CS 506, Spring 21, Midterm
Prof. Nassimi, CS Dept, NJIT

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First Last

Duration: 2 Hours

Thurs March 4, 4:00-6:00 pm

Instructions:

- Answer all questions on the exam papers in the spaces provided
- Exam is closed books closed notes. No reference material is allowed.
- No electronic device, no calculator, is allowed.

	Earned Points	Max
1		10
2		15
3		15
4		15
5		15
6		15
7		15
Sum		**100**

1. (a) Given the sets U, A, B.

$$U = \{1,2,3,4,5,6,7\}, \quad A = \{1,3,5\}, \quad B = \{1,4,5,7\}.$$

Find each of the following:

$A \cup B = \{1,3,4,5,7\}$

$A \cap B = \{1,5\}$

$B - A = \{4,7\}$

(b) Use algebraic method to prove

$$(A \cup B) - (A \cup C) \quad = (A - C) \cup (B - C)$$

$(A \cup B) - (A \cup C)$ set difference

$(A \cup B) \cap \overline{(A \cup C)}$ De Morgan's law

$(A \cup B) \cap (\bar{A} \cap \bar{C})$

$(A \cap \bar{A}) \cup (A \cap \bar{C}) \cup (B \cap \bar{A}) \cup (B \cap \bar{C})$

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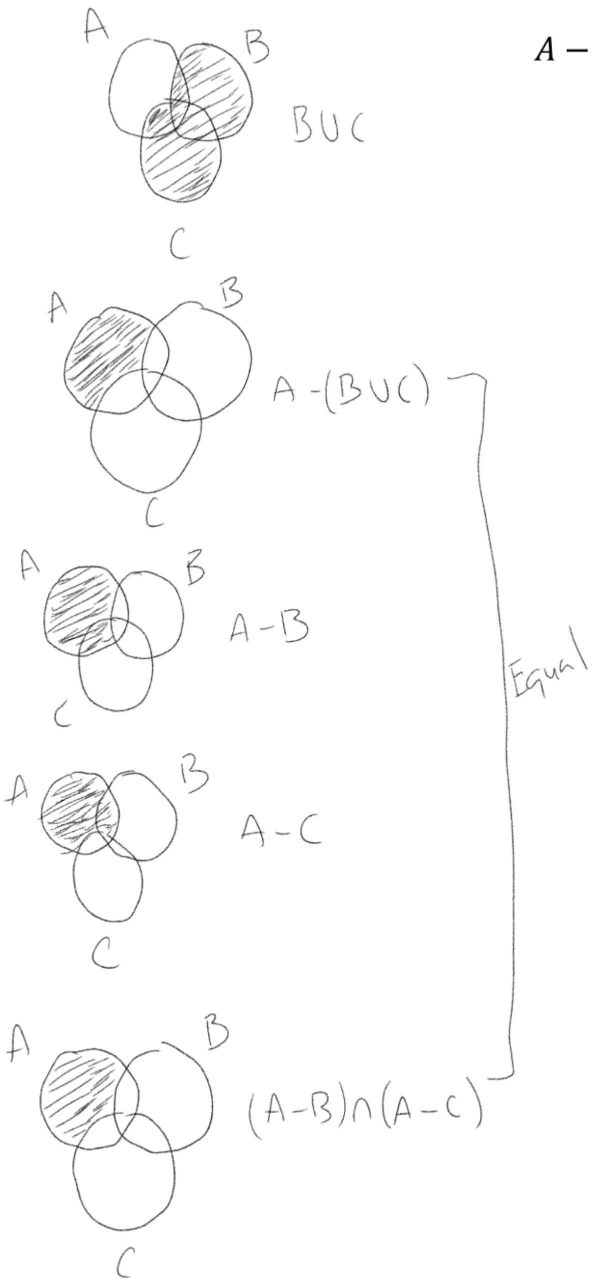
2. Use both a Venn diagram and algebraic method to prove the following set equality.

$$A - (B \cup C) = (A - B) \cap (A - C)$$



$A - (B \cup C)$ set difference

$A \cap \overline{(B \cup C)}$ De Morgan's law

$A \cap (\overline{B} \cap \overline{C})$ Distributive law

$(A \cap \overline{B}) \cap (A \cap \overline{C})$ set difference

$(A - B) \cap (A - C)$

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3. (a) Use a truth table to show the following propositions are equivalent.

$$A \to B$$
$$\neg B \to \neg A$$
$$\neg(A \wedge \neg B)$$

$\neg(A \wedge \neg B) = \neg A \vee B$

A	B	$A \to B$	$\neg A$	$\neg B$	$\neg B \to \neg A$	$\neg(A \wedge \neg B)$
T	T	T	F	F	T	T
T	F	F	F	T	F	F
F	T	T	T	F	T	T
F	F	T	T	T	T	T

(b) Find the negation of each of the following propositions, where A, B, C are some propositional functions of (x, y).

$p \to q$

$= \neg p \vee q$

$\neg(\neg p \vee q)$

$p \wedge \neg q$

$(i)\quad \forall x \forall y, \quad if\ (A\ and\ B)\ then\ C$

$\exists x \exists y$, if $(A \wedge B) \wedge \neg C$

$(ii)\quad \forall x \forall y, \quad if\ (A\ and\ \neg C)\ then\ \neg B$

$\exists x \exists y$, if $(A \wedge \neg C) \wedge B$

Are the above two propositions equivalent? Explain.

Yes, $(A \wedge B) \wedge \overline{C}$ commutative

$= (B \wedge A) \wedge \overline{C}$ associative

$= B \wedge (A \wedge \overline{C})$ commutative

$= (A \vee \overline{C}) \wedge B$

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4. Suppose the domain of x and y are integers. Determine true/false value of each of the following propositions. Provide reasoning.

(a) $\exists x\, \forall y,\ if\ (x > y)\ then\ (x^2 < y^2)$

This is true, for $x \leq -1$

$x = -1$
$y = -5$ $-1 > -5,\ (-1)^2 < (-5)^2$

$(b)\ \forall x\, \forall y,\quad if\ \ (x < y)\ then\ (x^2 < y^2)$

This is false, for $x \leq -1$

$x = -1$
$y = 1$ $-1 < 1,\ (-1)^2 \nless (1)^2$

Find the negation of each of the above propositions.

(a) $\neg S: \forall x\, \exists y,$ if $(x > y) \wedge (x^2 \geq y^2)$

This is false, S: true, $\neg S$: false

(b) $\exists x\, \exists y,$ if $(x < y) \wedge (x^2 \geq y^2)$

This is true, S: false, $\neg S$: true

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5. (a) We know that $\sqrt{2}$ is **irrational**. Now, use **contradiction** method to prove that for any integer k, the product $(k\sqrt{2})$ will not be integer.

Prove contrary that $K\sqrt{2}$ will be an integer

So, $K\sqrt{2} = \frac{i}{j}$ we know K is an integer, $\frac{P}{P}$

$\sqrt{2} = \frac{i/j}{P/P} = \frac{iP}{jP}$, this is rational. This contradicts $\sqrt{2}$ is irrational, so, $K\sqrt{2}$ will not be an integer

(b) For the above case, can the product $(k\sqrt{2})$ be rational? Explain.

No, K is a rational number because integers are rational. As in the above proof $\sqrt{2}$ will make the product irrational.

(c) Suppose the domain of n is positive integers. Prove by **contrapositive** method:

If n^2 is not divisible by 9, then n is not divisible by 3.

First show Contrapositive form:

If n is divisible by 3, then n^2 is divisible by 9.

Then provide the Proof:

Suppose, $n = 3K$, for some integer K

Then, $n^2 = (3K)^2 = 9K^2 = 3(3K^2)$

$3K^2$ is an integer, so

n^2 is divisible by 9

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6. Consider the following recurrence equation.

$$F_n = \begin{cases} 3, & n = 1 \\ 5, & n = 2 \\ 3\,F_{n-1} - 2\,F_{n-2}, & n \geq 3 \end{cases}$$

(a) Compute and tabulate F_n for $n = 3$ to 5. (Show the computation.)

n	1	2	3	4	5
F_n	3	5	9	17	33

$3(5) - 2(3) = 9 \quad n=3$

$3(9) - 2(5) = 27 - 10 = 17 \quad n=4$

$3(17) - 2(9) = 51 - 18 = 33 \quad n=5$

(b) Prove by induction that

$$F_n = 2^n + 1, \quad n \geq 1$$

Base cases: $2^1 + 1 = 3$ so base is true

$2^2 + 1 = 5$ so base is true

Suppose $F_m = 2^m + 1$, for all $m = n$ where $n \geq 3$

Suppose,

$3F_{n-1} = 2^{n-1} + 1$

$2F_{n-2} = 2^{n-2} + 1$

Then,

$F_n = 3F_{n-1} - 2F_{n-2}$

$= 2^{n-1} + 1 - (2^{n-2} + 1)$

$= 2^n\left(\frac{1}{2} - \frac{1}{2^2}\right)$

$= 2^n\left(\frac{1}{2} - \frac{1}{4}\right)$

$= 2^n\left(\frac{1}{4}\right)$

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7. A relation has the following matrix.

$$A = \begin{bmatrix} 1 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 1 \end{bmatrix}$$

By direct observation of the matrix, decide if the relation satisfies each of the following properties. Provide a reasoning for each answer.

- Reflexive?

No, $A[2,2] = 0$

- Symmetric?

No, $A[1,3] = 1$ but $A[3,1] = 0$

- Antisymmetric?

Yes, $\forall x \forall y$ where $x \neq y$, if $A[x,y] = 1$ then $A[y,x] = 0$

- Transitive? (Use direct observation of the matrix, not matrix multiplication.)

No, $A[1,3] = 1$ and $A[3,2] = 1$ but $A[1,2] = 0$

Use matrix multiplication to decide if the following relation is transitive, and explain why.

$$A = \begin{bmatrix} 1 & 0 & 1 \\ 1 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \times \begin{bmatrix} 1 & 0 & 1 \\ 1 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \overset{A^2}{\begin{bmatrix} 1 & 0 & 1 \\ 1 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix}}$$

No, $A[2,3] = 0$ and $A^2[2,3] = 1$