Student: Christopher Campbell	**Instructor:** Ferdinand Hallare	**Assignment:** 2.5 HW - Additive Rules
Date: 01/15/24	**Course:** MTH 283 - N01B Spring 2024	

Suppose that in a senior college class of 500 students, it is found that 193 smoke, 200 drink alcoholic beverages, 224 eat between meals, 78 smoke and drink alcoholic beverages, 61 eat between meals and drink alcoholic beverages, 84 smoke and eat between meals, and 22 engage in all three of these bad health practices. If a member of this senior class is selected at random, find the probability that the student (a) smokes but does not drink alcoholic beverages; (b) eats between meals and drinks alcoholic beverages but does not smoke; (c) neither smokes nor eats between meals.

(a) Let S be the event that a student smokes. Let D be the event that a student drinks alcoholic beverages. Let E be the event that a student eats between meals. Write the notation for the event that the student smokes but does not drink alcoholic beverages.

$S \cap D'$

If A and B are two events, then $P(A \cup B) = P(A) + P(B) - P(A \cap B)$. If A and B are mutually exclusive, then $P(A \cup B) = P(A) + P(B)$.

If A and A' are complementary events, then $P(A) + P(A') = 1$.

Since $S \cap D$ and $S \cap D'$ are mutually exclusive, $P(S) = P(S \cap D') + P(S \cap D)$.

If an experiment can result in any one of N different equally likely outcomes, and if exactly n of these outcomes correspond to event A, then the probability of event A is $P(A) = \frac{n}{N}$.

For the event that a student smokes, n = 193 and N = 500.

Use these values of n and N to find P(S).

$$P(S) = \frac{n}{N} = 0.386$$

Similarly, for event $S \cap D$, N = 500 and n = 78.

Use these values of n and N to find $P(S \cap D)$.

$$P(S \cap D) = \frac{n}{N} = 0.156$$

Next, use P(S) = 0.386 and $P(S \cap D) = 0.156$ to find $P(S \cap D')$.

$$P(S \cap D') + P(S \cap D) = P(S)$$
$$P(S \cap D') + 0.156 = 0.386$$
$$P(S \cap D') = 0.230$$

Therefore, the probability that a randomly selected student smokes but does not drink alcoholic beverages is 0.230.

(b) Write the notation for the event that the student eats between meals and drinks alcoholic beverages but does not smoke.

$E \cap D \cap S'$

Note that events $E \cap D \cap S$ and $E \cap D$ are given in the problem statement. These two events can be used to find $P(E \cap D \cap S')$ using the following equation.

$$P(E \cap D \cap S') = P(E \cap D) - P(E \cap D \cap S)$$

Determine $P(E \cap D)$. First, find the number of students that drink alcoholic beverages and eat between meals.

n = 61

Use this value of n and N = 500 to find $P(E \cap D)$.

$$P(E \cap D) = \frac{n}{N} = 0.122$$

Similarly, for $E \cap D \cap S$, there are $n = 22$ students that engage in all three bad health practices. Find $P(E \cap D \cap S)$.

$$P(E \cap D \cap S) = \frac{n}{N}$$
$$= 0.044$$

Use these values to calculate $P(E \cap D \cap S')$.

$$P(E \cap D \cap S') = P(E \cap D) - P(E \cap D \cap S)$$
$$= 0.122 - 0.044$$
$$= 0.078$$

Therefore, the probability that a randomly selected student eats between meals and drinks alcoholic beverages but does not smoke is 0.078.

(c) Write the notation for the event that the student neither smokes nor eats between meals.

$S' \cap E'$

For any two events A and B, $(A \cap B)' = A' \cup B'$ and $(A \cup B)' = A' \cap B'$. Thus, the event that the student neither smokes nor eats between meals can also be written as $(S \cup E)'$.

First, find $P(S \cup E)$ using additive rules. $P(S \cup E)$ can be written using the formula shown below.

$$P(S \cup E) = P(S) + P(E) - P(S \cap E)$$

From part (a), $P(S) = 0.386$. Find $P(E)$ and $P(S \cap E)$.

There are 224 students that correspond to event E, and 84 students that correspond to event $S \cap E$.

Calculate $P(E)$ and $P(S \cap E)$, using these two values and $N = 500$.

$$P(E) = \frac{224}{500} \qquad P(S \cap E) = \frac{84}{500}$$
$$= 0.448 \qquad = 0.168$$

Use $P(S) = 0.386$, $P(E) = 0.448$, and $P(S \cap E) = 0.168$ to calculate $P(S \cup E)$.

$$P(S \cup E) = P(S) + P(E) - P(S \cap E)$$
$$= 0.386 + 0.448 - 0.168$$
$$= 0.666$$

Subtract this value from 1 to find $P((S \cup E)')$.

$$P((S \cup E)') = 1 - 0.666$$
$$= 0.334$$

Therefore, the probability that a randomly selected student neither smokes nor eats between meals is 0.334.