Advanced Calculus II: Assignment 4

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Problem 1.

Problem 2.

Problem 3.

Show that f is partial differentiable with respect to any $(a, b) \in \mathbb{R}^2$:

Not continuous: Observe that f is defined for every point in $\mathbb{R}^2$ except for $(0, 0)$. Hence, we can approach $(0, 0)$ on any line. Thus, let $x = y^2$. Then, we have,

$$\begin{aligned} f(y^2, y) &= \frac{y^2 y^2}{(y^2)^2 + y^4} \\ &= \frac{y^4}{y^4 + y^4} \\ &= \frac{y^4}{y^4(1+1)} \\ &= \frac{1}{2} \end{aligned}$$

Thus, $\lim_{(y^2,y)\to(0,0)} f(x, y) = \frac{1}{2}$.

Now approach on the line $x = 0$. This yields,

$$\begin{aligned} f(0, y) &= \frac{(0)y^2}{(0)^2 + y^4} \\ &= \frac{0}{y^4} \\ &= 0 \end{aligned}$$

Hence, $\lim_{(0,y)\to(0,0)} f(x, y) = 0 \neq \lim_{(y^2,y)\to(0,0)} f(x, y)$.

We can see that the limit as we approach $(0, 0)$ is different depending upon the line we approach it from. Hence f is not continuous at $(0, 0)$.

Problem 4.

Problem 5.