

Solutions to Chapter 10 SULT Problems

1. ${}_6p_{34:64} = \frac{\ell_{40}}{\ell_{34}} \cdot \frac{\ell_{70}}{\ell_{64}} = \frac{99338.3}{99593.8} \cdot \frac{91082.4}{95082.5} = \boxed{0.95547}$
2. ${}_7p_{56:62} = \frac{\ell_{63}}{\ell_{56}} \cdot \frac{\ell_{69}}{\ell_{62}} = \frac{95534.4}{97651.2} \cdot \frac{91936.9}{95940.6} = \boxed{0.93749}$
3. ${}_{19}p_{34:46} = \frac{\ell_{53}}{\ell_{34}} \cdot \frac{\ell_{65}}{\ell_{46}} = \frac{98181.8}{99593.8} \cdot \frac{94579.7}{98957.6} = \boxed{0.94221}$
4. ${}_{12}p_{62:62} = \frac{\ell_{74}}{\ell_{62}} \cdot \frac{\ell_{74}}{\ell_{62}} = \frac{86627.6}{95940.6} \cdot \frac{86627.6}{95940.6} = \boxed{0.81528}$
5. ${}_{27}p_{40:48} = \frac{\ell_{67}}{\ell_{40}} \cdot \frac{\ell_{75}}{\ell_{48}} = \frac{93398.1}{99338.3} \cdot \frac{85203.5}{98783.9} = \boxed{0.81094}$
6. ${}_9q_{48:58} = 1 - {}_9p_{48:58} = 1 - 0.94781 = \boxed{0.05219}$, where
 ${}_9p_{48:58} = \frac{\ell_{57}}{\ell_{48}} \cdot \frac{\ell_{67}}{\ell_{58}} = \frac{97435.2}{98783.9} \cdot \frac{93398.1}{97195.6} = 0.94781$
7. ${}_{22}q_{53:47} = 1 - {}_{22}p_{53:47} = 1 - 0.80692 = \boxed{0.19308}$, where
 ${}_{22}p_{53:47} = \frac{\ell_{75}}{\ell_{53}} \cdot \frac{\ell_{69}}{\ell_{47}} = \frac{85203.5}{98181.8} \cdot \frac{91936.9}{98874.5} = 0.80692$
8. ${}_{15}q_{48:52} = 1 - {}_{15}p_{48:52} = 1 - 0.91863 = \boxed{0.08137}$, where
 ${}_{15}p_{48:52} = \frac{\ell_{63}}{\ell_{48}} \cdot \frac{\ell_{67}}{\ell_{52}} = \frac{95534.4}{98783.9} \cdot \frac{93398.1}{98326.2} = 0.91863$
9. ${}_{38}q_{45:42} = 1 - {}_{38}p_{45:42} = 1 - 0.52055 = \boxed{0.47945}$, where
 ${}_{38}p_{45:42} = \frac{\ell_{83}}{\ell_{45}} \cdot \frac{\ell_{80}}{\ell_{42}} = \frac{67614.6}{99033.9} \cdot \frac{75657.2}{99229.8} = 0.52055$
10. ${}_{10}q_{62:35} = 1 - {}_{10}p_{62:35} = 1 - 0.92364 = \boxed{0.07636}$, where
 ${}_{10}p_{62:35} = \frac{\ell_{72}}{\ell_{62}} \cdot \frac{\ell_{45}}{\ell_{35}} = \frac{89082.1}{95940.6} \cdot \frac{99033.9}{99556.7} = 0.92364$
11. ${}_{25}\overline{p}_{30:46} = {}_{25}p_{30} + {}_{25}p_{46} - {}_{25}p_{30:46} = 0.98114 + 0.91083 - 0.89365 = \boxed{0.99832}$, where
 ${}_{25}p_{30:46} = \frac{\ell_{55}}{\ell_{30}} \cdot \frac{\ell_{71}}{\ell_{46}} = \frac{97846.2}{99727.3} \cdot \frac{90134}{98957.6} = 0.89365$
12. ${}_5\overline{p}_{64:37} = {}_5p_{64} + {}_5p_{37} - {}_5p_{64:37} = 0.96692 + 0.99752 - 0.96452 = \boxed{0.99992}$, where
 ${}_5p_{64:37} = \frac{\ell_{69}}{\ell_{64}} \cdot \frac{\ell_{42}}{\ell_{37}} = \frac{91936.9}{95082.5} \cdot \frac{99229.8}{99476.7} = 0.96452$
13. ${}_{28}\overline{p}_{34:51} = {}_{28}p_{34} + {}_{28}p_{51} - {}_{28}p_{34:51} = 0.96332 + 0.79149 - 0.76246 = \boxed{0.99235}$, where
 ${}_{28}p_{34:51} = \frac{\ell_{62}}{\ell_{34}} \cdot \frac{\ell_{79}}{\ell_{51}} = \frac{95940.6}{99593.8} \cdot \frac{77927.4}{98457.2} = 0.76246$

$$14. \quad {}_{10}p_{\overline{52:65}} = {}_{10}p_{52} + {}_{10}p_{65} - {}_{10}p_{52:65} = 0.97574 + 0.90086 - 0.87901 = \boxed{0.99759}, \text{ where} \\ {}_{10}p_{52:65} = \frac{\ell_{62}}{\ell_{52}} \cdot \frac{\ell_{75}}{\ell_{65}} = \frac{95940.6}{98326.2} \cdot \frac{85203.5}{94579.7} = 0.87901$$

$$15. \quad {}_5p_{\overline{67:52}} = {}_5p_{67} + {}_5p_{52} - {}_5p_{67:52} = 0.95379 + 0.99094 - 0.94515 = \boxed{0.99958}, \text{ where} \\ {}_5p_{67:52} = \frac{\ell_{72}}{\ell_{67}} \cdot \frac{\ell_{57}}{\ell_{52}} = \frac{89082.1}{93398.1} \cdot \frac{97435.2}{98326.2} = 0.94515$$

$$16. \quad {}_{14}q_{\overline{59:38}} = {}_{14}q_{59} \cdot {}_{14}q_{38} = 0.09298 \cdot 0.01113 = \boxed{0.00103}$$

$$17. \quad {}_6q_{\overline{42:61}} = {}_6q_{42} \cdot {}_6q_{61} = 0.00449 \cdot 0.03019 = \boxed{0.00014}$$

$$18. \quad {}_{17}q_{\overline{36:52}} = {}_{17}q_{36} \cdot {}_{17}q_{52} = 0.01342 \cdot 0.06498 = \boxed{0.00087}$$

$$19. \quad {}_{18}q_{\overline{39:51}} = {}_{18}q_{39} \cdot {}_{18}q_{51} = 0.01964 \cdot 0.06622 = \boxed{0.0013}$$

$$20. \quad {}_5q_{\overline{59:35}} = {}_5q_{59} \cdot {}_5q_{35} = 0.01906 \cdot 0.00219 = \boxed{0.00004}$$

$$21. \quad {}_7E_{65:54} = {}_7p_{65:54} \cdot v^7 = 0.92538 \cdot 0.71068 = \boxed{0.65765}, \text{ where} \\ {}_7p_{65:54} = \frac{\ell_{72}}{\ell_{65}} \cdot \frac{\ell_{61}}{\ell_{54}} = \frac{89082.1}{94579.7} \cdot \frac{96305.8}{98022.4} = 0.92538$$

$$22. \quad {}_8E_{35:63} = {}_8p_{35:63} \cdot v^8 = 0.9398 \cdot 0.67684 = \boxed{0.63609}, \text{ where} \\ {}_8p_{35:63} = \frac{\ell_{43}}{\ell_{35}} \cdot \frac{\ell_{71}}{\ell_{63}} = \frac{99169.4}{99556.7} \cdot \frac{90134}{95534.4} = 0.9398$$

$$23. \quad {}_{10}E_{55:41} = {}_{10}p_{55:41} \cdot v^{10} = 0.95855 \cdot 0.61391 = \boxed{0.58846}, \text{ where} \\ {}_{10}p_{55:41} = \frac{\ell_{65}}{\ell_{55}} \cdot \frac{\ell_{51}}{\ell_{41}} = \frac{94579.7}{97846.2} \cdot \frac{98457.2}{99285.9} = 0.95855$$

$$24. \quad {}_{29}E_{43:31} = {}_{29}p_{43:31} \cdot v^{29} = 0.87069 \cdot 0.24295 = \boxed{0.21153}, \text{ where} \\ {}_{29}p_{43:31} = \frac{\ell_{72}}{\ell_{43}} \cdot \frac{\ell_{60}}{\ell_{31}} = \frac{89082.1}{99169.4} \cdot \frac{96634.1}{99695.8} = 0.87069$$

$$25. \quad {}_5E_{67:38} = {}_5p_{67:38} \cdot v^5 = 0.95126 \cdot 0.78353 = \boxed{0.74534}, \text{ where} \\ {}_5p_{67:38} = \frac{\ell_{72}}{\ell_{67}} \cdot \frac{\ell_{43}}{\ell_{38}} = \frac{89082.1}{93398.1} \cdot \frac{99169.4}{99433.3} = 0.95126$$

$$26. \quad {}_{11}E_{\overline{64:57}} = {}_{11}p_{\overline{64:57}} \cdot v^{11} = 0.99496 \cdot 0.58468 = \boxed{0.58173}, \text{ where} \\ {}_{11}p_{\overline{64:57}} = {}_{11}p_{64} + {}_{11}p_{57} - {}_{11}p_{64:57} = 0.8961 + 0.95146 - 0.8526 = 0.99496, \text{ where} \\ {}_{11}p_{64:57} = \frac{\ell_{75}}{\ell_{64}} \cdot \frac{\ell_{68}}{\ell_{57}} = \frac{85203.5}{95082.5} \cdot \frac{92706.1}{97435.2} = 0.8526$$

$$27. \quad {}_{32}E_{\overline{31:46}} = {}_{32}p_{\overline{31:46}} \cdot v^{32} = 0.99201 \cdot 0.20987 = \boxed{0.20819}, \text{ where}$$

- $_{32}P_{\overline{31:46}} = {}_{32}P_{31} + {}_{32}P_{46} - {}_{32}P_{31:46} = 0.95826 + 0.80849 - 0.77474 = 0.99201$, where
 ${}_{32}P_{31:46} = \frac{\ell_{63}}{\ell_{31}} \cdot \frac{\ell_{78}}{\ell_{46}} = \frac{95534.4}{99695.8} \cdot \frac{80006.2}{98957.6} = 0.77474$
28. ${}_9E_{\overline{64:51}} = {}_9P_{\overline{64:51}} \cdot v^9 = 0.9986 \cdot 0.64461 = \boxed{0.64371}$, where
 ${}_9P_{\overline{64:51}} = {}_9P_{64} + {}_9P_{51} - {}_9P_{64:51} = 0.92464 + 0.98148 - 0.90752 = 0.9986$, where
 ${}_9P_{64:51} = \frac{\ell_{73}}{\ell_{64}} \cdot \frac{\ell_{60}}{\ell_{51}} = \frac{87916.8}{95082.5} \cdot \frac{96634.1}{98457.2} = 0.90752$
29. ${}_{15}E_{\overline{59:33}} = {}_{15}P_{\overline{59:33}} \cdot v^{15} = 0.9991 \cdot 0.48102 = \boxed{0.48059}$, where
 ${}_{15}P_{\overline{59:33}} = {}_{15}P_{59} + {}_{15}P_{33} - {}_{15}P_{59:33} = 0.89372 + 0.99151 - 0.88613 = 0.9991$, where
 ${}_{15}P_{59:33} = \frac{\ell_{74}}{\ell_{59}} \cdot \frac{\ell_{48}}{\ell_{33}} = \frac{86627.6}{96929.6} \cdot \frac{98783.9}{99629.3} = 0.88613$
30. ${}_{34}E_{\overline{52:33}} = {}_{34}P_{\overline{52:33}} \cdot v^{34} = 0.97413 \cdot 0.19035 = \boxed{0.18543}$, where
 ${}_{34}P_{\overline{52:33}} = {}_{34}P_{52} + {}_{34}P_{33} - {}_{34}P_{52:33} = 0.58638 + 0.93746 - 0.54971 = 0.97413$, where
 ${}_{34}P_{52:33} = \frac{\ell_{86}}{\ell_{52}} \cdot \frac{\ell_{67}}{\ell_{33}} = \frac{57656.7}{98326.2} \cdot \frac{93398.1}{99629.3} = 0.54971$
31. $A_{\overline{46:56 \setminus 16}}^{\frac{1}{16}} = A_{46:56} - {}_{16}E_{46:56} \cdot A_{62:72} = 0.27244 - 0.40517 \cdot 0.49811 = \boxed{0.07062}$, where
 ${}_{16}E_{46:56} = {}_{16}P_{46:56} \cdot v^{16} = 0.88444 \cdot 0.45811 = 0.40517$, where
 ${}_{16}P_{46:56} = \frac{\ell_{62}}{\ell_{46}} \cdot \frac{\ell_{72}}{\ell_{56}} = \frac{95940.6}{98957.6} \cdot \frac{89082.1}{97651.2} = 0.88444$
32. $A_{\overline{52:52 \setminus 5}}^{\frac{1}{5}} = A_{52:52} - {}_5E_{52:52} \cdot A_{57:57} = 0.26814 - 0.7694 \cdot 0.3284 = \boxed{0.01547}$, where
 ${}_5E_{52:52} = {}_5P_{52:52} \cdot v^5 = 0.98196 \cdot 0.78353 = 0.7694$, where
 ${}_5P_{52:52} = \frac{\ell_{57}}{\ell_{52}} \cdot \frac{\ell_{57}}{\ell_{52}} = \frac{97435.2}{98326.2} \cdot \frac{97435.2}{98326.2} = 0.98196$
33. $A_{\overline{56:56 \setminus 23}}^{\frac{1}{23}} = A_{56:56} - {}_{23}E_{56:56} \cdot A_{79:79} = 0.31559 - 0.20734 \cdot 0.6754 = \boxed{0.17555}$, where
 ${}_{23}E_{56:56} = {}_{23}P_{56:56} \cdot v^{23} = 0.63684 \cdot 0.32557 = 0.20734$, where
 ${}_{23}P_{56:56} = \frac{\ell_{79}}{\ell_{56}} \cdot \frac{\ell_{79}}{\ell_{56}} = \frac{77927.4}{97651.2} \cdot \frac{77927.4}{97651.2} = 0.63684$
34. $A_{\overline{58:68 \setminus 12}}^{\frac{1}{12}} = A_{58:68} - {}_{12}E_{58:68} \cdot A_{70:80} = 0.43407 - 0.42585 \cdot 0.63234 = \boxed{0.16479}$, where
 ${}_{12}E_{58:68} = {}_{12}P_{58:68} \cdot v^{12} = 0.76477 \cdot 0.55684 = 0.42585$, where
 ${}_{12}P_{58:68} = \frac{\ell_{70}}{\ell_{58}} \cdot \frac{\ell_{80}}{\ell_{68}} = \frac{91082.4}{97195.6} \cdot \frac{75657.2}{92706.1} = 0.76477$
35. $A_{\overline{69:69 \setminus 5}}^{\frac{1}{5}} = A_{69:69} - {}_5E_{69:69} \cdot A_{74:74} = 0.50826 - 0.69565 \cdot 0.59225 = \boxed{0.09626}$, where
 ${}_5E_{69:69} = {}_5P_{69:69} \cdot v^5 = 0.88784 \cdot 0.78353 = 0.69565$, where
 ${}_5P_{69:69} = \frac{\ell_{74}}{\ell_{69}} \cdot \frac{\ell_{74}}{\ell_{69}} = \frac{86627.6}{91936.9} \cdot \frac{86627.6}{91936.9} = 0.88784$
36. $A_{\overline{37:37 \setminus 38}}^{\frac{1}{38}} = A_{37:38}^{\frac{1}{38}} + A_{37:38}^{\frac{1}{38}} - A_{\overline{37:37 \setminus 38}}^{\frac{1}{38}} = 0.03746 + 0.03746 - 0.07085 = \boxed{0.00407}$, where
 $A_{\overline{37:37 \setminus 38}}^{\frac{1}{38}} = A_{37:37} - {}_{38}E_{37:37} \cdot A_{75:75} = 0.14083 - 0.11489 \cdot 0.60912 = 0.07085$, where
 ${}_{38}E_{37:37} = {}_{38}P_{37:37} \cdot v^{38} = 0.73363 \cdot 0.15661 = 0.11489$, where
 ${}_{38}P_{37:37} = \frac{\ell_{75}}{\ell_{37}} \cdot \frac{\ell_{75}}{\ell_{37}} = \frac{85203.5}{99476.7} \cdot \frac{85203.5}{99476.7} = 0.73363$

37. $A_{\overline{31:31:34}}^1 = A_{\overline{31:34}}^1 + A_{\overline{31:34}}^1 - A_{\overline{31:31:34}}^1 = 0.01647 + 0.01647 - 0.03232 = \boxed{0.00062}$, where
 $A_{\overline{31:31:34}}^1 = A_{\overline{31:31}} - {}_{34}E_{31:31} \cdot A_{\overline{65:65}} = 0.10832 - 0.17131 \cdot 0.44366 = 0.03232$, where
 ${}_{34}E_{31:31} = {}_{34}p_{31:31} \cdot v^{34} = 0.89999 \cdot 0.19035 = 0.17131$, where
 ${}_{34}p_{31:31} = \frac{\ell_{65}}{\ell_{31}} \cdot \frac{\ell_{65}}{\ell_{31}} = \frac{94579.7}{99695.8} \cdot \frac{94579.7}{99695.8} = 0.89999$
38. $A_{\overline{37:47:42}}^1 = A_{\overline{37:42}}^1 + A_{\overline{47:42}}^1 - A_{\overline{37:47:42}}^1 = 0.04756 + 0.12151 - 0.14976 = \boxed{0.01931}$, where
 $A_{\overline{37:47:42}}^1 = A_{\overline{37:47}} - {}_{42}E_{37:47} \cdot A_{\overline{79:89}} = 0.186 - 0.04695 \cdot 0.77186 = 0.14976$, where
 ${}_{42}E_{37:47} = {}_{42}p_{37:47} \cdot v^{42} = 0.36442 \cdot 0.12884 = 0.04695$, where
 ${}_{42}p_{37:47} = \frac{\ell_{79}}{\ell_{37}} \cdot \frac{\ell_{89}}{\ell_{47}} = \frac{77927.4}{99476.7} \cdot \frac{45995.6}{98874.5} = 0.36442$
39. $A_{\overline{49:49:12}}^1 = A_{\overline{49:12}}^1 + A_{\overline{49:12}}^1 - A_{\overline{49:49:12}}^1 = 0.0168 + 0.0168 - 0.03324 = \boxed{0.00036}$, where
 $A_{\overline{49:49:12}}^1 = A_{\overline{49:49}} - {}_{12}E_{49:49} \cdot A_{\overline{61:61}} = 0.23652 - 0.53031 \cdot 0.38333 = 0.03324$, where
 ${}_{12}E_{49:49} = {}_{12}p_{49:49} \cdot v^{12} = 0.95236 \cdot 0.55684 = 0.53031$, where
 ${}_{12}p_{49:49} = \frac{\ell_{61}}{\ell_{49}} \cdot \frac{\ell_{61}}{\ell_{49}} = \frac{96305.8}{98684.9} \cdot \frac{96305.8}{98684.9} = 0.95236$
40. $A_{\overline{63:63:9}}^1 = A_{\overline{63:9}}^1 + A_{\overline{63:9}}^1 - A_{\overline{63:63:9}}^1 = 0.05155 + 0.05155 - 0.09987 = \boxed{0.00323}$, where
 $A_{\overline{63:63:9}}^1 = A_{\overline{63:63}} - {}_9E_{63:63} \cdot A_{\overline{72:72}} = 0.41288 - 0.56048 \cdot 0.55847 = 0.09987$, where
 ${}_9E_{63:63} = {}_9p_{63:63} \cdot v^9 = 0.86948 \cdot 0.64461 = 0.56048$, where
 ${}_9p_{63:63} = \frac{\ell_{72}}{\ell_{63}} \cdot \frac{\ell_{72}}{\ell_{63}} = \frac{89082.1}{95534.4} \cdot \frac{89082.1}{95534.4} = 0.86948$
41. $A_{\overline{43:53}} = A_{43} + A_{53} - A_{43:53} = 0.13859 + 0.21582 - 0.24039 = \boxed{0.11402}$
42. $A_{\overline{47:47}} = A_{47} + A_{47} - A_{47:47} = 0.16577 + 0.16577 - 0.21727 = \boxed{0.11427}$
43. $A_{\overline{30:40}} = A_{30} + A_{40} - A_{30:40} = 0.07698 + 0.12106 - 0.13709 = \boxed{0.06095}$
44. $A_{\overline{59:69}} = A_{59} + A_{69} - A_{59:69} = 0.27852 + 0.41285 - 0.44972 = \boxed{0.24165}$
45. $A_{\overline{51:61}} = A_{51} + A_{61} - A_{51:61} = 0.1978 + 0.30243 - 0.33344 = \boxed{0.16679}$
46. $\ddot{a}_{\overline{36:46:20}} = \ddot{a}_{\overline{36:46:10}} + {}_{10}E_{36:46} \cdot \ddot{a}_{\overline{46:56:10}} = 8.0559 + 0.60239 \cdot 7.974 = \boxed{12.85936}$, where
 ${}_{10}E_{36:46} = {}_{10}p_{36:46} \cdot v^{10} = 0.98124 \cdot 0.61391 = 0.60239$, where
 ${}_{10}p_{36:46} = \frac{\ell_{46}}{\ell_{36}} \cdot \frac{\ell_{56}}{\ell_{46}} = \frac{98957.6}{99517.8} \cdot \frac{97651.2}{98957.6} = 0.98124$
47. $\ddot{a}_{\overline{30:40:20}} = \ddot{a}_{\overline{30:40:10}} + {}_{10}E_{30:40} \cdot \ddot{a}_{\overline{40:50:10}} = 8.0747 + 0.60683 \cdot 8.0337 = \boxed{12.94979}$, where
 ${}_{10}E_{30:40} = {}_{10}p_{30:40} \cdot v^{10} = 0.98846 \cdot 0.61391 = 0.60683$, where
 ${}_{10}p_{30:40} = \frac{\ell_{40}}{\ell_{30}} \cdot \frac{\ell_{50}}{\ell_{40}} = \frac{99338.3}{99727.3} \cdot \frac{98576.4}{99338.3} = 0.98846$
48. $\ddot{a}_{\overline{40:50:20}} = \ddot{a}_{\overline{40:50:10}} + {}_{10}E_{40:50} \cdot \ddot{a}_{\overline{50:60:10}} = 8.0337 + 0.5972 \cdot 7.9044 = \boxed{12.75421}$, where

$${}_{10}E_{40:50} = {}_{10}p_{40:50} \cdot v^{10} = 0.97278 \cdot 0.61391 = 0.5972, \text{ where}$$

$${}_{10}p_{40:50} = \frac{\ell_{50}}{\ell_{40}} \cdot \frac{\ell_{60}}{\ell_{50}} = \frac{98576.4}{99338.3} \cdot \frac{96634.1}{98576.4} = 0.97278$$

$$49. \ddot{a}_{44:54:\overline{20}} = \ddot{a}_{44:54:\overline{10}} + {}_{10}E_{44:54} \cdot \ddot{a}_{54:64:\overline{10}} = 7.9986 + 0.589 \cdot 7.7953 = \boxed{12.59003}, \text{ where}$$

$${}_{10}E_{44:54} = {}_{10}p_{44:54} \cdot v^{10} = 0.95942 \cdot 0.61391 = 0.589, \text{ where}$$

$${}_{10}p_{44:54} = \frac{\ell_{54}}{\ell_{44}} \cdot \frac{\ell_{64}}{\ell_{54}} = \frac{98022.4}{99104.3} \cdot \frac{95082.5}{98022.4} = 0.95942$$

$$50. \ddot{a}_{43:43:\overline{20}} = \ddot{a}_{43:43:\overline{10}} + {}_{10}E_{43:43} \cdot \ddot{a}_{53:53:\overline{10}} = 8.0531 + 0.60174 \cdot 7.9652 = \boxed{12.84608}, \text{ where}$$

$${}_{10}E_{43:43} = {}_{10}p_{43:43} \cdot v^{10} = 0.98018 \cdot 0.61391 = 0.60174, \text{ where}$$

$${}_{10}p_{43:43} = \frac{\ell_{53}}{\ell_{43}} \cdot \frac{\ell_{63}}{\ell_{53}} = \frac{98181.8}{99169.4} \cdot \frac{98181.8}{99169.4} = 0.98018$$

$$51. \ddot{a}_{64|74} = \ddot{a}_{74} - \ddot{a}_{64:74} = 10.6649 - 9.8423 = \boxed{0.8226}$$

$$52. \ddot{a}_{53|53} = \ddot{a}_{53} - \ddot{a}_{53:53} = 16.4678 - 15.1318 = \boxed{1.336}$$

$$53. \ddot{a}_{33|43} = \ddot{a}_{43} - \ddot{a}_{33:43} = 18.0895 - 17.7176 = \boxed{0.3719}$$

$$54. \ddot{a}_{40|40} = \ddot{a}_{40} - \ddot{a}_{40:40} = 18.4578 - 17.6283 = \boxed{0.8295}$$

$$55. \ddot{a}_{40|50} = \ddot{a}_{50} - \ddot{a}_{40:50} = 17.0245 - 16.5558 = \boxed{0.4687}$$

$$56. \ddot{a}_{\overline{38:38}} = \ddot{a}_{38} + \ddot{a}_{38} - \ddot{a}_{38:38} = 18.6777 + 18.6777 - 17.9104 = \boxed{19.445}$$

$$57. \ddot{a}_{\overline{66:76}} = \ddot{a}_{66} + \ddot{a}_{76} - \ddot{a}_{66:76} = 13.2557 + 9.9674 - 9.1358 = \boxed{14.0873}$$

$$58. \ddot{a}_{\overline{52:52}} = \ddot{a}_{52} + \ddot{a}_{52} - \ddot{a}_{52:52} = 16.6606 + 16.6606 - 15.369 = \boxed{17.9522}$$

$$59. \ddot{a}_{\overline{50:60}} = \ddot{a}_{50} + \ddot{a}_{60} - \ddot{a}_{50:60} = 17.0245 + 14.9041 - 14.2699 = \boxed{17.6587}$$

$$60. \ddot{a}_{\overline{54:64}} = \ddot{a}_{54} + \ddot{a}_{64} - \ddot{a}_{54:64} = 16.2676 + 13.8363 - 13.1352 = \boxed{16.9687}$$

$$61. \ddot{a}_{\overline{53:63:\overline{10}}} = \ddot{a}_{53:\overline{10}} + \ddot{a}_{63:\overline{10}} - \ddot{a}_{53:63:\overline{10}} = 8.036 + 7.896 - 7.8272 = \boxed{8.1048}$$

$$62. \ddot{a}_{\overline{37:37:\overline{10}}} = \ddot{a}_{37:\overline{10}} + \ddot{a}_{37:\overline{10}} - \ddot{a}_{37:37:\overline{10}} = 8.0905 + 8.0905 - 8.0733 = \boxed{8.1077}$$

$$63. \ddot{a}_{\overline{48:58:\overline{10}}} = \ddot{a}_{48:\overline{10}} + \ddot{a}_{58:\overline{10}} - \ddot{a}_{48:58:\overline{10}} = 8.0645 + 7.9854 - 7.9431 = \boxed{8.1068}$$

$$64. \ddot{a}_{\overline{45:55:10}} = \ddot{a}_{45:10} + \ddot{a}_{55:10} - \ddot{a}_{45:55:10} = 8.0751 + 8.0192 - 7.987 = \boxed{8.1073}$$

$$65. \ddot{a}_{\overline{54:54:10}} = \ddot{a}_{54:10} + \ddot{a}_{54:10} - \ddot{a}_{54:54:10} = 8.0281 + 8.0281 - 7.9496 = \boxed{8.1066}$$

$$66. \ddot{a}_{\overline{56:66:20}} = \ddot{a}_{56:20} + \ddot{a}_{66:20} - \ddot{a}_{56:66:20} = 12.6271 + 11.7622 - 11.39468 = \boxed{12.99462}, \text{ where}$$

$$\ddot{a}_{56:66:20} = \ddot{a}_{56:66:10} + {}_{10}E_{56:66} \cdot \ddot{a}_{66:76:10} = 7.7199 + 0.52578 \cdot 6.9892 = 11.39468, \text{ where}$$

$${}_{10}E_{56:66} = {}_{10}p_{56:66} \cdot v^{10} = 0.85645 \cdot 0.61391 = 0.52578, \text{ where}$$

$${}_{10}p_{56:66} = \frac{\ell_{66}}{\ell_{56}} \cdot \frac{\ell_{76}}{\ell_{66}} = \frac{94020.3}{97651.2} \cdot \frac{83632.9}{94020.3} = 0.85645$$

$$67. \ddot{a}_{\overline{62:62:20}} = \ddot{a}_{62:20} + \ddot{a}_{62:20} - \ddot{a}_{62:62:20} = 12.2145 + 12.2145 - 11.45758 = \boxed{12.97142}, \text{ where}$$

$$\ddot{a}_{62:62:20} = \ddot{a}_{62:62:10} + {}_{10}E_{62:62} \cdot \ddot{a}_{72:72:10} = 7.7357 + 0.52927 \cdot 7.0321 = 11.45758, \text{ where}$$

$${}_{10}E_{62:62} = {}_{10}p_{62:62} \cdot v^{10} = 0.86213 \cdot 0.61391 = 0.52927, \text{ where}$$

$${}_{10}p_{62:62} = \frac{\ell_{72}}{\ell_{62}} \cdot \frac{\ell_{72}}{\ell_{62}} = \frac{89082.1}{95940.6} \cdot \frac{89082.1}{95940.6} = 0.86213$$

$$68. \ddot{a}_{\overline{31:31:20}} = \ddot{a}_{31:20} + \ddot{a}_{31:20} - \ddot{a}_{31:31:20} = 13.0384 + 13.0384 - 12.99173 = \boxed{13.08507}, \text{ where}$$

$$\ddot{a}_{31:31:20} = \ddot{a}_{31:31:10} + {}_{10}E_{31:31} \cdot \ddot{a}_{41:41:10} = 8.0833 + 0.60888 \cdot 8.0614 = 12.99173, \text{ where}$$

$${}_{10}E_{31:31} = {}_{10}p_{31:31} \cdot v^{10} = 0.9918 \cdot 0.61391 = 0.60888, \text{ where}$$

$${}_{10}p_{31:31} = \frac{\ell_{41}}{\ell_{31}} \cdot \frac{\ell_{41}}{\ell_{31}} = \frac{99285.9}{99695.8} \cdot \frac{99285.9}{99695.8} = 0.9918$$

$$69. \ddot{a}_{\overline{65:65:20}} = \ddot{a}_{65:20} + \ddot{a}_{65:20} - \ddot{a}_{65:65:20} = 11.892 + 11.892 - 10.9097 = \boxed{12.8743}, \text{ where}$$

$$\ddot{a}_{65:65:20} = \ddot{a}_{65:65:10} + {}_{10}E_{65:65} \cdot \ddot{a}_{75:75:10} = 7.5934 + 0.49822 \cdot 6.6563 = 10.9097, \text{ where}$$

$${}_{10}E_{65:65} = {}_{10}p_{65:65} \cdot v^{10} = 0.81155 \cdot 0.61391 = 0.49822, \text{ where}$$

$${}_{10}p_{65:65} = \frac{\ell_{75}}{\ell_{65}} \cdot \frac{\ell_{75}}{\ell_{65}} = \frac{85203.5}{94579.7} \cdot \frac{85203.5}{94579.7} = 0.81155$$

$$70. \ddot{a}_{\overline{44:54:20}} = \ddot{a}_{44:20} + \ddot{a}_{54:20} - \ddot{a}_{44:54:20} = 12.9526 + 12.7154 - 12.59003 = \boxed{13.07797}, \text{ where}$$

$$\ddot{a}_{44:54:20} = \ddot{a}_{44:54:10} + {}_{10}E_{44:54} \cdot \ddot{a}_{54:64:10} = 7.9986 + 0.589 \cdot 7.7953 = 12.59003, \text{ where}$$

$${}_{10}E_{44:54} = {}_{10}p_{44:54} \cdot v^{10} = 0.95942 \cdot 0.61391 = 0.589, \text{ where}$$

$${}_{10}p_{44:54} = \frac{\ell_{54}}{\ell_{44}} \cdot \frac{\ell_{64}}{\ell_{54}} = \frac{98022.4}{99104.3} \cdot \frac{95082.5}{98022.4} = 0.95942$$