Advanced Calculus II: Assignment 3

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Problem 1.

Let $L_1, L_2, L_3 \in L(\mathbb{R}^n, \mathbb{R}^m)$. Then all of these functions are linear transformation from $\mathbb{R}^n$ to $\mathbb{R}^m$. We will now use these properties to show that $L(\mathbb{R}^n, \mathbb{R}^m)$ is a vector space:

1. Associativity of addition

$$\begin{aligned}(L_1 + L_2)(x) + L_3(x) &= L_1(x) + L_2(x) + L_3(x) \\ &= L_1(x) + (L_2 + L_3)(x)\end{aligned}$$

2. Commutativity of addition

3. Identity element of addition

   Let L_0 be the function that assigns the 0 vector in $\mathbb{R}^m$ to every vector in $\mathbb{R}^n$. We must first show that this is a linear transformation.

   Let $u, v \in \mathbb{R}^n$ and let $c \in \mathbb{R}$. Then,

$$L_0(u + v) = 0 = L_0(u) + L_0(v)$$

   and,

$$L_0(cu) = 0 = c0 = cL(u)$$

   Thus $L_0 \in L(\mathbb{R}^n, \mathbb{R}^m)$. Now to show that it is the identity element of addition in that set:

$$\begin{aligned}(L_0 + L_1)(x) &= L_0(x) + L_1(x) \\ &= 0 + L_1(x) \\ &= L_1(x)\end{aligned}$$

4. Inverse elements of addition

Problem 2.

Problem 3.

Problem 4.

Problem 5.