Solutions to Chapter 10 SULT Problems

1. $A_{\overline{43:53}} = A_{43} + A_{53} - A_{43:53} = 0.13859 + 0.21582 - 0.24039 = \boxed{0.11402}$

2. ${}_8q_{\overline{38:52}} = {}_8q_{38} \cdot {}_8q_{52} = 0.00478 \cdot 0.01721 = \boxed{0.00008}$

3. $\ddot{a}_{47|47} = \ddot{a}_{47} - \ddot{a}_{47:47} = 17.5189 - 16.4374 = \boxed{1.0815}$

4. $\ddot{a}_{58|68} = \ddot{a}_{68} - \ddot{a}_{58:68} = 12.6456 - 11.8845 = \boxed{0.7611}$

5. $A_{\frac{1}{34:34}\backslash\overline{15}|} = A_{34:34} - {}_{15}E_{34:34} \cdot A_{49:49} = 0.1235 - 0.47228 \cdot 0.23652 = \boxed{0.0118}$, where
   ${}_{15}E_{34:34} = {}_{15}p_{34:34} \cdot v^{15} = 0.98182 \cdot 0.48102 = 0.47228$, where
   ${}_{15}p_{34:34} = \dfrac{\ell_{49}}{\ell_{34}} \cdot \dfrac{\ell_{49}}{\ell_{34}} = \dfrac{98684.9}{99593.8} \cdot \dfrac{98684.9}{99593.8} = 0.98182$

6. ${}_{17}E_{53:40} = {}_{17}p_{53:40} \cdot v^{17} = 0.90992 \cdot 0.4363 = \boxed{0.397}$, where
   ${}_{17}p_{53:40} = \dfrac{\ell_{70}}{\ell_{53}} \cdot \dfrac{\ell_{57}}{\ell_{40}} = \dfrac{91082.4}{98181.8} \cdot \dfrac{97435.2}{99338.3} = 0.90992$

7. ${}_9p_{\overline{57:66}} = {}_9p_{57} + {}_9p_{66} - {}_9p_{57:66} = 0.96495 + 0.90622 - 0.87446 = \boxed{0.99671}$, where
   ${}_9p_{57:66} = \dfrac{\ell_{66}}{\ell_{57}} \cdot \dfrac{\ell_{75}}{\ell_{66}} = \dfrac{94020.3}{97435.2} \cdot \dfrac{85203.5}{94020.3} = 0.87446$

8. ${}_8E_{61:41} = {}_8p_{61:41} \cdot v^8 = 0.94886 \cdot 0.67684 = \boxed{0.64223}$, where
   ${}_8p_{61:41} = \dfrac{\ell_{69}}{\ell_{61}} \cdot \dfrac{\ell_{49}}{\ell_{41}} = \dfrac{91936.9}{96305.8} \cdot \dfrac{98684.9}{99285.9} = 0.94886$

9. ${}_{10}p_{65:65} = \dfrac{\ell_{75}}{\ell_{65}} \cdot \dfrac{\ell_{75}}{\ell_{65}} = \dfrac{85203.5}{94579.7} \cdot \dfrac{85203.5}{94579.7} = \boxed{0.81155}$

10. ${}_7p_{64:64} = \dfrac{\ell_{71}}{\ell_{64}} \cdot \dfrac{\ell_{71}}{\ell_{64}} = \dfrac{90134}{95082.5} \cdot \dfrac{90134}{95082.5} = \boxed{0.89863}$

11. $A_{\overline{34:34}} = A_{34} + A_{34} - A_{34:34} = 0.09226 + 0.09226 - 0.1235 = \boxed{0.06102}$

12. $\ddot{a}_{39|49} = \ddot{a}_{49} - \ddot{a}_{39:49} = 17.196 - 16.7423 = \boxed{0.4537}$

13. ${}_{13}q_{52:47} = 1 - {}_{13}p_{52:47} = 1 - 0.9401 = \boxed{0.0599}$, where
   ${}_{13}p_{52:47} = \dfrac{\ell_{65}}{\ell_{52}} \cdot \dfrac{\ell_{60}}{\ell_{47}} = \dfrac{94579.7}{98326.2} \cdot \dfrac{96634.1}{98874.5} = 0.9401$

14. $\ddot{a}_{56:56:\overline{20}|} = \ddot{a}_{56:56:\overline{10}|} + {}_{10}E_{56:56} \cdot \ddot{a}_{66:66:\overline{10}|} = 7.9125 + 0.56911 \cdot 7.5351 = \boxed{12.2008}$, where
   ${}_{10}E_{56:56} = {}_{10}p_{56:56} \cdot v^{10} = 0.92702 \cdot 0.61391 = 0.56911$, where

$${}_{10}p_{56:56} = \frac{\ell_{66}}{\ell_{56}} \cdot \frac{\ell_{66}}{\ell_{56}} = \frac{94020.3}{97651.2} \cdot \frac{94020.3}{97651.2} = 0.92702$$

15. ${}_{6}p_{\overline{59:67}} = {}_{6}p_{59} + {}_{6}p_{67} - {}_{6}p_{59:67} = 0.97576 + 0.94131 - 0.91849 = \boxed{0.99858}$, where
${}_{6}p_{59:67} = \frac{\ell_{65}}{\ell_{59}} \cdot \frac{\ell_{73}}{\ell_{67}} = \frac{94579.7}{96929.6} \cdot \frac{87916.8}{93398.1} = 0.91849$

16. ${}_{20}q_{\overline{34:54}} = {}_{20}q_{34} \cdot {}_{20}q_{54} = 0.01578 \cdot 0.11625 = \boxed{0.00183}$

17. ${}_{5}E_{43:63} = {}_{5}p_{43:63} \cdot v^5 = 0.96662 \cdot 0.78353 = \boxed{0.75738}$, where
${}_{5}p_{43:63} = \frac{\ell_{48}}{\ell_{43}} \cdot \frac{\ell_{68}}{\ell_{63}} = \frac{98783.9}{99169.4} \cdot \frac{92706.1}{95534.4} = 0.96662$

18. $A_{\overline{63:63}} = A_{63} + A_{63} - A_{63:63} = 0.32785 + 0.32785 - 0.41288 = \boxed{0.24282}$

19. ${}_{27}q_{\overline{37:51}} = {}_{27}q_{37} \cdot {}_{27}q_{51} = 0.04417 \cdot 0.1874 = \boxed{0.00828}$

20. ${}_{28}E_{\overline{40:49}} = {}_{28}p_{\overline{40:49}} \cdot v^{28} = 0.98865 \cdot 0.25509 = \boxed{0.25219}$, where
${}_{28}p_{\overline{40:49}} = {}_{28}p_{40} + {}_{28}p_{49} - {}_{28}p_{40:49} = 0.93324 + 0.82996 - 0.77455 = 0.98865$, where
${}_{28}p_{40:49} = \frac{\ell_{68}}{\ell_{40}} \cdot \frac{\ell_{77}}{\ell_{49}} = \frac{92706.1}{99338.3} \cdot \frac{81904.3}{98684.9} = 0.77455$

21. ${}_{22}p_{31:35} = \frac{\ell_{53}}{\ell_{31}} \cdot \frac{\ell_{57}}{\ell_{35}} = \frac{98181.8}{99695.8} \cdot \frac{97435.2}{99556.7} = \boxed{0.96382}$

22. $\ddot{a}_{68:78:\overline{20}|} = \ddot{a}_{68:78:\overline{10}|} + {}_{10}E_{68:78} \cdot \ddot{a}_{78:88:\overline{10}|} = 6.7412 + 0.33136 \cdot 4.8588 = \boxed{8.35121}$, where
${}_{10}E_{68:78} = {}_{10}p_{68:78} \cdot v^{10} = 0.53975 \cdot 0.61391 = 0.33136$, where
${}_{10}p_{68:78} = \frac{\ell_{78}}{\ell_{68}} \cdot \frac{\ell_{88}}{\ell_{78}} = \frac{80006.2}{92706.1} \cdot \frac{50038.6}{80006.2} = 0.53975$

23. ${}_{6}E_{\overline{47:59}} = {}_{6}p_{\overline{47:59}} \cdot v^6 = 0.99983 \cdot 0.74622 = \boxed{0.74609}$, where
${}_{6}p_{\overline{47:59}} = {}_{6}p_{47} + {}_{6}p_{59} - {}_{6}p_{47:59} = 0.99299 + 0.97576 - 0.96892 = 0.99983$, where
${}_{6}p_{47:59} = \frac{\ell_{53}}{\ell_{47}} \cdot \frac{\ell_{65}}{\ell_{59}} = \frac{98181.8}{98874.5} \cdot \frac{94579.7}{96929.6} = 0.96892$

24. ${}_{8}q_{\overline{59:50}} = {}_{8}q_{59} \cdot {}_{8}q_{50} = 0.03643 \cdot 0.01401 = \boxed{0.00051}$

25. $A_{\frac{1}{\overline{39:39}\setminus\overline{8}|}} = A_{39:39} - {}_{8}E_{39:39} \cdot A_{47:47} = 0.1537 - 0.66988 \cdot 0.21727 = \boxed{0.00816}$, where
${}_{8}E_{39:39} = {}_{8}p_{39:39} \cdot v^8 = 0.98971 \cdot 0.67684 = 0.66988$, where
${}_{8}p_{39:39} = \frac{\ell_{47}}{\ell_{39}} \cdot \frac{\ell_{47}}{\ell_{39}} = \frac{98874.5}{99387.3} \cdot \frac{98874.5}{99387.3} = 0.98971$

26. ${}_{45}E_{35:34} = {}_{45}p_{35:34} \cdot v^{45} = 0.59462 \cdot 0.1113 = \boxed{0.06618}$, where
${}_{45}p_{35:34} = \frac{\ell_{80}}{\ell_{35}} \cdot \frac{\ell_{79}}{\ell_{34}} = \frac{75657.2}{99556.7} \cdot \frac{77927.4}{99593.8} = 0.59462$

27. ${}_{14}q_{62:40} = 1 - {}_{14}p_{62:40} = 1 - 0.86017 = \boxed{0.13983}$, where
${}_{14}p_{62:40} = \dfrac{\ell_{76}}{\ell_{62}} \cdot \dfrac{\ell_{54}}{\ell_{40}} = \dfrac{83632.9}{95940.6} \cdot \dfrac{98022.4}{99338.3} = 0.86017$

28. ${}_{18}p_{\overline{54:41}} = {}_{18}p_{54} + {}_{18}p_{41} - {}_{18}p_{54:41} = 0.90879 + 0.97627 - 0.88722 = \boxed{0.99784}$, where
${}_{18}p_{54:41} = \dfrac{\ell_{72}}{\ell_{54}} \cdot \dfrac{\ell_{59}}{\ell_{41}} = \dfrac{89082.1}{98022.4} \cdot \dfrac{96929.6}{99285.9} = 0.88722$

29. ${}_{18}p_{\overline{33:31}} = {}_{18}p_{33} + {}_{18}p_{31} - {}_{18}p_{33:31} = 0.98824 + 0.98986 - 0.97822 = \boxed{0.99988}$, where
${}_{18}p_{33:31} = \dfrac{\ell_{51}}{\ell_{33}} \cdot \dfrac{\ell_{49}}{\ell_{31}} = \dfrac{98457.2}{99629.3} \cdot \dfrac{98684.9}{99695.8} = 0.97822$

30. ${}_{16}q_{38:52} = 1 - {}_{16}p_{38:52} = 1 - 0.92946 = \boxed{0.07054}$, where
${}_{16}p_{38:52} = \dfrac{\ell_{54}}{\ell_{38}} \cdot \dfrac{\ell_{68}}{\ell_{52}} = \dfrac{98022.4}{99433.3} \cdot \dfrac{92706.1}{98326.2} = 0.92946$

31. ${}_{7}p_{57:55} = \dfrac{\ell_{64}}{\ell_{57}} \cdot \dfrac{\ell_{62}}{\ell_{55}} = \dfrac{95082.5}{97435.2} \cdot \dfrac{95940.6}{97846.2} = \boxed{0.95684}$

32. ${}_{38}E_{31:45} = {}_{38}p_{31:45} \cdot v^{38} = 0.6296 \cdot 0.15661 = \boxed{0.0986}$, where
${}_{38}p_{31:45} = \dfrac{\ell_{69}}{\ell_{31}} \cdot \dfrac{\ell_{83}}{\ell_{45}} = \dfrac{91936.9}{99695.8} \cdot \dfrac{67614.6}{99033.9} = 0.6296$

33. $\ddot{a}_{61:61:\overline{20}|} = \ddot{a}_{61:61:\overline{10}|} + {}_{10}E_{61:61} \cdot \ddot{a}_{71:71:\overline{10}|} = 7.7738 + 0.53774 \cdot 7.1371 = \boxed{11.6117}$, where
${}_{10}E_{61:61} = {}_{10}p_{61:61} \cdot v^{10} = 0.87593 \cdot 0.61391 = 0.53774$, where
${}_{10}p_{61:61} = \dfrac{\ell_{71}}{\ell_{61}} \cdot \dfrac{\ell_{71}}{\ell_{61}} = \dfrac{90134}{96305.8} \cdot \dfrac{90134}{96305.8} = 0.87593$

34. $\ddot{a}_{58|58} = \ddot{a}_{58} - \ddot{a}_{58:58} = 15.3901 - 13.8266 = \boxed{1.5635}$

35. ${}_{5}p_{69:56} = \dfrac{\ell_{74}}{\ell_{69}} \cdot \dfrac{\ell_{61}}{\ell_{56}} = \dfrac{86627.6}{91936.9} \cdot \dfrac{96305.8}{97651.2} = \boxed{0.92927}$

36. ${}_{12}q_{33:61} = 1 - {}_{12}p_{33:61} = 1 - 0.90743 = \boxed{0.09257}$, where
${}_{12}p_{33:61} = \dfrac{\ell_{45}}{\ell_{33}} \cdot \dfrac{\ell_{73}}{\ell_{61}} = \dfrac{99033.9}{99629.3} \cdot \dfrac{87916.8}{96305.8} = 0.90743$

37. ${}_{14}E_{\overline{54:52}} = {}_{14}p_{\overline{54:52}} \cdot v^{14} = 0.99762 \cdot 0.50507 = \boxed{0.50387}$, where
${}_{14}p_{\overline{54:52}} = {}_{14}p_{54} + {}_{14}p_{52} - {}_{14}p_{54:52} = 0.94576 + 0.95621 - 0.90435 = 0.99762$, where
${}_{14}p_{54:52} = \dfrac{\ell_{68}}{\ell_{54}} \cdot \dfrac{\ell_{66}}{\ell_{52}} = \dfrac{92706.1}{98022.4} \cdot \dfrac{94020.3}{98326.2} = 0.90435$

38. $A_{\frac{1}{44:54}:\overline{5}|} = A_{44:54} - {}_{5}E_{44:54} \cdot A_{49:59} = 0.2507 - 0.77152 \cdot 0.3079 = \boxed{0.01315}$, where
${}_{5}E_{44:54} = {}_{5}p_{44:54} \cdot v^{5} = 0.98467 \cdot 0.78353 = 0.77152$, where
${}_{5}p_{44:54} = \dfrac{\ell_{49}}{\ell_{44}} \cdot \dfrac{\ell_{59}}{\ell_{54}} = \dfrac{98684.9}{99104.3} \cdot \dfrac{96929.6}{98022.4} = 0.98467$

39. $A_{\overline{46:46}} = A_{46} + A_{46} - A_{46:46} = 0.15854 + 0.15854 - 0.20817 = \boxed{0.10891}$

40. ${}_5q_{37:61} = 1 - {}_5p_{37:61} = 1 - 0.97385 = \boxed{0.02615}$, where
${}_5p_{37:61} = \dfrac{\ell_{42}}{\ell_{37}} \cdot \dfrac{\ell_{66}}{\ell_{61}} = \dfrac{99229.8}{99476.7} \cdot \dfrac{94020.3}{96305.8} = 0.97385$

41. $\ddot{a}_{63:73:\overline{20}|} = \ddot{a}_{63:73:\overline{10}|} + {}_{10}E_{63:73} \cdot \ddot{a}_{73:83:\overline{10}|} = 7.2874 + 0.43449 \cdot 5.9276 = \boxed{9.86288}$, where
${}_{10}E_{63:73} = {}_{10}p_{63:73} \cdot v^{10} = 0.70774 \cdot 0.61391 = 0.43449$, where
${}_{10}p_{63:73} = \dfrac{\ell_{73}}{\ell_{63}} \cdot \dfrac{\ell_{83}}{\ell_{73}} = \dfrac{87916.8}{95534.4} \cdot \dfrac{67614.6}{87916.8} = 0.70774$

42. $A_{\overline{36:46}} = A_{36} + A_{46} - A_{36:46} = 0.10101 + 0.15854 - 0.17811 = \boxed{0.08144}$

43. $\ddot{a}_{40:40:\overline{20}|} = \ddot{a}_{40:40:\overline{10}|} + {}_{10}E_{40:40} \cdot \ddot{a}_{50:50:\overline{10}|} = 8.0649 + 0.60453 \cdot 8.0028 = \boxed{12.90283}$, where
${}_{10}E_{40:40} = {}_{10}p_{40:40} \cdot v^{10} = 0.98472 \cdot 0.61391 = 0.60453$, where
${}_{10}p_{40:40} = \dfrac{\ell_{50}}{\ell_{40}} \cdot \dfrac{\ell_{50}}{\ell_{40}} = \dfrac{98576.4}{99338.3} \cdot \dfrac{98576.4}{99338.3} = 0.98472$

44. $A_{\overline{\frac{1}{66:66}\backslash:9}|} = A_{66:66} - {}_9E_{66:66} \cdot A_{75:75} = 0.45947 - 0.52937 \cdot 0.60912 = \boxed{0.13702}$, where
${}_9E_{66:66} = {}_9p_{66:66} \cdot v^9 = 0.82123 \cdot 0.64461 = 0.52937$, where
${}_9p_{66:66} = \dfrac{\ell_{75}}{\ell_{66}} \cdot \dfrac{\ell_{75}}{\ell_{66}} = \dfrac{85203.5}{94020.3} \cdot \dfrac{85203.5}{94020.3} = 0.82123$

45. $\ddot{a}_{44|54} = \ddot{a}_{54} - \ddot{a}_{44:54} = 16.2676 - 15.7353 = \boxed{0.5323}$

46. ${}_{49}p_{\overline{36:32}} = {}_{49}p_{36} + {}_{49}p_{32} - {}_{49}p_{36:32} = 0.61481 + 0.73434 - 0.45148 = \boxed{0.89767}$, where
${}_{49}p_{36:32} = \dfrac{\ell_{85}}{\ell_{36}} \cdot \dfrac{\ell_{81}}{\ell_{32}} = \dfrac{61184.9}{99517.8} \cdot \dfrac{73186.3}{99663.2} = 0.45148$

47. ${}_{45}E_{\overline{32:41}} = {}_{45}p_{\overline{32:41}} \cdot v^{45} = 0.92529 \cdot 0.1113 = \boxed{0.10298}$, where
${}_{45}p_{\overline{32:41}} = {}_{45}p_{32} + {}_{45}p_{41} - {}_{45}p_{32:41} = 0.82181 + 0.58071 - 0.47723 = 0.92529$, where
${}_{45}p_{32:41} = \dfrac{\ell_{77}}{\ell_{32}} \cdot \dfrac{\ell_{86}}{\ell_{41}} = \dfrac{81904.3}{99663.2} \cdot \dfrac{57656.7}{99285.9} = 0.47723$

48. ${}_6q_{\overline{69:39}} = {}_6q_{69} \cdot {}_6q_{39} = 0.07324 \cdot 0.00356 = \boxed{0.00026}$

49. $A_{\overline{\frac{1}{38:38}\backslash:7}|} = A_{38:38} - {}_7E_{38:38} \cdot A_{45:45} = 0.14713 - 0.70498 \cdot 0.19942 = \boxed{0.00654}$, where
${}_7E_{38:38} = {}_7p_{38:38} \cdot v^7 = 0.99198 \cdot 0.71068 = 0.70498$, where
${}_7p_{38:38} = \dfrac{\ell_{45}}{\ell_{38}} \cdot \dfrac{\ell_{45}}{\ell_{38}} = \dfrac{99033.9}{99433.3} \cdot \dfrac{99033.9}{99433.3} = 0.99198$

50. ${}_6E_{\overline{30:66}} = {}_6p_{\overline{30:66}} \cdot v^6 = 0.99989 \cdot 0.74622 = \boxed{0.74614}$, where
${}_6p_{\overline{30:66}} = {}_6p_{30} + {}_6p_{66} - {}_6p_{30:66} = 0.9979 + 0.94748 - 0.94549 = 0.99989$, where
${}_6p_{30:66} = \dfrac{\ell_{36}}{\ell_{30}} \cdot \dfrac{\ell_{72}}{\ell_{66}} = \dfrac{99517.8}{99727.3} \cdot \dfrac{89082.1}{94020.3} = 0.94549$