Advanced Mathematical Statistics: Assignment 1

Chris Hayduk

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Problem 3.4.

a) $\mathbf{y}_1 = \begin{bmatrix} 3.497900 \\ 2.485475 \\ 1.782875 \\ 1.725450 \\ 1.645575 \\ 1.469800 \end{bmatrix}$

Then we have,

$$\begin{aligned} \text{proj}_{\mathbf{1}} \mathbf{y_1} &= \frac{\mathbf{y_1'1}}{\mathbf{1'1}}\mathbf{1} \\ &= \frac{12.607075}{6}\mathbf{1} \\ &= (2.10117917)\mathbf{1} \\ &= \begin{bmatrix} 2.10117917 \\ 2.10117917 \\ 2.10117917 \\ 2.10117917 \\ 2.10117917 \\ 2.10117917 \end{bmatrix} \end{aligned}$$

b) We have $\overline{x}_1 = \frac{1}{6}(\sum_{i=1}^{6} x_{1_i}) = 2.10117917$

Thus,

$$\begin{aligned} \mathbf{d}_1 = \mathbf{y}_1 - \overline{x}_1\mathbf{1} &= \mathbf{y}_1 - (2.10117917)\mathbf{1} \\ &= \begin{bmatrix} 1.39672083 \\ 0.38429583 \\ -0.31830417 \\ -0.37572917 \\ -0.45560417 \\ -0.63137917 \end{bmatrix} \end{aligned}$$

Furthermore, $L_{\mathbf{d}_1} = \sqrt{\mathbf{d}_1'\mathbf{d}_1} = 1.7167461$

The sample standard deviation is given by

$$\begin{aligned}\sqrt{\frac{1}{6}\sum_{i=1}^{6}(y_{1_i} - \overline{x}_1)^2} &= \sqrt{\frac{1}{6}\mathbf{d}_1'\mathbf{d}_1} \\ &= \frac{1}{\sqrt{6}}\sqrt{\mathbf{d}_1'\mathbf{d}_1} \\ &= \frac{1}{\sqrt{6}}L_{\mathbf{d}_1} \\ &= \frac{1}{\sqrt{6}}(1.7167461) \\ &\approx 0.70085866\end{aligned}$$

Thus, the sample standard deviation of $\mathbf{y}_1$ is given by $\frac{1}{\sqrt{6}}L_{\mathbf{d}_1}$.

c) $L_{\mathbf{y}_1 - \overline{x}_1\mathbf{1}} = L_{\mathbf{d}_1} = \sqrt{\mathbf{d}_1'\mathbf{d}_1} = 1.7167461$

$L_{\mathbf{y}_1} = \sqrt{\mathbf{y}_1'\mathbf{y}_1} = 5.425582$

$L_{\overline{x}_1\mathbf{1}} = \sqrt{\overline{x}_1\mathbf{1}'\overline{x}_1\mathbf{1}} = 5.14681682$

d) $\mathbf{y}_2 = \begin{bmatrix}0.623\\0.593\\0.512\\0.500\\0.463\\0.395\end{bmatrix}$

Then we have,

$$\begin{aligned}\text{proj}_{\mathbf{1}}\,\mathbf{y_2} &= \frac{\mathbf{y_2'1}}{\mathbf{1'1}}\mathbf{1} \\ &= \frac{3.086}{6}\mathbf{1} \\ &= (0.51433)\mathbf{1} \\ &= \begin{bmatrix}0.51433\\0.51433\\0.51433\\0.51433\\0.51433\\0.51433\end{bmatrix}\end{aligned}$$

We have $\overline{x}_2 = \frac{1}{6}(\sum_{i=1}^{6} x_{2_i}) = 0.51433$

Thus,

$$\mathbf{d}_2 = \mathbf{y}_2 - \overline{x}_2\mathbf{1} = \mathbf{y}_2 - (0.51433)\mathbf{1}$$
$$= \begin{bmatrix} 0.10867 \\ 0.07867 \\ -0.00233 \\ -0.01433 \\ -0.05133 \\ -0.11933 \end{bmatrix}$$

Furthermore, $L_{\mathbf{d}_2} = \sqrt{\mathbf{d}_2'\mathbf{d}_2} = 0.187305$

The sample standard deviation is given by

$$\begin{aligned}\sqrt{\frac{1}{6}\sum_{i=1}^{6}(y_{2_i} - \overline{x}_2)^2} &= \sqrt{\frac{1}{6}\mathbf{d}_2'\mathbf{d}_2} \\ &= \frac{1}{\sqrt{6}}\sqrt{\mathbf{d}_2'\mathbf{d}_2} \\ &= \frac{1}{\sqrt{6}}L_{\mathbf{d}_2} \\ &= \frac{1}{\sqrt{6}}(0.187305) \\ &\approx 0.0764671\end{aligned}$$

Thus, the sample standard deviation of $\mathbf{y}_2$ is given by $\frac{1}{\sqrt{6}}L_{\mathbf{d}_2}$.

$L_{\mathbf{y}_2 - \overline{x}_2\mathbf{1}} = L_{\mathbf{d}_2} = \sqrt{\mathbf{d}_2'\mathbf{d}_2} = 0.187305$

$L_{\mathbf{y}_2} = \sqrt{\mathbf{y}_2'\mathbf{y}_2} = 1.27370$

$L_{\overline{x}_2\mathbf{1}} = \sqrt{\overline{x}_2\mathbf{1}'\overline{x}_2\mathbf{1}} = 1.25985$

e) $\cos\theta = \frac{\mathbf{d}_1'\mathbf{d}_2}{L_{\mathbf{d}_1}L_{\mathbf{d}_2}} = \frac{0.28686869}{1.7167461(0.187305)} \approx 0.89212910878$

$\implies \theta = \arccos(0.89212910878) \approx 0.4687601612$

Problem 3.7.

Problem 3.8a.

Problem 3.11.

Problem 3.12.

Problem 3.13.

Problem 3.14.

Problem 3.16.

Problem 3.17.

Problem 3.18.