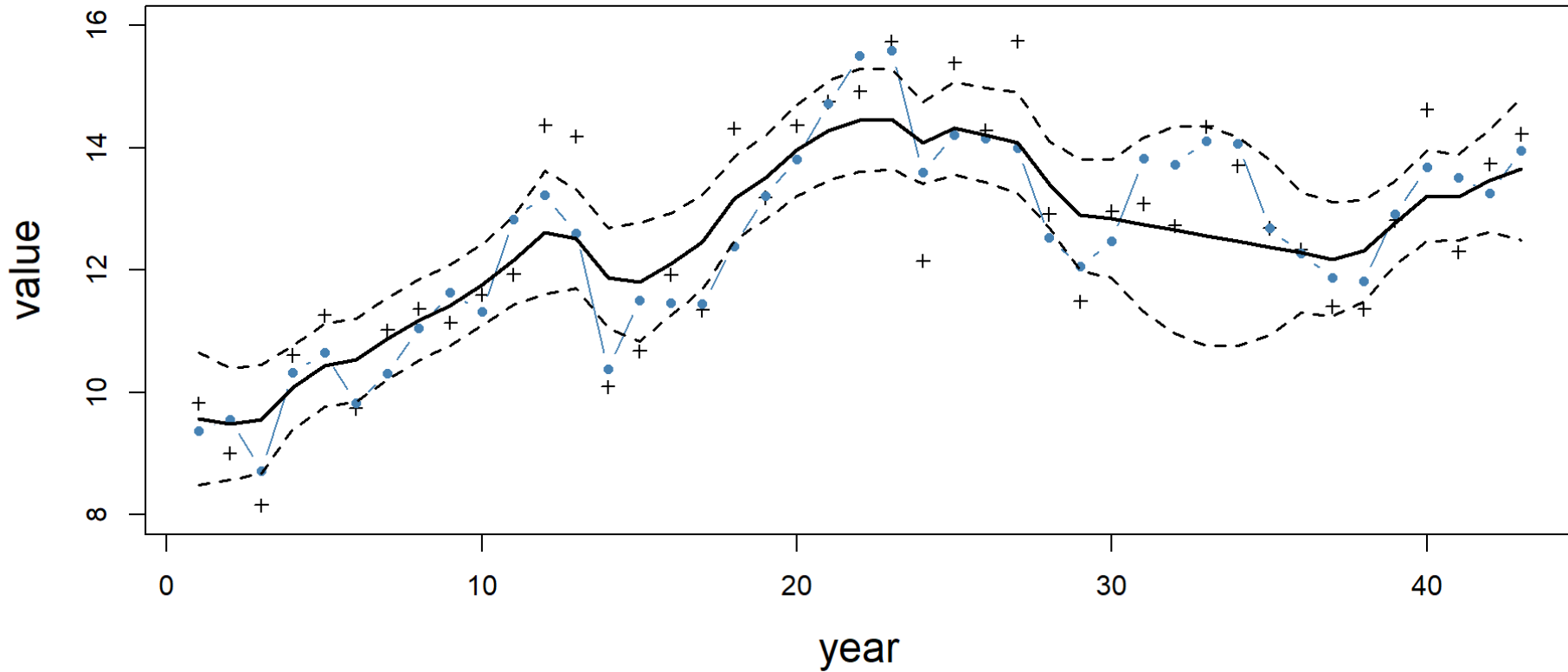
Remove some data and try it out

```
1 opt = nlminb(obj$par, obj$fn, obj$gr)
Optimizing tape... Done
iter: 1 value: 118.6808 mgc: 15.7428 ustep: 1
iter: 2 mgc: 1.065814e-14
iter: 1 mgc: 1.065814e-14
Matching hessian patterns... Done
outer mgc: 35.42793
iter: 1 value: 138.4644 mgc: 1.200569 ustep: 1
iter: 2 mgc: 8.187895e-16
iter: 1 mgc: 8.187895e-16
outer mgc: 10.50914
iter: 1 value: 118.3067 mgc: 2.883293 ustep: 1
iter: 2 mgc: 1.720846e-15
iter: 1 mgc: 1.720846e-15
outer mgc: 14.04101
iter: 1 value: 103.1652 mgc: 1.775114 ustep: 1
iter: 2 mgc: 2.774752e-15
1 sdr = sdreport(obj)
iter: 1 mgc: 7.674417e-15
outer mgc: 6.21457e-06
iter: 1 value: 77.98846 mgc: 0.003107419 ustep: 1
iter: 2 mgc: 6.550316e-15
outer mgc: 0.01087376
```

```
iter: 1  value: 77.92398 mgc: 0.00311364 ustep: 1
iter: 2  mgc: 7.327472e-15
outer mgc: 0.01086775
iter: 1  value: 77.967 mgc: 0.003107419 ustep: 1
iter: 2  mgc: 5.606626e-15
outer mgc: 0.04381543
iter: 1  value: 77.94548 mgc: 0.00311364 ustep: 1
iter: 2  mgc: 6.661338e-15
outer mgc: 0.04383615
iter: 1  value: 77.95622 mgc: 0.002488365 ustep: 1
```

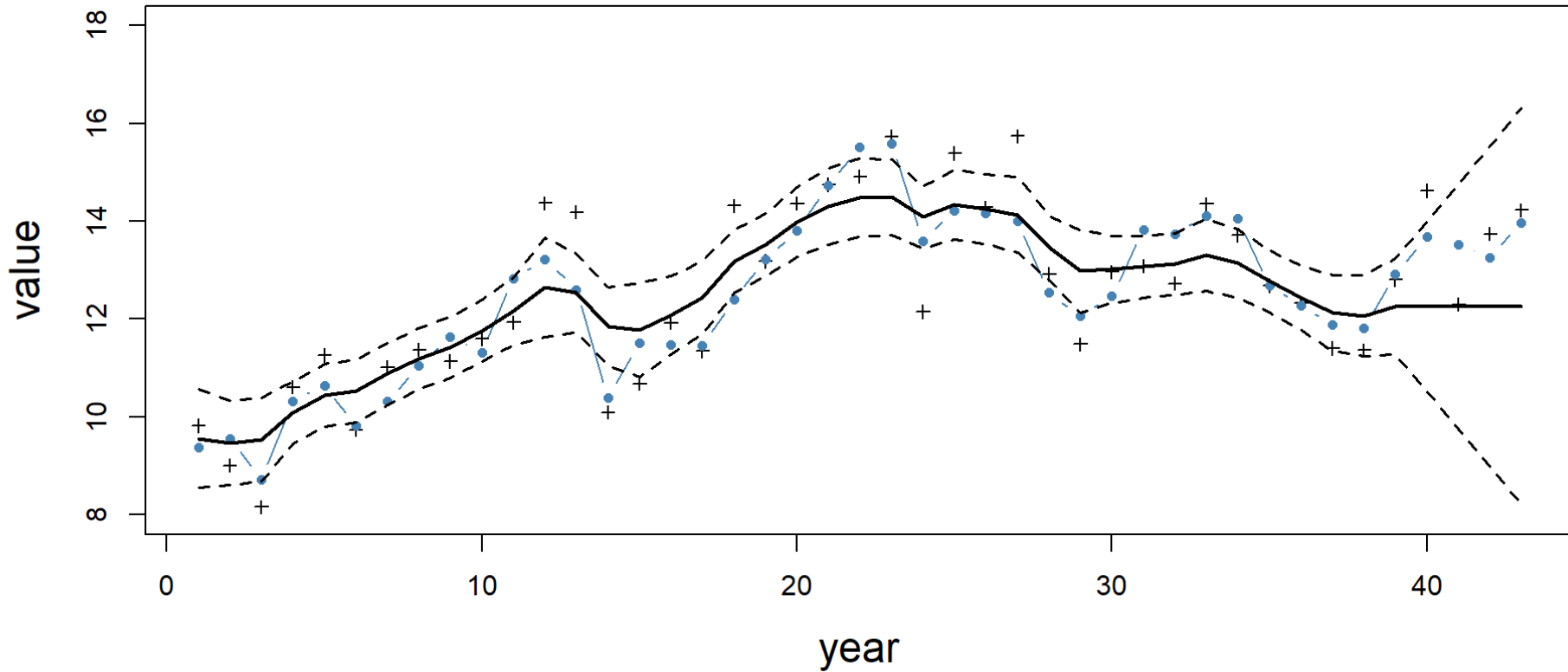
Visualize the hierarchical model fit - remove data years 31-35

λ_{true} (points) vs. y_{obs} (x's) vs. λ_{est} (black)



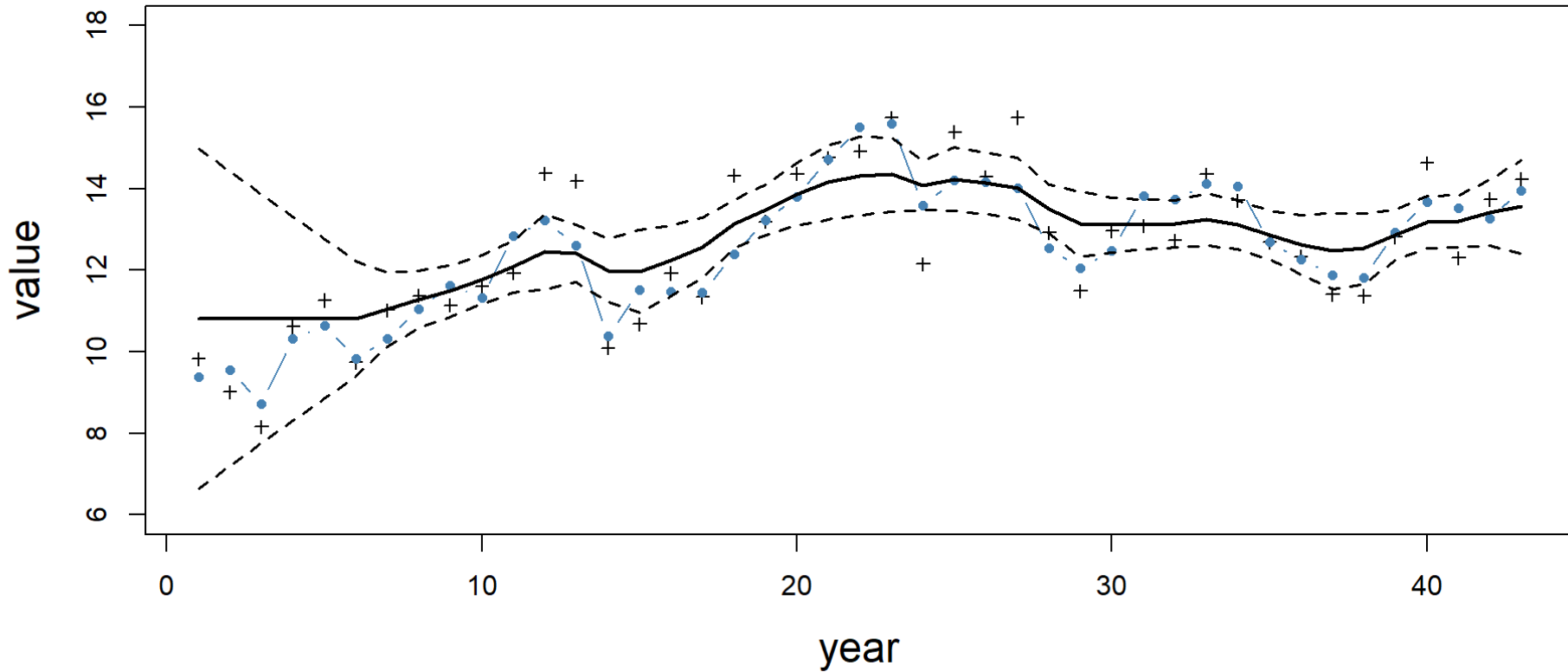
Visualize the hierarchical model fit - remove last 3 years

λ_{true} (points) vs. y_{obs} (x's) vs. λ_{est} (black)



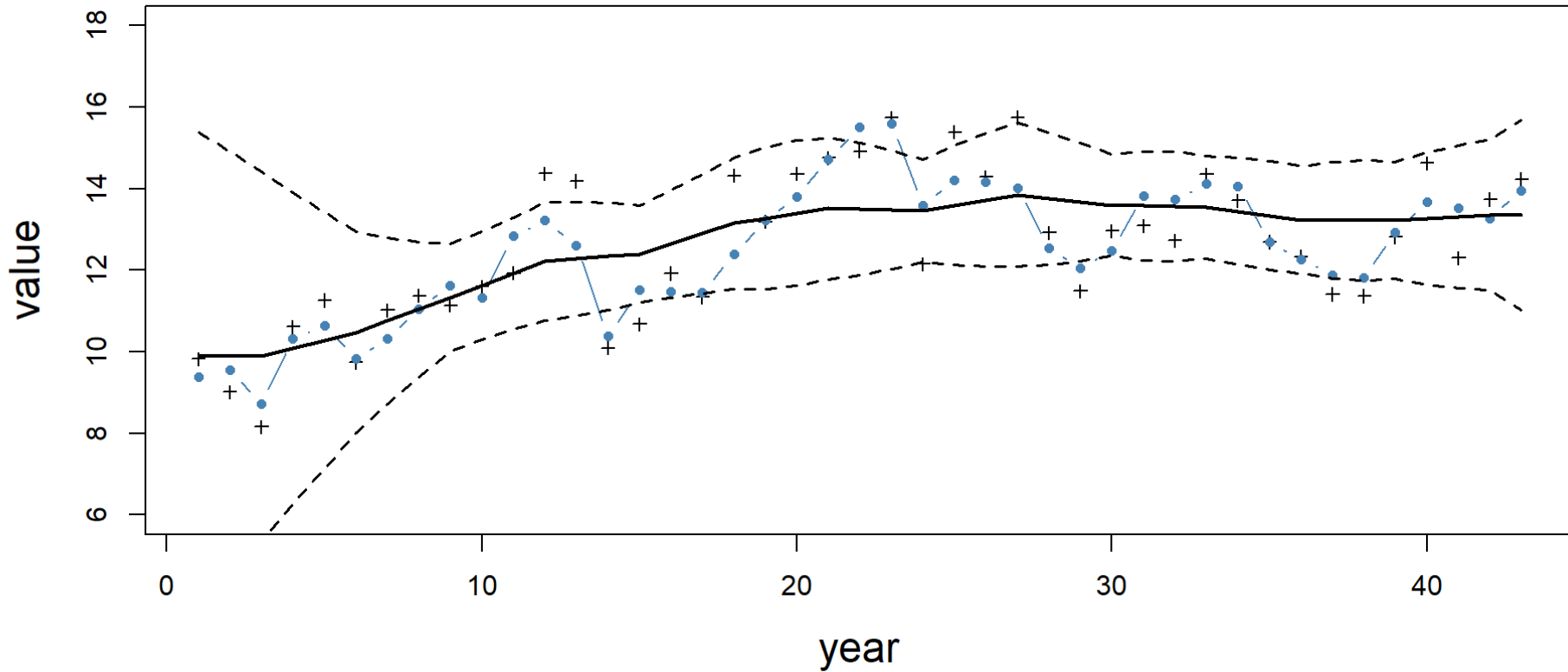
Visualize the hierarchical model fit - remove first 5 years

λ_{true} (points) vs. y_{obs} (x's) vs. λ_{est} (black)



Visualize the hierarchical model fit - data every third year

λ_{true} (points) vs. y_{obs} (x's) vs. λ_{est} (black)



Concluding remarks

- We have skipped over a lot of important theory and math.
- If n was large and we repeated this simulation experiment many times what do we expect?

Concluding remarks

- We have skipped over a lot of important theory and math.
- If n was large and we repeated this simulation experiment many times what do we expect to happen?
- The maximum likelihood estimator is *consistent*.
- $\hat{\theta}_{\text{mle}} \xrightarrow{\text{p}} \theta_0$.
- Most of what we have done here can be done entirely within TMB using a special `SIMULATE{ }` block.
- There are other ways to do this.

Useful references

- Royle and Dorazio. 2008. Hierarchical modeling and inference in Ecology.
- Kéry and Schaub. 2012. Bayesian population analysis: A hierarchical perspective
- Holmes, Scheuerell, and Ward. 2021. Applied Time Series Analysis for Fisheries and Environmental Sciences.
<https://atsa-es.github.io/atsa-labs/index.html#authors>

