Advanced Calculus II: Assignment 2

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Problem 1.

Problem 2.

Let $A = \mathbb{I}$, the set of irrational numbers.

Since $\mathbb{Q}$ is countable, $\mathbb{R}$ is uncountable, and $\mathbb{R} = \mathbb{Q} \cup \mathbb{I}$, we have that $\mathbb{I}$ must be uncountable. If it were not, then $\mathbb{R}$ would be the union of two countable sets and would hence be countable as well, a contradiction.

Now observe that $\mathbb{Q}$ is dense in $\mathbb{R}$. That is, for every $x_1, x_2 \in \mathbb{R}$ with $x_1 < x_2$, $\exists y \in \mathbb{Q}$ such that $x_1 < y < x_2$. Since $\mathbb{I} \subset \mathbb{R}$, we see that $\mathbb{Q}$ divides $\mathbb{I}$ in such a manner.

Problem 3.

Let $C \subset \mathbb{R}^p$ be open and suppose C is connected.