Chris Guarino
HW1

Study Module 1: Sets

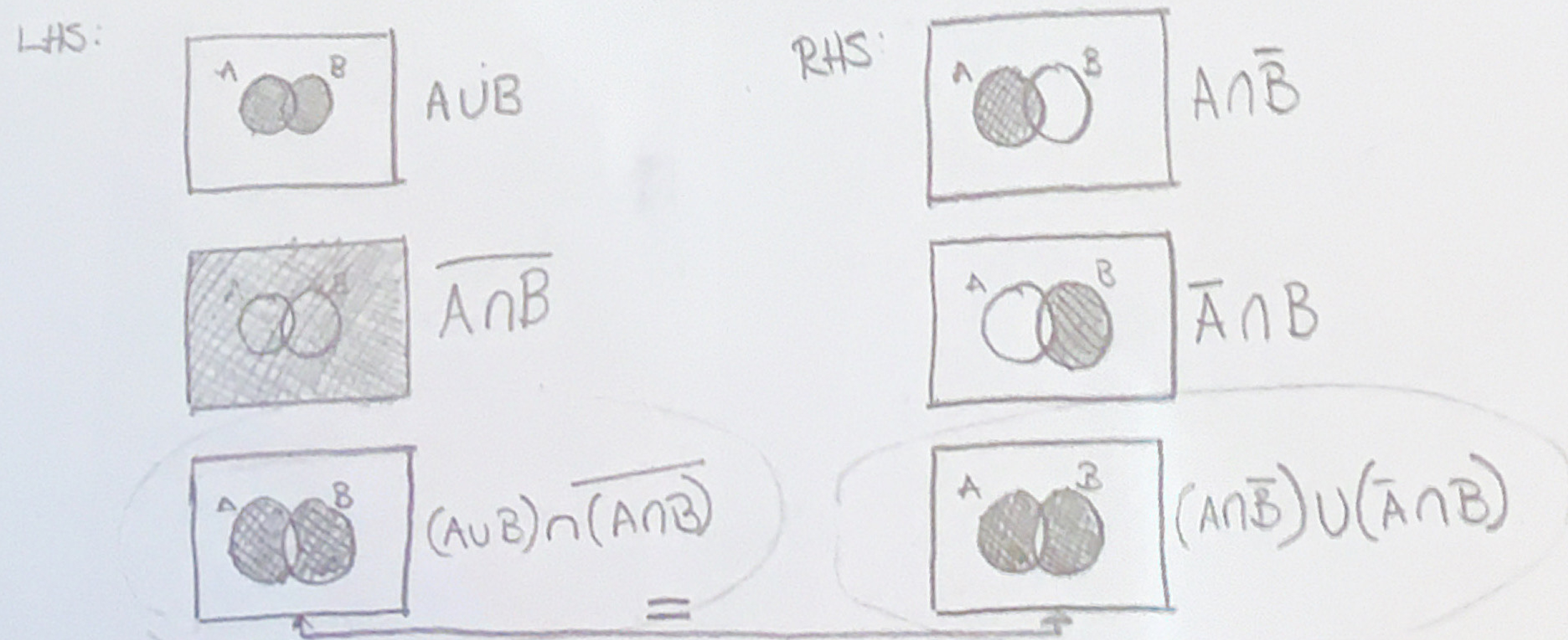
1. Let A = {1, 3, 5} and B = {2, 3, 4}. Determine each of the following sets.

- $A \cup B \rightarrow \{1,2,3,4,5\}$
- $A \cap B \rightarrow \{3\}$
- $A - B \rightarrow \{1,5\}$
- $B - A \rightarrow \{2,4\}$
- $(A \cup B) - (A \cap B) \rightarrow \{1,2,4,5\}$
- $(A - B) \cup (B - A) \rightarrow \{1,2,4,5\}$

2. Prove the following set-equality, first by Venn Diagram, and then by algebraic method.

$$(A \cup B) \cap \overline{(A \cap B)} = (A \cap \overline{B}) \cup (\overline{A} \cap B)$$

Venn Diagram Proof:



Algebraic Proof:

LHS

a. $(A\cup B)\cap(\overline{A}\cup\overline{B})$ - DeMorgans
b. $((A\cup B)\cap\overline{A})\cup((A\cup B)\cap\overline{B})$ - Distributive
c. $((\overline{A}\cap A)\cup(\overline{A}\cap B))\cup((\overline{B}\cap A)\cup(\overline{B}\cap B))$ - Distributive
d. $(\emptyset\cup(\overline{A}\cap B))\cup((\overline{B}\cap A)\cup\emptyset)$ - Complement
e. $(\overline{A}\cap B)\cup(\overline{B}\cap A)$ - Identity
f. $(A\cap\overline{B})\cup(\overline{A}\cap B)$ - Communitive

$(A\cap\overline{B})\cup(\overline{A}\cap B) = (A\cap\overline{B})\cup(\overline{A}\cap B)$ ✓

Chris Guarino
HW1

3. Prove each of the following set equalities by algebraic method. In the proofs, you may use the basic rules listed in the table above.

(a) Combination Rule:

$(A \cap B) \cup (A \cap \overline{B}) = A$

a. $A \cap (B \cup \overline{B}) = A$ – Distributive Reverse
b. $A \cap U = A$ – Complement
c. $A = A$ – Identity ✓

(b) Absorption Rule:

$A \cup (A \cap B) = A$

a. $(A \cap U) \cup (A \cap B) = A$ – Replace A with $A \cap U$
b. $A \cap (B \cup U) = A$ – Distributive Reverse
c. $A \cap U = A$ – Bound
d. $A = A$ – Identity ✓

(c)

$A \cup (\overline{A} \cap B) = A \cup B$

a. $(A \cup \overline{A}) \cap (A \cup B) = A \cup B$ – Distributive
b. $U \cap (A \cup B) = A \cup B$ – Complement
c. $A \cup B = A \cup B$ – Identity ✓

$A \cap (\overline{A} \cup B) = A \cap B$

a. $(A \cap \overline{A}) \cup (A \cap B) = A \cap B$ – Distributive
b. $\varnothing \cup (A \cap B) = A \cap B$ – Complement
c. $A \cap B = A \cap B$ – Identity ✓

(d)

$(A \cap \overline{B}) \cup (A \cap B) \cup (\overline{A} \cap B) = A \cup B$

a. $A \cup (\overline{A} \cap B) = A \cup B$ – Combination Rule
b. $(A \cup \overline{A}) \cap (A \cup B) = A \cup B$ – Distributive
c. $U \cap (A \cup B) = A \cup B$ – Complement
d. $A \cup B = A \cup B$ – Identity ✓

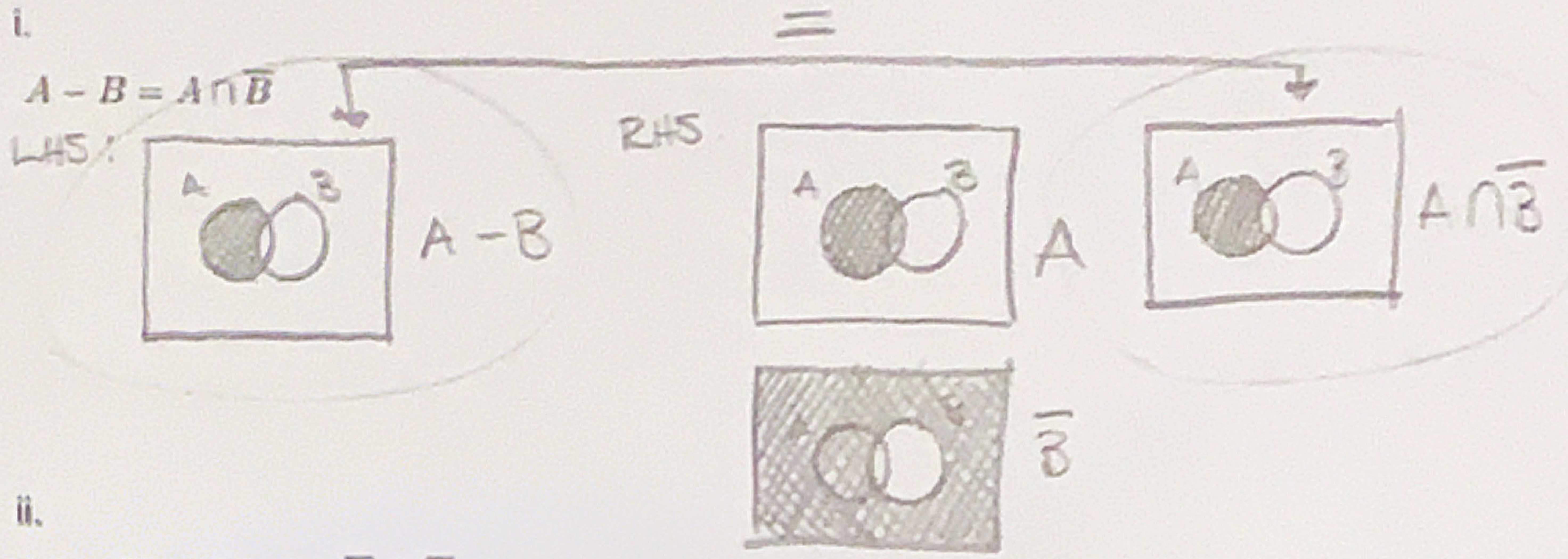
Chris Guarino

HW1

4. (a) Use Venn Diagrams to prove each of the following.

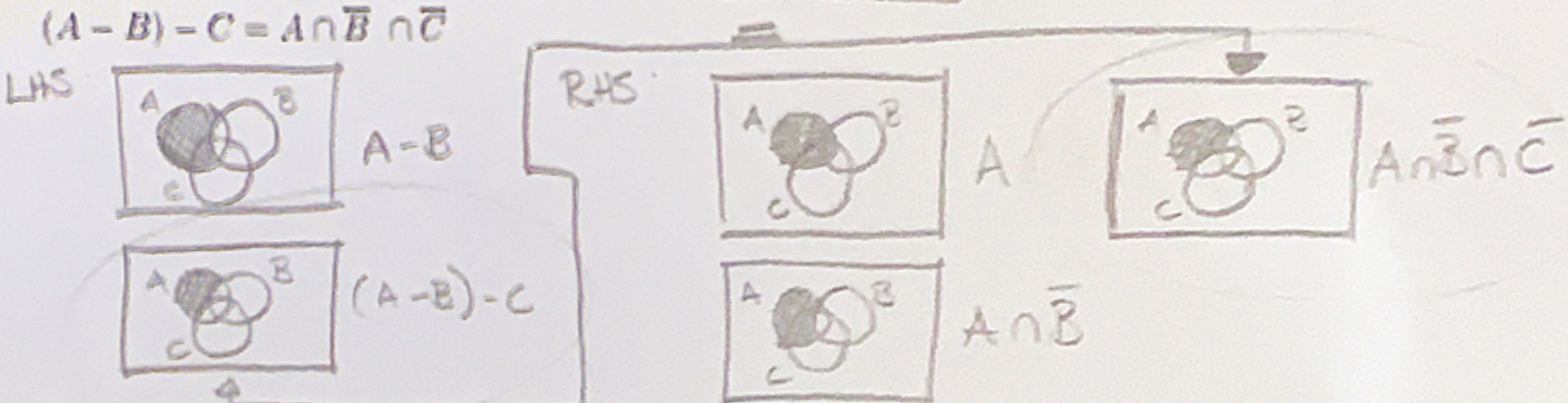
i.

$A - B = A \cap \overline{B}$



ii.

$(A - B) - C = A \cap \overline{B} \cap \overline{C}$



(b) Use the algebraic method to prove each of the following set equalities.

i.

$A - (B - C) = (A \cap \overline{B}) \cup (A \cap C)$

a. $A - (B \cap \overline{C}) = (A \cap \overline{B}) \cup (A \cap C)$ − Replace Set Subtraction

b. $A \cap \overline{(B \cap \overline{C})} = (A \cap \overline{B}) \cup (A \cap C)$ − Replace Set Subtraction

c. $A \cap (\overline{B} \cup \overline{\overline{C}}) = (A \cap \overline{B}) \cup (A \cap C)$ − DeMorgan

d. $A \cap (\overline{B} \cup C) = (A \cap \overline{B}) \cup (A \cap C)$ − Complement

e. $(A \cap \overline{B}) \cup (A \cap C) = (A \cap \overline{B}) \cup (A \cap C)$ − Distributive

ii.

$\overline{(A - B)} = \overline{A} \cup B$

a. $\overline{(A \cap \overline{B})} = \overline{A} \cup B$ − Replace Set Subtract

b. $\overline{A} \cup \overline{\overline{B}} = \overline{A} \cup B$ − DeMorgans

c. $\overline{A} \cup B = \overline{A} \cup B$ − Complement

iii.

$A \cap \overline{(A \cap B)} = A - B$

a. $A \cap (\overline{A} \cup \overline{B}) = A - B$ − DeMorgans

b. $(A \cap \overline{A}) \cup (A \cap \overline{B}) = A - B$ − Distributive

c. $\emptyset \cup (A \cap \overline{B}) = A - B$ − Complement

d. $A - B = A - B$ − Replace Set Subtract Reverse

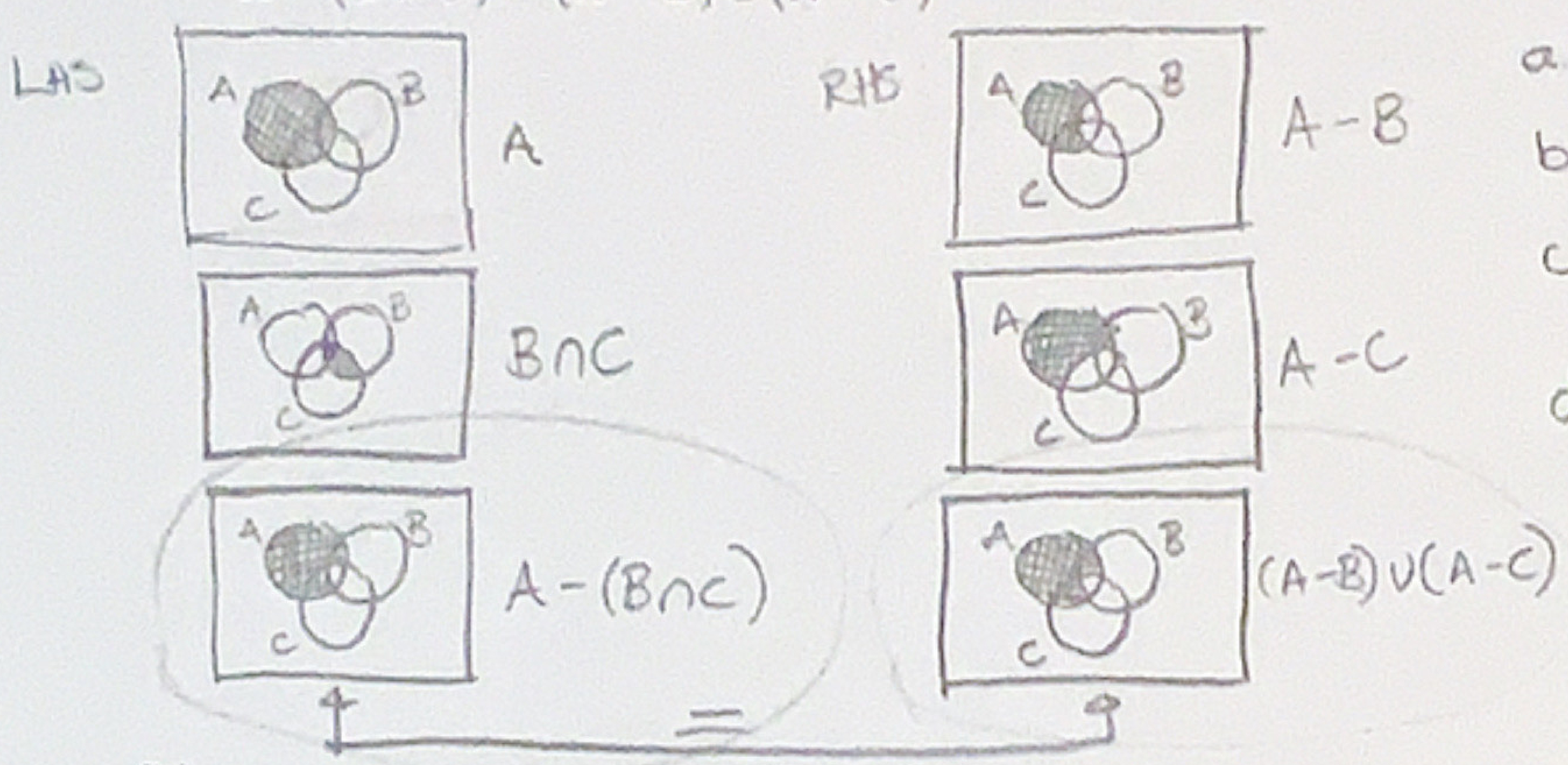
Chris Guarino

HW1

5. Prove each of the following set equalities both by Venn Diagram and by algebraic method.

(a)

$A-(B\cap C)=(A-B)\cup(A-C)$



a. $A\cap(\overline{B\cap C})=(A-B)\cup(A-C)$
└ Replace Set Subtraction

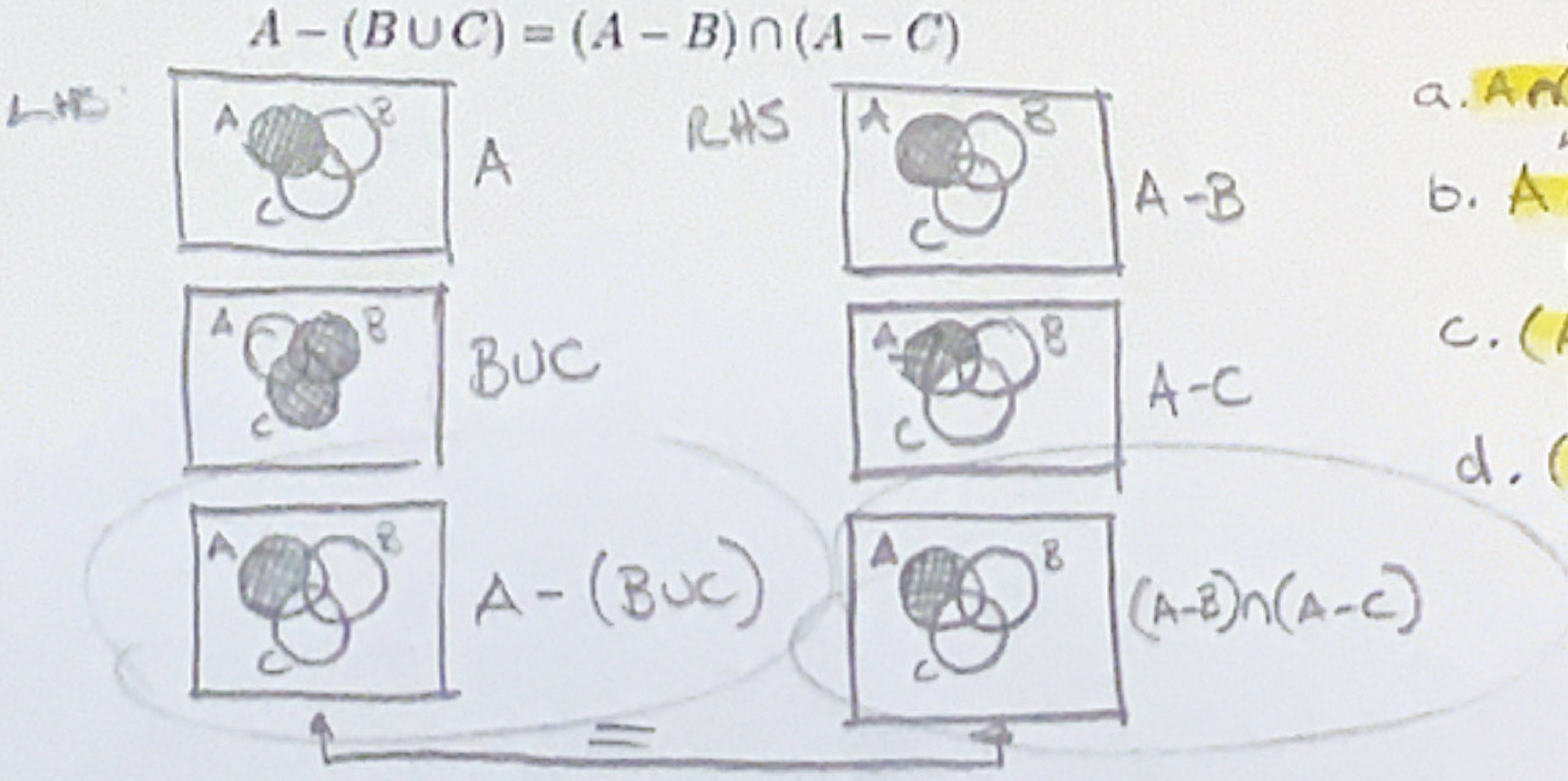
b. $A\cap(\bar{B}\cup\bar{C})=(A-B)\cup(A-C)$
└ DeMorgans

c. $(A\cap\bar{B})\cup(A\cap\bar{C})=(A-B)\cup(A-C)$
└ Distributive

d. $(A-B)\cup(A-C)=(A-B)\cup(A-C)$
└ Reverse Replace Set Subtraction ✓

(b)

$A-(B\cup C)=(A-B)\cap(A-C)$



a. $A\cap(\overline{B\cup C})=(A-B)\cap(A-C)$
→ Replace Set Subtraction

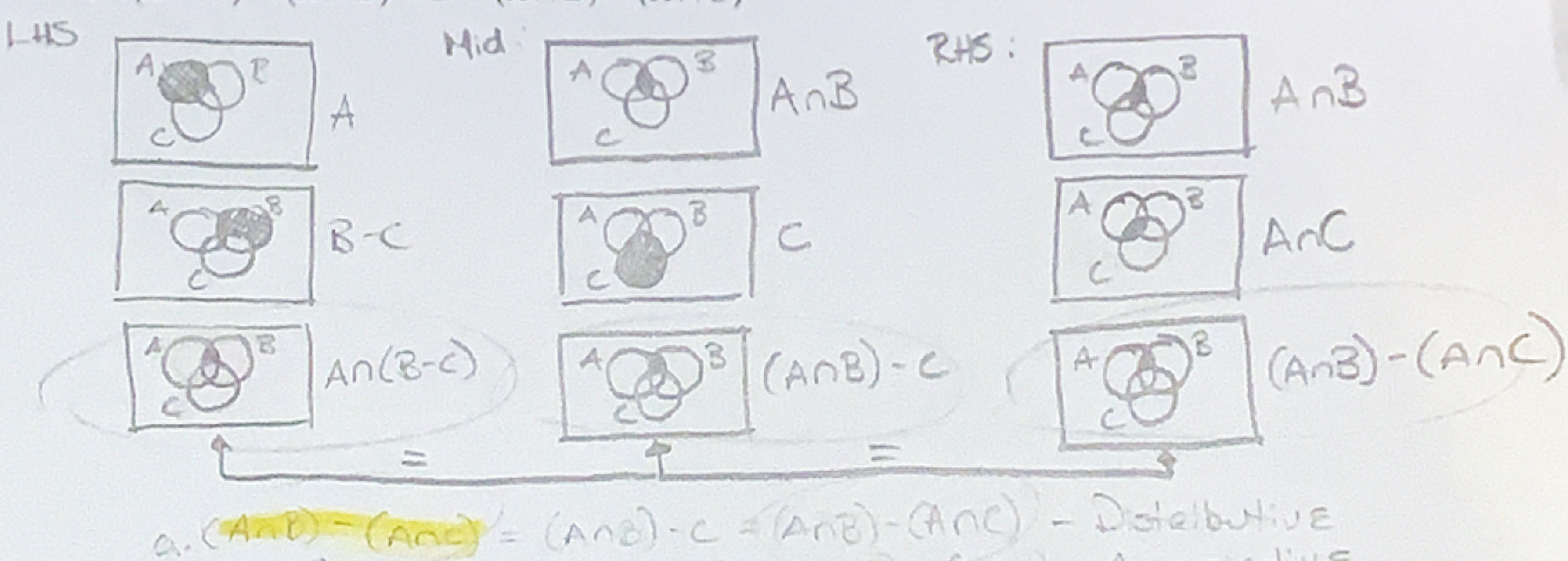
b. $A\cap(\bar{B}\cap\bar{C})=(A-B)\cap(A-C)$
└ DeMorgans

c. $(A\cap\bar{B})\cap(A\cap\bar{C})=(A-B)\cap(A-C)$
└ Distributive

d. $(A-B)\cap(A-C)=(A-B)\cap(A-C)$
└ Reverse Replace Set Subtraction ✓

(c)

$A\cap(B-C)=(A\cap B)-C=(A\cap B)-(A\cap C)$



a. $(A\cap B)-(A\cap C)=(A\cap B)-C=(A\cap B)-(A\cap C)$ - Distributive

b. $(A\cap B)-(A\cap C)=A\cap(B-C)=(A\cap B)-(A\cap C)$ - Associative

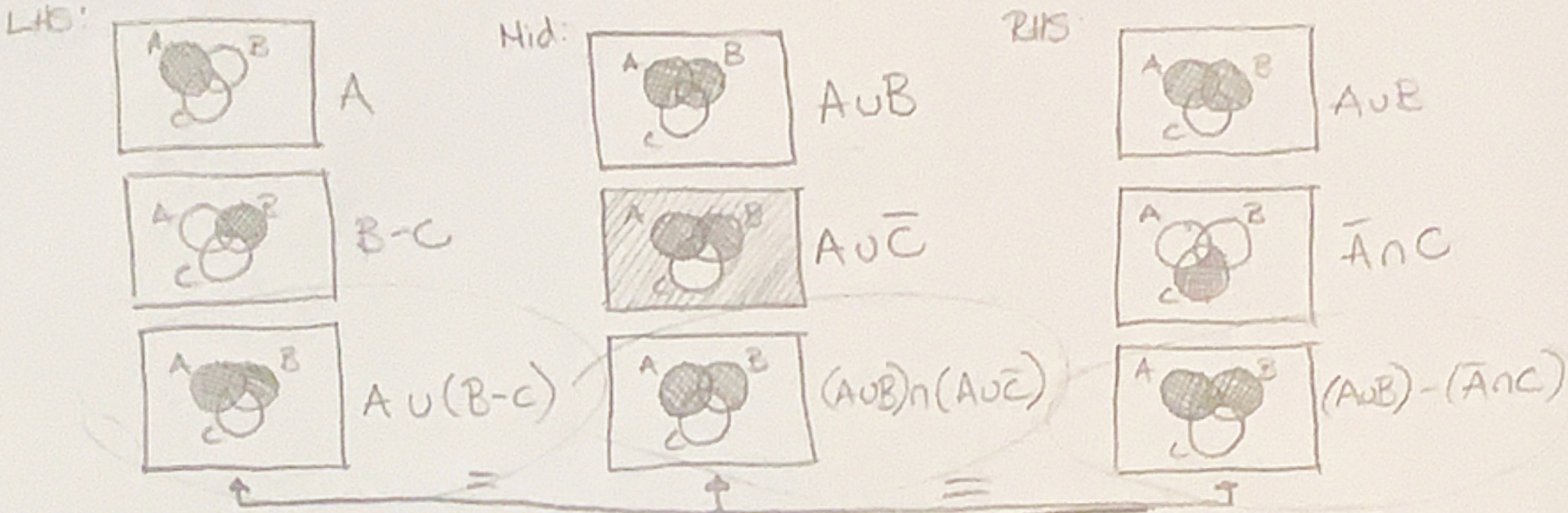
c. $(A\cap B)-(A\cap C)=(A\cap B)-(A\cap C)=(A\cap B)-(A\cap C)$ - Distributive ✓

Chris Guarino

HW1

(d)

$$A \cup (B - C) = (A \cup B) \cap (A \cup \overline{C}) = (A \cup B) - (\overline{A} \cap C)$$



a. $A \cup (B-C) = (A\cup B) \cap (A \cup \overline{C}) = (A \cup B) \cap (\overline{\overline{A} \cap C})$ - Replace Set Subtraction.

b. $A \cup (B-C) = (A\cup B) \cap (A \cup \overline{C}) = (A \cup B) \cap (\overline{\overline{A}} \cup \overline{C})$ - DeMorgans

c. $A \cup (B-C) = (A\cup B) \cap (A \cup \overline{C}) = (A \cup B) \cap (A \cup \overline{C})$ - Complement

d. $A \cup (B \cap \overline{C}) = (A\cup B) \cap (A \cup \overline{C}) = (A \cup B) \cap (A \cup \overline{C})$ - Replace Set Subtraction

e. $(A \cup B) \cap (A \cup \overline{C}) = (A\cup B) \cap (A \cup \overline{C}) = (A \cup B) \cap (A \cup \overline{C})$ - Distributive ✓