

# Stock-recruitment and reference points

Feedback policy design

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# The Dark Arts of feedback policy design



“Did You Ever Hear The Tragedy Of Darth Plagueis The Wise?” *Star Wars: Episode III - Revenge of the Sith*

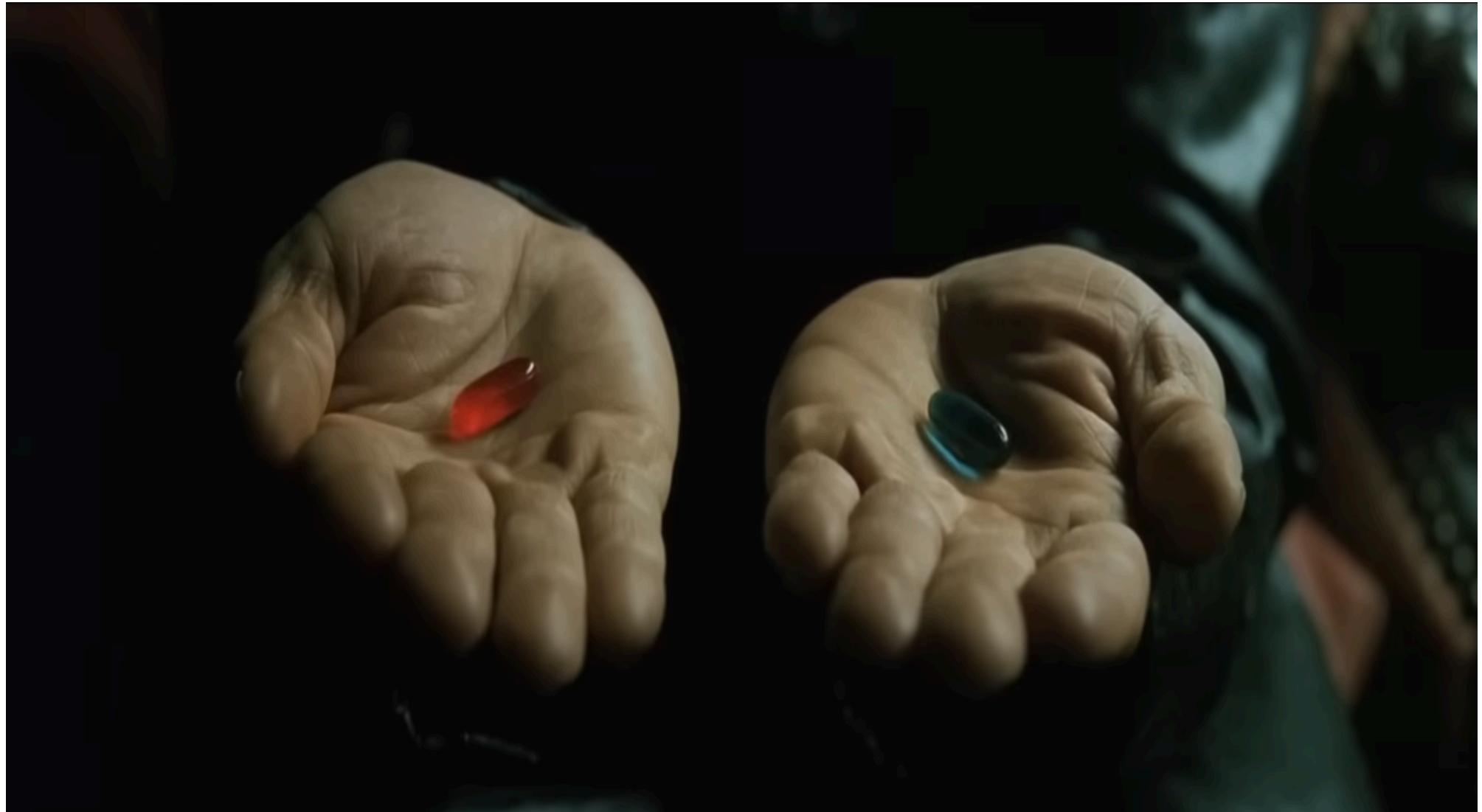
# Objectives

- A description of feedback control that we would share with grandma
- Understand what a harvest control rule is and why such rules are useful for managing natural resources
- Understand the importance of objectives: yield vs. HARA utility
- Introduce the omniscient manager
- Approximation in policy space
  - Approximate linear and rectilinear harvest control rules

# Feedback policies (harvest control rules)

- Feedback policies (also called harvest control rules) are agreed-upon, transparent, mathematical equations that adjust harvest in response to changes in stock size
  - Should help meet explicit management objectives (e.g., stable yield, rebuilding, consistent catches)
  - Stock size may change due to environmental variability or fishing pressure
  - Managers often implement them by setting total allowable catches (TACs)
- Some agencies (e.g., NOAA, DFO) use default harvest control rules
  - These use estimates like  $F_{\text{msy}}$  and  $B_{\text{msy}}$  (or proxies)
  - Those values are plugged into a standard rule structure adopted by the agency
  - Often justified as being “precautionary”
  - Analysts run simulations to explore tradeoffs using tools like Management Strategy Evaluation (MSE)
- You can set up and explore these policies using the tools you already have

# Is there another way?



“You take the red pill – you stay in Wonderland, and I show you how deep the rabbit hole goes.” Morpheus in *The Matrix*.

# Multi-stage decision problems are hard

“From a teleological point of view the particular numerical solution of any particular set of equations is of far less importance than the understanding of the *nature* of the solution.”

— Richard Bellman

“In view of all that we have said in the foregoing sections, the many obstacles we appear to have surmounted, what casts the pall over our victory celebration? It is the curse of dimensionality, a malediction that has plagued the scientist from the earliest days.”

— Richard Bellman

“Turning to the succor of modern computing machines, let us renounce all analytic tools”

— Richard Bellman

“Problems worthy of attack prove their worth by fighting back.”

— Paul Erdős

# Feedback policy design

- There exists a rich history and literature of developing feedback policies to regulate human exploitation in managed systems
- Recommended reading:
  1. Walters 1986, chapter on feedback policies
  2. Hilborn and Walters 1992, chapter on designing effective fisheries management systems
  3. Quinn and Deriso 1999, chapter 11 on optimal harvesting
  4. Walters and Martell 2001, chapters 2-4
  5. Williams et al. 2002. Analysis and management of animal populations, chapters 21, 22, 24, 25

Walleye example: <https://github.com/ChrisFishCahill/managing-irf-complexity/tree/main>

# Many ways to solve the harvesting problem

- Equilibrium and simulation based  $F_{MSY}$  and  $MSY$

Feedback control (dynamic fishing policies), which has direct linkages to adaptive management

- *Optimal control*: solve for exact optimal actions assuming perfect state knowledge
  - dynamic programming
  - analytical solutions (rare)
- *Open-loop optimization*: simulate dynamics forward in time, choose the full time series of actions
  - omniscient manager, estimate all exploitation rates  $U_t$  under known future recruitment
  - approximation in policy space: estimate within a constrained rule class such as a fixed harvest control rule
- *Reinforcement learning*: learn or approximate optimal policies from interaction with environment, beyond the scope of this class

# Harvesting theory, dynamic programming, and optimal solutions

- A large body of theoretical work on optimal harvesting strategies for dynamic populations shows a consistent result: **optimal policies tend to reduce harvest when population size declines, and increase it when population size grows** (e.g., Walters 1975; Walters and Hilborn 1978).
- These results often come from dynamic programming approaches, where the optimal action is derived by solving a recursive optimization problem. Many such solutions recommend adjusting harvest as a **simple function of biomass or abundance** (e.g., Clark 1974; Clark and De Pree 1979; Mangel 1984; Moxnes 2002).
- This is surprising—because complex systems might seem to require complex strategies—but much of the work from the 1970s and 1980s showed that **simple feedback policies** often perform well or are even optimal (e.g., Walters 1986; Hilborn and Walters 1992).
- This body of work forms a large part of the theoretical basis of the *de facto* precautionary rules used by e.g., NOAA, DFO, and others.

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  - Why are they precautionary?

# Feedback policy design: the autopilot analogy

- Think of a harvest control rule like designing an autopilot system
- The goal: create a **mathematical rule** that links **measurements from the environment** (e.g., altitude, wind, speed) to **actions** (e.g., turning, accelerating)
- This rule is a **feedback policy** – it responds to where the system is, and tries to steer it toward a desired outcome
- In the plane, the “environment” is physics; in fisheries, it’s the population dynamics of the targeted population

# From autopilot to fisheries management

- In aircraft, sensors report wind speed, altitude, heading, etc.
  - In fisheries, we use **surveys** and **stock assessments** to estimate stock size and trends
- A good autopilot increases lift if the plane is dropping, or turns right if wind pushes it left
- Similarly, a good harvest policy reduces catch if stock size is low, or allows more harvest if stock size is high
- Both systems aim to use feedback to **guide** the system toward its goal (safe flight or sustainable harvest or whatever goal you want)
- Designing a “controller” for a nonlinear dynamical system.
  - Beyond this point, logic starts to fray: many counter-intuitive results

# On the need for explicit objectives

- Like autopilot, harvest control rules need clear objectives to function
- Without agreed-upon objectives, feedback policy design stalls
- Defining objectives is hard

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  - Defining objectives is hard
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- Many fisheries lack explicit objectives
  - We'll use two provisional performance criteria to explore tradeoffs:
    - Maximum Average Yield (MAY)
    - HARA utility (favoring consistency in catches)
  - These objectives bracket a range from aggressive yield-maximizing policies to preferences for stable catches and thus help to highlight **tradeoffs** associated with different policy options

# Maximum Average Yield (MAY) objective

- MAY is the stochastic analog of MSY and aims to maximize long-term average yield
- Optimal policies often resemble a **fixed escapement rule**:
  - No harvest below a biomass threshold
  - Harvest aggressively above that threshold
- Benefits: high average yield from dynamic populations
- Drawbacks: can cause **fisheries closures** when biomass drops below threshold
- Yield-focused, but **variable and risky** from a harvester's perspective
- Regardless of our individual thoughts on this objective, we can probably agree that this isn't your grandad's MSY and harvesting at rates above it is unwise

# HARA utility objective

- HARA = Hyperbolic Absolute Risk Aversion
  - A utility function used to model risk-averse preferences for catch
  - Values **predictability** over high but uncertain outcomes
- Annual utility:

$$\text{utility}_t = \text{yield}_t^{\text{upow}}, \quad 0 < \text{upow} \leq 1$$

- upow → 1.0 utility seeks to maximize yield
- Lower upow → stronger aversion to variability in catch
- Interpretation:  
A harvester with upow = 0.6 would prefer a guaranteed catch of ~\$24,600 over a 50/50 gamble between \$0 and \$100,000
- Often between 0.3-0.6 for many harvesters

# Estimating your own risk aversion

- Imagine you're indifferent between:

- A 50/50 chance of **0** or **H** fish
- A guaranteed catch of **X** fish

- Then:

$$X^{\text{upow}} = 0.5 \cdot H^{\text{upow}} \Rightarrow \left(\frac{X}{H}\right)^{\text{upow}} = 0.5$$

- Solving for **upow**:

$$\text{upow} = \frac{\ln(0.5)}{\ln(X) - \ln(H)}$$

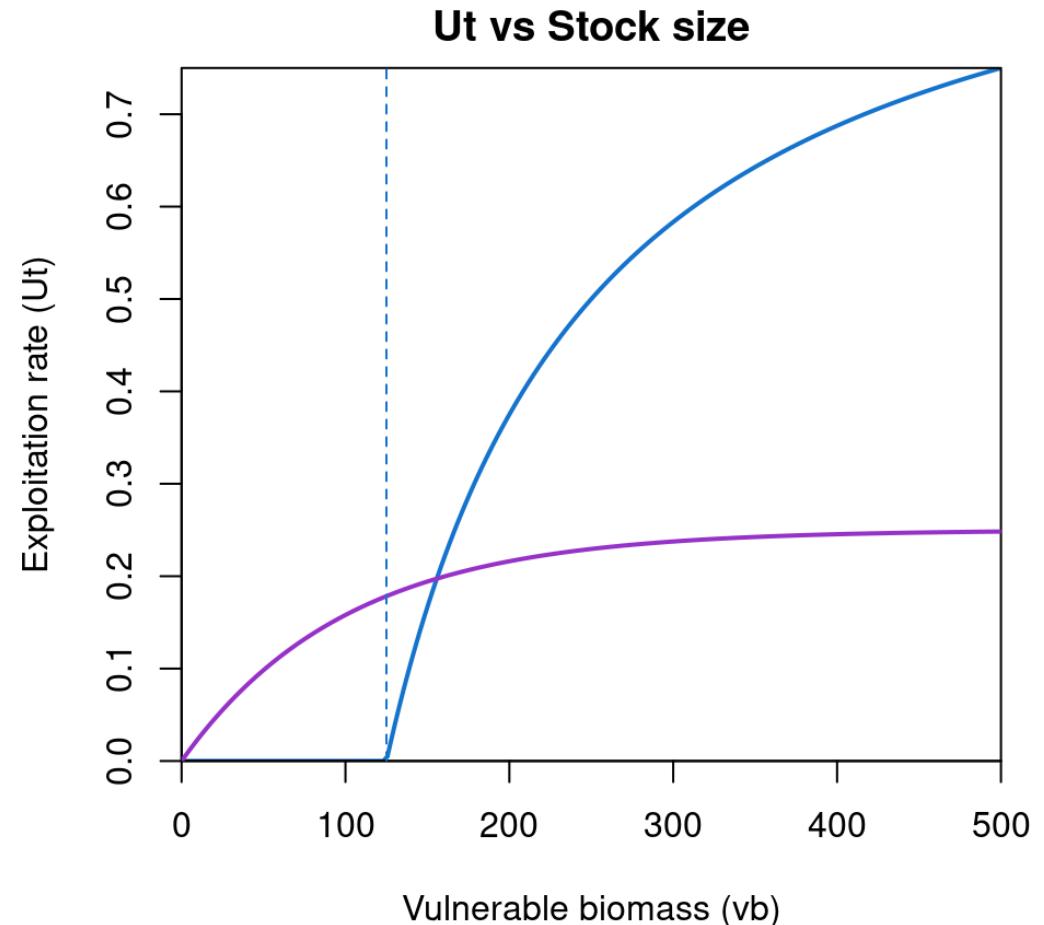
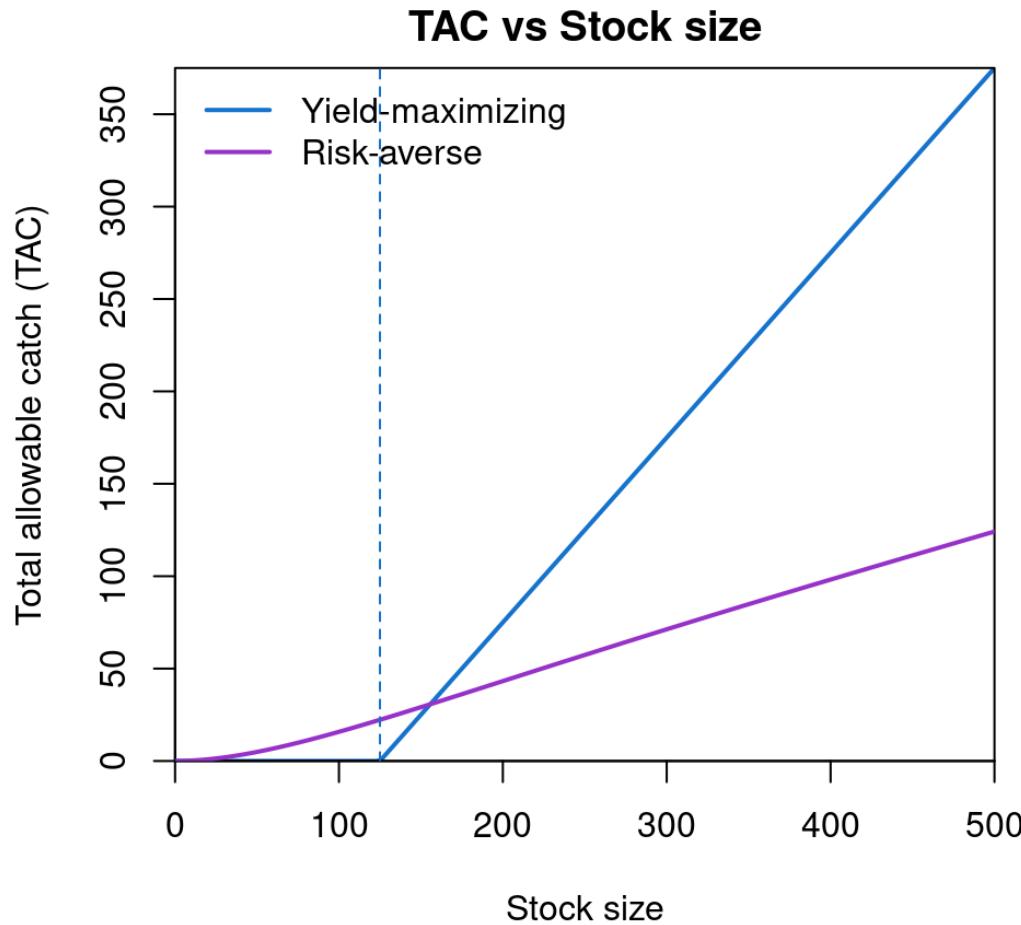
# Estimating your own risk aversion

- In R

```
1 get_upow <- function(X, H) {  
2   # X = guaranteed catch  
3   # H = high value of 50/50 gamble (other outcome is 0)  
4   log(0.5) / (log(X) - log(H))  
5 }  
6  
7 # Example: Cahill is indifferent between 2 guaranteed and  
8 # a 50/50 gamble between 0 or 10 walleye  
9 get_upow(X = 2, H = 10)
```

```
[1] 0.4306766
```

# MAY vs. HARA: some general themes



- $U_t = TAC / \text{Stock Size}$

see also Hilborn and Walters 1992; Walters and Parma 1996; Walters and Martell 2004

# Output vs. direct U control in fisheries management

- ***Output control.*** Set a quota or total allowable catch each year.
  - In this case, stock assessments attempt to estimate total abundance and recruitment.
- ***Input or direct exploitation rate control.*** Develop regulations to directly control exploitation rate.
  - In this management-chain, we worry about growth, vulnerability, M, and the intercept of the stock-recruitment relationship.
    - time/area closures
    - direct monitoring of exploitation rate through tagging studies
    - good estimates of total abundance not necessarily required
  - Assume that F to be implemented is without error, e.g. via effort controls with known catchability
  - There are many tactics for effective harvest regulation.

# Output vs. direct U control

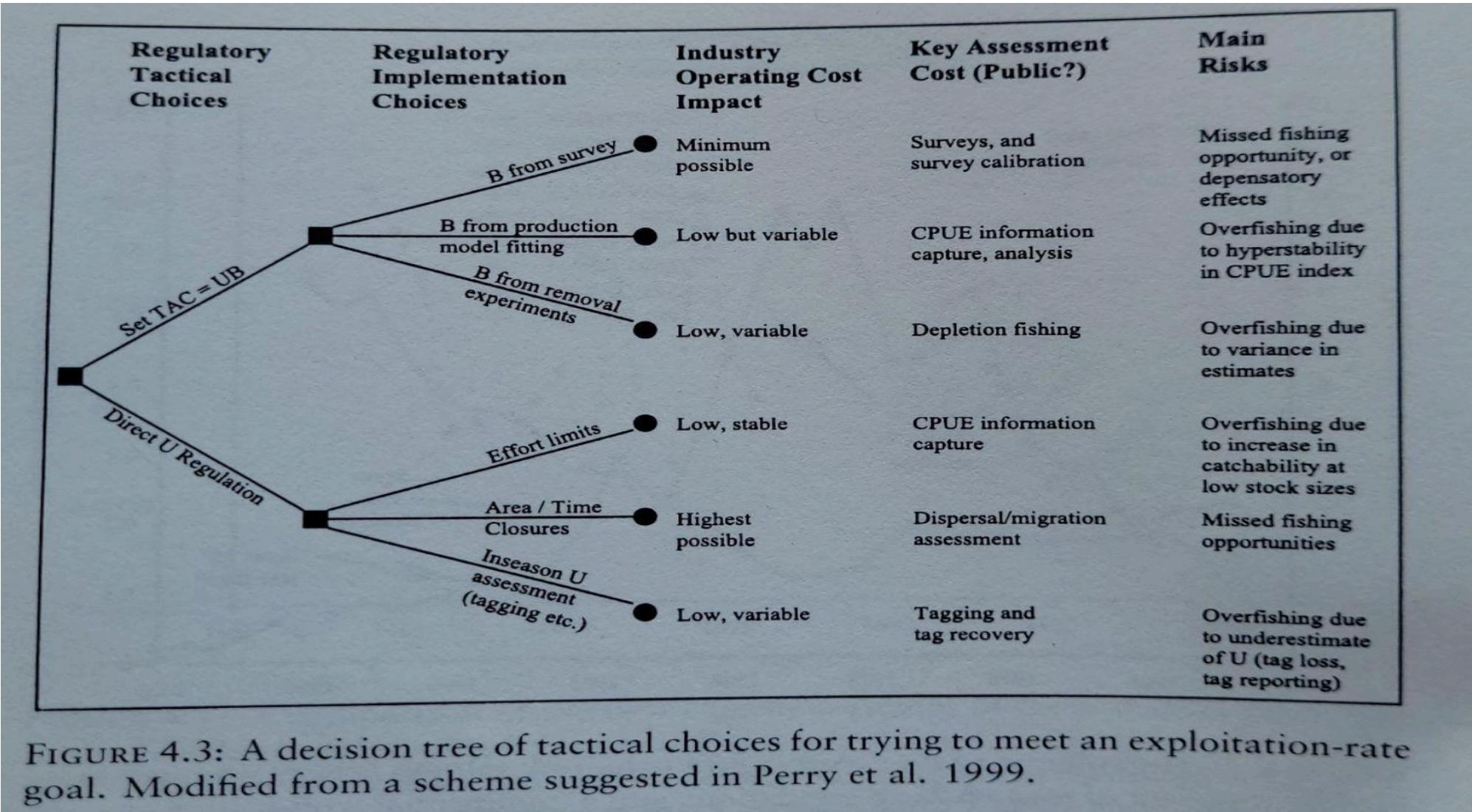
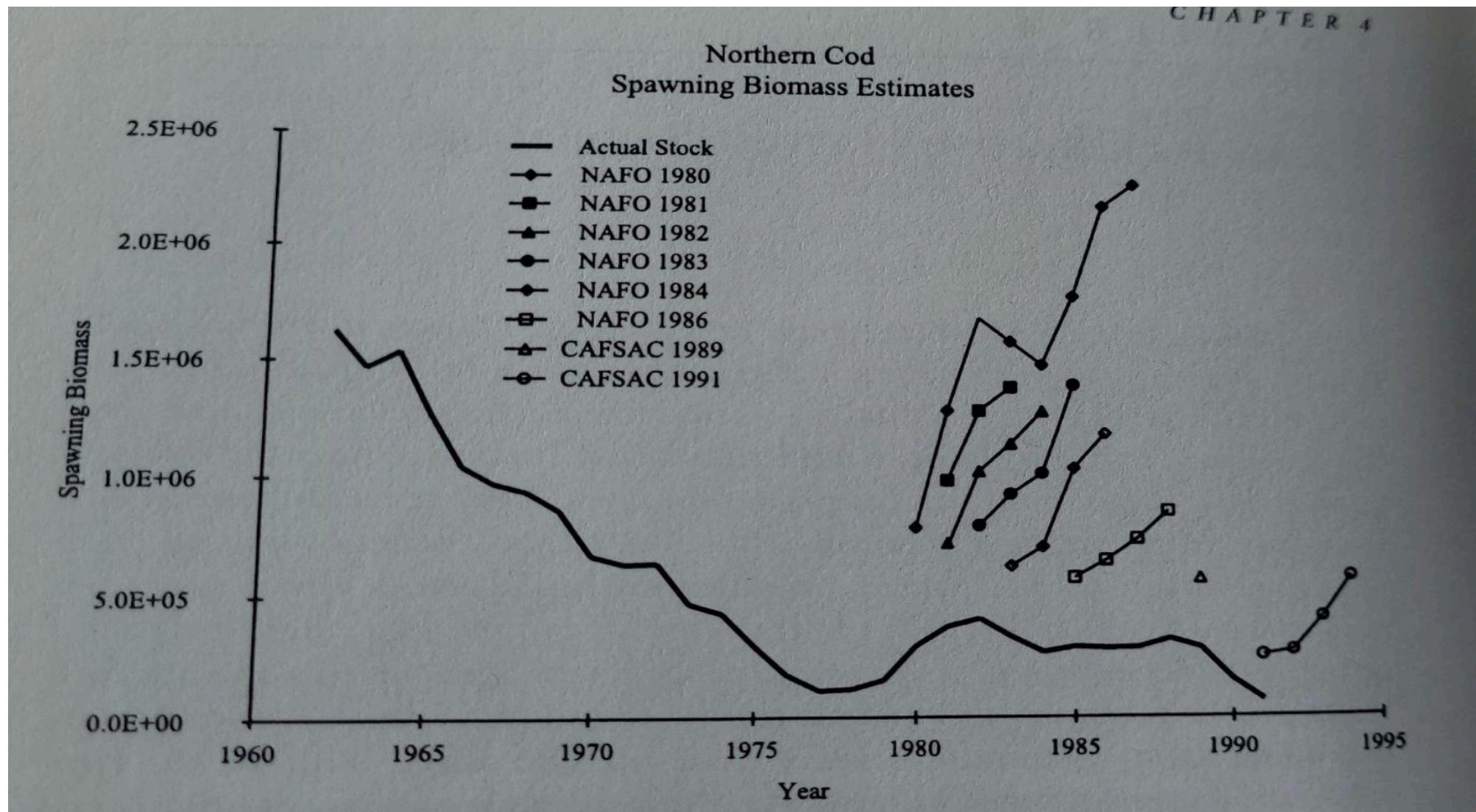


FIGURE 4.3: A decision tree of tactical choices for trying to meet an exploitation-rate goal. Modified from a scheme suggested in Perry et al. 1999.

# Can output control go wrong?



On the need to consider assessment error. Walters and Martell 2004; Chapter 4

# The Omniscient Manager

- Let us pose the following thought-experiment
  - *Suppose a manager knew future recruitment, mortality, and system dynamics. What sequence of instantaneous fishing mortality  $F_t$  would they set to maximize long-term objectives?*
  - What would you do if you knew what was going to happen?

# The Omniscient Manager

We assume the manager knows all future recruitment deviations  $w_t$ , and chooses fishing mortalities  $\{F_t\}_{t=2}^T$  **up front** to maximize utility.

$$\max_{\{F_t\}} \frac{1}{T} \sum_{t=2}^T (Y_t)^{\text{upow}}$$

where  $T$  is the time horizon,  $\text{upow}$  describes risk aversion, and  $Y_t$  is yield at time  $t$ . Also note that population dynamics are governed by our standard age-structured model (see earlier presentations).

Assumptions:

- In our case, fishing starts in year two
- All future stochastic deviations  $w_t$  are **known** (perfect foresight)
- $F_t$  is **not adaptive** – it's selected all at once
- This is an **open-loop** benchmark for best possible management under full information

Even though the omniscient manager finds the optimal  $\{F_t\}$ , this policy is not implementable in real systems. It assumes we know future surprises.

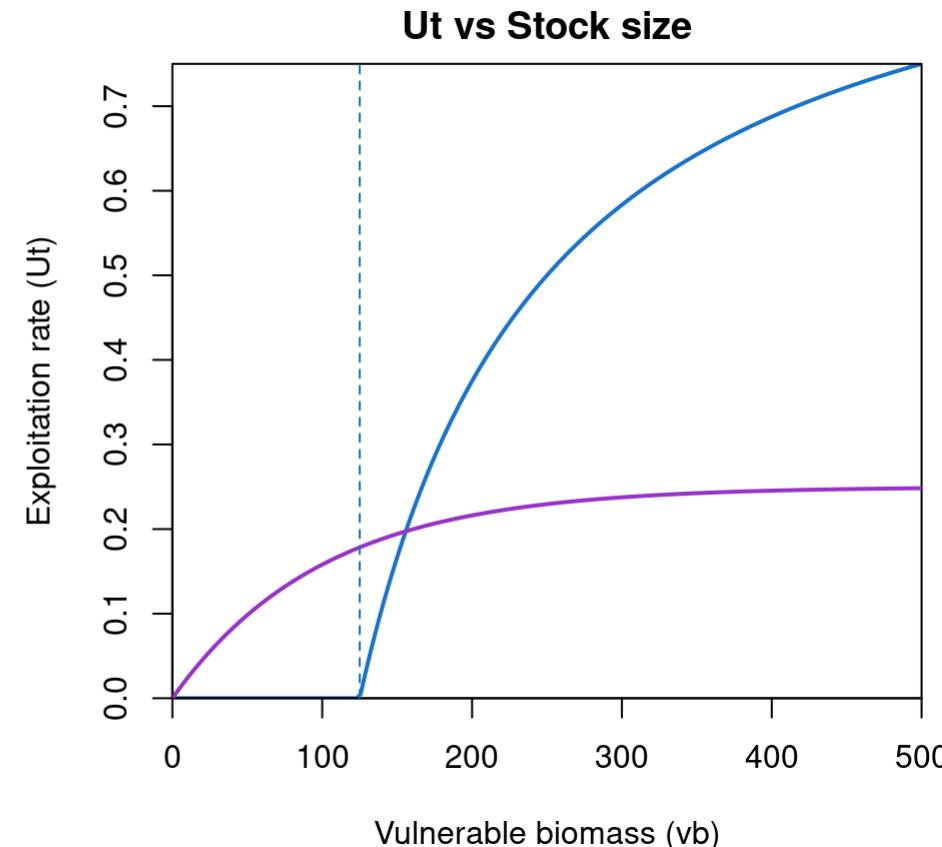
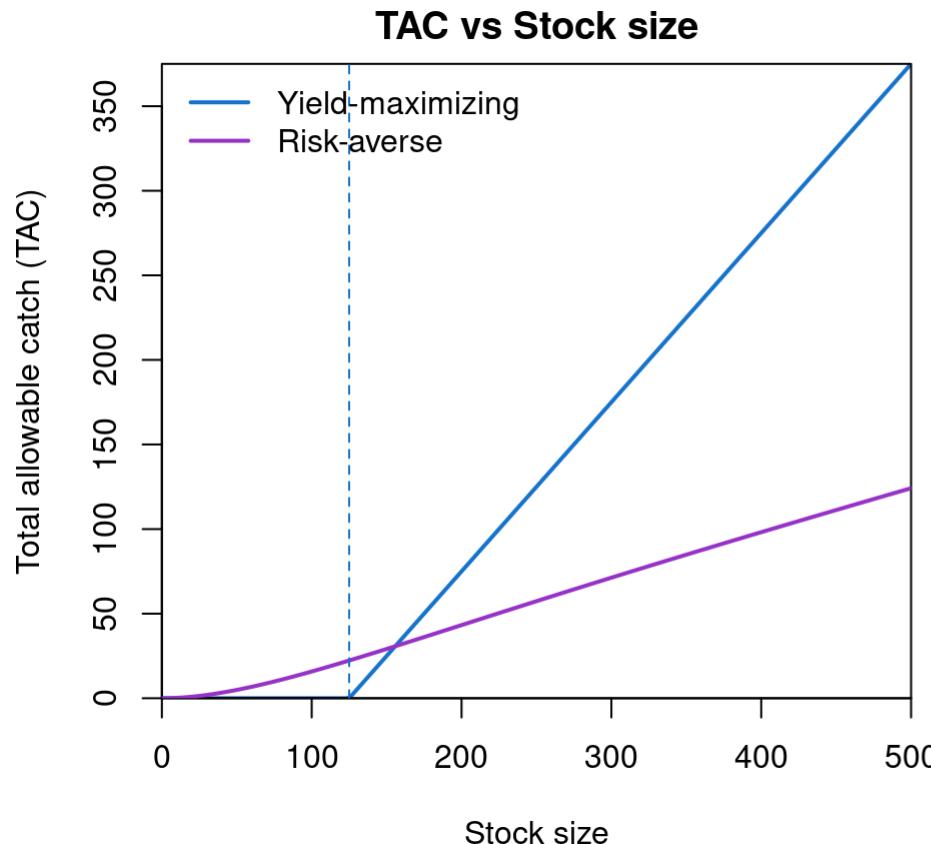
This approach is almost infinitely adaptable: can you think of some things you might want to test?

# Exercise: The Omniscient Perch Manager

People keep asking us how to assess and manage perch 😊

Let's play a game and see how an Omniscient Manager might manage our yellow perch with stochastic recruitment 🎭

Specifically, do the standard theoretical findings hold true with perch:



# Feedback control

- Surely we are not omniscient
- We cannot implement an omniscient manager policy (too complex)
- Can we instead approximate these answers using simpler feedback policies that adapt to observed outcomes?

# Approximation in policy space

- Specify harvest control rules that determine  $U_t$  directly as a function of stock size
- Estimate parameters of these rules to maximize an objective over a finite time horizon (e.g. yield, HARA utility)
- Use **smooth approximations** (e.g. softplus) to ensure differentiability and compatibility with numerical optimization

see also Walters and Hilborn 1978; Walters 1986; Hilborn and Walters 1992

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- Recognize that the approach differs under:
  - **Input control:** where  $U_t$  is implemented and  $\text{TAC}_t$  is implied
  - **Output control:** where  $\text{TAC}_t$  is implemented and  $U_t$  is implied

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  - Compare harvest strategies
  - Evaluate robustness under uncertainty, implementation, or observation error
  - Explore trade-offs between yield, variability, and risk

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  - Evaluate robustness under uncertainty, implementation, or observation error
  - Explore trade-offs between yield, variability, and risk
- Very much NOT estimating population dynamics, but rather the parameters of a feedback rule that controls or modulates the dynamics of the population(s) in question 

see also Walters and Hilborn 1978; Walters 1986; Hilborn and Walters 1992

# Why smooth approximations?

- Harvest control rules often involve non-differentiable functions like  $\max(0, x)$
- These create problems for automatic differentiation optimization
- Smooth approximations (e.g. **softplus**) resolve this:

$$\max(0, x) \approx \frac{1}{\beta} \log(1 + e^{\beta x})$$

- Benefits:
  - Fully differentiable with respect to all parameters
  - Retains shape of original function as  $\beta \rightarrow \infty$
  - Compatible with automatic differentiation tools (e.g. ADMB, TMB, RTMB)

# Smooth approximation of linear HCR

Linear control rule (on total catch):

$$TAC_t = \max(0, c \cdot (VB_t - LRP))$$

Converted to an exploitation rate:

$$U_t = \frac{1}{VB_t} \cdot \max(0, c \cdot (VB_t - LRP))$$

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Smooth approximation using softplus:

$$U_t = \frac{1}{VB_t} \cdot \frac{1}{\beta} \cdot \log(1 + \exp[\beta \cdot c \cdot (VB_t - LRP)])$$

Here,  $c$  controls the slope,  $\beta$  controls the smoothness, and  $LRP$  is the lower reference point.  $VB_t$  is the biomass vulnerable to fishing at time t. This form smoothly ramps up the catch rate as biomass exceeds LRP.

When does this form assume harvest occurs?

# Ramped harvest control rule (HCR)

We define a control rule with a ramp up to a capped exploitation rate:

$$U_t = \begin{cases} 0, & VB_t \leq \text{LRP} \\ c \cdot (VB_t - \text{LRP}), & \text{LRP} < VB_t < B_{\text{trigger}} \\ u_{\text{cap}}, & VB_t \geq B_{\text{trigger}} \end{cases}$$

with slope:

$$c = \frac{u_{\text{cap}}}{B_{\text{trigger}} - \text{LRP}}$$

- $VB_t$ : observed vulnerable biomass
- LRP: lower reference point
- $B_{\text{trigger}}$ : biomass where full exploitation begins
- $u_{\text{cap}}$ : maximum exploitation rate
- $c$ : slope of the ramp

# Smooth approximation of ramped HCR

**Step 1: smooth ramp-up from zero (softplus):**

$$soft_{ramp}(VB_t) = \frac{1}{\beta} \log(1 + \exp[\beta \cdot c \cdot (VB_t - \text{LRP})])$$

This approximates:

$$\max(0, c \cdot (VB_t - \text{LRP}))$$

**Step 2: smooth upper cap (soft-min):**

$$U_t = u_{\text{cap}} - \frac{1}{\beta} \log(1 + \exp[\beta \cdot (u_{\text{cap}} - soft_{ramp})])$$

This approximates:

$$\min(u_{\text{cap}}, soft_{ramp})$$

# Final exploitation rate and notes

Step 3: convert to fishing mortality

$$F_t = -\log(1 - U_t)$$

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Notes:

- $\beta$  controls **smoothness** of transitions
  - higher  $\beta \rightarrow$  sharper transitions (closer to rectilinear form)
- This form is **differentiable** and works well in optimization
- If harvest occurs gradually over the year, apply to average biomass using Pope's approximation:

$$VB_t \cdot e^{-M/2}$$

# Exercise

- Play with input control scripts for linear and rectilinear harvest control rules
- Explore how objective (yield vs. HARA) changes feedback policy shape
- Where does the maximizing yield objective cap mortality?
- What does the rectilinear rule do that the linear rule does not?
- Does observation error change your answers? If so, how?

# Thank you



Chris going to sleep for eternity. *Nosferatu* 1922.

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