Dynamical Systems: Homework 6

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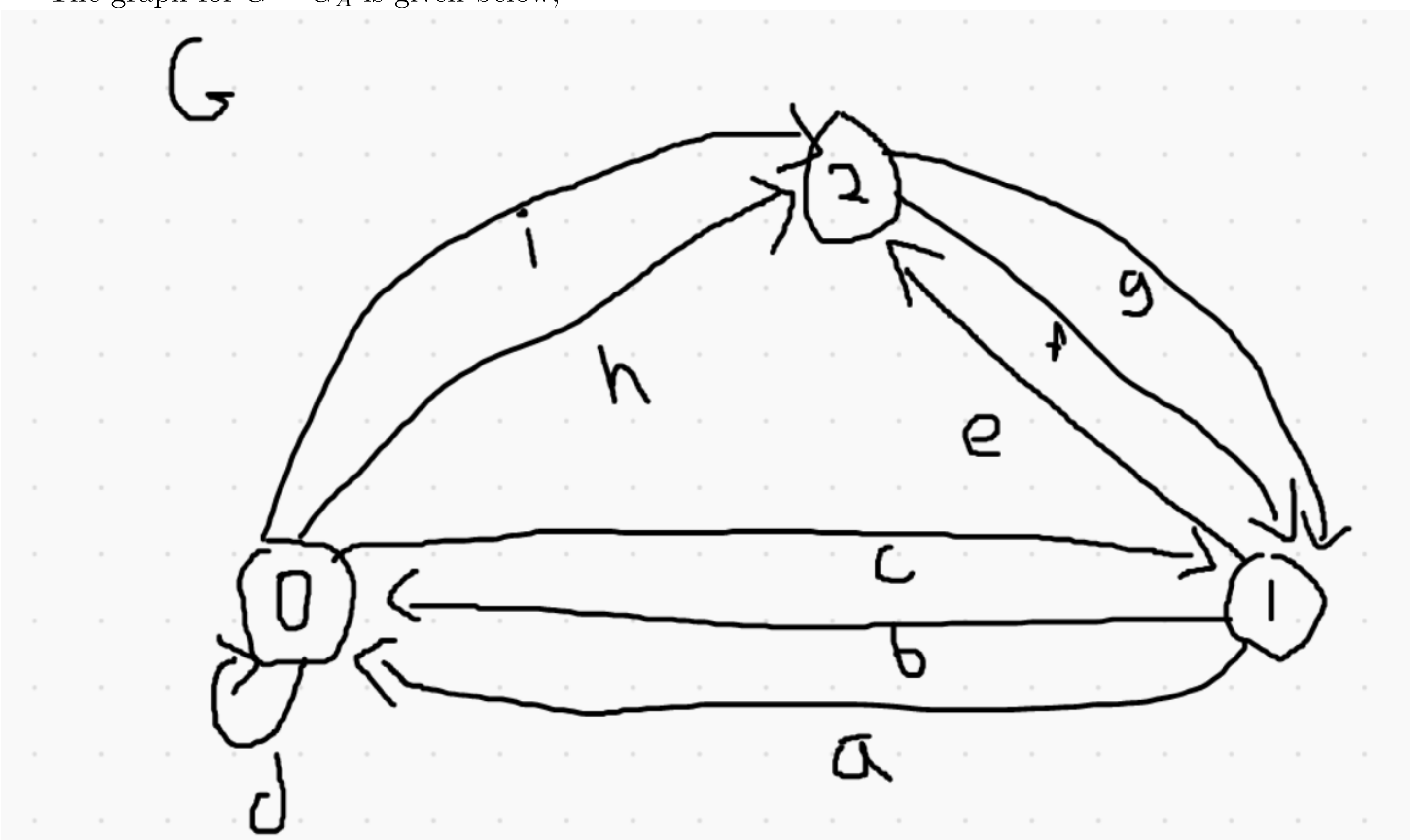
Problem 1.

The graph for $G = G_A$ is given below,



We have labeled the edges above in order to more easily construct the matrix for the shift space, which consists of showing which edges are "permissible" following the presence of a given edge. Hence, the shift space, X, associated with G is given by the following matrix,

$$X = \begin{array}{c} \\ a \\ b \\ c \\ d \\ e \\ f \\ g \\ h \\ i \end{array} \begin{array}{c} \begin{array}{ccccccccc} a & b & c & d & e & f & g & h & i \end{array} \\ \begin{pmatrix} 0 & 0 & 1 & 1 & 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 1 & 0 & 0 & 0 & 1 & 1 \\ 1 & 1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 \end{pmatrix} \end{array}$$

Problem 2.

Define $h : X \to Y$ as $h(00) = 1$, $h(01) = 0$, and $h(10) = 0$. We have defined h over every possible block of size 2 in the Golden Mean Shift, and each 2 block has one corresponding size 1 block in the even shift. Hence, h is a well-defined function from $X \to Y$. Moreover, the pre-image of 0 in X is $\{01, 10\}$ and the pre-image of 1 is $\{00\}$. Thus, all of the pre-images are finite so h is finite to one.

Now we need to show that h is surjective.

Problem 3.

1. **Theorem:** Let (X, d) be a compact metric space and let $f : X \to X$ be continuous. If f is transitive and if $\text{Per}(f)$ is dense in X, then f has sensitive dependence on initial conditions. That is, f is chaotic (since it satisfies all three conditions as stated by Devaney).

   This theorem helps to simplify the requirements for f to be choatic, since now we need to only check 2 conditions rather than 3.

2. **Theorem (Sharkovskii's Theorem):** Let $f : \mathbb{R} \to \mathbb{R}$ be continuous with a periodic point of prime period 3. Then there exist periodic points of any other period.

   This results is surprising because of how simply stated, yet powerful it is. We only need to know that 1 periodic point exists with period 3 in order to assert that there are periodic points of every other period.

3. **Theorem:** If $\mathcal{L}$ is a language, then there exists a unique subshift X such that $\mathcal{L}(X) = \mathcal{L}$

   In some sense, the subshift X captures all of the information contained in the language.

4. **Theorem:** J_f and F_f are completely invariant sets. That is, $f(J_f) = J_f = f^{-1}(J_f)$ and $f(F_f) = F_f = f^{-1}(F_f)$

   Both the Julia set, J_f, and the Fatou set, F_f, are closed under the application of f or f^{-1}. That is, if we apply f or f^{-1} to any point $x \in J_f$, then $f(x), f^{-1}(x) \in J_f$. The same holds true for the Fatou set.

Problem 4. Bonus problem

Suppose X is a shift space that is topologically conjugate to a subshit of finite type Y. Then there exists a sliding block code $\phi : X \to Y$ such that ϕ is invertible.