Dynamical Systems: Homework 5

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Problem 1.

Let F_f be the Fatou set of $f : C \to C$, a polynomial with degree $d \geq 2$.

Problem 2.

Fix $w \in W^s(z)$. We know that,

$$|f^n(z) - f^n(w)| \to 0 \text{ as } n \to \infty$$

That is, for every $\epsilon > 0$, there exists a natural number N such that for all $n > N$, we have that,

$$|f^n(z) - f^n(w)| < \epsilon$$

Let $\epsilon' < \epsilon/2$ and fix $w' \neq w \in B(w, \epsilon')$. Then, by the continuity of polynomials, we have

$$\begin{aligned} |f^n(z) - f^n(w)| &= |f^n(z) - f^n(w') + f^n(w') - f^n(w')| \\ &\leq |f^n(z) - f^n(w')| + |f^n(w') - f^n(w')| \end{aligned}$$

Problem 3.

We have that $f'(z) = 2z$. Hence, $\lim_{z \to \infty} f'(z) = \infty$. Since $f(2)$ is a fixed point, we have $f^n(z) \to \infty$ for $z > 2$. Now for $z < -2$, we have that $f(z) > 2$. Hence, $f^n(z) \to \infty$ for $z < 2$ as well. Thus, the basin of attraction for ∞ is $W^s(\infty) = \overline{\mathbb{C}} \setminus [-2, 2]$

Problem 4.

Problem 5. Bonus problem