

# Advanced Mathematical Statistics: Assignment 2

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October 3, 2019

## Problem 4.2.

a)  $f(x_1, x_2) = \frac{1}{2\pi\sqrt{1.5}} \times \exp\left\{-\frac{2}{3}\left[\left(\frac{x_1}{\sqrt{2}}\right)^2 + (x_2 - 2)^2 - \left(\frac{x_1}{\sqrt{2}}\right)(x_2 - 2)\right]\right\}$

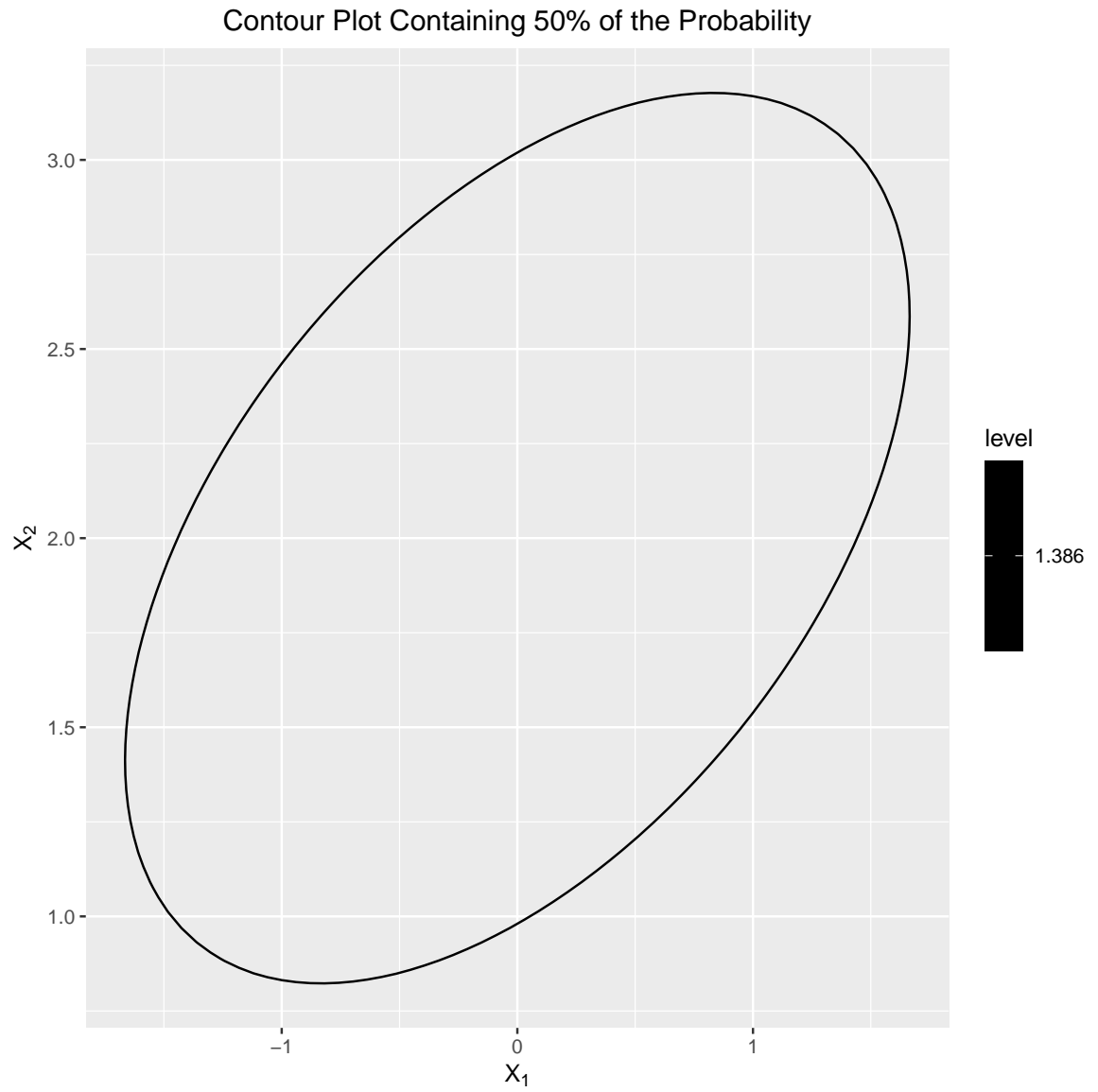
b) We have that  $\rho_{12} = 0.5 \implies \sigma_{12} = 0.5(\sqrt{\sigma_{11}\sigma_{22}}) = 0.5(\sqrt{2})$ . So,

$$\begin{aligned} & (\mathbf{x} - \mu)' \Sigma^{-1} (\mathbf{x} - \mu) \\ &= \begin{bmatrix} x_1 - \mu_1 & x_2 - \mu_2 \end{bmatrix} \frac{1}{\sigma_{11}\sigma_{22} - \sigma_{12}^2} \begin{bmatrix} \sigma_{22} & -\sigma_{12} \\ -\sigma_{12} & \sigma_{11} \end{bmatrix} \begin{bmatrix} x_1 - \mu_1 \\ x_2 - \mu_2 \end{bmatrix} \\ &= \begin{bmatrix} x_1 & x_2 - 2 \end{bmatrix} \frac{1}{2 - (0.5\sqrt{2})^2} \begin{bmatrix} 1 & -0.5\sqrt{2} \\ -0.5\sqrt{2} & 2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 - 2 \end{bmatrix} \\ &= \begin{bmatrix} \frac{2}{3}x_1 + (-\frac{\sqrt{2}}{3})(x_2 - 2) & (-\frac{\sqrt{2}}{3})(x_1) + \frac{4}{3}(x_2 - 2) \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 - 2 \end{bmatrix} \\ &= x_1 \left( \frac{2}{3}x_1 + \left( \frac{-\sqrt{2}}{3} \right) (x_2 - 2) \right) + (x_2 - 2) \left( \left( \frac{-\sqrt{2}}{3} \right) x_1 + \frac{4}{3}(x_2 - 2) \right) \\ &= \frac{2}{3}x_1^2 + \left( \frac{-\sqrt{2}}{3} \right) (x_2 - 2)x_1 + (x_2 - 2) \left( \frac{-\sqrt{2}}{3} \right) x_1 + \frac{4}{3}(x_2 - 2)^2 \\ &= \frac{2}{3}x_1^2 + \frac{-2\sqrt{2}(x_2x_1 - 2x_1)}{3} + \frac{4}{3}x_2^2 - \frac{16}{3}x_2 + \frac{16}{3} \end{aligned}$$

c)  $c^2 = \chi_2^2(0.5) \approx 1.386294$ . So we take

$$\frac{2}{3}x_1^2 + \frac{-2\sqrt{2}(x_2x_1 - 2x_1)}{3} + \frac{4}{3}x_2^2 - \frac{16}{3}x_2 + \frac{16}{3} = 1.386294$$

to be the surface of the ellipsoid containing 50% of the probability. The graph for this can be seen below.



**Problem 4.3.**

- a)
- b)
- c)
- d)
- e)

**Problem 4.4.**

**Problem 4.5.**

Problem 4.6.

Problem 4.7.

Problem 4.10.

Problem 4.11.

Problem 4.12.

Problem 4.13.

Problem 4.14.

Problem 4.15.

Problem 4.16.

Problem 4.17.

Problem 4.18.

Problem 4.19.

Problem 4.20.

Problem 4.21.