Some thoughts on radians. A full circle is arbitrarily defined to be 360 degrees, and all other angles are measured relative to that: half a circle is 180 degrees, one third of a circle is 120 degrees, etc. Similarly, a full circle is defined to be 2π radians, and all other angles are measured relative to that: half a circle is π radians, one third of a circle is $\frac{2}{3}\pi$ radians, etc. Radians are less arbitrary / more fundamental than degrees because of the following property. Take a fragment of a circle (i.e. draw straight lines from any two points on the edge into the centre, giving the shape of slice of cake or pizza). Dividing the length of the curved edge by the radius gives the angle at the point (the angle 'subtended at the middle') in radians. Doing this for a whole circle gives 2π radians because the circumference of a circle is 2π times the radius.

Finding the period of oscillatory behaviour. If you see $\sin(\omega t)$ written, where ω has some particular value in your case, you can think about re-writing it as $\sin(2\pi t/T)$. Comparing the two shows that $\omega = 2\pi/T$. This is a helpful form however, because you can see that when $t = T$, the argument of the sine function is 2π, corresponding to one full cycle of the oscillation: we're back to we started. This means T is the *period* of the oscillation. Re-arranging the expression $\omega = 2\pi/T$ shows that the period T is given by $2\pi/\omega$. Sometimes people include an explicit factor of π in omega, for example writing $\sin(\frac{2}{3}\pi \text{ per week } t)$ – here $\omega = \frac{2}{3}\pi$ per week, so $T = 2\pi/(\frac{2}{3}\pi \text{ per week}) = 3$ weeks.

A bit of terminology: a function of the form $y = a^x$, where a can be anything, is referred to as *an* exponential function. The case where a has the value $e = 2.718\ldots$ is referred to as *the* exponential function.

A worked example, using exponentials and logs. The incidence for a particular epidemic, I, begins increasing with time t as follows: $I(t) = 1 \text{ per day} \times 10^{t\times 2 \text{ per month}}$. Starting from $t = 0$ and assuming continued exponential growth,

- What is the incidence after one week? (ANS: first, 1 year = 12 months = 365.25 days = 365.25 days / (7 days per week) = (365.25/7) weeks; so one week = $(7 \times 12/365.25)$ months. Then,

$$\begin{aligned} I(1 \text{ week}) &= 1 \text{ per day} \times 10^{1 \text{ week}\times 2 \text{ per month}} \\ &= 1 \text{ per day} \times 10^{(7\times 12/365.25)\text{months}\times 2 \text{ per month}} \\ &= 1 \text{ per day} \times 10^{7\times 12\times 2/365.25} \\ &= 2.9 \text{ per day (2 s.f.)} \end{aligned} \tag{1}$$

- How long would it take, to reach an incidence of 1000 per day? (ANS: $1\frac{1}{2}$ months)

- What is the doubling time? (ANS: taking natural logs (i.e. log base e) of both sides, we see this is $\frac{\ln 2}{2\ln 10}$ months = 0.15 months (2 s.f.) $\approx 4\frac{1}{2}$ days)

- What is 'the exponential growth rate', with its usual meaning? Hint, first rewrite the expression $I(t)$ so that the exponential uses base e instead of base 10. (ANS: $10^x = \exp(\ln(10^x)) = \exp(\ln(10) \times x)$, so $I(t) = 1 \text{ per day} \times e^{t\times 2\times \ln(10) \text{ per month}}$. The exponential growth rate is therefore $2 \times \ln(10)$ per month = 4.6 per month $(2s.f.)$.)