Advanced Calculus II: Assignment 1
Chapter 2 - A Taste of Topology

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Problem 12 on p. 126.

(a)

(b)

(c) A rearrangement of (p_n) where f is a surjective function does not necessarily preserve the limit ℓ. For example, take $(p_n) = \frac{1}{n}$. This sequence converges to 0.

Now let $f(n) = \begin{cases} 1 & \text{n is odd} \\ 2 & \text{n} = 2 \\ f(n-2)+1 & \text{n is even and n} > 2 \end{cases}$

Call this new sequence $q_k = p_{f(k)}$. Then we have that (q_k) contains all terms in the original sequence (p_n) and, for every odd term m, $q_m = 1$. Thus, the even terms of (q_k) converge to 0 while the odd terms converge to 1. Since we have two subsequences in (q_k) that converge to different limits, (q_k) does not converge.

Problem 44 on p. 128.

Problem 76 on p. 131.

Problem 1 on p. 147.