

# Image Based Rendering with Depth Cameras: How Many are Needed?

Christopher Gilliam, James Pearson, Mike Brookes and Pier Luigi Dragotti  
Communications & Signal Processing Group  
Imperial College London

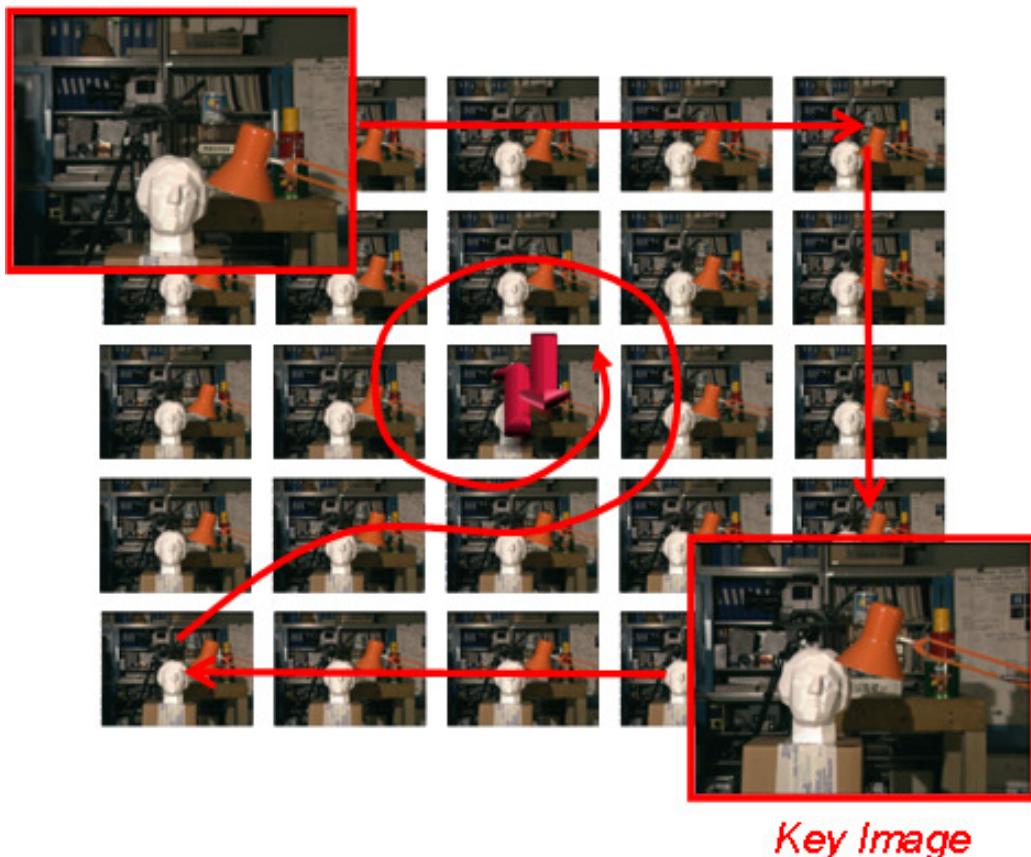
30th March 2012



# Motivation

Image Based Rendering (IBR)  $\implies$  Rendering new viewpoints of a scene from a multi-view image set

*Key Image*



Courtesy of James Pearson [1]

↗ Trade-off between geometric information and image density

# Motivation

Obtaining geometric information:

- Passive Depth Sensing
  - ⇒ Large computational complexity and inconsistent
- Active Depth Sensing
  - ⇒ Low-cost, fast and reliable depth cameras



Mesa Imaging's SwissRanger™



Microsoft's Kinect

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⇒ Multi-View Color and Depth Camera System

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Density of color cameras examined using the concept of the plenoptic function<sup>[2,3,4]</sup>

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Therefore combine RGB and Depth:

- ⇒ Multi-View Color and Depth Camera System
- ⇒ How many depth cameras are required?
- ⇒ Interplay between: Number of Color Cameras, Number of Depth Cameras and Scene Geometry

# Overview

- The Plenoptic Function and Plenoptic Sampling
  - Current Theory
  - Spectral Analysis of a Slanted Plane
  - Adaptive Sampling Algorithm
- Multi-view Depth Images
  - Concept → Plenoptic style framework
  - Spectral Analysis for Multi-view Depth Images
  - Results for synthetic and real scenes
- Conclusions

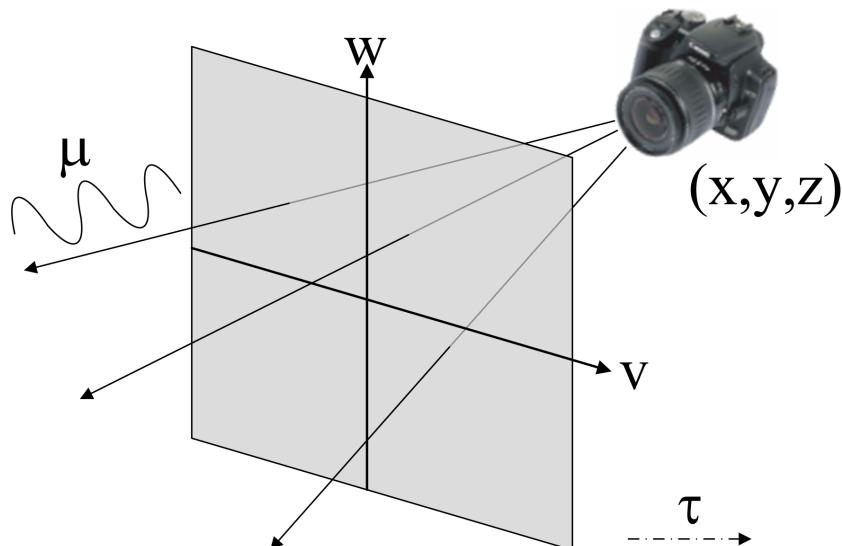
# The Plenoptic Function

IBR in more detail:

- Images sample a set of light rays from the scene to the camera
- New rendering interpolated from captured light rays
- Light ray modelled using the 7D *Plenoptic Function* [2]

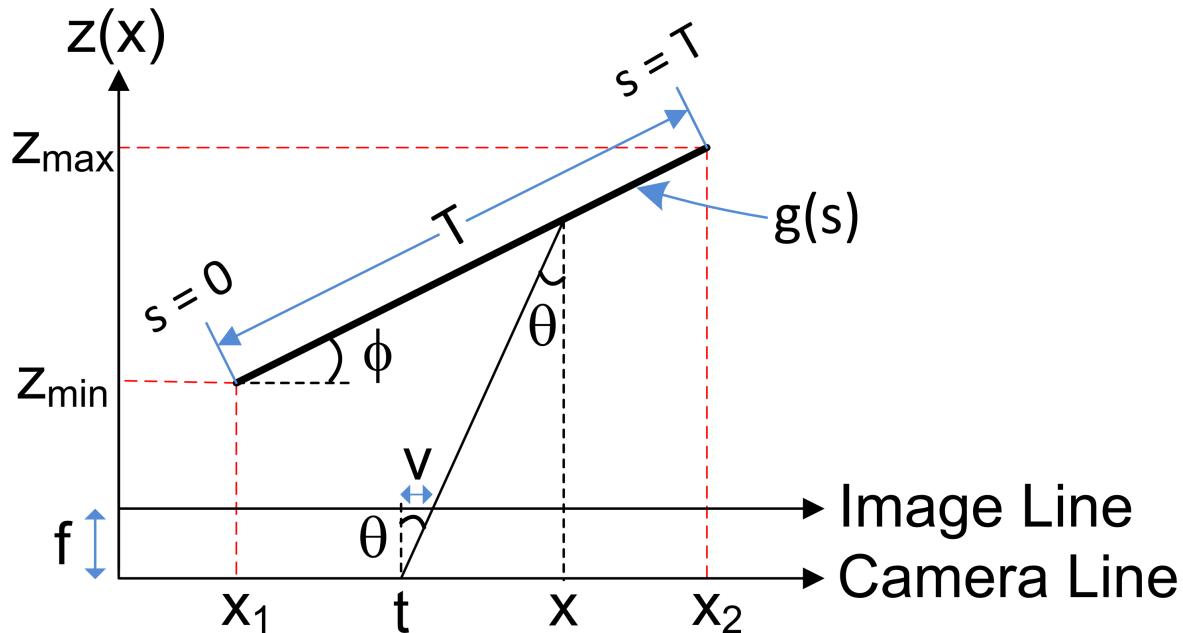
↳ IBR viewed as the Sampling and Reconstruction of the Plenoptic Function

↳ Camera density from spectral analysis of Plenoptic Function<sup>[3]</sup>



- Camera centre location  $(x, y, z)$ ,
- Pixel Coordinate  $(v, w)$ ,
- Wavelength  $\mu$ ,
- Time  $\tau$ .

# Slanted Plane Geometry



Functional Scene Model [4]:

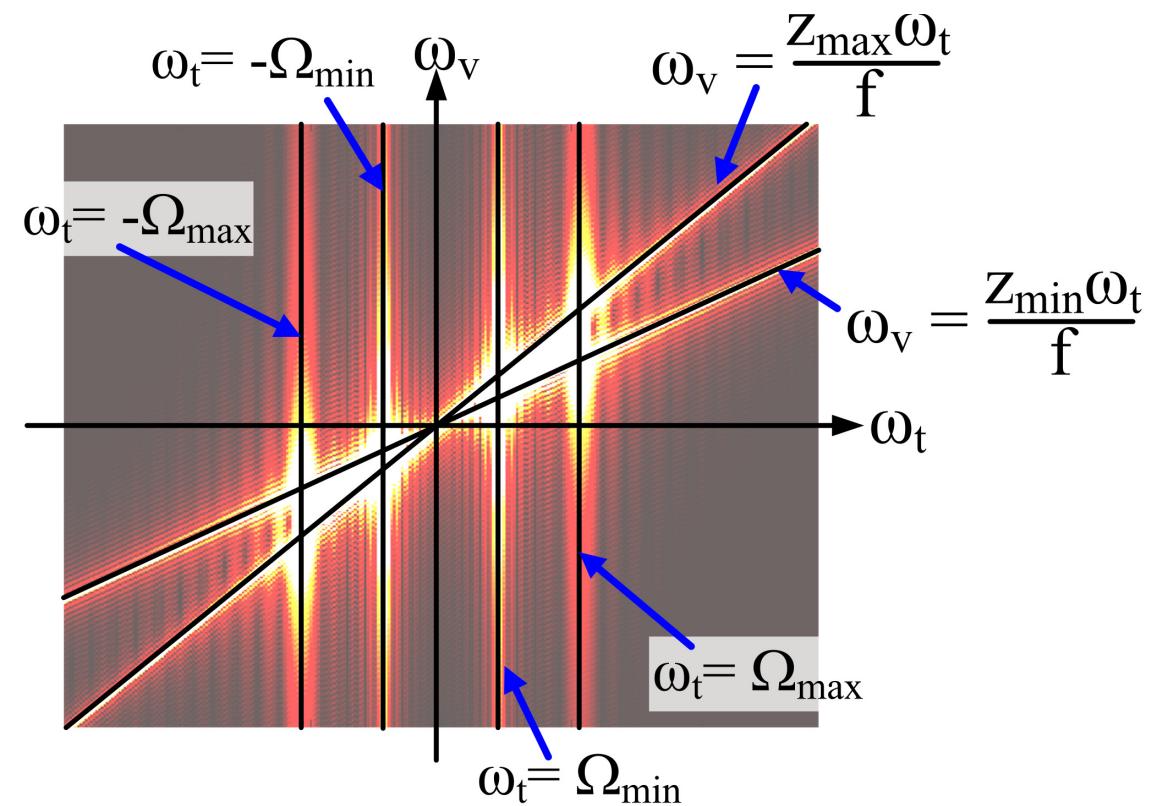
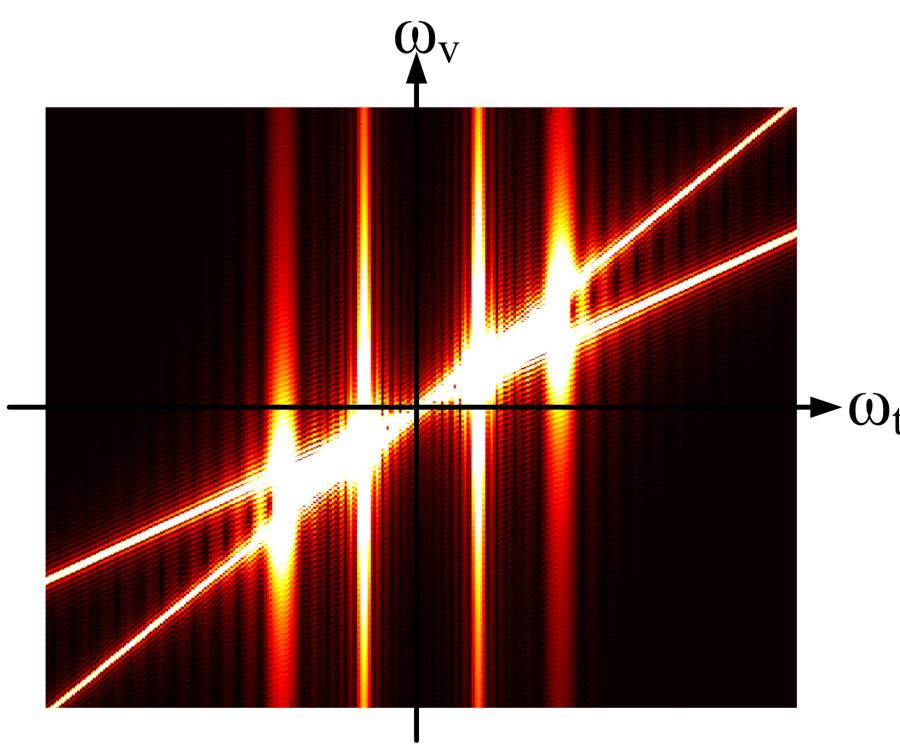
- $s$  is the Curvilinear Coordinate
- $x$  is the Projection of  $s$  onto  $t$
- $\phi$  is the Slant of the Plane
- $\theta$  is the viewing angle
- $f$  is the focal length

Bandlimited Texture Signal Pasted to Scene Surface,  $g(s)$

Constraints:

- 2D Plenoptic Function  $\implies p(t, v)$
- Finite Field of View (FoV) for the Cameras  $\implies v \in [-v_m, v_m]$
- Finite Plane Width  $\implies s \in [0, T]$

# Slanted Plane Geometry

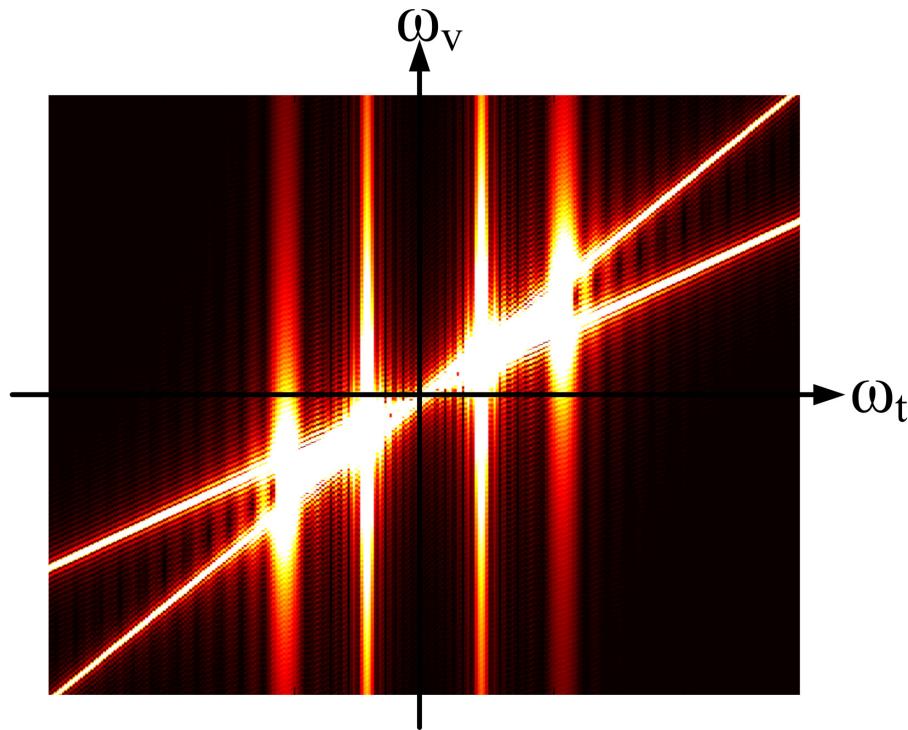


Spectral Analysis:

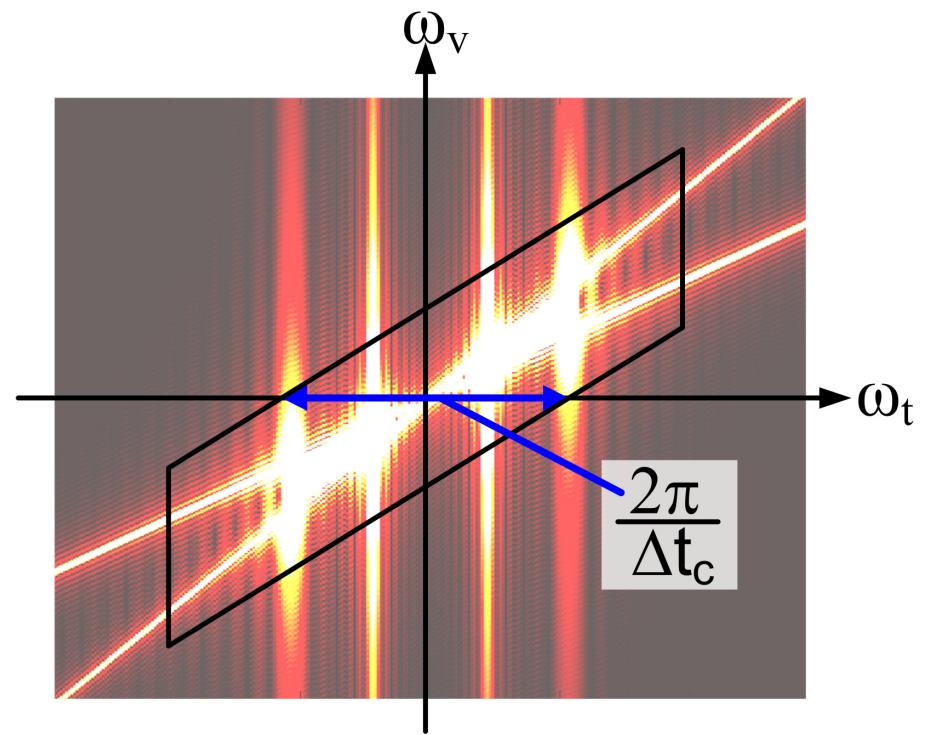
Close-form expression for the plenoptic spectrum of a slanted plane<sup>[5]</sup>

↗ Characterised by lines relating to the depth and warped texture signal

# Slanted Plane Geometry



(a) Example Spectrum



(b) Its Essential Bandwidth

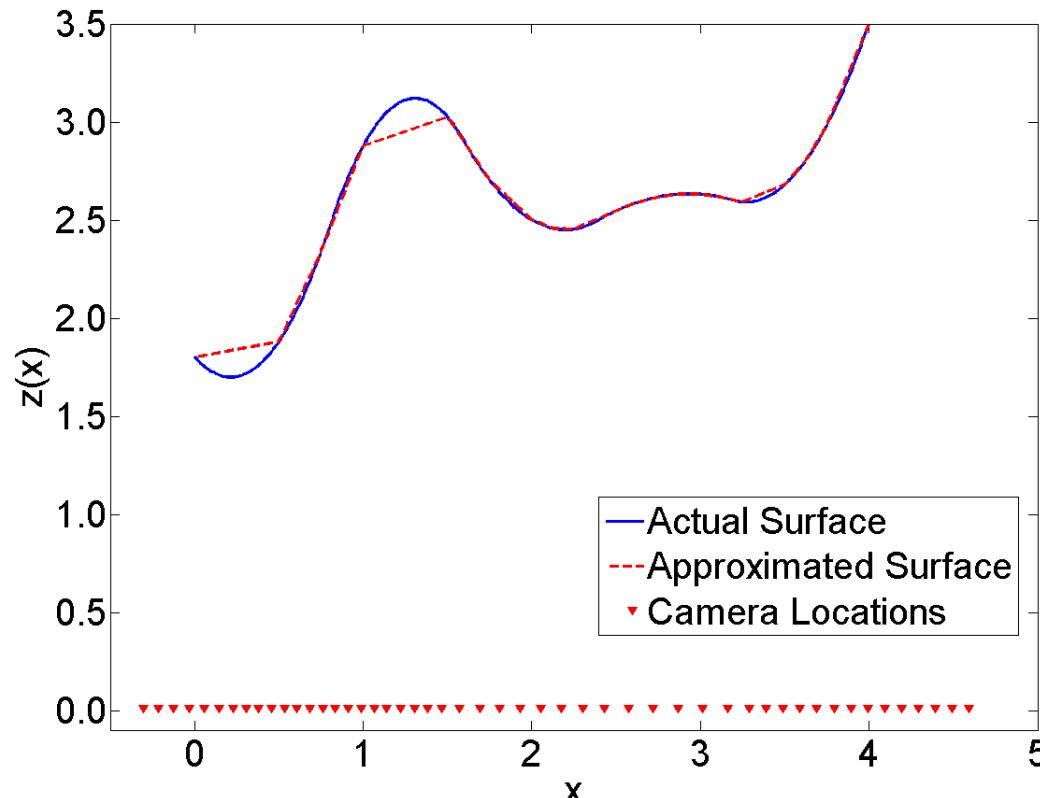
Spectral Analysis:

Close-form expression for the plenoptic spectrum of a slanted plane<sup>[5]</sup>

Spectrum's support  $\implies$  Maximum camera spacing,  $\Delta t_c$

# Sampling Realistic Scenes

Smoothly varying scene surface with bandlimited texture:

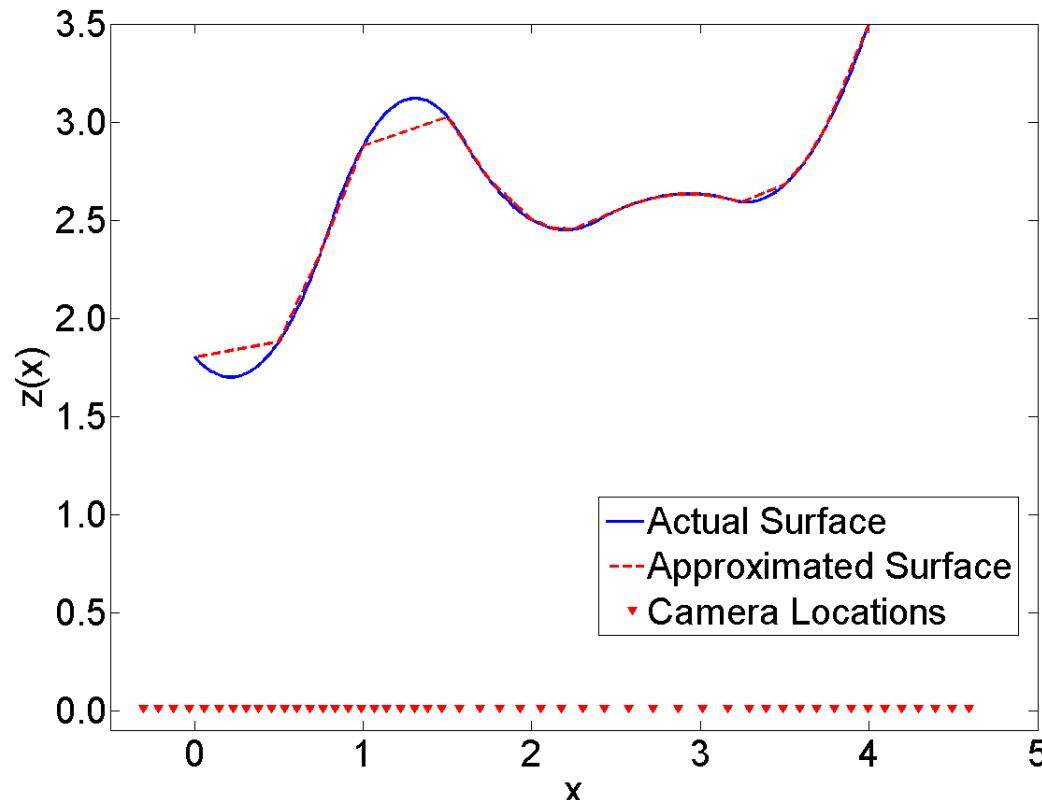


Adaptive Sampling Algorithm<sup>[6]</sup>:

- Slanted plane as a basis element  $\implies$  Construct more complicated scenes.
- Approximate the scene surface with a set of slanted planes.
- Non-uniformly sample the plenoptic function based on the approximation.

# Sampling Realistic Scenes

Smoothly varying scene surface with bandlimited texture:



Limitation:

Approximation of the surface requires knowledge of the full scene geometry

↗ Need to determine the scene geometry...

# Multi-View Depth Images

Use a similar concept to the 2D plenoptic function:

↪ Treat multi-view depth images as samples of a 2D function  $q(t, v)$

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$q(t, v) \implies$  inverse depth of the scene at camera location  $t$  and pixel position  $v$

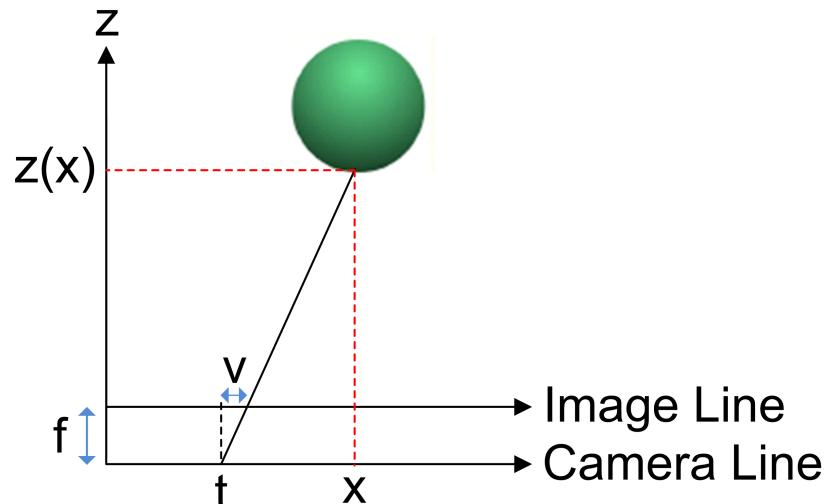
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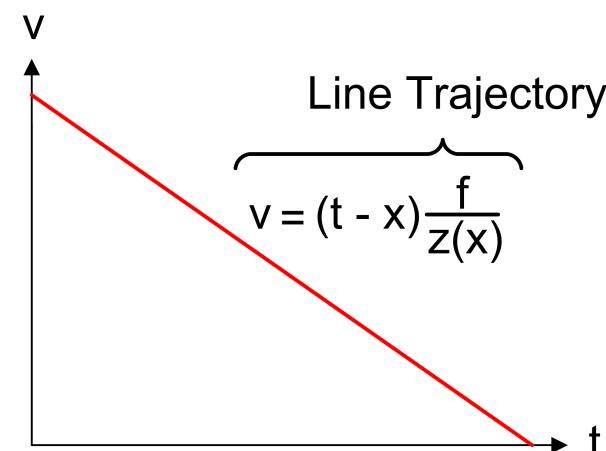
↪ Treat multi-view depth images as samples of a 2D function  $q(t, v)$

$q(t, v) \implies$  inverse depth of the scene at camera location  $t$  and pixel position  $v$

Similar to the Epipolar Plane Depth Image (EPDI)<sup>[7]</sup>:



(a) Scene



(b)  $q(t, v)$

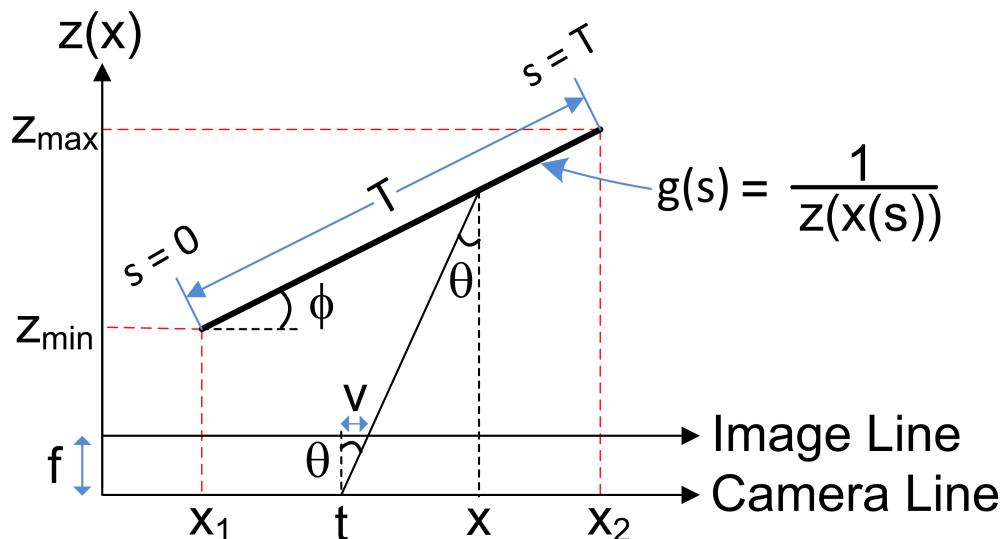
↪ Point in the scene,  $(x, z(x)) \implies$  Line in  $q(t, v)$

↪ Intensity along the line =  $1/z(x)$

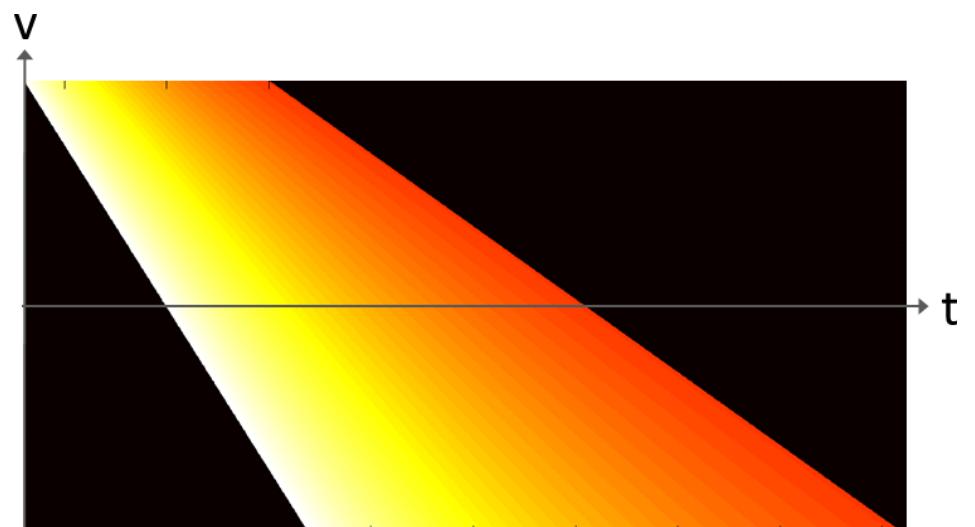
# The Pantelic Function for a Slanted Plane

We term  $q(t, v)$  the Pantelic Function:

Pantelic  $\implies$  A slight abuse of the Greek  $\pi\alpha\nu$  meaning *all* and  $\tau\eta\lambda\epsilon$  meaning *at distance*



(a) The Scene



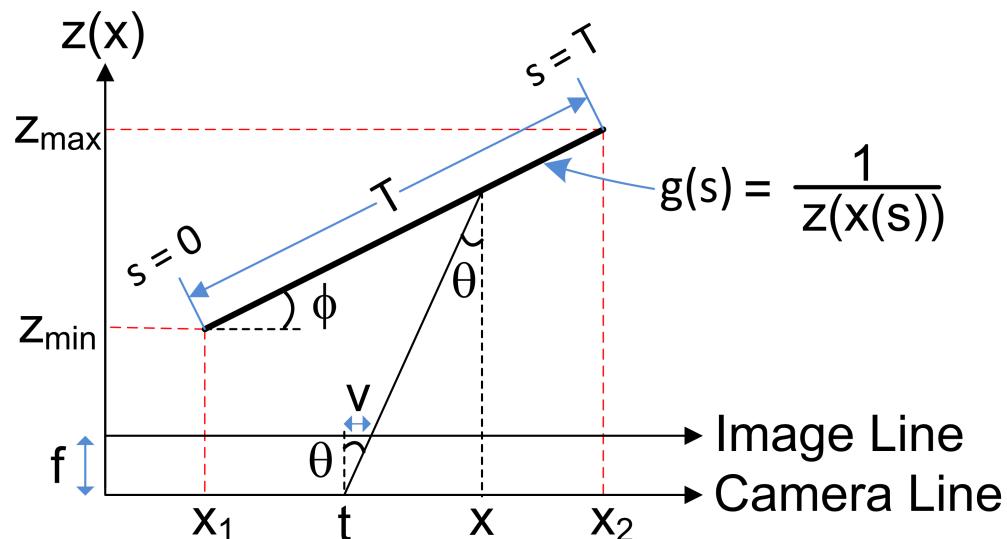
(b) Its Pantelic Function

$\hookrightarrow g(s)$  is a false texture equivalent to the inverse depth of the scene

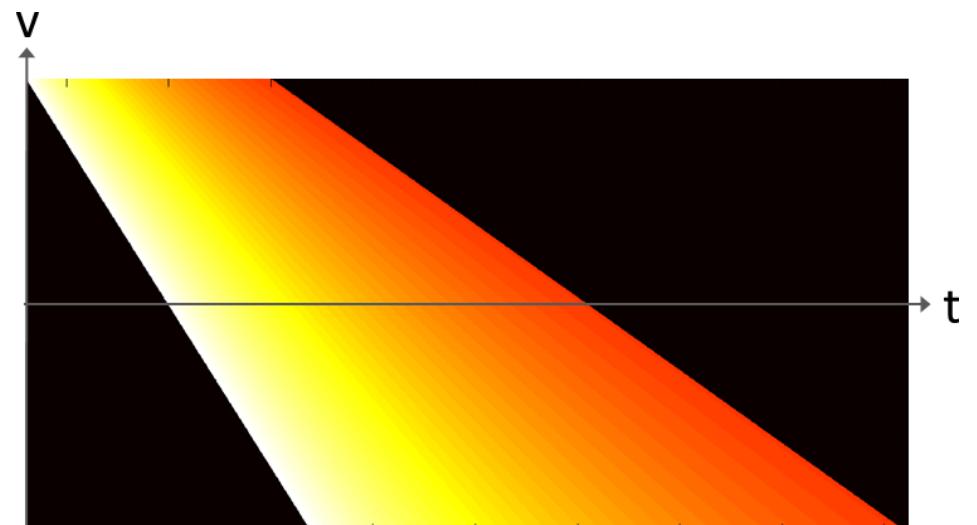
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(a) The Scene



(b) Its Pantelic Function

Therefore like the plenoptic function:

⊕ Determine number of depth cameras needed through spectral analysis

# Evaluating the Spectrum for a Slanted Plane

Fourier Transform of Pantelic Function:

$$\int_{t=-\infty}^{t=\infty} \int_{v=-\infty}^{v=\infty} q(t, v) e^{-j(\omega_t t + \omega_v v)} dv dt$$

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Changing from  $t$  to  $x$ :

$$\Rightarrow \int_{x=-\infty}^{x=\infty} \int_{v=-\infty}^{v=\infty} \hat{q}(x) \left[ 1 - v \frac{z'(x)}{f} \right] e^{-j(\omega_v - z(x) \frac{\omega_t}{f}) v} e^{-j\omega_t x} dv dx$$

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Finite Plane Width Constraint:

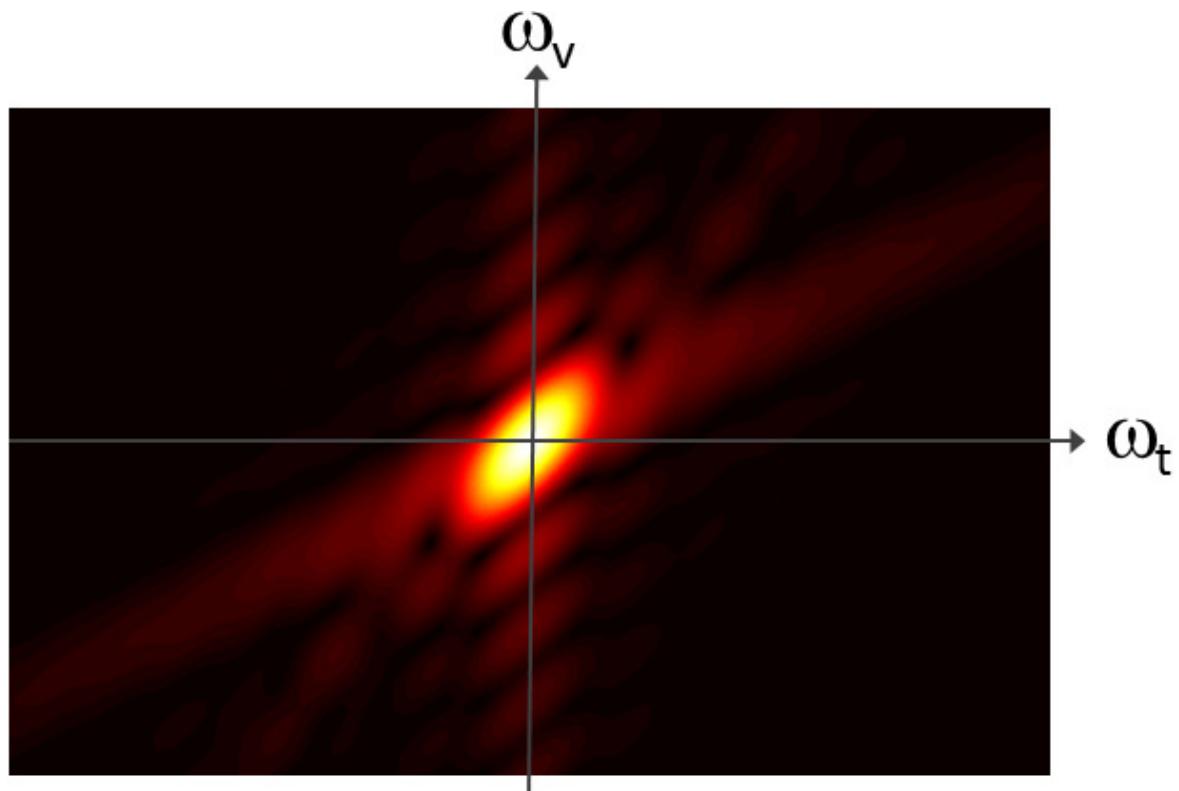
$$\Rightarrow \int_{s=0}^{s=T} g(s) \cos(\phi) \int_{v=-v_m}^{v=v_m} \left[ 1 - v \frac{\tan(\phi)}{f} \right] e^{-j(\omega_v - s \frac{\sin(\phi) \omega_t}{f}) v} e^{-j\omega_t \cos(\phi)s} dv ds$$

# The Result...

## The Spectrum

$$Q_S = j 2v_m \left( \frac{\text{sinc}(a)}{z_{\max}\omega_t} e^{-j(a-b)c} - \frac{\text{sinc}(b)}{z_{\min}\omega_t} \right) - j \frac{2v_m^2}{f} e^{jbc} \int_b^a \frac{\text{sinc}(\Omega)}{(\omega_v v_m - \Omega)^2} e^{-j\Omega c} d\Omega$$

Where  $a = \omega_v v_m - \omega_t \frac{z_{\max} v_m}{f}$  ,  $b = \omega_v v_m - \omega_t \frac{z_{\min} v_m}{f}$  ,  $c = \frac{-f}{\tan(\phi)v_m}$



### Structure:

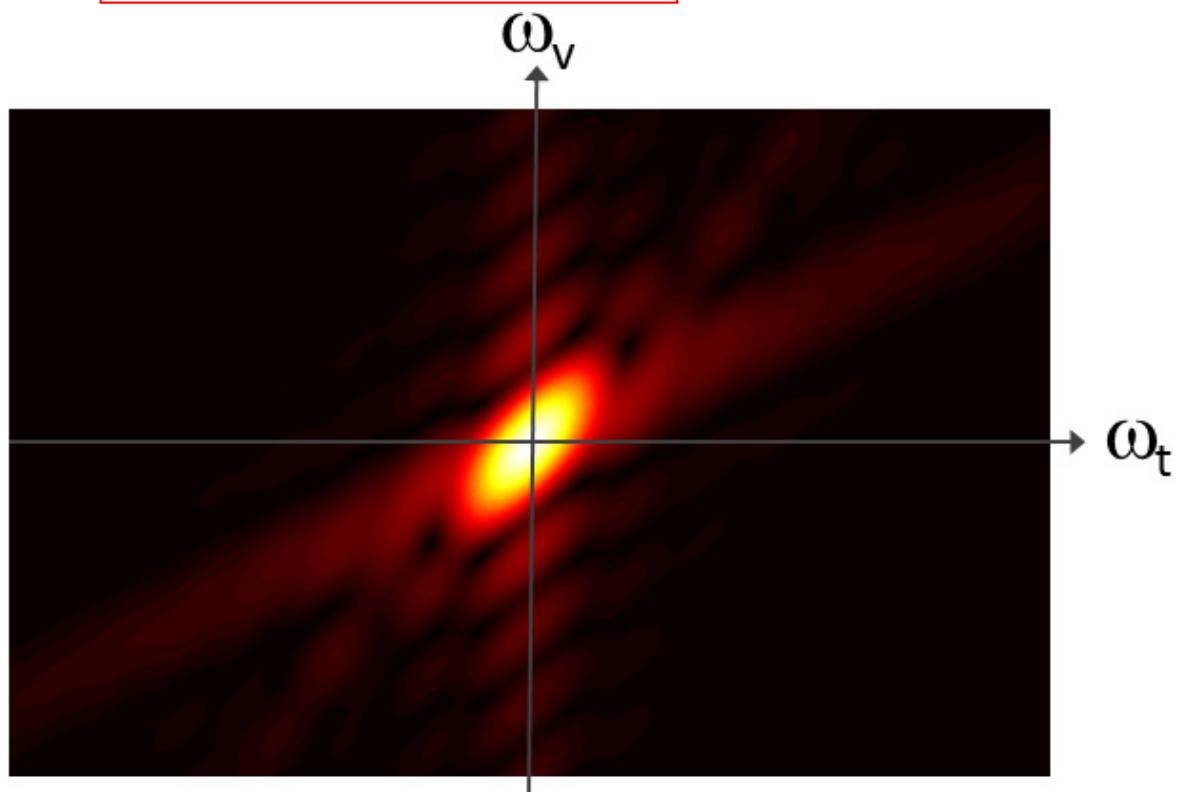
- Lines relating to  $z_{\min}$  and  $z_{\max}$
- Spectrum concentrated around the origin

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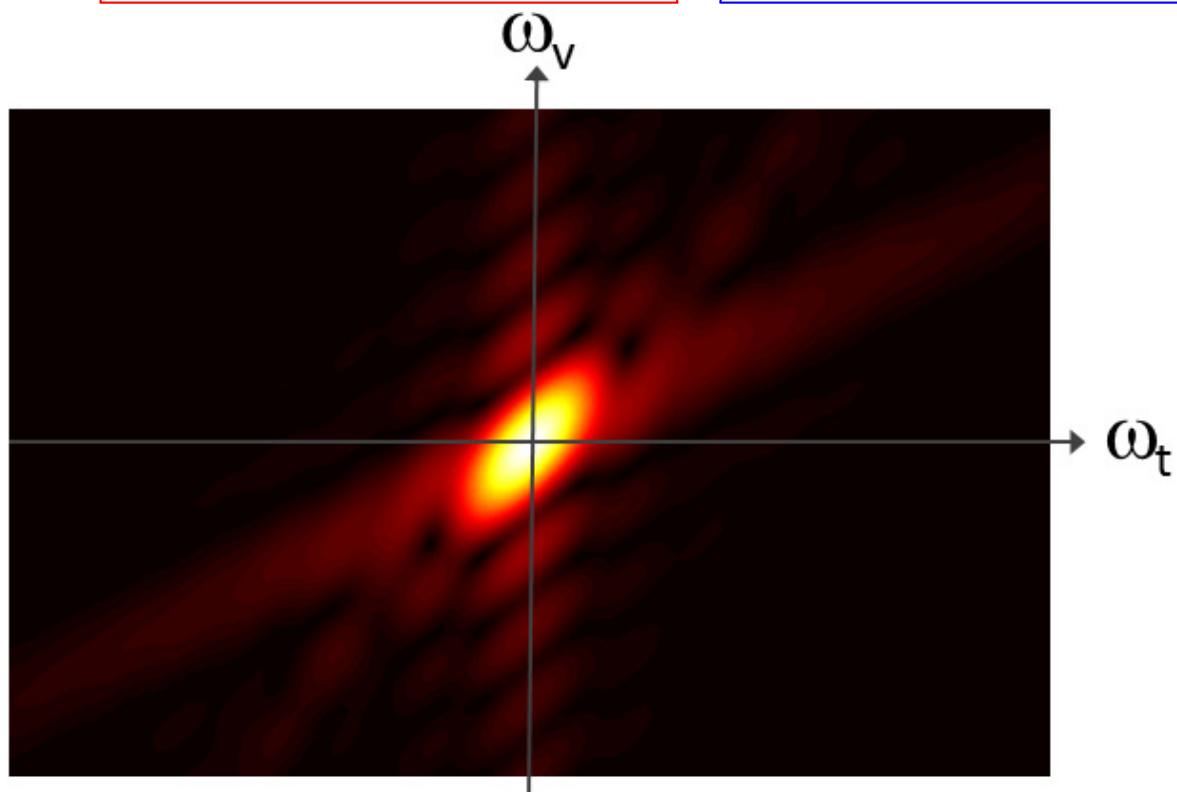
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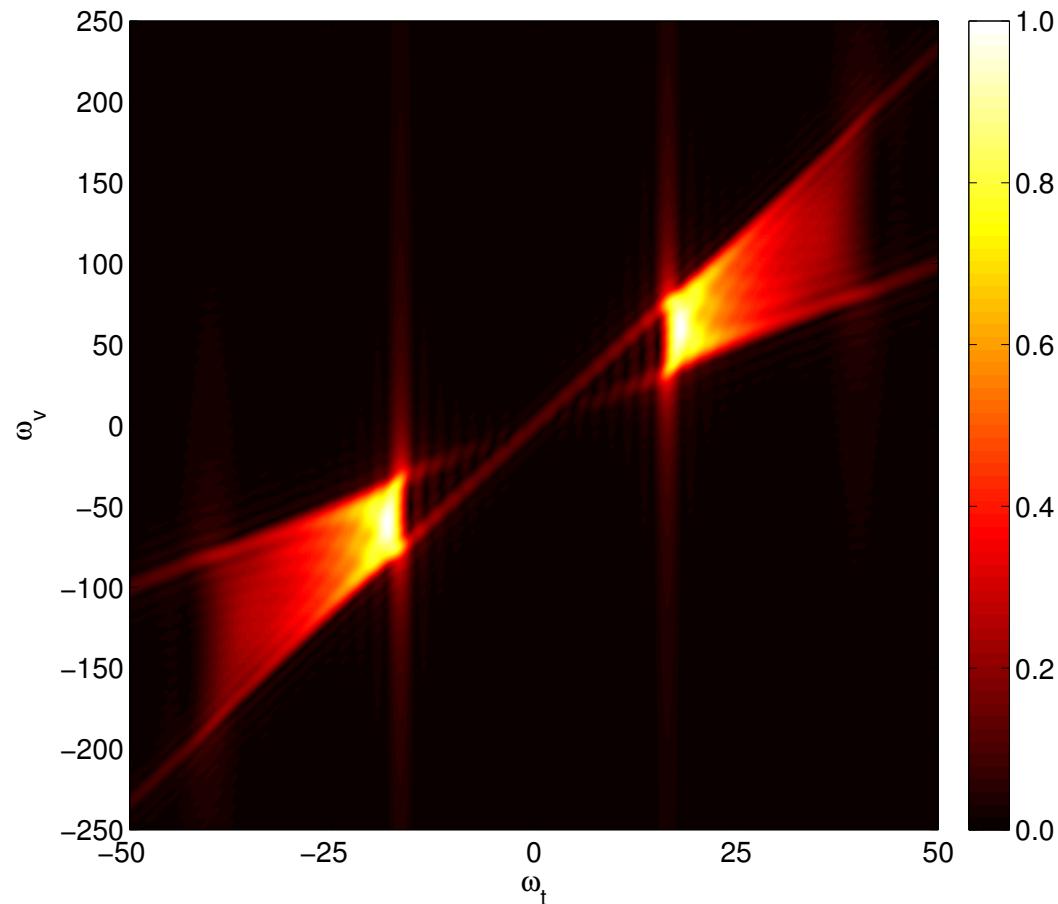


### Structure:

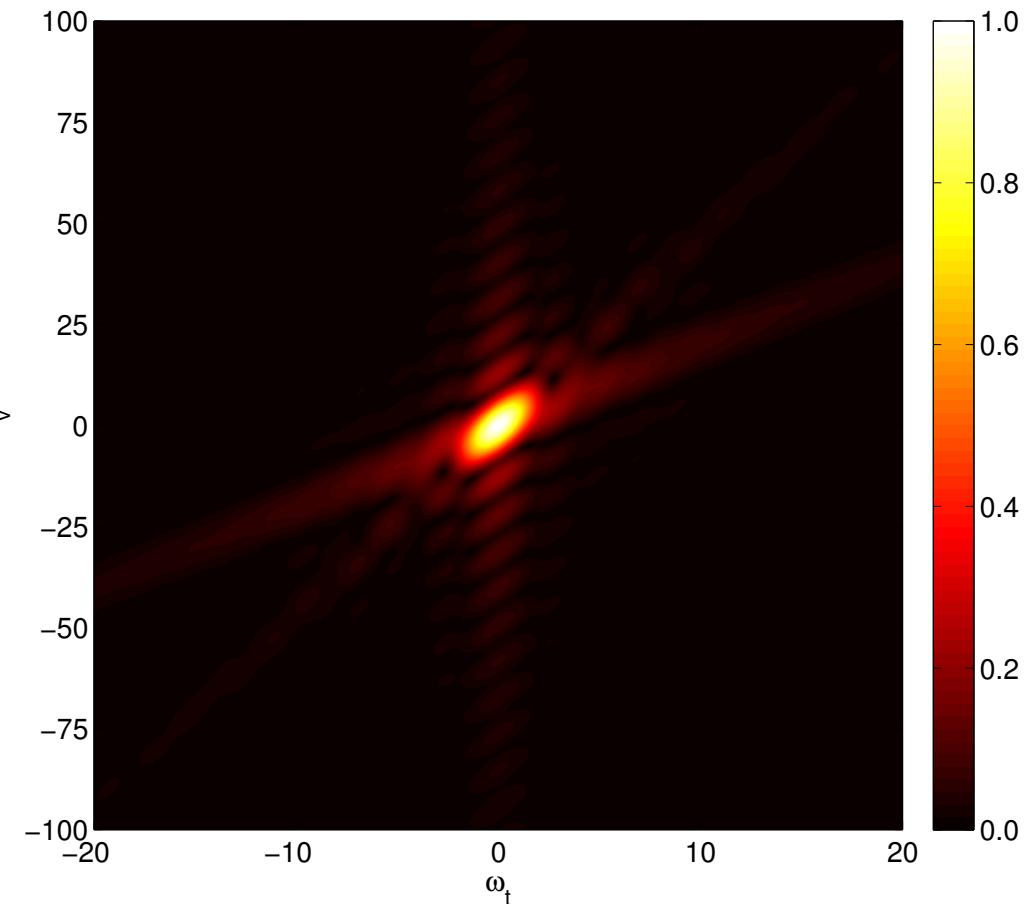
- Lines relating to  $z_{\min}$  and  $z_{\max}$
- Spectrum concentrated around the origin

# The Result...

Comparison between the Plenoptic and Pantelic Spectrum:

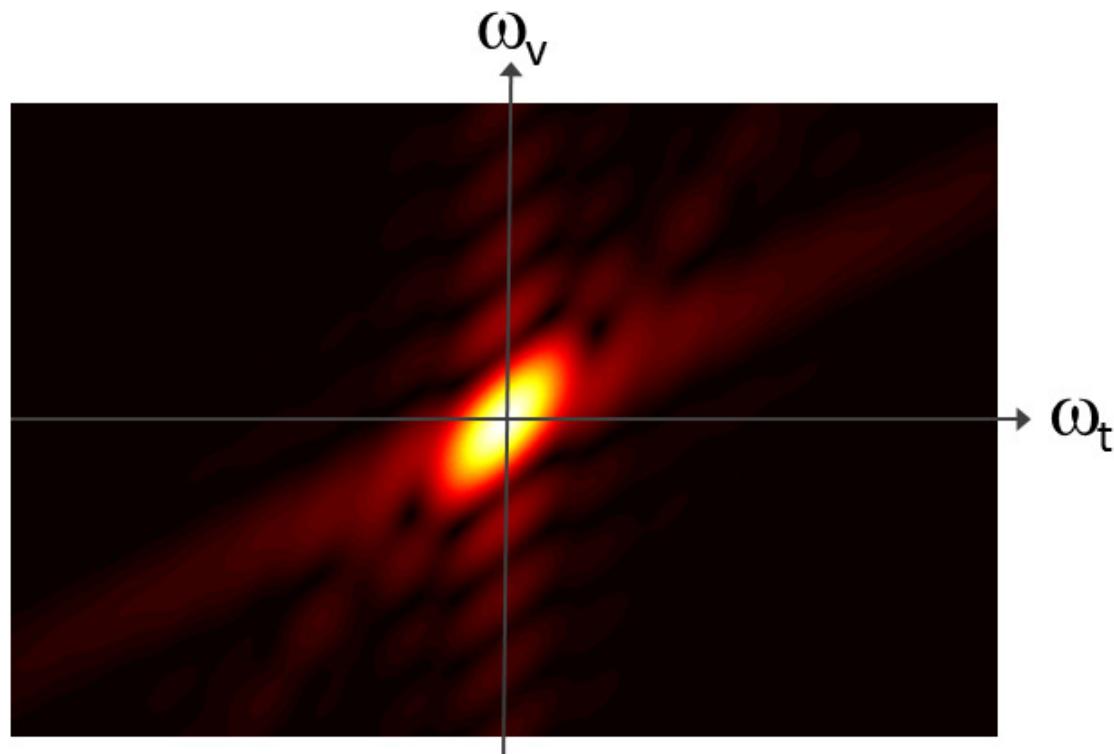


(a) Plenoptic Spectrum



(b) Pantelic Spectrum

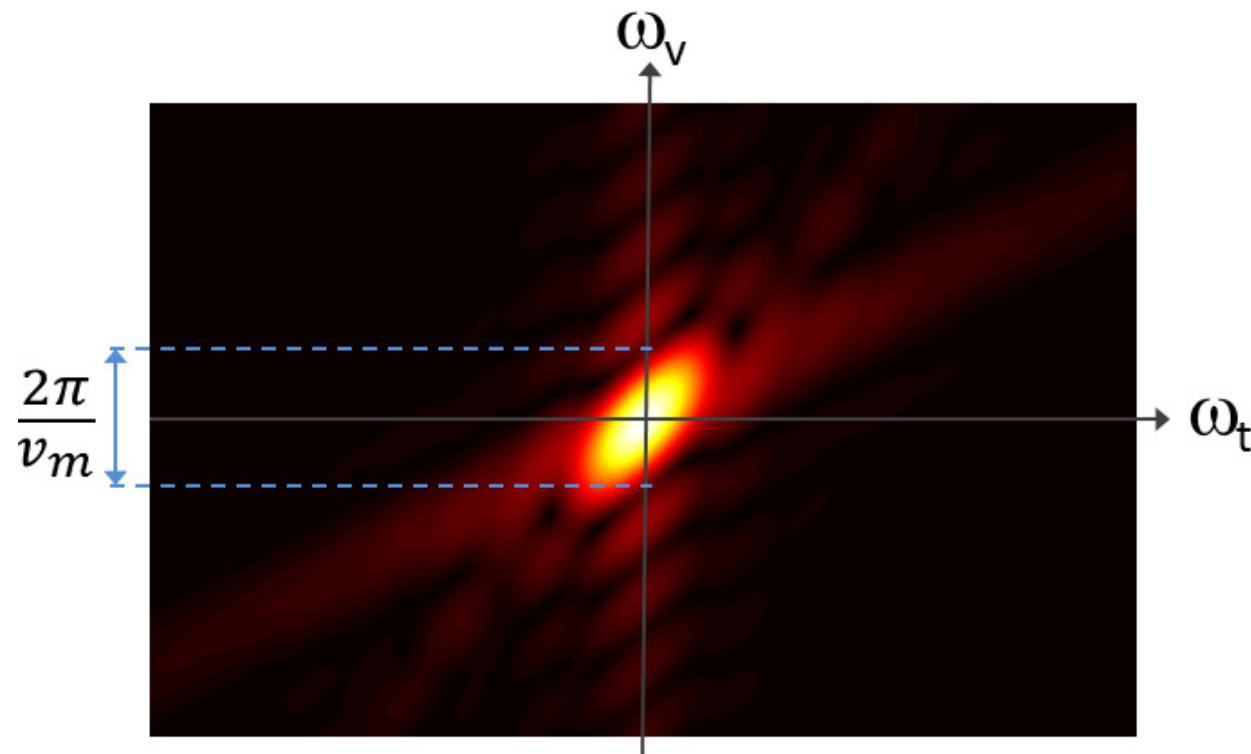
# Estimating the Essential Bandwidth



The spectrum along  $\omega_t = 0$ :

$$Q_S(0, \omega_v) = \frac{2v_m}{\tan(\phi)} \ln \left( \frac{z_{max}}{z_{min}} \right) \left[ \text{sinc} (\omega_v v_m) - j \frac{v_m \tan(\phi)}{f} \text{sinc}' (\omega_v v_m) \right]$$

# Estimating the Essential Bandwidth

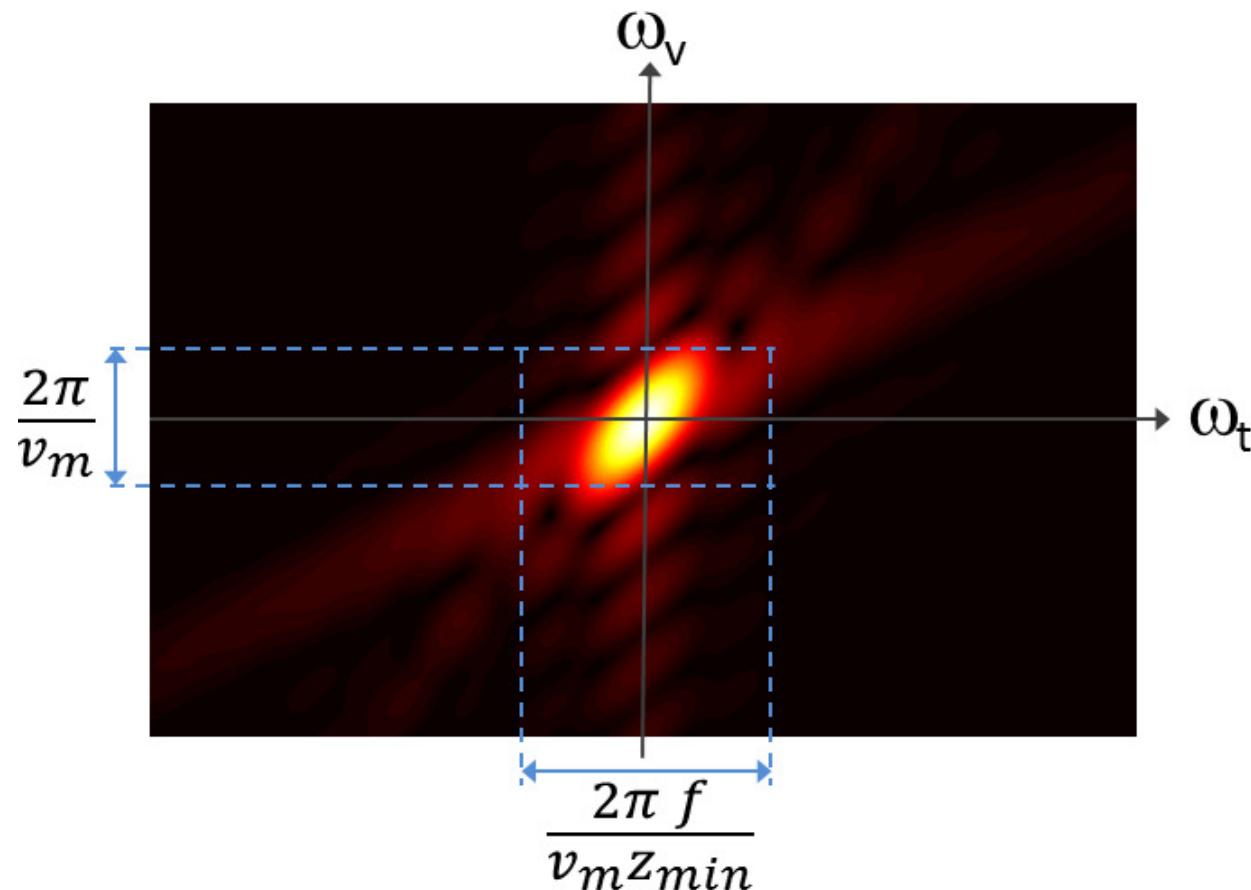


Estimate the essential bandwidth along  $\omega_t = 0$  as:

$$Q_S(0, \omega_v) \sim \text{sinc}(\omega_v v_m)$$

↗ EB of sinc function = width of main lobe,  $\frac{2\pi}{v_m}$

# Estimating the Essential Bandwidth



Project onto  $\omega_t$ -axis using:

$$\omega_v = \omega_t \frac{z_{min}}{f}$$

# Depth Cameras vs Colour Cameras

Maximum spacing between depth cameras for a slanted plane:

$$\Delta t_d = \frac{z_{min} v_m}{f}$$

Maximum spacing between colour cameras for a slanted plane:

$$\Delta t_c = \frac{2\pi z_{opt} v_m}{v_m \Omega_t T |\sin(\phi)| + 2\pi f}, \quad \text{where} \quad \Omega_t = \frac{\omega_s f}{f \cos(\phi) - v_m |\sin(\phi)|} + \frac{2\pi}{T},$$

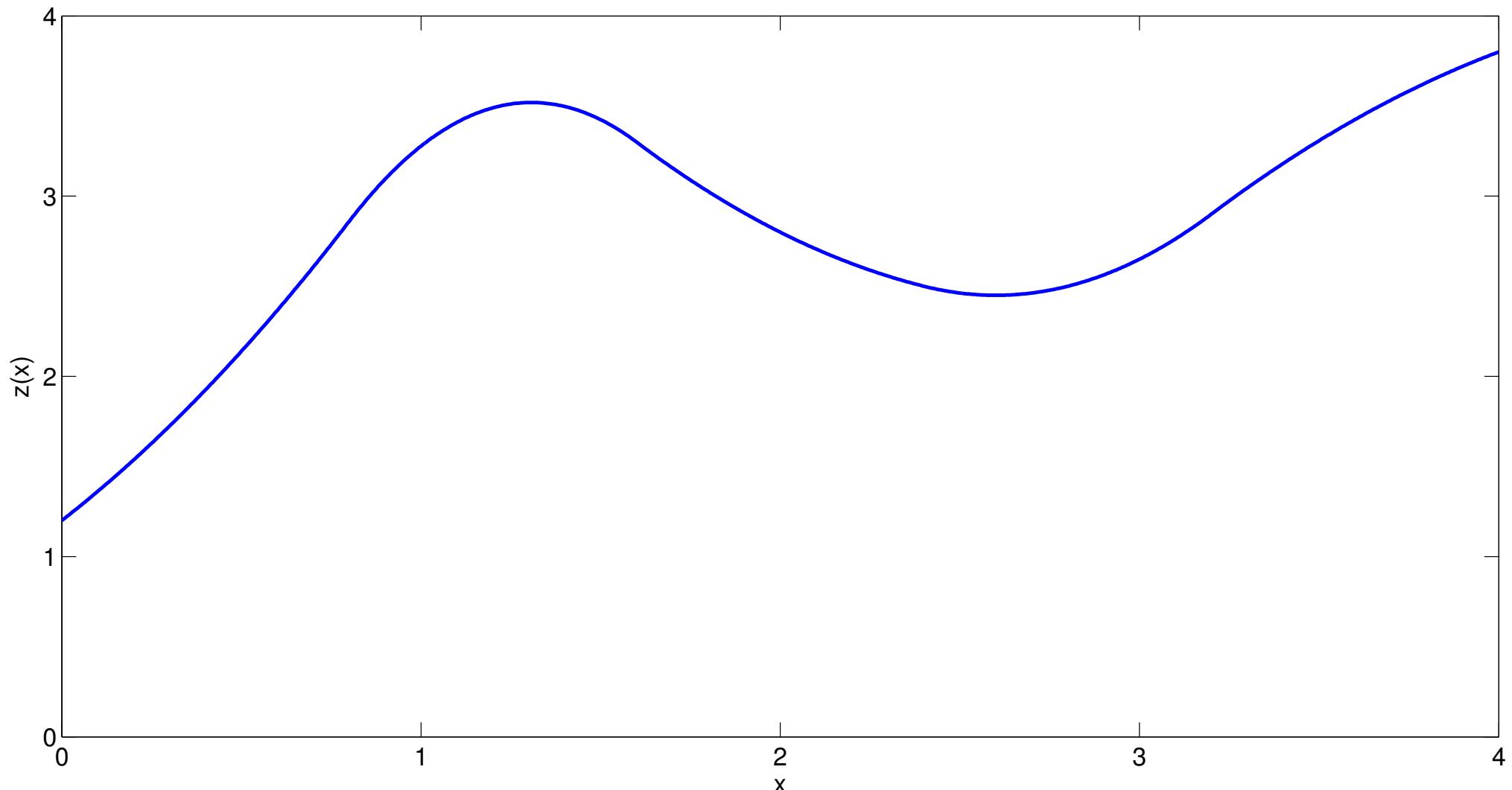
and  $z_{opt}$  is the average depth of the scene.

Observations:

- A point in the scene will be seen by at least two depth cameras
- Assuming  $\omega_s > 1 \text{ rad/m}$   $\implies$  Fewer depth cameras required than color cameras
- If plane fronto-parallel ( $\phi = 0$ )  $\implies \Delta t_c = \Delta t_d$

# Results: Synthetic Scene

Sample and reconstruct plenoptic function  $\implies$  Adaptive sampling algorithm<sup>[6]</sup>

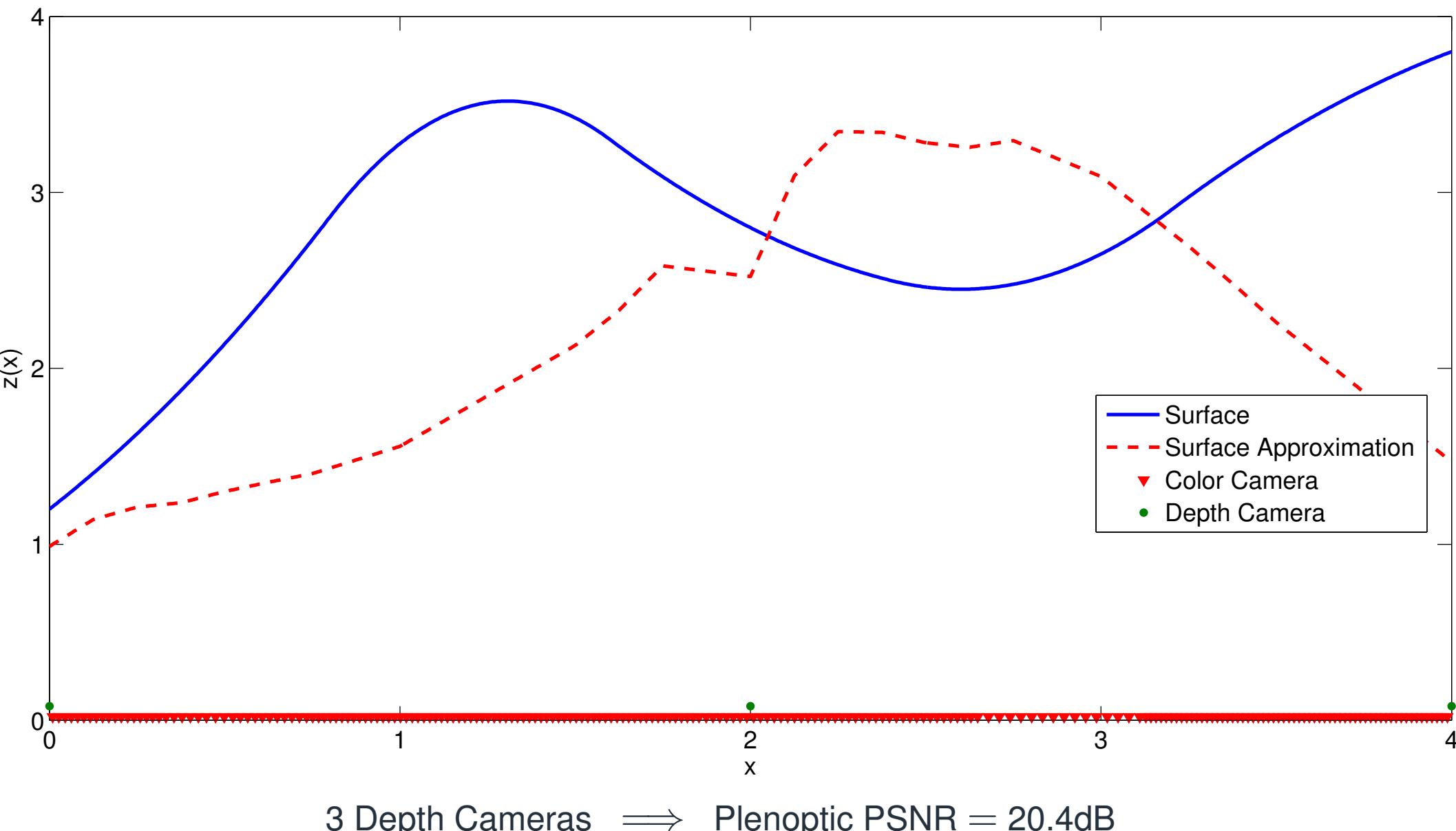


Piecewise quadratic surface with sinusoidal texture

↗ Estimate scene geometry from the Pantelic function

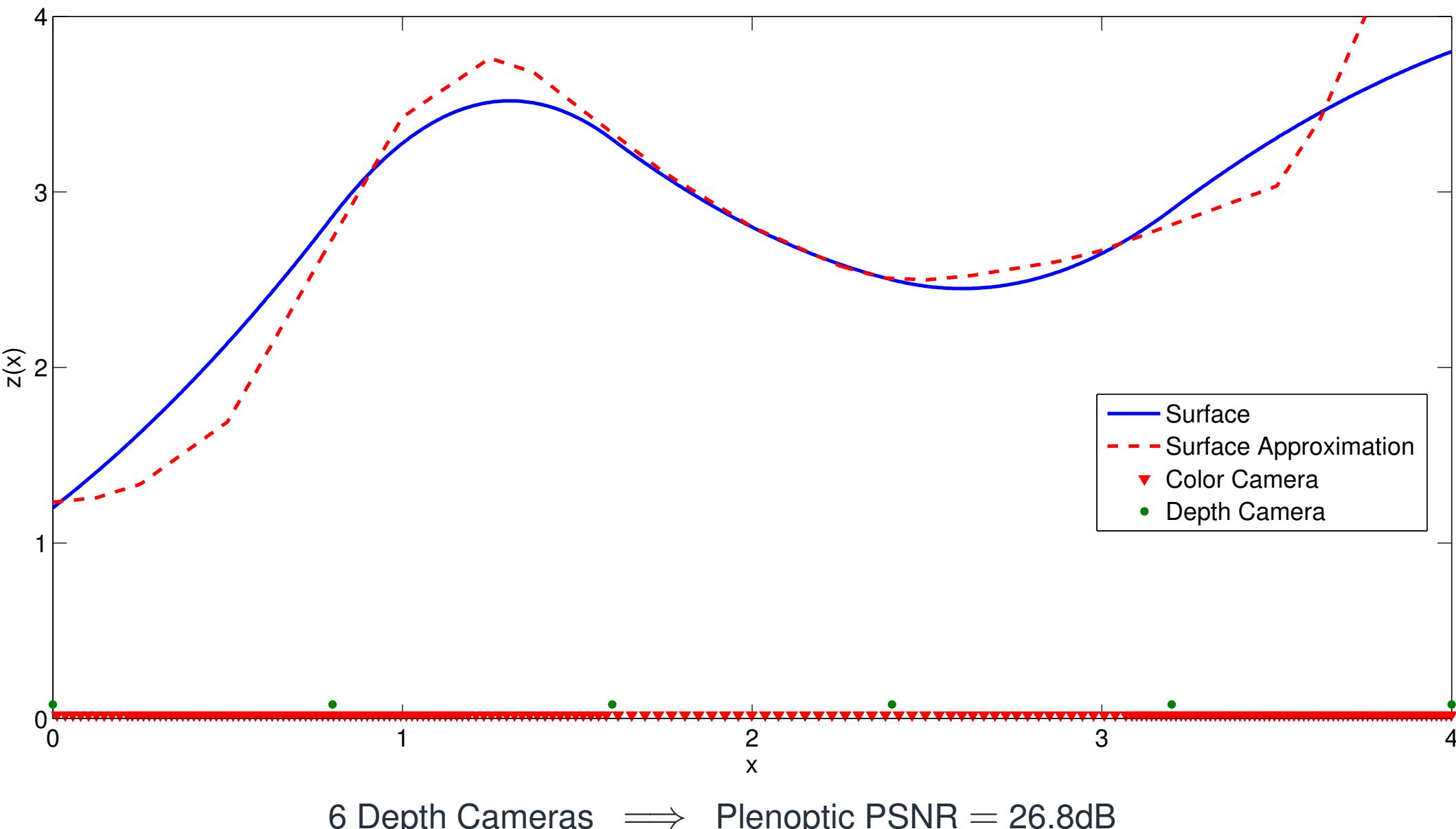
# Results: Synthetic Scene

Analysing the position of the 250 color cameras:



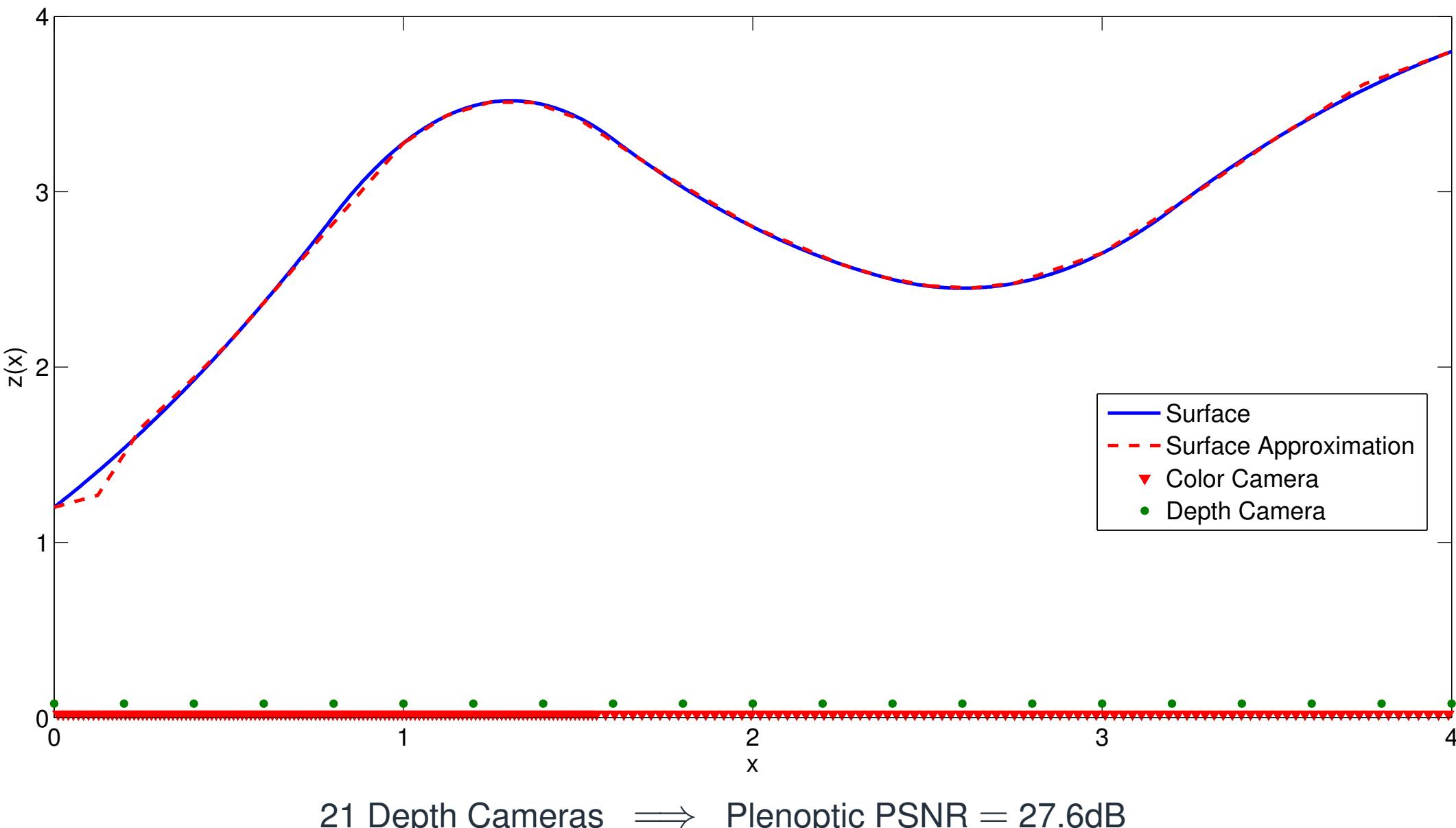
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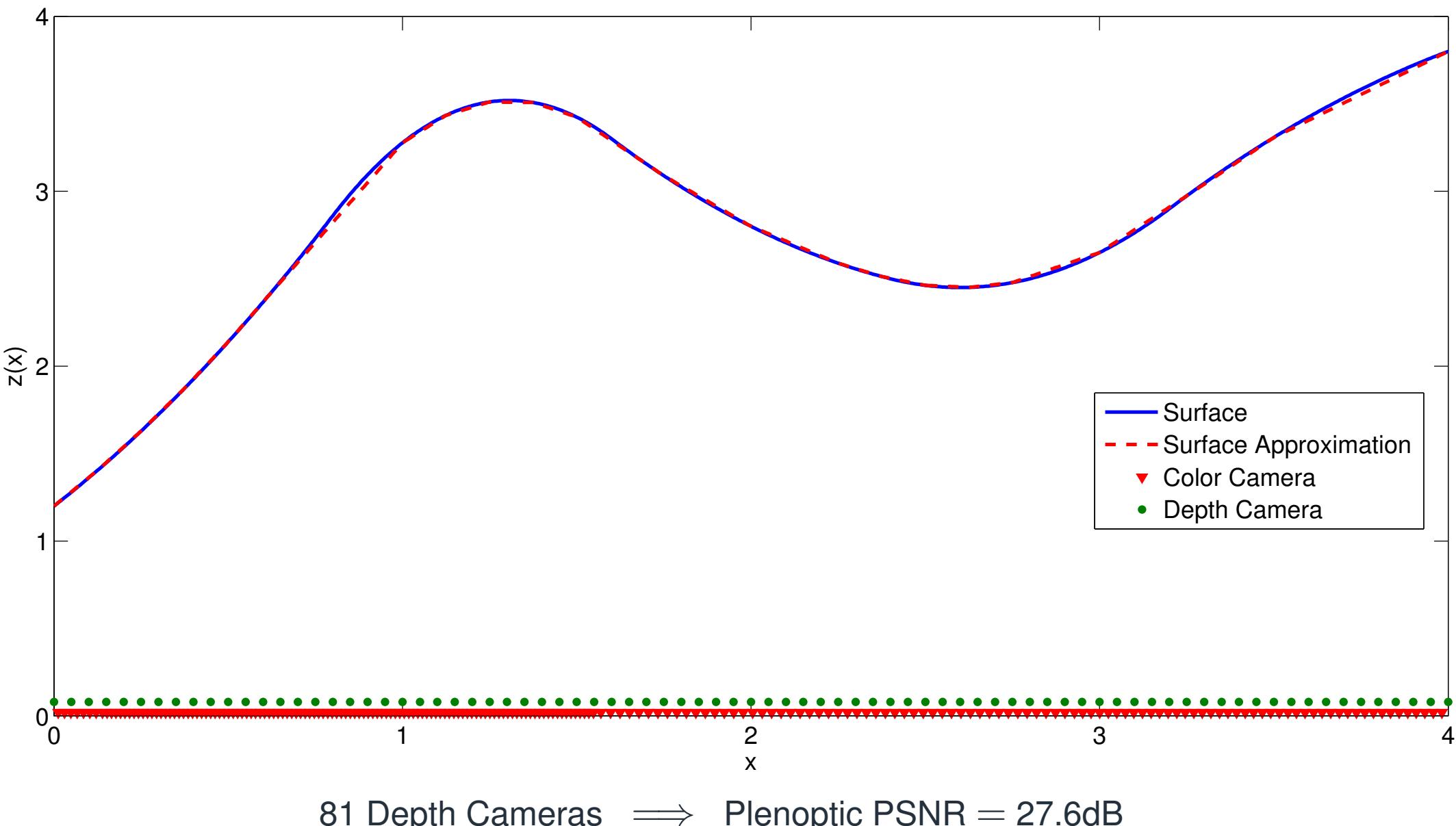
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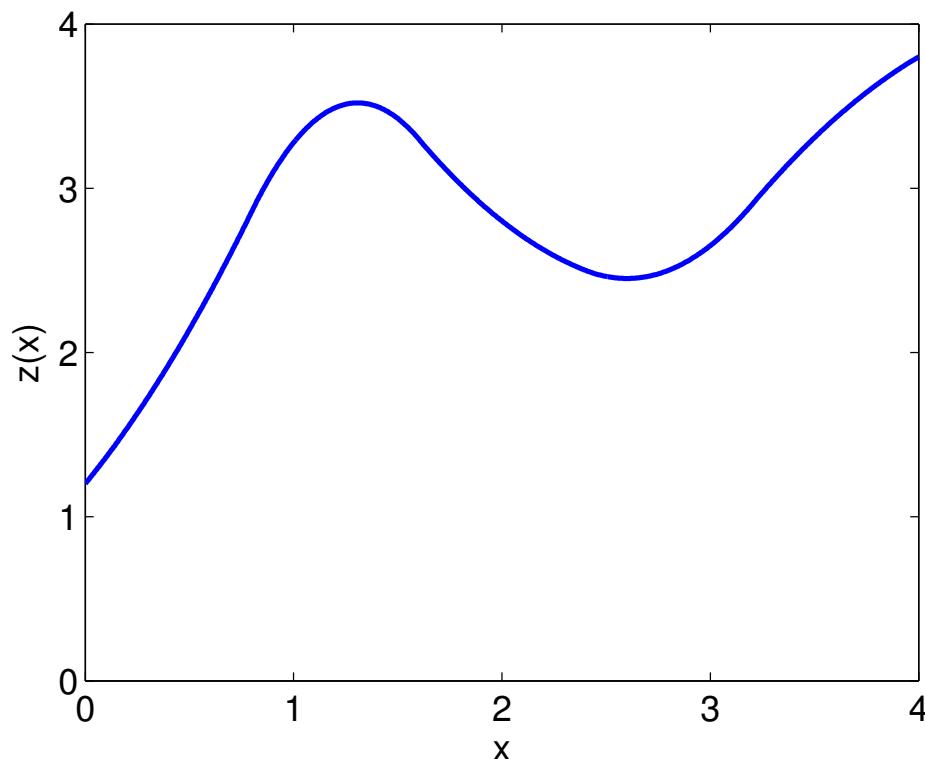
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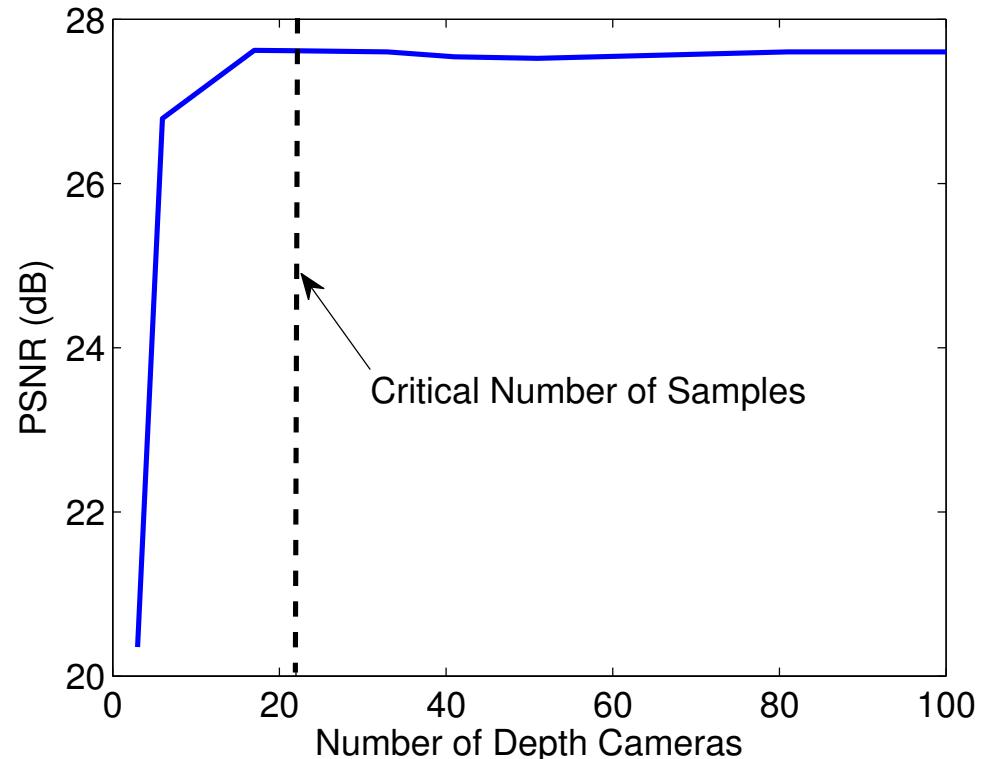
# Results: Synthetic Scene

Approximating the number of depth cameras with

$$\Delta t_d = \frac{z_{min} v_m}{f}$$

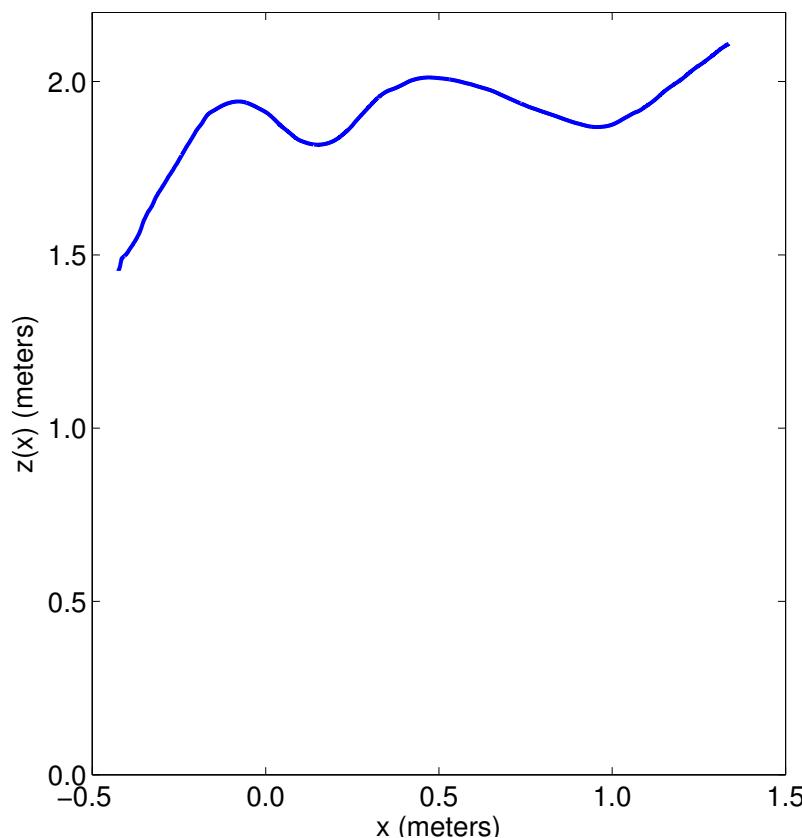


(a) Piecewise quadratic surface with sinusoidal texture



(b) PSNR of the reconstructed Plenoptic Function

# Results: Real Scene



(a) Scene Geometry



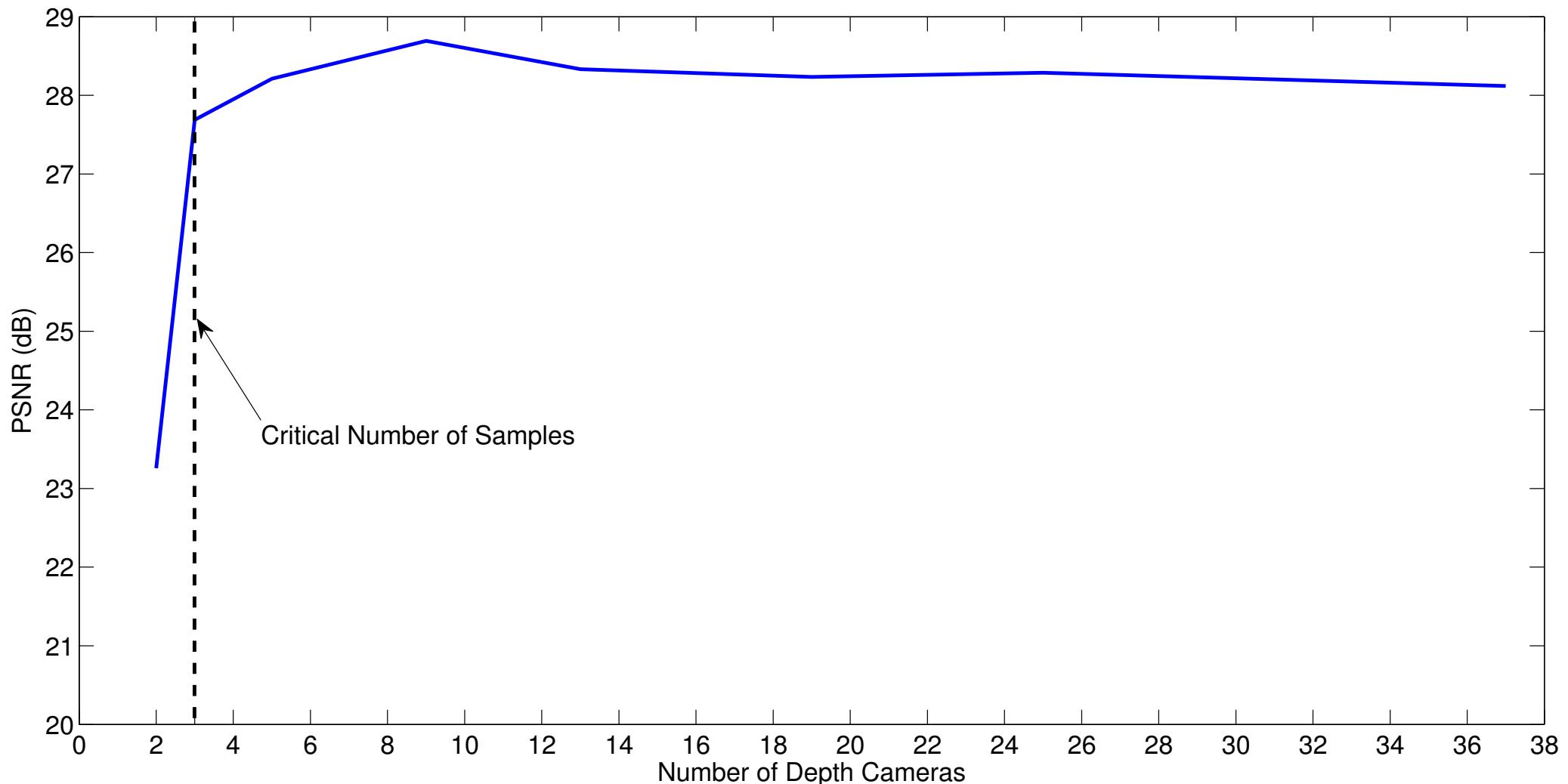
(b) Acquiring the Data

↗ Initial Image Set = 73 Color and Depth Images (1cm apart)

➡ Color Images 3008 by 2000 pixels      ➡ Depth Images 374 by 248 pixels

# Results: Real Scene

PSNR of Plenoptic Function as the Number of Depth Cameras Increases:



↗ Number of Color Images = 37

# Conclusions

- Presented a framework, similar to the plenoptic function, for multi-view depth images.
- Extended the spectral analysis of a slanted plane to multi-view depth images, assuming finite field of view and scene width.
- Maximum depth camera spacing for a slanted plane depends on the minimum depth and the camera characteristics.
- Empirical results, for synthetic and real scenes, show that fewer depth images are required than colour images for IBR view synthesis.

# References

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