

Dynamical Systems: Homework 2

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September 24, 2020

Problem 1.

We have,

$$\Sigma = \Sigma_b = \{X = (x_k)_{k=-\infty}^{\infty} : x_k \in \{0, \dots, b-1\}\}$$

and,

$$d(x, y) = d_{\theta}(x, y) = \begin{cases} 0 & \text{if } x = y \\ \theta^{\min\{|k| : x_k \neq y_k\}} & \end{cases}$$

where $0 < \theta < 1$.

Need to show that (Σ, d) is compact.

Let us show that (Σ, d) is closed and bounded, and then prove that this gives us compactness.

Consider $C_{-1,1} = \{x_{-1}, x_0, x_1\}$, $x \in \Sigma$. There are b^3 possibilities.

For $C_{-2,2}$, we have

$$C_{-2,2} = \cup_{y \in C_{-1,1}}$$

Note that $\Sigma = \Sigma_b = \{g : \mathbb{Z} \rightarrow \{0, \dots, b-1\} \mid g \text{ function}\}$

Problem 2.

Problem 3.

(a)

(b)

Problem 4.