A Harvest Control Rule Simulation Framework for Alberta Walleye Fisheries

Christopher L. Cahill and Carl J. Walters

20 March 2022

TODO

- parellel ordering of MAY / HARA
- Carl's point about including precautionary rules
- proofread – check citations
- finish by end of fiscal

Purpose

The purpose of this tutorial is to develop a management strategy evaluation or simulation framework that can be used to evaluate harvest control rules for any of the Alberta Walleye fisheries assessed in Cahill et al. 2021 using either yield or catch-based objectives.

Goals

- Provide readers with some background on the development of policies for fisheries management
- Understand what a harvest control rule is and why such rules is useful for managing natural resources
- Understand how the code is pulling results from the stock assessments and using these as inputs into our harvest control rule simulation program
- Work through the model structure of the harvest control rule simulator, including calculations of maximum average yield and HARA utility
- Develop intuition for the catch and release mortality components of the simulation model
- Understand the tabular or grid search approach to "optimizing" simple linear harvest control rules

Background

Harvest control rules are agreed-upon, transparent, and repeatable mathematical equations that help managers alter harvest levels in response to changes in stock size to achieve some explicit goal. Changes in population size may be due to either environmental stochasticity or the effects of fishing. In Alberta, harvest control rules may be particularly useful for informing special harvest license allocations given some agreed upon and explicit management objectives and Fall Walleye Index Netting (FWIN) survey data. The development of explicit harvesting rules is important because a finding of Cahill et al. 2021 was that many Alberta Walleye lakes appeared underharvested relative to common fisheries reference points.

Fisheries science has a rich history of using harvest control rules or feedback policies to design effective resource management systems, and there are some key references that will help you better understand many of the issues that we will discuss in this tutorial. These references are:

1. Walters 1986, chapter on feedback policies
2. Hilborn and Walters 1992, chapter on designing effective fisheries management systems
3. Quinn and Deriso 1999, chapter 11 on optimal harvesting
4. Walters and Martell 2001, chapters 2-4

In general, you will find that it is much more difficult to design effective fisheries management systems and harvest control rules than it is to simply assess a particular fishery. This is because what we are actually doing is attempting to design a control rule (a "controller") for a nonlinear dynamical system.

A nonmathematical description of the problem that we would share with Grandma

You can think of the harvest control rule problem like trying to design an autopilot system (or a control rule or policy) that keeps a plane air born–our goal in this case is to create a mathematical rule that relates measurements from the environment taken by sensors (altitude, jet speed, wind speed, wind direction, etc.) to actions the plane can take to alter its course of movement (increase speed, decrease speed, turn left/right/up/down). The point here that really matters is that we are trying to design a feedback policy, which relates where we are (in the environment) to what we want to achieve.

In this case, the "environment" simply refers to the physical equations governing the flight of a plane. In our fisheries situation, it refers to all of the equations and ecological processes describing the dynamics of a harvested population. Whereas there are sensors on the plane which tell us where the plane is and what the state of the environment is outside of that plane, we use surveys and sometimes stock assessments to determine the state of a population in fish and wildlife management settings.

In the oversimplified case of the autopilot system, the goal is not to crash. Intuitively, this might mean that a good feedback policy for the plane would seek to orient the plane to the right if the wind pushed it to the left, and try to maneuver upward if the plane were decreasing in altitude for whatever reason. In a fishery, the goal might be to let the stock size recover if it has been overfished, or to harvest more if the stock size were thought to be high.

Harvesting theory, dynamic programming, and optimal solutions

A great deal of theoretical work on optimal harvesting strategies for dynamic populations shows that optimal policies generally decrease harvest rate when population size declines, and increase it as population size increases. These findings have often come from dynamic programming solutions, and the solutions for these particular problems are often simple functions of harvestable abundance or biomass. This finding appears general and was surprising–it was generally believed that complex policies would be required to manage complex ecological systems. However, work on harvesting theory during the 1970-80s showed that simple policies performed well or were even optimal in many situations (e.g., Walters 1975; Walters and Hilborn 1978; Mangel XXXX; Clark XXXX; Moxnes XXXX).

A limitation of dynamic programming and the related methods discussed above is that these approaches can only be applied to relatively simple models and management objectives. The age-structured Bayesian Estimation of Recruitment Trends in Alberta ('BERTA) model is too complex to use with such methods. Consequently, we will instead use a simulation technique known as approximation in policy space to find good harvest control rules (see methods in Moxnes XXXX; Edwards and Dankel 20XX; new fancy book) for the BERTA models.

We briefly hit on this above, but there is one more difficulty facing us and that is that there are multiple (often conflicting) objectives in resource management. We note that the management strategy simulation approach we use in this tutorial can cope with this issue quite nicely if folks have explicit objectives.

On the need for explicit objectives

In the above toy airplane autopilot example, we wouldn't get very far if we didn't agree up front that the goal was to keep the plane flying within some (acceptable) flight parameters. The same is true for designing harvest control rules for renewable resources: if we do not agree upon how to manage a population a priori, we will not get very far. If we want to design effective fisheries management systems, we at least need to agree upon how best to manage those fisheries. This is easier said than done, and individuals have dedicated their entire careers toward developing structured decision making programs to meaningfully engage relevant stakeholders and develop such objectives.

For Alberta Walleye, there are presently no agreed upon *explicit* objectives for fisheries management. Believe it or not, this is pretty common for many fisheries. Thus, we will introduce two concepts–Maximum Average Yield (MAY) and Hyperbolic Absolute Risk Aversion Utility (HARA)–as starting points for Alberta Walleye fisheries. As stated above, objectives should be set through something like a structured decision making program (see Edwards and Dankel 20XX). It is unlikely that Cahill and Walters (or any manager or researcher for that matter) should be setting the objectives for Alberta fisheries.

Our approach here is to specify two provisional, albeit conflicting objectives to demonstrate how Alberta Walleye fisheries could be managed given explicit objectives. Later on we will simulate across a range of policy options to find policies that do well in our simulations given these provisional objectives. We discuss both of these objectives in the following section.

Maximum Average Yield (MAY) objective

Maximum Average Yield is the stochastic analog of Maximum Sustainable Yield (MSY). In a harvested population, this is the highest yield that can be taken on average from a population experiencing average environmental conditions. While there are many criticisms of MSY or MAY (e.g., Larkin 19XX), such yield-maximization policies demonstrate how much yield can be achieved from a given fishery if a MAY policy was adopted. When you simulate yield maximization policies or attempt to solve this problem using dynamic programming, what you find is that the optimal policy often is some type of "fixed escapement" policy. That is, there is a lower limit reference point below which no harvesting occurs, and above this lower limit reference point you tend to harvest at a rate near F_{msy}, or the instantaneous fishing mortality rate thought to achieve MSY.

While such policies maximize the average yield that can be taken from dynamic, uncertain, or even unpredictable fish populations, they have the downside that they result in increased variability to harvesters in terms of fisheries closures as stock size drops below a fixed lower limit reference point. Remember, if the stock drops below the lower limit reference point the MAY policy shuts off harvesting completely.

Hyperbolic Absolute Risk Aversion (HARA) utility objective

Hyperbolic Absolute Risk Aversion for catch (herein-after, HARA) is an objective that values consistency in catch through time from the perspective of a harvester. It is critical to include risk-averse utility for catch as a performance measure representing the interests of stakeholders and harvesters so as to explicitly consider stakeholder interests. The simplest way to do this is to calculate utility each year using a hyperbolic absolute risk averse utility function, which boils down to the following equation:

$$utility_t = catch_t^{pp} \tag{1}$$

In this case, $catch_t$ is simply yield in year t, and the power term pp is a risk aversion parameter that for most people is near 0.3-0.5. You can empirically assess pp by asking people to choose between two income options: a 50:50 gamble between 0 and 100,000 dollars, or a sure income of around 30,000 dollars. The

expected value of the income outcome of the 50:50 gamble is 50,000 (i.e., 1/2 * 100,000), but most people will choose a guaranteed income of around 30,000 dollars vs. the 50:50 chance at winning the 100,000 dollars (myself included).

This scenario implies $pp = 0.57$. You can calculate someone's pp by asking how much guaranteed reward (catch, income) they would need to receive before they were unwilling to take the "unsure" bet–so if you said your $X = 20,000$ dollars (or catch of fish), your pp would be

$$pp = \frac{ln(0.5)}{(ln(20) - ln(100))} = 0.43 \tag{2}$$

And note that we have substituted "20" for 20,000 and "100" for 100,000 because the math works out to be the same. When we ask fishermen what X they would need to not fish, typical answers have been in the 20,000-30,000 range, i.e. having some income or catch is much more important than maximizing average income if that means having highly variable income or catch. When framed this way, HARA utility is a powerful tool for capturing aversion to variability in catch.

When HARA utility is maximized as an objective, it tends to do two things relative to the MAY objective discussed above. First, the HARA objective tends to shift the lower limit reference point toward zero. This is because if we "shut off" fishing we lose a great deal of utility, which I hope makes sense. The second thing that this objective tends to do when you undertake formal optimization (i.e., dynamic programming) is that it generally speaking lowers the rate at which you increase harvest as stock size increases. We will demonstrate this via some simple R code shortly and via simulation later on.

Simulating some simple (fake) MAY vs. HARA policies

We picked these two objectives (MAY, HARA) to represent management tradeoffs (see Walters and Martell 2001). MAY represents maximum yield that say a corporation exploiting many populations at once might choose to maximize expected catch from a collection of populations, while HARA represents a better policy for a risk averse fisher or angler. To harvest under a harvest control rule that was developed to achieve MSY or MAY is to concede that a fishery will need to be closed for potentially many years if stock sizes drop below the lower limit reference point. This is particularly problematic for fisheries with highly variable recruitment dynamics like Alberta Walleye, as closures might be triggered simply due to natural variability.

Conversely, to harvest under a HARA objective and corresponding policy is to harvest at a lower rate but to continue to harvest when biomass drops below the lower limit reference point required to achieve MAY. These are key lessons from theoretical studies of harvested populations (e.g., see Walters and Hilborn 1976; 1978; Walters 1986), and we will demonstrate these principles in excruciating detail as we proceed with our Alberta Walleye example below.

Let's plot it in R

If you are a visual learner, you might have an easier time with the following hypothetical depiction of fixed escapement MAY vs. a HARA policy in R.

Let's load some packages:

```
# packages
library(tidyverse)
```

And then simulate and plot some dead simple harvest control rules (I made these up):

```r
vB <- seq(from=0.1, to =100, length.out = 100) # biomass vulnerable to fishing
b_lrp <- c(0, 30) # lower limit reference point
c_slopes <- c(0.2, 1) # slope of harvest control rule

TAC_HARA <-  c_slopes[1] * (vB - b_lrp[1]) # total allowable catch HARA
U_HARA <- TAC_HARA / vB # exploitation rate HARA

TAC_MAY <-  c_slopes[2] * (vB - b_lrp[2])
TAC_MAY[which(TAC_MAY < 0 )] <- NA # Set negative values to NA (don't harvest below blrp)
U_MAY <- TAC_MAY / vB

# make a big data frame
data_HARA <- data.frame(vB, TAC = TAC_HARA,  U = U_HARA, policy = "HARA")
data_MAY <- data.frame(vB, TAC = TAC_MAY,  U = U_MAY, policy = "MAY")
data <- rbind(data_HARA, data_MAY)

data %>%
  ggplot(aes(x=vB, y = TAC, group = policy))+
  geom_line(aes(linetype = policy), lwd = 1.25) +
  ylab("Total allowable catch (kg/ha)") +
  xlab("Biomass of finned critters vulnerable to fishing (kg/ha)") +
  ggtitle("Hypothetical HCRs: Total Allowable Catch vs. Vulnerable Biomass")
```


In this example, there is a lower limit reference point b_{lrp} and a slope at which you increase harvest as stock size increases (c_{slope} in the code). Next we can plot exploitation rate as a function of stock size for these simple examples using this code:

```
data %>%
  ggplot(aes(x=vB, y = U, group = policy))+
  geom_line(aes(linetype = policy), lwd = 1.25) +
  ylab("Exploitation rate (TAC / Vulnerable Biomass)") +
  xlab("Biomass of finned critters vulnerable to fishing (kg/ha)") +
  ggtitle("Hypothetical HCRs: Exploitation Rate vs. Vulnerable Biomass")
```

Hypothetical HCRs: Exploitation Rate vs. Vulnerable Biomass
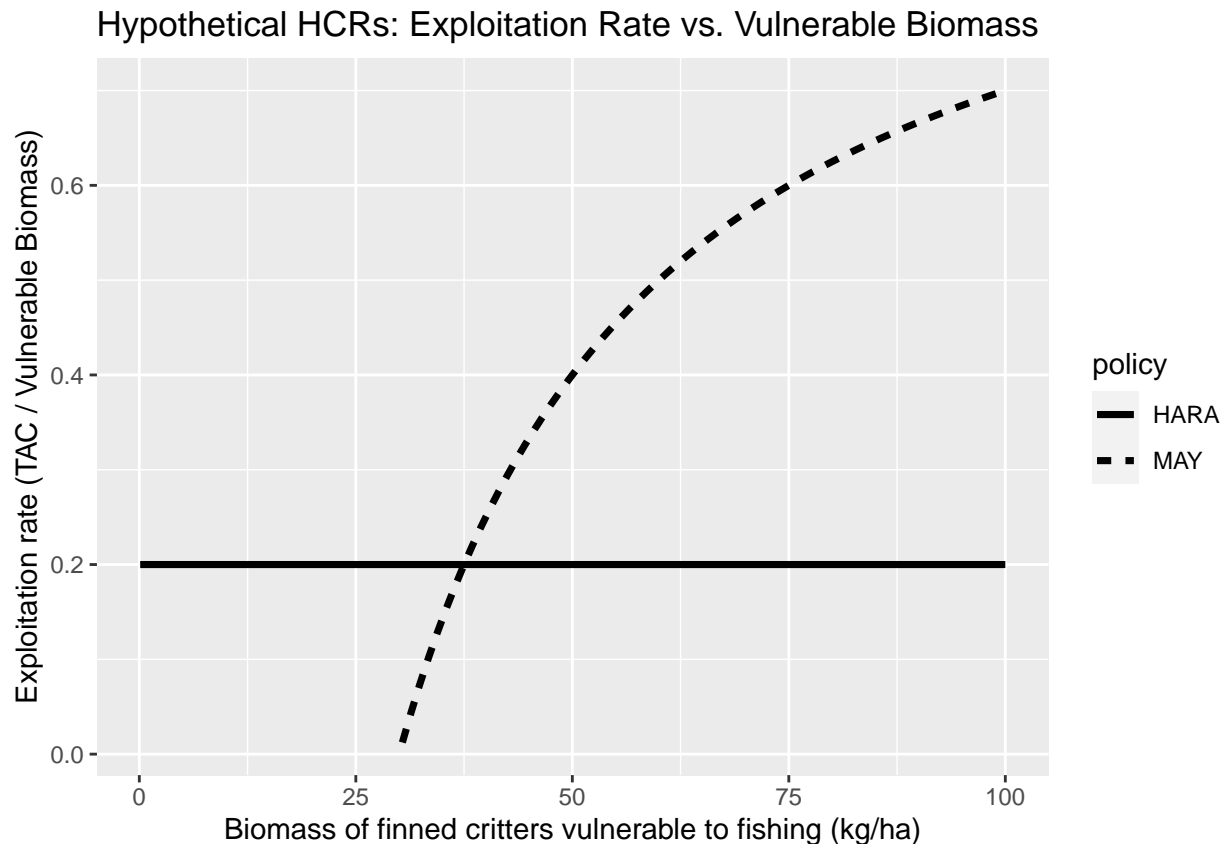
Think about these last two plots a bit–it is important to wrap your head around what this is showing you. Most of the harvest control rule simulation script that we will go through below is shifting the intercepts and slopes of these lines from Figure 1 to find simple linear policies that maximize specific objectives like MAY or HARA utility.

Thinking through one more harvest control rule

While the polices above arise from explicit objectives and dynamic programming solutions (again, take my word for it or go read all that stuff I referenced at the beginning of the tutorial), we will take this opportunity to work through one more policy that you might encounter in fisheries. This is DFO's so-called precautionary harvest control rule (DFO XXXX), which appears to be loosely based on work from Restrepo XXXX. This policy has been adopted for fisheries throughout Canada (and regardless of the management objectives of those fisheries). The DFO policy specifies a rectilinear harvest control rule, and while it is often unclear what is specifically supposed to be precautionary about this rule we will simulate it anyway to show you what it is doing.

The DFO policy increases exploitation rate from zero to U_{MSY} as biomass increases from a lower limit reference point to an upper limit reference point. These reference points are typically set at $0.4 \cdot B_{MSY}$ and $0.8 \cdot B_{MSY}$. Note that U_{MSY} and B_{MSY} typically need to be estimated in an assessment model.

In our simple example, we will say $U_{MSY} = 0.2$, and that $B_{MSY} = 30$ kg/ha. Thus, we would specify the DFO harvest control rule as follows:

```
U_MSY <- 0.2
B_MSY <- 30
b_lrp <- 0.4*B_MSY
u_lrp <- 0.8*B_MSY

U_DFO <- U_MSY * (vB - b_lrp) / (u_lrp - b_lrp)
U_DFO[which(U_DFO < 0 )] <- NA # Set negative values to NA
U_DFO[which(U_DFO > U_MSY)] <- U_MSY # Set negative values to NA

# calculate the TAC as U*vB
data_DFO <- data.frame(vB, TAC = U_DFO*vB,  U = U_DFO, policy = "DFO")
data <- rbind(data, data_DFO)
```

We can then plot it like we did before:

```
data %>%
  filter(policy == "DFO") %>%
  ggplot(aes(x=vB, y = TAC, group = policy))+
  geom_line(aes(linetype = policy), lwd = 1.25) +
  ylab("Total allowable catch (kg/ha)") +
  xlab("Biomass of finned critters vulnerable to fishing (kg/ha)") +
  ggtitle("Hypothetical HCRs: Total Allowable Catch vs. Vulnerable Biomass")
```
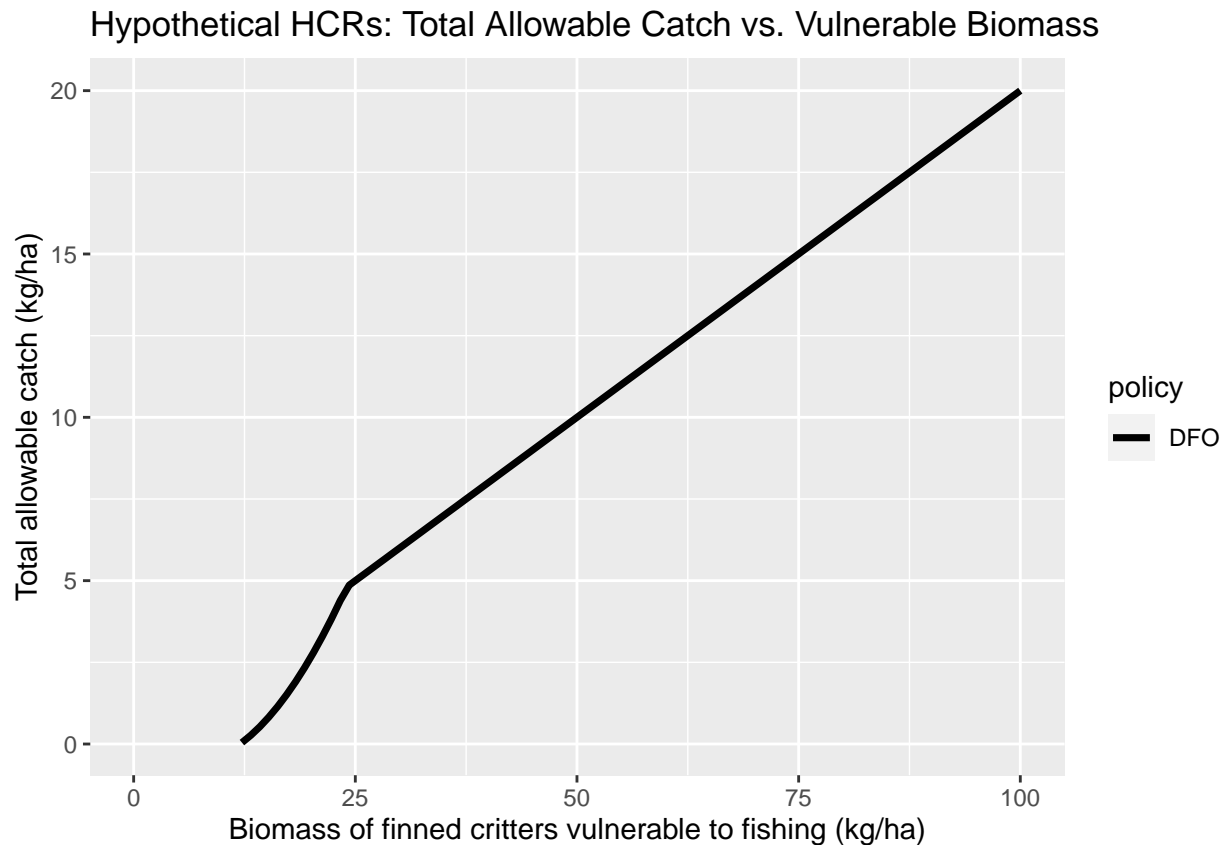
This plot is admittedly a bit weird looking, but that's just how this policy plays itself out. We can also plot the exploitation rate vs. vulnerable biomass for this policy as well:

```
data %>%
  filter(policy == "DFO") %>%
  ggplot(aes(x=vB, y = U, group = policy))+
  geom_line(aes(linetype = policy), lwd = 1.25) +
  ylab("Exploitation rate (TAC / Vulnerable Biomass)") +
  xlab("Biomass of finned critters vulnerable to fishing (kg/ha)") +
  ggtitle("Hypothetical HCRs: Exploitation Rate vs. Vulnerable Biomass")
```
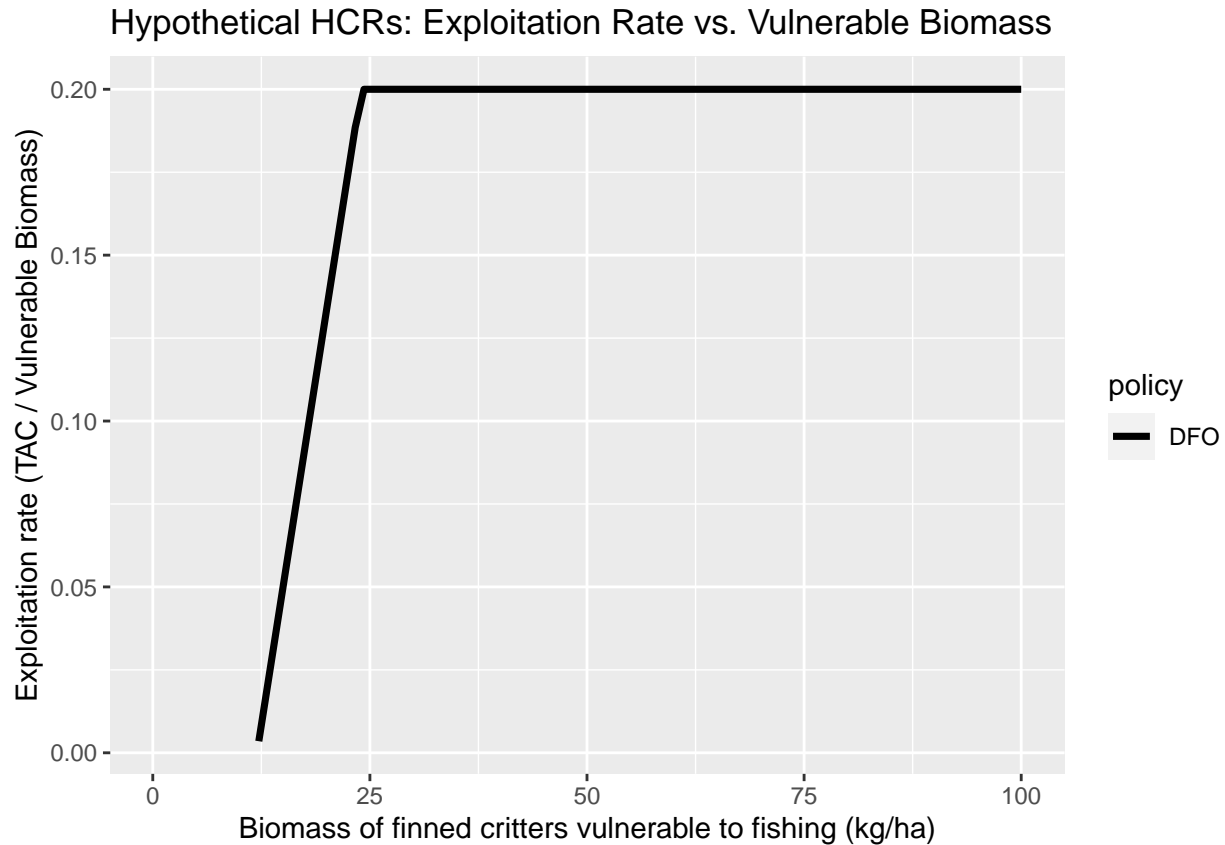

And you can see where the name "rectilinear" harvest control rule comes from. So in this example, there's no harvest below some lower limit reference point, and then we "cap" the maximum harvest rate at U_{MSY}.

Remember, this is the harvest control rule used for most fisheries managed via a harvest control rule by DFO. What does this rectilinear rule imply from a HARA or MAY perspective? What does it imply for other objectives?

Is there a biological basis for the $0.4 \cdot B_{MSY}$ and $0.8 \cdot B_{MSY}$ cutoff values?

How do we use our Bayesian assessment models to inform harvest control rules?

Before we begin working through the harvest control rule simulation script, we need to understand what we currently have in terms of the analyses we have completed up to this point. In previous tutorials we have estimated a straightforward age-structured stock assessment model for Alberta Walleye. We have saved the posteriors from these model fits for each lake in their own .rdata files in the `fits` folder on the Github for this project.

The posteriors that we have saved capture uncertainty in Walleye population dynamics given our model structure, priors, and data for a given lake. Thus, we can use results from these assessments to initialize a harvest control rule simulation, and run these simulations for many random draws from the posterior to characterize uncertainty in our policy development. We will undertake a specific type of management strategy evaluation known as retrospective harvest control rule simulation. All this means is that once we initialize our simulation model using our stock assessment results, we will repeat recruitment anomaly sequences w_t some number of times into the future. We will play around with this a bit, but the primary goal here is to develop harvest policies that are robust to the recruitment dynamics that were estimated in Cahill et al. 2021.

Conceptually, the way we will do this is as follows:

1. Initialize our population dynamics simulation model using assessment model results for some lake.
2. Project this model into the future using (nearly) the same equations as in Cahill et al. 2021.
3. For each policy we want to test the performance of, we initialize our simulation model using some number of random draws of the posterior, run the simulation forward in time while implementing that specific policy for each random draw, and record average performance in terms of our objectives over years of our simulation and number of draws.
4. Look across all policies we tested, and look up the policy that maximized average yield or HARA utility, respectively.
5. Go for a long run and drink tasty beer, never think about designing controllers for nonlinear dynamical systems again.
6. Upon recovering, go back and deal with the rectilinear policy, and see how well the DFO rule works for Alberta Walleye relative to the simple rules above.

This will get a little tedious as we work through code, but the key point to remember here is that we are trying to create a rule for managers that says "if our objective is x and our net catch is y, how many fish should we allocate in year t?" We will relax a variety of assumptions around this (assessment interval, survey variability, and catch and release mortality), but the main point is we are trying to come up with an equation that helps folks achieve some objective(s) given a net catch.

Additionally, while we are only going to use two conflicting objectives, it is straightforward to do this for any specific objective(s) you want. Phrased differently, if you have explicit multi-criteria objectives and some weighting scheme you could easily adapt this script to test the performance of such an fancy objective.

The `hcr.R` simulation script

The `hcr.R` script works like the `run.R` script in our earlier tutorials. The `get_hcr()` function takes in some inputs, reads in a specific Bayesian model fit from a given lake, extracts the relevant stuff that we need to parameterize an age structured model, and then runs our simulations forward in time for some number of draws and records performance of that simulation. It then saves the simulation output as a .rds file with a unique file name (based on whatever simulation choices you chose) in a `sim/` folder. The full function is available here:

https://github.com/ChrisFishCahill/managing-irf-complexity/blob/main/r-code/hcr.R#L17-L359

This function was primarily developed to test the simple linear harvest control rules (see MAY vs. HARA examples above), but it can also be used to test the DFO-style rectilinear harvest control rule. For now, we'll ignore the rectilinear rule entirely and just focus on the simple linear rules. Once we work through the linear harvest control rules and get a handle on how this works, we'll loop back at the end of this tutorial deal with the rectilinear policy. Eventually we will be able to compare the performance of the linear vs. rectilinear harvest control rules for Alberta Walleye fisheries.

Before we start jumping through this function line by line, I want you to look at the following lines of code below the function here.

Please note you don't have to know what all of this is doing yet. The main reason to show you this is that there is eventually going to be stuff in our R environment (a bunch of simulation control parameters and our Bayesian model fits in list form) that our `get_hcr()` function is going to interact with. If I didn't show you this up front, you might get confused as we work through the function. So keep that in mind.

The first chunk of code in `get_hcr()` is the following:

```r
get_hcr <- function(which_lake = "lac_ste._anne", ass_int = 1,
                    sd_survey = 0.05, d_mort = 0.3,
                    rule = c("linear", "rectilinear")) {
# many more lines of code...
```

These are inputs you can change depending on your what you want. The arguments that you can play around with are:

- `which_lake`, a string that specifies the lake name that you want to run. This needs to be lowercase and have underscores in place of spaces.
- `ass_int`, describes the interval at which we will assess the fishery.
- `sd_survey`, describes the amount of noise in the survey that we base our harvest control rules off of.
- `d_mort`, the discard mortality rate for released fish.
- `rule`, specifies between linear or rectilinear harvest control rules.

The next few lines of the function specify a bunch of additional simulation parameters:

```r
#-----------------------------------------------------------------
# initialize stuff
#-----------------------------------------------------------------
n_draws <- hcr_pars$n_draws
n_repeats <- hcr_pars$n_repeats
retro_initial_yr <- hcr_pars$retro_initial_yr
retro_terminal_yr <- hcr_pars$retro_terminal_yr
n_sim_yrs <- hcr_pars$n_sim_yrs
ages <- hcr_pars$ages
n_ages <- length(ages)
initial_yr <- hcr_pars$initial_yr
initial_yr_minus_one <- hcr_pars$initial_yr_minus_one
ret_a <- hcr_pars$ret_a
Ut_overall <- hcr_pars$Ut_overall
sbo_prop <- hcr_pars$sbo_prop
rho <- hcr_pars$rho
sd_wt <- hcr_pars$sd_wt
psi_wt <- hcr_pars$psi_wt
n_historical_yrs <- length(retro_initial_yr:retro_terminal_yr)
grid_size <- hcr_pars$grid_size
```

These lines of code are looking up values from a tagged list called `hcr_pars`. I set the values in this function later on in the script in these lines:

You can change these if you want, but you'll notice that I haven't put these explicitly in the function call for `get_hcr()`. I did this because you should probably know what you are doing if you want to change these parameters. This might seem annoying, but the reason I did this is because if I needed to change any of

these values I needed all the following code in this function to update automatically. This was the best way I could think to do that, though I'm sure there are other (and perhaps more elegant) solutions.

I don't want you to worry too much about these for the moment. You may not like these initial values. So be it. The point of coding it this way is so that we can loop through and run many different simulations with many different input parameters.

The next code block is just taking the `which_lake` string, and using it to grab the appropriate Bayesian model fit (which eventually will be stored in our R environment from the fits folds):

```
#---------------------------------------------------------------------
# subset lake-specific posterior from all fits
#---------------------------------------------------------------------
lake_str <- gsub(" ", "_", which_lake)
fit_idx <- grep(lake_str, names(fits))
fit <- fits[[fit_idx]]
rec_ctl <- ifelse(grepl("bh", names(fits)[fit_idx]), "bh", "ricker")
```

The first line makes sure there are no spaces in the variable `which_lake`. The next line determines which of the fits (in the R environment) corresponds to the `which_lake` we wanted to run the simulation for. We then read in the fit corresponding to our lake, and then declare a control variable called `rec_ctl` indicating whether the fitted Bayesian model used a Beverton-Holt (i.e., "bh") or Ricker stock-recruit model (i.e., "ricker"). Sweet.

The next ten or so lines are used for the rectilinear harvest control rule, and so we will ignore this for now. Note this code is only triggered when rule = "rectilinear":

https://github.com/ChrisFishCahill/managing-irf-complexity/blob/main/r-code/hcr.R#L49-L62

And so we will ignore them for now for sanity purposes. Again, we will come back to this later on.

Lines 64-144 or so are pretty annoying. Quite literally all these lines of code are doing is extracting relevant values from our Bayesian model fits that we are going to use in our simulations. Some of these terms were read into our Bayesian model as data, while others were explicitly estimated as parameters (and thus values will vary per each draw of the posterior).

I am using the tidybayes package to manage Bayesian model output. There are many other ways to do this, but I like this way, and so now you have to learn it if you want to figure out what I did. Let's assume for now that `n_draws = 1`. The next chunk of code is:

```
# extract estimated and derived parameters from BERTA
devs <- fit %>%
  spread_draws(Ro, ln_ar, br) %>%
  sample_draws(n_draws)

draw_idx <- unique(devs$.draw) # which posterior rows did sample_draws() take?
```

This piece of code takes a specific model fit, pipes it into the function `spread_draws()` to access this fit's posterior, which is then passed to `sample_draws()`. The `sample_draws()` function randomly samples some number of draws `n_draws` from the posterior (i.e., rows of the posterior where columns are parameter names and rows are parameter values). Note also that we are only to grabbing three parameters in this case–R_0, $ln(ar)$, and br. These values are stored in the object `devs`.

We then run one more line of code that figures out which specific draw(s) or rows of the posterior we randomly selected, and we store this value as `draw_idx`. This is important when we sample more than one parameter because we want our sample values to come from the same (randomly selected) rows of the posterior, which preserves co-variation among the parameter estimates used in our simulation (cool).

11

Next we basically do the same thing for all the other stuff we want to draw in from our Bayesian model fit, and we make sure we sample the same rows of the posterior as we did above via the `filter()` call:

```
w_devs <- fit %>%
  spread_draws(w[year]) %>%
  filter(.draw %in% draw_idx) %>% # force same .draw to be taken as above
  mutate(year = year + initial_yr_minus_one) # make years 1980-2028

# extract nta from stan
nta_stan <- fit %>%
  spread_draws(Nat_array[age, year]) %>%
  pivot_wider(names_from = age, values_from = Nat_array) %>%
  filter(.draw %in% draw_idx) %>%
  mutate(year = year + initial_yr_minus_one)

# which columns have ages:
age_cols <- which(!is.na(str_extract(
  string = colnames(nta_stan),
  pattern = "[0-9]|10[0-9]"
)))

# rename age columns to correct ages 2-20
colnames(nta_stan)[age_cols] <- ages
```

So here we are extracting the recruitment anomalies w_t's and the estimated numbers at age by year matrix. We then rename the Stan fit indeces to make them intelligible. Remember, Stan begins its indeces at 1, so we have to do this extra step if we want Walleye ages to range from 2-20 and years to range from 1980-2028.

We then extract even more stuff from the posterior to use in our management strategy evaluation simulator:

```
# extract MAP estimate of Ro from posterior (for indexing bmin)
Ro_summary <- rstan::summary(fit, pars = "Ro")$summary
Ro_map <- Ro_summary[, "mean"]

# extract the age structure corresponding to 1990
nta_init <- nta_stan %>% filter(year == retro_initial_yr)

# extract w estimates from 1990-2015
w_devs <- w_devs %>%
  filter(year %in% retro_initial_yr:retro_terminal_yr)
```

Here you'll note that the code is a little different, and that we are extracting a summary value for R_0 rather than an entire posterior or some random subset of that posterior. In particular, we are extracting the most probable single estimate of R_0 for use a little later on.

You'll note we are also subsetting our Bayesian posterior model fits a bit. Instead of using the full 1980-2018 numbers at age matrix and w_t values, we'll subset these estimates to correspond to years 1990-2015. We do this for two reasons.

First, most of the Fall Walleye Index Netting surveys had information on Walleye age structure and recruitment back to at least 1990. While there were some systems where survey data carried information on recruitment and age structure back to the early 1980s, our choice of 1990 seemed to be a safe early year bound based on our data explorations.

Second, recruitment is usually difficult to estimate until cohorts of fish have been present in a fishery for a few years, which means recruitment estimates from recent years (i.e., 2015-2018) are generally unreliable (see Hilborn and Walters 1992; Walters and Martell 2001).

As our main goal was to design harvest control rules robust to the recruitment variation present in these systems, we felt it was best to use this 1990-2015 range to initialize our age-structured simulation below because these were the estimates that we could trust the most given our data and models.

Next, we join all of that stuff into one tibble:

```r
# join all those devs into one big tibble called "post"
suppressMessages(
  post <- left_join(w_devs, nta_stan,
    join_by = c(.draw, year)
  )
)

suppressMessages(
  post <- left_join(post, devs,
    join_by = .draw
  ) %>% arrange(.draw)
)
```

Note, the `supressMessages()` function is just telling the join commands to shut up and not print a bunch of garbage to the console while we run this chunk of code. I personally don't like it when my programs back sass me, and so I suppress their output here.

Next up we extract a bunch of leading parameters from the Bayesian model fit. These are terms that were read in as data, and thus these are not changing on each draw of the posterior. So, we'll just bring them in based on a single draw of the posterior:

```r
# extract leading parameters from stan to get vbro
leading_pars <- fit %>%
  spread_draws(
    Lo_report[age],
    l_a_report[age],
    v_a_report[age],
    v_f_a_report[age],
    f_a_report[age],
    w_a_report[age],
    M_a_report[age]
  ) %>%
  filter(.draw %in% draw_idx[1]) # indexed on 1 bc not changing
leading_pars$age <- ages # correct ages
w_a <- leading_pars$w_a_report
f_a <- leading_pars$f_a_report

v_survey <- leading_pars$v_a_report
v_fish <- leading_pars$v_f_a_report
Lo <- leading_pars$Lo_report
M_a <- leading_pars$M_a_report
vbro <- sum(Lo * v_survey * w_a)
sbro <- sum(f_a * Lo)

# put some extra stuff in post
post$sbro <- sbro
post$Ro_map <- Ro_map
post$vbro <- vbro # biomass vulnerable per recruit *surveys*
post$vbo <- sum(Ro_map * Lo * v_fish * w_a) # biomass vulnerable to *fishing*
```

So basically we are just extracting a bunch of the stuff from our fit that we discussed in detail during the previous tutorial If you don't remember, these terms are:

- `w_a` is a vector of relative weight at ages.
- `f_a` is a vector of relative fecundity at ages.
- `v_survey` is a vector describing Walleye vulnerability to the survey gear at each age.
- `v_fish` is a vector describing Walleye vulnerability to fishing at each age.
- `Lo` is survivorship at age in the unfished condition.
- `M_a` is instantaneous natural morality age age (based on the Lorenzen equation).
- `vbro` is vulnerable biomass per recruit in the unfished condition.
- `sbro` is spawners per recruit in the unfished condition.

and we then put the stuff we need into the `post` tibble for use later on. Remember: this is just us reading in relevant information, parameters, data, whatever from the BERTA model fits and setting it up to use with our simulations below.

Now that once we have read in all of the stuff that we need to run the simulations, we are going to set up the simple linear policies for which we want to test performance in terms of how well they achieve the MAY and HARA objectives.

The way this works is that for each policy we want to test, and then for each draw of the posterior we want to simulate across, we'll run the model some number of years into the future. We'll then record the expected performance of our policies and pick the best policy overall for a specific objective. The linear policies we showed above have two parameters: b_{lrp} and *cslope*, which correspond to the x-axis intercept (the fixed escapement point below which fishing is shut off) and the slope of the line (or the rate at which harvesting increases as stock size increases), respectively.

We'll plot a few of these in R to remind you what is going on:

```r
b_lrp <- seq(from = 0, to = 45, length.out = 25) # lower limit reference point
c_slopes <- seq(from=0.01, to = 1.0, length.out = length(b_lrp)) # slope of hcr

# solve c_slopes*(vb-b_lrp) for vb = 0
yints <- -b_lrp*c_slopes

my_lines <- data.frame(yints, b_lrp, c_slopes)
my_lines$policy <- as.factor(1:nrow(my_lines))
my_lines$vB <- seq(from = 0, to = 100, length.out = nrow(my_lines))
my_lines$TAC <- seq(from=0, to = 60, length.out = nrow(my_lines))

my_lines %>%
  ggplot(aes(x=vB, y = TAC))+
  geom_abline(mapping = aes(intercept = yints, slope = c_slopes,
              group = policy))  +
  scale_x_continuous(limits = c(0, 50)) +
  scale_y_continuous(limits = c(0, 20)) +
  ylab("Total allowable catch (kg/ha)") +
  xlab("Biomass of finned critters vulnerable to fishing (kg/ha)") +
  ggtitle("Visualizing a few possible linear policies")
```

Visualizing a few possible linear policies
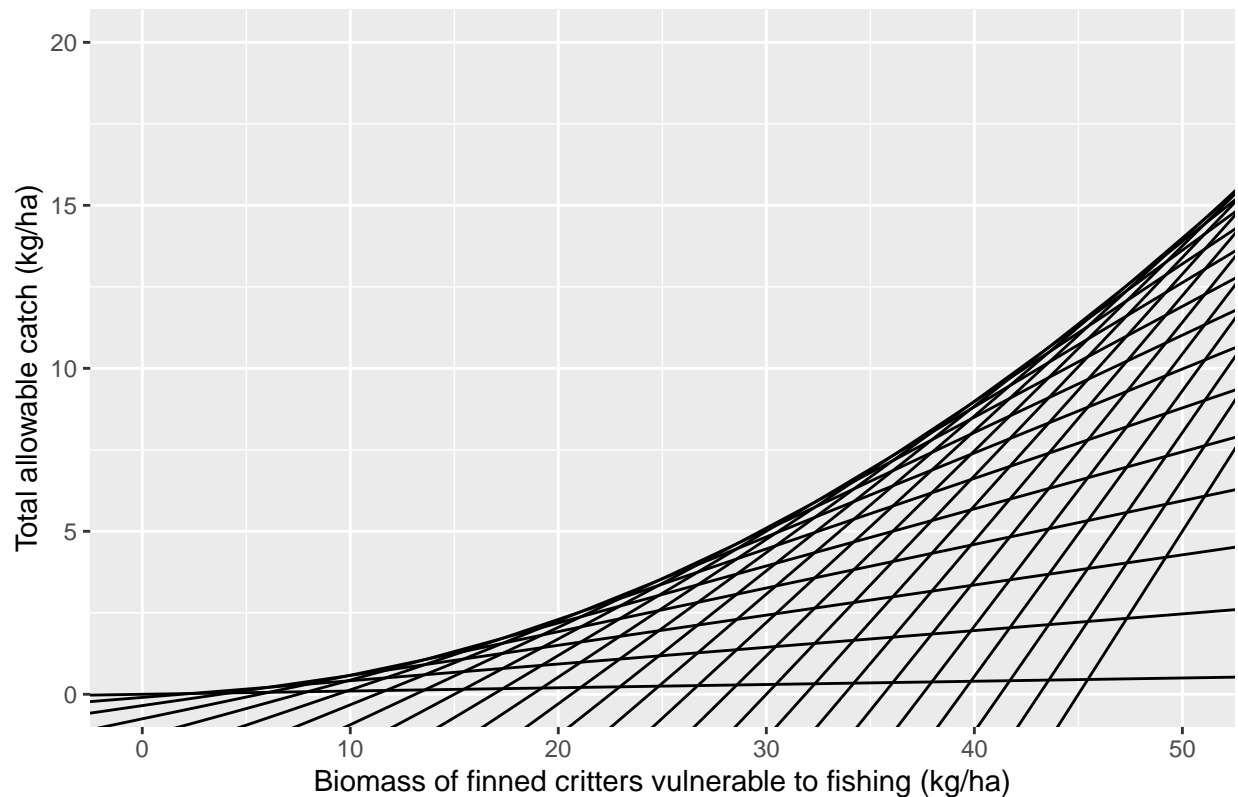
Each of these lines has a specific lower limit reference point b_{lrp} and a slope to that particular line c_{slope}. What we need to do then, is test each of these policies against our simulation and see which one wins in terms of achieving our objective(s).

Note that this is not all possible linear policies. I just took a few and plotted them to give you a sense of what I am talking about—there are too many to plot on this single graph. What we really need to do is take a range of plausible b_{lrp} and c_{slope} values, and then test each possible combination of these values.

You can, however, visualize or think about the solution space of all possible linear harvesting policies in terms of a matrix:

```r
# get all combinations of blrp, cslope
my_grid <- expand.grid(b_lrp = b_lrp, c_slope = c_slopes)

my_grid %>%
  ggplot(aes(x = b_lrp, y = c_slope)) +
  geom_point(size=0.5) +
  geom_point(data = my_lines, aes(x = b_lrp, y = c_slopes),
             colour = "steelblue") +
  xlab("possible lower limit reference points") +
  ylab("posible cslope values") +
  ggtitle("The policy space for possible simple linear HCRs")
```

The policy space for possible simple linear HCRs
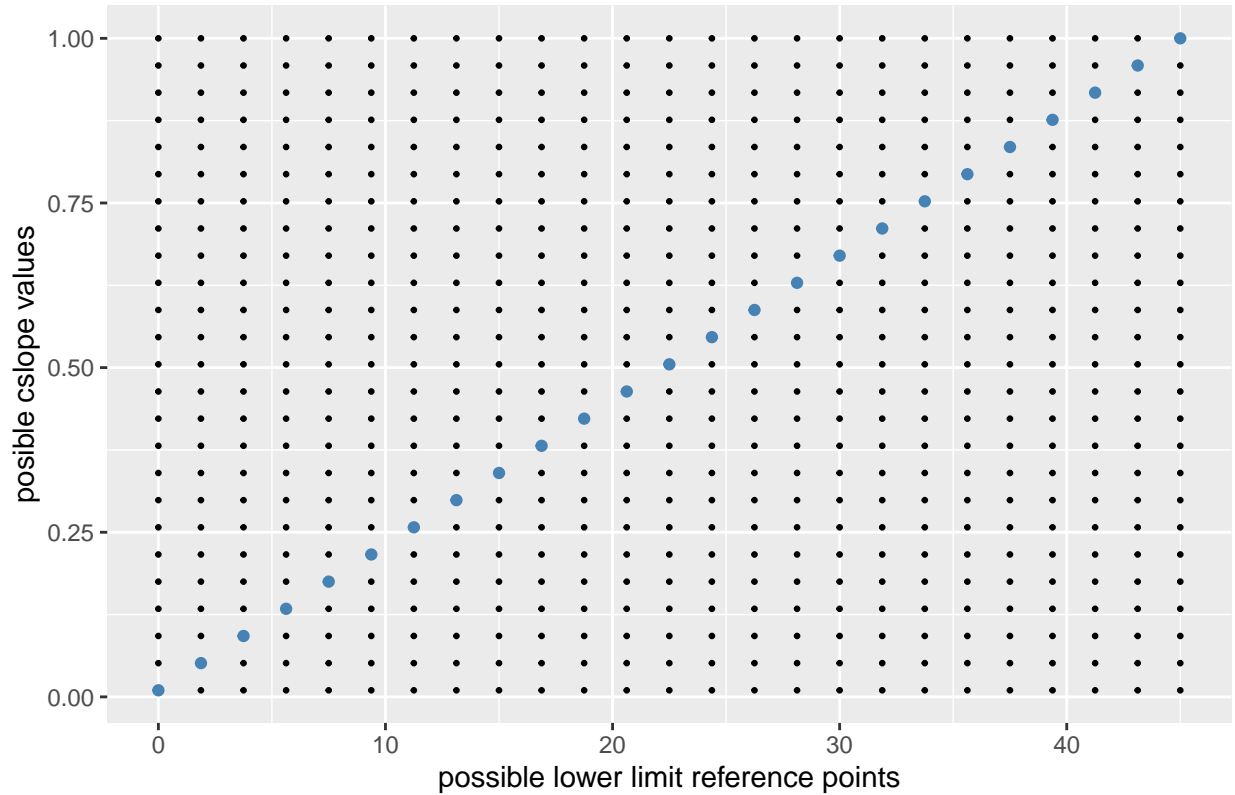

```
# blue points identify the subset of policies from the previous line plot
# black points are additional policies that weren't plotted on the previous plot
```

The key thing to recognize is that each possible combination of b_{lrp} and c_{slope} defines a single harvest policy (i.e., each point on the plot). We want to test the performance of each of these policies, so we need to figure out a range of policies to iterate across for each lake.

The next chunk of simulation code does declares a possible range of b_{lrp} and c_{slope} values:

```
bmin_max_value <- Ro_map * sum(Lo * v_fish * w_a)
```

Here, we are just setting the maximum b_{lrp} value at B_0, which is the biomass vulnerable to fishing in the unfished condition and is which calculated as the sum product of survivorship at age, fishing vulnerability at age, and relative weight at age:

$$B_0 = R_0 \cdot \sum_{age=2}^{age=20} L_{o_a} \cdot v_{f_a} \cdot w_a \tag{3}$$

It should make intuitive sense that we would say that the lower limit reference point shouldn't be higher than B_0, or at least I hope this is intuitive. The next chunk of code in this hog of a function is:

```
if (which_lake == "calling lake") {
    bmin_max_value <- 100
```

16

```
    } else {
    bmin_max_value <- ifelse(bmin_max_value > 75, 75, bmin_max_value)
}
c_slope_seq <- seq(from = 0.01, to = 1.0, length.out = round(grid_size / 2))
bmin_seq_low <- seq(from = 0, to = 20, length.out = round(grid_size / 2))
bmin_seq_high <- seq(from = 20.5, to = bmin_max_value,
                     length.out = grid_size - length(bmin_seq_low))
bmin_seq <- c(bmin_seq_low, bmin_seq_high)
```

First, we are setting up a simple conditional statement to handle `bmin_max_value` for situations where it gets too high. This is ugly, and required different condition handling for calling lake vs. other systems for reasons we can talk about in our tutorial in person.

Next, we are setting up a sequence of c_{slope} values. Then we do something weird relative to what I have shown you. If you look closely, we are setting up two b_{lrp} sequences, and then smushing them together.

If you read code, you'll see that we are stacking half of the lower limit reference points for our policies in the range of 0-20, and half in the range 20.5-bmin_max_value.

Let's actually visualize what this is actually doing by picking some values for the `grid_size` and `bmin_max_value` parameters and just plotting it:

```
grid_size = 75
bmin_max_value <- 50 #let's just pick this to show what's happening
c_slope_seq <- seq(from = 0.0, to = 1.0, length.out = round(grid_size / 2))
bmin_seq_low <- seq(from = 0, to = 20, length.out = round(grid_size / 2))
bmin_seq_high <- seq(from = 20.5, to = bmin_max_value,
                     length.out = grid_size - length(bmin_seq_low))
bmin_seq <- c(bmin_seq_low, bmin_seq_high)
# get all combinations of blrp, cslope
my_grid <- expand.grid(b_lrp = bmin_seq, c_slope = c_slope_seq)

my_grid %>%
  ggplot(aes(x = b_lrp, y = c_slope)) +
  geom_point(size=0.25) +
  xlab("lower limit reference points") +
  ylab("cslope values") +
  ggtitle("Linear HCR policies to test")
```

Linear HCR policies to test
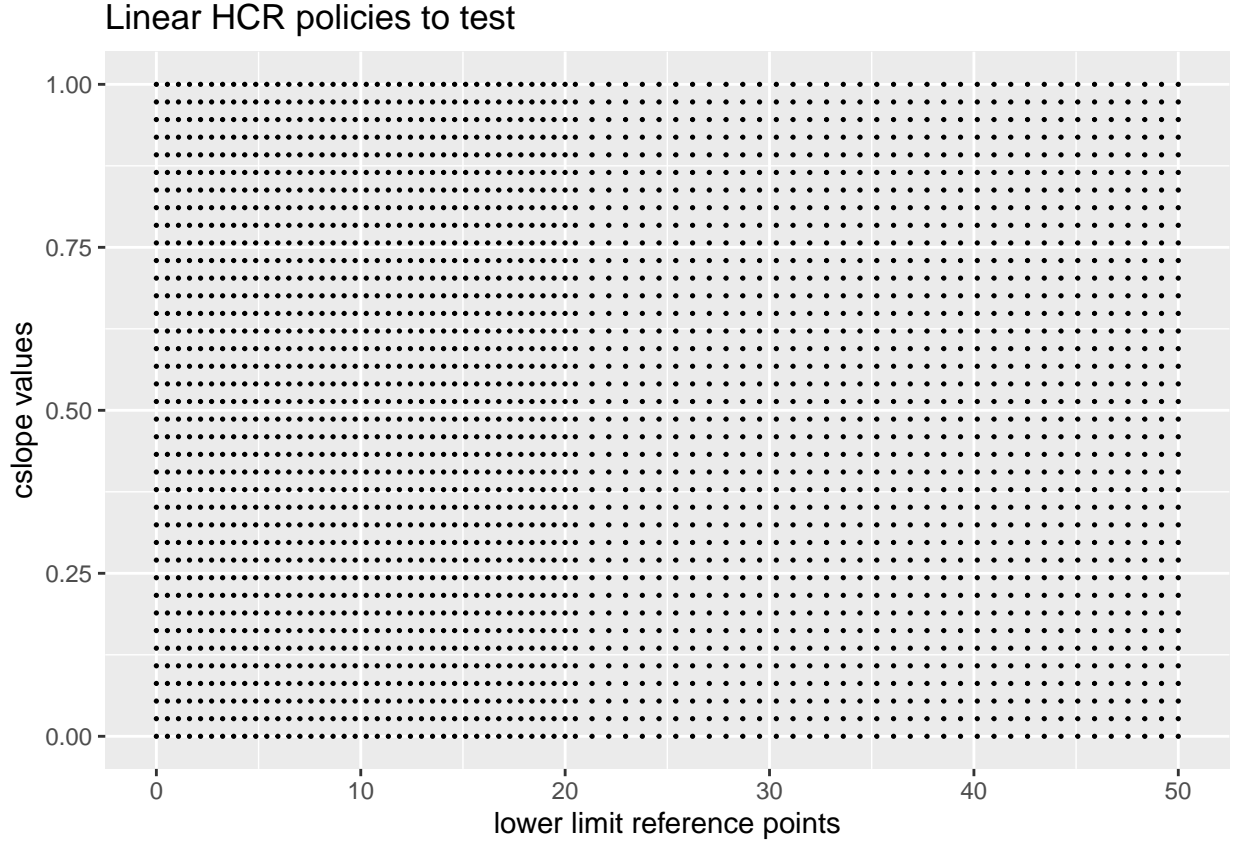
If you look closely you can see that this code is stacking more points (policies) into the lower portion of this plot. We did this for computational purposes. We wanted more resolution at low stock sizes for the harvest control rule simulation, and wanted to maintain some sort of computational efficiency. Thus, we chose more policies to test in the lower portion of the above plot.

Remember, the specific linear policies for a lake depend on a given lake's estimated R_0 (see above) and won't necessarily correspond to these points here. This plot is just giving you a general idea of what that damn code is doing.

So now we basically just need to loop through each of these policies, run our simulation (for some number of draws from the posterior, and some number of years into the future. We jump into this bit of the simulation script now.

```
tot_y <- tot_u <- prop_below <- TAC_zero <-
  matrix(0, nrow = length(c_slope_seq), ncol = length(bmin_seq))
yield_array <- vB_fish_array <-
  array(0, dim = c(length(c_slope_seq), length(bmin_seq), n_sim_yrs))
wt_seqs <- matrix(NA, nrow = n_sim_yrs, ncol = n_draws)
```

This is just us creating a bunch of empty matrices and a couple of 3d arrays that we will fill below in the loops of misery (below). If you are not used to arrays, you can think of a 3 dimensional array like a book where each page is a matrix with rows and columns.

We need 3d arrays for some of our model output. For example, saving output for MAY or HARA utility for each year in our simulation and for each c_{slope}, b_{lrp} combination requires three dimensions.

The loops of misery

Here's where the simulation actually starts to happen (finally). Everything up to this point was just reading in data or model output and setting it up to run our simulation. So now we just need to run our simulation over some finite time horizon for each policy (determined in this case by the c_{slope}, b_{lrp} combinations) and for each draw of the posterior (which was set by the n_{draws} parameter.

```
#------------------------------------------------------------------------
# run retrospective simulation for each cslope, bmin, draw, and sim yr
#------------------------------------------------------------------------
for (i in seq_along(c_slope_seq)) {
  c_slope <- c_slope_seq[i]
  for (j in seq_along(bmin_seq)) {
    b_lrp <- bmin_seq[j]
    set.seed(24) # challenge each bmin, cslope combo with same set of rec seqs
    wt_re_mat <- matrix(rnorm(n = n_sim_yrs * n_draws, mean = 0, sd = sd_wt),
                        nrow = n_sim_yrs, ncol = n_draws) # generate random deviates
    for (k in seq_len(n_draws)) {
      # pick a single draw
      sub_post <- subset(post, post$.draw == unique(post$.draw)[k])
      # lots more code below...
```

In words, this code is doing the following:

- For each c_{slope} i

- For each b_{lrp} j

- Set the random number generator seed (ensures all simulations are comparable)

- Generate a matrix of random deviates called `wt_re_mat`, with dimensions of the number of simulation years and the number of draws from the posterior

- For each draw of the posterior k

- Set up a bunch of stuff based on this specific draw to use in your simulation (not shown – code below)

So as we have been saying, we are going to run our simulation for each of these things.

The next bit of code takes a specific draw of the posterior, and uses it to set some more parameters and variables before we begin to loop through each year of our simulation, t:

```
# set leading parameters from sampled draw
rec_a <- exp(sub_post$ln_ar[1])
rec_b <- sub_post$br[1]
Ro <- sub_post$Ro[1]
sbo <- Ro * sbro
vbo <- Ro * vbro
```

Here we are plucking out the recruitment α and β parameters and the average number of recruits in the unfished condition R_0 from one draw of the posterior. We then calculate two derived variables `sbo` and `vbo`, which correspond to the total spawner biomass in the unfished condition and the biomass vulnerable to fishing in the unfished condition, respectively. We'll need these for our age structured calculations below.

Setting up the recruitment anomaly sequences

The next thirty lines of code are where a bunch of critical stuff happens. In particular, this is where we set the unpredictable recruitment anomaly sequences that we test our policies against. Recall that we have w_t values for 1990-2015. This means we have 26 years of estimated recruitment anomalies.

Given these estimates, there are many ways one could proceed. We could, for example, simply repeat the estimated w_t sequence some number of times into the future. In fact, we did this for pink shrimp problem we were working on over the fall. For the pink shrimp problem, we had longer time series of trustworthy recruitment estimates (e.g., 40 + years) and the shrimp don't live as long as a walleye. For this pink shrimp management strategy simulation tool, we just put the recruitment anomaly sequences end-to-end-to-end many times, and then tested performance of the various policies we were interested in.

At first we tried to do this with Alberta walleye. We repeated the recruitment series that we estimated, but it turned out this didn't work well because there were not enough recruitment estimates to make the harvest control rule simulation reliable. Thus, we had to figure out a workaround for this issue.

While we could not simply repeat the historical sequences for the walleye problem, it is critical to note there is still useful information in the historical w_t sequences that can be exploited to inform our policy simulation. For example the historical 26 year estimated w_t sequence provides at least some information on the relative frequency of strong and weak year classes, year class magnitude, and on the autocorrelation structure in recruitment for a given lake.

Because we only have 26 years of recruitment estimates for a fish that lives to around 15-20 years, about the only thing we can be sure of is that the next 26 years will almost certainly not be an exact repeat of the last 26. Given these ideas, we came up with the following rules to generate plausible future recruitment patterns for Alberta walleye:

1. We will start by generating a repeating historical sequence of w_t as $w_{t_{historical}}$ exactly how we did for shrimp problem above. So we just repeat the estimated w_t values `n_repeats` or 8 times x 26 years of reliable recruitment estimates = 208 total simulation years.

2. Next, we'll create an entirely new autocorrelated and noisy series of where the simulation parameters for this sequence are assumed similar among lakes, call this $w_{t_{sim}}$. We'll show the math below.

3. For the first 26 years of simulation, simply set w_t to the first 26 estimated values in $w_{t_{historical}}$.

4. For year 27-208, calculate the remaining w_t values for the simulation as

$$w_t = \psi_{w_t} \cdot (1 - w_{t_{historical}}) \cdot w_{t_{sim}}, \text{ where } \psi_{w_t} = 0.5 \tag{4}$$

These rules resulted in qualitatively similar patterns to the estimated recruitment anomalies for Alberta walleye. All this jiggery-pokery does is set the first 26 years to the estimated w_t values from the assessment models, and then sets the remaining w_t values (i.e., years 27-208) as a weighted average of the repeated historical sequence and our simulated sequence $w_{t_{sim}}$. The weighting between the estimated and simulated recruitment anomalies is given by the parameter ψ_{w_t}. Again, this gave us recruitment anomaly sequences that looked qualitatively similar to the values we estimated.

Note we set the $w_{t_{sim}}$ values as follows:

$$w_{t_{sim}} \sim \text{Normal}(0, \sigma_{wt}) \text{ if t} = 1 \tag{5}$$
$$w_{t_{sim}} = \rho \cdot w_{t-1_{sim}} + wt_{re}, \text{ where } wt_{re} \sim \text{Normal}(0, \sigma_{wt}) \text{ if t} > 1 \tag{6}$$

To do this, we first generated an anomaly sequence where we simply repeated the estimated w_t values `n_repeats` times. In all our simulations we set this at 8 repeats, and so we our simulations are 8 x 26 = 208 years in length. A simple rule of thumb here is to run the simulation at least ten times the generation length of the fish you are working on, which for Walleye is probably close to 200+ years.

The reason you simulate this long into the future is to ensure the initial condition or state that you start your simulation at does not influence the best harvest control rule choice. Phrased differently, we are trying to find a stationary and approximately optimal harvest control rule that works reasonably well regardless of the intial conditions.

We first extract the w_t values from the posterior, and then do some calculations:

```
# repeat the historical recruitment series
wt_historical <- sub_post$w
wt_bar <- mean(wt_historical)
wt_historical <- wt_historical - wt_bar # correct nonzero wt over initialization period
wt_historical <- rep(wt_historical, n_repeats) # 8 x 26 year sequence of values
```

Here we are mean-centering the historical recruitment series by calculating and then subtracting off `wt_bar` or the average from each historical w_t sequence during 1990-2015. We do this so that we don't do anything silly like assume the recruitment was trending upward or downward in the initialization period. This avoids a hidden or implicit assumption that productivity was shifting through time (i.e., we need to make sure that when we select our 1990-2015 w_t values that they were not larger or smaller on average than our whole model fitting period (from 1980-2018).

Once we correct the `wt_historical` sequence by subtracting `wt_bar`, we just repeat the sequence `n_repeats` number of times. This preserves the historical frequency and relative strength of strong and weak year classes that we estimated in the assessment model. Note that we are limited to the historical 26 year reference period for our management strategy simulations. Phrased differently, we cannot "see" any longer term patterns or event occurrences than have been visible during the reference period. In this regard, we are to some extent beholden to our historical experiences with these a given stock or fishery. Up to now we have specified the w_t sequences identically to how we did for the shrimp problem, but here is where things change.

One thing that we can be absolutely sure about is that the next 25 years will NOT just be a repeat of the last 25, and whatever random pattern we challenge the HCRs with will give best HCR for which we can still look retrospectively at how manager would have behaved over the historical period given the historical anomaly sequence, if we just make the first 25 simulated wt(t)'s be the historical ones. I am getting reasonable looking future patterns (same wt standard deviation, occurrence of fairly regular peaks) when I set the w(t) pattern using the following rules:

we then created a second

these, we then created a

with autocorrelation parameter ρ equal to the average autocorrelation among recruitment estimates among lakes, which was 0.6 (we estimated it).

We are doing our best to preserve the historical frequency of high and low year classes. . .

Similarly, the σ_{w_t}

e just need to preserve the observed historical frequency and severity of strong and weak year classes, i.e. avoid any stupid assumption about things being more or less variable into the future. So for each lake, repeat the sequence estimated for that lake for the 1990-2015 period. We can "see" back to 1990 for all those lakes (including Newell and Pigeon, just had low recruitments not highly uncertain ones), and cannot see after 2015 or 6 since only weak observations of younger fish. That gives us a 25 year reference period for each lake, will just have to warn them that we can't see any longer term patterns or event occurrances than have been visible during the reference period.

junk writing

A bewildering array of harvest control rule shapes and forms exists (see review in Deroba and Bence 20XX). These rules relate how much should be allocated for harvest relative to the biomass or number of critters
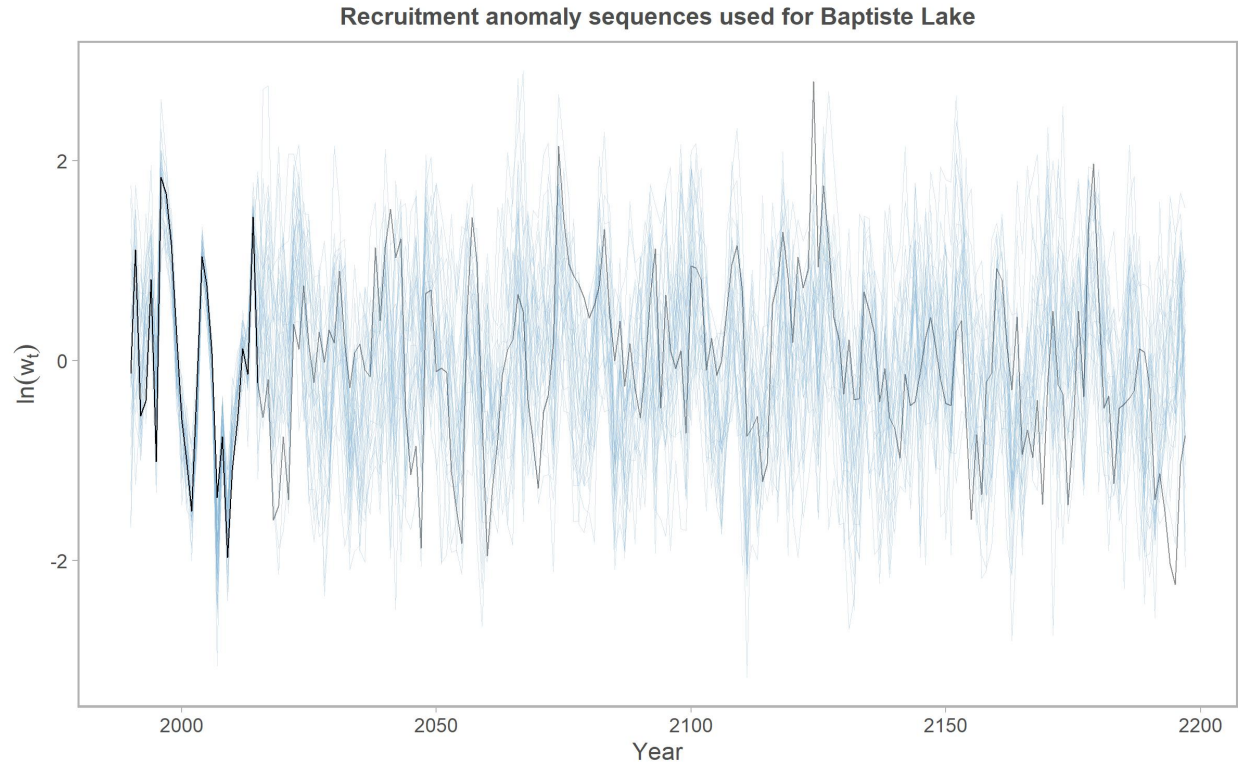
Figure 1: Examples of estimated and simulated recruitment anomalies w_t that we use for harvest control rule development. Thick black line represents one estimated recruitment anomaly sequence from 1990-2015, while the light grey line shows the evolution of the simulated recruitment sequence into the future. Note the simulated values are modeled as an average of the estimated sequence during 1990-2015 and an autocorrelated process that we specified to give recruitment series that were qualitatively similar to those we estimated in the Bayesian models. Light blue lines show different recruitment anomaly trajectories.

vulnerable to harvesting. Here is an example of a simple harvest control rule: After examining the data and model outputs for Alberta walleye in detail it seemed more appropriate to model recruitment anomalies into the future as a weighted average between the original estimated w_t values and another autocorrelated time series.

Early optimization work on harvesting theory shows that optimal rules given simple models and single objectives typically decrease harvest rate as population size decreases and increase harvest rate as population size increases (citations). It doesn't have to be complicated. Many rules, such as the generic rule proposed in Fisheries and Oceans Canada (DFO) 20XX, suggest that rectilinear harvest control rules are "precautionary." However, what *specifically* is precautionary about such rules is rarely

rarely specify what is precautionary about the specific rule beyond references to Restrepo XXXX.

(and

Given these considerations, we will primarily focus on the simulation evaluation of simple linear harvest control rules. If it isn't broke, don't fix it sort of thing.

and repeat this process many times (e.g., 30-100 random draws). Each time we record the performance of a policy

each of some number (e.g., say 30 or 100) of random values from our assessment posteriors, run our model into the future.

Resources:

Cahill et al. 2021 CJFAS.

Department of Fisheries and Oceans 2006. A harvest strategy compliant with the precautionary approach.

Deroba and Bence XXXX. Harvest control rules.

Edwards and Dankel 20XX. Fisheries Management Science.

Gelman et al. 2013. Bayesian Data Analsyis.

Larkin. Epitaph on Maximum Sustainable Yield.

Moxnes XXXX.

Restrepo XXXX. Precautionary stuff.

Walters and Hilborn 1976. Adaptive control of fishing systems.

Walters and Hilborn 1978. Ecological optimization and adaptive management.

Hilborn and Walters 1992. Quantitative Fisheries Stock Assessment.

Walters 1986. Adaptive Management of Renewable Resources.

Walters and Martell. 2001. Fisheries Ecology and Management.