Core Mathematics in the MPH and MSc in Epidemiology Courses, Imperial College London

Session 1: Manipulating Numbers

Chris Wymant
c.wymant@imperial.ac.uk

October 2014

Do please email me any typos you find. If anything isn't clear, for preference tell me in person.

Motivation

"As a matter of fact all epidemiology, concerned as it is with variation of disease from time to time or from place to place, must be considered mathematically, however many variables are implicated, if it is to be considered scientifically at all. *To say that a disease depends on certain factors is not to say much, until we can also form an estimate as to how largely each factor influences the whole result.* And the mathematical method of treatment is really nothing but the application of careful reasoning to the problems at hand." Sir Ronald Ross, 1911. The italics are my own. Despite the quote singling out epidemiology, MPH students should feel the relevance of the quote as much as MSc students!

Science is about understanding how the things around us interact with each other and affect each other. Wherever possible we want to know not just that one thing depends on another, but *how*, and *how much*: that means maths. To understand quantitatively a relationship you're interested in – for example, how the number of people infected with a disease depends on certain factors – you need to know something about maths, which is hopefully obvious and hopefully why you're attending this optional course.

A Note on Examples/Applications

In teaching this course we will endeavour to give many examples of how the tools we'll be talking about help us to understand problems in public health and epidemiology; however, presenting an application for each and every mathematical rule we discuss would both be very slow, and result in some fairly contrived examples.

Just to broaden the scope for a moment: once you have identified a mathematical topic as being relevant to your work, then it's a very useful skill as a scientist (or health economist) to be able to to sit down and learn that bit of maths in its own right, turning off just momentarily that part of your brain saying "how does this apply to my example, how does this apply to my example, how does this apply to my example". For example, later in these first two sessions on the basics, I'll quickly explain why $x^a \times x^b = x^{a+b}$, and it's best for you to approach this without the idea fixed in your head that x is some particular quantity – prevalence, the concentration of a drug in one's bloodstream, whatever – but just to understand it as a mathematical rule in generality, where x can be anything at all. You'll get a deeper understanding that way.

1 What is a Function?

A function takes a number a gives you a number back. The number you give it is called the function's *argument*. You can give it lots of different numbers (usually any number, but a few functions are more fussy) and can get lots of different numbers back; we use this to describe the dependence of one quantity on another. We visualise functions with a graph/plot; see session 5. A function can be described in words: for example "whatever number you give me, I give you it back plus two", or "whatever number you give me, I give you one back four times as big", but for brevity and clarity it's better to put a label on "whatever number you give me". Let's call it x.[1] Then the function "whatever number you give me, I give you it back plus two" can be written more concisely as $x + 2$; the function for multiplication by 4 can be written as $4 \times x$ etc.

So we call the number going into the function x; what do we call the function itself? When you first encountered functions at school, it's quite possible you saw one, or the other, or both of two traditional notations:

- with x the independent variable – something we can choose freely – we say y is the dependent variable, depending on x.
- $f(x)$, read out loud as "f of x", meaning the function f is a function of x.

Sometimes the notations are mixed: you might see $y = y(x)$ or $y = f(x)$ written to mean y is a function of x. However it's important not to build your understanding of something based on what particular letter is used – don't understand functions as y always depends on x and that's as far it goes; rather one quantity depends on another, and different symbols can be used in different contexts. In other courses you'll want to know how the number of infected people I changes with time t, and the notation $I(t)$ may or may not be used – so long at you understand I changes with t that's all that matters.

This touches on a general point: in one self-contained piece of maths, you can use any letter for any quantity, provided you're consistent, and it makes no difference to the result. For example, if you want to solve $f(x) = C$ for x (i.e. find the value of x for which this is true) and you've already seen the solution of $f(y) = C$, you can take that solution and just replace y by x.

2 Recap: Working with fractions

Functions are about doing things with numbers, so let's dive into some basic things we want do with numbers. I'm assuming you're all familiar with how the $+$, $-$ and $\times$ operators work. As for division, a common error made by people who haven't done maths for a long time is failure to understand the following:

$$\frac{a+b}{c} = \frac{a}{c} + \frac{b}{c} \qquad \text{but} \qquad \frac{a}{b+c} \neq \frac{a}{b} + \frac{a}{c} \tag{1}$$

[1] Historical aside: the traditional use of x to represent the unknown in equations we're solving is why X-rays are called X-rays. When first discovered they were known to be radiation, but beyond that their nature was unknown.

The easiest way to convince yourself that these are true is to try choosing values of a, b and c and to evaluate the fractions; keeping in mind the division of a number of cakes between a number of people never goes amiss. Note also:

$$a \times \frac{b}{c} = \frac{a \times b}{c} \tag{2}$$

$$\frac{a}{b} \div c = \frac{a}{b \times c} \tag{3}$$

or combining these last two,

$$\frac{a}{b} \times \frac{c}{d} = \frac{a \times c}{b \times d} \tag{4}$$

DISCUSS EXAMPLES IN CLASS

3 Operator Precedence ('BIDMAS' for Grown-ups)

To evaluate $3 \times 2 + 2$ does one do the multiplication first (giving 8) or the addition first (giving 12)? The former. This is because the multiplication operator has a *higher precedence* than the addition operator: its operation comes first. At school this is sometimes taught using the mnemonic 'BIDMAS': brackets have highest precedence, then indices (raising things to powers), then division, then multiplication, then addition and finally subtraction. Taken literally, this is wrong: $3 - 2 + 1$ is 2, not 0, because the subtraction is done before the addition, contrary to what is suggested by 'BIDMAS'. The subtraction and addition operators have the same precedence, and so the order in which the operations are performed is simply from left to right. Likewise, division and multiplication have the same precedence as each other. Indices have higher precedence still: $2 + 3^2 \times 2 = 2 + 9 \times 2 = 2 + 18 = 20$. Brackets have the highest precedence of all because their raison d'être is to allow manual specification of the intended order of evaluation, overriding the other rules. e.g. $(2 + 3)^2 = 5^2 = 25$.

DISCUSS EXAMPLES IN CLASS

4 Playing with Powers

A note before starting: for brevity we usually suppress the $\times$ symbol between two letters and also between a letter and a number (but not between two numbers). For example,
$a \times b = ab$,
$3 \times x = 3x$,
but $2 \times 3 \neq 23$.
Exceptions: a number of functions have a name with multiple letters: $\sin(x)$, $\cos(x)$, $\tan(x)$, $\log_a(x)$, $\ln(x)$, $\exp(x)$; this does not mean those letters are being multiplied together – it's just one function. Sometimes you'll see the brackets dropped with these functions: $\sin(x)$ as $\sin x$, and this doesn't mean the sine function multiplied by x (which doesn't make sense), it still means x is the argument of the sine function.

Hopefully you already know what squaring something means: $x^2 = x \times x$. More generally, raising a number to a power a means considering a factors of that number all multiplied together:
$x^3 = x \times x \times x$
$x^a = \underbrace{x \times x \times \ldots x}_{a \text{ factors}}$

Simply by using the definition of what raising to a power means, we can derive some important results. Firstly, raising a product to a power:

$$\begin{aligned}(x \times y)^a &= \underbrace{(x \times y) \times (x \times y) \times \ldots (x \times y)}_{a \text{ factors of } (x \times y)} \\ &= \underbrace{x \times x \times \ldots x}_{a \text{ factors}} \times \underbrace{y \times y \times \ldots y}_{a \text{ factors}} \\ &= x^a \times y^a \qquad (5)\end{aligned}$$

In the middle we used the following property of multiplication: $x \times y = y \times x$, i.e. order doesn't matter. Therefore in a product of multiple numbers one can swap around the numbers however one desires: $a \times b \times c \times d = c \times a \times d \times b$ etc. By extension of the rule $(x \times y)^a = x^a \times y^a$, we have

$$(x \times y \times z \times \ldots)^a = x^a \times y^a \times z^a \times \ldots \qquad (6)$$

To see why this is true, you can start with the rule for just x and y and then define y to be a product of two other numbers.

Next, multiplying two different powers of the same number: $x^a \times x^b$:

$$\begin{aligned}x^a \times x^b &= \underbrace{x \times x \times \ldots x}_{a \text{ factors}} \times \underbrace{x \times x \times \ldots x}_{b \text{ factors}}, \\ &= \underbrace{x \times x \times \ldots x}_{(a+b) \text{ factors}} \\ &= x^{a+b} \qquad (7)\end{aligned}$$

Now, multiplying by a number then dividing by that same number does nothing. Therefore when multiplying by x a times and dividing by x b times, b of the a multiplications get cancelled out:

$$\begin{aligned}x^a \div x^b &= \frac{\overbrace{x \times x \times \ldots x}^{a \text{ factors}}}{\underbrace{x \times x \times \ldots x}_{b \text{ factors}}}, \\ &= \frac{\overbrace{x \times x \times \ldots x}^{b \text{ factors}} \times \overbrace{x \times x \times \ldots x}^{(a-b) \text{ factors}}}{\underbrace{x \times x \times \ldots x}_{b \text{ factors}}}, \\ &= \underbrace{x \times x \times \ldots x}_{(a-b) \text{ factors}} \\ &= x^{a-b} \qquad (8)\end{aligned}$$

Another useful result.

So we know $x^a \div x^b = x^{a-b}$. Choosing $b = a$ gives $x^a \div x^a = x^0$; this should clearly equal 1, since any number[2] divided by itself is 1. This tells us that any number[3] raised to the power 0 gives 1.

We can define what is meant by a *negative* power using our understanding that $x^0 = 1$ and $x^a \times x^b = x^{a+b}$. Together, these imply that $x^a \times x^{-a} = x^0 = 1$, which means that x^{-a} is the *reciprocal* of x^a, i.e. 1 divided by it:

$$x^{-a} = \frac{1}{\underbrace{x \times x \times \ldots x}_{a \text{ factors}}} \tag{9}$$

Recalling our definition of x^a, we can obtain a result for $(x^a)^b$:

$$\begin{aligned}(x^a)^b &= \underbrace{(\underbrace{x \times x \times \ldots x}_{a \text{ factors of } x}) \times (\underbrace{x \times x \times \ldots x}_{a \text{ factors of } x}) \times \ldots}_{\text{b factors of the bracket, each containing } a \text{ factors of } x}, \\ &= \underbrace{x \times x \times \ldots x}_{(a \times b) \text{ factors of } x}, \\ &= x^{ab} \end{aligned} \tag{10}$$

From the definition of x^a, x^1 is simply x. Then since $(x^a)^b = x^{ab}$, we know that $(x^{1/n})^n = x^1 = x$, which defines

$$x^{1/n} = \sqrt[n]{x} \tag{11}$$

as the number that gives x when multiplied together n times. For $n = 2$ we say the *square* root, for $n = 3$ the *cube* root, for greater n simply the nth root. For even n there are always two solutions – plus-or-minus the same thing – because the product of an even number of negative numbers is positive.
e.g. $4^{1/2} = \pm 2$ because $2 \times 2 = 4$ and $(-2) \times (-2) = 4$.
c.f. $8^{1/3} = 2$, because $2 \times 2 \times 2 = 8$ but $(-2) \times (-2) \times (-2) \neq 8$.

DISCUSS EXAMPLES IN CLASS

5 Scientific Notation a.k.a. Standard Form

A figure is in standard form when it is expressed in the following way:
$a \times 10^b$,
where $1 \leq a < 10$ and b is an integer (a whole number). Examples:

- $568,000,000 = 5.68 \times 10^8$
- $0.00014 = 1.4 \times 10^{-4}$

The number system you all know and love is *decimal*, or base-ten: there are ten different digits including zero, and so we need to start using two digits from the number ten upwards. This means that the interpretation of b – the power of ten required in standard form – is the number

[2] Any number except zero: 0/0 is undefined
[3] Any number except zero: 0^0 is undefined

of positions one has to move the first digit to the right before it touches the decimal point sign (which is usually not written for whole numbers, but is lurking there implicitly nonetheless). Numbers in standard form are easily multiplied, divided and raised to powers, because we can use the rules for manipulating powers from the previous section to manipulate the powers of ten. To add or subtract numbers in standard form it's easiest to take one of them *out* of standard form if necessary, so that they're both multiplied by the same power of ten. For example:

$$\begin{aligned} 8 \times 10^7 - 3 \times 10^5 &= 800 \times 10^5 - 3 \times 10^5 \\ &= (800 - 3) \times 10^5 \\ &= 797 \times 10^5 \\ &= 7.97 \times 10^7 \end{aligned}$$

DISCUSS EXAMPLES IN CLASS

6 Decimal Places and Significant Figures

Most of the time, when we quote a number to someone, neither of us are interested in knowing the number to an arbitrary amount of precision. In 2013, the official population of Greater London was 8,416,535; however unless you need to carefully account for every individual (perhaps if you're keeping track of who is voting, for example) you might only care that it's 8.42 million, or about 8 million, etc. according to taste. This means *rounding* numbers appropriately.

Usually, the extent to which we round a number depends on how big it is. In the above example knowing the population of Greater London to the nearest whole number (of course, it has to be a whole number) is too much information and we want to round it more. However if I asked 'how many metres tall are you?', you wouldn't round to the nearest whole number of metres, because that's too little information in this case. Therefore what we usually want is a reasonable amount of *relative* precision – the level at which we're rounding the number should be set relative to the size of the number itself. This is where the concept of significant figures, henceforth sig figs, is useful.

A number rounded to n sig figs means the nth digit in the number, going from left to right and ignoring any leading zeroes (e.g. for 0.000456 you start at the digit 4), is rounded. Rounding to n decimal places is easier: the nth digit after the decimal place is rounded. Either way, digits $1, 2, 3, 4$ get rounded down; $5, 6, 7, 8, 9$ get rounded up; and 0 doesn't change when rounded.

DISCUSS EXAMPLES IN CLASS

Getting an idea of the implied level of precision from the way the number is given:

- '180 minutes' could mean (a) anything between 175 and 185 minutes, or (b) anything between 179.5 and 180.5 minutes; it depends on the context (we can't tell if it's 2 s.f. or 3 s.f.).
- 'Three hours' should mean anything between two-and-a-half and three-and-a-half hours.
- 'Half a day' probably means anything from a quarter of a day to three-quarters of a day.

6.1 Combining Numbers with Different Precision (Sometimes Tricky...)

This touches upon the subject of combining uncertainties in different quantities, which can be hard, and I suspect will not be tested in modules outside of the statistical and Bayesian ones. I mention it here for completeness, because if you're ever working with numbers outside of this masters, and your answer matters, you should be aware of these issues.

When multiplying and dividing numbers with different numbers of sig figs, as a rule of thumb quote your answer to the least number of sig figs from all the numbers one started with:

$$\begin{aligned} 20\,(2\,\text{s.f.}) \times 122.44\,(5\,\text{s.f.}) &= 2400\,(2\,\text{s.f.}) \quad \text{(fine)} \\ &= 2448.8\,(5\,\text{s.f.}) \quad \text{(misleading)} \end{aligned}$$

If lives depend on your giving an irreproachably correct answer, there's nothing for it but to carefully consider what are the largest and smallest possible answers based on the sig figs of your input numbers. For example, what is $20\,(1\,\text{s.f.})/10\,(1\,\text{s.f.})$? Using the rule of thumb, one would simply say $2\,(1\,\text{s.f.})$, which means anything between 1.5 and 2.5. However, the largest possible value for the answer is when the numerator in the fraction is as large as it can be and the denominator as small as it can be: based on their respective sig figs, this is $25/5 = 5$. The smallest possible value for the answer comes from the reverse situation, which is $15/15 = 1$. So in this case, we can't rule out the answer being as small as 1 or as large as 5: a fairly different conclusion from saying $2\,(1\,\text{s.f.})$ based on the rule of thumb. The large difference here arises because of the smallest possible number of sig figs for the inputs – just one each – and from my choosing a number whose first digit was 1, for which 1 sig fig accuracy means a large 50% uncertainty (to be compared to a number beginning with the digit 9, for which 1 sig fig accuracy means a smaller $0.5/9 = 6\%$ uncertainty).

Adding and subtracting numbers may also require checking how large or small the answer could be. For example $80\,(1\,\text{s.f.}) + 1\,(1\,\text{s.f.})$ can be anything between 75.5 and 86. Quoting this answer as $80\,(1\,\text{s.f.})$ is therefore not correct. (The error may however be small enough not to make any difference, but there's nothing wrong with giving the right answer, right?)

7 Some Common Symbols and Their Meanings

The mathematical symbols in Table 1 frequently crop up in descriptions of scientific or real-world problems; it's best to memorise them.

Symbol	Meaning	Examples
$<$	less than	$1 < 2$ (true); $2 < 2$ (false); $3 < 2$ (false)
$>$	greater than	$1 > 2$ (false); $2 > 2$ (false); $3 > 2$ (true)
$\leq, \leqslant$	less than or equal to	$1 \leq 2$ (true); $2 \leq 2$ (true); $3 \leq 2$ (false)
$\geq, \geqslant$	greater than or equal to	$1 \geq 2$ (false); $2 \geq 2$ (true); $3 \geq 2$ (true)
$\ll$	much less than (subjective, context dependent). It's usually based on the relative difference, rather than the absolute difference.	$0.001 \ll 1$ (usually true); $9.001 \ll 10$ (usually false)
$\gg$	much greater than (subjective, context dependent)	$1 \gg 0.001$ (usually true); $10 \gg 9.001$ (usually false)
$\pm$	plus or minus	See the discussion in the text.
$\simeq, \approx$	approximately equal to (subjective, context dependent)	For a *total* budget for a national health emergency, $£1 \approx £2 \approx £0$: in each case you've got effectively no money to act. If these three quantities represent instead the average amount donated by each person in the country, the three cases are *not* roughly similar, they are dramatically different. Context!
$\propto$	proportional to. This means if one thing increases by a given factor, the other increases by that same factor (e.g. if one doubles, the other doubles). Equivalently, their ratio is constant. y is proportional to x means $y = mx$ for some constant c.	Without a multi-buy special offer (e.g. buy two get one free), the total price for multiple identically priced items is proportional to the number of items bought.
$\ldots$	Used to replace something the author hopes is obvious.	1+2+3+…+8 = 1+2+3+4+5+6+7+8
$\sum$	'sum over', 'the sum of'	$\sum_{i=1}^{4} i = 1 + 2 + 3 + 4$
$\prod$	'multiply together', 'the product of'	$\prod_{i=3}^{5} i = 3 \times 4 \times 5$
!	factorial: the product of all numbers between 1 and the number in question, inclusive	$9! = 9 \times 8 \times 7 \ldots \times 2 \times 1$
%	per cent, literally one per hundred.	See the discussion in the text.
$\implies$	implies. $A \implies B$ means 'B is true if A is true', or 'A is a sufficient condition for B'.	$x = 2 \implies x^2 = 4$
$\therefore$	'therefore'. Based on everything that has been said so far, we can draw the conclusion that follows.	$x < -2$, $y = -1$, $z > 3$, $\therefore x < y < z$. I think $\therefore$ I am.
$\vert$	'given that'	The probability of wet pavements in the morning $\vert$ we're in a drought $<$ the probability of wet pavements in the morning $\vert$ it rained in the night.
$\vert\ldots\vert$	the 'modulus' or 'absolute value' of something. $\vert x\vert = \begin{cases} x, & \text{if } x \geq 0 \\ -x, & \text{if } x < 0 \end{cases}$	$\vert 2\vert = 2$; $\vert -2\vert = 2$
$\int$	the integral sign	See the session devoted to integration.

Table 1: Some common symbols, their meanings, and examples.

Discussion of the percent symbol. Sometimes the % symbol is exactly equivalent to the number '0.01', particularly if dealing with dimensionless (unitless) numbers constrained to be between 0 and 1; e.g. 'a 5% chance' means a probability of $5 \times 0.01 = 0.05$. Other times it isn't: compare 'the average weight increased by 0.05' and 'the average weight increased by 5%' – in the former case we are compelled to ask '0.05 what?'. Here the meaning of % is to show that the associated figure is a *relative* measurement – the increase is 0.05 times whatever the value was before. Note that a murky point of interpretation is percentage changes in quantities that are themselves naturally expressed as percentages: 'prevalence was 5% then increased by 10%' – does that give a prevalence of 5.5% or 15%? If you're the one saying it, be explicit; if someone else says it, get them to clarify.

Two meanings of the ± symbol. Sometimes it means 'plus or minus' in a perfectly precise and literal sense. For example $\sqrt{2} = \pm 2$: here the answer is $+2$ or -2, because both give 4 when squared. Other times it means 'probably somewhere in between the plus value and the minus value', indicating uncertainty in a measurement. For example, an incidence said to be '$10,000$ per year plus or minus $1,000$', written $(10,000 \pm 1,000)$ year^{-1}, means $10,000$ is our estimate and the uncertainty is $1,000$ in each direction (bigger or smaller).

You may encounter the symbols in Table 2 but, in the author's opinion, less commonly. You probably don't need to memorise them – just look them up if you need them. Here they are for reference.

Discussion of real numbers. Specifying that one is dealing with *real* numbers explictly excludes the possibility of *imaginary* numbers. Since no number on the real number line (the one-dimensional continuous line stretching from negative infinity to positive infinity) gives -1 when squared, the imaginary number i is simply defined to be the square root of -1. This is far beyond the scope of this course, and indeed your masters – you should take for granted that all numbers you're dealing with are real. It's totally fine to know simply these two points: all real numbers give something greater than or equal to zero when squared, and so square-roots (and fourth-roots, and sixth roots, etc.) should only be used on positive numbers or zero, provided you want a real answer, which you always will here. i is also used to mean other things in other contexts, so if you do see it, assume it means something else and not $\sqrt{-1}$.

Note that some symbols exist with slashes through them to negate the symbol's meaning: $\neq$ means not equal to, $\notin$ means not in, $\not\Longrightarrow$ means 'does not imply' (which is not the same as 'implies that the following is false').

7.1 Subscript Indices: *Vectors*

Sometimes it is convenient to collect together numbers into a single object. In this case, one symbol is used for the object, and a subscript index denotes which number in the object we are talking about. For example, if I am going to make a number of measurements of something traditionally called x, I would probably call the value of my first measurement x_1, the value of my second measurement x_2 etc. Now, a *vector* is really just a collection of numbers[4], so we can consider our collection $(x_1, x_2, \ldots)$ to be a vector. The notation for vectors in typeset text is usually bold font:

$$\mathbf{x} = (x_1, x_2, \ldots) \tag{12}$$

and in handwritten text, the impracticality of bold font means squiggly underlining $\underset{\sim}{x}$ and over-arrows $\vec{x}$ are used, to taste.

[4] An alternative way of thinking about a vector is a single number together with a *direction.* This is equivalent, because n numbers can be thought of as a set of coordinates for a point in n-dimensional space; the point can also be specified by its distance from the origin and what direction it is in.

The subscript can itself be a variable: just as x_1 is the first element of $\mathbf{x}$, x_i is the ith element. It can be useful to refer to the elements of a vector non-specifically like this. For example, a concise way of writing the sum of all elements of a vector is the left-hand side below:

$$\sum_i x_i = x_1 + x_2 + \ldots \tag{13}$$

Note that contrary to the example shown in Table 1, I have not explictly stated the lowest and highest values of i that the sum should contain. Instead, it is implicit that the sum should contain (or the sum should 'run over') all of the values of i there are. If $\mathbf{x}$ contains n elements, then i in the left-hand side of Eq. 13 is understood to run from 1 to n.

Acknowledgement

I wrote these notes while funded by ERC Advanced Grant PBDR-339251.

Symbol	Meaning	Examples
‰	per mille; literally one per thousand.	5‰ = five per mille = 0.005
$\Longleftarrow$	is implied by. $A \Longleftarrow B$ means 'A is true if B is true', or 'B is a sufficient condition for A'	$x^2 = 4 \Longleftarrow x = 2$
$\Longleftrightarrow$	implies and is implied by. $A \Longleftrightarrow B$ means 'B is true if and only if A is true', or 'A is a necessary and sufficient condition for B'.	$x^2 = 4 \Longleftrightarrow x = \pm 2$
$\because$	because	$2x^2 \geq x^2$ $\because$ for all $x \neq 0$, $2x^2 > x^2$; but for $x = 0$, $2x^2 = x^2$
$(\ldots, \ldots)$	All numbers between, but not including, the two numbers specified.	$(1, 2)$ means all numbers between 1 and 2 not inclusive.
$[\ldots, \ldots]$	All numbers between and including the two numbers specified.	$[1, 2]$ means all numbers between 1 and 2 inclusive.
$\{\ldots\}$	The set of numbers specified in between the curly braces.	$\{1, 2, 4, 7\}$ the set of numbers 1, 2, 4, and 7.
$\in$	is in, is an element of	p is a probability $\Longrightarrow p \in [0, 1]$
$\mathbb{Z}$	The set of all integers (whole numbers): $\mathbb{Z} = \{\ldots - 3, -2, -1, 0, 1, 2, 3, \ldots\}$	$x \in \mathbb{Z}$ means x is a whole number.
$\mathbb{R}$	The set of all real numbers, including all possible numbers in between any two integers.	$x \in \mathbb{R}$ means x is a real number. See the discussion in the text.
$:=$	is defined to be. Used for things that are true because we define them to be so, not because we have derived them from other results.	For a triangle with a side O opposite an angle θ, and hypotenuse H, $\sin\theta := O/H$.
$\equiv$	Two meanings. The first is identical to ':='. The second is 'is identical to', denoting an equality which is always true and not just in the current context (though many people just use the regular equals sign for this).	$2(x + 1) \equiv 2x + 2$ (this is true regardless of the value of x).
$\subset$	is a subset of, is contained by	$\{1, 2\} \subset \{1, 2, 3\}$
$\supset$	is a superset of, contains	$(1, 2) \supset (1.1, 1.9)$
$\cup$	the union of	$\{1, 2, 3\} \cup \{3, 4, 5\} = \{1, 2, 3, 4, 5\}$
$\cap$	the intersection of	$\{1, 2, 3\} \cap \{3, 4, 5\} = \{3\}$
$\forall$	for all	$2x^2 > x^2 \ \forall\ x \neq 0$ means multiplying x^2 by 2 makes it larger for all x except $x = 0$.
$\mp$	Minus or plus. It's only used when the $\pm$ symbol also appears in the same equation; the meaning is that when one of them is $+$, the other is $-$, and vice versa.	$-1 \times (\pm 1) = \mp 1$

Table 2: Some less common symbols, their meanings, and examples.