CS 506, Spring 2021, Final Exam
Prof. D. Nassimi, CS Dept, NJIT

Name: Travis Virgil
First Last

Duration: 2.5 Hours

Tues. May 11, 6:00-8:30 pm

Instructions:

1. This exam is closed-book, closed notes, no use of internet, but a simple calculator allowed.
2. Answer all questions on the exam papers in the spaces provided. You may attach extra sheets if needed. Make sure everything is in order.
3. If you don't have a printer, you can write your answers on plain sheets of paper. Please start each question on a separate sheet, and mark your answers clearly with question number.
4. You may use a calculator to do the arithmetic calculations, but make sure to include all intermediate work.
5. Upload a PDF file by the due time on Canvas (same way as a homework).

	Earned Points	Max
1		20
2		20
3		20
4		20
5		20
Sum		**100**

Please Read and Sign:

You have prepared your answers honorably and have worked independently. The work you submit is entirely your own work, carried out in full honor.

Name (Printed): Travis Virgil

Signature: Travis Virgil

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1. The following program finds the Max and Min of an array of n elements.

```
Find ( dtype A[ ], int n, dtype Max, dtype Min) { // Assume n ≥ 1.
if (n == 1) { Max = Min = A[0]; return };
Find (A, n − 1, Max, Min);  // Find Max and Min of the first n-1 elements
if ( A[n − 1] > Max )        // Compare the last element again the max
    Max = A[n − 1];              // Update the max
else if (A[n − 1] < Min )   // Compare the last element against the min
    Min = A[n − 1];              // Update the min
}
```

(a) Let $f(n)$ be the worst-case number of key-comparisons for the above algorithm. Write a recurrence equation for $f(n)$.

$$f(n) = \begin{cases} 0, & n = 1 \\ f(n-1) + 1, & n \geq 2 \end{cases}$$

(b) Find the exact solution by repeated substitution method.

$$\begin{aligned} f(n) &= 1 + f(n-1) \\ &= 1 + 1 + f(n-2) \\ &= 1 + 1 + 1 + f(n-3) \\ &= 1 + 1 + 1 + \ldots + 1 + f(1) \\ &= n - 1 \end{aligned}$$

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2. Find the exact solution of each recurrence. Make sure you compute the constants.

(a)

$$F_n = \begin{cases} 5\,F_{n-1} - 6\,F_{n-2}, & n \geq 2 \\ 0, & n = 0 \\ 5, & n = 1 \end{cases}$$

$F_n - 5F_{n-1} + 6F_{n-2} = 0$

$r^n - 5r^{n-1} + 6r^{n-2} = 0$

$r^{n-2}(r^2 - 5r + 6) = 0$

$r^2 - 5r + 6 = 0$

$(r-2)(r-3) = 0$

$r_1 = 2,\ r_2 = 3$

$F_n = A2^n + B3^n$

$F_0 = 0 = A + B \qquad A = -B$

$F_1 = 5 = 2A + 3B$

$5 = -2B + 3B$

$5 = B$

$A = -5$

$F_n = -5 \cdot 2^n + 5 \cdot 3^n$

(b)

$$F_n = \begin{cases} 4\,F_{n-1} - 4\,F_{n-2}, & n \geq 2 \\ 1, & n = 0 \\ 6, & n = 1 \end{cases}$$

$F_n - 4F_{n-1} + 4F_{n-2} = 0$

$r^n - 4r^{n-1} + 4r^{n-2} = 0$

$r^{n-2}(r^2 - 4r + 4) = 0$

$r^2 - 4r + 4 = 0$

$(r-2)(r-2) = 0$

$F_n = nr^n,\ r = 2 \qquad (r-2)^2 = 0$ two roots

$P(nr^n) = nr^n - 4(n-1)r^{n-1} + 4(n-2)r^{n-2}$

$P(nr^n) = r^{n-2}[nr^2 - 4(n-1)r + 4(n-2)]$

$P(n2^n) = 2^{n-2}[4n - 8(n-1) + 4(n-2)] = 0$

$F_n = A2^n + Bn2^n$

$F_0 = 1 = A$

$F_1 = 6 = 2A + 2B$

$6 = 2 + 2B$

$4 = 2B$

$2 = B \qquad$ So, $A = 1,\ B = 2$

$F_n = 2^n + 2n \cdot 2^n$

$= 2^n(1 + 2n)$

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3. (a) What is the number of ways to place 5 **distinct** balls (numbered 1 to 5) in 3 **distinct** holes {A, B, C}? Provide a short explanation.

Because each ball is distinct, each ball has 3 possible holes, {A, B, C}.

$b_1 \cdot b_2 \cdot b_3 \cdot \ldots \cdot b_K$

$= 3 \cdot 3 \cdot 3 \cdot \ldots \cdot 3 = 3^K$, where $K = 5$

$= 3^5 = 243$

(b) What is the number of ways to place 5 **identical** balls in 3 distinct holes {A, B, C}? Show the combinatorial derivation, together with a brief explanation.

5 balls plus 2 dividers is 7 places. So,

$$C(3+5-1,\ 3-1) = C(7,2) = \frac{7!}{2!\,5!} = 21$$

Positions: 1 2 3 4 5 6 7

Line up: $\underbrace{0\ 0}_{A}\ |\ \underbrace{0\ 0}_{B}\ |\ \underbrace{0}_{C}$

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4. A poker hand is a random drawing of 5 cards out of a deck of 52 cards.
A **full-house** is a hand with three of one kind and two of a second kind. Symbolically we show this as (3, 2).

Suppose the first **two** cards of a poker hand are already known to be **two aces.** (The suits do not matter.) Given this knowledge, derive the conditional probability of ending up with a full-house after receiving 3 more cards. Derive the number of ways (W), and then the probability (P). Provide a brief explanation.

- Get one more ace, and two cards of a second kind.

$$W_1 = C(2,1) \cdot C(11,1) \cdot C(4,2)$$

$$= \frac{2!}{1!1!} \cdot \frac{11!}{1!10!} \cdot \frac{4!}{2!2!} = 2 \cdot 11 \cdot 6 = 132$$

- Get no more aces, and three cards of a second kind.

$$W_2 = C(11,1) \cdot C(4,3)$$

$$= \frac{11!}{1!10!} \cdot \frac{4!}{3!1!} = 11 \cdot 4 = 44$$

$$W = 132 + 44 = 176$$

- Total number of hands after the first two cards

$$U = C(50,3) = \frac{50!}{3!47!} = \frac{50 \cdot 49 \cdot 48}{3 \cdot 2 \cdot 1} = 19600$$

- Probability

$$P = \frac{W}{U} = \frac{176}{19600} \approx \frac{1}{111}$$

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5. (a) Use the iterative GCD algorithm to find the greatest-common-divisor (gcd) of integers $X = 100$, $Y = 19$ and the constants (s, t), where $\gcd = sX + tY$.

X	Y	$q = \lfloor \frac{x}{y} \rfloor$	$r = x - qy = x \bmod y$
100	19	5	5
(1,0)	(0,1)		(1,-5)
19	5	3	4
(0,1)	(1,-5)		(-3,16)
5	4	1	1
(1,-5)	(-3,16)		(4,-21)

$s^r = s^x - (q s^y)$

$t^r = t^x - (q t^y)$

$\gcd = 4 \cdot 100 + 19 \cdot (-21) = 1$

$\gcd = 1$

(b) Use the above results to find $19^{-1} \bmod 100$.

$\gcd = 4 \cdot 100 + 19 \cdot (-21) = 1$

So, $19^{-1} \bmod 100 = 79$

(c) Use exponentiation method to find $7^{-1} \bmod 15$. Provide an explanation.

$x^{-1} = x^{\phi(n)-1} \bmod n$

$\phi(n) = (p-1)(q-1)$, since n is not prime

$= (5-1)(3-1) = 4(2) = 8$

$7^{8-1} \bmod 15 = 7^7 \bmod 15 = 13$